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Preface

This, the 57th volume of the TRANSACTIONS of the American Institute of Electrical Engineers, is the first to be published under the revised publication policy adopted during 1937. Under this policy, the TRANSACTIONS contains only finally reviewed papers and correlated discussions approved by the AIEE technical program committee for presentation at national conventions or District meetings. Because of the transition from the former to the new publication policy, the content of this volume is somewhat less than normal.

Included in this volume are papers and related discussions presented at the following AIEE meetings and conventions:

1. 1938 winter convention, New York, N.Y., January 24–28: all papers presented except those published in the 1937 volume under the former publication policy.
2. North Eastern District meeting, Lenox, Mass., May 18–20: all papers approved for presentation.
3. 1938 summer convention, Washington, D.C., June 20–24: all papers presented, except nine that will appear in the 1939 volume, and two that were published in the 1937 volume under the former publication policy.
4. 1938 Pacific Coast convention, Portland, Ore., August 9–12: one paper; others will appear in the 1939 volume.

The present publication policy also provides for the publication of a large proportion of the approved TRANSACTIONS papers and discussions month by month in a segregated TRANSACTIONS section of the monthly issues of ELECTRICAL ENGINEERING; extra sheets are printed at the same time for ultimate binding in the annual TRANSACTIONS volume. The papers and discussions so published during the year 1938 comprise pages 1 to 712, inclusive, of this volume; pages 713 to 796, inclusive, did not appear in ELECTRICAL ENGINEERING.

Since the adoption of the new publication policy involved a change from a program in which all papers were published in advance of presentation at an Institute meeting or convention to a program in which they all were published after such presentation, there was of necessity a transition period of several months during which papers and discussions were not completely correlated. Therefore, the first portion of this volume, up to page 492, contains: (1) discussions of some papers that were published in the 1937 TRANSACTIONS volume under the previous publication policy; (2) papers published before the related discussions became available, in order to maintain a TRANSACTIONS section of reasonable proportions in the monthly issues of ELECTRICAL ENGINEERING; and (3) discussions of these papers published separately in subsequent pages. From page 493 on, all discussions published have been correlated with their respective papers.

Continuing the practice of recent years, full correlation of all material in this volume has been accomplished through the medium of the multientry reference index beginning on page 797. A reference to any of the several subject entries for any technical paper will lead directly to the paper and to any discussion on that paper published during 1938.

Erratum

Page 47, fourth line below equation 4, first mathematical expression should read "$(S\delta) > 2.0$," not "$(S\delta) < 2.0$."

Electrical Studies of Living Tissue — II

Correlation Between Tissue Response and Voltage Distribution

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ALTHOUGH opinion differs as to whether tissue response to electrical stimulation results directly from imposed electrical quantities or indirectly from accompanying chemical changes, there is general agreement that the response occurs when a definite threshold value of some changing condition within the tissue has been reached. Bishop[1] supports the belief that response results when a definite polarization is developed within the tissue; Hill[2] associates response with a threshold value of cathode potential. We present here evidence to show that response occurs when a definite internal voltage has been built up in the tissue. Our conclusions are based on data obtained from oscillograms of this internal voltage.

In 1907 LaPique[3] established the relations between the intensity and duration of currents producing minimal responses in tissue; he described the process of excitation as analogous to the charging of a leaky condenser. Since then numerous investigators[6–9] have demonstrated that tissue possesses capacitance. Hill[2] and Blair[4] have shown that an equivalent circuit with properly proportioned constants could be used to explain the intensity duration curve obtained by LaPique.[3] In a previous paper[5] we have demonstrated the effects of tissue capacitance on the shape of the voltage waves across tissue during the passage of current impulses of rectangular wave shape.

Using the voltage wave shapes as criteria of circuit constants, this work has been extended here to define the character of the equivalent circuits. We find that for rectangular waves of current 0.001 to 0.0001 second in duration and from 1 to 10 times the rheobase (the rheobase current is the minimum current that will produce a response, that is, it is the current represented by the ordinate of the horizontal asymtote to the intensity-duration curve) value in intensity that human and frog tissue behaves electrically as a simple circuit consisting of one condenser and one or more resistors. The magnitude of tissue impedance remains constant throughout the duration of the impulse.

The correlation between tissue response and the voltage distribution in the tissue has been determined and is here described on the basis of the voltage distribution in the equivalent circuits.

General Method

In determining the electrical characteristics of tissue, a current of rectangular wave shape was applied and the accompanying voltage wave across the tissue was recorded. The numerical values of the components of the impedance thus indicated were calculated. They were confirmed by the passing of the current impulse through a suitable circuit—and equivalent circuit—containing the necessary resistances and capacitances to give a duplication of the voltage wave.

To obtain current impulses of rectangular wave shape suitable for the electrical analysis of tissue a special apparatus was developed. This apparatus and its particular advantages have been described by the authors.[5]

Electrical Characteristics of Human Tissue

We have previously[5] described the characteristics of human tissue that have led us to correlate the tissue responses with the voltage distribution in equivalent circuits. These characteristics are: (1) minimal response occurs in human tissue when the potential across the tissue reaches a certain threshold value; (2) the voltage required to produce a minimal response in different human beings may be different, but for a given individual it is a constant, regardless of the duration of the impulse; (3) the current required to produce a minimal response is a function of the duration of the impulse. The time intensity relations obtained by the authors are similar to those obtained by other investigators.

Our point of departure from previous investigations of

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[1] For all numbered references see list at end of paper.

Figure 1. Minimal response voltage waves for shocks of different durations

this kind was in recording the voltage wave across the tissue at the time of stimulation. Although the currents required to produce responses are a function of duration of the impulses, the accompanying voltages are constant for each subject.

Electrical Characteristics of Frog Tissue

The methods by which the measurements were made on the frog have been described by the authors.[5] The voltage wave of frog tissue is of distinctly different shape from that obtained across human tissue, indicating a different type of equivalent circuit. The dissymmetry between the voltage produced across frog tissue and that produced across a resistor, although only slight, indicates that the frog's tissue is not purely resistive. The circuit that most nearly expresses the electrical equivalent of frog's tissue is shown in figure 3; that of human tissue in figure 7.

Figure 1 shows 3 superimposed voltage waves produced by current impulses of different duration causing minimal responses. In contrast to the result obtained in human tissue, the peak voltages at minimal response in the frog are not constant. The relations of peak voltages to currents at minimal response for different durations have been shown.[5]

The differences between the shapes of the curves showing the relation of peak voltages to duration in human and frog tissue can be explained on the basis of the difference in the nature of the equivalent circuits of tissue. It is evident from the waves shown in figure 1 that the voltage does not drop to zero at the instant that the current is stopped. Furthermore this residual voltage appears to be constant regardless of the intensity and duration of the impulse producing minimal responses. In order to determine more accurately the magnitude of this residual voltage the sensitivity of the amplifier supplying the cathode ray oscillograph was increased so that the upper part of the voltage wave was moved off the oscillograph screen and the residual voltage appeared approximately 2 centimeters high. A series of minimal-response shocks was then administered and the corresponding residual voltages photographed. Figure 2 shows 3 such voltages for shocks of different durations. This residual voltage, when referred to the equivalent circuit, figure 3, is that which appears across the parallel part at the instant the current is shut off.

Our interpretation of these findings is that minimal response of frog tissue occurs when, and only when, the voltage across the parallel portion has been built up to a definite value.

Correlation of Equivalent Circuits to Living Tissue

An explanation of the shapes of the curves for the time intensity relations at minimal response and also the corresponding curves for peak voltages of different tissues is afforded here on the basis of the voltage produced in the equivalent circuit by the passage of current.

In the case of frog's tissue let figure 3 represent the equivalent circuit; R_1 and R_2 the number of ohms in the series and parallel portions of this circuit; C the capacity in farads; i_1 the instantaneous current in C; i_2 the instantaneous current in R_2; I the current supplied to the circuit; T its duration. A current of rectangular wave shape supplied to this circuit will produce a voltage wave corresponding to that of figure 1 and shown here in a diagram as figure 4. The distance *ab* (figure 4) represents the voltage across R_1 produced by the current I. When the current is shut off the voltage across R_1 disappears giving a reduction in voltage of *cd*. For a rectangular current wave the rise in voltage (*ab*) is equal to the drop in voltage *cd*. When the current is first applied the condenser acts as a short circuit on the resistor R_2; consequently the voltage E_2 is zero and at this instant E_1 is equal to the total circuit voltage E. After the current has flowed for some time t a voltage builds up across the condenser. The magnitude of this voltage is determined by the current I, the resistance R_2, the capacity C, and the duration of the current impulse. The voltage E_2 can be expressed by the equation

$$E_2 = IR_2(1 - e^{-\frac{t}{R_2C}}) \qquad (1)$$

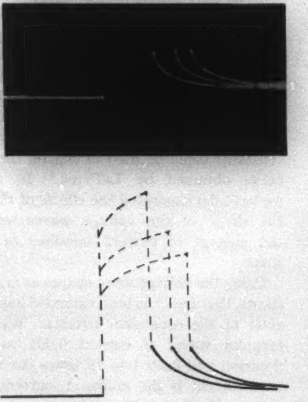


Figure 2. Threshold voltages in frog tissue produced by minimal response shocks

The total voltage across the circuit produced by a rectangular current wave is:

$$E = E_1 + E_2 = IR_1 + IR_2(1 - e^{-\frac{t}{R_2C}}) \tag{2}$$

The first term of this equation (IR_1) is represented by the distance *ab*, figure 4; the second term $IR_2(1-e^{-\frac{t}{R_2C}})$ is represented by the vertical distance between a horizontal line drawn through *b* and the curve *bc*. Since the distance *ab* is equal to *cd*, the distance *de* is equal to E_2 at the instant the current impulse is shut off.

It has been shown experimentally that E_2 at the instant the current is shut off is constant for impulses producing minimal response. Therefore for such impulses

$$E_2 = K = IR_2(1 - e^{-\frac{T}{R_2C}}) \tag{3}$$

$$I = \frac{K}{R_2(1 - e^{-\frac{T}{R_2C}})} \tag{4}$$

Equation 4 is that for the minimal-response current curve. When T is infinite the term

$$e^{-\frac{T}{R_2C}}$$

becomes zero and I is equal to K/R_2, this particular value of I being the rheobase. As the duration of the applied current is decreased T becomes shorter and the value of I must necessarily increase to fulfill the requirements of equation 4.

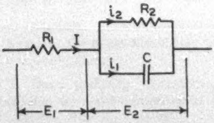
The voltage across frog's tissue for an impulse of a rectangular current wave can be expressed as

$$E = IR_1 + IR_2(1 - e^{-\frac{t}{R_2C}}) \tag{5}$$

but the second term, which becomes

$$IR_2(1 - e^{-\frac{T}{R_2C}})$$

at the end of any minimal-response impulse is always constant regardless of the duration T. A minimal-response impulse of short duration therefore, requires a large value of I to keep this term constant and the total peak voltage E must consequently be higher for short impulses because of the increase in IR_1.



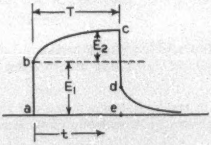


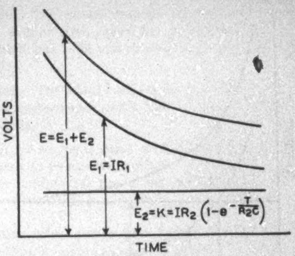
Figure 3 (left). Voltages in the equivalent circuit of frog tissue

Figure 4 (right). Voltage wave for frog tissue

If the peak value of $E_2 = K$ (value at the end of the impulse) for impulses of various duration are plotted against duration as in figure 5, it will be represented as a horizontal line. The first term of equation 5 (IR_1) must take the form as plotted on the same axes. The total voltage across the tissue producing the minimal response will be the sum of these 2 terms which gives a curve corresponding to the total voltage E.

Since the voltage across the parallel part of the circuit attains a definite peak value at the instant any minimal response impulse is shut off, it is evident that the peak current i_2 which flows through R_2 (figure 3) must build up to a definite peak value for each impulse causing minimal response. If this final peak value of i_2 is plotted against the duration of the impulse, it will be constant as shown in figure 6. If the impulse duration is exceedingly long so that the condenser current is zero, this peak value of current i_2 will be equal to the total current I which in this case is equal to the rheobase K/R_2.

Figure 5. Peak voltages for shocks of different durations (minimal response)



From equation 4 it has been shown that the current required to produce a minimal response is

$$I = \frac{K}{R_2(1 - e^{-\frac{T}{R_2C}})}$$

(when K is the threshold voltage of the parallel circuit) and that the rheobase current, $I_{Rh} = K/R_2$. Since $I = i_1 + i_2$ and, at the instant prior to "shut off" of the current, the peak value of $i_2 = K/R_2$, then the current i_1 which flows through the condenser at this instant is

$$i_1 = I - i_2$$

$$= \frac{K}{R_2(1 - e^{-\frac{T}{R_2C}})} - \frac{K}{R_2}$$

When T is infinite the first and second terms of this equation are equal and i_1 is zero. For impulses of shorter duration the value of i_1 prior to shutting off I, will be

$$i_1 = \frac{K}{R_2(e^{\frac{T}{R_2C}} - 1)}$$

From these facts it follows that if the duration were exceedingly short the current i_1 would necessarily be infinite. The final value of i_1 for each shock of minimal response when plotted against duration is shown in figure 6. The sum of the currents i_1 and i_2 are also shown. The resultant curve expresses the time intensity relation for minimal response.

On representing the equivalent circuit of human tissue by the diagram and notations of figure 7, equation 1 (previously applied to figure 3) will represent the voltage produced across the circuit (or tissue) by a rectangular current wave. We have shown experimentally that this peak voltage is constant for any impulse producing minimal response. The current necessary to produce minimal response is therefore expressed by equation 4. The curves shown in figure 1 can be derived from equation 2. The equation which expresses the voltage necessary to produce a minimal response in frog tissue with a rectangular current wave, is also applicable to human tissue. In this particular case R_1 is zero and

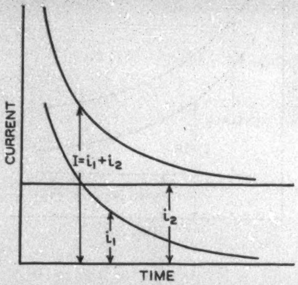


Figure 6. Peak current at instant of shut-off for minimal response shocks

$$E = IR_2(1 - e^{-\frac{T}{R_2C}}) = K$$

Therefore the differences in the minimal-response voltage curves of frog and human tissue can be attributed to and explained by the differences existing in the equivalent circuits representing these tissues.

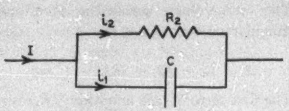


Figure 7. The equivalent circuit of human tissue

It must be remembered that an equivalent circuit of living tissue is not physiological in nature. It is used in the same manner as the equivalent circuits of other conductors of electricity, such as transformers or motors. It is used here only as a means of explaining the correlation between some of the electrical and physiological characteristics of living tissue. Other equivalent circuits could be used for this purpose. These other circuits since they are equivalent circuits naturally reveal the same correlations.

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A New High-Speed Distance-Type Carrier-Pilot Relay System

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Synopsis: This paper outlines the reasons for application of pilot relay equipment, particularly of the carrier-current type. A one-cycle carrier-pilot-relay scheme is described which now makes available the desirable high-speed and back-up characteristics of step-type distance protection combined with the 100 per cent simultaneous tripping feature possible with a pilot circuit. Impedance and over current supervised single-phase directional elements are utilized for controlling carrier on phase and ground faults, respectively.

Introduction

THE ADVANTAGES and necessity of rapid fault clearing have been long recognized. Improved engineering means have been gradually made available for accomplishing this object so that starting with circuit breaker operating times of a half second and relay times ranging up to several seconds 10 years ago, developments have made available up to the present day standard 8-cycle breakers and relays operating in one cycle, on a 60-cycle basis. In some instances breaker times as low as 3 cycles have been obtained.

The chart in figure 1 illustrates the gain in clearing time of transmission line faults available with increased speeds of relays and circuit breakers. Clearing times from 30 to 120 cycles are common with the 24-cycle breakers and induction-type relays using time as a basis for selectivity. A major improvement is generally available in systems of any complexity by going to high-speed relays and again by improving the breaker speed to 8 cycles. The further gain available by the addition of carrier pilot is of particular importance when the maximum clearing time for a fault anywhere in line section is the determining factor in major features of system design.

A number of such situations listed herewith are indicative of factors now leading to the application of carrier pilot protection on numerous systems.

1. *Stability.* Simultaneous clearing improves system stability and increases the loads which may be safely carried over parallel interconnecting lines.

2. *Quick Reclosing.* Simultaneous tripping is essential to fast reclosing, the combination being particularly effective in increasing stability with single tie lines.

3. *Shock to System.* System shock, evidenced by voltage dips and dropping of synchronous load is lessened by fast clearing.

4. *System Design Flexibility.* Desirable system arrangements are made possible by carrier relaying which could not be relayed with sufficient speeds to permit their use otherwise.

5. *Growth of Faults.* The more serious 3-phase and double ground faults generally originate as line-to-line or single ground and with sufficient speed of clearing the spreading to other phases is greatly reduced.

6. *Ground Relaying Improved.* On systems where high-speed ground relaying is not feasible otherwise, carrier pilot provides an ideal solution.

7. *Out-of-Synchronism.* The carrier channel provides means for preventing operation of protective relays due to power swings or out-of-synchronism conditions while still clearing faults during such conditions.

8. *Simultaneous Faults.* The added basis for discrimination makes possible superior relay performance under the condition of simultaneously occurring faults.

9. *Joint Use.* From an economic point of view joint use of the carrier channel for point to point communication, or for control or remote metering may indicate the use of carrier pilot where thr relaying requirements alone would not justify it.

Step-type distance-relay protection is the use of 2 or preferably 3 impedance or distance measuring elements set for successively greater distances and successively longer times. The action of this principle relay protection is readily shown for a simple series connection of line sections with sources only at the ends. While practical systems are seldom so simple, the flexibility of setting provided by this arrangement in obtaining both high-speed and back-up protection has proved its adaptability to a large proportion of the systems in this country and Canada. Approximately 2,000 relays operating on this principle are in use. The importance of this point must be stressed since the provision of adequate and selective back-up protection is frequently a more difficult relay application problem than the high-speed protection.

Figure 2 illustrates the "step" characteristic tripping time versus fault location graph of a 3-zone impedance relay on a single series system. The breaker time must be added to the time here indicated to get the clearing time as shown in figure 1. The tripping characteristic of figure 2 is obtained by the use of relay elements responsive separately to 3 quantities, namely:

1. Distance of fault from the breaker.
2. Direction of fault from the breaker.
3. Time from the beginning of fault.

Three of the distance elements are required, one directional element and one timing element having 2 separately adjustable time contacts to obtain the characteristic of figure 2. The contacts on these elements are arranged in the trip circuit as shown in figure 3. Considering breaker number 1, faults to the left do not close the directional contact *D*. In the first zone, the directional contact *D* is closed together with all 3 distance elements *Z*1, *Z*2, and *Z*3. The breaker is tripped through *D* and *Z*1 without awaiting the closure of the timer contacts, *T*2 or *T*3. Thus, for the first zone the tripping time is simply the operating time of the high-speed directional and distance elements, normally under one cycle. The second

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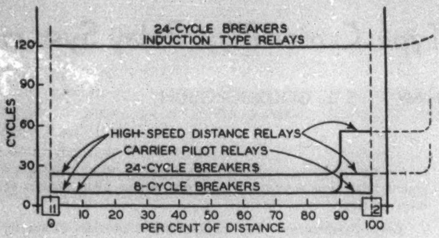

Figure 1. Reduction of clearing times by modernization

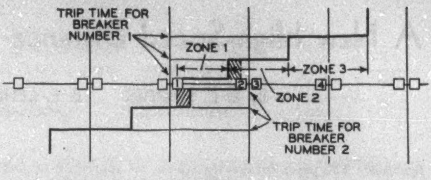

Figure 2. Chart of step-type distance relays

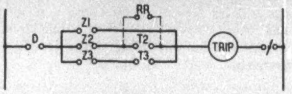

Figure 3. Elementary diagram of distance-relay carrier-current scheme

and third elements would also trip after a time delay, for faults in this region, but this is of no consequence since the first element has tripped the breaker.

The second zone is beyond the distance setting of the first zone element and $Z1$ therefore remains open for faults in this region, the tripping taking place through contacts D, $Z2$, and $T2$. The prime function of the second zone is to trip breaker number 1 for faults between zone 1 and breaker number 2. This distance is quite short, about 10 per cent of the section length, but cannot be entirely eliminated due to the practical accuracies of distance relays, current and potential transformers and application data. However, the second zone element also has a very important function in opening the breaker number 1 with suitable time delay in case of faults occurring to the right of breaker 2 on bus or lines, which are not properly cleared due to the failure or absence of apparatus to perform this function. Such failure may be in breakers, trip coils, or relays, or there may be small unprotected sections on which the expected frequency of fault occurrence does not warrant relay protection.

The third zone extends through the end of the next adjacent section so that breaker number 1 backs up breaker 3 for faults anywhere in the section 3–4. The second and third zones provide selectivity between the back-up protection of adjacent sections without the necessity of greater time settings in successive sections since the third zone for one section selects with the second zone for the next section.

The addition of carrier pilot to the scheme extends the zone of high-speed tripping up to breaker number 2, as shown dotted in figure 2 and eliminates the tripping time indicated by the shaded area. This is accomplished by a contact on the carrier receiving or blocking relay labeled RR in figure 3. This relay is controlled by the fault detectors and carrier current so that its contacts only close if the fault is within the protected section. The trip circuit is completed through D, $D2$, and RR, and also slightly faster through D and $Z1$ except for faults in the remote end zone.

The addition of carrier current pilot with separate relays for its control would involve a bulky and expensive addition to the relay equipment. It is a major purpose of this paper to show that the requirements for carrier control overlap the requirements of the distance relay elements and have made possible the combination of carrier pilot action with the step-type distance protection with very little added relay equipment.

Most of the variations in manner of utilizing the carrier pilot channel, which were considered in the earlier stages of development have been discarded in favor of the scheme which may be termed rather generally, "Intermittent With Normally Blocked Trip Circuit." The carrier is used in a straight telegraph manner, that is, unmodulated, and the principal implications of the name are:

1. That the carrier is transmitted, at least for the relaying function only at times of fault or test.

2. That advantage is taken of the inherently faster operating possibility of fault detector relays, which need not integrate effect over time, as compared with directional elements which must integrate forces over a definite time to obtain a correct operating indication. Because of this inherent difference carrier may be started and the normally blocked trip circuit may always be transferred by the fault detectors to control by carrier while the directional elements are making up their minds which way the power flows.

After the carrier is started by the fault detectors, the control of carrier and hence tripping is on a directional basis. If both directional indications point in, carrier transmission is stopped at both ends, the receiver relay closes its contacts RR and tripping is permitted.

With this general picture in mind, the requirements of the carrier controlling elements may be stated.

1. Fault detectors for starting carrier should be separate from those for tripping under carrier control, and should be set for a lower operating value, for both phase and ground faults. This is to insure that the carrier starting element at one end of the line always functions when the tripping element at the other end of the line operates. The third zone and second zone distance elements have the necessary relationship for carrier starting and tripping fault detectors, and substantially equivalent action must be provided for ground faults.

2. A directional element is required which will close a contact as rapidly as possible when and only when the fault power through the breaker is flowing into the line. Closure of this contact is necessary only if the fault lies somewhere within the protected sections. The second zone trip circuits of a high-speed distance relay possess this characteristic since at least one of them (for the faulty phase) will close through D and $Z2$ whenever the fault is within the pro-

tected section and none will close for faults back of the breaker. Also, none will close due to load conditions on the unfaulted phases since the distance elements will not be operated by loads.

Thus, the third zone element $Z3$ may be used for carrier starting and the second zone trip circuit D and $Z2$ may be used as the directional combination for controlling carrier after it is started by the fault detectors. Since these elements are available in the high-speed distance relay, they are used together with the corresponding ground elements as illustrated in the wiring diagram figure 4 and schematic diagram figure 5, showing the arrangement of a highly successful operating scheme built on the principles just outlined.

The upper part of schematic diagram figure 5 comprises the tripping circuits and the lower part of the carrier control circuits. The conventional distance type trip paths are: First zone D and $Z1$; second zone D, $Z2$, and $T2$; third zone D, $Z3$, and $T3$. The carrier-controlled tripping path is composed of D, $Z2$, and RRP contacts. For ground protection a carrier controlled trip circuit can be set up through the contacts D_0 and I_{02} of the ground relay and the carrier controlled contact RRG. The contacts RRP and RRG are on the blocking relay controlled by the carrier signal.

The contacts $Z3$ (A, B, and C) in the lower part of figure 5 serve to start the transmission of the carrier signal for phase faults and the contact I_{03} performs the same function for ground faults. These carrier start contacts $Z3$ are on the same fault detector elements as the tripping contacts $Z3$ in the upper part of the diagram. The ground carrier start contact I_{03} is operated by an overcurrent element separate from that which operates the tripping contact.

Normally, with phase and ground carrier start contacts open the grid of the master oscillator tube is connected through the resistors R_1, R_2, and R_3, the auxiliary switch coils CS, normally shorted by a $Z3$ back contact, and the resistors R_{CS} to the negative side of the battery. Under this condition the tube cannot oscillate. However, upon closure of any of the $Z3$ contacts or the ground start contact the grid is connected to the positive bus and the tube is thereby caused to oscillate.

The stopping of the carrier signal is controlled by the tripping contacts D and $Z2$ for phase faults and D_0 and I_{02} for ground faults. Closure of contacts D and $Z2$ energizes the auxiliary switch CSP which closes its contacts CSP one of which connects the point P to the negative bus. For phase faults carrier is started by $Z3$ contact which makes the grid positive and is stopped by CSP contact which connects the point P and with it the grid back to negative. However, if the ground start contact I_{03} also closes, for example, a phase to phase to ground fault, the point G and the grid are made positive and it is then impossible for the CSP contact to control the grid. In this case the carrier is stopped by the closure of the ground contacts D_0 and I_{02} energizing the CSG auxiliary switch to close the CSG contact which directly connects the grid to

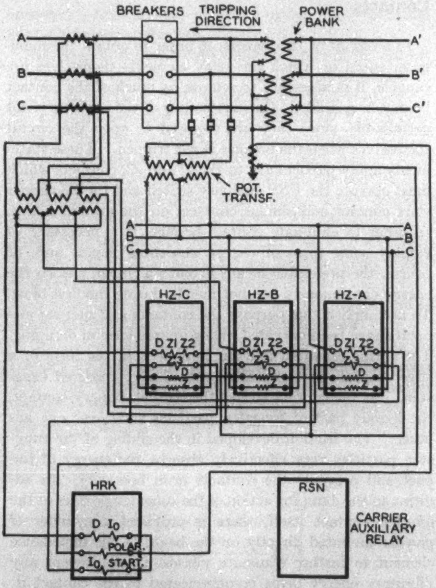
BREAKERS
TRIPPING DIRECTION
POWER BANK
A
B
C
A'
B'
C'
POT. TRANSF.
A
B
C
A
B
C
HZ-C
HZ-B
HZ-A
D Z1 Z2
Z3
D
Z
D Z1 Z2
Z3
D
Z
D Z1 Z2
Z3
D
Z
HRK
D Io
POLAR.
Io START.
RSN
CARRIER AUXILIARY RELAY

Figure 4. A-c schematic connections for distance-relay carrier scheme

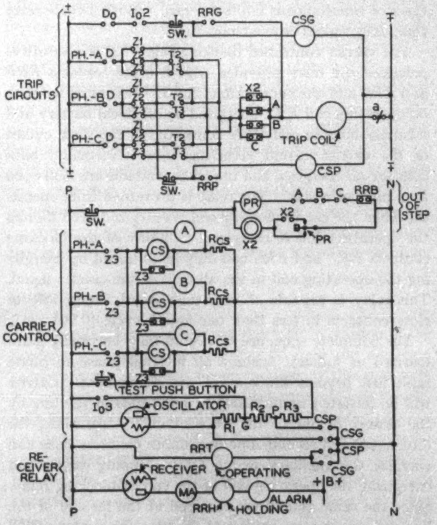
D0
Io2
RRG
SW.
CSG
TRIP CIRCUITS
PH.-A D
PH.-B D
PH.-C D
Z1
Z2
Z3
T2
T3
X2
A
B
C
TRIP COIL
a
CSP
SW.
RRP
P
N
PR
A B C RRB
OUT OF STEP
SW.
X2
X2
PR
CARRIER CONTROL
PH.-A
PH.-B
PH.-C
Z3
CS
A
B
C
RCS
TEST PUSH BUTTON
Io3
OSCILLATOR
R1
G
R2
P
R3
CSP
CSG
CSP
CSG
RE-CEIVER RELAY
RRT
RECEIVER
OPERATING
MA
RRH
HOLDING
ALARM
B+
P
N

Figure 5. Schematic d-c connections of distance-relay carrier scheme

negative. Thus, it is seen that during faults involving ground, the action of the ground relays in starting and stopping carrier is given preference over the phase relays.

The purpose of this arrangement whereby the ground relays are given superiority in both the starting and stopping of the carrier signal is to prevent possible incorrect indications of the phase relays due to load currents and the flow of positive- and negative-sequence currents during external ground faults on systems with a multiplicity of grounding banks. On the other hand, in some cases it may be desirable to give the phase relays preference over the ground relays or to have no preference at all. Any one of these conditions can be obtained by a simple change in connections. If the grounding sources are of high impedance, the phase relay during an external fault may be operated in one direction by load current and the ground relays in the opposite direction at each end due to ground current. Without ground preference the line would trip out since each end has an internal fault indication. The use of single-phase directional elements with impedance supervision is very effective in reducing the possibility of load indication overriding faults on poorly grounded systems as only one-third of the load is opposed to the fault current on the active directional element. The ground preference scheme is provided in the standard system because most faults are grounds and it expedites ground clearing. The objection to ground preference action of blocking the phase relays in certain cases of 2 line-to-ground faults or simultaneous ground faults at 2 locations on a line does not apply to this system since the distance protection is still available to protect for this rare condition. Grounds are very frequent and authentic cases of simultaneous faults are rare. With 2-cycle relay time, they should be even more rare.

The carrier controlled blocking relay *RR* is a sensitive polarized d-c relay provided with 2 make contacts *RRP* and *RRG* and one back contact *RRB*. It is provided with an operating coil *RRT* energized by the local battery and a carrier holding coil *RRH* connected in the plate circuit of the carrier-current receiving tube. Normally, both coils are de-energized and the make contacts are held open by a magnetic bias. The relay is prevented from operating when the carrier holding coil is energized even though the operating coil is energized. Closure of the blocking contacts *RRP* and *RRG* can only be obtained by energizing the operating coil in the absence of the carrier signal. This relay is capable of very high speed, being able to close contacts in less than one-fourth cycle (0.004 sec.).

The complete sequence of events may be briefly summarized as follows: Assume an internal phase to phase fault just beyond the zone of the *Z*1 element. Carrier will be initiated immediately at both ends of the line by the closure of one of the *Z*3 contacts. Meanwhile, the directional and second-zone impedance contacts close and energize the auxiliary switch *CSP*, stopping carrier and energizing the operating coil of the carrier-blocking relay. Since the same action has occurred at the far end of the line, no carrier is received and the blocking contact *RRP* is closed completing the trip circuit through *D* and *Z*2. If the fault had been external to the section, then tripping could not have occurred, since the carrier holding coil *RRH* would have been energized by carrier from the far end.

If we assume a phase-to-phase-to-ground fault, the ground carrier start contact I_0 will also close, making the point *G* positive and it will be impossible for the phase relays to remove carrier through the *CSP* contacts. However, the ground tripping contacts (D_0, I_{02}) will close energizing the *CSG* auxiliary relay whose contacts can remove carrier.

Operating Time

Very satisfactory tripping times are obtained with this carrier current pilot relay scheme. The minimum times are, of course, obtained for faults within the reach of the *Z*1 elements and they are approximately 0.7 to 1.25 cycles (0.011 to 0.021 second). For faults in the end zones the tripping time is slightly longer because of the time consumed by the auxiliary element associated with the carrier and varies from 1.5 to 3.0 cycles (0.025 to 0.05 sec.), depending upon the magnitude of the short circuit current and voltage. The time consumed by the auxiliary switch *CSP* in removing carrier and included in the above time is approximately 0.3 cycle. This adds very little to the overall time in tripping, but during quick reversals of power where carrier must definitely be established at one end before it is removed from the other end of the line, this small time delay provides a positive margin of safety.

Contacts

In a circuit of this nature, in order to obtain the maximum speed of which the relay elements themselves are capable, it is necessary to remove as much of the contact bounce as possible. Current loading of the contacts is not permissible, since they are required to open the circuit themselves when the breaker is not tripped. These statements apply particularly to the *D* and *Z*2 contacts which must operate the *CSP* auxiliary switch and the *Z*3 carrier start contact and similar contacts on the ground relays. In order to eliminate contact bounce, it is necessary to employ some form of energy absorbing means and, of course, the preferable means is one which can absorb the energy continuously and not merely during the first blow. To accomplish this purpose the contacts and moving elements have been provided with a unique form of damping. The spring-mounted moving contact is in the form of a silver capsule partially filled with finely powdered tungsten. As the contact strikes the rigid stationary contact, the loosely packed tungsten particles slide over one another. The friction developed in the sliding of the tungsten particles very effectively absorbs the energy of impact and prevents the contacts from bouncing. In addition to the damping action of the tungsten powder in the moving contact itself, there is provided a cylinder of powder mounted directly on the beam of the impedance element to further eliminate vibration and prevent any vibratory energy being communicated to the contact itself. A similar arrangement is used on the directional element.

Out of Step

It is often desirable to prevent the operation of relays during out-of-step conditions as the system may pull back into step and if the system does not stabilize itself, it is then desirable to separate at locations where frequency control for synchronizing is available. The carrier signal between the 2 ends of the line provides a means of preventing tripping during out-of-step conditions without impairing the ability to trip with reasonable speed for internal faults occurring during out-of-step conditions.

The fundamental difference between a 3-phase fault and an out-of-step condition is that a fault suddenly reduces the voltages and increases the current, whereas during the approach of an out-of-step condition the voltage and current changes are comparatively gradual. For a 3-phase fault the distance elements are all operated practically simultaneously while during out of step the most sensitive distance element *Z*3 operates first, followed by *Z*2 and then *Z*1. As the system returns toward the in-phase position, the elements reset in the opposite order, that is, *Z*1, *Z*2, *Z*3.

To prevent tripping during out of step, it is only necessary to arrange for the closure of the 3 *Z*3 contacts and the receiver relay back contact to operate an additional blocking relay to open the trip circuit. This blocking relay must have a slight time delay so that it does not open the trip circuit before tripping on a 3-phase fault can occur. On the other hand, it must open the trip circuit during an out-of-step condition before the second element *Z*2 is operated. A time delay of 3 to 4 cycles (60-cycle basis) has been found satisfactory.

Referring to the schematic diagram figure 5, the out-of-step blocking contact is designated *X*2. In parallel with it are 3 contacts, *A*, *B*, *C*, which are the back contacts on the auxiliary switches *A*, *B*, *C*, operated by the *Z*3 contacts of the distance relays. The make contacts, *A*, *B*, *C* of these switches are in series with the back contact *RRB* of the receiver-blocking relay to energize the coil *PR* of a pendulum-type time-delay relay, whose lower contacts make and energize the coil of the *X*2 blocking relay. Every time that all 3 of the *Z*3 carrier start contacts close, the contacts *A*, *B*, *C*, and *X*2 open the trip circuit after 3 to 4 cycles. If the electrical center is inside the section, then when the two voltage sources are 180 degrees out of phase, the directional and impedance elements at each end of the line will be closed, removing carrier and allowing the contact *RRB* to open, de-energizing the pendulum relay *PR*. The blocking contact *X*2, however, does not immediately reset because its coil *X*2, is energized by the alternate closing of the bottom and top contacts *PR* due to the oscillation of the pendulum. After the amplitude of vibration of the pendulum has decreased to a certain value, it will not strike either of its contacts and *X*2 will reset. This time delay is adjustable and should be longer than the time during which both directional elements "point in," which depends upon the length of the "slip cycle" of the system.

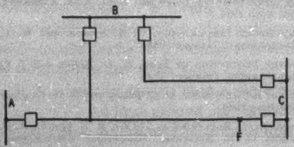

Figure 6

It is desirable to clear internal faults occurring during an out-of-step condition, but it is not so essential to be able to clear at high speed. The ground-relay trip circuit is not blocked by the out-of-step relay *X*2 and can, of course, trip instantly. On phase-to-phase faults one or 2 of the *Z*3 contacts will reset when the system swings in phase, thus allowing one of the back contacts *A*, *B*, or *C* to complete the trip circuit without waiting for the reset of *X*2. On a 3-phase fault, however, none of the *Z*3 contacts will reset and, consequently, tripping will not occur until after the expiration of the *X*2 time delay. The reset of *X*2 is made possible by the opening of the receiver relay back contact *RRB*. It will be noted from the diagram that the back-up tripping through *D*, *Z*3, and *T*3 is shown blocked by the out-of-step contacts in which case time delay back-up protection on 3-phase faults during out of step is not possible. It is arranged, however, so that the *T*3 connection can be made on the other side of the out-of-step contacts, and in this case tripping on out of step cannot be prevented for a period longer than the time setting of *T*3.

The *power supply* for the carrier set should be unfailing. This usually means battery supply with the high-voltage direct current obtained from either of 2 inverted rotary converters with automatic-transfer equipment, supplying 115-volt 60-cycle power to the power pack of the transmitter-receiver. Recent tube developments make possible a transmitter-receiver of moderate power operating directly from 125-volt battery, without power pack.

The *communication channel* for relaying must be reliable and for this reason wave traps or tuned choke coils are used. These choke coils insert enough resistance in the line to nullify the effect of impedance changes due to switching operations on the system. All taps or dead-end connection of noticeable length should be checked as an open line. One-quarter wave length long is a short circuit at carrier frequency. If *f* is the carrier frequency, the wave length is (186,000/*f*) miles for open wire line.

Supervision

It is desirable to periodically check the condition of the carrier set to determine its ability to send and receive a carrier signal. A means for accomplishing this manually is provided in the test push button connected in parallel with the carrier-start relay contacts. A milliammeter and an alarm element similar to the receiver blocking relay are connected in the receiver tube plate circuit. Depressing the test push button sends a carrier signal which operates the alarm relays at each end whose contacts operate a bell or other signal device. If the carrier set is not functioning, no signal is heard when the test push button is operated and this, of course, indicates trouble

Figure 7. Two carrier-relay sets being tested on artificial transmission system

which must be investigated. The alarm receiver element has a minimum operating value in excess of the minimum value required to block the receiver relay from operating so that an indication of impending trouble can be obtained before actual failure occurs.

A periodic manual check every 8 hours is generally considered sufficient. However, if desired, an automatic periodic check can be obtained by arranging the necessary time delay relays to initiate carrier at one end of the line, transmit to the other end, and then re-transmit to the first end. If the carrier fails in either direction, an alarm is sounded.

Figure 6 is more or less typical of certain existing lines which must be protected. The application of carrier current protection alone to such a line will not operate in all cases, as the fault current flow may not be into the line at all three terminals. For example, with a fault at *F* and a high impedance source at *B*, a portion of the fault current will flow over the parallel line between *B* and *C*. The direction being out at *B* carrier will be transmitted, blocking all terminals. With distance protection added, breaker *C* will trip on the first zone setting and as soon as the breaker has cleared *B* and *A* will trip through the carrier relays This will be somewhat faster than the second zone time step of the impedance protection as there is no time added to allow for long time clearing. Tripping occurs immediately after the first breaker opens, while the timers must be set longer than the maximum relay plus arcing time.

The carrier starting element is required to start transmission on the occurrence of a fault in the range of the relay setting. For grounds either the line residual current may be used or grounding transformer neutral current. In the latter case, auxiliary switches on the breakers should be used to prevent starting when breaker is open, this is essential on three terminal lines where the line may be energized with one terminal open.

All through the development, the idea has been kept in mind to prevent tripping a good line and to clear a faulted line at the highest practical speed consistent with the above requirement. Greater tripping speed is possible, but with less security against errors. In the testing of various sets of equipment on the laboratory artificial line several hundred faults have been applied with no failures to operate correctly, which could not be traced to some defects in connection or associated apparatus.

Conclusions

1. Zone-arranged distance relays having separate impedance measuring elements for each zone contain inherently the necessary carrier controlling and tripping circuits for the intermittent-normally blocked carrier pilot relay system.

2. The zoned distance type carrier-pilot relay system affords the economic possibility of installing high-speed distance relays at one stage in the development of a project and adding the carrier later when simultaneous clearing of all faults is economically justified.

3. With the 3-zone impedance-type carrier-pilot scheme protection of the best type available prior to the carrier relay development is afforded during intervals when the carrier equipment is out of service.

4. Spare or "transfer" breakers equipped with this relay complement may be switched into use on carrier pilot or high-speed distance relayed lines using the same distance relays in either case.

5. The 3-zone distance-type carrier-pilot scheme having a first zone independent of carrier provides proper clearing in most cases of multiterminal lines where fault power may flow *out* at one or more terminals during an *internal* fault on the protected line.

6. Experience in over 2,000 installations of the zoned distance arrangement both with and without carrier pilot has shown it to be more flexible in obtaining selectivity of back-up protection with the balance of the system relaying, than any other known arrangement.

7. Greatest flexibility in testing arrangements and in substitution and addition of elements to meet individual system relay requirements is afforded by individual switchboard mountings of phase and ground relays and auxiliaries. This has proved generally more satisfactory than the mounting of all elements in a single case with greatly restricted availability of interelement connections for test and interconnection with special elements.

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Some Engineering Features of Petersen Coils and Their Application

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THE TUNED-REACTANCE method of grounding a system neutral was first disclosed by Professor W. Petersen of Germany in 1917 (German patent number 304,823). It has been used extensively in Europe and other parts of the world and the number of installations in the United States recently has been increasing rapidly, which suggests that there is a place for the Petersen coil in American grounding practice. Tracing briefly the installations of Petersen coils in this country, the first one was made in a 44-kv system in 1921. Operating records on this system showed that the Petersen coil had improved service considerably. A second installation of 2 coils was made in a 140-kv system in 1931. Again expected improvements in service were substantiated. Further installations were made in 33-kv systems in 1935 and 1936. Although favorable reports on service continuity were given on all these installations, up until 1937, a period of approximately fifteen years, only a total of five coils were installed in systems in the United States. During the same period approximately 1,700 Petersen coils were installed in systems in other parts of the world.

In recent months, 12 more Petersen coils have been purchased for installation in systems in various parts of this country. When these installations are completed, it will bring the total number of Petersen coils in the United States to 17, protecting more than 3,000 miles of line of voltages ranging from 33 kv to 230 kv. Further installations are being contemplated by operating companies. A summary of the various applications of Petersen coils in this country is listed in table I.

It is the purpose of this paper to discuss how electric service may be substantially improved with Petersen coils. Furthermore, a résumé of some of the practical design and application features of Petersen coils will be given, based upon the experience obtained from the previously mentioned installations.

The Place for the Petersen Coil in Grounding Practice

Possibly one of the reasons there is renewed interest in the Petersen-coil method of grounding a system neutral is the fact that it has proved to be a simple, effective, and relatively inexpensive means for eliminating the short-circuit currents and service interruptions usually associated with ground faults on overhead systems when the neutral is arranged in any other way.

It is generally known that a system operated with its neutral isolated from ground may be subjected to transient overvoltages due to arcs to ground during ground faults. Such a disturbance may not confine itself to the original faulty circuit but may spread to remote sections of the system and involve other phases. Experience has shown that the possibility of such trouble increases as system voltage and mileage increase. Grounding the system neutral solidly or through relatively low impedance will reduce the transient overvoltages, but in arranging the neutral in this way a momentary flashover to ground on one conductor becomes a short circuit which must be cleared by de-energizing the circuit. Thus, with either the neutral grounded or isolated, an interruption to service may occur.

The Petersen coil or tuned reactance method of grounding eliminates the transient overvoltages of the isolated neutral system and the short circuits of the grounded neutral system by extinguishing automatically the arc at the point of fault. Thus, one advantage of the Petersen-coil method of grounding from the viewpoint of those interested in maintaining service is that it eliminates the outages associated with flashes to ground usually encountered when the system neutral is otherwise arranged. Ground faults usually cause the majority of outages on overhead transmission lines as may be seen by referring to the latest report[1] of the subcommittee on grounding of the protective devices committee. This report gives information obtained from 27 systems which shows that the percentage of the total faults involving ground varied from a minimum of 30 per cent of a maximum of 97 per cent, with an average of 69 per cent. Thus, it may be reasoned that the elimination of the outages associated with ground faults will result in a substantial improvement in service on many systems.

Ground faults may be caused by lightning, wind, sleet, tree branches, birds, etc. A 5-year average of the probable causes of ground faults on systems ranging from 26 kv to 220 kv has been given in a recent AIEE paper.[2] It shows that 73 per cent of the ground faults are probably due to lightning and the rest to "other causes." The Petersen coil is capable of quenching flashes to ground from all transitory causes. It prevents the flash to ground from developing into a short circuit which disrupts service.

The Petersen Coil

The fundamental theory of the Petersen coil has been thoroughly treated in various papers. Therefore, this discussion will be limited to some of the practical aspects of an application.

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[1] For all numbered references, see list at end of paper.

Table I. Petersen Coils in the United States

Power Company	Date of Installation	Number of Coils	System Voltage (Kv)	Approximate Miles of Protected Overhead Line	Line-Supporting Structures	Remarks
Alabama Power Company	1921	1	44	93	Wood poles	81 miles of overhead ground wire
Georgia Power Company	1929*	1	38	129	Wood poles and steel towers	97 miles wood poles, no ground wires
Consumers Power Company	1931	2	140	275	Steel towers	All but 40 miles of line without ground wires
Central Maine Power Company	1935	1	33	560	Wood poles	No ground wires
Public Service Company of Indiana	1936	1	33	160	Wood poles	No ground wires; 4,250 feet 3-conductor cable
Public Service Company of Denver, Colorado	1937	1	95	187	Wood poles and steel towers	31 miles of wood poles; 156 steel towers; no ground wires
Public Service Company of Indiana	1937	4	33	1,300	Wood poles	No ground wires.
Metropolitan Edison Company	1937	3	66	320	Wood poles and steel towers	127 miles wood poles; 194 steel towers
Public Service Company of New Hampshire	1937	1	33	120	Wood poles	No ground wires
Consolidated Gas and Electric Light and Power Company	1937	1	33	30	Wood poles	No ground wires; 3,000 feet 3-conductor cable
Southern California Edison Company	1938	2	230	260	Steel towers	2 overhead ground wires
	Total	17				

* Coil formerly on Alabama Power Company.

The quenching of an arc by the mechanical separation of contacts as in an oil circuit breaker is so universally used and understood that often a mental hurdle has to be cleared before it can be conceived that an arc can be extinguished otherwise. The Petersen coil puts out an arc without the aid of any moving parts by limiting the current in a flash to ground to a relatively few amperes and keeping the rate of rise of the recovery voltage low.

Theoretically, the leading charging current of the system is neutralized by the lagging inductive current from the coil which results in zero current in the fault. In practice, this is approximated as closely as conditions will permit.

The current in the fault consists of several components as shown in figure 1. Leading current is derived from the inherent capacitance in the lines, insulators, transformers, and other connected apparatus. Capacitance is usually considered to be a constant. Actually it has been observed to vary with weather conditions and corona. Measurements have been taken on a Petersen-coil-equipped system which show that the capacitive reactive current increased approximately 5 per cent during stormy weather. Corona also increases the charging current.

The lagging component of current comes from the Petersen coil. The coil reactance is high which permits the system neutral to be almost fully displaced during a ground fault.

With complete neutralization of the charging current, there is still a small amount of current remaining in the fault. This is an in-phase component of current and is derived from the resistance in the series and shunt paths in both the capacitive circuits and the Petersen coil. Corona loss will also increase the in-phase component of current. There is also harmonic current in the fault. The Petersen-coil impedance is too high for harmonic currents to return in any appreciable magnitude through the system neutral so they must return to the system through the system capacitance. On those systems which have installed Petersen coils to date, the third harmonic has been almost completely absent. The fifth harmonic has been present in some instances but there has been no occasion for compensating for anything other than the fundamental component of charging current. Harmonic currents of any appreciable magnitude higher than the fifth have not been observed.

The oscillogram on figure 2 shows the current in a Petersen coil and in the arc which was taken during a staged test on a 33-kv system. The harmonics are plainly visible in the arc current. System phase-to-phase voltages are also shown. It is to be noted that the fault produced no change in these voltages with the result that the connected load on the system was unaware of the disturbance.

Whatever fundamental frequency in-phase component of current is present in a ground fault is in phase with the line-to-neutral voltage of the faulty phase and is easily interrupted. In considering arc extinction, the magnitude of the residual current alone is not the only consideration. The magnitude of the recovery voltage at the time the current passes through zero is far more important. Should the current in a ground fault lead or lag behind the neutral voltage of the grounded phase by approximately 90 degrees, and an interruption be attempted at a current zero, there is full line-to-neutral voltage available to re-establish the arc. This condition obtains when the neutral is isolated or solidly grounded. If the fault current and voltage are in phase, the voltage phases through zero at approximately the same time the current does, and accordingly, the recovery voltage is low and the arc easily ex-

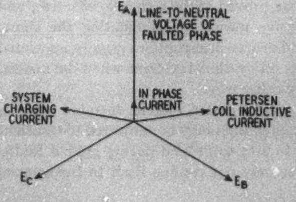

E_A
LINE-TO-NEUTRAL VOLTAGE OF FAULTED PHASE
IN PHASE CURRENT
SYSTEM CHARGING CURRENT
PETERSEN COIL INDUCTIVE CURRENT
E_C
E_B

Figure 1. Vector diagram showing relation of the components of current in a ground fault on a system with Petersen coil

tinguished. This latter case occurs when a Petersen coil is in tune with a system. The significance of the part the recovery voltage takes in arc extinction may be demonstrated by staged tests on a Petersen-coil-equipped system by applying arcing faults with the coil detuned so that there is a predominance of either leading or lagging current in the fault as has been done on several occasions.

Successful arc extinction has occurred on a system[9] equipped with Petersen coils with the magnitude of the in-phase component of current as high as 50 amperes, and there is reason to believe that this current is not the outside limit that can be successfully handled.

Factors Influencing Characteristics of a Petersen Coil

The insulation on a Petersen coil is dictated by the voltage to which it is subjected. As it is a grounding impedor of special characteristics, designed for location between the system neutral and ground, it is subjected to dynamic neutral-to-ground voltages during ground faults.

In Europe, the Petersen coil has been insulated for the system line-to-line voltage and no protective equipment has been provided. In this country, Petersen coils have been insulated for system leg voltage and in addition, protective equipment has been furnished which is an integral part of the coil, consisting of either a station-type Thyrite lightning arrester or Thyrite resistor. An internal view of a Petersen coil showing the Thyrite resistor is given on figure 3.

Petersen coils installed in this country have all been constructed with iron cores except the one for the Alabama Power Company. This latter coil was a coreless reactor and service experience with it showed that it produced surges on switching operations, making it necessary to short circuit it during switching. It was found that unequal pole opening on one of the system oil circuit breakers put the coil and system into series resonance. Series resonance may tend to occur when there is an unbalance in the system voltage to ground, but no ground fault is present. Trouble of this nature has been entirely absent on all subsequent installations and was eliminated by constructing the Petersen coils with iron cores. These cores are specially designed to saturate on overvoltages. Saturation changes the coil reactance and destroys resonance. The protective equipment included is for the purpose of rendering harmless the transient overvoltages which might reach the coil through the transformer in the neutral of which it is connected.

The current rating of a Petersen coil is established by the system charging current. The coil is designed to pass an inductive reactive current which is equal in magnitude to the system charging current in the 2 healthy conductors when the third conductor is grounded and the system neutral is at leg voltage above ground.

The charging current in a line-to-ground fault on a system is not subject to precise calculation as there are many factors to be considered which cannot be readily analyzed. The location of the equivalent ground plane; the effect of the supporting poles or towers on the transmission lines; substation structures, and trees along the right-of-way; the variation in sag, angles and bends in the lines, etc., and capacitance in connected apparatus, all influence the capacitance to ground and make it extremely difficult to predetermine exactly what the charging current will be.

Experience has shown that the charging currents calculated by the aid of Maxwell's coefficients should be used with caution, especially on the medium voltage lines. Measurements on several 33-kv systems have given charging currents as high as 50 per cent in excess of calculations for the line configurations in use. For medium voltage circuits with overhead lines, the measured charging current has varied between 4.8 and 6.7 amperes per 100 miles of line per 10 kv of voltage. For high-voltage circuits, Maxwell's coefficients have given the correct current within 10 per cent if corrections for corona are included.

Ammeter measurements of system charging current in solid conductor-to-ground faults will usually give currents in excess of requirements for the Petersen coil. Such measurement includes harmonic currents. The Petersen coil functions on the fundamental component of current only and accordingly due allowance must be made for the harmonic currents.

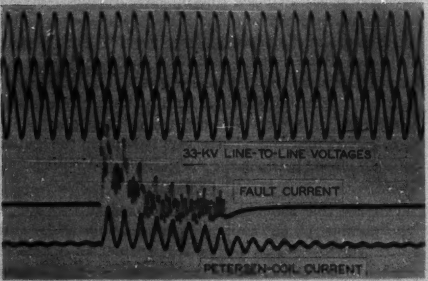


Figure 2 (left). Oscillogram showing the extinction of arcing fault by Petersen coil

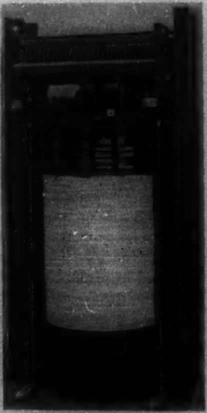


Figure 3 (right). Interior view of 33-kv Petersen coil showing Thyrite shunting resistor

The Petersen coil winding is tapped so that its inductive reactance may be matched with the capacitive reactance of the system. Additional taps are usually included so that variations in system capacitance resulting from switching or system growth may also be compensated. A current range on a coil greater than 3 or 4 to 1, however, is usually considered inadvisable because with a wider current range, a sufficient degree of saturation cannot be obtained in the iron core. If a greater range is required, 2 or more coils are recommended with the current requirements proportioned among them and provision made for switching them in and out of service as required. The taps on the coil are brought to a suitable position switch, the handle of which in the later coils is located on the side of the tank where it is convenient for adjustments as shown on figure 4. Taps are provided so that accurate tuning can be readily obtained. The number of these has varied in different coils from 10 to 25. The taps are so proportioned that the current output of a coil may be adjusted in increments varying between five and ten per cent of the current rating.

The thermal characteristics of a coil are usually established by whether or not the system has means for locating and segregating permanent ground faults. Petersen coils with 10-minute time ratings are recommended for systems that must remove all ground faults promptly. Time ratings shorter than 10 minutes are discouraged as a multiplicity of temporary faults may occur in rapid succession and overheat a shorter time rated coil. Petersen coils with continuous time ratings should be used where the ground fault is likely to remain on the system for a considerable time. Experience has also shown that when permanent faults have to be located by cut-and-try methods, time ratings of at least one hour are advisable.

Petersen coils are either furnished with conservators or gas-seals. A coil with a conservator is shown on figure 4.

General Application

Petersen coils may be applied to any transmission system or network. System voltage is no limitation on the application. Theoretically, one Petersen coil is sufficient to protect all parts of an entire system which are metallically interconnected. However, if the system is extensive, it is advisable to install several small coils rather than one large one. The combined current ratings of the coils should equal the total charging current of the system. With several coils located in selected sections of the system, the loss by switching a section out of service will not appreciably affect the protection for the remainder of the system. On systems with a limited number of miles of line, Petersen coils for compensation of charging currents under 5 amperes usually prove to be uneconomical.

Petersen coils may be applied to systems which are operated at present with the system neutral grounded or isolated. If the system neutral has been grounded, a check must be made to determine if all apparatus connected between the lines and ground, such as, for example, transformers and lightning arresters, are insulated for the system line-to-line voltage. Power transformers with graded insulation may be used with Petersen coils in their neutrals, provided the neutral insulation is suitable for line-to-neutral voltage. If the system neutral has been isolated, then usually there is little question but what the connected apparatus is suitable.

Petersen coils are designed for location between the system neutral and ground. The neutral of any convenient power transformer bank can be used for this purpose or, if none is available, grounding transformers will have to be installed. Attention should be called to the fact that during ground faults, the Petersen coil imposes an additional burden on the power transformer. This usually is not serious unless operation for extended periods is contemplated with one line conductor solidly grounded. In the latter case, a reduced power load on the transformer may be advisable; the exact amount to be specified by the transformer manufacturer. If grounding transformers are to be used, the additional burden imposed by the Petersen coil should be considered in determining its characteristics. On large networks, the use of a multiplicity of coils tends to decrease the Petersen coil burden on individual power transformer banks.

The exact location of a Petersen coil in an area to be protected is immaterial as far as the proper functioning of the coil is concerned. It is desirable, however, from an operating viewpoint, to have it in a centrally located position preferably where the transmission lines radiate to the various parts of the system so that the usual switching operations will not separate the coil from a major portion of the system.

Petersen coils will not function properly if the system on which they are installed is metallically connected to a solidly grounded-neutral system because the solid-neutral grounds in effect short circuit the Petersen coil. Likewise, an interconnection with an isolated neutral system requires retuning of the coil. Two systems, each properly compensated with Petersen coils can be interconnected without any adjustments of the coils. Tuning is not critical. On medium-voltage systems, different sections may be switched out up to possibly 25 to 30 per cent of the total mileage of the system and still obtain successful arc extinction without retuning for the new conditions.

Systems interconnected through autotransformers are in effect metallically connected together and if Petersen coils are to be used, they must be designed to compensate for the charging current of all the lines metallically connected to both sides of the autotransformers. It is inadvisable to use Petersen coils on such systems, however, if the transformation ratio of the autotransformer is appreciable because of the dynamic overvoltage that may be impressed on system apparatus during ground faults resulting from neutral displacement.

Systems with transmission lines of appreciable length of different voltage ratings on the same supporting structures may require special attention because of the mutual capacity effects between the lines of different voltage ratings.

Petersen coils should not be considered as a substitute for lightning protective equipment in substations and switch yards. It is a service rather than an apparatus protective device and should not be considered otherwise.

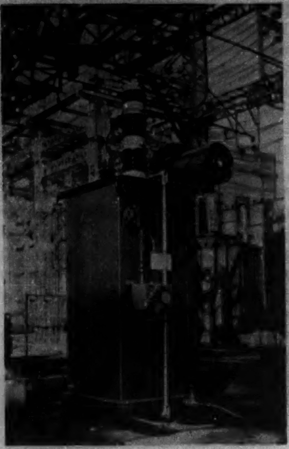


Figure 4. View of Petersen coil installed on 33-kv system

Petersen Coils and Ground Relays

An American solution of the arcing-ground problem has been to ground the system neutral solidly or through a relatively low impedance and install a selective system of ground relays. Consequently, the question is often raised as to the merits of applying Petersen coils to a system so equipped.

The addition of a Petersen coil will prevent the ground relays from functioning on all transitory ground faults, and thus eliminate many unnecessary line outages. If the fault is persistent, means may be provided for regrounding the neutral so that the ground relays can function to clear the faulty section. Thus, the Petersen coil may be considered to be a supplement to the ground relays; a device which prevents them from operating on all except permanent ground faults.

A small percentage of the ground faults on a system are a result of damage to the system insulation and must be located and segregated from the rest of the system by oil circuit breakers. The most positive way of locating such faults on a Petersen coil equipped system which has been proposed to date is to use a system of ground relays which responds to ground faults when the system neutral is directly grounded. The reactive current passed by a Petersen coil during a ground fault is insufficient to operate a selective system of ground relays but by arranging to bypass the Petersen coil and ground the neutral directly, when a permanent fault occurs, the relay system can be made to function when it is needed. This requires a suitable single-pole switch in parallel with the coil with control equipment which will close the switch after the ground fault has persisted for a definite predetermined time. Operating experience with Petersen coils to date indicates that approximately 15 seconds is ample time for a ground fault to clear if it is going to be extinguished automatically.

On a system grounded at one point, the relaying for ground faults is the simplest because the flow of fault current is all in one direction. The addition of a Petersen coil to such a system is a very simple procedure.

On a network grounded at several points, the relaying for ground faults becomes more involved because on occasions the network may be divided into sections. Neutral ground points should be located in each section if ground relaying is to be obtained under all conditions. For like reasons, Petersen coils should be located in each individual section of the system if protection is to be obtained at all times. On such a system, single-pole switches for short-circuiting the Petersen coils should be furnished for making the ground relays operative. Additional solid-neutral ground points over and above those in which the Petersen coils are located may be needed for selective relaying. Suitable switches in these neutrals functioning on zero-sequence voltage will provide these additional ground points. The far ends of radial feeders from a network may be relayed on zero-sequence voltage rather than zero-sequence current when there is an infeed of power at the far end. In this case a faulty radial feeder would clear in cascade, the near end first on residual overcurrent. This would leave the line with the neutral ungrounded for a brief interval of time, but the possibility of arcing grounds occurring is nil because the fault would be a direct ground.

In Europe, attempts have been made to relay on the uncompensated in-phase component of current in case of permanent ground faults. This requires a watt type of relay which is usually not very difficult to manufacture. However, as the magnitude of this in-phase current is small to begin with and varies with seasonal changes, the characteristics of the individual current transformers used with the relays must be matched very closely if selective relaying is to be obtained. The best information available indicates that these watt relays are used for fault indication rather than for tripping oil circuit breakers. On the occurrence of the fault, the station operators determine which feeder has relays that show an infeed of fault current at both ends. This circuit is then tripped by hand. This is also in keeping with the established practice in many parts of Europe of operating with one conductor grounded for considerable periods of time until it is convenient to remove the faulty feeder from the circuit. Operating with one conductor grounded for extended periods in this country will probably seldom be resorted to on overhead systems because of the possible hazards to safety, communication, etc.

Description of Systems on Which Petersen Coils Are Installed and Their Service Record

The Alabama Power Company Petersen coil was located on a 93-mile section of their 44-kv system. The lines were carried on wood poles, of which all but 12 miles were equipped with one overhead ground wire at the peak of the poles. All poles had down wires. During a period of slightly over 2 years this coil operated approximately 360 times, about 90 per cent of which were recorded as correct. The first year's operation indicated an 83-per-cent reduction in the number of interruptions over that obtained for the previous year.

This coil was kept in service about 4 years on this system. At the end of this time, the system outgrew the coil and it was removed from service. In March 1929, this coil was installed in the Marietta transmission substation of the Georgia Power Company's 38-kv system.

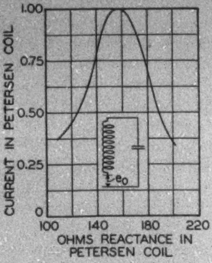


Figure 5. Tuning curve with no conductor grounded

This system has 129 miles of overhead line, 97 of which are wood poles without down wires or ground wires. On June 4, 1931, immediate initial reclosure was applied to the feeder circuit breakers. This company reports that the latter method of protection is so superior to any other method yet tried for improving service to radial feeders that little time has since been spent in analyzing the Petersen-coil performance, and data on its performance were not available at the time of writing this paper. It appears from other information sent us on this system that most of the tripouts from lightning resulted from phase faults and if this is the case, then the Petersen coil will prove to be of little use to them in improving service continuity.

The Consumers Power Company's 2 Petersen coils are installed on a 275-mile steel-tower section of their 140-kv system. All but about 87 miles of this system is single circuit and about 140 miles is without ground wires.

The operating record on this system shows that approximately 70 per cent of all the faults that occurred in 5 years' time were cleared without oil-circuit-breaker operation. In this period 242 faults were recorded with 171 extinguished by the Petersen coil.

The Central Maine Power Company's Petersen coil is installed on their 33-kv system. Five hundred and sixty miles of line of wood poles with pin insulators are protected with one coil. No overhead ground wires or down wires on the poles are used. In the first 14 months after installation, this coil operated 54 times and cleared all ground faults, which were not permanent faults, without a service interruption. On this extensive system, before the coil was installed, simultaneous faults in different parts of the system were quite numerous. Since the Petersen coil has been in service, this type of trouble has been mostly absent.

The Public Service Company of Indiana has 5 Petersen coils protecting more than 1,500 miles of 33-kv lines. These lines are supported on wood poles with pin insulators and, in general, are without ground wires or down wires. These coils are located in their Newcastle, Edwardsport, Lenore, Dresser, and Bedford Stations. The last 4 named coils, at the time of writing this paper, have not been in service long enough to have established a service record. The Newcastle coil has been in service for a longer period and the operating record has been watched with keen interest.

The 33-kv system in which the Newcastle Petersen coil is located is normally not interconnected with any of the other 33-kv lines. Approximately 80 miles of overhead line and 4,250 feet of 3-conductor cable constitute the usual operating set-up although on occasion a total of 160 miles of overhead line may be protected by the coil.

The operating record shows that 91 per cent of the faults on this system were cleared without a tripout. This operating record does not extend entirely through the severe part of the lightning season. Based on the record available, however, lightning caused approximately 25 per cent of all the faults. The ground faults which were successfully cleared resulted from wind, sleet, tree limbs, and lightning. This company reports that it is extremely well pleased with its Petersen-coil equipment.

The Public Service Company of Colorado has one coil in a 187-mile system operating at 95 kv. This line has 31 miles of wood poles and 159 miles of steel towers without ground wires. The neutral of this system was solidly grounded before the Petersen coil was installed. The operating record, month by month, on this system after the Petersen coil was installed, has been steadily improving. At first, a number of outages occurred which were apparently the result of somewhat defective insulation and incorrect relay operations. The latest monthly operating reports, however, indicate that these conditions have been remedied because approximately 73 per cent of all of the faults in these later months have been cleared by the Petersen coil. A detailed report of this installation has been given in a companion paper at this midwinter convention.[13]

The other installations of Petersen coils which have been made recently have not been in service long enough to have a report on their record. A summary of some of the system characteristics of these latter Petersen-coil installations, however, is given in table I.

The question is often raised as to the performance to be expected from a Petersen coil on a wood-pole line. The reasoning was that owing to the high insulation value of wood, most of the faults from lightning would involve more than one phase as the flashover potential between phases would be lower than from one phase to ground. The operating records on the several installations of Petersen coils on wood-pole lines previously mentioned show definitely that ground faults from lightning are obtained. One possible explanation of this is the fact that most wood-pole lines are guyed every few structures which provides a low-impedance path to ground. Also it is to be remembered that lightning is not the only cause of trouble. Studies, also previously mentioned, showed that lightning caused about three-quarters of all line interruptions and on medium-voltage lines, the proportion of faults attributed to lightning is often less. Although the operating experience with Petersen coils on wood-pole lines during lightning seasons has been good, the probability is that it would have been even better if every pole on the lines in question had down wires and low footing resistance.

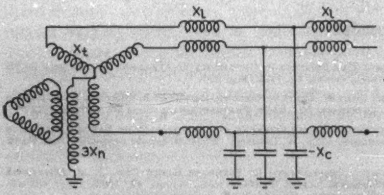
Determination of the Optimum Tuning of a Petersen Coil

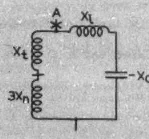
When a Petersen coil is installed in a system, it is necessary to match the inductive reactance of the coil with the capacitive reactance of the system. Taps are provided on the coil winding so that its reactance can be adjusted to meet the system requirements.

The proper setting of a Petersen coil may be determined by calculation if the charging current of the system has been accurately determined. Usually it is desirable to check at least one adjustment by test at the time the coil is put in service. These tests may be conducted during normal conditions (no conductor grounded) or with one conductor solidly grounded.

During normal conditions of operation, there is usually some slight unbalance in the system phase-to-ground voltages. This unbalance is sufficient to cause a small amount of current to flow continuously through the coil. For tuning tests, the coil ohms are varied and the current through the coil noted for each tap position. A typical plot of the coil current as a function of the ohms reactance in the coil taken on a 33-kv system is shown on figure 5. Correct tuning is obtained on the tap that results in maximum amperes.

The shape of this curve may be explained by understanding that the inductive reactance of the coil and the capacitive reactance of the system form a series resonant circuit with a voltage applied to it resulting from the normal system unbalance in voltage as shown on figure 5. When the coil is in tune with the system, the inductive reactance of the coil balances the capacitive reactance of the system so that the current flowing is at a maximum and is limited only by the losses in the circuit. The current diminishes as the coil is detuned because the impedance of the resonant circuit increases.





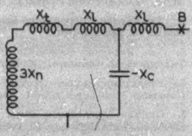


Figure 6. Zero-sequence impedance of Petersen-coil-equipped system with no ground fault

Figure 6A (lower left). Zero-sequence impedance for ground fault at (A)

Figure 6B (lower right). Zero-sequence impedance for ground fault at (B)

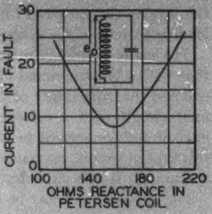


Figure 7. Tuning curve with one conductor grounded

For those familiar with symmetrical components, Petersen-coil action results when the zero-sequence impedance of a system is infinite. Accordingly, during ground faults, there will be no fault current in the positive- or negative-sequence impedance circuits and for calculations, the zero-sequence impedance circuit need only to be considered.

Tuning on normal unbalance in system voltage is expressed in symmetrical components by referring to the system zero-sequence impedance as represented on figure 6. Neglecting X_l and X_t which are small compared with X_n and $-X_c$ and designating the different capacitances to ground of the 3 phases as X_{c1}, X_{c2}, and X_{c3} the following equations may be written:

$$E + E_0 - I_{c1} X_{c1} = 0 \tag{1}$$

$$a^2E + E_0 - I_{c2}X_{c2} = 0 \tag{2}$$

$$aE + E_0 - I_{c3}X_{c3} = 0 \tag{3}$$

$$E_0 = -I_nX_n \tag{4}$$

$$I_n = -(I_{c1} + I_{c2} + I_{c3}) \tag{5}$$

$$I_n = \frac{E(X_{c2}X_{c3} + a^2X_{c1}X_{c3} + aX_{c1}X_{c2})}{-X_{c1}X_{c2}X_{c3} + X_n(X_{c2}X_{c3} + X_{c1}X_{c3} + X_{c1}X_{c2})} \tag{6}$$

tuning results when

$$X_n = \frac{X_{c1}X_{c2}X_{c3}}{X_{c2}X_{c3} + X_{c1}X_{c3} + X_{c1}X_{c2}} \tag{7}$$

When the capacitances of the 3 phases to ground are equal

$$X_n = \frac{1}{3} X_c \tag{8}$$

Tuning tests with one conductor solidly grounded are made by recording the current in the fault with the Petersen coil in different tap positions. The current in the fault is the resultant of the fundamental-frequency lagging current passed by the coil, and the leading charging current of the system, plus the in-phase components which result from leakage and harmonics. A typical plot of the fault current as a function of the ohms reactance in the coil is shown on figure 7. Correct tuning is obtained on the tap that results in minimum current in the fault as the lagging current passed by the coil just neutralizes the leading current of the system.

The shape of this curve on figure 7 results from the fact that with a line-to-ground fault on a system, the Petersen coil and the capacitance of the system form a shunt resonant circuit with a voltage equal to the system line-to-neutral voltage applied to it as shown on figure 7. With this type of circuit, resonance is obtained when the current in the fault is at a minimum.

The fault current may be expressed as

$$I_f = \frac{E}{X_0} \qquad (9)$$

For a fault at the terminals of the transformer in the neutral of which the Petersen coil is located (location *A*, figure 6*A*) the zero sequence impedance is equal to

$$X_0 = \frac{3X_n(X_l - X_c + X_t)}{3X_n + X_l + X_t - X_c} \qquad (10)$$

Varying the magnitude of X_n, a curve is obtained such as is shown on figure 7.

In equation 10 neglecting X_l and X_t, resonance obtains when

$$X_n = \frac{1}{3} X_c \qquad (11)$$

This shows that provided X_c is a constant, either method of tuning gives the same tap position for resonance as may be checked by comparing equations 8 and 11.

Corona has the effect of increasing the effective diameter of the line conductors which in turn increases the system capacitance and when corona is present, a lower ohmic setting will be obtained for correct tuning during solid ground faults. When corona is present, the results obtained from fault tuning rather than those from the normal leakage current through the coil should be used.

It has also been observed that the system capacitance changes slightly with changes in the weather. It is known that the apparent capacitance of pin insulators increases when they are wet or covered with sleet. In all cases noted, these changes have caused a shift of less than 5 per cent in the tuning so it has been of little practical significance.

Tuning is independent of fault location. A Petersen coil in tune to a system for a ground fault at any particular location is correctly tuned for a ground fault at any other location. For a ground fault at the terminals of the transformer of the system (location *A*) shown on figure 6 the zero-sequence impedance is as given by equation 10. For a ground fault at the far end of the system at location *B*, the zero-sequence impedance is (figure 6*B*)

$$X_0 = \frac{(-X_c)(3X_n + X_t + X_l)}{3X_n + X_t + X_l - X_c} + X_l \qquad (12)$$

Correct tuning is obtained when X_0 is infinite. This occurs when the denominator of equations 10 and 12 are zero. Since the denominators of the 2 equations are alike, correct tuning in both cases is obtained.

It is to be noted that as the ground-fault location progresses away from the coil, the impedance increases and the current through the coil decreases. The charging current also decreases in like proportion so that correct tuning is always maintained.

The question often arises as to the accuracy needed in tuning a Petersen coil to a system. Field tests have shown repeatedly that the best results are obtained when the coil is adjusted to within a few per cent of the exact tuning necessary. However, on medium-voltage systems, successful arc extinction has been obtained through a relatively wide range in reactance in the coil. Field tests on several 33-kv systems have shown that successful arc extinction would still be obtained if 25 to 30 per cent of the system was switched out of service. On high-voltage lines, a somewhat closer tuning is indicated to be desirable.

Summary

Service experience with Petersen coils on transmission systems and networks in this country has demonstrated conclusively certain facts concerning these protective devices which may be stated as follows:

1. There is definitely a place for Petersen coils in American system-protective practice.
2. The Petersen coil will prevent the interruptions to service usually associated with transitory flashovers to ground.
3. The Petersen coil may be applied to any system insulated for full neutral displacement.
4. Petersen coils do not lose their effectiveness if used on a system when different sections are switched out of service unless the system capacitance is changed more than 25 to 30 per cent.
5. Surges during switching operations encountered on the first Petersen-coil application have not recurred in later applications.
6. By the use of Petersen coils, it is possible that from 70 to 80 per cent of all faults may be cleared without the opening of a single oil circuit breaker, and without any dip in the phase-to-phase voltage, thus improving system stability and service continuity.

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The Thyratron Motor at the Logan Plant

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MEMBER AIEE

Introductory

THE QUEST for a simple variable speed a-c motor has been long and, heretofore, not particularly successful. Consequently ever since the thyratron motor was mentioned during the early years of the present decade, it has aroused a great deal of interest among engineers, both because of its operating principles and because it appears to offer the possibility of a really effective and economical variable-speed alternating-current drive. It has been of particular interest to central-station engineers.

Quite a number of types of a-c drive are in use today in central station service and many more have been designed and discarded or never applied. The difficulties of the problem have not been ones of principle. On paper it may appear easy to develop such a drive having a minimum of special parts or a minimum assembly of standard units. Yet when these are tried in practice, some apparently insignificant detail—an oil ring or a rotating bearing—develops unexpected trouble during operation and definitely limits the new design. The squirrel-cage motor has set a criterion in ruggedness and simplicity that it has so far been impossible to equal.

In central-station service, the tendency has been to reduce to a minimum the number of auxiliaries requiring variable speed. In recent years studies of heat cycle, indicating the desirability of using steam turbines for the larger auxiliaries have further limited the use of variable speed motors. Nevertheless, those auxiliaries which still require varying speed, such as boiler feed pumps and draft fans, are among the largest, from the point of view of energy consumption, in the plant. They also require the utmost in reliability and ease of control and because they are often on practically a 24-hour basis of operation, it is desirable that variable speed be obtained without sacrificing efficiency.

The reasons for adopting this type of motor were given in the *Electrical World* of July 6, 1935, and need not be repeated here. The advantages appeared sufficiently attractive so that the American Gas and Electric Company urged the completion of developments on this motor and were in close contact with the manufacturer as the work progressed. In a development of this kind, it is particularly essential that the operating requirements are clearly brought to the attention of the designers. Their concentration on the theoretical aspects of the problem sometimes leads them to overlook homely but nevertheless important details. Dust tight distributor and tube compartments, safety interlocks on tube cells and cubicle doors, segregation of power and control terminals in the cubicle, forced air cooling, overspeed device, tube removal under load, and tube failure alarm are examples of the results of this co-operation. As in all new designs many minor difficulties arose which had to be overcome but nothing which could be considered basic.

Description of Equipment

The motor is rated 400 horsepower, 2,300 volts, 3-phase, 60-cycles, 40 degrees centigrade and was designed to drive an induced draft fan having a speed range from 625 to 350 rpm. It closely resembles a standard synchronous motor with a rotating field, except that it has two independent star-connected stator windings installed in the same slots. Unlike a synchronous motor, however, the field terminals, after being brought out through slip rings, connect to the neutral points of the star windings. As shown in figure 4, the six remaining power leads connect in a special arrangement to a series of 18 type *FG*-118 thyratron tubes, each having a continuous rating of 12.5 amperes and a maximum rating of 75 amperes.

The accessory equipment includes a three-phase current-limiting reactor, a thyratron-tube cubicle, and a speed-

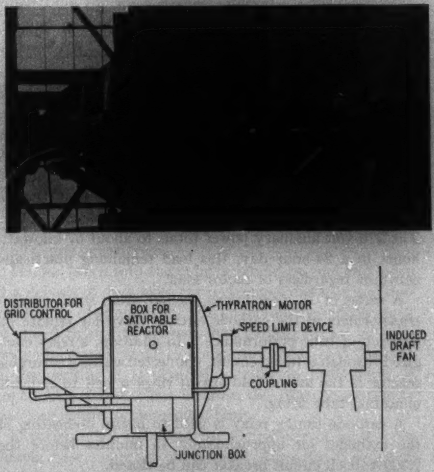

Figure 1. Thyratron-controlled 400-horsepower motor, Appalachian Electric Power Company—Logan plant

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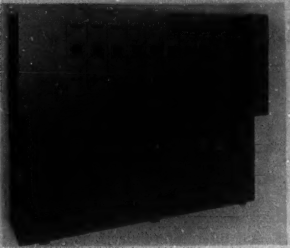

Figure 2. Thyratron cubicle showing spare tube compartment, Appalachian Electric Power Company—Logan plant

control panel. Each tube is mounted in an individual removable cell so as to facilitate replacement during operation. Tube cooling ducts and blowers, heater and grid transformers, and other accessories are all assembled within the cubicle.

The speed-control panel contains a small wound-rotor motor functioning as a phase shifter for the purpose of effecting the desired speed changes. On this panel are also mounted a line ammeter, the 2,300-volt primary oil circuit breaker control switch and the auxiliary power control switch. The motor, cubicle, and speed-control panel are shown in figures 1, 2, and 3 respectively.

Accessory Devices

The 18 thyratron tubes consume about three and one-half kw at full load with another one and one-half kw consumed in the various auxiliary transformers and relay coils in the cubicle. Forced air cooling is used to dissipate the tube losses. The air stream is directed at the base of the tubes and leaves at the top. For winter operation, the cooling system is operated partly closed and is supplemented by strip heaters. In the summer time it operates entirely open and without the heaters. Thermostatically controlled dampers regulate the output of the strip heaters as well as the proportion of recirculated and fresh air.

About 800 watts are consumed in excitation losses in the stator of the small wound-rotor motor which acts as a phase shifter for speed control, but since this is mounted on the speed-control panel which is remote from the cubicle, it does not contribute to the heat generated there. The 220-volt auxiliary power totals to about 57 kilowatt-hours in a 24-hour day, this load remaining practically constant regardless of motor load.

A series of 18 glow lamps, excited from small current transformers in the cathode circuits serve for remote indication of tube performance. In addition, a relay energized through a tuned circuit, connected across a saturable reactor in the motor field circuit rings a bell to indicate abnormal tube firing.

A cathode timing relay is used to insure preheating of the cathodes for approximately 15 minutes before the 2,300-volt oil circuit breaker can be closed.

Figure 5 shows the distributor which is mounted on the motor shaft to obtain commutation of energy between thyratron tubes. The distributor segments are connected to the grids through intermediate transformers. Two sets of segments are used, one for normal speed and a supplementary set effective at low speed by utilizing the inherent commutating action of the three-phase power supply to obtain a high starting torque. Transfer between sets is made automatically at about 150 rpm by means of a relay operating from a current transformer in the stator circuits.

An ingenious arrangement of current transformers is utilized to act on the grid circuits and cause a phase shift independently of the speed controller and thus limit tube current. Due to this feature the motor can be accelerated from standstill to a preset speed by throwing it directly across the line without danger of damage. It is thus inherently self-protecting, the overload relays on the oil circuit breaker acting solely as backup protection. Only twice rated full load current is taken under this condition.

For best operation the cathode heater potential should be lowered slightly as the load increases, since this tends to maintain a constant tube temperature. This is accomplished by inserting one winding of a saturable reactor in series with the heater circuit, the other winding being in series with the motor field. A third or saturating winding is excited from direct current. This saturable reactor serves a dual purpose, its other function being to initiate an alarm when a tube fails, as mentioned above.

Tube Disconnecting Feature

During the development of the control equipment it became evident that facilities would have to be provided for tube replacement while the motor was in operation. Since the anode potential is 2,300 volts, safety to personnel had to be taken into consideration. Means had also to be provided for interrupting the anode supply before the tube was removed and this required a 2,300-volt disconnecting switch capable of interrupting the normal tube current, since it was not considered advisable to interrupt

Figure 3. Speed-control panel for thyratron-controlled motor, Appalachian Electric Power Company—Logan plant

the anode circuit at the separable contacts of the individual tube cell. This presented a problem on account of the limited space available. It was successfully solved by developing the fuse type of disconnecting switch shown in figure 6*A*. The circuit is interrupted from the outside of the cubicle by the key which unlatches the door. The individual cells cannot ordinarily be unlocked without opening the respective disconnecting switch. However, when all tubes are to be inspected while the motor is out of service, a master key permits access to all of the cells without releasing the disconnecting switches. A thyratron tube with filament continuously heated is kept available at all times in a spare cell. (See figure 2).

Operating Principles

The operating principles of the motor have been fully described elsewhere (Electrical Engineering, November 1934). For the sake of completeness a brief review with reference to figure 4 may be helpful.

The motor, although structurally resembling a synchronous machine, functions like a d-c series motor and has characteristics closely resembling such a machine. The field winding and each phase of each stator winding are traversed by a direct current, although like values of current in the stator coils differ in time phase.

A thyratron is a mercury-vapor hot-cathode rectifier tube. A positive grid voltage is essential to start the tube conducting, but once the current flows, the grid has no further effect and conduction will continue until the anode becomes negative with reference to the cathode. If the applied grid voltage is alternating, a shift in phase between the anode and grid voltages will delay the initiation of anode-current flow, decrease the amount of energy passed during the positive half cycle, and hence decrease the speed of the motor. From this it should be clear that the motor speed is independent of system frequency.

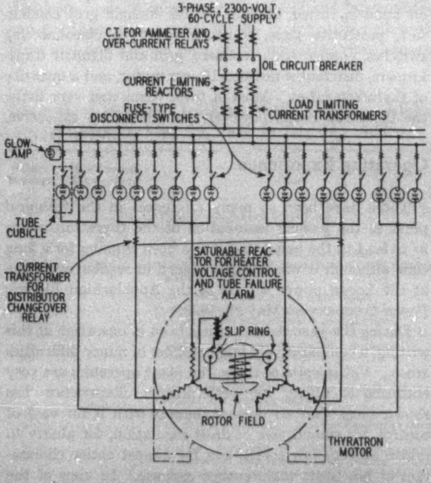
3-PHASE, 2300-VOLT, 60-CYCLE SUPPLY
C.T. FOR AMMETER AND OVER-CURRENT RELAYS
OIL CIRCUIT BREAKER
CURRENT LIMITING REACTORS
FUSE-TYPE DISCONNECT SWITCHES
LOAD LIMITING CURRENT TRANSFORMERS
GLOW LAMP
TUBE CUBICLE
CURRENT TRANSFORMER FOR DISTRIBUTOR CHANGEOVER RELAY
SATURABLE REACTOR FOR HEATER VOLTAGE CONTROL AND TUBE FAILURE ALARM
SLIP RING
ROTOR FIELD
THYRATRON MOTOR

Figure 4. Power connections for thyratron-controlled motor, Appalachian Electric Power—Logan plant

An open grid circuit will prevent tube conduction; therefore a distributor mounted on the motor shaft and connected into the grid circuits can control the time of firing of the tubes as a function of rotor position. Thus "commutation" can be attained. By connecting in parallel three tubes which receive their supply from a three-phase source, polyphase energy may be utilized for one stator phase subject to the control of the distributor. If the phase shifting and distributing features are connected in series, any tube can be controlled to fire at the proper time and for a desired interval. Thus commutation and controlled rectification with concomitant speed variation may be obtained within a single tube.

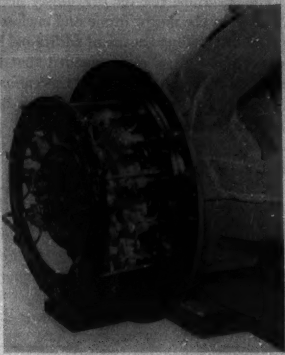

Figure 5. Distributor on thyratron-controlled motor, Appalachian Electric Power Company—Logan plant

Installation Considerations

Figure 7 shows a typical arrangement of connections between the motor and its control equipment. It is evident that there is an advantage in keeping the cubicle close to the motor. However, it is also necessary that it be installed in as cool a place as possible in order to limit the tube temperatures to the recommended upper limit of 55 degrees centigrade.

The air taken into the cubicle should be cool and clean as a deposit of dust may lessen creepage distance and lead to a failure of equipment. All covers in the cubicle must be sealed tightly to prevent dust gaining access to parts having inherently small clearance to ground. At the time of this writing it is planned to relocate the cubicle at Logan to a cooler place because the tube temperatures are close to the recommended limit.

Experience has indicated the desirability of having the cubicle base mounted on resilient supports in order to minimize shock. Rubber mounting pads under the four corners are being used for this purpose.

Since the motor has characteristics similar to a d-c series motor, precautions against loss of load, however remote, must be taken and a speed-limit device interlocked with the primary breaker control should be installed. An additional safeguard is provided in a limit stop which is placed on the speed control dial so that the rated speed cannot be exceeded. (See figure 3.)

The saturable reactor previously described is mounted directly on the motor frame and enclosed in a sheet metal box, thus avoiding the use of additional high voltage power connections. It is recommended that the distributor selector relay be also mounted in this cabinet, thus materially lessening the number of control wires between the motor and cubicle.

Spare Parts

Since there are 18 duplicate tubes with their accessories, the number of different types of device is not very large and consequently the number of spare parts to be kept on hand is rather limited. These include grid condensers, protective gaps, resistors, fuse-type disconnecting switches, copper-oxide rectifiers, grid and filament transformers, distributor brushes, blower motor, and a quantity of thyratron tubes. Most of these items cost very little; the thyratron tubes, however, are still rather expensive.

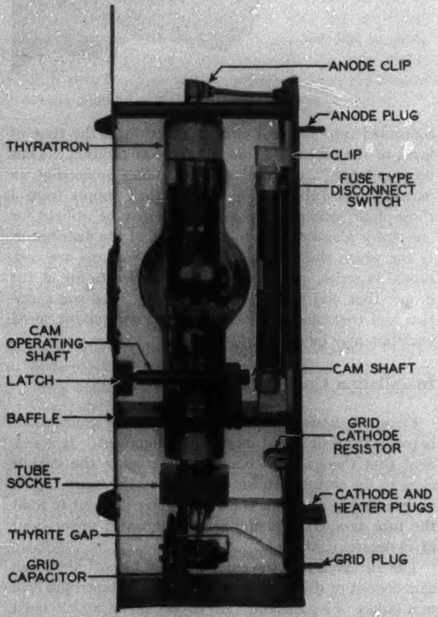

Figure 6. Thyratron tube cell showing 2,300-volt fuse-type disconnect switch

The door latch rotates the eccentric thus releasing pressure on a flat copper contact strip. The spring retracts the conductor through a section of fuse powder, thereby breaking the circuit. This switch can be used indefinitely since the conductor does not ordinarily fuse—only a slight pitting appears at its tip after a number of operations. To recondition, an operation similar to refilling a standard fuse is required, taking only a few minutes

Table I. Thyratron Tube Failures, May 29, 1936 to July 31, 1937

Failure Number	Tube Service Hours of Operation	Time of Failure	Nature of Failure
1	191	While starting	Cathode heater burned open
2	679	While starting	Cathode heater burned open
3	787	While starting	Cathode heater burned open
4	1,079	During preliminary heating before starting	Cathode heater burned open
5	2,362	During preliminary heating before starting	Cathode heater burned open
6	2,862	While starting	Cathode heater burned open Grid destroyed
7	792	During preliminary heating before starting	Cathode heater burned open
8	3,576	During operation	Apparent short circuit in tube. Protective gap and capacitor destroyed
9	4,509	During operation	Cathode heater burned open
10	4,628	During preliminary heating before starting	Cathode heater burned open
11	5,440	During operation	Tube ceased firing for no apparent cause
12	5,829	During preliminary heating before starting	Cathode heater burned open
13	5,829	While starting	Tube ceased firing for no apparent cause
14	3,550	During operation	Tube ceased firing for no apparent cause
15	6,400	During operation	Cathode heater burned open
16	742	During preliminary heating before starting	Cathode heater burned open
17	7,106	During operation	Cathode heater burned open
18	2,053	During operation	Interior of tube completely destroyed
19	7,560	During operation	Tube ceased firing for no apparent cause
20	8,272	During operation	Cathode heater burned open

Operating Experience

There have been so many references in the technical press to the pioneer installation of the thyratron motor as to lead to the belief that it has been running for a long time although it was actually placed in regular operation at the Logan power station of the Appalachian Electric Power Company on May 29, 1936.

During the year the motor has been in operation at this writing, a comparatively large number of minor difficulties arose. Yet in spite of these, the plant operators are very enthusiastic about its performance. Experience has shown them that no drive compares with it for ease of control, for smoothness of draft regulation, for ability to rotate at very low speed, and for almost entire elimination of fan blade maintenance expense. In view of the fact that plant operators are pretty hard-headed and tend to look with suspicion on new "gadgets," this response would seem convincing evidence of its success from a practical standpoint.

The most serious difficulty has been with the thyratron tubes themselves. In 14 months of operation, 20 tubes have failed, almost all of them due to the opening of the cathode heater circuit. Table I summarizes these failures and table II troubles from other causes. It is to be noted that the actual hours of motor operation represent a service availability of 84 per cent. This includes shutdown from all causes such as boiler and fan maintenance. Interruptions due to the motor alone took only three and one-half per cent of the operating time and an availability of 96$^1/_2$ per cent may be considered a not unsatisfactory record for an entirely new type of control.

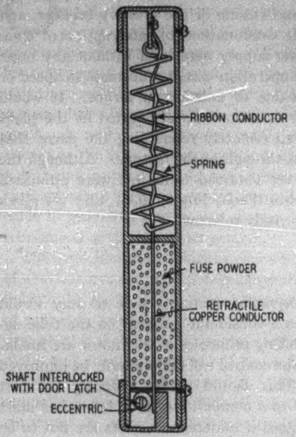
RIBBON CONDUCTOR
SPRING
FUSE POWDER
RETRACTILE COPPER CONDUCTOR
SHAFT INTERLOCKED WITH DOOR LATCH
ECCENTRIC

Figure 6A. 2,300-volt fuse-type disconnect switch for thyratron-controlled motor

Table II. Operating Troubles With Thyratron Motor Accessories, May 29, 1936 to July 31, 1937

Failure Number	Interruption Time—Hours	Cause
1		Filament transformer burned out
2	192	Field coil burned out
3	84	Field coil burned out
4	$^1/_2$	Fuse type disconnect switch burned open
5	20	Ground on one stator phase due to cable rubbing against dowel pin
6		Fuse type disconnect switch pulled apart
7	1$^1/_2$	Protective gap and grid capacitor destroyed
8	$^1/_2$	Thyratron tube and protective gap destroyed
9	3$^1/_2$	Fuse type disconnect switch arced to ground. Replaced. Again arced to ground. Replaced
10	4	Ventilating fan motor burned out
11		Field coil showed evidence of charring
12		During boiler shutdown, new set of field coils installed, and bearings changed
13		Fuse type disconnect switch burned open
14		3 tubes fired irregularly. Brush on distributor made poor contact
15		Bearings worn due to erratic magnetic center Evidence of electrolysis causing bearing pitting. Corrected by placing brushes in contact with both ends of shaft, connecting together and grounding

Total: 306 boiler-hours lost.
Total hours of operation: 8,376.

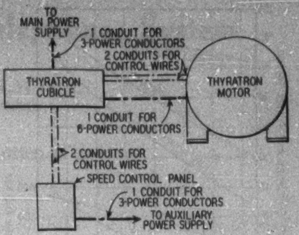
TO MAIN POWER SUPPLY
1 CONDUIT FOR 3-POWER CONDUCTORS
2 CONDUITS FOR CONTROL WIRES
THYRATRON CUBICLE
THYRATRON MOTOR
1 CONDUIT FOR 6-POWER CONDUCTORS
2 CONDUITS FOR CONTROL WIRES
SPEED CONTROL PANEL
1 CONDUIT FOR 3-POWER CONDUCTORS
TO AUXILIARY POWER SUPPLY

Figure 7. Thyratron-controlled motor—typical arrangement of equipment and conduits

Discussion of Operating Troubles

Many of the difficulties encountered are of a nature which would make it extremely improbable that they will be encountered again. For instance the trouble with three field coils was evidence that the motor field was being subjected to unexpected surge voltages and that its original insulation was inadequate. When the entire set of coils was replaced by ones having greater insulation, no further trouble of this type manifested itself.

The pitting of the bearings and the unexpected thrust due to unbalanced magnetic forces causing excessive bearing lining wear and coupling troubles were completely cured by the installation of brushes bearing on the shaft and connected to ground.

Failure of parts due to arcing to ground have been corrected where possible by providing greater clearance and the experience gained from this can of course be incorporated in subsequent designs. The fuse-type disconnect switches leave some room for improvement but they were an afterthought in the initial installation and probably a better type of device will be available for subsequent installations.

The grounding of a stator phase lead by rubbing against a metal dowel pin could have occurred in any machine.

Auxiliary transformers, protective gaps, resistors, condensers, blower motors, etc., are always possible sources of trouble. Filament and grid transformers operate at 2,300 volts and clearances must be kept large to avoid creepage due to infiltration of conducting dust which is always present in a power plant. No doubt some of the smaller devices will occasionally fail from time to time, but they are comparatively inexpensive to replace and mostly of such a nature as not to cause an immediate shutdown in case of failure. Even these can be improved by careful attention to the requirements.

The thyratron tubes leave the biggest room for improvement. A large number of failures has resulted either while starting or during the preliminary heating period and tube designers should consider these facts. Without a doubt the fragility of the cathode heaters can be corrected. The very persistance of this type of failure should be a clue to the remedial measures necessary. Steps have already been taken in this direction by the use

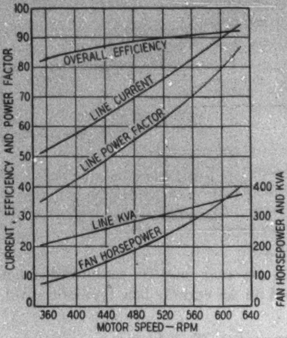

Figure 8. Performance characteristics of thyratron motor as determined by test

of flexible instead of solid connections at the cathode heater terminals within the tube.

Operation With Defective Tubes

Although defective tubes are replaced almost immediately during operation, it is interesting to note that no harmful effects occur if several tubes cease firing simultaneously. During factory tests as many as four tubes were removed simultaneously in various combinations, from different parts of the circuit, the only noticeable effect being a drop in speed and a slight irregularity in rotation. The motor will even start single phase (six tubes removed) although, of course, it will not attain full speed under this condition.

Tests were made with the cathode heater supply and grid control circuits suddenly interrupted to simulate blowing of fuses. Speed remained normal during the 15-second interval permitted by the cathode timing relay before shuttting down the motor. Low primary voltage, sudden opening and closing of the primary breaker, and almost all possible combinations of interruption of main and auxiliary power supply were tried without any noteworthy change in operation except a decrease in speed or irregular rotation due to unbalanced torque. It would go hard with any operator who attempted at the plant any operation even remotely resembling the paces that the motor was put through at the factory. Although the characteristic curves obtained from test were published previously (*Electrical World*, July 6, 1935), they are given here (figure 8) for ready reference.

Conclusion

Whether the thyratron motor is here to stay would seem to depend to a considerable extent on the tube designer. The operating principles of the motor are fundamentally sound; the control will undoubtedly be improved and simplified. This should aid in reducing the first cost of the motor to a competitive level, but tubes must be made more rugged if maintenance costs are not to be excessive.

A New Single-Channel Carrier Telephone System

By H. J. FISHER
Membership Application Pending

M. L. ALMQUIST
MEMBER AIEE

R. H. MILLS
ASSOCIATE AIEE

ON OPEN-WIRE lines where the growth is not rapid, there is frequently need for adding telephone circuits one at a time. When the type-*D* single-channel carrier telephone system was developed a few years ago it became possible to meet this need without stringing additional wires.[1] More than 500 of these systems have been placed in service in the Bell System plant. A new single-channel carrier telephone system, known as the type *H*, has recently been developed and is now being applied. This new system offers improved performance, and also, because of its lower cost, is applicable to providing service over shorter distances than were economical with the earlier system.

The type-*H* system, which is characterized by a number of new features and special developments, is applicable not only to the needs of telephone companies but also to those of railroads, power systems, and oil companies.[2] In the first place it is designed to operate either on a-c or on d-c plate and filament supply. A repeater is available to extend the range of operation. Through the use of specially designed but simple filters the system can be employed on circuits which are equipped with bridged telephone stations at intermediate points as is frequently the case in railroad operation.

A unique feature is the use of opposite sidebands of the same carrier frequency for opposite directions of transmission. The upper sideband is used in one direction and the lower sideband in the other, the carrier being suppressed. For the modulators and demodulators copper-oxide "varistors" are employed in place of vacuum tubes. The amplifiers are single stage, employ pentode tubes and are stabilized in performance by feedback. The filters are simplified in construction by the use of coils with a new type of core material and by improved designs of paper condensers.

The size of the new terminal has been so reduced that it occupies less than 40 per cent of the space required for a type-*D* terminal, as indicated in figure 1. The equipment may be mounted on racks as is customary in telephone offices, or a complete terminal or repeater may be mounted in a small cabinet.

Single-channel carrier systems have been used in the Bell System principally for short open-wire toll circuits. Thus, the type *D* systems are for the most part between 50 and 200 miles in length. The type-*H* system, since it includes a repeater can be used for greater distances, and due to its lower cost is economical for shorter distances.

General Description of System

Basic System

The basic system consists of two terminals one of which is refered to as an "east" terminal and the other as a "west" terminal, as indicated in figure 2. The two terminals differ only in minor respects, the differences being due to the fact that at one terminal the upper sideband is transmitted and the lower sideband is received, while at the other terminal the reverse takes place. In order to simplify co-ordination between various types of carrier systems operating on the same pole line, the frequencies between 7,400 and 10,150 cycles are transmitted in the east to west (or north to south) direction, and the fre-

TYPE D CARRIER TERMINAL
TYPE H CARRIER TERMINAL

Figure 1. Installation of type-H carrier telephone system at Charlotte, N. C.

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1. For all numbered references, see list at end of paper.

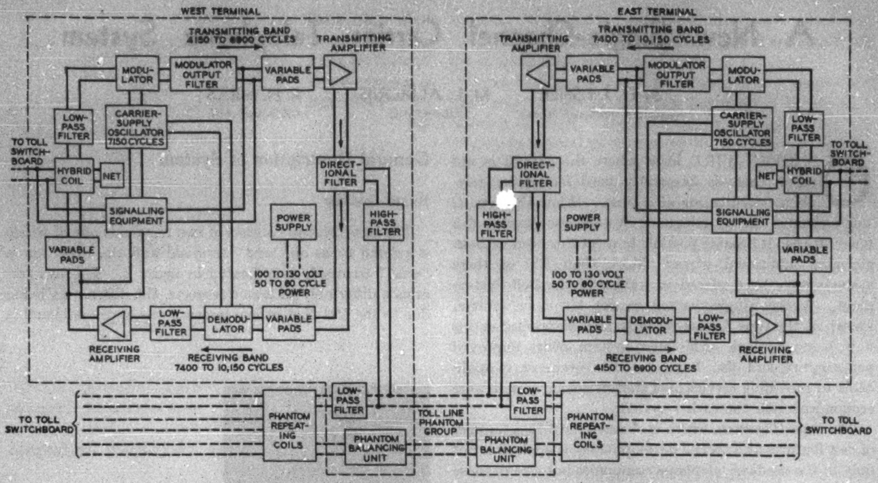
WEST TERMINAL
TRANSMITTING BAND 4150 TO 6900 CYCLES
TRANSMITTING AMPLIFIER
MODULATOR
MODULATOR OUTPUT FILTER
VARIABLE PADS
LOW-PASS FILTER
CARRIER-SUPPLY OSCILLATOR 7150 CYCLES
TO TOLL SWITCHBOARD
HYBRID COIL
NET
DIRECTIONAL FILTER
SIGNALLING EQUIPMENT
POWER SUPPLY
HIGH-PASS FILTER
VARIABLE PADS
100 TO 130 VOLT 50 TO 60 CYCLE POWER
LOW-PASS FILTER
DEMODULATOR
VARIABLE PADS
RECEIVING AMPLIFIER
RECEIVING BAND 7400 TO 10,150 CYCLES
EAST TERMINAL
TRANSMITTING BAND 7400 TO 10,150 CYCLES
TRANSMITTING AMPLIFIER
VARIABLE PADS
MODULATOR OUTPUT FILTER
MODULATOR
CARRIER-SUPPLY OSCILLATOR 7150 CYCLES
LOW-PASS FILTER
TO TOLL SWITCHBOARD
DIRECTIONAL FILTER
NET
HYBRID COIL
HIGH-PASS FILTER
POWER SUPPLY
SIGNALLING EQUIPMENT
VARIABLE PADS
100 TO 130 VOLT 50 TO 60 CYCLE POWER
VARIABLE PADS
DEMODULATOR
LOW-PASS FILTER
RECEIVING AMPLIFIER
RECEIVING BAND 4150 TO 6900 CYCLES
TO TOLL SWITCHBOARD
PHANTOM REPEATING COILS
LOW-PASS FILTER
PHANTOM BALANCING UNIT
TOLL LINE PHANTOM GROUP
LOW-PASS FILTER
PHANTOM BALANCING UNIT
PHANTOM REPEATING COILS
TO TOLL SWITCHBOARD

Figure 2. General schematic of type-H system applied to side circuit of phantom group

quencies between 4,150 and 6,900 cycles in the west to east (or south to north) direction. The frequency allocation of the type *H* system and those for the type-*D* and the three-channel type-*CS* system are shown in figure 3. All three types may be operated on the same pole line.

The circuit arrangement is given in greater detail in figure 4, which shows a schematic diagram of one terminal, with the exception of the power supply circuit. Each terminal is made up of a transmitting branch which includes a modulator, a receiving branch which includes a demodulator, and a hybrid coil to combine the two branches into a two-wire voice frequency circuit. The carrier is generated by a vacuum tube oscillator which supplies both the modulator and demodulator. The output of the modulator, which is of the copper-oxide type, consists principally of the two sidebands. The desired sideband is selected by the modulator output filter and an amplifier raises the level to that desired for transmission over the line. The demodulator is also of the copper-oxide type; its output consists principally of the two sidebands, one of which is a reproduction of the original voice-frequency input. This is selected by means of a low-pass filter and applied to a voice-frequency amplifier which provides the necessary receiving gain. Adjustable pads serve as a means for adjusting the transmitting and receiving gains. The characteristics and functions of the various filters are described later.

For signaling over the carrier circuit, a 1,000-cycle signaling system is employed. The method is similar to that used on other types of circuits, but includes a number of simplifications. The number of tubes has been reduced, and the vibrating relays used in the older type circuits have been replaced by copper oxide rectifiers and simple d-c relays requiring little maintenance. When 20-cycle ringing current is received from the switchboard the frequency of the carrier supply oscillator is shifted by 1,000 cycles and its output is interrupted at a 20-cycle rate, and applied to the input of the transmitting amplifier. At the receiving end this appears at the input of the signal receiving circuit as a 1,000-cycle current interrupted at a 20-cycle rate. It is then demodulated in a copper-oxide rectifier and the resulting 20-cycle current is amplified and applied to a second rectifier the output of which is connected to a d-c relay. Operation of this relay causes 20-cycle current from a local source to be sent toward the switchboard. Freedom from false operation on speech is obtained by tuning the receiving circuit to the rate of interruption and by providing a slow operating relay such that the signal must persist for several tenths of a second before 20-cycle current is passed to the switchboard.

The power supply circuit, which is shown in figure 5, operates from an a-c source of 100 to 130 volts, 50 to 60 cycles and requires about 50 watts. It supplies a-c for the filament supply, 160 volts d-c for plate supply and 24 volts d-c for relay operation. Provision is also made for direct operation from 24-volt and 130-volt central office batteries.

The equipment for a terminal is mounted on two panels. One of these, shown in figure 6, is $15^{1}/_{4}$ inches by 19 inches in size and contains all the apparatus except the line filter circuit. The second panel, shown in figure 7, is $3^{1}/_{2}$ inches by 19 inches and contains the line filters, and other equipment which is required for balancing purposes.

Repeater

A schematic of the repeater is shown in figure 8. The amplifiers are the same in design as the transmitting

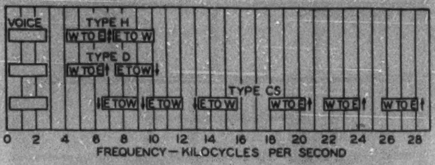

Figure 3. Frequency allocation

amplifier at the terminals. Two sets of line filters are required, and are identical with those used at the terminals. The a-c power supply circuit is substantially the same as the one used at the terminals except that the 24-volt supply for relay operation is omitted. The power required for operation is about 35 watts.

The complete repeater consists of three panels—the repeater panel, which is $10^1/_2$ inches by 19 inches, and two line filter and balancing panels. The repeater panel is shown in figure 9.

Transmission Performance

Line Considerations

In outlining the transmission performance of the system, it is necessary to consider the characteristics of the lines as well as those of the equipment. At carrier frequencies the line losses and the variations in these losses with weather are considerably greater than at voice frequencies. This is illustrated in figure 10 which shows the attenuation-frequency characteristics for four commonly employed gauges of open-wire line under dry and wet weather conditions, respectively, at a temperature of 68 degrees Fahrenheit. These curves are for 12-inch spaced copper wires equipped with the type of insulators ordinarily employed for toll circuits. Changes in temperature also affect the attenuation, the loss increasing as the temperature rises. The variations due to changes in temperature are, however, smaller than those due to changes from dry to wet weather and are more likely to occur gradually. It is apparent that the variations in attenuation with weather increase with the length of the system, and that more frequent adjustments will be required on the longer systems to maintain the over-all circuit net loss within given limits.

The carrier frequency attenuation of cable circuits is much greater per unit length than that for open-wire lines. Hence, the losses introduced by comparatively short sections of cable, either at entrances to offices or at intermediate points are of considerable importance. In addition to the attenuation of the cable itself, there are large reflection losses at the junction of the cable and the open wire, due to the difference between the characteristic impedance of the open wire and that of the non-loaded cable. The carrier terminal and repeater have been designed to have the same impedance as the average of the open-wire line facilities, so the same considerations apply

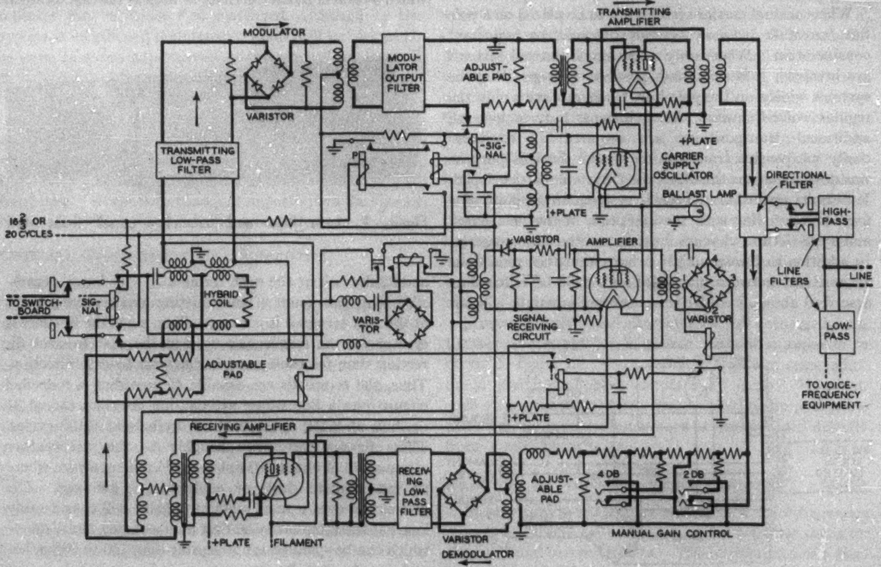
MODULATOR
VARISTOR
MODULATOR OUTPUT FILTER
ADJUSTABLE PAD
TRANSMITTING AMPLIFIER
+PLATE
SIGNAL
P
CARRIER SUPPLY OSCILLATOR
BALLAST LAMP
DIRECTIONAL FILTER
HIGH-PASS
TRANSMITTING LOW-PASS FILTER
16⅔ OR 20 CYCLES
+PLATE
TO SWITCHBOARD
SIGNAL
HYBRID COIL
VARISTOR
VARISTOR
AMPLIFIER
1
2
3
4
VARISTOR
LINE FILTERS
LINE
LOW-PASS
SIGNAL RECEIVING CIRCUIT
ADJUSTABLE PAD
+PLATE
TO VOICE-FREQUENCY EQUIPMENT
RECEIVING AMPLIFIER
RECEIVING LOW-PASS FILTER
ADJUSTABLE PAD
4 DB
2 DB
MANUAL GAIN CONTROL
+PLATE
FILAMENT
VARISTOR
DEMODULATOR

Figure 4. Schematic of type-H terminal

at the junction of the cable and the carrier equipment. As an example of the magnitude of these effects, the insertion loss at 8,000 cycles of two miles of non-loaded 16-gauge cable is approximately 6 decibels, or about the same as that of 60 miles of 104-mil open wire.

By applying carrier loading that has been developed for this purpose, the attenuation loss of such a cable can be reduced to about one decibel at 8,000 cycles. In addition, the reflections at the terminals of the cable will be reduced to satisfactory low values by virtue of the impedance matching properties of the loading. This method of treatment has the important advantage that it improves the transmission characteristics in both the voice and carrier range.

In cases where substantial transmission margins exist, it is sometimes practicable to use impedance matching transformer networks at the cable terminals, as a substitute for carrier loading, with economies that are proportional to the length of the cables. At the present stage of development, this so-called transformer treatment is much less satisfactory in the voice-frequency range than in the carrier-frequency range. Certain inherent limitations in simple transformer treatment result from the fact that the ratio of the (non-loaded) cable impedance to the open-wire impedance varies widely over the frequency band to be transmitted, and the transformer impedance ratio that is optimum at carrier frequencies is distinctly disadvantageous in the voice frequency range. The choice of optimum transformer ratio for the complete transmission band may thus involve different compromises for different sets of conditions and service requirements.

Where several carrier systems are to be placed on a pole line, crosstalk between systems becomes an important consideration. Where only a few single-channel systems are involved, it is sometimes possible, by separating the systems widely on the pole line, to operate with only the regular voice-frequency transpositions, but, in general, additional transpositions are required. A comparatively inexpensive transposition system for this purpose was designed at the time the type-*D* system was developed. It permits operation of type-*H* systems on all pairs of a four crossarm line with the exception of the pole pairs, and type-*C* three-channel systems on the top crossarm. In addition to transposing it is important that reflections at junctions between open-wire and cable be reduced as described above, in order that near-end crosstalk will not through reflection appear as crosstalk at the distant terminal of the system.

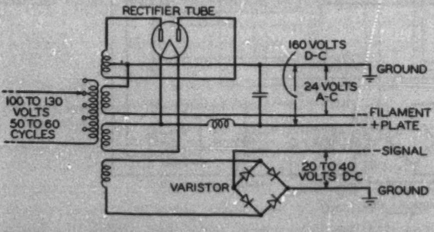

Figure 5. Schematic of a-c power supply for terminal

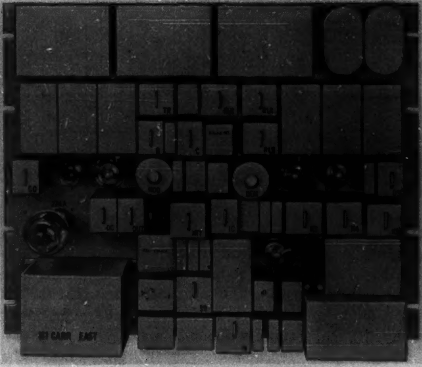

Figure 6. Terminal panel—front view

RANGE OF OPERATION

The terminals and repeaters have sufficient load carrying capacity so that they may be operated at an output level 16 decibels above that at the transmitting toll switchboard. About 19 decibels transmitting gain and 14 decibels receiving gain are available at each terminal, of which a total of 20 decibels may be used at the east terminal and 22 decibels at the west terminal. The lower permissible loop gain (sum of transmitting and receiving gain) at the east terminal is not controlling, since the line loss is greater for the frequencies used in the east to west direction than for those used in the west to east direction. Thus, the terminals are capable of providing a 9-decibel circuit over a line whose attenuation does not exceed 31 decibels at 8,150 cycles and 29 decibels at 6,150 cycles. These figures correspond roughly to the wet-weather attenuation of about 280 miles of 104-mil open wire where no intermediate cable or equipment is involved. The presence of even a small amount of cable will considerably increase the attenuation so that in most cases, the distance which can be spanned is not greater than 150 to 200 miles, and may be even less.

Figure 7. Line filter and balancing panel—front view

Where greater distances are to be covered, an inter-

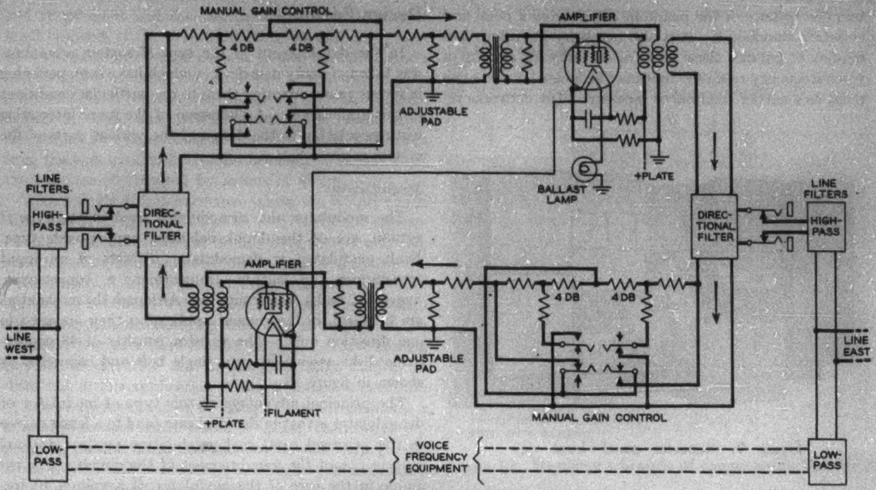

Figure 8. Schematic of repeater

mediate repeater may be added. The repeater has a useful gain of about 23 decibels in each direction, with some flexibility as to allocation of gains between the two directions of transmission. More than one intermediate repeater can, of course, be employed, although as the system is lengthened maintenance effort will be increased, as more frequent adjustments of the over-all net loss will be required to compensate for the variations in line attenuation with weather. No provision is made for a pilot channel such as is generally provided on the long multichannel systems, and adjustments of over-all net loss must be made manually. Also, no provision has been made for equalizing the variation in line attenuation with frequency. These factors are not important on the shorter circuits for which the system has primarily been designed.

Over-All Transmission Characteristics

The circuit provided by a type-*H* system without a repeater has a band width of about 2,750 cycles, extending from about 250 to 3,000 cycles. This is somewhat wider than that for the type-*D* system. The introduction of repeaters will tend to narrow the band somewhat. Representative frequency characteristics are shown in figure 11. One set of characteristics is for a typical circuit without a repeater, and the other for a circuit including two repeaters. It is assumed that the line conditions are such that no large reflection effects are present.

The band width is limited principally by the characteristics of the band filters. The small differences in the characteristics for the two directions of transmission are due partly to differences in the filters and partly to the fact that the attenuation of the line increases with frequency and is greater at the 3,000-cycle point in the east-to-west channel than for the 3,000-cycle point of the west-to-east channel. As the circuit is increased in length this difference tends to increase.

Variations in over-all circuit net loss are due largely to variations in the loss of the high frequency line. For a circuit 200 miles long these may amount to ± 3 decibels. The key-controlled pads which are included at each terminal and repeater are provided for making adjustments to compensate for these variations. Variations due to the equipment are small in comparison with the line variations. The transmitting gain at a terminal may vary ± 0.5 decibel and the receiving gain ± 0.3 decibel for variations of ± 10 volts in the a-c supply. With a more stable a-c supply or when operated from regulated plate and filament batteries such as are employed in the larger telephone offices, these variations will be less than half the figures given above. With suitable maintenance it should be possible to maintain the over-all circuit net loss within ± 2 decibels of its normal value.

A representative load characteristic, as measured with 1,000-cycle current for a system without a repeater is shown in figure 12. On a repeatered system some additional limiting of high inputs may occur. However, even on repeatered systems, there should be no noticeable distortion for input volumes such as are obtained directly from a switchboard.

Reactions on Voice-Frequency Circuit

In superimposing a carrier system on a voice-frequency circuit, line filters are added to provide separate paths for the voice and carrier circuits. The introduction of a filter in one side of a phantom group requires the addition of a network in the other side of the phantom group to maintain the balance of the phantom circuit from a noise and crosstalk standpoint. It is also necessary for return loss reasons to balance these units in the network circuits of voice-frequency repeaters that may be located at the same point as a carrier terminal or repeater. The networks required to take care of these conditions are included with the carrier system.

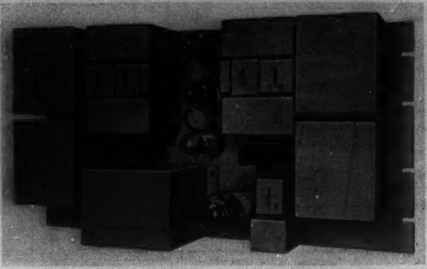

Figure 9. Repeater panel—front view

In some cases it is desired to apply the type-*H* system to circuits equipped with bridged telephone stations at intermediate points. Such arrangements are common on railroad communication systems, and occur to a small extent in the Bell System. In such cases, excessive interference into the carrier system due to talking at the way station, and into the way stations due to talking on the carrier system, is likely to occur unless suitable filters are provided at each way station. A simple filter for this purpose has been developed for use with the 501-type subscribers set, and work is proceeding on a similar filter for use with other types of subscribers sets.

The line filters and the filters for use at way telephone stations each introduce a loss of about 0.15 decibel to the through voice-frequency transmission. Where the voice-frequency circuit is equipped with repeaters and return loss conditions permit, these additional losses may be taken care of by readjusting the voice-frequency repeater gains. In extreme cases, particularly where a considerable number of filters are to be added, it may be necessary to resort to other means of improving the transmission on the voice-frequency circuit, such as loading of incidental cables or the addition of a voice-frequency repeater.

On circuits equipped with way stations, selective signaling by means of selectors is sometimes used. Such signaling systems are generally arranged to apply an "answer back" tone to the line when a station has been called to indicate to the calling party that the selector has operated. This tone contains a considerable amount of high-frequency currents so that it is necessary to modify the selector circuit to filter out the high frequencies. The modification is a simple one and makes the answer back tone inaudible on the carrier circuit.

Design Features

In the development of the type-*H* system advantage was taken of many new devices which have been perfected in recent years, adapting them to the particular conditions of this application. A discussion of the more interesting features relating to the design of the various parts of the system is given below.

MODULATORS

The modulator and demodulator used in the type-*H* system are of the double-balanced copper-oxide type. Each modulator or demodulator consists of an input transformer, an output transformer, a copper-oxide "varistor" and a carrier supply. Although the modulators are bilateral, in the present application they are used in one direction only. The varistor consists of 48 copper-oxide disks assembled on a single bolt and connected as shown in figure 13.

The principal advantage of this type of modulator or demodulator is that in the ideal case (and to a lesser degree in the practical case) each modulation product appears only in one of the four branches of the circuit. For example in the case of the modulator, if a voltage of frequency V is applied to the input and a voltage of frequency C is applied by the carrier supply circuit, resulting products of modulation will appear in the ideal case as shown in figure 13. It is obvious that the only unwanted products in the output which cannot be suppressed by filters or balance are those which are of the frequency $(C \pm AV)$ which for some values of A and V fall in the frequency range of the desired sideband $(C + V)$ or $(C - V)$.

These components, however, are normally more than 50 decibels weaker than the sideband and are not noticeable. Of course, the term AV represents not only odd harmonics of V but odd order intermodulation products such as $(2V_1 \pm V_2)$. The relative amplitudes of the $(C \pm AV)$ terms increase with load in a similar manner to that for the distortion products of an amplifier as the overload point is approached, and the effect on articulation is the same.

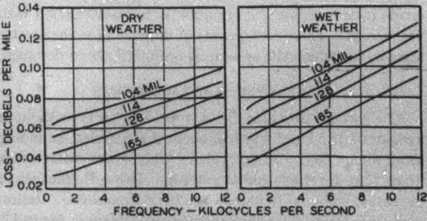

Figure 10. Attenuation of open-wire side circuits; 12-inch spaced copper wire with double-petticoat glass insulators

For the actual case the modulator balance is not perfect and all products and the original frequencies do appear in all branches of the circuit including the output. However, the balance in most cases is greater than 30 decibels and the filter requirements are helped to that extent over some portions of the frequency range. This is particularly helpful in connection with suppressing the carrier from the output and input since it lies only about 200 cycles from the pass band and it would be costly to obtain all of the suppression required by means of filters.

With a single disk in each arm, taking the factory run of disks and making no attempt to select units the balance for many assemblies would be less than 15 decibels. By selecting units, this balance could be improved to any desired amount. In the present design, however, to save the cost of selection 12 disks were used per arm to obtain the better balance resulting from the averaging of the characteristics of a large number of disks. There is some sacrifice in efficiency due to using the large number of disks but in this application it was of minor importance.

The averaging obtained by using 12 disks in each arm is helpful in several other respects. First, the normal impedance, transmission, and balance do not vary greatly from unit to unit. Secondly, although each disk has a negative coefficient of resistance vs. temperature and there is a variation in the coefficient among disks, the average coefficient of 12 disks chosen at random will be very nearly equal to the average of any other 12 disks chosen at ramdom and the balance between arms will, therefore, remain practically constant with temperature, even though the impedance and efficiency vary. A similar advantage is obtained in the case of aging and a good balance is obtained throughout the life of the equipment.

OSCILLATOR

As mentioned previously the same carrier (suppressed) frequency is used for transmission in both directions, thus requiring a single oscillator instead of two as in the type-*D* system. The principal requirement for an oscillator for this use is that its frequency remain stable under the variations in power supply voltage and temperature which will occur. The new design which is shown schematically in figure 14, provides a degree of stability such that no operating adjustments will be required due to these factors. Relatively high stability with changes in temperature is obtained by balancing the positive capacitance-temperature coefficient of the copper-oxide load and the mica tuning condenser against the negative coefficient of the paper tuning condenser.

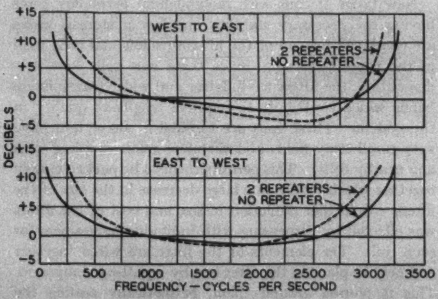

Figure 11. Representative over-all circuit net loss characteristic of system

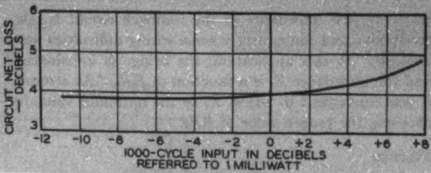

Figure 12. Representative system load characteristic

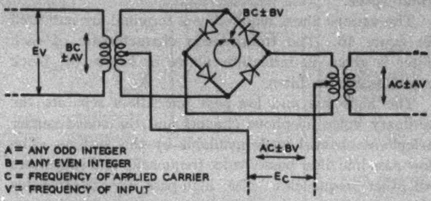

Figure 13. Simplified schematic of copper-oxide modulator

The stability of frequency with plate voltage variations is about $5/10^6$ parts per volt. This is adequate and was obtained without the use of an expensive tuning inductance. The coil used, which also serves as output and feedback transformer has a ratio of reactance to resistance of about 20 and is an air-core solenoid potted in a copper can.

AMPLIFIERS

Both the receiving and transmitting amplifiers employ a single pentode with about 9 decibels feedback. For this amount of feedback, the variations of gain and impedance due to power supply variations are reduced to at least one-third of the amount of the variation obtained without feedback, and the load carrying capacity is increased about one decibel.

The two amplifiers differ in that the frequency range transmitted is different and in that the output transformer of the receiving amplifier also acts as an inequality ratio hybrid coil to separate the receiving signaling circuit from the two-wire voice circuit. The two circuits are shown in figure 4. In each case, the feedback is accomplished by means of a bridge circuit in the output and a series connection in the input. This can be more readily seen from figure 15, which is a simplified circuit representing both amplifiers. There is a considerable saving in circuit elements as compared to the familiar resistance bridge feedback connection. The output power loss due to shunt arms of the resistance bridge is eliminated. Furthermore, the impedance of the feedback circuit is relatively low, and consequently some wiring difficulties were avoided. In this application, the bridge is unbalanced, and the impedance Z_0 is a function of KR_0. As a result, it was convenient to adjust Z_0 to the optimum value by choosing the proper value of KR_0.

FILTERS

Filters constitute an important part of the type-*H* carrier system. They represent about 30 per cent of the cost of the terminal and occupy about 25 per cent of the total space.

The various filters required at a terminal are indicated in figure 16. The transmission characteristic of each filter is given in miniature above or below the block representing the filter.

The high-pass and low-pass line filters separate the ordinary voice telephone channel and the added carrier telephone channel made available by this system. The low-pass line filter passes voice frequencies and suppresses all other frequencies. The high-pass line filter passes the carrier frequencies and suppresses the voice frequencies. Each filter offers a high impedance to the frequencies passed by the other, and bridges off only a very small part of the energy of these frequencies.

The remaining filters are associated with the carrier terminal proper, where they serve to separate the transmitting and receiving paths and suppress unwanted frequencies. The voice frequencies pass through the hybrid coil and the transmitting low pass filter to the modulator. This filter limits the path between the hybrid coil and the modulator to voice frequencies only. Modulation of the voice with the carrier frequency of 7.15 kilocycles produces two sidebands extending from 4.15 to 6.90 kilocycles and from 7.40 to 10.15 kilocycles. At an east terminal, the upper sideband is transmitted, and the modulator output filter passes this sideband and suppresses the lower sideband, together with other unwanted modulation products. In this manner it limits the load on the amplifier to the desired sideband. The transmitting directional filter offers further suppression to frequencies lying outside this band. The receiving directional filter will not pass this band but has a high impedance to these frequencies. The high-pass line filter passes all frequencies above roughly 3.5 kilocycles and, therefore, this band passes through it readily and out onto the line for transmission to the distant terminal. Transmission from a west terminal is identical in principle but here the lower sideband is passed by the modulator output filter and transmitting directional filter while the upper sideband is suppressed.

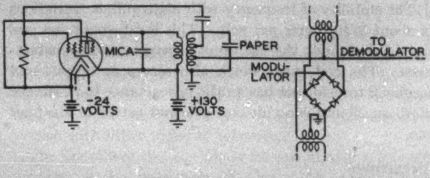

Figure 14. Schematic diagram of oscillator circuit

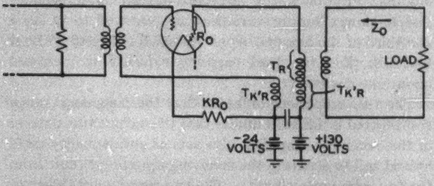

Figure 15. Simplified schematic of amplifier

It is apparent from figure 16 that the received sideband coming in on the line from the distant repeater or terminal is operated upon by the filters in a reverse manner from that described above for the transmitted sideband. The incoming frequencies are directed through the receiving directional filter to the demodulator, where modulation with the original carrier reproduces the voice frequencies together with other modulation products. The desired voice-frequency band is then separated from these products by the receiving low-pass filter.

In addition to performing the function of selecting desired and rejecting undesired currents, a filter, if operating in parallel with another as in the case of the directional filters or the line filters, should offer a high impedance to the transmitted currents of the other and thus prevent an excessive drain of these currents. Since these filters are designed for operation in parallel, either filter operated without the other would have somewhat different electrical characteristics.

The economies and reduction in size of these filters as compared to those of the type-*D* system are due to several factors. Improved paper condensers having the required stability are used. Compared to mica condensers, these condensers are less costly and smaller for a given capacity. Moreover, at a very slight increase in cost paper condensers are used which will withstand 1,000-volt line surges. Most of the condensers of the type-*D* system were designed for only 500-volt protection.

New types of coils with molybdenum permalloy cores having the necessary stability and low hysteresis losses are employed instead of the air-core solenoidal coils used in the type-*D* system. Since these new coils have less dissipation, the filters have flatter and wider transmission bands which contribute to improve telephone quality in the system. These coils are toroidal in shape and have very small stray fields and therefore small coupling with any nearby coils. This permits them to be packed closely together which results in a large decrease in the size of the filters. A further reduction in size and cost of the filters was effected by dispensing with individual containers for each coil. The elements of the filter are wired together and held in place in the filter can by a potting compound. This is poured around them, hermetically sealing the whole assembly.

The small size of the elements and the very low coupling

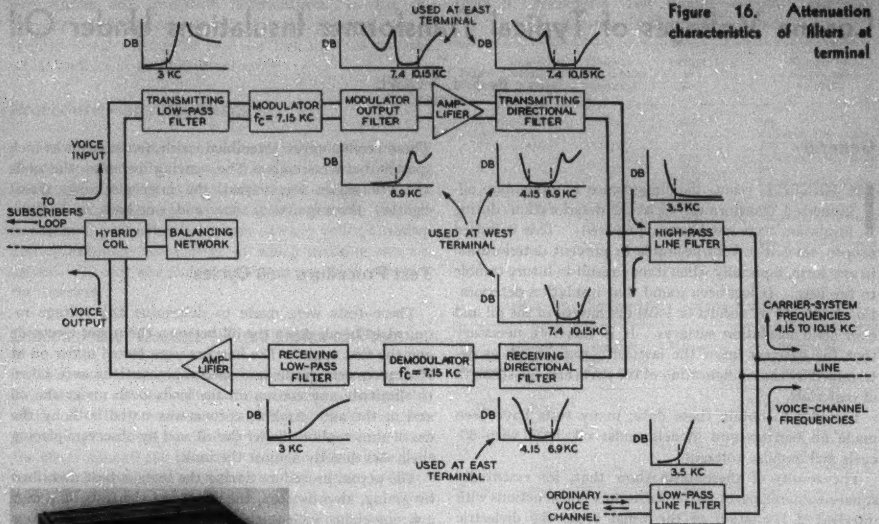
USED AT EAST TERMINAL
DB
3 KC
DB
7.4 10.15 KC
DB
7.4 10.15 KC
TRANSMITTING LOW-PASS FILTER
MODULATOR f_C = 7.15 KC
MODULATOR OUTPUT FILTER
AMP-LIFIER
TRANSMITTING DIRECTIONAL FILTER
VOICE INPUT
DB
6.9 KC
DB
4.15 6.9 KC
DB
3.5 KC
TO SUBSCRIBERS LOOP
HYBRID COIL
BALANCING NETWORK
USED AT WEST TERMINAL
HIGH-PASS LINE FILTER
DB
7.4 10.15 KC
VOICE OUTPUT
CARRIER-SYSTEM FREQUENCIES 4.15 TO 10.15 KC
LINE
AMP-LIFIER
RECEIVING LOW-PASS FILTER
DEMODULATOR f_C = 7.15 KC
RECEIVING DIRECTIONAL FILTER
VOICE-CHANNEL FREQUENCIES
DB
3 KC
DB
4.15 6.9 KC
USED AT EAST TERMINAL
DB
3.5 KC
ORDINARY VOICE CHANNEL
LOW-PASS LINE FILTER

Figure 16. Attenuation characteristics of filters at terminal

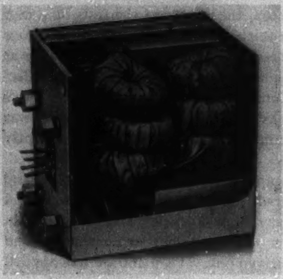

Figure 17. Filter (with cover cut away) showing general method of assembly

between them permit the assembly of more than one filter in the same can. For example, by a careful placing of the elements it was possible to place the transmitting and receiving low-pass filters and the modulator output filter in one can approximately three and one-fourth inches by four and one-fourth inches by four and one-fourth inches in size. A photograph of a high- and low-pass line filter with the can cut away is shown in figure 17. The filters for a terminal of this system require 70 square inches of mounting space or about one-fifth of that required for those of a type-*D* system.

Conclusions

The development of the type-*H* system is another step in extending the use of carrier systems. Improvements in performance and simplifications which are effective in reducing its cost as compared with the type-*D* system which it supersedes have been obtained. Reduction in size and provision for operation on a-c supply simplify its installation, particularly in outlying offices where suitable d-c power supply is not ordinarily available. Its portability makes it well suited to providing additional circuits required in cases of emergency. The type-*H* system is expected to have a large application in the Bell System telephone plant, and in addition to provide carrier circuits for the communication systems of other companies.

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Corona Voltages of Typical Transformer Insulations Under Oil

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ASSOCIATE AIEE

General

IN RECENT years, the importance of designing oil-immersed transformers to avoid deterioration during insulation tests has been emphasized. This is indeed proper, since it is very desirable to prevent deterioration in any form, especially when it may result in future trouble to the user. It has been found that insulation deterioration is caused by corona or local discharges in the oil and over solid insulation surfaces. It is therefore necessary that the designer know the laws of corona in oil, as determined by the configuration of the parts and the strength of materials.

In order to obtain these data, many tests have been made on barriers and models under oil, both with 60-cycle and impulse voltages.

The results of these tests show that, for essentially square-cornered parts, as in transformer constructions with interleaved insulation of the usual type, the dielectric strength varies approximately as the barrier thickness to the two-thirds power. These tests show that the ratio of impulse-voltage corona to 60-cycle corona is in the order of 2.2.[1,2] They also show that entrapped air within the insulation structure may greatly reduce the dielectric strength, and that the use of vacuum is very helpful in avoiding this difficulty.

In obtaining the data, two series of tests were made and a description of the test specimens, procedure and data follow.

Description of Barriers—First Series of Tests

The barriers tested were of three thicknesses, two and one-eighth inch, four and one-eighth inch, and seven inches. In order to simulate conditions in a transformer, an additional one-quarter-inch duct was placed between the barriers and the upper electrodes, making total separations of two and three-eighths, four and three-eighths, and seven and one-quarter inches. The barriers were all constructed with fullerboard sheets and ducts, similar to figure 1. Angles were interleaved as shown. The bottom electrode was a large plate. The top electrode consisted of five dummy coils of the same dimensions. The five coils were used in order to more closely simulate actual conditions in a transformer.

Each dummy coil was made of a three-eighths-inch-thick fullerboard sheet covered with lead foil, taped, and varnished, to a total thickness of about one-half inch. These coils were assembled with five-sixteenths-inch spacers between coils. The spacing between the coils and the angles was varied, the intervals being three-eighths, three-quarters, one and one-half, and three inches.

Test Procedure—60 Cycles

These tests were made to determine the voltage required to break down the oil between the upper electrode and the first angle. The barriers were tested under oil at 70 degrees centigrade, and special precautions were taken to eliminate any corona on the leads both under the oil and in the air. Audible corona was noted both by the use of a microphone under the oil, and by observers placing their ears directly against the tank.

The actual procedure during the tests is best described by giving, step by step, the results of the tests on a four and one-eighth-inch-thick barrier, with the dummy coils three inches from the first angle. After the barrier was immersed in the oil, it was shaken and the upper electrodes pounded. Considerable air was removed by this process. Then 125 kv was applied. Corona, which was present as shown by intermittent clicks, stopped abruptly after three minutes. When the voltage was removed, some air and gas escaped. Next 150 kv was applied and corona again appeared, gradually becoming weaker and at longer intervals, say 30 or 40 cycles, and disappearing. The voltage was removed, and the barrier shaken, and more bubbles were driven out. Then 180 kv was applied for five minutes, with no corona appearing during the last four minutes. At 200 kv, deterioration set in as indicated by a weak steady buzz. The voltage was immediately removed and then 150 kv, 180 kv, and 195 kv were applied without a sign of corona. Above 200 kv, corona appeared regularly, and thus 200 kv was determined as the corona voltage for this arrangement.

In every case, when bubbles were present, it was found that either the air had not all been removed, or that there

Table I. Test Results—60 Cycles—Oil Temperature Approximately 70 Degrees Centigrade

Barrier Number	Separation Dummy Coils to Plate	Distance Dummy Coils to Angle	Corona Point-Kv RMS	Maximum One-Minute Hold
1	$2^{3}/_{8}$	$^{3}/_{8}$	180	225
2	$2^{3}/_{8}$	$^{3}/_{4}$	170	240
3	$2^{3}/_{8}$	$1^{1}/_{2}$	150	250
4	$2^{3}/_{8}$	3	130	
8	$4^{3}/_{8}$	$^{3}/_{8}$	290	360
9	$4^{3}/_{8}$	$^{3}/_{4}$	280	
10	$4^{3}/_{8}$	$1^{1}/_{2}$	260	350
16	$4^{3}/_{8}$	3	200	
18	$7^{1}/_{4}$	$^{3}/_{4}$	360	466

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[1] For all numbered references, see list at end of paper.

was corona and insulation deterioration. In the determination of the corona point, it was required that a slightly lower voltage be held for five minutes without either bubbles or audible disturbance.

Procedure—Impulse Tests

Two series of tests were made, all using approximately $1^1/_2$x40 positive full waves. One series was made to failure, by applying at least 100 surges at each voltage setting. Another series was made by removing the barrier from the oil after 100 surges at each voltage setting, and determining the voltage at which marking was observed. The air was removed as far as possible by shaking the barriers.

Discussion of Results—First Series of Tests

These results are of importance because they show how entrapped air affects the insulation strength of an assembly under oil. They are also of importance in showing how the strength of oil is affected by the distance between the electrode and the barriers. Some of the values given by table I are shown on figure 2. This figure indicates that for a given arrangement the breakdown or corona voltage varies as the two-thirds power of the separation between the dummy coils and plate. The values also indicate that neither the 60-cycle nor impulse breakdown are much affected by the distance *B* between the coils and the first barrier for these particular tests, but this is not necessarily true in general, as shown by later tests. Table II also shows that insulation may stand hundreds of surges above the corona voltage as established by insulation marking. They also show that the impulse ratio for corona voltages is in the order of 2.1, but later tests on samples more closely similar to actual transformer construction, show a ratio of 2.2 or over.

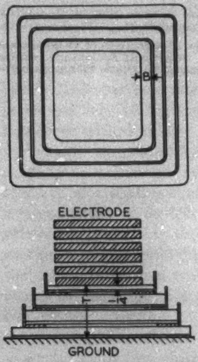

Figure 1. Sketch showing typical arrangement of barriers in the tests. B was three-eighths, three-quarters, one and one-half, and three inches. T was two and three-eighths, four and three-eighths, and seven and one-quarter inches

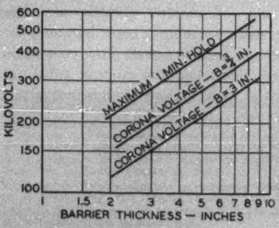

Figure 2. Relations between maximum one-minute hold or corona voltage and barrier thickness. Data taken under oil at approximately 70 degrees centigrade and with 60-cycle voltages

Table II. Breakdown Test Results—Impulse Tests—Oil Temperature 76 Degrees Centigrade

Barrier Number	Separation Dummy Coils to Plate	Distance Electrode to First Angle	Voltage in Kv	Number of Surges and Remarks
1	$4^3/_8$	$^3/_8$	870	100
			920	Failed on 59th Surge
2	$4^3/_8$	$^3/_8$	870	260—Slightly marked
			920	Failed on 187th Surge
3	$4^3/_8$	$^3/_8$	870	230
			920	Failed on 80th Surge
4	$4^3/_8$	$^3/_8$	870	Failed on 240th Surge
5	$4^3/_8$	$^3/_4$	820	446
			870	Failed on 161st Surge
6	$4^3/_8$	$^3/_4$	870	293
			920	Failed on 136th Surge

Table III. Endurance—Corona-Voltage Tests

Barrier Number	Separation Dummy Coils to Plate	Distance Electrode to First Angle	Voltage in Kv	Number of Surges and Remarks
12	$4^3/_8$	$^3/_8$	790	100—No marks
			830	100—Marked
7	$4^3/_8$	$^3/_4$	720	100—No marks
			790	110—No marks
			850	110—Marked
8	$4^3/_8$	$^3/_4$	790	100—No marks
			820	100—Marked
10	$4^3/_8$	3	590	100—No marks
			620	100—Marked
11	$4^3/_8$	3	590	100—No marks
			620	100—Marked

Table IV. Comparison of Corona Values—60 Cycle and Impulse

Separation Dummy Coils to Plate	Distance Electrode to First Angle (Inches)	Corona Voltage		Ratio Impulse to 60 Cycles
		60 Cycle Crest-Kv	Impulse Kv $1^1/_2$ x 40 Pos. Wave	
$4^3/_8$	$^3/_8$	410	830	2.0
$4^3/_8$	$^3/_4$	396	835	2.1
$4^3/_8$	3	282	620	2.14

Description of Barriers—Second Series of Tests

Since the preceding tests confirmed the necessity of removing all air, it was decided to make another series of tests, impregnating and filling the models with oil under vacuum. The insulation around the coil conductor has considerable bearing upon the dielectric strength, and so the new models were made with dummy coils of copper ribbon taped to 0.120 inch thickness with paper. The distance from the insulated coil to the first angle was from three-eighths to three-quarters inch. The barrier thickness was two and five-eighths inches.

Test Procedure—60 Cycles and Impulse

In these tests, the voltage was raised in steps, holding the voltage for one minute at each step, until corona appeared as indicated by the formation of fine bubbles. A similar procedure was followed in obtaining impulse data, except that 100 positive wave impulses were applied at each voltage step. The impulse voltages were chopped by a gap at approximately three microseconds.

Table V. Results of Second Series of Tests

60-Cycle Corona	Impulse Corona
285	917
281	920
Average—283	918

The impulse ratio—impulse voltage at approximately 3 microseconds to 60 cycle crest voltage—is approximately 2.29. Corona at 60 cycles was not indicated until the breakdown voltage was reached.

Discussion of Results—Second Series of Tests

The results of these tests show how effectual the application of vacuum is in obtaining the full insulation strength of a transformer without the necessity or risk of carefully building the voltage up, and making so-called "bubble runs." They show clearly that large high-voltage transformers will have their maximum dielectric strength immediately when put in service, if filled with oil under vacuum. It also shows that the ratio of impulse to 60-cycle corona is of the same order as the impulse ratio of breakdown previously given, namely, 2.2 or more.

Conclusions

1. These tests, and previous experiences with models of transformers, show that corona can be avoided in transformers, both for 60 cycles and for impulses. It also shows that an average ratio between the impulse and 60-cycle crest voltage corona point of insulation similar to actual transformer insulation is 2.2.

2. These tests show the importance, both from a 60-cycle and impulse viewpoint, of the removal of all entrapped air. It is shown to be desirable to fill transformers with oil under vacuum, both for factory tests and at the time of installation. Tanks for many transformers of the larger sizes and higher voltages are made suitable for the application of vacuum.

3. These tests show that in some cases insulation will stand hundreds of impulses slightly above the corona point. This situation is similar to that found in 60-cycle tests, in which insulation withstands a higher value for one minute than the "marking" value.

Acknowledgment

The data covered in this paper were obtained over a period of years, and to those who helped in obtaining it, especially Mr. Carl Alsing and Mr. W. L. Teague, acknowledgment is due.

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Recent Advances in Resistance Welding

DURING the past few years fundamental concepts of resistance welding have not been greatly changed, progress being limited mostly to refinements in the art of welding procedure, design of machines and auxiliary equipment, and broader applications to industry in general. Advances may be classified under the headings of time, pressure, and current control which are fundamental to all resistance welders, materials to be welded including their preparation for welding, and means for placing the parts in the welding machine.

Perhaps the most noteworthy improvements have to do with the accuracy of the time period of current application. Controls whereby the primary current of the welding transformer is fed directly through either thyratron or ignitron tubes are coming into general use since they make practicable perfect timing of current application in exact periods of one-half cycle or larger multiples thereof. The maximum root mean square current capacity of these tubes has also been steadily increased up to 10,000 amperes at present for continuous welding operations. Not only have these exact tube controls been developed for use with much higher currents than were formerly used, but at the same time they have also made possible satisfactory commercial welding of materials which can only be welded in these short time intervals at high current densities. An auxiliary development to purely electronic timing is heat control which is accomplished by suitably delaying the ignition point of each tube for each half cycle of power current conducted and consequently cutting down the weld power for each half cycle. This has been used as a substitute, but usually as an auxiliary to the tap switch on the welding transformer for critical welding heat adjustments.

Probably stimulated by the performance of electronic control, very marked improvements in the operating speed of spot welding contactors has taken place. Also a large number of devices have been developed for more accurately controlling these contactors. Sending a current pulse of very brief duration or a series of impulses to the contactor magnet coil as determined by the rate of charge or discharge of a condenser, the inductance of a coil, or the operation of sensitive relays controlled by clock motors has allowed more satisfactory operation of magnetic contactors while refinements in the operation of mechanical contactors as synchronously co-ordinated with the welding circuits has also been considerable. Air-operated weld timers are also proving quite satisfactory on some classes of work and multiple control units are being used on automatic spot welders for controlling contactors and the motion of air operated heads and their dwell.

Welding machines have undergone marked improvement in the last few years, and this improvement is undoubtedly partly due to the more precise control now available. Pressure means on press welders such as air-cushion chambers in combination with cam action and the elimination of pounding are decided steps forward. The use of initial and final pressures on projection welding is also quite common. Better synchronization of machine operation and control makes possible fully automatic spot and projection welders with semiautomatic feed and discharge of parts to and from the welding position. Flash welders afford much better flash protection and the jaw movements whether co-ordinated by cam, spring, air, or hydraulic action or combination thereof have been much improved. Much higher capacities of transformer are being used and kilovolt-ampere capacities considerably over 1,000 are not at all unusual. Means for adjusting the welding current in smaller steps are more often provided especially for nonferrous materials. Some machines are also equipped with milling cutters which machine the work piece at their contact points prior to welding which reduces the possibility of variable power input into the weld area in successive welding operations. The use of automatically operated resistance welders for accurate annealing temperatures as controlled by photoelectric cells focused on individual parts has reached a high degree of development.

Multitransformer and multielectrode machines have come into extensive use during the last three years. Multielectrode machines such as the hydromatic spot welder using either one or more transformers has made possible very rapid spot welding on large assemblies and in addition has made possible individual quality control for each weld. Multitransformer machines are used to insure quality in individual weld spots and also to insure the proper current distribution on projection welding platens or large flash welding machines.

In order to speed up spot welding operations on strip materials wheel type electrodes are used under conditions similar to seam welding, but with longer periods of current interruption and consequent speeds as high as 120 feet per minute.

Special machines such as portable spot welders used for large assemblies of sheet iron parts and light frameworks are more popular than ever and are finding many more applications outside of the automobile industry where they were first put to use. In order to reduce the weight of the gun and lessen operator fatigue on large portable welders these are being built of aluminum or magnesium alloys in the hydromatic type so that the pressure cylinder is only about one inch in diameter instead of four or five. Plier type welders which are really miniature portable spot welders are also used for making electrical connections on meters, radios, or similar apparatus. Even percussion welders of which there are relatively few are increasing in number and the discharge of a bank of con-

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Personnel of the subcommittee on resistance welding of the AIEE committee on electric welding: C. L. Pfeiffer, *chairman;* Malcolm Thomson, Carroll Stansbury, C. E. Heitman, J. W. Dawson, F. E. Taylor, H. A. Woofter, R. L. Browne, H. E. Stoddard, and Herman Lemp.

densers or its equivalent into the primary of a resistance welding transformer finds applications on very critical welding setups.

Interrupted spot welding sometimes known as "persistence" welding is a relatively new method of welding very heavy sheets up to one inch in thickness. A series of welding current impulses is sent through the electrodes of a conventional spot welding setup giving the weld area time to heat up at the point of fusion but allowing the surface in contact with the electrodes to remain relatively cool. Such a set-up operates most consistently and with the best results by using electronically and automatically controlled timing periods, water-cooled electrodes and materials which spot weld readily such as low-carbon steel.

Practically all users of resistance welding machines now use either die-type electrodes or electrode tips made of special electrode materials. The use of these materials in die-type electrodes in combination with satisfactory cooling media makes possible the extension of resistance welding to a wide variety of special shapes including many not of sheet form. In addition to the tungsten-copper combinations and the cadmium copper alloy there are now a number of other electrode alloys available such as silver-tungsten and those which consist principally of copper, but contain small percentages of chromium, molybdenum, cobalt, beryllium, silicon, or zirconium.

The industries in which the application of resistance welding has made considerable strides and are using more welding equipment are radio, aircraft, electric machinery and equipment, refrigeration, metal sash and window frame, automobile, metal wares, telephone, farm machinery, railroad equipment, and metal furniture.

In the past, frequent criticism has been heard because many users of and even some manufacturers of resistance welding apparatus have operated on a decidedly rule-of-thumb basis. Attempts are being made to put resistance welding on something approaching a quantitative, scientific basis and some progress has been made by the introduction of various simple meters and measuring devices, especially adapted to the peculiar requirements, such as means for measuring the time of current application, electrode pressure meters, and instantaneous type current meters employing a gaseous glow lamp as the measuring element. Some tables have also been published showing the proper electrode area, time, current, and pressure to use on specific material of given dimensions. It is also felt that more effort should be expended on standardization of transformer and secondary circuit performance of welding machines and that more coordination along these lines between manufacturers of this equipment is desirable.

Temperature Limits for Short-Time Overloads for Oil-Insulated Neutral Grounding Reactors and Transformers

By V. M. MONTSINGER
FELLOW AIEE

THE PROTECTIVE DEVICE committee is working on the problem of preparing standards for neutral grounding devices. One of the most important points up for consideration is the selection of the temperature limits for various times of operations of the devices.

At the present time grounding transformers are designed to withstand a one-minute short circuit (duration of fault) without exceeding a temperature of 160 degrees centigrade.

The increasing use of oil-immersed neutral grounding devices—reactors, transformers, and Petersen coils—where the fault may last several minutes makes it necessary to standardize safe temperature limits for the longer periods of time.

In the case of grounding transformers the 160-degree-centigrade temperature limit is based on all heat being stored in the copper (neglecting the insulation) during the one-minute period. Obviously, where the time of fault lasts several minutes the maximum temperature reached will be influenced considerably by some loss of heat during the heating up period, especially if the winding has oil ducts between all coils. In other words, both heat storage and dissipation should be taken into consideration.

On the other hand, since many neutral grounding devices carry no load except during the fault, windings with no oil ducts between coils can and are sometimes used. In this case very little loss of heat will take place in the interior of the coil stack during the heating up period and the maximum temperature can be based on all heat being stored in the copper and its insulation. Again in this case the rate of cooling after the fault is removed is very much less than it is for coils with oil ducts. In fact, as shown later the cooling period of coils with no oil ducts is the controlling factor so far as aging of the insulation is concerned. Coils with oil ducts will be termed "open type" coils and coils with no oil ducts will be termed "closed type" coils.

It is the purpose of this paper to calculate the temperatures for various lengths of faults—up to 30 minutes—that will produce the same aging of insulation as is produced by 160 degrees centigrade for a period of one-minute fault—the present standard for grounding transformers.

I. Aging of Insulation

It is obvious that short-time temperature limits should be based upon the aging of the insulation. By aging is meant the mechanical deterioration (tensile strength) since electrical deterioration does not take place[1] until the material has become quite brittle and well carbonized.

To estimate the relative aging of insulations under short-time overload conditions, three steps must be taken, namely:

1. Calculation of the temperature rise of the winding until the load is removed.
2. Calculation of the cooling of the windings after the load is removed.
3. Integration of the area represented by the heating curve, taking into account both temperature and time.

Two types of coils are considered. First, coils of open type construction. Second, coils of closed type construction.

II. Heating Curve of Open Type Coils

Figure 1 shows a typical heating curve until conditions become constant of an oil-immersed transformer winding. This curve was based on the average of three curves having final copper rises in excess of the oil of 27, 42.5, and 86 degrees centigrade given on figure 5 of a previous AIEE paper.[2]

III. Cooling Curves of Open Type Coils

The cooling of windings of the open type construction has been calculated by an empirical formula shown later since, as was pointed out, in the AIEE paper,[3] the standard cooling curve equation $\theta_t = \theta_0 \, e^{-\beta t}$ does not apply to the usual open type oil-immersed winding. The cooling curves in terms of watts per pound, figure 2, were based on the formula (equation 8, reference 3):

$$\theta = 1.95\, W_c^{0.7} (1 - e^{-Lt}) \qquad (1)$$

where

θ = cooling in degrees centigrade
W_c = watts per pound of bare copper
= $2.16 \times 10^{-6} D^2$ (at 25 degrees centigrade) where D = amperes/square inch
e = 2.718
L = $0.106\, W_c^{0.3}$
t = time in minutes

To use the cooling curves shown in figure 2 under transient heating conditions (i. e., before the rise becomes constant) the temperature rise at the instant load is removed must be obtained in terms of watts per pound of bare copper for constant conditions. In other words, the watts per pound that would give the temperature rise (at the instant load is removed) for constant conditions must be found. Such a curve is shown in figure 3 for a typical

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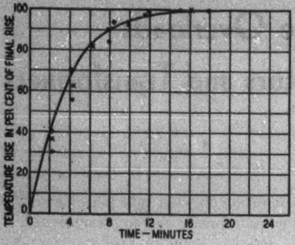
[1] For all numbered references, see list at end of paper.



Based on three curves (27 degrees, 42.5 degrees, and 86 degrees copper over oil- final rises) figure 5 of reference 2

Based on curve with:
- • 27-degree final rise
- × 42.5-degree final rise
- o 86-degree final rise

Figure 1. Temperature rise of horizontal disk coils. Copper rise over oil until conditions become constant

coil stack.* For example, if, when the load or fault is removed, the temperature rise (over oil) is 55 degrees, 100 watts per pound (W_c) should be used in calculating the cooling by equation 1.

IV. Relative Aging of Insulation for Various Overloads

Several years' experience with grounding transformers has indicated that 160 degrees centigrade for one minute is satisfactory. As a working base it is therefore assumed that 160 degrees centigrade is safe for a one-minute load. It has been shown[1] that the rate of aging doubles for each 8 degrees centigrade increase in temperature or conversely is halved for each 8-degree decrease in temperature, and that the general form of the equation for expressing aging (or relative aging) as a function of both time and temperature can be written:

$$A = t_0\, e^{0.088 T_2} \qquad (2)$$

where

A = units of aging
t_0 = time in minutes
e = 2.718
T_2 = temperature in degrees centigrade

In its present form equation 2 gives only relative aging units of insulation resulting from a continuous temperature T_2 applied for a length of time to, and represented by the rectangular area of the time-temperature graph. Later an empirical constant is added for the purpose of giving some idea of how long the insulation will last in service.

The equation for integrating temperature areas in the form of triangles and trapezoids is of the form

$$A = t_0 \left[\frac{e^{0.088 T_2} - e^{0.088 T_1}}{0.088\,(T_2 - T_1)} \right] \qquad (3)$$

where T_2 and T_1 are the maximum and initial temperatures respectively of a triangle, but for a trapezoid T_2 and T_1 represent the vertical parallel sides—t_0 being the base of both triangles and trapezoids. See figure 7. See appendix for derivation of equation 3 and examples worked out.

Figure 4 shows the temperature limits for various durations of overloads on open type coils starting at 25 degrees that give approximately the same degree of aging obtained where the temperature reaches 160 degrees centigrade at the end of one minute. Figures 5 and 6 show similar curves except that the starting temperature is 75 degrees centigrade.

The methods used in integrating the temperature curve areas are illustrated in the appendix.

The results of these calculations are shown in table I.

Table I. Relative Aging of Open Type Coils

Time of Overload (Minutes)	Initial Temp. 25 Deg.		Initial Temp. 75 Deg.	
	Maximum Temp. (Deg. Cent.)	Units of Aging	Maximum Temp. (Deg. Cent.)	Units of Aging
1	160	380,000	160	778,000
5	149	405,000	149	770,000
10	139	410,000	138	799,000
30	112	400,400	119	787,400

It will be seen from table I that the amount of aging where the starting temperature is 75 degrees is approximately double that where the starting temperature is 25 degrees centigrade. The reason for this may be explained as follows: with a lower initial (oil) temperature, a greater overload is required to reach a given temperature limit than is required with a higher initial (oil) temperature. The higher the overload (or losses), the steeper the heating-up curve will be. Also, the higher the rise (over oil) at the time overload is removed, the faster in degrees per minute will be the cooling. (It is assumed that the oil temperature does not materially change during the entire period of heating and cooling.)

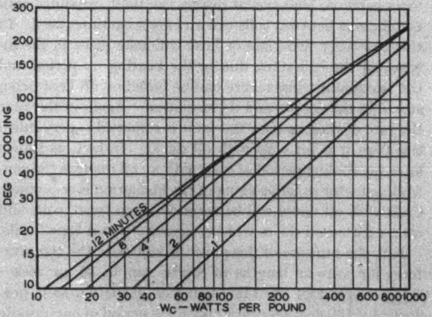


Figure 2. Cooling after shutdown of coils (with oil ducts)

Calculated by equation 1

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* This curve was determined by tests on typical coils. The equation of the curve is

$\theta = 1.38\, W_c^{0.8}$

where θ is the temperature rise over average oil temperature and W_c is the watts per pound of bare copper. In all cases where temperature rise is mentioned, it means the copper rise in excess of the oil temperature.

The result is that with the steeper heating curves and faster cooling curves (with the lower initial temperature and greater overloads) the areas near the top of the heating and cooling curves are smaller than they are with the higher initial temperature and lower overloads. This difference in the areas can clearly be seen in the curves shown in figures 4 and 5. Since most of the aging occurs near the peak temperatures the size of the top areas is very important and hence the aging is less for the lower initial temperature. This means that to obtain the same aging during the overload period, for 25-degree initial temperature, approximately 8-degrees higher temperature could be allowed than for 75-degree initial temperature, since, it has been assumed that aging doubles for each 8-degree increase in temperature. In fact, the difference would be more than 8 degree because this does not take into account the aging during the time when no overloads are on, during which time it would, of course, be greater for a continuous temperature of 75 degrees than for a continuous temperature of 25 degrees centigrade.

Based upon the assumption that 160 degrees centigrade is safe for one minute and upon the values shown in table I, it would seem that the following temperature limits (or some of these limits) could be standardized for oil-immersed grounding neutral devices of open coil construction.

Table II. Temperature Limits for Open Type Coils

Overload Time (Minutes)	Degrees Centigrade
1	160.0
2	157.5
3	155.0
5	150.0
10	140.0
20	125.0
30	120.0

For closed type coil (with no oil ducts) the temperature limits, for a given initial temperature, are obviously lower than for open type coils for the reason that the cooling period is much longer than for open type coils. Based on the rate of aging being halved for each 8-degree decrease in temperature, to keep the aging on a coil stack with no oil ducts to the same value as for an open type coil reaching 160 degrees in one minute, the temperature should not exceed approximately 135 degrees centigrade. On the other hand, there is very little difference in aging whether the heating up period is one minute, five minutes or ten minutes, since most of the aging takes place after the load is removed.

Since most, if not all, neutral grounding devices of the Petersen coil type operate only when there is a fault and where the starting temperature would be at room temperature, it would seem that devices with closed type coils could operate during a fault at somewhat higher temperatures than could devices with open type coils, which permit of carrying some load continuously if occasion demands it. In other words, it may be safe to use the temperature limits given in table III for coils of the closed type. This is a question that should be given consideration when selecting temperature limits for standardization purposes.

HEATING AND COOLING OF CLOSED TYPE COILS (NO OIL DUCTS)

In calculating the time-temperature curves of the closed type coils the heating-up curves were taken as approximately straight lines (except for the slight increase in resistance with increasing temperatures) since there is very little, if any, dissipation of heat from the interior of the coil stack during the short periods of time under consideration. Also, both the temperature rises and cooling curves were based on the thermal capacity of both the copper and intervening insulations. The thermal capacity of oil-immersed fibrous insulations is, by volume, approximately one-half that of copper.[3]

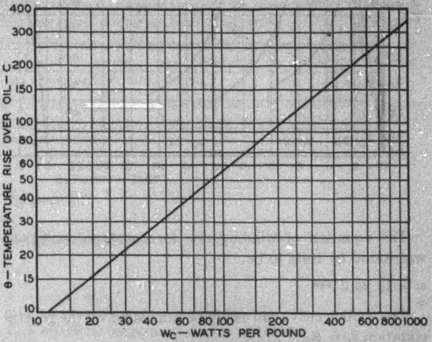

Figure 3. Temperature rise versus watts per pound of coil stack with oil ducts

Equation $\theta = 1.38\ W_c^{0.8}$

The cooling after load is removed was calculated by the standard cooling curve equation given in the first part of this paper. The size of conductor used in a closed type coil stack* on which tests were made to determine the curve of watts per pound versus temperature rise (for constant conditions) was 0.400 by 0.040 inch having 0.021-inch two sides thickness of insulation.

From the above

$$\beta = \frac{W_c 0.676 \dfrac{a}{A + a}}{\theta_0}$$

$$= \frac{0.259\ W_c}{\theta_0}$$

where a = 0.400 by 0.040 inch
and A = 0.421 by 0.061 inch

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* Consisting of 19 coils with no oil ducts.

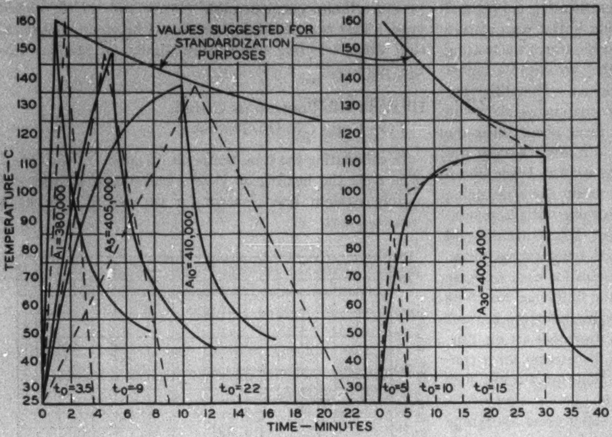

Figure 4. Various durations of loads and temperatures of oil-immersed windings (with coil ducts) to give approximately the same degree of aging of the insulation

Initial temperature T_1 = 25 degrees centigrade

Formulas shown in figures 5 and 6

The equation of winding rise for constant conditions was found by tests to be

$$\theta_0 = 18.25\ W_c^{0.64}$$

The cooling curve can easily be calculated by the general equation $\theta_t = \theta_0 e^{-\beta t}$.

The aging for a load attaining 160 degrees centigrade in one minute with a 25-degree initial temperature is approximately 3,290,000 units.

For a load attaining 152 degrees centigrade in 20 minutes the aging is approximately 3,000,000 units.

If it is assumed that 160 degrees is safe for one minute for closed type coils, calculations (for 25-degree initial temperatures) indicate that the following temperature limits could be permitted for 5, 10, 20, and 30 minutes.

Table III. Temperature Limits for Closed Type Coils

Overload Time (Minutes)	Degrees Centigrade
1	160
5	158
10	155
20	150
30	145

Since the aging in the above cases is in the order of 3,000,000 units, it is roughly 7.5 times that of an open type coil stack with 25-degree initial temperature.

It should be understood that the temperature limits obtained by the methods used in this paper are at best only approximations. Different designs will have different shapes of heating and cooling curves, although it is believed that the kind of coils chosen for these calculations are fairly well representative of the type of coils used. However, these calculated values should be more dependable than values arbitrarily chosen without any supporting data.

V. Life Expectancy

The life of any neutral grounding device obviously depends not only on the temperature limits chosen but also

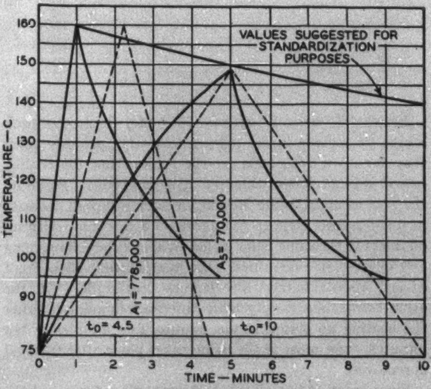

Figure 5. Various durations of loads and temperatures of oil-immersed windings (with coil ducts) to give approximately the same degree of aging of the insulation

Initial temperature T_1 = 75 degrees centigrade
T_2 = 160 and 149 degrees centigrade

$$A = t_0 \left[\frac{e^{0.088T_2}\ e^{0.088T_1}}{0.088\,(T_2 - T_1)}\right]$$

on the frequency of operation, which, of course, is unpredictable for this kind of service.

The following formula has been given[1] for estimating the life of insulation when operating continuously at various temperatures:

$$Y = 7.15 \times 10^4 e^{-0.08x} \qquad (4)$$

where

Y = life in years
x = temperature degrees centigrade
e = 2.718

Equation 4 gives the following years of life for 90, 95, 100, and 105 degrees centigrade temperatures:

Table IV

Temperature (Deg. C.)	Years—Life (Approximate)
90	26
95	17
100	11
105	7

If we choose 26 years as a reasonable life, this means that the equivalent of 90 degree continuous temperature should not be exceeded.

By equation 2 the units of aging for 26 years at 90 degrees is

$$A = (26 \times 365 \times 24 \times 60)\, e^{0.088 \times 90}$$
$$= 37{,}600{,}000{,}000 \text{ units}$$

With an initial temperature of 25 degrees the number of permissible operations per day would be (approximately):

$$\frac{37{,}600{,}000{,}000}{26 \times 365 \times 395{,}000} = 10 \text{ times for open type coils,}$$

or,

$$\frac{37{,}600{,}000{,}000}{26 \times 365 \times 3{,}000{,}000} = 1.3 \text{ times for closed type coils.}$$

These numbers of operations appear to be more than would ever occur under service conditions.

Conclusions

As a result of the study given in the paper the following conclusions have been drawn:

1. These data should make it possible to establish fairly dependable temperature limits for short-time overloads for oil-immersed neutral grounding devices.

2. For neutral grounding devices having open type coil construction, where the time exceeds one minute, most of the aging of insulation takes place during the heating-up period.

3. For neutral grounding devices having closed type coil construction, most of the aging takes place after the overload is removed; that is, during the cooling period.

4. For neutral grounding devices that carry no load, except during a fault, the temperature limits given in table III should be safe for standardization purposes.

Appendix

Derivation of equation 3:

Let

A = aging units
T = temperature, degrees centigrade
T_2 = maximum temperature degrees centigrade
T_1 = some lower temperature—see figure 7
t_0 = time in minutes

When the aging doubles for each 8-degree increase in temperature, the exponential constant is 0.088. Therefore:

$$A = \int_0^{t_0} e^{0.088T}\,dt$$

$$T = T_1 + (T_2 - T_1)\frac{t}{t_0}$$

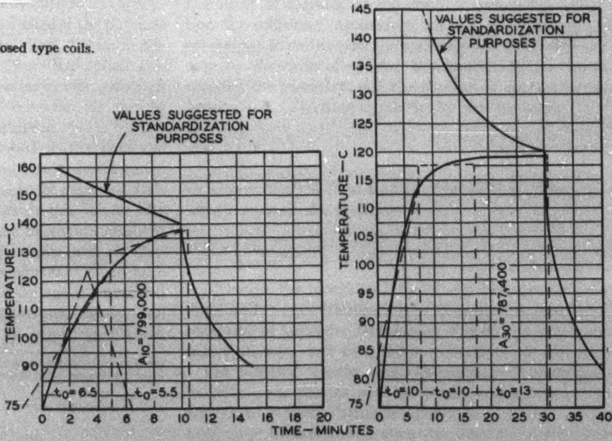

Figure 6. Various durations of loads and temperature of oil-immersed windings (with coil ducts) to give approximately the same degree of aging of the insulation

Initial temperature T_1 = 75 degrees centigrade

$A = t_0 e^{0.088 T_2}$ (for rectangles)

Formula shown in figure 5 used for triangles (T_1 = 75 degrees)

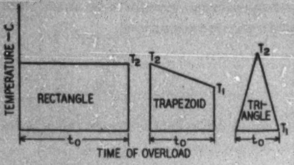

Figure 7. Temperature areas applicable to integration by:

Equation 2 for rectangles

Equation 3 for trapezoids and triangles

$$A = \int_0^{t_0} e^{0.088\left[T_1+(T_2-T_1)\frac{t}{t_0}\right]}\,dt$$

$$= e^{0.088T_1}\int_0^{t_0} e^{0.088(T_2-T_1)\frac{t}{t_0}}\,dt$$

$$= \frac{e^{0.088T_1}t_0}{0.088\,(T_2-T_1)}\left[e^{0.088(T_2-T_1)}-1\right]$$

$$= t_0\left[\frac{e^{0.088T_2}-e^{0.088T_1}}{0.088\,(T_2-T_1)}\right] \qquad (3)$$

Examples

1. Five Minute Load

For illustrative purposes the aging for the five-minute load, shown in figure 4, will be worked out in detail.

The temperature rise (found by trial) is 149 − 25 = 124 degrees centigrade. By figure 1 the rise in five minutes is 75 per cent of constant and at, say, three-minutes is 55 per cent of constant. Then the rise at three minute period (of the five-minute load) should be $\frac{55}{75}$ = 73.3 per cent of 124 degrees or 91 degrees centigrade. 91 + 25 = 116 degrees actual temperature. The other points on the curve are worked out similarly.

As the cooling is a function of the watts per pound of copper at the instant load is removed, it is necessary to convert the rise in terms of watts per pound. A rise of 124 degrees represents a loss of 275 watts per pound by figure 3.

By figure 2 the cooling in one minute is 45 degrees. 149 − 45 = 104 degrees temperature at the six-minute point. The cooling for two minutes is 70 degrees and 149 − 70 = 79 degrees centigrade temperature at the seven-minute point.

The next step is to resolve the temperature curve into a triangle of approximately the same area, especially the top portion where the aging is most severe.

The triangle shown by dashes has a base of nine minutes and a peak temperature of 149 degrees centigrade.

By substituting in equation 3 and letting $T_2 = 149$, $T_1 = 25$, and $t_0 = 9$,

$$A = 9\left[\frac{e^{0.088\times149}-e^{0.088\times25}}{0.088\,(149-25)}\right]$$

$$= 9\left(\frac{490{,}000}{10.9}\right) = 405{,}000 \text{ units.}$$

2. Thirty-Minute Load.

The aging for the 30-minute load shown in figure 4 is as follows

$$A_{15} = 15e^{0.088\times112} \qquad = 282{,}000$$

$$A_{10} = 10\left[\frac{e^{0.088\times112}-e^{0.088\times100}}{0.088\,(112-100)}\right] = 116{,}000$$

$$A_5 = 5\left[\frac{e^{0.088\times90}-e^{0.088\times25}}{0.088\,(90-25)}\right] = \quad 2{,}400$$

400,400 total units.

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3. Cooling of Oil-Immersed Transformer Windings After Shutdown, V. M. Montsinger. AIEE Proceedings, volume 36, part 1, 1917, page 711.

A New Correlation of Sphere-Gap Data

By D. W. VER PLANCK
ASSOCIATE AIEE

Synopsis: The similarity principle for electrical discharges is used to correlate the most recent AIEE sphere-gap data for both positive and negative impulses. Empirical formulas are developed from which the entire range of data can be calculated to within about 1.5 per cent. Some practical results of this study are:

1. Spark-over curves for each value of spacing to diameter ratio are straight lines when plotted to logarithmic scales, at least to within the certainty of the present data.

2. A spark-over voltage chart covering the entire useful range of sphere sizes, spacings, and air densities can be constructed easily.

3. New air-density correction factors are derived which differ appreciably from those previously published.

Introduction

THE SPHERE GAP, widely used for measuring high voltages, is not an absolute standard; that is, its performance cannot be calculated from fundamental considerations. The standard calibration is based on measurements mainly in terms of low-voltage standards through the use of potential transformers or potential dividers. The curves for the eight standard sphere sizes are commonly regarded as independent entities each relying for its accuracy on a separate testing program. Knowledge of an orderly relationship between the curves would fulfill a definite need. It is the purpose of this paper not only to show that there is indeed such a relation but that it is of surprisingly simple form. A direct calculation by classical electrostatic theory is not feasible because for air the puncture strength, or maximum gradient just prior to spark-over, different for each electrode configuration and spacing, is known only through the reverse calculation from experimentally determined spark-over curves. Nor is the present knowledge of the physical mechanism of sparking sufficiently exact to form the basis for accurate calculations. Thus the sphere-gap calibrations are the result of direct experiments, although empirical formulas have been of considerable use in smoothing the data and for extrapolation.

Of the empirical methods developed in the past the best known, due to Peek,[1] gives good results in the range for which it was intended, but, as might be expected, fails when used for extrapolation far outside this range. Its basis is an empirical formula for the maximum gradient prior to spark-over involving sphere radius and air density, but not spacing as it probably should. When both spheres are insulated the spark-over voltage is calculated from this gradient by classical electrostatic theory, and when one sphere is grounded, by an empirical method. The AIEE standard air-density correction factors were calculated from Peek's gradient formula.

Bellaschi and McAuley[2] have smoothed and extrapolated their data by making use of their discovery that for each constant ratio of sphere gap opening to sphere diameter plots of average gradient against the logarithm of sphere diameter are essentially straight lines for sphere diameters above 25 centimeters. Later Bellaschi[3] elaborated on this method giving empirical formulas with numerical values for their parameters.

The new method which is the subject of this paper, while empirical, appears to have a wider range of usefulness and greater simplicity than methods published heretofore. It is based on the principle of similarity for electrical discharges, a principle well established in the field of physics, but whose power seems not to have been fully appreciated by engineers in this country.

The Principle of Similarity for Electrical Discharges

The principle of similarity for electrical discharges, a generalized form of Paschen's Law, states that in a given gas "Geometrically similar systems will discharge at the same potential if the products of their leading dimension and mass of gas per cubic centimeter are the same; or the discharge potential is a function of the product of density and a dimension for similar systems."[4] This law originally deduced from, and since substantiated by direct experimental evidence, the extent of which is mentioned below, has also been shown by Townsend[5] to result from his theory of sparking as the result of cumulative ionization by collision. Applied to an isolated symmetrical sphere gap it means that the sparking voltage V depends only on the ratio of spacing to diameter (S/D), which specifies the geometry, and the product of spacing by air density ($S\delta$). Written symbolically this becomes

$$V = F\left(S\delta, \frac{S}{D}\right) \quad (1)$$

Or alternatively if D instead of S is taken as "the leading dimension," the equation is

$$V = \Phi\left(D\delta, \frac{S}{D}\right) \quad (2)$$

Although the functions F and Φ can be expressed only graphically or empirically, it is evident that the number of independent variables determining V is not three as commonly thought, but two, the combinations $S\delta$ and S/D.

For practical sphere gaps other ratios of dimensions describing the shanks and the position of the gap with respect to grounded surfaces should be included. However, since little is known about the effects of these other

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1. For all numbered references, see list at end of paper.

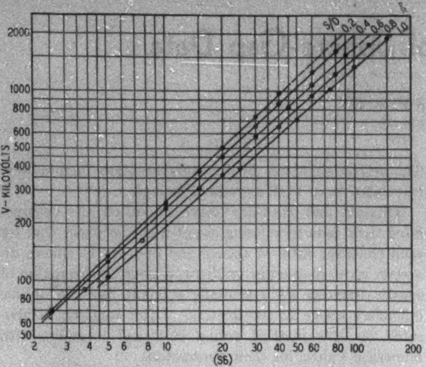

Figure 1. Lines for which S/D is constant

Data from AIEE Revised Standards for Negative Impulse

Table I. Parameters for Equation 3

One sphere grounded: $2 < (S\delta) < 200$; $5 < (D\delta) < 200$

	For Negative Impulse and 60 cycles		For Positive Impulse	
S/D	K	α	K	α
0.1	30.10	0.941	30.15	0.941
0.2	29.95	0.940	30.15	0.940
0.3	29.65	0.931	30.15	0.931
0.4	29.25	0.917	30.15	0.917
0.5	28.55	0.904	30.15	0.900
0.6	27.45	0.895	30.10	0.884
0.7	26.21	0.891	29.70	0.872
0.8	24.96	0.888	29.00	0.865
0.9	23.69	0.886	28.00	0.860
1.0	22.44	0.885	27.00	0.855

Table II. Comparison of Calculated and Standard Spark-Over Voltages, Negative Impulse and 60 Cycles

Upper figures are calculated; lower figures are from AIEE Standards[8]

S/D	D							
	6.25	12.5	25	50	75	100	150	200
0.1			71.3	136.9	201	263	385	504
			72	136	200	261	383	506
0.2		70.9	136.0	261	382	501	733	960
		70.8	136	260	380	504	736	973
0.3	53.2	101.5	193.5	369	538	703	1,026	1,341
	53.8	100.6	192	367	528	700	1,040	1,346
0.4	67.8	128.0	242	456	662	861	1,249	1,626
	68.8	127.0	241	451	653	862	1,254	1,635
0.5	80.0	149.7	280	524	756	981	1,415	1,835
	81.0	147.5	278	519	751	985	1,410	1,857
0.6	89.6	166.6	310	576	828	1,072	1,540	1,992
	90.3	164.5	309	573	827	1,084	1,552	2,027*
0.7	97.6	181.1	336	623	894	1,155	1,657	2,141
	98.2	179.5	338	615	890	1,163	1,662	2,169*
0.8	104.2		357	661	947	1,222	1,752	2,263
	105.5		362	651	945	1,234	1,760	2,293*
0.9			374	691	989	1,276	1,828	2,359
			379	681	992	1,295	1,820	2,395*
1.0			387	716	1,024	1,321	1,892	2,440
			393	707	1.025	1,338	1,900	2,475*

* Extrapolated by AIEE, not used in determining parameters.

geometric ratios, except that in practice they are usually small, they are ignored here. Necessarily F and Φ depend on whether both spheres are insulated or one is grounded and if so on polarity as well. Except for density, whose primary significance in this connection is that it is inversely proportional to the mean free path, other properties of the air, such as its humidity, have no appreciable effect on V within the range of S/D used in measurement practice.

Researches on the similarity principle for spheres are well summarized by Whitehead,[4] Thomson,[6] and Bowker.[7] V has been shown to depend only on the product $(S\delta)$, with S/D constant, independently of S, temperature and pressure. With hydrogen and nitrogen the temperature was carried to over 800 degrees centigrade at pressures from 0.25 to 2 atmospheres. In air the law holds up to at least 300 degrees centigrade, and at room temperature from pressures of a few millimeters to about 10 atmospheres, above which there is a departure. The spheres used were small, from 1 to 2.5 centimeters in diameter, and S/D was usually less than 0.2 although in some cases as high as 0.5. Taking $\delta = 1$ for normal atmospheric conditions of 760 millimeters and 25 degrees centigrade, the maximum $(S\delta)$ attained was about 2.0 for which approximately $V = 60$ kv.

Correlation of the Data

As a consequence of the usual testing procedure, sphere-gap data are customarily presented as separate curves or tables of V vs. S, $(\delta = 1)$, for each size of sphere. Variations in δ are taken into account by means of tabulated correction factors. Three independent variables S, D, and δ are thus considered in determining V. However, as shown here by the similarity principle, V is really a function of only two independent variables, the combinations $(S\delta)$ and S/D. A function of two variables is not hard to tabulate, but it is even easier to show graphically on a chart. A number of forms of chart were tried in an effort to find one that would be not too large and still include the desired range of variables, easy to read accurately, and easy to construct.

In the course of this study it was discovered that, to within the certainty of the best data available, curves of V vs. $(S\delta)$ for constant S/D plotted to logarithmic scales are *straight lines* over the entire useful range of sphere gap

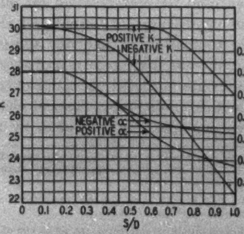

Figure 2. Parameters in the formula $V = K(S\delta)^\alpha$

One sphere grounded, both impulse polarities

$2 < (S\delta) < 200$
$5 < (D\delta) < 200$

Table III. Comparison of Calculated and Standard Spark-Over Voltages, Positive Impulse

Upper figures are calculated; lower figures are from proposed AIEE standards[9]

S/D	D 6.25	12.5	25	50	75	100	150	200
0.3	54.1	103.2	196.8	375	547	715	1,043	1,364
	54.3	101.7	196	374	548	715	1,055	1,364
0.4	69.9	131.9	249	470	682	888	1,288	1,677
	69.3	132.3	252	474	687	888	1,293	1,671
0.5	84.1	156.9	293	546	787	1,020	1,468	1,902
	81.8†	158.0	296	547	786	1,024	1,453	1,896
0.6			330	609	871	1,123	1,607	2,073
			334	605	870	1,124	1,597	2,069*
0.7			360	660	939	1,207	1,719	2,209
			364	655	939	1,209	1,718	2,220*
0.8			387	705	1,001	1,284	1,823	2,339
			390	698	999	1,284	1,824	2,360*
0.9			407	740	1,048	1,342	1,902	2,437
			409	732	1,046	1,344	1,902	2,464*
1.0			423	766	1,083	1,385	1,958	2,505
			426	738†	1,081	1,390	1,944	2,543*

* Extrapolated by AIEE, not used in determining parameters.
† Not used in determining parameters.

testing. For examples, see figure 1. These curves are described by the empirical equation

$$V = K(S\delta)^{\alpha} \qquad (3)$$

Where K and α are parameters depending only on the ratio S/D and the polarity. The variation of these parameters with S/D is shown in table I and figure 2, for one sphere grounded and both impulse polarities.

The source of the data for 60-cycle and negative impulse voltages is the partial revision of AIEE Standards No. 4 published in July 1936,[8] and that for positive impulse voltages is a part of the same revision as yet unpublished.[9] The revision is based mainly on the extensive researches of Bellaschi and McAuley[2] and of Meador.[10] Points at desired values of S/D not appearing in the tables were obtained from plots. Using values of S/D for each tenth up to 1.0, K and α were adjusted to best fit the data subject to the condition that curves of K and α against S/D should be smooth. A large scale logarithmic plot was used to obtain preliminary values of K and α which were then refined by successive calculations using logarithms and a computing machine.

The data used and the corresponding calculated values are shown together in tables II and III for the range of conditions defined by $(S\delta) > 2.0$; $6.25 < D < 200$; $S/D < 1.0$; and $\delta = 1.0$. As to accuracy, a comparison of the calculated values and the data for the 116 points given in the tables reveals that 83 agree to better than 1 per cent, 28 to between 1 and 1.5 per cent, and 3 to between 1.5 and 1.9 per cent. Two values for positive impulse are disregarded entirely because they are so far out of line as to be almost certainly in error.

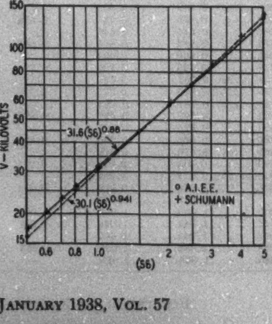

Figure 3. Comparison of formulas for use above and below $(S\delta) = 2.0$, $S/D < 0.2$

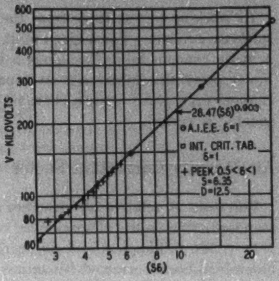

Figure 4. Agreement of Peek's low-density tests with AIEE Standard values for $\delta = 1$ and with the new formula for $S/D = 0.508$

When $(S\delta) < 2.0$ calculations using the parameters of table I no longer fit the data so closely, the departure at $(S\delta) = 1.0$, $S/D = 0.1$ being nearly 5 per cent. In this region the true curves to logarithmic scale depart appreciably from linearity although the curvature is still not great. In the range $0.4 < (S\delta) < 2.0$; $S/D < 0.2$, the formula

$$V = 31.6(S\delta)^{0.88} \qquad (4)$$

gives a good approximation to data collected by Schumann[11] for a uniform field, $S/D = 0$, and to the few AIEE data in this range. These data, equation 4, and the line for $(S\delta) < 2.0$ and $S/D = 0.1$ are plotted for comparison in figure 3.

The data discussed so far are for values of $(S\delta)$ when $\delta = 1$, that is for standard atmospheric conditions. Test data for $\delta \neq 1$ and $(S\delta) > 2.0$ are not easily found in the literature. Some reported by Peek[1] for 12.5-centimeter spheres, one grounded, spaced 6.35 centimeters ($S/D = 0.508$) at relative air densities down to about 0.5 are plotted in figure 4. There is close agreement between these points and the line defined by equation 3 with the parameters for $S/D = 0.508$ read from figure 2.

Sphere-Gap Spark-Over Chart

The sphere-gap function, F in equation 1, can be mapped to the best advantage on a chart whose co-ordinates are V and $(S\delta)$ to logarithmic scales. Curves of V against $(S\delta)$ for constant S/D, spaced at convenient intervals, are then, to the present certainty of the data, the perfectly straight lines whose parameters K and α have been given. Curves of constant $(D\delta)$ can be drawn through points easily located on the constant S/D lines. In addition to simplicity of construction the logarithmic scales give compactness together with constant accuracy for all parts of the chart. A chart for negative impulses and 60 cycles is shown to a small scale in figure 5.

In working out settings from the chart the following rules are evident. For a given fixed gap with varying air density, changes take place along a line of constant *S/D*. For a particular sphere size at a particular air density, changes take place along a curve of constant (*Dδ*) as the spacing is varied.

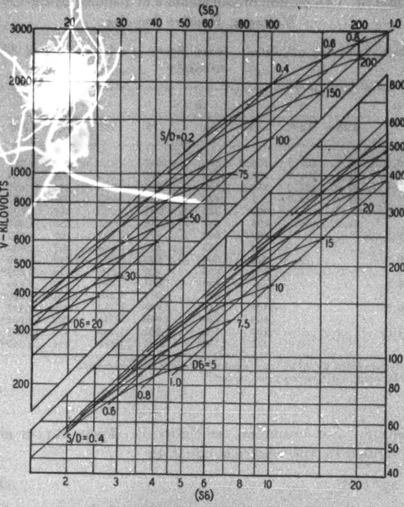

Figure 5. Sphere-gap spark-over chart

One sphere grounded; negative impulse and 60 cycles

Air-Density Correction Factor

For tests at relative air densities other than unity the standard table values of spark-over voltage, which are for $\delta = 1$, 760 millimeters and 25 degrees centigrade, have heretofore been corrected by means of the multiplying factors given in AIEE Standards No. 4. These factors, which are identical with those calculated by Peek[1] and those recommended by the I.E.C.[12] are tabulated for each size of sphere and for $0.50 < \delta < 1.10$. No account is taken of any effect due to changes in *S/D*, nor of polarity since they are for 60 cycles.

The method advocated here is based on the similarity law and contained in the chart, figure 5. For a given *S/D* the spark-over voltage for any δ is read from the curve of *V* vs. ($S\delta$), which may have been found originally for some other δ. If a chart is not available, then with the aid of the empirical formula 3 the spark-over voltages of a particular sphere gap at two different densities are related by

$$V_2 = V_1 \left(\frac{\delta_2}{\delta_1}\right)^{\alpha} \qquad (5)$$

The air-density correction factor for use with the standard values is thus simply $(\delta)^{\alpha}$, and depends not on *D* but on *S/D* and the polarity. The new factors for each polarity and one sphere grounded are shown in table IV. That they should depend on the polarity of the unsymmetrical gap is not surprising when it is remembered that positive and negative discharges have distinctly different properties. A comparison with the present standard factors shows for negative impulses or 60 cycles the possibility of a maximum difference between them of about 1 per cent at $\delta = 0.9$ increasing to about 5 per cent at $\delta = 0.5$. In general, the direction of the difference depends on both *D* and *S/D*. When $\delta < 1.0$ the new factors for

Table IV. Air-Density Correction Factors; One Sphere Grounded

Where they differ, factors for positive impulse are printed below

δ	2.0 < (Sδ) < 200									0.4 < (Sδ) < 2.0
	S/D 0.2 and less	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	0.2 and less
0.50	0.521	0.524	0.530	0.534 0.536	0.538 0.541	0.539 0.546	0.540 0.550	0.541 0.551	0.541 0.553	0.543
0.55	0.570	0.573	0.579	0.582 0.584	0.585 0.589	0.586 0.593	0.588 0.596	0.589 0.598	0.589 0.600	0.591
0.60	0.618	0.621	0.626	0.630 0.631	0.633 0.636	0.634 0.640	0.635 0.643	0.636 0.644	0.636 0.646	0.638
0.65	0.667	0.670	0.674	0.678 0.679	0.680 0.683	0.681 0.687	0.682 0.689	0.683 0.690	0.683 0.692	0.685
0.70	0.715	0.717	0.721	0.724 0.725	0.727 0.729	0.728 0.732	0.729 0.734	0.729 0.736	0.729 0.737	0.731
0.75	0.763	0.765	0.768	0.771 0.772	0.773 0.775	0.774 0.778	0.775 0.780	0.775 0.781	0.775 0.782	0.776
0.80	0.811	0.813	0.815	0.817 0.818	0.819 0.821	0.820 0.823	0.821 0.824	0.821 0.825	0.821 0.826	0.822
0.85	0.859	0.860	0.862	0.864	0.865 0.866	0.866 0.868	0.866 0.869	0.866 0.870	0.866 0.870	0.867
0.90	0.906	0.907	0.908	0.909 0.910	0.910 0.911	0.911 0.912	0.911 0.913	0.911 0.913	0.911 0.914	0.911
0.95	0.952	0.953	0.954	0.954	0.955	0.956	0.956	0.956 0.957	0.956 0.957	0.956
1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.05	1.047	1.046	1.046	1.045	1.045 1.044	1.044 1.043	1.044 1.043	1.044 1.043	1.044 1.043	1.044
1.10	1.094	1.093	1.091	1.090	1.089 1.088	1.089 1.087	1.088 1.086	1.088 1.085	1.088 1.085	1.088

6.25-centimeter spheres are smaller than the old over the whole range of spacing, while for 100-centimeter spheres they are always larger. The possible differences between the new positive impulse factors and those now in Standards No. 4 are even greater, about 1.3 per cent at $\delta = 0.9$.

Concluding Remarks

The method given here for calculating the standard sphere gap spark-over voltages has an accuracy of better than 1.5 per cent, which is well within the tolerances in this field of measurement. AIEE Standards No. 4 specifies an accuracy of 2 per cent for tests carried out in accordance with its instructions, while in the revised standards this figure has been increased to 3 per cent in recognition of the difficulty of attaining the ideal conditions in practice.

It is possible that in many cases the new air-density correction factors are enough different from those now in use to materially reduce the discrepancies between the results of the various standardizing laboratories.

It is suggested that future standardization tests be guided by the facts brought out here; that in accordance with the similarity principle the basic variables are S/D and $(S\delta)$; and that with S/D held constant the relation of V to $(S\delta)$ is a very simple one. Even though more precise tests may show the empirical formula 3 not quite exact, the curves of constant S/D to logarithmic scale must be so nearly linear that they can be firmly established using only four or five wisely chosen sphere sizes.

List of Symbols

V = Spark-over voltage in kilovolts crest.
D = Diameter of sphere in centimeters.
S = Spark-over distance between spheres in centimeters.
δ = Relative air density; unity for 760 millimeters and 25 degrees centigrade
K = Empirical coefficient depending on S/D.
α = Empirical exponent depending on S/D.
F, Φ = Functions of unknown mathematical form.

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Electrical Equipment for Modern Urban Surface Transit Vehicles

By S. B. COOPER

Synopsis: This paper reviews modern urban surface transit developments, traces the reasons for the changes, and outlines some of the problems involved and their solution.

THE CHANGING conditions that have produced such marked developments in public urban transportation vehicles during the past ten years have naturally brought about corresponding changes and developments in the electrical propulsion equipment.

During the period from 1926 on, the increasingly serious competition from the private automobile and the gas bus made it more and more evident that the transit industry could not continue to compete successfully with these newer forms of transportation while still using the tools and methods developed during the previous decade with little change up to that time other than detail improvements.

The need became increasingly apparent about this time for vehicles which would do several things:

1. Provide a faster, smoother ride.
2. Provide a more attractive vehicle from the standpoint of passenger appeal—finish, seats, lights, quietness, etc.
3. Reduce operating costs.

These requirements led to a greatly accelerated rate of development in cars, trolley coaches, and gas electric busses, and in the electrical equipment for each of these vehicles.

The first major efforts of this period in improvements in cars lay along the lines of:

1. Reduced weight.
2. Lower floors and steps.
3. Reduction in unsprung weight.
4. Reduction in noise.
5. Improved braking.
6. Increased acceleration.
7. Automatic acceleration.
8. Better appearance.

These objectives led to a period of intensive development from 1927 to 1930 during which were introduced *WN* double reduction, single reduction, and worm type gear units, which in turn called for and made possible the 300 volt, high-speed spring-supported motor now so generally accepted. New ideas were introduced along truck and body lines, largely to take advantage of features made possible by the new motor and drive developments. Also the need for more rapid acceleration brought about the introduction of the variable automatic controls of different types—it being generally felt that the higher rates and the general adoption of one man operation made automatic acceleration very desirable if not essential.

This period of intensive development, with aims sometimes paralleling and sometimes in conflict, together with a growing realization of the urgent need for some degree of standardization, no doubt had a part in the organization during 1930 of the Presidents' Conference Committee.

Other papers deal with the history and accomplishments of this committee; suffice it to say that the results obtained in the past two and a half years fully justify its activity.

Following the early period of investigation and research by the committee, the problem presented to the designers of electrical equipment was about as follows:

1. To produce a 55-horsepower motor lighter and smaller than anything so far achieved.
2. A control equipment which would permit automatic acceleration rates up to 4.75 miles per hour per second so smooth that even standing passengers would not be able to detect individual notches.
3. The motor and control equipment to be so designed as to take the major share of braking duty away from the shoe on the wheel, and to be capable of producing in conjunction with the air-actuated wheel brakes and the magnetic track brakes smooth braking rates up to 4.75 miles per hour per second in service applications and up to 8 or 9 miles per hour per second in emergency.

Standardized mounting and housing arrangements were called for, with means for recovery of accelerating and braking resistor losses for car heating when required.

Two different equipments are now available to produce these results. It is felt that detailed description of the apparatus and functioning is beyond the scope and purpose of this paper—these details are available to those interested. The electrical manufacturing companies have produced equipment fulfilling the objectives established by the committee. The 55-horsepower 300-volt motors weigh about 700 pounds each in contrast to 2,185 pounds for an axle-hung type motor of corresponding capacity. The motor barrel diameter is 15 $^{3}/_{4}$ inches and the length over the housings about 25 inches. Figure 1 shows a motor of this type; such motor designs have been made possible by the use of high-temperature insulation, improved antifriction bearings, and perhaps most of all by developments in armature and commutator design and construction which permit peripheral speeds way beyond anything considered possible ten years ago.

It happens that during the period from 1926 to 1930 there was also a rapid and intensive development in a-c motors for steam-railroad electrification, leading up to the Pennsylvania electrification program of 1931. Many of the improvements, processes, and principles worked out

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in connection with these a-c motors were found directly applicable to d-c motors as well. This contributed greatly to the development of the light-weight high-speed motors which we have today for cars, trolley coaches, and gas and oil electric busses.

For instance the technique of application of antifriction bearings to railway motors underwent tremendous development during this period. The recognition of the necessity for careful choice and maintenance of adequate internal clearances in these bearings under railway motor conditions might be cited. The growing use of cylindrical roller bearings is at least in part due to the ease and accuracy with which the internal clearance after assembly can be measured.

The extreme importance of keeping lubricant *in* and dirt *out* of these bearings has become more generally understood. The use of the steel labyrinth type of seal without rubbing or wearing surfaces has aided greatly in the accomplishment of these objectives without the necessity for frequent renewals due to wear on any type of rubbing surfaces at the high speeds used in these bearing applications. Another important improvement in this respect, especially from the maintenance point of view is the use of "cartridge" type bearing construction which permits the removal, storage, and replacement of complete armatures without opening the bearing enclosures and the resultant risk of getting dirt into the bearings.

The a-c motor development also added greatly to our knowledge on such subjects as permissible limits in armature-field ampere-turn ratios, pole widths and shapes, pole-tip shapes and spacings, etc., all of which has contributed to a marked degree to the design of the d-c motor as well.

Noise reduction has become increasingly important; noises in electrical machines can be generally grouped into three classes—windage, magnetic, and bearing. As the result of careful study, air paths and air foil shaping of surfaces directing air flow are better understood and proper attention to these details has resulted in marked improvement. Intensive research has led to a better understanding of the sources of and remedies for magnetic noise. The use of skewed slots, or more recently, careful selection of number of slots in combination with proper pole shape and spacing, have both been found effective in reducing or eliminating magnetic noises. Bearing noise is of course largely a matter of proper control of bearing clearances and lubrication.

Figure 1. Motor for PCC car

The urge for smaller and lighter machines for a given rating forced the designers years ago to look for insulating materials which would stand operating temperatures beyond the limits of treated cotton insulation. This led to the use of mica and asbestos in various forms, and tremendous improvements have been made in the technique of manufacture and application of mica and asbestos insulation to railway motors. Unfortunately, both of these materials are natural products, found in only comparatively limited quantities in grades suitable for electrical insulation; like all natural products their characteristics are not uniform, and there is always the question of the ultimate exhaustion of available sources of supply. In addition, there are certain inherent limitations in these materials that at least so far have not been entirely overcome. For instance, both mica and asbestos are mechanically weak and therefore require, for practical manufacturing reasons, the use of certain carrier materials to add mechanical strength and stability during handling and application. Mica, being inherently a flaky material, is usually built up on a paper or silk tape carrier, and is somewhat difficult to apply satisfactorily to small conductors. Asbestos tapes are necessarily built up with cotton or silk cross threads to give mechanical strength, and there are certain definite limits in minimum thickness below which asbestos insulation cannot be obtained.

There has recently been developed a new material for high temperature insulation which affords very great promise—glass tape. This is a fabric type tape woven from threads of spun glass. It has most astonishing flexibility and strength, can be made in almost any required thickness down at least to 5 mils and of course has wonderful heat resisting and dielectric properties. Being a synthetic product its properties can be closely controlled, and the raw materials from which it is made are available in almost unlimited quantities. It is now beginning to be used in these types of high-speed railway motors, and it holds great promise as a means of still further improving designs.

A most important factor in the design and production of these small light-weight motors is the very large increase in limiting peripheral speeds of armatures and commutators as compared to previous practices. Here again the fundamental development work carried out on the a-c motor was directly applicable to these small d-c motors. It is not so very many years ago that 7,000 feet per minute on armatures and 5,000 on commutators were considered top limits. Now we are going up to 12,000 and 9,500, respectively. A number of factors are involved in this increase. Improved bearings and better understanding of bearing application, improved methods and materials for banding and wedging, better methods of dynamic balancing, and perhaps most important of all, very marked advance in technique of design and building commutators have all contributed their share to the final results.

Armatures of the type used in these applications are balanced to within 20 inch-square ounces. The two planes at which corrective balance weights are applied are 11 inches apart, and the radius at which weights are applied is $4^1/_2$ inches, so it can be seen that the limiting weight tolerance is about two-fifths ounce.

Commutator improvements have been principally along the lines of better understanding of the forces acting on the bars at high speeds and temperatures, and the absolute necessity of maintaining a practically true cylindrical surface throughout the entire range of operating speeds and temperatures. We have come to realize that a very large percentage of our commutation and current collection problems are mechanical in nature, and to successfully meet them we must keep an intimate and continuous contact between the bars and the brush. When a real commutator expert speaks of a rough commutator he probably means one where the variation in radius from one bar to the next may be as great as four ten-thousandths of an inch.

A commutator is made up of a steel spider and V rings, mica bushing and V ring insulation, and copper and mica bars. All these materials have widely divergent mechanical and temperature characteristics, and their behavior under repeated cycles of speed, heating, and cooling is decidedly different. Successful solution of the problems involved requires the use of very high grade alloy steels for the bushings and V rings, extraordinarily careful selection and treatment of the copper and mica, and extremely close control of and tolerances in the entire process of building and seasoning. Seasoning has been found to be an absolutely essential final step in commutator manufacture. Each one is put through repeated cycles of spinning under heat, grinding and tightening until the entire structures acquires the necessary mechanical stability and the ability to "stay put" in service.

Figure 2. Accelerator for PCC car control

While the subject of commutation is under discussion another related important factor should be mentioned. Under all conditions, but particularly at the high speeds here involved, commutator and brush performance and life are very seriously affected by the presence of dirt in the ventilating air.

Earlier ventilated railway-type motors drew their incoming air directly from the space under the vehicle, with all the street dirt, wheel wash, and whatever else might be present. All this dirt and moisture is extremely bad for commutator and brush performance and life, and also for all the insulation in the machine. In the PCC car where partial or full skirting tends to carry the turbulent swirl of dirty air along under the car, it was felt that improved life and reduced maintenance of brushes, commutator, and coil insulation fully justified an effort to obtain relatively clean dry air for motor ventilation. Flat ducts are built into the body bolster just above the motors. Air is drawn into louvred openings at the outer ends of these ducts, outside the car skirt and practically at floor level. Flexible bellows connect these ducts to the motors.

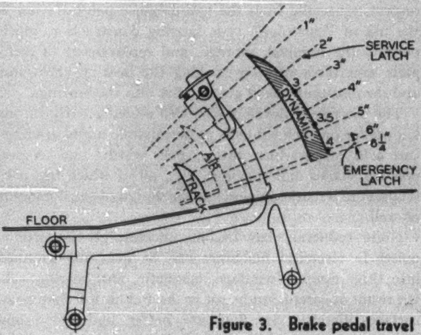

Figure 3. Brake pedal travel

The marked reductions finally achieved in weight and size of these types of motors sometimes lead purchasers to expect corresponding reductions in cost. From the very sketchy outline given above of some of the major problems of these designs and of their solutions, it must be clear that the reductions in weight and size have been possible only by the substitution of higher grade materials and very great increases in labor and tool cost per pound of material used. Price per pound may under some circumstances be a very useful index when applied to the same type of design and construction, but may be wholly misleading if radical differences in design and manufacturing processes are involved.

An important part of the research work carried on by the Presidents' Conference Committee was an investigation of the maximum rates of acceleration and braking and the maximum changes of rate which could be comfortably tolerated by standing passengers. These research tests disclosed some very surprising results, and indicated that rates much beyond those previously considered possible were entirely acceptable, provided they were maintained sufficiently smoothly. Previously existing forms of control varied from 8 or 9 notches in standard *K* types to 14–18 in the then existing automatic types. These gave fairly comfortable accelerations at rates up to 2.5 or 3 miles per hour per second except for an occasional "bump" at transition, but were too rough at accelerations higher than this.

In an eight-notch *K* control, tractive effort increments of the order of 50 per cent of the previously existing value are quite common. In the earlier forms of automatic control this was reduced to about 20 per cent. In the present controls developed for PCC cars the percentage increment in tractive effort per notch is far too small to be perceptible even to standing passengers. Experience has indicated that after the number of notches is increased to about 22, further increases add nothing to passenger comfort, and the selection of number of notches becomes entirely a matter of the limitations of the type of contact device selected.

Actual street experience with the new cars has indicated quite definitely that it is possible for a street car to accelerate and brake as rapidly as a private automobile in the hands of an expert driver, and more rapidly than an average car in the hands of an average driver.

The success of the control equipments developed for these cars may be ascribed to the use of sufficiently large number of notches to eliminate jerks or surges in the acceleration, to the use of actuating means sufficiently sensitive and responsive to maintain smoothly the preselected rates of acceleration and braking, and particularly to means provided to ensure prompt and uniform response of the equipment in dynamic braking.

In one form of equipment the variable resistor device is called an "accelerator" (figure 2) and consists of a cylindrical assembly of resistors, tapped at 97 points, each tap connecting to a spring finger. Successive fingers are pressed into butt type contact with a copper bus by the action of a roller cam carried on an arm rotated by a 32-volt pilot motor through a small worm gear. A sensitive vibrating contact type of limit relay controls the direction and speed of the pilot motor, and thus controls the rate of change of resistance in the main motor circuits. The successive steps are so small in magnitude that individual increments of tractive effort are exceedingly small—far too small to be evident to passengers.

The limit relay setting is changed at the will of the operator by cams on both the brake and the master controllers which are in turn actuated by the foot pedals (or handles on cars arranged for hand instead of foot control) at the operator's position. Thus the one limit relay serves to regulate the automatic acceleration and the automatic dynamic brake, and also to control a function known as "spotting" which is further described below.

The brake controller is actuated from the operator's brake pedal and regulates all three forms of braking, dynamic, track, and air. It consists essentially of three parts mounted on a common shaft. A set of cam-actuated master-controller-type fingers establishes the proper control circuits for the change from motoring to braking. A brush arm rotating around a small fixed commutator regulates the series resistance in the track brake circuit. The third element is a self-lapping type of brake valve. (Figure 3.) The first three inches of pedal travel varies the rate of dynamic braking from a very low minimum rate up to about three miles per hour per second. At the same time the air brake valve is set for a pressure corresponding to the dynamic rate, but this air is blocked off from the brake cylinders by a "lockout" magnet valve which is energized as long as dynamic braking is in effect. When the speed is reduced to the point where dynamic brake begins to fade out, the lockout magnet is de-energized and permits air at the pressure already established at the self-lapping valve to flow to the brake cylinders without conscious action or further pedal manipulation on the part of the operator. The dynamic brake does not fade out suddenly, nor does the pressure in the brake cylinders build up instantly. There is an appreciable and almost equal time interval for both of these functions. As a result, the blend from dynamic to air braking is normally exceedingly smooth and free from any sense of sharp change.

Figure 4. One-hundred-twenty-five-horsepower trolley-coach motor

Pressure of the brake pedal from 3 inches to 6 inches travel superimposes a graduated amount of track braking on top of the dynamic or air brake already obtained, so that rates higher than three to three and one-half miles per hour per second are obtained partially from dynamic and partially from track braking, with air coming in automatically as the dynamic fades out at low speeds.

Depression of the brake pedal to the "full" or emergency positions cuts out the lockout magnet and gives full application of dynamic, track, and air brakes, with automatic application of sand to the rails.

In the other form of equipment available, the variable resistor element consists of a special form of resistors with about 150 taps connected to the bars of a fixed commutator. A brush arm is rotated around this commutator by an air engine, and an ingenious arrangement of transfer contactors permits the use of this variable element twice during acceleration and four times during braking by switching fixed blocks of resistance in or out of the circuit in series with the variable element. In this equipment also a sensitive type of limit relay controls the rate of change of resistance both in motoring and in braking.

In earlier forms of dynamic braking some difficulty was experienced in getting prompt and uniform response at various speeds at the time of application of braking. During dynamic braking the motors are acting as series generators, with voltage varying over a wide range from maximum to minimum speed. In order to quickly establish a given rate of braking the value of resistance in circuit at the time of brake application should therefore be definitely related to speed. For lack of a better name, the means of accomplishing this has been called "spotting" and it is incorporated in both forms of equipment, although somewhat different methods of accomplishing it are used in the two schemes. In general, spotting establishes the correct value of resistance in the circuit, so that either braking or reapplication of power is promptly and smoothly established following a period of coasting.

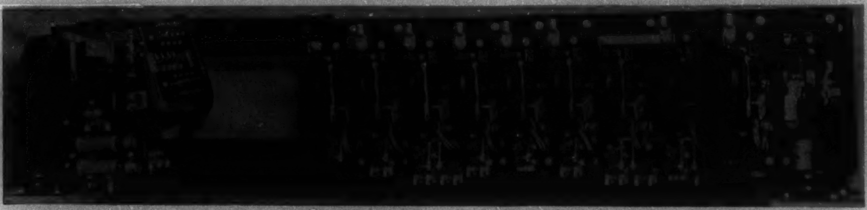

Figure 5. Trolley-coach control panel

Early experience with certain of the experimental cars indicated the desirability of more than one form of storage braking, that is, independent of power supply. It was felt that the high speeds of which the new cars were to be capable could not be used effectively unless the operator was fully confident of his abilty to obtain good braking even with bad rail and with an interrupted power supply. This led to the development of a battery excited track shoe. The presence of a battery on the car for this purpose naturally led to the use of the battery for control energy, so that in effect three forms of storage brake are available—in other words even with power off the line or the trolley off the wire, dynamic, track, and air brakes would all function.

The use of a battery required some means of charging. This led to the use of a motor generator set, which was then made to perform the additional functions of driving the air compressor and the 1,250-cubic-feet-minute blower used in conjunction with car heating and ventilation, as described in other papers on this program.

It is evident that the electrical equipment as well as other features of the PCC car incorporate many features radically different from previous practices in the streetcar field, and while troubles have been encountered, none of them may be said to be of a fundamental nature, and all of them have been or are being overcome. At least two of the largest operators have expressed themselves as having experienced less trouble with these cars than any lot of new cars they ever placed in service.

Trolley-Coach Equipment

The modern trolley coach may be said to have had its beginning with the Salt Lake installation in 1928. Its growth has been phenomenal—relatively slow at first but now increasingly rapid. There are now in service or on order 1,270 trolley coaches in the United States in addition to "all-service" type vehicles which operate either as trolley coaches or gas electrics.

The earliest motors used were 50 horsepower, but with increasing weights of coaches and increasing severity of service 65-horsepower motors weighing 785 pounds each became standard—two on 40 passenger coaches and one on 30 passenger.

Control equipments of the variable automatic type have become pretty well standardized for two motor equipments and a simpler nonautomatic type for the 30 passenger single motor coach since these are generally used for lighter service in smaller cities where high rates of acceleration are not so important.

Within the last two years there has been an increasing demand for lighter and simpler equipment for 40-passenger coaches (36 to 44). Development of a suitable single-bowl axle to handle the output of a single motor of sufficient size to give adequate performance on coaches weighing from 17,500 to 20,000 pounds has brought about the development of such motors. There are now in service or on order in the United States 447 trolley coaches using 125-horsepower single-motor equipments.

(Figure 4.) This motor rates 125 horsepower one hour rating and weighs 1,100 pounds, a weight of only 8.8 pounds per horsepower. As in the case of the PCC car motor, this result is partly due to the use of high-temperature insulation and improved designs of roller bearings, but principally to developments in design and manufacture of high-speed armatures and commutators. This motor is particularly noteworthy in this respect as it must necessarily be designed for 600 volts while the PCC motor is wound for 300 volts, two in series on 600.

Another feature of this equipment which makes the motor design even more remarkable is the unusually wide spread between full field and short field curves. There has been a growing conviction for some time in the minds of certain operators that the period of constant-current acceleration is too short, i. e., that acceleration should be carried to higher speed before striking the short field motor curve. The new single-motor 125-horsepower equipment is supplied in two forms, one with a relatively small amount of field shunting, in a single step, and one with three steps of field shunting with a final shunting of over 50 per cent. With the latter arrangement constant-current acceleration is carried up to a speed of 25 miles per hour as compared to 17 miles per hour with the single point shunting. This feature not only permits better schedules but is of great value in passing slower moving vehicles in the street. It also gives higher balancing speeds; while these higher balancing speeds may not often be reached on level runs, the entire speed curve is raised and on hilly routes higher speeds may be maintained on grades.

The single-motor equipment, because of the absence of series-parallel control, has a slightly higher power consumption than the two-motor equipment. In many applications it has been found that the increase in power cost is more than offset by the lower weight and cost, greater simplicity, and reduced maintenance and inspection cost of the equipment. In some cases, where unit cost of power is particularly high or for other reasons, the two-motor equipment may be preferred. Each case must be considered on its own merits and the final selection made by the purchaser on the basis of his own conditions.

(Figure 5.) The development of the single-motor equipment has brought about marked improvement in control equipments. These are now available in forms in which all the equipment except current collectors, motor, and resistor is mounted on a single panel, adapted for either side or rear pocket mounting, and completely wired at the factory. This greatly simplifies installation by the coach builder, and inspection and maintenance by the operator.

It has been found that because of the torque cushioning effect of the tires, a smaller number of notches than on cars gives equally smooth accelerations. The new equipments have 13 or 15 notches including one or three steps of field shunting. Automatic acceleration is universally used on 40 passenger coaches. Marked improvement has been made in simplification of the equipment by improved means of obtaining automatic progression.

In one form of control now available, the progression is entirely by means of electrical interlocking and a sensitive-type limit relay. This has been made possible by the use of "split" operating coils having two sections—a "pull in" winding and a "holding" winding. This scheme greatly reduces the number of interlocks required, and along with the development of a more sensitive current relay removes the obstacles which originally made it necessary to depart from progression interlocking on small capacity equipments—i. e., our inability to put enough interlock fingers on the relatively small contactors required for this size equipment, and the lack of a sufficiently quick response relay.

Another form of control recently developed for single-motor coaches obtains its automatic progression by means of winding up a spring which drives a camshaft through an electromechanical escapement mechanism between the spring and the camshaft.

Some of the features worked out in connection with the PCC car are finding their place in trolley-coach practice as well. In some cases, accelerating-resistor losses are being used for coach heating; in others, definite provision is being made to bring cleaner drier ventilating air to the motors.

The trolley coach is meeting with enthusiastic public response, and is proving itself a vehicle of very great value in that field between the heavy trunk-line route served by street cars and the lighter-service routes served by gas busses.

Gas-and Diesel-Electric Bus Equipment

Another interesting field for the application of electric propulsion equipment is the drive or transmission for gas and Diesel busses. It has been found that mechanical transmissions for large size busses in frequent-stop service are not altogether satisfactory. Frequent acceleration up to the limit of engine capacities imposes severe strains and heavy wear on transmission, clutch, and rear end, and the mechanical labor, time lost, and roughness incident to frequent to gear shifting are serious disadvantages from the standpoint of operator and passenger.

The development of the transit-type bus with rear compartment or under-floor-mounted engines has tended to make the mechanical transmission problems more difficult, and the introduction of Diesel-type engines with their less favorable speed flexibility has also tended to emphasize the advantages of electric drive.

Fundamentally and internal-combustion-type engine performs best as a constant-speed, constant-load prime mover. In urban transit vehicles the power and speed requirements are continually varying over a wide range. The problem presented to the electrical designer is to attempt to reconcile these conflicting characteristics of prime mover and vehicle.

At starting the demand is for high tractive efforts at low speed, which electrically means high current at low voltage; as the speed of the bus increases the current demand gradually falls off and the voltage rises, until at the balancing or free running condition, a low current at high voltage is required. It has been found that these requirements can best be met with a shunt generator with a moderately drooping voltage characteristic, so that the combined motor and generator speed-tractive effort curve approximates a rectangular hyperbola—thus giving very nearly constant horsepower and constant speed loading to the engine. When starting from standstill with the engine idling, a small amount of battery excitation is applied to the shunt field to assist the generator in quickly picking up its load as the engine accelerates. This is cut off by a voltage re-

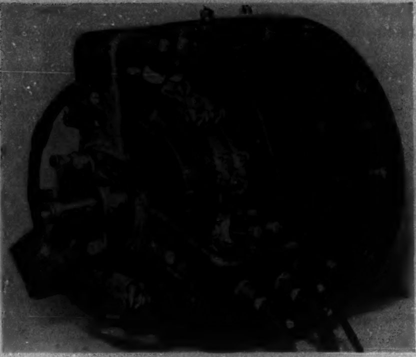

Figure 6. Gas-electric-bus generator

lay, as over excitation of the field for too long a period tends to hold down the engine speed. The prompt pickup charteristic assists in giving electric drive one of its major advantages over a gear shift. With electric drive the engine speed is quickly raised from idling to full speed where it is maintained—thus giving full engine speed and horsepower throughout the acceleration period. This is in marked contrast to a shifting gear mechanical transmission, where each full start consists of three or four successive engine accelerations from idling to full speed, interrupted twice or three times by periods of declutching, shifting, and reclutching so that the average engine speed and power during acceleration is materially below full value, with resultant loss in over-all rate of acceleration.

As bus sizes and weights increase and as service conditions become more severe as a result of increased traffic congestion and the application of busses to heavier-service routes, the tendency is toward larger, more powerful engines. The electrical designer is constantly being called on to provide equipment for larger engine capacities, but of course, is also under continually increasing pressure to produce smaller, lighter, and cheaper motors and generators. Everything that has been said earlier in this paper about the advances in design and manufacturing procedure applies to the machines for electric drive of busses with special emphasis, because electric drive is fundamentally heavier and more expensive than the mechanical drive which it replaces.

In this connection it should be pointed out that, contrary to beliefs apparently held in some quarters, electric drive can never put any more horsepower at the rear wheels than it gets out of the engine. Too often in discussing the performance to be expected from a given bus with electric drive, standard engine curves, taken under conditions vastly different from those which will exist with the equipment as installed in the bus are regarded as showing the engine output under operating conditions. It is important in predicting the performance of busses that account be taken of such items as auxiliaries—fans, battery charging generators, air compressors, etc., and that proper reduction in power delivered to the generator be made. The particular carburetor type and jets with which the equipment is to be used should also be used in making dynamometer tests.

Generators, motors, and control equipment are available for a range of 110 to 180 horsepower in gross engine output, for both single-reduction and double-reduction axles. Figures 6 and 7 show typical generators and motors for gas-or-oil-electric-drive applications.

A particularly interesting form of gas-electric bus is the "all-service" vehicle, developed and widely used on the Public Service Co-ordinated Transport in New Jersey. This vehicle is capable of operation either as a gas-electric or as a trolley coach. As usually built it has a single generator and two motors. The motors are connected in parallel for operation from the generator and in series for operation from the 600-volt trolleys. This vehicle has obvious advantages of extreme flexibility—it may operate over parts of its route where either local ordinances or low traffic density preclude the use of overhead wire, while retaining the advantages of trolley coach operation over heavily traveled sections of the route. It is also available for chartered or other service on unwired routes.

The only essential difference in its electrical equipment from a straight gas-electric is in the requirement of operation from 600-volt trolley, and the provision of current collectors and suitable changeover devices from engine to trolley operation.

We have attempted in this review to give a picture of the recent advances in the design and manufacture of electrical equipment for these public urban transit vehicles—not that these results are final—but rather as a survey of what has been done, and an indication of the direction in which further progress can be expected.

Figure 7. Gas-electric-bus motor

Cities exist and will probably continue to grow—public transportation is an absolutely essential service and must continue to expand and its methods must progress. We confidently expect that electric propulsion will continue to occupy a vitally important place in this field.

Application of Modern Electric Vehicles to Urban Transportation

By C. M. DAVIS
ASSOCIATE AIEE

THE URBAN transportation problem involves the movement of large numbers of city dwellers to and from work, school, or the shopping districts. These movements are frequently concentrated into a few hours in the morning and evening, and are handled largely through city streets, or, in the larger cities, on subway or elevated lines. The corporations handling this business, commonly called "mass transportation," operate under franchises from local authorities. These franchises usually specify minimum service over specified routes.

Transportation by private automobile or in public taxi cabs involves a large number of vehicles for a relatively small number of passengers transported. It would be impossible for our larger cities to function without the organized transportation systems of rapid transit, street railways, or busses. Not only would the cost of daily transportation be greatly increased for the great majority of riders, but the cities themselves would have to be rebuilt, if highways were to be provided of adequate capacity to handle, in small capacity vehicles, the present users of urban transportation systems.

What, then, is the ideal transportation system for our cities? It is one that will move the traveling public at the highest schedule speed consistent with safety; with a minimum of waiting time; with reasonable comfort; with the least interference with other traffic; and at the lowest cost, including fixed charges on the capital investment, consistent with the foregoing requirements.

Urban transportation is handled at the present time by four classes of service — rapid transit lines operating on private rights of way, over or under city streets; surface street railways; trolley coach lines; and gas busses. Each has a proper field which is not only the most logical, but the most economical from the community standpoint, though existing systems do not in every case justify this assumption.

In the small city, where the available traffic on any individual route is limited to 100 or 200 passengers per hour in the direction of maximum movements, which can be handled by small vehicles on 15- or 20-minute headways, the gas bus may be the logical type to use, since it involves a lower investment, and in some respects, lower operating costs than other forms of transit. Busses seating 20 passengers and carrying ten standees, on 15-minute headways, can carry away from a given area 120 passengers per hour, and the installation cost per mile of route will be very small compared to other types of service. With the cheaper types of gas busses of this size, a route giving 15-minute service, may be equipped including garage facilities, for $3,000 to $4,000 per mile. When pioneering into new territory in small or medium sized cities, this is probably the logical equipment to use.

However, routes in many cities are operated with such vehicles on long headways, where a more attractive service might substantially increase riding, and amply justify larger and more attractive units, more frequent headways, or both.

Larger gas busses, seating 30, on 15-minute headways, could handle 180 passengers per hour, and would require an investment of $8,000 to $9,000 per mile of route, including spares and garage. On the same headway, 40 passenger units can carry 240 passengers per hour, and will have an installation cost of approximately $12,000 per route mile.

But at about this point of traffic density, the trolley coach comes into the economic picture. These vehicles have a body and chassis similar to those of the highest grade gas busses, but instead of the engine, clutch, transmission, radiator, gas tank, and battery, they use electric motors and control and take their power from a pair of trolley wires, through poles sufficiently long to permit the vehicle to move 12 to 14 feet to either side of the wires. The simplicity and long life of this type of equipment effects lower maintenance and depreciation as compared with gas busses. Electric power is usually cheaper than gasoline. The overload capacity of its motors, an unlimited supply of power available, and the smoothness of its application make it possible to accelerate at rates far in excess of the most powerfully engined gas bus, and to make faster schedule speeds in frequent stop service.

However, the installation cost of a trackless trolley system is much greater than one operated by gas busses. Its use involves a pole line carrying four trolley wires that may cost $5,000 to $10,000 per route mile to erect, and an investment in substation facilities of from 30 to 40 kw for each vehicle operated, costing $1,500 to $2,000. On 15-minute headways, with 30 passenger vehicles, a trolley-coach system will have an installation cost per route mile about $6,000 higher than a gas-bus system, and with 40 passenger vehicles on five-minute headways, the difference in investment would be around $12,000 per route mile in favor of the gas bus.

But the economic life of the trolley coach should be appreciably longer than that of the gas bus, so its depreciation rate would be less. The line work may be assumed at five per cent or less annual depreciation, and the substation not more than four per cent. Even on this basis, the gas bus system will have an advantage in annual fixed charges for interest and depreciation of from $400 to $700 per route mile. This, however, amounts to but 0.6 cent to 0.8 cent per vehicle mile operated.

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On the other hand, operating records from companies using both types of vehicles indicate that, in the heavier classes of urban service, trolley coaches will save enough in equipment maintenance, power, and garage accounts as compared to gas busses in similar service, to much more than offset their higher fixed charges, and that they will attract more traffic by giving a smoother, quieter, more comfortable, and faster ride to users of the service.

Cost of service, therefore, including fixed charges, will usually be lower for a trolley-coach system if vehicles seating 30 passengers or more are to be used on headways of 15 minutes or less. This is true for new routes. It is even more apparent when an existing street-railway route is changed over to rubber-tired vehicles, for the substation and much of the distribution system is already paid for and installed, so the excess cost of the trolley-coach system over gas busses will be much less than was indicated in preceding calculations, and the relative costs of service are more certain to favor the electrically propelled coach.

As an alternative, where physical, political, or financial factors dictate the use of large-capacity motor busses instead of trolley coaches, the former may be equipped with electric drive, and obtain advantages in faster acceleration, and hence higher schedule speed, greater quietness and smoothness of operation, and a reduction in engine fumes. The electric drive consists of a generator directly connected to the engine, delivering power to a motor geared to the driving axle. Its use eliminates the clutch and gear transmission of the usual mechanical drive, and results in lower maintenance cost. It greatly reduces physical effort on the part of the driver, and provides a faster and more comfortable ride for the passengers.

Diesel engines with mechanical drive, in frequent stop bus service, have developed many mechanical difficulties that are minimized or eliminated when electric drive is installed. In fact, it appears that the successful employment of Diesel engines in passenger busses in urban service is conditioned upon the use of electric transmission. A number of such Diesel-electric busses have been put into operation in the past year, and they are showing such substantial reductions in fuel costs, as compared to gas busses, that it is safe to say many more installations will be made.

One large transportation company has introduced a distinctly new type of rubber tired vehicle using electric drive. This is the so-called "all-service vehicle," nearly 500 of which are in service, and which is primarily a trolley coach, but equipped with gas-electric drive for routes where no trolley wires are available, for use on detours, or for excursion work on Sundays or holidays when not all of the fleet are needed on trolley routes. When beneath the wires, the two motors used on these vehicles are operated in series; when power comes from the engine, the two are placed in parallel across a low voltage generator. They are normally operated as trolley coaches, but under certain conditions have greater flexibility than the straight electric coach.

The standard type of 40-passenger gas busses, so far built in this country, can carry in rush hours, with reasonable comfort, not over 70 passengers, and probably cannot operate at less than 40 second headways without interfering with one another. This means a street capacity of 90 units per hour, carrying 6,300 people in the direction of maximum traffic.

Trolley coaches, seating 40, where they are confined to city streets, are built to larger dimensions and wider aisles, and can carry loads 20 per cent to 25 per cent heavier, so on the same headways they might move 7,600-7,800 passengers per hour.

Under such conditions, the investment required would be about $245,000 per route mile for the gas busses, and $275,000 per mile for the trolley-coach system. From an economic standpoint, a street-railway system would be preferable, despite the expense of track construction, which, in paved streets, may cost $100,000 per mile.

A modern street car, seating 60, can readily handle 120 rush-hour passengers, so 53 cars per hour could carry away the 6,300 assumed for the busses, and 65 per hour equal the capacity of 90 trolley coaches. The rail system would have a lower investment in vehicles, shop facilities, line and substation than the trolley coach, but the cost of track would bring the total investment required up to $320,000, 30 per cent above that of the gas bus, and 15 per cent higher than the trolley-coach system. Due to differences in depreciation rates, the fixed charges on the three systems would be approximately equal. Operating costs on the rail system would be very much lower, due to the smaller number of vehicles, even though their cost per car mile would be appreciably higher. The total cost of service for the rail system, under these conditions, may be assumed as $22,000 less per route mile per year than the trolley coach system, and $35,000 less than the gas bus system, besides causing less street congestion for other vehicles using the street.

As the density of traffic decreases, the advantage of the large-capacity rail car becomes less apparent, and to handle rush-hour traffic requiring 2-minute service with trolley coaches, or 3-minute service with rail cars, the costs of service are equal. Again, it might be pointed out that where tracks are already installed and in good condition, the economic dividing line between the rail and trackless service would be in a much lighter traffic field.

If the street railway be used to its maximum capacity, operating on the 40-second headways assumed for the busses, it could handle 10,800 people per hour in the direction of maximum movement, or three parallel streets with rail lines could handle as much as five parallel-bus, or four trolley-coach routes, leaving 33 per cent to 67 per cent more street space available for other traffic. The relative cost of service would be even more favorable to the rail than in the example to which we previously referred, despite an installation cost for the street railway of nearly $500,000 per route mile.

The foregoing discussion is based on the use of the most modern type of large capacity street cars, equipped to operate at schedule speeds as high as even the trolley coach can perform. Such cars have been developed by the industry within the past three years and are operating in Brooklyn, Chicago, Washington, Baltimore, Pittsburgh, Los Angeles, and other cities. As compared to the slow and antiquated equipment that constitutes the bulk of the rolling stock in most cities today, any type of new vehicle, even gas busses, may be justified on economic grounds and earning power, but we believe that in many cases where gas busses have replaced rail service, it would have been better economics to use modern street cars or trolley coaches.

Now we come to the only system that can handle the volume of traffic of our largest cities, the rapid transit systems operating on private rights of way. These are expensive to build, and their carrying charges are enormous, but so is their carrying capacity. The latest system of this type, the four-track municipal subways built and equipped by the New York Board of Transportation, is arranged to handle 11-car trains, each having a capacity of 3,080 passengers, on 90-second headways, or 123,000 passengers per hour on the express tracks alone. Including the local tracks, the total one-way capacity is probably in excess of 180,000 per hour. The older Interborough lines, using smaller cars and shorter trains, now regularly carry 100,000 to 120,000 per hour at the peaks. Even the latter figure equals the capacity of 11 street railway routes, or of 19 streets filled with busses, and the municipal installation may, at no distant date, carry 50 per cent more. Its cost for construction and equipment will probably average $12,000,000 per route mile, but high as this appears at first glance, it is very low in terms of community benefits compared to many other expenditures intended to expedite the movement of passenger traffic.

Chicago spent $22,000,000 a mile to build a two-level river front marginal way, the well-known Wacker Drive, which has an effective capacity of 2,500 automobiles per hour in each direction, or perhaps 5,000 passengers. Fairmont Parkway in Philadelphia, Michigan Avenue in Chicago, and the Sixth Avenue Extension in New York, roads costing from $14,000,000 to $19,000,000 a mile, are other examples of city expenditures, of greater magnitude than subway costs, to provide faster movement for a relatively small number of people.

The New Jersey State Highway from the Holland Tunnel to Newark Airport cost nearly $6,000,000 per mile to build. Its capacity for auto riders is about 4,500 per hour in each direction, and if one lane in each direction were given over to busses, it would still carry less than 10,000 people per hour, so 12 to 18 such highways would have to be provided to equal the subway usefulness.

Many street widening projects have been undertaken in New York, Brooklyn, Detroit, Boston, and Pittsburgh to relieve vehicular congestion and to expedite traffic, that averaged $6,000,000 per mile, including condemnation of property, and even so, the movement of vehicles through them is less rapid than that of the subway trains.

No form of surface transportation in metropolitan areas can approximate the speed of rapid transit trains. No possible expenditures can be made that will permit the movement of an equal number of people in small, individual vehicles, autos, or busses as fast, or as economically as can the subway.

As compared to its nearest competitor in efficient handling of the heaviest traffic, the street railway using the most advanced equipment, the subway costs twice as much to build, as surface lines of an equal hourly capacity. If we neglect the cost of the street in which the rails are laid. On the face of things, it would appear that difference in first cost penalizes the users of the subway in high fixed charges. This is not a fact, however. Of the $12,000,000 referred to as the subway cost, only $3,000,000 or 25 per cent, went for cars, shops, tracks, power distribution system, signals, lights, etc., on which depreciation must be set up, and the other $9,000,000 went into real estate and construction, in other words, into nondepreciable items. Moreover, the depreciation rate on the track and on the cars is lower than for surface equipment, and calculations indicate that if both systems were financed by 50-year bonds carrying a sinking fund that would amortize the principal in that period, and adequate depreciation set up for both, the fixed charges on 11 surface lines would almost exactly equal that of the subway, which would be $850,000 per route mile per year.

The operating cost of the rapid-transit system per car mile is less than that of the surface line, and very much less per passenger handled. While it is impossible for either the privately operated or municipal subways to show an adequate return on the investment on a five-cent fare, and some of the surface rail and bus systems appear to be doing so, the conditions are not comparable.

The average rider on the surface travels not over two or three miles. The average subway ride is at least four times as long. If the surface lines had to carry their passengers the same distances as do the subway trains, instead of picking up a series of overlapping loads, their revenue per car mile or bus mile would decrease in inverse ratio to the longer ride and would fail to equal their out-of-pocket operating expense, whereas the subways operate for between 50 per cent and 60 per cent of their passenger revenue.

Aside from questions of operating economics, it would disrupt the business life of the community if those using public transport had to take two and one half times as long to get to work and return, which is about the ratio of schedule speeds on the surface to that of the subway.

Density of rush-hour traffic to and from the business areas is the final measuring stick that should govern the type of transportation service in every community. It is generally agreed that for our largest cities rapid-transit lines, subway or elevated, are an absolute necessity to the proper functioning of the community life. In addition, there is every economic justification for retaining and re-equipping a very considerable amount of surface rail transportation in these cities, and the rail service should be supplemented with trolley coaches and gas busses.

In the larger cities which are functioning without rapid transit facilities, it would appear that the modern rail car is still the most suitable vehicle for the heaviest traffic routes, but many existing rail lines should be changed to trolley coach operation. In many smaller communities, even up to perhaps 500,000 population, it is probably good engineering economics to gradually discard the rails and supplant them with trolley coaches and gas busses.

In all cities large enough to require mass transportation, it appears certain that electrically propelled vehicles, whether rail cars or trolley coaches, should be used for the great bulk of the service, having the ability to provide a faster, safer, more comfortable, and more economical form of passenger transportation than any other form now used in public service.

The PCC Street Car

By C. F. HIRSHFELD
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Synopsis: This car is the result of a collective effort to modernize and improve the street car. Its development was preceded by extensive investigation directed toward determining the characteristics of conventional cars and the ways in which and extents to which these should be modified to approach the ideals of street-railway executives and of the riding public, respectively. Car and equipment designs were then produced. These represent the best technical approach to these ideals which could be reached within the economic limitations which exist. The car has been made more agile, has been given better appearance and better performance in many respects which affect the passenger. In spite of the addition of equipment not used in the older cars and of many refinements of design it is obtainable at a price which appears to meet the economic requirements. Over 500 of these new vehicles are already in use in seven cities of this country.

THE VEHICLE which has become known as the PCC car is the product of several years of intensive research and development. The effort was initiated and sponsored by executives of leading street-railway companies. They realized the extent to which basic improvements in street cars had lagged behind the development of other vehicles available for both private and public transportation, and the undesirable effect this was having upon street-railway revenues. They believed that street cars must continue in use on certain routes and under certain conditions but they realized that the conventional street car fell far short of requirements.

These executives formed Electric Railway Presidents' Conference Committee as an instrumentality through which a modernized car might be developed. Abbreviation reduced this lengthy name to Presidents' Conference Committee and ultimately to PCC.

The assignment given the engineering staff of PCC was to determine whether the street car could be improved radically and, if so, how. One of its first endeavors was the determination of what were considered the faults in, and shortcomings of the conventional cars; and the characteristics which it was thought an ideally perfect car should possess. In this part of the investigation street-car operators and street-car users were consulted. In fact, interviews were also had with individuals who might have been passengers but were not.

One result of this investigation was the enumeration of a number of objectives of physical character. The principal ones were:

1. Great agility. This means ability to maintain high schedule speed in the face of frequent stops.
2. Modern appearance.
3. Low noise level, both inside and outside the car.
4. Good riding quality. This refers to the absence of high-frequency vibrations, the absence of repetitive low-frequency movements of objectionable sorts, and the absence of jerks or jolts.
5. Good ventilation.
6. Good illumination.
7. Minimum wear and tear of road bed.

Another result of the investigation was an enumeration of certain economic factors within the limitations of which the desired physical characteristics had to be obtained. These economic factors were:

1. Low first cost.
2. Low operating cost.
3. Low maintenance expense.

It is interesting to note that, as in the case of many of the older industries, little factual information of a technical character was available with respect to the performance of conventional equipment. The performance of representative cars in different respects had to be determined in order to establish what may be called points of departure. Frequently it was even necessary to develop instruments and test methods. Outstanding examples are furnished by the problems presented in attacking such important factors as riding quality and noise level. In a few cases it was found desirable to experiment on human beings to determine certain characteristics as, for example, their tolerances for different kinds of motion.

In the later design of new equipment the ideals determined by these early investigations served as the goals to be approached as nearly as technical ability and economic considerations permitted. An account of the methods adopted and, to some extent, the degree of success achieved forms the bulk of this paper.

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1. For all numbered references, see list at end of paper.

Great Agility

It is well known that the schedule speed attainable with street cars operating with frequent stops is determined almost entirely by the number of stops, the average length of stop, and the speed of starting and of stopping. The maximum attainable speed is of little consequence except in the case of long, uninterrupted runs.

The number of stops is a function of the passenger capacity of the car and the characteristics of the route on which it is operating. Once the size of car is determined, the designer has practically no control over the number of stops. The designer can however control the length of stop to a limited extent. For example, ease of entering and of leaving the car naturally tend to speed up passenger movement and thus to decrease the length of stop required to load or unload a given number of passengers.

This has been recognized in this case. Steps have been made to approach the proportions used in houses and office buildings to the extent that certain limiting conditions permitted. Effective illumination has been provided for the stepwells and the adjacent street area so as to facilitate mounting and demounting. Moreover, the illumination is so arranged as not to produce glare in the eyes of the moving passengers. The flow of passengers into and out of cars was studied and the PCC design provides as far as possible for natural flow, that is movement along the lines most naturally followed by the passenger. The seating arrangement is such as to provide the maximum practicable amount of open floor space in the front half of the car so as to permit ready movement toward the rear and ready access to the central exit doors. Stanchions and handrails are so arranged that there shall always be a handhold of some sort within easy reach of any passenger so as to encourage passenger movement while the car is in motion. Moreover, the handholds are so arranged that they afford a convenient hold for persons of all heights. Every effort was made to shorten the time of stops by these and similar methods. However, at best, the possible improvement of schedule speed by such methods is decidedly limited.

The greatest possibilities lie in increased speed of starting and stopping. Anyone who has watched the conventional street car during its slow and deliberate start will have observed the way in which the more agile automotive vehicles glide ahead of it, pre-empt its track and hold it down to a sedate but unacceptable rate of progress. Anyone who has watched the operation of the car in traffic will have observed how the operator is continuously throwing off power and applying his brakes in anticipation of a possible collision or the approach to an intersection at which he must stop. If the street car is to make reasonably high schedule speeds under modern conditions it must be capable of starting and stopping at rates comparing favorably with those of automotive traffic.

All this is well-nigh axiomatic. But a very vital question presents itself when one has arrived at this point. How rapidly can a standing human being be put into motion or brought to rest without excessive danger of accident? When this work was undertaken no one seemed to know the answer. It was decided to determine the tolerance of standing human beings for accelerating forces.

The results of this investigation have been published elsewhere.[1] Suffice it to record here that experiments were performed on standing passengers who were not holding onto any form of support. It was found they could tolerate the forces required to take them from a state of rest to an acceleration of 4.75 miles an hour per second in one second and to maintain the higher rate for two seconds or more. It was necessary only that the force be steadily applied; that is, there may be no marked and sudden variation in force during the entire process.

Tests on conventional cars showed that the forces applied to the passengers during acceleration fell far short of meeting this requirement, even when the automatic types of control were used. A variation of 50 to 100 per cent in very short time periods was not at all uncommon.

A performance specification was prepared which called for a constant increase of acceleration from zero to 4.75 miles an hour per second in one second, the maintenance of the higher value for two seconds and the maximum practicable approach to constant power input thereafter. After some experience this was modified to permit a lower rate of increase for a very short time at the start of the accelerating process. It was found that this arrangement was more acceptable to the passengers.

Equipment has been produced which approximates this performance specification very closely. It is now in use on over 500 cars in regular service in seven different cities and appears to produce results acceptable to the passengers. Certainly the car is much better able to take and hold its place in a stream of traffic.

With the starting process settled, there remained the method to be used for stopping. Experience with experimental cars showed there was no value in quick starting ability unless equal or greater stopping ability were provided. An operator would not avail himself of the ability to move fast unless certain that he could stop sufficiently quickly to avoid a collision with vehicles or pedestrians.

The maximum braking rate attainable with wheel to rail adhesion is well known and was not found adequate. Consequently magnetic track brakes were included in the braking equipment. The braking force produced by these devices is obviously independent of the coefficient of adhesion between wheels and rails and may be made additive to braking produced by the wheels.

The car is equipped with three different braking systems. These are: dynamic brakes dependent upon the use of the motors as generators; air-operated wheel-tread brakes; and magnetic track brakes. A service stop is normally made with the dynamic brakes supplemented by the tread brake as the dynamic brake "fades." At any time the operator can bring into use the track brake to increase the braking effect.

Normal or service braking is almost the reverse of the starting sequence. The deceleration increases at a constant rate to a desired value. After that it is maintained constant for the desired period and finally is decreased at a constant rate to zero value as the car comes to rest. The maximum deceleration attainable is between 8 and 9 miles an hour per second, depending upon rail condition and other variables. This has proved sufficiently high to yield safe operation and that feeling of security on the part of the operator which is necessary to the attainment of high schedule speed.

Acceleration and braking are produced by foot pedal operation as in an automobile. The operator need not know how the pedals operate the control equipment. He need only know that further depression of either one of them produces more of the effect which that one controls, and that too sudden application or release of pressure will produce uncomfortable jolts. The mechanism is capable of giving smooth performance if properly maintained. The necessary manipulation is simple and easily learned.

The balancing speed of the car is about 40 miles per hour. While of little significance in congested urban traffic, this speed is of value in the outlying districts where it can be used.

Modern Appearance

The airplane and the automobile have given the public certain ideas with respect to the appearance of an up-to-date vehicle. They have created what may be called a style. In the light of this style the conventional street car has a decidedly antiquated appearance and the public attributes antediluvian characteristics. It is obviously desirable that a modernized car should appear as such to the public which is expected to accept and patronize it.

The car has been given automotive appearance with respect to the major lines and all obvious fittings and decoration. Front and rear vestibule hoods are formed of metal as in the case of the automobile, and to similar lines. The letterboard has been corrugated to break its flat appearance and to produce an idea of motion. It runs by easy curves into the roof structure, just as the corresponding part does in the automobile. The windshield and the vestibule corner posts are enlarged versions of good automotive practice. The over-all height of the car has been materially decreased and a suggestion of "streamlining" has been introduced. The windows are arranged to produce group effects and they have rounded corners.

It is important to note that the matter of appearance was not treated separately. Innovations have been made with an eye to reduction in weight and to the production of a vehicle mechanically better adapted to its task. There should therefore be some lasting value in the changed appearance.

Low Noise Level

The greater part of the noise made by conventional street cars originates at the wheel-rail contact and at metal-to-metal contacts in the trucks. These noises are not only broadcasted at the points of origin but are also transmitted through the metal of the truck to the car body which in turn serves as a huge sounding board.

The general principles adopted for the suppression of noise may be described as maximum practicable suppression at the source and the minimum practicable transmission from that source. The steel tire was retained of necessity. But it is supported in rubber within the wheel in such a way that noise generated at the contact with the rail is absorbed in that rubber to the maximum practicable extent. The truck is so designed as to substitute rubber articulation for metal-to-metal contacts. Thus wear and accumulation of lost motion, with resultant noise, are prevented and rubber is used to break the otherwise continuous metal paths through which the sounds would be transmitted. The use of rubber in this way has been carried to the extent of eliminating steel car springs completely and using specially designed rubber springs instead.

These and other departures and refinements have both materially reduced the noise level and made the remaining noise of more acceptable quality.

Good Riding Quality

When this work was undertaken there was little information available with respect to the tolerance of human beings for vibrations of different magnitudes and frequencies, and oriented in different directions. An attempt was made to obtain sufficient information to serve as a guide in the design of a better car. The results have been published[2–4] and need not be detailed here.

Two types of vibration or disturbance came up for consideration. These may be called high frequency and low frequency although there is no sharp demarcation. The high-frequency disturbances are practically all in the audible range; that is above 20 to 25 per second. The low-frequency disturbances are of the order of 0.2 to 5 per second.

Space does not permit a lengthy explanation nor a detailed account of the methods used to bring about acceptable conditions. The high-frequency vibrations were almost completely eliminated by abandoning the use of the conventional leaf spring and substituting the rubber spring referred to above; and by designing certain parts of the structure stiffer than had been customary. The lower-frequency vibrations and disturbances of an objectionable sort were minimized by the design of the springing system and by a radically new method of suspending the axle in the truck.

The results obtained have been checked by adequate instruments throughout the entire development. There is therefore no question regarding the extent of the improvement. Recording accelerometers adapted to measure accelerations in the three principal directions show the floor of this car to vibrate very little, through very small amplitudes, and at very low frequencies. Such movements as do occur are characterized by low accelerations so that the record is a wavy instead of a jagged line. The conventional cars give very jagged records, showing numerous high and low frequencies of large amplitude superposed so as to produce "beats" which further aggravate the situation.

Closely associated with good riding quality is reasonable freedom from jolts caused by operation of motor control and of brakes. This has been referred to above. The equipment provided is such that a moderately careful operator can produce remarkably smooth operation without having to possess exceptional skill.

Good Ventilation

The ventilating and heating arrangements are combined into one system. Motor and brake control resistors are enclosed in a compartment under the center of the car where they are protected from the weather. A motor-driven fan blows over these resistors the quantity of air required to maintain them at a safely low operating temperature. During the heating season more or less of this air, as required, is blown into the car after being warmed by passing over the resistors. What is not needed to maintain the car temperature is wasted through thermostatically controlled dampers.

The air handled by the fan is drawn from the car interior through ducts which form part of the car structure. This air enters the car by different routes. Part of it enters by leakage around doors and at other places. Part of it enters through a roof ventilator so designed that the quantity passed increases with car speed. During the heating season part of it is blown in by the fan as above described and is in reality recirculated.

The total quantity of air passing through the car interior varies with conditions. The minimum aimed at is 1,200 cubic feet per minute. This is, in fact, a very generous supply when considered in terms of building ventilation.

Good Illumination

Illuminating engineers have consistently urged greater intensity of artificial illumination in the interest of less eye fatigue and greater health. Vehicle illumination has commonly been of the order of 2 to 5 foot candles on the passenger's reading plane. Office illumination has seldom been greater than 10 foot candles on the table or desk top although recent installations have shown a tendency toward at least doubling this figure.

Recognizing that the public of this country reads extensively while riding and that at best there will be a certain amount of uncontrollable movement of the printed surface with respect to the eye, it seems reasonable to conclude that a high degree of illumination should be provided in mass transportation vehicles. A value of 15 foot candles on the reading plane of the seated passenger was chosen in this case, with provision for raising it to 20 if later found desirable.

Two difficulties are encountered when attempting a high intensity of illumination in a street car. First, the ceiling is necessarily low and it is difficult to install thereon or therein sources of illumination capable of giving the desired standard of lighting without producing at the same time most objectionable and nerve racking glare. This difficulty has been at least partly surmounted by the development of new types of fixtures and methods. Completely indirect lighting is capable of producing a technically satisfactory result but it is undesirably expensive both as to first cost and as to operation. Lens types and louver types of lighting fixtures have thus far offered the most satisfactory solutions.

The second difficulty arises from the fact that the human eye adjusts itself rather slowly when one goes from an area of high illumination intensity to one of low intensity. Every passenger must ultimately alight from the car. If this involves stepping into a poorly illuminated street from a brightly illuminated car the passenger is temporarily almost incapable of seeing sufficiently clearly to guard against accident. The intensity chosen, combined with a reasonably high illumination of the street surface outside the exit door by means of car lighting, appears to be within safe limits.

Minimum Wear and Tear of Road Bed

The action of a rolling vehicle upon a road bed is not yet completely understood in a technical sense. It is known that the wear and tear increases with increasing weight of the unsprung parts and with increasing weight of the vehicle, all other things being equal. It is also known that it increases at a rapid rate if the assembly of masses and springs happens to be resonant to a frequency that can be imposed by the road bed, as for example rail joints or pavement joints. It is also known that violent movements of the vehicle, such as "nosing" and "galloping" tend to rapid destruction of the road bed.

In the present case all these facts were taken into consideration in the design of the vehicle. The completely unsprung weight consists of the wheel tread combined with part of the disk of the wheel whereas in the conventional car it consists of the entire axle assembly and a large fraction of the motor. The weight of the trucks, including the rather heavy magnetic track brakes, is somewhat less than that of conventional trucks without such brakes. The weight of the complete vehicle is 33,000 pounds against figures of the order of 38,000 to 45,000 for the later cars of conventional design. It is believed that refinement of design may bring the weight to a figure nearer 30,000 pounds. The present weight is achieved with steel construction. If the economics of the situation should ultimately show that the extensive use of lightweight alloys can be justified, the total weight could be reduced drastically; in fact a reduction of two to three tons is not inconceivable.

Resonance and violent movements have been minimized by careful attention to the dynamics of the assembly and by certain innovations in truck design, including the new suspension of the axle in the truck referred to above.

Economic Considerations

It has been stated that the technical improvements that have been reviewed briefly in the preceding paragraphs had to be achieved within certain economic limitations. Obviously a technically perfect car would be of no value to the industry if it did not also possess such characteristics as to make it economically usable. The following paragraphs are devoted to these economic factors.

Low First Cost

The street car is only one of several types of vehicle available for mass transportation. Each type has certain physical and economic characteristics which determine its proper place in a well-co-ordinated transportation system. The street railway is operated on and with structures which necessitate a considerable investment; fixed charges therefore are very significant in its economy. Its compensating advantage is that where traffic density is sufficiently high the unit cost of handling passengers is lower than in the case of any other form of surface transportation. Thus the economic field of the street car under given circumstances and in comparison with other public vehicles, such as busses, is circumscribed by the extent to which fixed charges can be limited. It follows that low first cost of vehicle is particularly important.

Much serious thought was given to the possible means of obtaining low first cost because it was known that a car which would give the desired performance would require more equipment and greater refinement of design than previously had been used in this field. That is to say, the problem was to develop a much better car saleable at a lower price. Experience in other industries pointed to standardization as one possible means. Here, however, further difficulty was injected by the very real physical differences and the limitations which exist upon the street railway properties in various cities. It is only necessary to mention by way of illustration the different track gauges in use and the limitations on external contour which result from unchangeable clearance conditions on the individual properties.

This phase of the problem was attacked by standardizing parts and sub-assemblies which, within limits, could be assembled to produce the different vehicles required by different cities while retaining most of the economics of standardized production. Thus, the body between corner posts is a standardized assembly usable without change on nearly all properties in this country and Canada. The front and rear vestibules are each standardized and are usable on all properties. A car made of the standardized body and these vestibules is usable on most properties. It is known as the "basic car." This has an overall length of 46 feet.

A property requiring a shorter car obtains it by dropping out one window panel. This shortens the car by 30 inches. A property requiring a longer car obtains it by putting a filler piece between one vestibule and the adjacent end of the car body. Such modifications of the basic car are already in use.

The standard arrangement is front entrance, center exit. It was known that other arrangements would be required on certain properties, including an additional rear exit door in one case. The structure is so designed as to make such modifications possible at minimum cost.

The truck presented a more difficult problem than did the car body. Track gauges in use vary from three feet six inches to five feet four and one-half inches. The first attempts to produce a truck design embodying the principles which had been found necessary, and still easily modifiable to fit all track gauges, were not completely successful. A design was evolved which could be used with very minor modifications on standard and wide gauges. Later a second and more flexible design was developed. This is now in use on one narrow gauge property. It can be modified very simply to adapt it to any track gauge.

As the solution now stands, the same major parts can be used in trucks of any gauge. Certain minor modifications are required. For example, the lengths of axles and of axle housings must vary with the gauge. However, the variations necessary in different parts are of such character as to entail a minimum of expense.

The method of standardization adopted has worked well in practice. Cars have been purchased at prices which make them economically usable in spite of the addition of many items and of the refined construction required to obtain the improved performance aimed at.

Low Operating Cost

The two principal factors in operating cost are platform labor and power. Probably the next most important factor is the cost of accidents; if that be included under this heading.

Provision has been made for reducing platform labor cost to a minimum through two factors, namely, providing for increased schedule speed and by designing the car for one-man operation. This does not mean that it was merely arranged for such operation; it was designed to make it possible for one man to perform all required functions with the maximum of convenience and the minimum of fatigue. Included in these provisions are foot operation of the car with respect to power and brakes; a comfortable, well placed and adequately adjustable operator's seat; a well ventilated and properly warmed operator's position; a conveniently located gang switch controlling all auxiliary functions, and numerous other details.

The power consumption per car mile is greater than that of the conventional car. The greater agility demanded brings this about as an unavoidable consequence. In addition, the requirement of maximum smoothness during acceleration further increased the power consumption through the elimination of the commonly used series-parallel type of control. The philosophy back of this step is that the performance of the car must be made acceptable to the public. If such performance requires increased power consumption the car must carry this burden. Should it prove economically unable to do so it no longer has any place on the street.

The increased power consumption is, however, offset more or less perfectly by the use of control resistor heat for car heating. In this way the annual peak demand is relatively reduced and, under the form of contract on which most street railways buy power, its cost is correspondingly reduced. It remains for experience to show to what extent this balances the increased power consumption.

It is still too early to draw conclusions with respect to accident hazards. The cars now in use are comparatively new instruments with characteristics quite different from those of their predecessors, but operated by men who were trained in the use of the older equipment. It is however certain that the use of the magnetic track brake has made stopping much more independent of the condition of the rail and has thus reduced one well-recognized hazard.

Low Maintenance Expense

It is almost axiomatic that "wear and tear" tends to increase with speed. However, it is not necessary that they do so, in an absolute sense. One may so design that maintenance is lower at high speed than with other designs at low speed. This has been attempted in this case.

The body of the car is a completely welded structure so that it acts as a unit in contrast to riveted structures which sooner or later act as a rather loosely knit assemblage of parts. Collision experience had thus far appears to prove that this body suffers less damage and is more easily repaired than was the case with the older type of structure.

Moreover, the body is made of an assemblage of standardized and interchangeable parts, just as is an automobile. Therefore, replacement parts may be both relatively cheap and relatively cheaply installed.

Most of the electric and other moving equipment is enclosed in ventilated compartments so that it is protected from the weather and given the best chance for uninterrupted operation. All this equipment is standardized and arranged for easy removal and replacement by similar parts. To a great extent the mounting dimensions of competitive makes of equipment are standardized. It follows that in many respects maintenance can be performed by replacement of a piece of equipment with an equivalent piece so that the primary investment can be returned to service immediately. The equipment which has been removed can be transported to a bench arranged for the easy and cheap performance of the necessary maintenance operations.

Propulsion motors are supplied with ventilating air which is taken from outside the sides or the car, carried through ducts within the car-body bolster structure, and delivered to the motors through bellows connections. In this way much of the dirt that ordinarily finds its way into street-car motors is excluded. The maintenance requirements should be correspondingly reduced.

The trucks of street cars are generally responsible for high maintenance costs if they are adequately maintained. The new trucks were designed to reduce maintenance requirements and to simplify the performance of such maintenance as is required. It has already been indicated that relatively moving metal-to-metal contacts have been avoided in so far as practicable. Where they are absolutely essential, as in journal bearings and gears, provision is made for flooded lubrication and the proportions are calculated for a life of 750,000 miles. In many cases necessary articulations are made by means of rubber so arranged that distortion of the rubber within very conservative limits permits the necessary relative movements of metal parts. The principle followed in the use of rubber is to have all parts with respect to which relative motion is required literally float in rubber.

The trucks are so designed that they consist of an assemblage of standardized and interchangeable parts, readily removable from the assembly as integral units. The favorable effect upon maintenance is obvious.

Finally, as already indicated, the trucks are mounted on resilient wheels. Experiment in the laboratory and on regular street-railway equipment in the street showed that this type of wheel reduced tremendously the destructive forces to which the parts of the truck are subjected. In spite of this fact, all truck parts have been designed for operation on conventional solid wheels, so that the resultant high factor of safety should be effective in minimizing maintenance expense.

Such figures as are available to the author indicate that in spite of all the radically new features in car body and trucks and in spite of all the unforeseen and unforeseeable errors that enter into the design, manufacture and use of new equipment of this sort, the maintenance expense on these cars is now of the same order of magnitude as that of the older cars in spite of their higher schedule speeds and in spite of the lack of experience in handling them on the part of both operating and maintenance personnel. This gives ground for the belief that when initial errors in design and fabrication are eliminated, and when the personnel involved becomes more familiar with this equipment, the maintenance expense will in fact be considerably lower than that characteristic of the older forms of rolling stock.

Such, in brief, is the history and status of development of the PCC car. It certainly represents both a major technical and economic advance over the conventional vehicle previously available. Even so, it must be considered only one more link in a practically endless chain of improvement. The heads of those who produced this product are already filled with visions of the further steps that might be taken. The present product is in reality a modernization and refinement of the older vehicle; radical in a few respects but still adhering fairly closely to the conventional. There is good reason for believing that much more radical changes are not only possible but would be economically valuable.

To the author, this development is interesting chiefly because it demonstrates the dollar value of the research method in an old and apparently completely developed line of endeavor. Now that the door to new designs has been opened, lengthy vistas, extending far into the future and apparently filled with promise, have come into view.

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A New A-C Network Analyzer

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Introduction

WHEN POWER systems were relatively small and simple, frequently comprising only one generating station with a radial distribution system, there was little need to study the effect of a proposed addition or change on an existing system. The growth and in recent years the interconnection of power systems have brought with them an increased quality of electric service and divers new devices for maintaining that service. In a power system of the size and complexity of many of those now in operation, many of the component parts inherently have the ability to profoundly affect the successful operation of the system. Consequently, it has become necessary for engineers to study their systems much more critically and to know a great deal more about them than heretofore. Fortunately, the technical background for such critical studies has kept pace with the increase in size and complexity of power systems. However, the calculations which constitute a large part of the work of making such a critical study, when carried out by long hand methods are generally laborious—frequently, sufficiently laborious to greatly curtail the scope of the study and at times to prevent the study being made altogether.

To simplify and shorten the work of making calculations of this sort, the a-c network analyzer has been developed. It is a tool which in the hands of a competent engineer becomes a very valuable aid in analyzing the performance of systems and apparatus. Moreover, a much more complete analysis of a power system and the interrelated performance of its component parts may be obtained from the analyzer data than can be gathered from actual system data and operating reports, which must of necessity be in some degree incomplete.

The analyzer affords a quick and effective means for studying major system disturbances in order to check and correlate available field observations, to determine what probably occurred, and to compare suggested remedies. It is useful also for observing in "slow motion" the sequence of events during abnormal system operation. Periodic checks with it of normal system operation and of operating instructions frequently show important changes brought about by the shift and growth of loads. Furthermore such checks often afford explanations for unusual observed operating phenomena. The comprehensiveness of the picture secured with the help of the analyzer of all the many factors which enter into the solution of a problem and which must be accurately weighed in fashioning its solution has created a feeling among those who have used the analyzer that all changes and additions to power systems involving major expenditures could profitably be critically studied with it.

From the historical point of view some of the first attempts to study power system problems were made with miniature three-phase systems using laboratory equipment,[1] some employing rotating machines as large as 600 kva at 23,000 volts[2] for the determination of stability and power limits. A general purpose three-phase miniature a-c transmission system for the practical solution of network and transmission problems employing 3.75 kva rotating generators was constructed and extensively used in the General Electric Company.[3,4] In all of these early arrangements rotating generators of appreciable power were used. Later, however, the method of symmetrical-component analysis of unbalanced three-phase systems suggested the use of single-phase artificial representations of such systems. The representation of generators by means of static phase shifting transformers and the use of transformer equivalent circuits was described by Messrs. H. H. Spencer and H. L. Hazen[6]. By applying these principles, the MIT network analyzer[9] was built jointly by the General Electric Company and the Massachusetts Institute of Technology. Another a-c calculating board of interesting design was also built and described by the Westinghouse Electric and Manufacturing Company.[10]

Design Fundamentals

Experience gained through the operation of the earlier analyzers showed that refinements in the general usability, degree of accuracy and reduction of size, and weight are desirable.

Usability may be considered from several standpoints. For example, in setting up the network on the analyzer a suitable number of circuit units should be available to adequately represent any power system. System quantities should be easily converted into analyzer quantities and vice versa. The adjustment of analyzer circuit units should be simple and connections of the units to form a network should be capable of being made with facility. The parts of the analyzer should be so arranged that visual checks of the connections and settings of the units can be readily made. It follows that all of these things apply equally well when adjusting an already formed network to a new set of conditions.

If the measuring system is to have maximum usability it should be such that data may be obtained from the analyzer as rapidly as it can be recorded, and that desired electrical quantities such as volts, amperes, watts, vars,

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[1] For all numbered references, see list at end of paper.

Figure 1. Network analyzer showing the general arrangement of cabinets

vector components of voltage and current as well as phase angles should be read directly. It must be easy to insert the measuring instruments in the network at the points desired and to read the instrument scales accurately and without eyestrain. The importance of obtaining correct data can hardly be overemphasized, as mistakes which arise either through faulty plugging and setting of units through improper reading of instruments or through fatigue, sometimes result in serious delays and unnecessary duplications of work.

A network analyzer should have a degree of accuracy at least equal to the accuracy of the constants of the systems which are to be reproduced in miniature. An even higher degree of accuracy is desirable in order that small differences in operating conditions and apparatus constants will produce the corresponding changes in associated network currents, voltages, and power flow. It is also desirable that a sufficient number of adjustable steps be provided on the circuit units that system constants may be set up with a good degree of accuracy. The instruments must have sufficiently low burdens that they will produce negligible changes in network currents and voltages. Moreover the departures of the analyzer units and instruments from their calibration should seldom be sufficiently large that correction of the measured results are necessary.

In view of the fact that the large number of circuit units generally occupy the greatest portion of the analyzer space, these units should be made small in order to reduce the analyzer size and weight. This can be achieved, as a preliminary design analysis showed, by using small network currents and voltages and by selecting an appropriate operating frequency. In this manner the power consumption is also minimized so that a small motor-generator set may be used. Furthermore, the over-all heat dissipation from the analyzer becomes small as is desired. In addition, standard high-quality low-cost switches, terminal plugs, etc., which, through years of use in radio and telephone fields have attained a high degree of durability and dependability, may be used.

The New A-C Network Analyzer

In the design of the analyzer described in this paper an effort was made to achieve all of the objectives outlined in the preceding section. In order to satisfy certain of these objectives many new developments were necessary, particularly the development of a new instrument system including amplifiers.

Before discussing in detail the essential design features and engineering problems it is perhaps well at this point to describe briefly the new analyzer recently placed in operation in the General Electric Company at Schenectady.

The general arrangement may be seen from figure 1 showing the center and one wing of the analyzer. It is assembled symmetrically with respect to the centrally located instrument and control cabinet. A circuit-connecting cabinet is situated on either side of this control cabinet and each wing is formed by four cabinets containing the adjustable circuit-element units. The distance from wing to wing is 26 feet and the depth is 11 feet. Power is supplied to the network through generator units placed in the upper part of the circuit-connecting cabinets and the instruments and control cabinet. This power is derived from a motor-generator set and phase balancer remotely located from, but controlled from the analyzer.

Each circuit unit terminates in a set of flexible cords and plugs on the connecting cabinets. By inserting these plugs in horizontally adjacent jacks of the vertical jack panel of these cabinets the circuit units may be connected together to form the desired network. The end jacks of each horizontal row of one jack panel are connected to the respective end jacks of the other jack panel, thus interconnecting them. The generator units are connected to rows of jacks located on the extreme ends of each jack panel.

Key switches are provided on the inclined panel of the instrument cabinet to connect any circuit unit or "jumper circuit" of the network to the instrument busses. Other switches for operating the instruments are provided on the vertical instrument panel so that the operator may

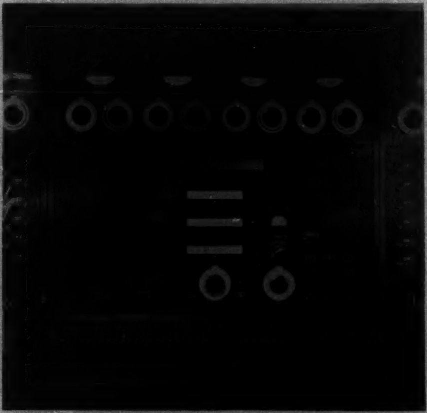

Figure 2. Central instrument and control panel showing generator units and instruments with their control switches

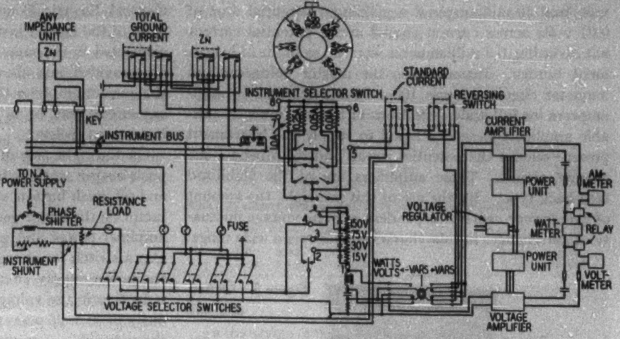
ANY IMPEDANCE UNIT
ZN
TOTAL GROUND CURRENT
ZN
INSTRUMENT SELECTOR SWITCH
8
7
6
5
10A
0.25A
0.05A
0.01A
STANDARD CURRENT
REVERSING SWITCH
KEY
INSTRUMENT BUS
CURRENT AMPLIFIER
POWER UNIT
AM-METER
TO N.A. POWER SUPPLY
PHASE SHIFTER
RESISTANCE LOAD
FUSE
VOLTAGE REGULATOR
WATT-METER
RELAY
INSTRUMENT SHUNT
4
3
2
1
150V
75V
30V
15V
WATTS VOLTS
-VARS
+VARS
POWER UNIT
VOLT-METER
VOLTAGE SELECTOR SWITCHES
VOLTAGE AMPLIFIER

Figure 4. Schematic circuit diagram of the instrument system connections

measure the magnitudes or vector components of voltage, current and power, and phase angles in any branch of a network without moving from in front of the instrument cabinet, a close view of which is shown in figure 2.

The master instruments consist of an ammeter, voltmeter, and wattmeter-varmeter. They have light-beam pointers in order to secure a short response time. The light beam pointers used in conjunction with eight-inch scales reduce eyestrain to a marked degree. In order that these instruments shall have very low burdens and so make an inappreciable change in the network currents, current and voltage amplifiers are used.

The dials and scales of all of the analyzer circuit units and instruments are marked in per cent. With this marking the multipliers for changing system quantities into analyzer quantities and vice versa are simpler than they would be if the analyzer scales were marked in actual volts, amperes, and ohms. Furthermore, as a great many system calculations are made using percentage quantities, data for these systems may be frequently transferred directly to the analyzer. The nominal or 100 per cent voltage is 50 volts and the nominal or 100 per cent current is 50 milliamperes. Consequently 100 per cent ohms on the analyzer corresponds to 1,000 ohms.

Figure 3. Rear view of two circuit-unit cabinets showing the wiring details

The wiring between sections of the analyzer is carried in the base of the cabinets as may be seen from figure 3, which shows a back view of the circuit-unit cabinets. The circuit units are mounted in individual compartments and may readily be removed for inspection.

All cabinets are of the unit type and might have been arranged in any other manner than that shown in figure 1 if the configuration of the room had required it.

The Instrument System

One of the most important parts of the network analyzer is the master-instrument group and its controls. The master instruments, the voltmeter, the ammeter and wattmeter-varmeter, are of the "light beam" type with eight-inch scales having 150, 100, and 150 divisions respectively. The "light beam" pointer is a luminous figure on the opalescent instrument scale consisting of a thin vertical line. These instruments were constructed from modified standard instrument elements by replacing the pointers with reflecting mirrors in the usual manner. Owing to the light-beam pointers and suitable moving system springs, the time of response of these instruments is a fraction of a second. The voltmeter and ammeter are of the moving-iron-vane type and the wattmeter-varmeter is a d-c milliammeter of the permanent-magnet moving-coil type whose d-c operating current is supplied by the wattmeter proper which is a sensitive torque balancing telemeter. In this telemeter-wattmeter a null method of measurement similar to that of the photo-electric recorder was used.[16] The a-c potential coils and the current coils of the instruments are connected in series so that all master instruments indicate simultaneously.

For the nominal analyzer voltage and current of 50 volts and 50 milliamperes, a maximum potential drop of 0.01 volts across current shunts and a potential current not exceeding 0.3 milliamperes were set as limits of instrument burden. Inasmuch as the master voltmeter and ammeter circuits require 100 milliamperes and 50 milliamperes for full scale deflections of the instruments, suitable amplifiers were developed to supply the instrument power—one for the potential circuit and the other for the current circuit. These amplifiers are of the stabilized feed-back type. By the use of this principle, the amount of gain is very nearly independent of plate voltage fluctuations and changes in tube characteristics over a wide range.

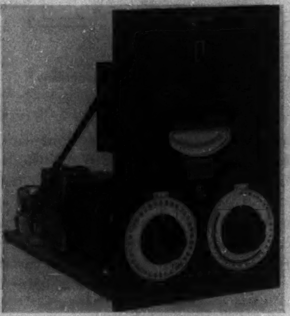

Figure 5. Generator unit

The accuracy of the amplifiers is further improved by a simple transformer-type checking circuit which automatically compares the amplifier input and output and introduces into the amplifier a voltage corresponding to any differential from the correct ratio of these two quantities, so that the amplification ratio is accurately maintained. The amplifiers have an accuracy of better than plus or minus 20 minutes in phase angle and plus or minus one-half per cent in magnitude. They are fully described in a paper by H. A. Thompson.[15]

A complete schematic circuit diagram showing the connections of the instruments, amplifiers, and the instrument switches is shown in figure 4. Referring to figure 2, the instrument-scale-selector switch, having seven positions corresponding to the most common voltage and current scales used, is installed just below the master instruments. By means of an auxiliary switch located on the right of the scale selector switch two additional low-voltage ranges are provided. On the left of the scale selector switch there are installed the watt-varmeter switch and the current-reversing switch.

By means of voltage switches, which are in the bottom row to the left of the master instruments, the voltages across any circuit unit or from either side of a circuit unit to ground, or from either side of a circuit unit to another point in the network, may be connected to the voltage amplifier. In addition, for measuring vector components of current, a reference voltage obtained from the instrument Selsyn (Selsyn is a registered trademark of the General Electric Company), which is below and to the right of the master instruments on the panel, may also be connected to the same amplifier. Three current switches are provided just above the voltage switches to connect the current amplifier through shunts to the ground bus or any circuit unit, or to the reference current circuit of the instrument Selsyn.

On the right side of the panel there are the start-stop push-button stations for the motor-generator set, the on-and-off push-button stations for the power-supply contactor on the motor-generator set control panel and for the amplifier-power and instrument-light switch. There is also a switch for changing taps on the autotransformer in the power supply circuit in order to obtain a convenient reduction in the voltages of all generator units simultaneously to 50 or 25 per cent voltage when faults are applied to the network. A miniature type voltmeter is supplied to indicate the analyzer supply voltage. A rheostat is provided to adjust the voltage held by the voltage regulator associated with the motor-generator set. To the left of the master instruments a second rheostat is provided to adjust the voltage and current of the instrument Selsyn.

The potential shunt or voltage divider and the current shunts for the master instruments shown in figure 4 are mounted in the scale-selector-switch housing. Because of the high impedance of the voltage divider, the capacity coupling to ground and to adjacent electrical circuits of the divider elements introduces currents which cause phase-angle shifts. It is necessary to have very close adjustment of phase angles to measure power at low power factors with the degree of accuracy sometimes required in stability studies when large machines are involved. In order to control these stray-capacity effects, adjustable trimmer capacitors have been connected across the resistance parts of the divider to equalize the phase-angle shifts on all taps. The effects of the capacity to ground have been made inappreciable by proper shielding of the divider, shunt leads and switches. Inasmuch as the voltage divider now has a permanent and constant leading phase-angle shift the current shunts have been wound inductively and made adjustable in inductance in order that the voltage drop of each shunt will have the proper phase position with respect to the input voltage of the potential amplifier for correct measurement of power at power factors approaching zero.

In the diagram of figure 4 only a typical circuit unit and corresponding key switch is shown; there are 300 key switches in the analyzer, one for each circuit.

The over-all accuracy of the master voltmeter and ammeter systems is within plus or minus 0.5 per cent of full

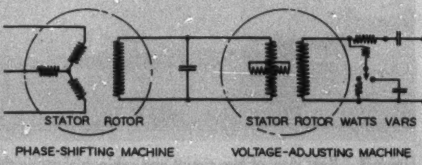

Figure 6. Schematic circuit diagram of generator unit

scale. The wattmeter-varmeter has the same accuracy when the phase angle between the current and voltage is not large. For very low power-factor measurements the most frequently used instrument connection has been carefully adjusted as described above to give accurate measurements of watts and vars at phase angles between the voltage and current approaching 90 degrees. For all other instrument connections there is a maximum instrument system phase angle displacement of the order of one degree.

The Analyzer Generator Units

Referring to figure 1, 12 adjustable single-phase power-source or generator units are installed in suitable compartments located at the top of the two plugging cabinets and the central instrument cabinet. These units are completely self-contained and may be inserted or removed from their compartments from the front. A photograph of one of these units is shown in figure 5.

Each generator unit consists of two power Selsyns, one having a three-phase stator and single-phase rotor for phase-angle control, the other having a two-phase stator and a single-phase rotor for voltage control. A schematic connection diagram is shown in figure 6. The three-phase stator winding of the phase shifting Selsyn is excited at 220 volts and 480 cycles through an autotransformer from the main power supply, the motor-generator set. The phase angle of the induced voltage in the single-phase rotor winding depends on the rotor angle. The rotor-output voltage is applied to one of the quarter-phase stator windings of the voltage-adjusting Selsyn. By turning the single-phase rotor of this machine the voltage may be varied from zero to a maximum at an essentially constant phase angle. As shown in the circuit diagram of figure 6, capacitors were used to compensate for magnetizing current and the equivalent single-phase reactance of the unit.

The full-load (100 per cent current) voltage regulation of the unit is approximately 2 per cent at unity power factor and essentially zero at zero power factor. The phase-angle shift for a corresponding change of current at zero power factor is approximately one degree at zero power factor and approximately zero at unity power factor. Because of this close regulation, a calibrated voltage dial was supplied with the voltage regulator instead of a voltmeter. By means of a gear transmission, the 90-degree mechanical rotor angle, from zero to full voltage, is magnified to an angle of approximately 300 degrees on the voltage dial, making the setting of the voltage more convenient.

The only direct-reading instrument mounted on the panel of this unit is a wattmeter-varmeter for measuring or adjusting the active and reactive voltampere output. This instrument has 150 per cent and 300 per cent voltage ranges and 100 per cent and 400 per cent current ranges. Switches are provided for changing scale multipliers and for short-circuiting the current coil when the generator-unit current exceeds 400 per cent. A switch is also supplied to disconnect the generator unit from the power supply.

Table I. Voltage-Wave-Shape Analysis of Analyzer Power Sources

Harmonic	Frequency (Cycles)	Voltage Components					
		Volts	Per Cent	Volts	Per Cent	Volts	Per Cent
At no load							
1	480	50.00	100.00	100.00	100.00	125.00	100.00
3	1440	0.13	0.26	0.5	0.5	0.66	0.525
5	2400	0.65	1.3	1.26	1.26	1.53	1.22
7	3360	0.028	0.056	0.057	0.057	0.068	0.054
With 0.25 ampere (500 per cent) at 70 per cent power factor							
1	480	50.00	100.00			125.00	100.00
3	1440	0.11	0.22			0.73	0.58
5	2400	0.62	1.24			1.68	1.34
7	3360	0.0275	0.055			0.066	0.053

In the design of the power supply motor-generator set and also of the analyzer generator units themselves, care was exercised to keep harmonics in the voltage to a minimum. With the complete analyzer equipment installed a voltage-wave-shape analysis was made at the output terminals of the generator units. The results are shown in table I.

The fifth-harmonic voltage is the largest but it does not exceed 1.4 per cent. Thus the analyzer generator-unit output voltage is very nearly sinusoidal.

The Circuit Units

For the representation of power systems by means of artificial miniature equivalent circuits there are generally used, in addition to generator units, adjustable resistors, reactors, capacitors, autotransformers, and mutual reactors. These may be combined into groups to form network branches. In the new analyzer there are five different types of circuit units as follows:

(*a*). Line-impedance units. These consist of adjustable resistors and reactors connected in series and are intended for representing lines or other circuits of relatively small impedance.

(*b*). Load-impedance units. These units also consist of adjustable resistors and reactors which may be connected either in series or in parallel. They differ from the line-impedance units in that their impedances are much higher.

(*c*). Capacitor units. The adjustable capacitors of these units are used to represent line capacity, and frequently to represent synchronous condensers as well as negative reactance such as occur in equivalent transformer circuits.

(*d*). Autotransformer units. The autotransformers are used to step up or down the voltage at points in the network.

(*e*). Mutual-transformer units. By means of these units magnetic coupling between circuits may be simulated. They are 1:1 ratio transformers. The desired degree of mutual coupling is obtained by connecting across the secondary a suitable impedance.

All units, with the exception of the last one which is not adjustable in itself, are mounted on individual 7- by 16-inch metal bases with calibrated front panels attached. These mounted units fit interchangeably into the compartments of the circuit-unit cabinets. The mutual-transformer units are mounted on racks back of the connection cabinet jack panels. Figure 7 shows a photograph of a line-impedance unit and a load-impedance unit while figure 8 shows a capacitor-unit and an autotransformer

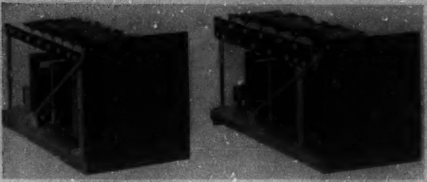

Figure 7. Line and load-impedance units

unit. An individual mutual-reactor unit is shown in figure 9.

In determining the proper number and the kind of units for the analyzer a great many systems were inspected and their constants tabulated. Studies which have been made on other analyzers were thoroughly reviewed. The constants of synchronous machines, transformers, and the like, were examined in order that the analyzer units would cover adequately the necessary ranges of impedance and fineness of steps. The following table shows the number and range of circuit units in the new network analyzer.

Table II. Number and Range of Circuit Units

Units	Number	Per Cent Range	Steps
Line-impedance units (series $R + jX$)	150	R 0–51 per cent	Continuous
		X 0–81 per cent	0.2 per cent
Load-impedance units	50	R 0–1,610 per cent	Continuous
(series or parallel $R + jX$)		X 0–1,605 per cent	5 per cent
Capacitor units (susceptance)	30	0–100 per cent	1 per cent
Autotransformer units	12	± 15 per cent.*	1 per cent
Mutual-reactor units	15		

* Values expressed in per cent of primary voltage.

The resistance and reactance part of each line-impedance unit and load-impedance unit is individually rated at a maximum of 62.5 volts (125 per cent nominal voltage) or 250 milliamperes (500 per cent nominal current) whichever is the limiting factor. Each capacitor of the capacitor units is rated at a maximum of 62.5 volts (125 per cent nominal voltage). The tapped resistances and capacitors have an accuracy of plus or minus one per cent on all taps. Small variable resistors have an accuracy of plus or minus three per cent. All reactors have an accuracy of plus or minus one per cent at a specified current and a change of reactance less than plus or minus two per cent over the full current range at any setting.

An ideal reactor should have a very small R/X ratio. However, there are obviously practical limits of R/X ratio below which a reactor tends to become unduly expensive and large. Thus the values of R/X given in table III of the circuit-unit reactors represent a compromise between reasonable cost and an acceptable R/X ratio.

In the design of the reactors, autotransformers, and mutual-reactance units, low losses and high permeability at the low flux densitites were desirable; consequently, nickel-iron-alloy magnetic cores were generally used.

The autotransformers are rated at 62.5 volts (125 per cent) and 250 milliamperes (500 per cent). The taps are accurate to plus or minus one per cent of the dial setting. The series reactance of these transformers is compensated with suitable series capacitors.

The mutual-transformer units are rated 62.5 volts (125 per cent) and 250 milliamperes (500 per cent). The two windings of this unit, a 1:1 ratio transformer, are wound together so that minimum leakage reactance is obtained. The relatively large capacity coupling resulting from the close spacing of the wires is compensated by means of a small reactor connected from one winding to the other.

The Motor-Generator Set

The motor-generator supplies three-phase, 480-cycle power at 440 volts through an autotransformer to the 12 analyzer generator units. Because of the low analyzer circuit voltage and currents used, the maximum 480-cycle input to the entire analyzer is less than 5 kva. The single-phase component is less than one kva.

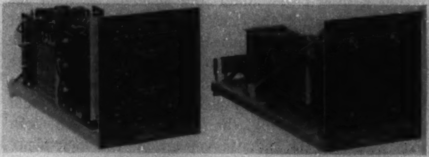

Figure 8. Capacitor and autotransformer units

In order to maintain substantially balanced voltages at the terminals of the three-phase generator with a single-phase load component, a phase balancer is connected in parallel with it. The phase balancer is directly coupled to the generator and rotates at synchronous speed. In addition to its voltage balancing action, the phase balancer absorbs a good deal of the small residual harmonic content of the voltage of the sine-wave generator. The phase balancer magnetizing current is compensated by means of suitable capacitors.

The complete motor-generator set assembly consists of

Table III. Typical R/X Ratios of Circuit-Unit Reactors

Circuit Unit	Reactance Setting (Per Cent)	R/X Ratio (Per Cent)
Line-impedance unit	0.2	8
	1.0	5.5
	10.0	3.5
	70.0	3.0
Load-impedance unit	5	5.0
	10	4.5
	100	3.5
	1,500	3.0

four machines, a 60-cycle driving motor, a 480-cycle alternator, a phase balancer, and an exciter, all directly coupled and mounted on a 20-inch-by-7-foot base.

In view of the small amount of 440-volt, 480-cycle power to be transmitted, the motor-generator set may readily be remotely installed from the analyzer. In the General Electric installation the distance is of the order of 200 feet and considerably larger distances are quite feasible.

Conclusions

The operation of the analyzer since it was completed has shown that the design has achieved the high degree of facility and speed of operation which was expected with a resultant saving roughly the equivalent of one operator. When the occasion arises the analyzer may be operated conveniently by a single operator. The over-all accuracy and accuracy of the component parts has been within the limits for which the equipment was designed.

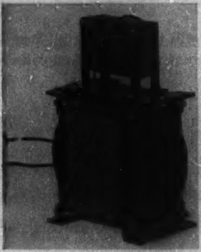

Figure 9. Mutual-reactance unit with compensating reactor

The following distinctive design features have contributed to the efficient analyzer performance:

1. The instruments and all selector-key switches are centrally arranged, so that the operator may take conveniently the desired readings of network quantities from one position in front of the instrument cabinet without assistance.
2. The concentration of the circuit unit connecting equipment in two cabinets facilitates the work of connecting the circuit units to form the network to be studied. In addition, the amount of transfer plugging, frequently a source of error, is materially reduced. With this arrangement the network busses may be formed on the plugging panels in the same relative positions which they have on the network diagram, affording easy checks of all connections.
3. The accurate light-beam-type instruments have a very short time of response and serve to measure voltage, current, watts, (vars) reactive voltamperes, and vector components of voltage and current with a minimum of eyestrain. In the measurement of these vector components and for reading phase angles an instrument Selsyn is used.
4. The instrument burdens have been reduced to very low values by the use of precision amplifiers of the stabilized type. Thus the introduction of the instruments in the network has a negligible effect on the existing distribution of voltage and current, resulting in a degree of accuracy of the measurements which precludes the need for correction of the measured results.
5. The close electrical tolerances to which all the component parts of the analyzer have been held, have resulted in the practical elimination of the use of correction tables for adjusting the network impedances to the values desired for the representation of the power network.

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The Properties of Three-Phase Systems Deduced With the Aid of Matrices

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ANY MATHEMATICAL process, even though fully developed in theoretical form, takes on more meaning, and thereby becomes more useful, if it is applied to familiar concepts. If in this application certain conclusions can be reached with ease which ordinarily are reached with difficulty, the utility of the mathematical process is still more conclusively demonstrated.

Three-phase circuits are very generally used. The simpler properties of such circuits have been considered, and have been taught in colleges for many years. There are, however, certain important properties of these systems, deduced in the following, which are not as widely known as they should be. The reason for this fact is simply that ordinarily the equations required to deduce these results get so cumbersome as the work progresses that the details hide the general properties of the system. Matrices are extremely effective in clearing away the complication of detail thus leaving the expressions for the general properties of the system more evident.

A further advantage of matrix nomenclature is that even to a beginner in the study of polyphase systems, after having mastered the matrix treatment of three-phase systems, the method of extending the details of the matrices to any other polyphase system will be evident.

The following treatment of three-phase systems shows the power of matrices, not only in simplifying the statement of the properties of circuits in general, but in determining the detailed properties of particular circuits. The method of manipulating matrices and of reaching certain conclusions from them is shown.

Preliminary

A nomenclature, convenient for all occasions when complex numbers, real numbers, vectors, and matrices are all used in the same discussion, is not easily established. The writer has found through his own investigations and through presenting the material to students that the following symbolism is simple and convenient: a matrix will be represented by the script symbols

$\mathfrak{U}, \mathfrak{V}, \mathfrak{Z}, \mathfrak{I}, \mathfrak{E}$, etc.,

a complex number or vector in the usual complex notation by

V, I, y, z, etc.,

and a scalar or real number by the absolute value as

$|V|, |I|, |y|, |z|$, etc.

This symbolism will be used in what follows.

The properties of matrices as given in the paper by Pipes[1] will be assumed, in as far as they are needed, in what follows. In addition, and in keeping with the nomenclature to be used in this discussion, the unit matrix will be designated as

$$\mathfrak{U}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

Other unit matrices which will be used below, formed by shifting the last column successively to the first column position, are

$$\mathfrak{U}_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \text{ and } \mathfrak{U}_3 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{2}$$

The following matrices will also be useful:

$$\mathfrak{U}_{13} = \mathfrak{U}_1 - \mathfrak{U}_2 \text{ and } \mathfrak{U}_{12} = \mathfrak{U}_1 - \mathfrak{U}_3 \tag{3}$$

Note that the matrices given by equation 3 are singular.

Three-Phase Delta Connection

(*a*) *General:* In matrix notation the voltage equations in terms of the cyclic currents for figure 1—assuming no mutual inductance or capacitance—are

$$\mathfrak{E} = \mathfrak{Z}\mathfrak{I} \tag{4}$$

where

$$\mathfrak{E} = (E_{12}, E_{23}, E_{31}, 0) \tag{5}$$

$$\mathfrak{I} = (I_{12}, I_{23}, I_{31}, I_4) \tag{6}$$

and

$$\mathfrak{Z} = \begin{pmatrix} \begin{matrix} z_{12}+z_{2B}+ \\ z_{BA}+z_{A1} \end{matrix} & -z_{2B} & -z_{A1} & -z_{AB} \\ -z_{2B} & \begin{matrix} z_{23}+z_{3C}+ \\ z_{CB}+z_{B2} \end{matrix} & -z_{3C} & -z_{BC} \\ -z_{A1} & -z_{3C} & \begin{matrix} z_{31}+z_{1A}+ \\ z_{AC}+z_{3C} \end{matrix} & -z_{AC} \\ -z_{AB} & -z_{BC} & -z_{AC} & \begin{matrix} z_{AB}+ \\ z_{BC}+z_{CA} \end{matrix} \end{pmatrix} \tag{7}$$

The current and voltage matrices are to be considered as one columned, but are written as in equations 5 and 6 as a matter of convenience.

Further, since the impedance matrix is not singular,

$$\mathfrak{I} = \mathfrak{Y}\mathfrak{E}$$

where the admittance matrix is the inverse of the impedance matrix of 7.

The phase currents at the generator are

$$\mathfrak{I}_{pg} = (I_{12}, I_{23}, I_{31}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \mathfrak{I} = \mathfrak{T}_{pg}{}^{\Delta}\,\mathfrak{I} \tag{8}$$

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[1] For numbered reference, see end of paper.

and the phase currents at the load are

$$\mathfrak{I}_{pl} = (I_{BA}, I_{CB}, I_{AC}) = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \mathfrak{I} = \mathfrak{T}_{pl}{}^{\Delta}\, \mathfrak{I} \qquad (9)$$

The line currents are

$$\mathfrak{I}_L = (I_{A1}, I_{B2}, I_{C3}) = \mathfrak{A}_{13}\mathfrak{I}_{pg} = \mathfrak{A}_{13}\mathfrak{I}_{pl} = \mathfrak{A}_{13}\mathfrak{T}_{pg}{}^{\Delta}\mathfrak{I} = \mathfrak{A}_{13}\mathfrak{T}_{pl}{}^{\Delta}\mathfrak{I} \qquad (10)$$

Note, as is evident from figure 1 and equations 8 and 9, that $\mathfrak{I}_{pg} \neq \mathfrak{I}_{pl}$ even though equation 10 at first glance makes them appear so. The fact that the transformation matrix $\mathfrak{A}_{13}$ is singular makes this relation possible. Note further that $\mathfrak{I}_{pg}$ or $\mathfrak{I}_{pl}$ *cannot* be determined from $\mathfrak{I}_L$ since $\mathfrak{A}_{13}$ is singular, thus pointing out a fact which is not immediately evident from the equations themselves.

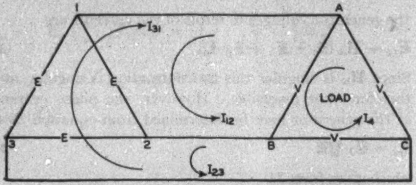

Figure 1. Three-phase delta connection

The line and phase voltages at the generator are

$$\mathfrak{E}_{Lg} = \mathfrak{E}_{pg} = \mathfrak{T}_{pg}{}^{\Delta}\, \mathfrak{E} = (E_{12}, E_{23}, E_{31}) \qquad (11)$$

and the terminal voltages of the generator, in terms of the voltage drops of line and load, are

$$\mathfrak{V}_{pg} = \mathfrak{E}_{pg} - \mathfrak{V}_{ig} = \mathfrak{V}_{pl} + \mathfrak{A}_{12}\mathfrak{V}_L \qquad (12)$$

where the load and line drops at the generator terminals are

$$\mathfrak{V}_{pg} = (V_{31}, V_{32}, V_{12}) \qquad (13)$$

the internal generator drops are

$$\mathfrak{V}_{ig} = \begin{pmatrix} z_{12} & 0 & 0 \\ 0 & z_{23} & 0 \\ 0 & 0 & z_{31} \end{pmatrix} \mathfrak{I}_{pg} = \mathfrak{Z}_g\, \mathfrak{I}_{pg} \qquad (14)$$

the line drops are

$$\mathfrak{V}_L = \begin{pmatrix} z_{A1} & 0 & 0 \\ 0 & z_{B2} & 0 \\ 0 & 0 & z_{C3} \end{pmatrix} \mathfrak{I}_L = \mathfrak{Z}_L\, \mathfrak{I}_L \qquad (15)$$

and the phase drops at the load are

$$\mathfrak{V}_{pl} = \begin{pmatrix} z_{AB} & 0 & 0 \\ 0 & z_{BC} & 0 \\ 0 & 0 & z_{CA} \end{pmatrix} \mathfrak{I}_{pl} = \mathfrak{Z}_l\mathfrak{I}_{pl} \qquad (16)$$

Other relations may be readily set up to handle any particular problem. Some special cases of interest are given in the following.

(*b*) *Balanced Voltages and Unbalanced Load Impedances.* In order to simplify the expressions and still retain enough of the circuit for present purposes, assume that all impedances of figure 1 are zero except the three load impedances. Then since the voltages are assumed as balanced—*acb* phase sequence—

$$\mathfrak{V}_{pl} = V_{BA}\,(1, a, a^2) \qquad (17)$$

where

$$a = \epsilon^{j\frac{2\pi}{n}} = \epsilon^{j\frac{2\pi}{3}} = \epsilon^{j120}$$

Also from equations 10 and 16

$$\mathfrak{I}_L = \mathfrak{A}_{13}\mathfrak{I}_{pl} = \mathfrak{A}_{13}\mathfrak{Y}_l\mathfrak{V}_{pl} \qquad (18)$$

But since

$$\mathfrak{Y}_l = \begin{pmatrix} \frac{1}{z_{AB}} & 0 & 0 \\ 0 & \frac{1}{z_{BC}} & 0 \\ 0 & 0 & \frac{1}{z_{CA}} \end{pmatrix} = \begin{pmatrix} y_{AB} & 0 & 0 \\ 0 & y_{BC} & 0 \\ 0 & 0 & y_{CA} \end{pmatrix}$$

equations 17 and 18 give

$$\mathfrak{I}_L = \begin{pmatrix} y_{AB} & 0 & -y_{CA} \\ -y_{AB} & y_{BC} & 0 \\ 0 & -y_{BC} & y_{CA} \end{pmatrix} V_{BA}\,(1, a, a^2)$$

which multiplied out gives

$$\mathfrak{I}_L = V_{BA}\,(y_{AB} - a^2 y_{CA}, -y_{AB} + a y_{BC}, -a y_{BC} + a^2 y_{CA}) \qquad (19)$$

But if the balanced voltage phase-sequence is changed to the *abc* sequence

$$\mathfrak{V}_{pl} = V_{BA}\,(1, a^2, a),$$

and

$$\mathfrak{I}_L = V_{BA}\,(y_{AB} - a y_{CA}, -y_{AB} + a^2 y_{BC}, -a^2 y_{BC} + a y_{CA}) \qquad (20)$$

Examining these two relations—19 and 20—makes it evident that, for any set of load impedances such that their phase angles, or the ratio x/r, is the same for all three impedances, the *magnitude of the line currents will not change* under a change in phase sequence of voltage; and that for any other impedance relation the line currents will change in magnitude with a change in the phase sequence of the voltages.

(*c*) *Unbalanced Voltages and Balanced Impedances.* Considering load impedances only, evidently

$$\mathfrak{Y}_l = y\mathfrak{A}_1 \qquad (21)$$

Further, if n' is located at the center of gravity of the load voltage triangle, and if

$$\mathfrak{V}_{N'} = (V_{N'A}, V_{N'B}, V_{N'C}) \qquad (22)$$

then

$$\mathfrak{V}_{pl} = (V_{BA}, V_{CB}, V_{AC}) = \mathfrak{A}_{12}\, \mathfrak{V}_{N'} \qquad (23)$$

But from 18, 21, and 23

$$\mathfrak{I}_L = \mathfrak{A}_{13}\, y\, \mathfrak{A}_1\, \mathfrak{A}_{12}\, \mathfrak{V}_{N'} = y\mathfrak{A}_{13}\, \mathfrak{A}_{12}\, \mathfrak{V}_{N'} \qquad (24)$$

which gives

$$\mathfrak{I}_L = y \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \mathfrak{V}_{N'} = 3y\, \mathfrak{V}_{N'} \qquad (25)$$

This relation shows immediately that the line currents are associated with the voltages to the center of gravity of the

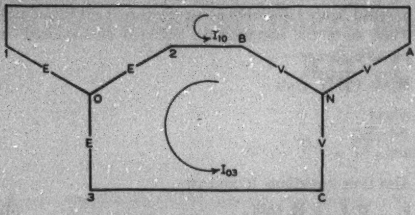

Figure 2. Three-phase wye connection

voltage triangle by the phase angle of the load impedances. Also note that the voltages to the center of gravity may be determined from

$$\mathfrak{V}_{N}' = \frac{1}{3y}\mathfrak{I}_L \tag{26}$$

(*d*) *Balanced Voltages and Balanced Impedances.* Once more considering the load only—*acb* phase sequence—

$$\mathfrak{I}_{pl} = I_{BA}(1, a, a^2)$$

$$\mathfrak{I}_L = \mathfrak{M}_{13}\mathfrak{I}_{pl} = I_{BA}\mathfrak{M}_{13}(1, a, a^2)$$

$$\mathfrak{I}_L = \sqrt{3}\,\epsilon^{j30}\,\mathfrak{I}_{pl} \tag{27}$$

Also since the relation between line and phase currents is now simply a complex number rather than a singular matrix

$$\mathfrak{I}_{pl} = \frac{\epsilon^{-j30}}{\sqrt{3}}\mathfrak{I}_L \tag{28}$$

Three-Phase Wye Connection

(*a*) *General.* The voltage equations in terms of cyclic currents for figure 2—assuming no mutual inductance or capacitance—are, in matrix form,

$$\mathfrak{E} = \mathfrak{Z}\mathfrak{I} \tag{29}$$

where

$$\mathfrak{E} = (E_{13}, E_{23}) \tag{30}$$

$$\mathfrak{I} = (I_{10}, I_{03}) \tag{31}$$

and

$$\mathfrak{Z} = \begin{pmatrix} z_{10}+z_{20}+z_{2B}+z_{BN}+z_{NA}+z_{A1} & -z_{02}-z_{2B}-z_{BN} \\ -z_{02}-z_{2B}-z_{BN} & z_{03}+z_{02}+z_{3C}+z_{CN}+z_{NB}+z_{2B} \end{pmatrix} \tag{32}$$

Of course, since $\mathfrak{Z}$ is nonsingular

$$\mathfrak{I} = \mathfrak{Y}\mathfrak{E}$$

The line currents of the wye system are

$$\mathfrak{I}_L = (I_{A1}, I_{B2}, I_{C3}) = \mathfrak{I}_{pg} = \mathfrak{I}_{pl} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{pmatrix}\mathfrak{I} = \mathfrak{T}_y\mathfrak{I} \tag{33}$$

The voltages are

$$\mathfrak{E}_{Lg} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{pmatrix}\mathfrak{E} \text{ and } \mathfrak{E} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\mathfrak{E}_{Lg} \tag{34}$$

and in terms of phase voltages

$$\mathfrak{E}_{Lg} = \mathfrak{M}_{13}\mathfrak{E}_{pg}$$

Also

$$\mathfrak{E} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}\mathfrak{E}_{pg} = \mathfrak{T}_{yt}\mathfrak{E}_{pg} \tag{35}$$

The subscript *t* will be used to denote the transpose of a matrix. The singular transformation matrices indicate that the generator phase voltages cannot be determined from $\mathfrak{E}$ or from the generator line voltages.

If the impedances of the system are written as diagonal matrices

$$\mathfrak{Z}_l = \begin{pmatrix} z_{AN} & 0 & 0 \\ 0 & z_{BN} & 0 \\ 0 & 0 & z_{CN} \end{pmatrix} \quad \mathfrak{Z}_L = \begin{pmatrix} z_{A1} & 0 & 0 \\ 0 & z_{B2} & 0 \\ 0 & 0 & z_{C3} \end{pmatrix} \quad \mathfrak{Z}_g = \begin{pmatrix} z_{01} & 0 & 0 \\ 0 & z_{02} & 0 \\ 0 & 0 & z_{03} \end{pmatrix} \tag{36}$$

the generator voltages in terms of the currents are

$$\mathfrak{E}_{Lg} = \mathfrak{M}_{13}(\mathfrak{Z}_l + \mathfrak{Z}_L + \mathfrak{Z}_g)\mathfrak{I}_{pg} \tag{37}$$

Since $\mathfrak{M}_{12}$ is singular this transformation is singular, and, therefore, not reversible. However, the phase currents at the generator may be determined from equation 33 as

$$\mathfrak{I}_{pg} = \mathfrak{T}_y\mathfrak{Y}\mathfrak{E} \tag{38}$$

which gives from 34

$$\mathfrak{I}_{pg} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{pmatrix}\mathfrak{Y}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\mathfrak{E}_{Lg}$$

Also from 38 and 35

$$\mathfrak{I}_{pg} = \mathfrak{T}_y\mathfrak{Y}\mathfrak{T}_{yt}\mathfrak{E}_{pg} \tag{39}$$

This equation cannot be used to determine $\mathfrak{E}_{pg}$ in terms of $\mathfrak{I}_{pg}$ because of the singularity of the transformation. Further investigation will show that no relation can be found for $\mathfrak{E}_{pg}$ in terms of $\mathfrak{I}_{pg}$ and the circuit impedances or admittances; i. e., given a set of wye-connected impedances and the line currents, the phase generator voltages of the source cannot be computed. There is an infinite set of these phase voltages which will satisfy the system.

(*b*) *Balanced Voltages and Unbalanced Load Impedances.* Using 39 and the fact that

$$\mathfrak{E}_{pg} = E_{10}(1, a, a^2)$$

for the 132 sequence, a relation corresponding to the delta circuit relation may be deduced; namely, that if the three-phase impedances have the same ratio x/r, a change of phase sequence will not affect the current magnitudes.

(*c*) *Unbalanced Voltages and Balanced Load Impedances.* Assuming all but load impedances as negligible, equation 32 reduces to

$$\mathfrak{Z} = \begin{pmatrix} 2z & -z \\ -z & 2z \end{pmatrix} = z\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \text{ and}$$

$$\mathfrak{Y} = \frac{1}{3}\begin{pmatrix} 2y & y \\ y & 2y \end{pmatrix} = \frac{y}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

Then from 39

$$\mathfrak{I}_L = \mathfrak{I}_{pg} = \frac{y}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}\mathfrak{E}_{pg} \tag{40}$$

If $E_{10} + E_{20} + E_{30} = 0$ this relation becomes

$$\mathfrak{I}_L = \mathfrak{I}_{pg} = y\,\mathfrak{E}_{pg}$$

A further property of this system may be deduced as follows: the phase drops at the load are

$$\mathfrak{V}_{pl} = \mathfrak{Z}_l \mathfrak{I}_{pg} = z \mathfrak{U} \mathfrak{I}_{pg} = z \mathfrak{I}_{pg} \qquad (41)$$

Further, in terms of the voltages to the center of gravity of the generator voltage triangle,

$$\mathfrak{E} = \mathfrak{T}_{yt} \mathfrak{E}_{pg}' = \mathfrak{T}_{yt} (E_{10}', E_{20}', E_{30}') \qquad (42)$$

and from 38 and 42

$$\mathfrak{I}_{pg} = \mathfrak{T}_y \mathfrak{Y} \mathfrak{T}_{yt} \mathfrak{E}_{pg}' = \frac{1}{3} y \mathfrak{T}_y \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathfrak{T}_{yt} \mathfrak{E}_{pg}' \qquad (43)$$

Hence from 41 and 43

$$\mathfrak{V}_{pl} = \frac{1}{3} \mathfrak{T}_y \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathfrak{T}_{yt} \mathfrak{E}_{pg}' = \frac{1}{3} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \mathfrak{E}_{pg}' \qquad (44)$$

But since $E_{10}' + E_{20}' + E_{30}' = 0$

$$\mathfrak{V}_{pl} = \mathfrak{E}_{pg}' \qquad (45)$$

which shows that the neutral or common point of the load voltages is at the center of gravity of the line voltage triangle.

(*d*) *Balanced Voltages and Impedances.* The voltages are—sequence *acb*—

$$\mathfrak{V}_L = \mathfrak{U}_{12} \mathfrak{V}_{pl} = \mathfrak{U}_{12} V_{NA} (1, a, a^2)$$

$$\mathfrak{V}_L = \sqrt{3}\, \epsilon^{-j30} \mathfrak{V}_{pl} \qquad (46)$$

Also the phase voltages may for this special case be determined from the line voltages as

$$\mathfrak{V}_{pl} = \frac{\epsilon^{j30}}{\sqrt{3}} \mathfrak{V}_L \qquad (47)$$

Further from 39 and the conditions here being considered

$$\mathfrak{I}_{pg} = y \mathfrak{E}_{pg} = y \mathfrak{V}_{pl} \qquad (48)$$

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A System of Electric Remote-Control Accounting

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MEMBER AIEE

THE ACCOUNTING system which will be outlined in this paper will be discussed mainly in its application to a department store. However, it will not require any great stretch of the imagination to visualize the application of the same system and devices to other fields, including factory production control and pay rolls, public-utility accounts receivable, and others.

The Punched-Card Method

The punched-card method of accounting by automatic machinery has had a steady growth since its use in the United States census tabulation of 1890. At the present time the method is used in the great majority of large corporations, and literally *billions* of cards are used every year in this country alone.

In its simplest elements, the method involves the preparation of cards through which are punched holes in certain coded positions to represent numbers, or, more rarely, letters of the alphabet. The cards after preparation are stacked in hoppers of automatic machines, through which they are fed one by one, and the coded numbers picked up by electrical or mechanical sensing devices. These sensing devices in turn control adding units or other devices which do the actual accounting. There are machines for sorting, adding, subtracting, and multiplying, and these have many kinds of automatic controls which it would be out of place to describe in detail here.

The system of remote-control accounting which it is proposed to describe is not a complete system in itself, but has been designed as an aid and adjunct to extend the field of economical application of the punched-card method.

The Economical Application of the Punched-Card Method

The punched-card method is usually found to be economical of application where large masses of numerical data must be handled accurately and totaled in numerous groupings. Small quantities of data may be handled well on the machines, but the costs of handling on punched cards are much more favorable if there is enough work to keep at least a full set of machines busy continuously. The speeds and capacities of the machines are so great (400 cards per minute for the sorters, 150 per minute for the tabulators) that large masses of data are required to keep the machines usefully employed.

It should be clear also that when the cards have once been prepared (i.e., punched and checked) it is a relatively easy matter to run off a new set of statistics from them by sorting them automatically into new groupings, and tabulating them in these groupings. The more groupings into which it is desired to tabulate and totalize the information, the smaller the fraction of the card cost and punching cost which should properly be allocated against each tabulation, and therefore the method shows up most favorably when there are numerous groupings to be handled.

The census, which was the first application of the punched-card method, still stands out as the ideal type of job in its great mass of data and numerous segregations desired.

It should not be inferred that where the number of segregations is small, the punched-card method is undesirable, because other factors than direct cost, such as speed and precision, may be controlling factors. Other things being equal, however, it is unquestionably true that the most favorable applications are as has been stated. By way of illustration we may note that some of the most successful, widely adopted, and long continued applications have been in the fields of insurance, chain groceries, factory production control, and payroll. In insurance, the cards are used for many forms of actuarial analysis as well as for handling the specific dollar accounting of the policy holders individually. In chain grocery warehousing, a single card can be used in turn as an on-order record, for making a tally list for checking merchandise received, as a record in a perpetual inventory, as an order to ship to an individual retail store, and as a billing medium to that store. Thereafter, with additional data of store identification and date of shipment punched in, it is available with other cards to furnish various analyses of store performance, turnover of merchandise, profit by manufacturer, etc., etc. It would perhaps become tedious to describe in a similar vein the various uses to which a punched card is put in the other applications which have been mentioned.

There are of course accounting problems which involve so few segregations of data that they are not suitable for solution by the punched-card method, and there are others in which the advantages and disadvantages neutralize; and this in spite of a large volume of data.

In one or the other of these last two classifications lie most of the phases of accounting and control in large department stores. There have been dozens of punched-card installations in department stores, but many of these after thorough trial have been discontinued. It is apparent then that the executives in this large field are receptive to the idea, and that there are many advantages in its appli-

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1. For all numbered references, see list at end of paper.

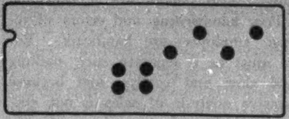


Figure 1. Punched token

cation, but these are just about offset by the disadvantages, which include the costs. Since this situation seemed favorable to the application of an auxiliary development, and further since the accounting problems of department stores are more standardized than, say, those of manufacturing plants, while yet offering comparable volume, the department store was selected as the field of the first experimental installation of remote-control accounting.

The magnitude of the department store accounting problem can be appreciated from a few statistics. There are approximately 300 department stores and departmentized specialty stores in the United States which do an annual business of over one million dollars per year, and the aggregate net sales of this group is about 4 billion dollars per year. An average store in this group would do about 13 million dollars of business per year. Using the percentage expense figures given in the Harvard Business School reports,[1] the payroll expense alone for accounts receivable, accounting office and credit, and marking, is in excess of two per cent of sales or $260,000 per year for

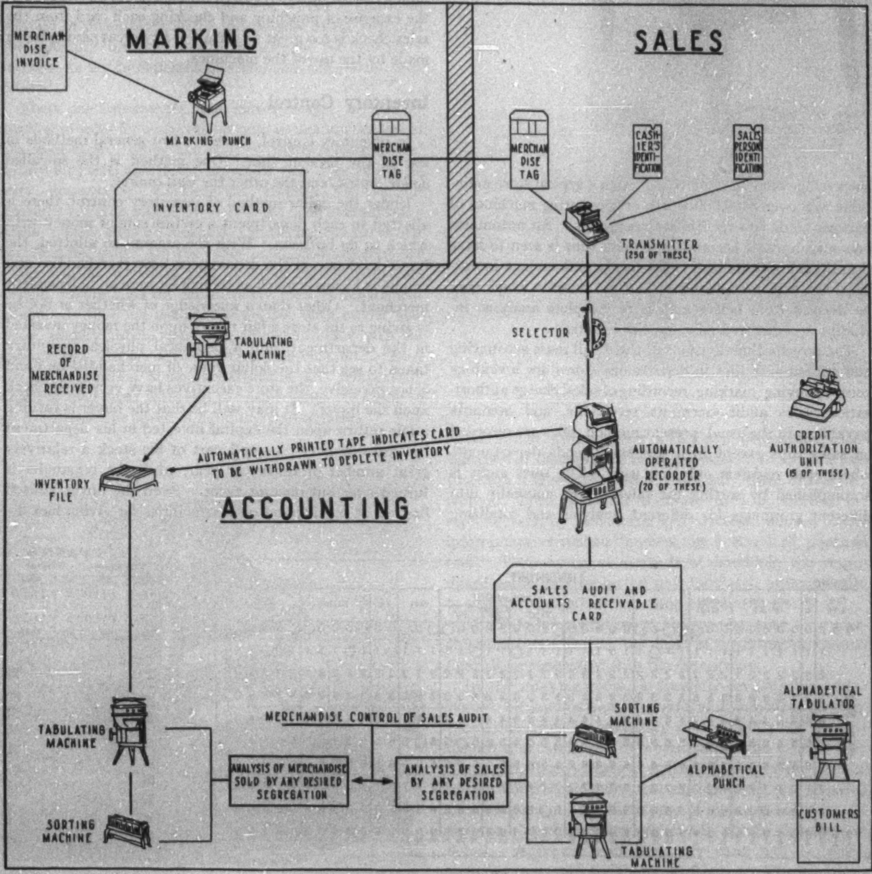

MARKING
SALES
MERCHANDISE INVOICE
MARKING PUNCH
MERCHANDISE TAG
INVENTORY CARD
MERCHANDISE TAG
CASHIER'S IDENTIFICATION
SALESPERSON IDENTIFICATION
TRANSMITTER (250 OF THESE)
SELECTOR
RECORD OF MERCHANDISE RECEIVED
TABULATING MACHINE
CREDIT AUTHORIZATION UNIT (15 OF THESE)
AUTOMATICALLY PRINTED TAPE INDICATES CARD TO BE WITHDRAWN TO DEPLETE INVENTORY
AUTOMATICALLY OPERATED RECORDER (20 OF THESE)
INVENTORY FILE
ACCOUNTING
SALES AUDIT AND ACCOUNTS RECEIVABLE CARD
MERCHANDISE CONTROL OF SALES AUDIT
SORTING MACHINE
ALPHABETICAL TABULATOR
TABULATING MACHINE
ANALYSIS OF MERCHANDISE SOLD BY ANY DESIRED SEGREGATION
ANALYSIS OF SALES BY ANY DESIRED SEGREGATION
ALPHABETICAL PUNCH
SORTING MACHINE
CUSTOMERS BILL
TABULATING MACHINE

Figure 2. Chart of operating procedure

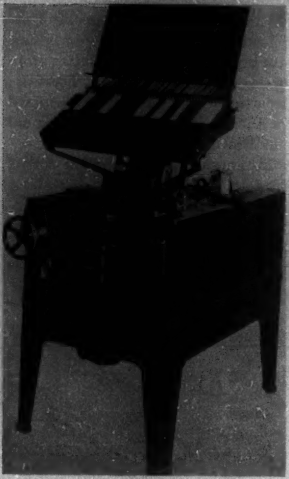


Figure 3. Duplex marking punch

the average store of the group. Such a typical store maintains well over $100,000 worth of accounting machines of various kinds to expedite its operations. An automatic or semiautomatic accounting system thus is seen to have considerable possibility of effecting actual savings in out-of-pocket expense, entirely aside from benefits which may be derived from better and more complete analysis, including speedier and more precise results.

The principal subdivisions of the detail mass accounting and related activities in department stores are inventory control, buying, marking, recording of sales, charge authorization, sales audit, accounts receivable, and accounts payable. In the usual present method, sales are recorded on sales slips except in a few small-goods departments where cash registers only are used. The sales audit is accomplished by sorting the sales checks manually into different groupings for different analyses and totalizing under each grouping. Lost checks are a continual source of worry. Poor handwriting and errors of recording by low-wage employees are handicaps. For each analysis there must be a separate sort, making the manual method slow and costly; and because of this a comprehensive analysis of sales is not economically feasible. A number of stores have already used punched-card machines in the sales audit, but approximately half of these stores have discontinued the punched-card method and gone back to their old plan. A considerable number of stores still continue to use punched cards in sales audit, but it is apparent from the history of the whole group that the benefits are not conclusive evidence of the superiority of that method. The obstacles to the use of punched cards in this work are that the average unit sale is small (about two dollars),[1] the number of groupings required for analysis is not large, and the expense of punching and checking each card from the sales check is too great to permit of any great saving being made by the use of the machines.

Inventory Control

In inventory control, there are two general methods in use at the present time. One method is the so-called *dollar control*, and the other the *unit control*.

Under the dollar method of inventory control, there is allotted to each department a certain sum of money with which to do business. With the amount so allotted, the buyer in charge of the department buys merchandise and conducts business much as though he were an independent merchant. Other than a knowledge of whether or not he is giving to the store a fair return upon the money invested in the department, and a periodical physical inventory taken to see that the dollar value of merchandise on hand is not excessive, the store executives have very little check upon the buyer. It may well be that the buyer is earning a fair return upon the capital invested in his department by "turning over" a small part of his stock a relatively great number of times per year, while the remainder is turned over but once or twice. Even the buyer himself has not at his disposal proper facilities for giving him de-

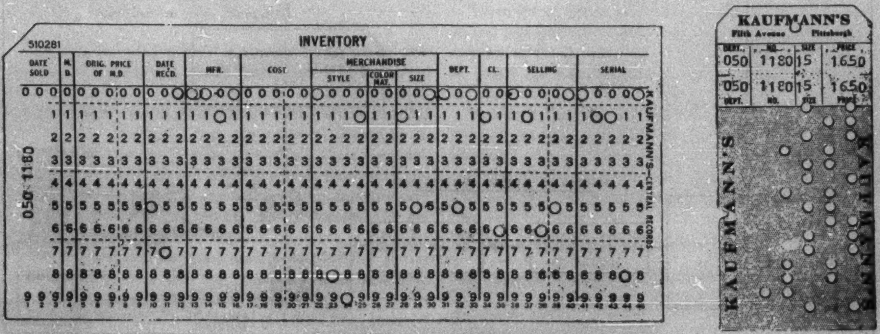


Figure 4. Unit inventory card and corresponding price tag

tailed knowledge of the merchandise in his department, and its turn-over.

Under the unit method of inventory control, a record is kept of each individual article of merchandise as it comes into the store. When each article of merchandise is sold, immediate note of the sale is made on the inventory records so that there is kept a running up-to-date record of the inventory on hand. Under this method, the executives and department heads may know at all times not only the dollar value of merchandise on hand but also just what the description is of this merchandise. They are able to determine what merchandise is moving and what is not, also what merchandise is being handled at a profit and what is not. Under this method, it is possible to keep the inventory at the lowest point consistent with having a sufficient amount and variety of merchandise on hand to meet the requirements of customers. This is the ideal method of inventory control, but because of the cost and the difficulty involved in the keeping of the necessary records, its use in department stores has necessarily been limited.

There are advantages and disadvantages in the recording of sales by cash register only as against the use of the sales book. Using the register, there is more opportunity for theft, and in the aggregate this is known to be very appreciable. Cash-register receipts for various amounts are soon scattered all over the floor in many sales areas. They offer no means of identifying the particular merchandise sold. On the other hand, cash registers are quick and do not waste time of either customer or sales person. Sales slips are more widely used in department stores. If they are used in connection with carrier systems, usually tubes, there is a time-consuming wait for change and receipt, aggravating to the customer. During busy seasons some departments are temporarily changed over from sales-slip to cash-register recording so as to increase the speed of handling transactions.

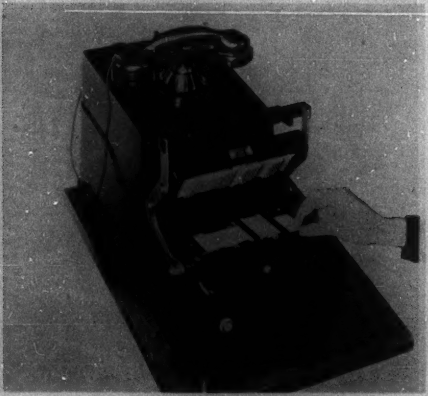


Figure 5. Central records transmitter

Outline of Operations of Remote-Control Accounting System

The factors involved in a sale are: (1) the customer, (2) the merchandise and its selling price, (3) the sales person, and (4) the cashier. In the system to be described, each of these factors is identified by a small punched tag, illustrated in figure 1. In a charge transaction, identification of the cashier is unnecessary. In a cash transaction, identification of the customer is unnecessary. In any one sale therefore it is necessary to identify only three of the four factors. These punched tags are used to control the reproduction, by mechanical and electrical means, of the information contained thereon, on a standard punched card. Then by usual methods of handling, the reproduced cards may be sorted and tabulated automatically to perform the major accounting operations.

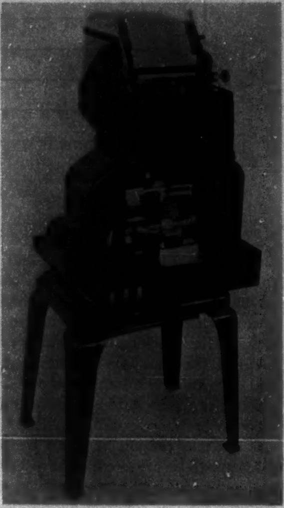


Figure 6. Central records recorder

The detailed operation of the system may be understood best by following the chart of operating procedure, figure 2, in connection with the description.

In the marking room, a specially-developed machine (figure 3) simultaneously prints and punches standard tabulating cards for inventory record, and price tags for display and mechanical reproduction at the time of sale. For goods whose value and character warrant the expense of unit control, an individual card and tag are prepared for each piece. For other types of merchandise there are variations in the system, which will not be described in detail. A typical unit inventory card and the corresponding price tag are shown in figure 4. The inventory cards are tabulated and listed, and totals checked against

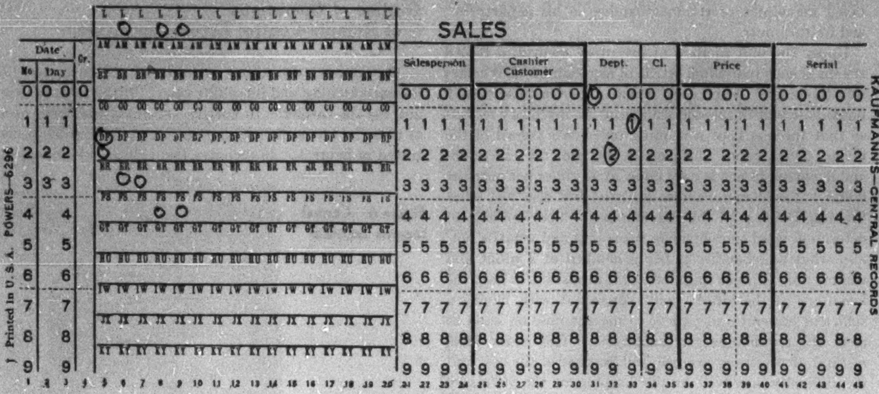


Figure 7. Sales record card

the invoices. It is to be noted that these cards and the price tags are both prepared by a single keyboard setup on the duplex punch illustrated in figure 3, and that a check of the inventory cards is also a check of the price tags. After checking, the inventory cards go to the stock file, and the price tags are attached to the merchandise and go with it to the sales area. An important point in the marking procedure is that the punching is done on a quantity basis, and not item by item. A shipment of merchandise may contain 500 items, and if 500 unit inventory cards are to be prepared, most of the information will be common to all, and but a single keyboard setup will be required for this common information. If this information is checked for one card, it is checked for all. There will be of course some information which is not common to all, but the keyboard changes made in one field of the card do not disturb those in other fields. When producing duplicate cards and tags, the marking punch shown in figure 3 has an output of 100 per minute.

Let us follow the tagged merchandise to the sales area. When a piece of merchandise is sold for cash, the merchandise tag is torn from its stub, which contains duplicate printing of the merchandise identification for use in case the merchandise is returned for credit later by the customer. The detached portion of the tag, along with a punched celluloid token bearing in code the identificaton number of the sales person, and a similar token for the cashier, are placed in a *transmitter*, illustrated in figure 5. When the top of the transmitter is closed (provided it contains all three of the tokens), through appropriate wiring and automatic selector switches it is immediately connected to an idle *recorder* (figure 6) in a central accounting office. All the information needed for a complete record of the sale is automatically transmitted from the punched tokens and reproduced on a standard punched card, and is also printed and totaled on the recording machine.

Transmitting the record of the sale takes five seconds, and at its completion the merchandise tag is automatically stamped with the date of sale, and the top of the transmitter opens. The stamped tag is given to the customer as a receipt, and it identifies the exact merchandise and the price at which it has been sold.

The sales record card produced at the central office is reproduced in figure 7. All the information punched on the right-hand half of the card is obtained from the transmitter. The date only, which occupies the extreme left-hand field on the card, is set up each morning on the recorder punches, and reproduces on all cards punched that day. The large alphabetical field in the left central part of the card is not used on cards which record cash transactions.

The sales audit is obtained from the sales cards by sort-

Figure 8. Charge authorizing unit

ing and tabulating them by machine in the usual way. This part of the work includes reports of cash by cashier, sales by sales person, sales by department, and for the entire store.

The merchandise piece number or serial number on the sales record card serves as a means of identifying, in the inventory file, the inventory card corresponding to the item sold. This card is withdrawn, and with similar cards serves as a basis for the analysis of merchandise sold by any desired segregation. The cards are "gang punched" with the date of sale. Among the reports which have been found most useful from these cards are analyses of fastest selling items and styles, turnover by manufacturer, turnover by department, number of items by size, gross profit by manufacturer and by department, and sales by price lines. All these analyses and many more are obtained without any further detail work, but merely by the operation of sorters and tabulators whose speeds are respectively 400 cards per minute and 150 cards per minute. There is a possibility of human error coming in during the operation of withdrawing the inventory cards from the file, but this has been practically eliminated by *totaling the serial numbers* of the sales record cards, which is done automatically, and if the totals check the probability of error is very small indeed.

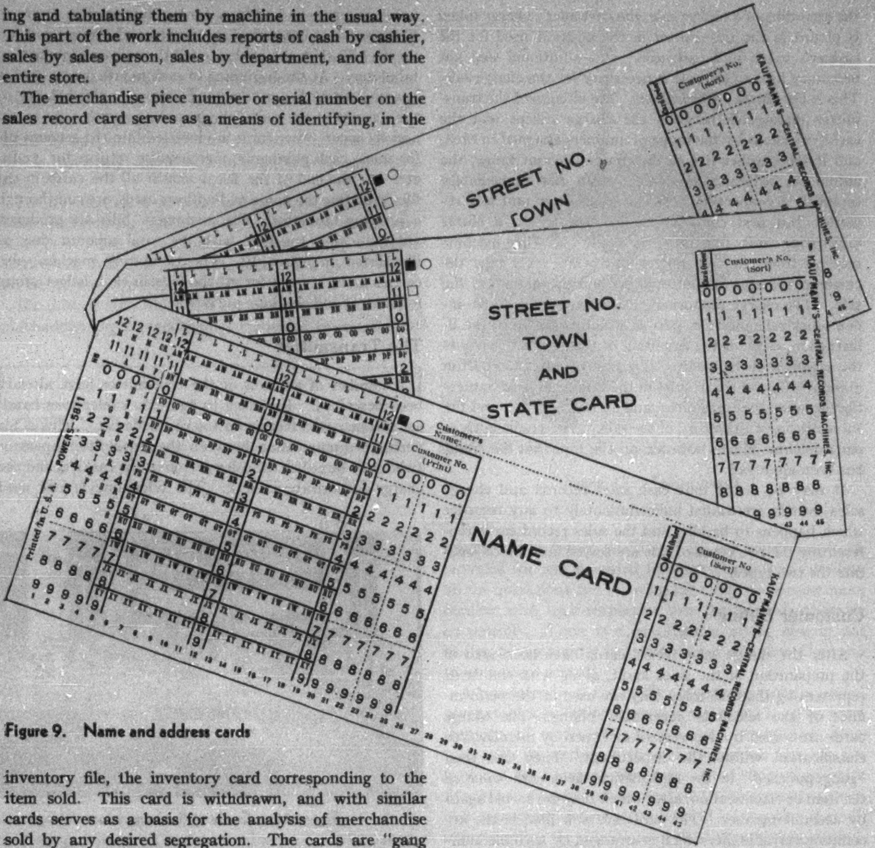


Figure 9. Name and address cards

The cards remaining in the file represent merchandise on hand, and highly useful and valuable information can be drawn from them. Reports which were regularly prepared from this file in a large experimental installation included a weekly summary of merchandise on hand; a monthly analysis by department of the age of merchandise on hand in dollar value and in pieces; an analysis of markdowns for the month by manufacturer, by price line and by reason; a monthly analysis of returns to manufacturer; a quarterly analysis of merchandise in each department classified by size, and size distribution for sales and inventory compared and brought into proper balance.

A file of "on order" cards is also maintained, and used as a basis for analysis of merchandise on order.

Charge Sale

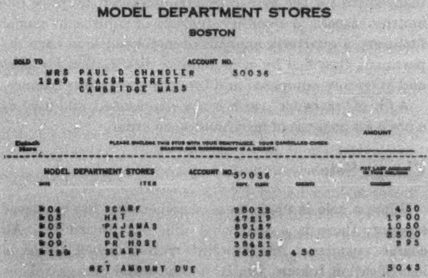
When a sale is made to a customer who has a charge account, there is a somewhat different procedure. All charge customers are supplied with punched celluloid identification tokens, similar to the usual charge tokens except that the account number is punched in code. In the recording of a charge sale, the customer's charge token is placed in the transmitter in the position used for the cashier's token in a cash sale. One additional step not necessary in the cash sale is necessary for the charge sale. This is the *charge authorization*. The circuits of the transmitter distinguish between the charge tokens and the cashiers' tokens by the range of numbers assigned to each, and if the number lies in the charge account range, the automatic switching equipment does not immediately establish a connection between transmitter and idle recorder, but first connects the transmitter to a *charge authorizing unit*, illustrated in figure 8. This machine receives from the transmitter and prints on a tape the customer's charge account number and the amount of the sale. The charge authorizing clerk refers to a visible-index quick-reference file, part of which is shown in the illustration, to see if the condition of the account warrants the approval of the credit. If so, she operates the button marked OK, and this results in the establishing of connection between the transmitter and an idle recorder, and the transmitting of the record of the sale. The credit authorizing machine makes notation on the tape that the credit has been approved.

It may be noted that cash sales records and charge sales records are routed indiscriminately to any recorder which happens to be idle, and the sales record cards representing the two types of sale are sorted later on to separate the two types.

Customer Billing

After the charge sales record cards have been used in the preparation of the sales audit, along with the cards representing the cash sales, they are used in the performance of the accounts receivable billing. The charge cards are sorted by department and then by merchandise classification within the department. They are then "gang punched" in the alphabetical field with *name* of the item or classification, after which they are sorted again by account number. The cards are now filed in the accounts receivable file, which is arranged by account number. During this work a dollar total by ledger groups is used as a control check.

The accounts receivable file for each account contains a series of name-and-address cards, illustrated in figure 9. These serve as index cards in the file, and also act to head up the bill when the cards are fed through an alphabetical tabulator. At the beginning of each month a new card is inserted for each active account, showing the balance of the account at the end of the preceding month. As transactions occur, other cards are inserted into the account file for sales, cash payments, merchandise returns for credit, etc. At the end of the fiscal month all the cards in the file, including the name-and-address cards, are run through a printing tabulator. The customers' bills are produced automatically, together with the total amount due, as illustrated in figure 10. The tabulating machine also carries along a total for all the bills in each ledger group for control and checking purposes.



MODEL DEPARTMENT STORES
BOSTON

SOLD TO
MRS PAUL D CHANDLER
1989 BEACON STREET
CAMBRIDGE MASS

ACCOUNT NO. 50036

Detach Here

PLEASE ENCLOSE THIS STUB WITH YOUR REMITTANCE. YOUR CANCELLED CHECK BEARING OUR ENDORSEMENT IS A RECEIPT.

AMOUNT

MODEL DEPARTMENT STORES

ACCOUNT NO. 50036

DATE	ITEM	DEPT. CLERK	CREDITS	CHARGES
04	SCARF	28032		4 50
05	HAT	47219		12 00
05	PAJAMAS	89127		10 50
08	DRESS	98026		25 00
09	PR HOSE	38421		2 95
18	SCARF	28032	4 50	
	NET AMOUNT DUE			50 45

PAY LAST AMOUNT IN THIS COLUMN

Figure 10. Automatically prepared customer's bill

The Transmitter

A number of features of the transmitter have already been described. Referring to figure 5, a telephone handset will be noted resting on top of the device. This is for verbal communication between the transmitter operator (who may be either the sales person or the cashier) and the charge-authorization clerk. The telephone is not used unless there is some question relative to the approval of the credit, and if the authorization clerk desires to converse, she operates a button which causes a light, shown at the left front of the transmitter, to glow. The telephone circuit is already connected by automatic switching, and the transmitter operator has only to lift up the telephone and start talking. In designing the system, it was felt that in the delicate business of passing on the credit standing of the customer, the importance of preserving good will required the flexibility of handling afforded at a distance only by telephonic communication.

Figure 11. Credit-authorizing room

Electrical interlocks are provided in the transmitter so that failure to put in one of the tokens, or putting in a token in the wrong position, will prevent the transmitter top from locking down when closed, and also prevent the operation of the selector switches and the sending of the record. Two or more transmitters can be connected to a single cable circuit in a "party-wire" relation, and when this is done a transmitter coming on a busy line merely waits until the termination of the transmission period of five seconds before it goes into action. This party-line feature makes it feasible to provide numerous receptacles, analogous to electric-light sockets, into which extra transmitters can be plugged to take care of peak loads occasioned by special sales or other causes. The plug-in feature is obviously useful also in maintenance and repair work.

The transmitter circuits are arranged so as to be inoperable if the wrong number of holes exists or apparently exists in any one of the tokens. Thus a scrap of paper accidentally covering one of the holes of a token would not result in an erroneous record.

The date of sale, which the transmitters stamp on each merchandise tag transmitted, is embossed by small steel stamps, and these have to be changed by hand each day to the new day's date.

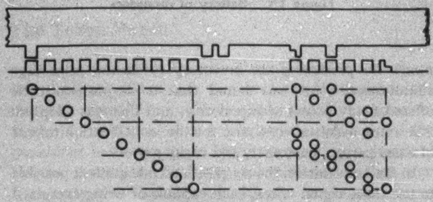


Figure 12. Code bar for punching dual code

Each transmitter contains a circuit which sorts the records and routes the charge sales to the credit authorizing department and the cash sales directly to the recorders.

The Credit Authorizing Machine

The credit authorizing unit, shown in figure 8, receives the record of the account number and amount of sale for a charge purchase. There are five button controls, namely for OK, reject, call back or telephone communication, and two supplementary controls for emergency release and for carriage return. A row of 20 lights in the front of the base is provided to indicate how many calls there are awaiting their turns for being passed upon for authorization. The telephone is at the left of the main unit.

The routing of sales to the credit authorizing machines differs from the indiscriminate method employed for the recorders of selecting any idle machine. Instead, each credit authorizing unit has routed to it by the selector switches only records involving accounts which lie within a specific limited range. The particular unit pictured in figure 8, for example, receives only accounts numbered from 220,000 to 239,999 inclusive, as is indicated by the large numbers just above the row of 20 lights. This routing is accomplished by allowing the first two digits of the customers' charge tokens to control the operation of selector switches which thereupon establish connection to the proper authorizing unit. The purpose of this selection is to reduce the size of the file which has to be provided for and operated by each clerk, thus speeding up the operations and allowing the clerks to become more familiar with their respective lists than would otherwise be possible. There is a considerable saving also in not having to maintain a large number of complete files, and to keep them posted.

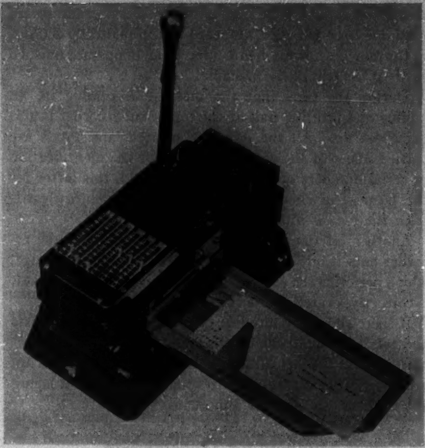


Figure 13. Token punch

A general view of the credit authorizing room in an experimental installation is shown in figure 11. The room contains 15 credit authorizing units around the sides, each with its section of the quick-reference file. In the center of the room are the desks of the supervisors, with telephone and tube communication to the accounts receivable department where more detailed information is available regarding the conditions of the accounts than is available in the quick-reference files.

The Recorder

The recorder is shown in figure 6. It is a combination of a punch adapted for remote-control operation, and a tabulator. A stack of blank cards is kept in the magazine at the front of the machine, as may be seen in the picture. A card is normally always kept in the punching position, between the punches and dies, and if there should by accident be no card there, then an electrical interlock will prevent the particular recorder from being selected by the automatic switching equipment to receive one of the records. In this event a buzzer is also sounded to notify the attendant. As the message comes in, in normal operation, the punching is not done step by step, but instead a set basket is set up with the entire message, and at the end all the punching is done at one stroke. The date of sale is left set up on the set basket all day, so that this does not have to be transmitted. When a sales record card has been punched, the card is automatically fed out of the punching position into a receiving hopper for finished cards at the rear of the machine.

During the time that the message is coming in, an adding and printing unit at the top of the recorder is also being set up. All the data are printed, and totals are carried along of the sales prices and of the identification or serial numbers of the merchandise sold, this latter for the purpose of checking withdrawals of inventory cards from the file, as already mentioned.

Another interlock which has been provided on the recorder cuts it out of circuit unless the traveling carriage, which performs the setting up operation on the set basket, is correctly located at the starting position.

If there should be any lost or mutilated cards, the lists and totals are available to check and to make possible replacement of the lost cards. However, since the cards are produced in the central records room, the chance of losing any of them can be made almost nil.

Coding of Numbers for Transmission

The usual coding of digits on punched cards involves merely punching in the proper one of ten positions numbered from 0 to 9 in each column. The use of such a system for transmitting would require a large number of relays and conductors, and would require a wide price tag, which is considered by retailers as a heavy handicap, because it spoils the appearance and attractiveness of the merchandise. It was therefore decided to adopt the expedient of multiple punching for the merchandise tag, using fewer than ten positions in each column.

It is possible to code the ten digits by the use of a minimum of four positions, using four single-hole and six double-hole codes. However, if a double-hole column had one hole accidentally or purposely covered over, there would result an incorrect transmittal, and this could not be tolerated. With five positions it is possible to have ten combinations of just two holes. However, in working out the transmitter circuits and the coding device in the punch for producing the inventory cards and price tags simultaneously, it was found that a six-position code offered many points of superiority, and this was adopted. The same code was used also for the identification tokens of sales person, customer, and cashier.

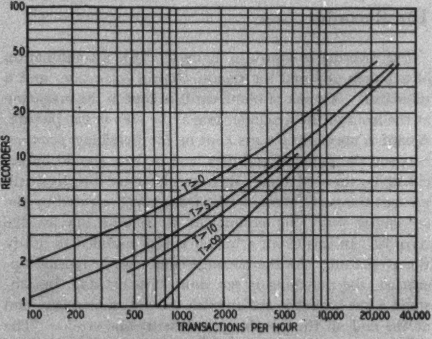


Figure 14. Traffic curves

Figure 15. Battery of recorders

In the transmitter, the six-position code made it possible to use three signal wires, each capable of being energized positively or negatively, but only two of them to be energized at one time. This offers 12 combinations, of which only 10 are needed. At the central office, a system of relays decoded the message so that the recording machine received the message over 10 signal wires in a single-hole code.

The Marking Punch

The design of the punch for the simultaneous preparation of the inventory card in single-hole code and the price tag in double-hole code seemed to offer some difficulty, and the first machine for the purpose embodied two separate punches with a set of coding relays connected between them. Subsequent thought, however, evolved a much simpler and more effective mechanical method of coding, which is embodied in the present machine shown in figure 3. This machine has a slide keyboard, and as a slide is moved by hand, a steel rod carrying a lug is moved over the tops of the punches below, in the corresponding column. For the part of the punch which serves to prepare the inventory card there is but a single lug, as shown in the left-hand portion of figure 12. A continuation of the same rod carries four lugs, of which one is partly cut away. The top of one of the punches is also partly cut away to match, and as may be seen by studying the right-hand half of figure 12, the same movement of the same slide and rod will produce a two-hole code in the six-position tag while producing at the same time the usual single-hole code in the inventory card.

The actual punching is accomplished by the lifting of the die-plate after the entire setup has been completed.

Printing units are provided for both inventory card and tag, and the setup is accomplished by but a single operation of the keyboard. The inventory card is imprinted only with characters which are needed in identifying it in the file, and the printing is done at one end so that space-saving vertical filing can be used. This is advantageous also in that the fingering does not wear or soften the long edges which are the ones used in the mechanical feeding of the cards.

In the punch, blank inventory cards are placed in a hopper at the front, and blank price tags at the rear. The finished cards and tags are discharged into hoppers at the middle of the machine. There is an indicator to show when one or the other of the magazines runs empty; a counter for the number of cards and tags; the operation may be controlled for a single card and tag or the machine may be made to turn them out continuously at the rate of 100 per minute.

The Token Punch

For punching the tokens, a small lever-set hand-operated punch, with a code bar similar to that used in the marking punch, is provided. This punch is shown in figure 13. When a token is prepared, a duplicate of the punchings is always made simultaneously on a standard tabulating card and filed away for reference, so that even if an error is made in the punching, it can be discovered and rectified later on.

Traffic Density on Central Records System

The number of transmitters required in the remote-control central records system described depends upon the area and arrangement of the selling space of the store, and the criteria are the convenience and saving of time of the sales employees and the customers, as compared to incremental cost.

The number of recorders required depends only on the peak rate of transactions and the time required for recording each transaction. The time required for each transaction is five seconds.

If each recorder were kept busy continuously, it could handle 12 transactions per minute or 720 transactions per hour. However, it is known that the transactions will not occur evenly throughout the hour, but will occur more or less at random. Sufficient additional transmitters have to be provided to take care of the brief peaks of traffic which will take place owing to this random distribution. It is impossible to avoid some possibility of delay in recording a transaction without providing a recorder for each transmitter, which is economically unfeasible. Some practical compromise must be reached between the two extremes mentioned.

The consideration of traffic density is a problem in the theory of probability similar to trunking and other traffic problems in the telephone industry. A solution applicable to the case at hand has been worked out and charted by E. C. Molina,[2] and a set of curves derived from his article is presented in figure 14. The upper curve of the chart, marked "$T > 0$," shows the relation which exists between the number of transactions per hour and the number of recorders required if the system is to be operated on a basis of one transaction out of every hundred on the average being delayed at all in transmitting. For example, if the rate is 10,000 transactions per hour, then 24 recorders would be required on this basis.

The second curve of the chart, marked "$T > 5$," indicates the relation of the number of transactions per hour and the number of recorders where one transaction out of each 100 would be delayed more than five seconds before transmitting. On this basis there would be required only 17 recorders to take care of 10,000 transactions per hour.

The third curve, marked "$T > 10$," represents the relation existing when one transaction out of 100 will meet with a delay of more than ten seconds before being transmitted.

Naturally, the number of recorders which can be depended on for continuous use will be one or two less than the total number installed, to allow for repairs and maintenance.

The numbers of the different units in the experimental installation were: 250 transmitters, 20 recorders, and 15 credit authorizing units, to handle a peak transaction rate of 9,000 per hour, of which about half were charge transactions. During very busy times, however, charge transactions of very small amount are allowed without specific reference to the credit authorization files.

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Line-to-Line Faults on A-C Network Feeders

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Synopsis

This paper deals with the magnitude of currents in a secondary a-c network due to a line-to-line fault on the high-voltage feeder, and gives the analysis of the double-unbalance circuit resulting when a network protector fails to open during such a fault, and one of the protector fuses blows. Results of field tests made on an actual network are included.

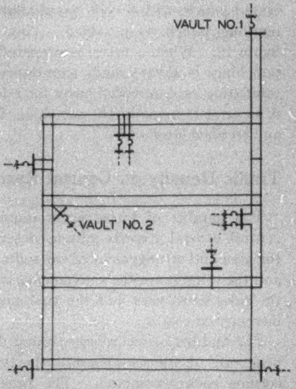
Introduction

IN a low-voltage network, the network protector is designed to open automatically for all faults on the high-voltage feeder supplying the network. In order to provide back-up protection in case a protector fails to open for a feeder fault, fuses are incorporated in the phase leads of the protector. The minimum rating of these fuses is limited to some extent because the fuses should not blow for faults on the low-voltage network. On the Commonwealth Edison Company a-c network, the fuses used on 300-kva transformer banks are rated at 1,500 amperes and those on 500-kva banks at 3,000 amperes.

With the rating of fuses limited by the restriction given above, it is desirable to determine whether or not the various types of primary faults will cause the fuses to blow, in case a protector fails to open for some reason or other. Calculations show that for three-phase faults the magnitude of the fault currents in a typical network, will always be great enough to blow the protector fuses. For line-to-ground faults on the primary feeder, the magnitude of the current through the fuses is well below the fuse rating, because the transformer bank is connected wye-delta. Hence, fuses cannot clear the transformer bank for line-to-ground faults. For line-to-line or two-line-to-ground faults, the analysis of the circuit is more complicated. These types of faults cause unequal currents to flow through the fuses and in many cases only one of the three fuses will blow. The purpose of this paper is to show the results of an investigation made to determine the magnitude of currents in a network for line-to-line faults. This investigation included field tests as well as an analytical study.

Figure 1 is a one-line diagram of a typical network system. The secondary of this system is four-wire, using single-conductor lead-covered cable, the neutral consisting of a copper conductor in parallel with its sheath and the sheaths of the phase cables. Figure 2 shows the flow of currents in the network due to a line-to-line fault on the primary feeder, after the feeder breaker at the station has opened. It will be noted that one fuse carries twice the value of current flowing in the other two fuses. It can be shown that the value of current in the fuse carrying the greatest current is equal in magnitude to the current which would be carried by the fuse for the case of a three-phase fault. In all cases this value of current is great enough to cause the fuse to blow. At the time this fuse blows the remaining two fuses will be intact since they carry only half the value of current. Figure 3 shows the circuit conditions resulting after the first fuse has blown. The current now flows through the two remaining conductors and returns over the neutral path. Since the impedance of the neutral path is comparatively high, the value of current flowing through the two fuses decreases in value after the first fuse blows. This reduced value of current in many cases will not be high enough to blow the two remaining fuses and the transformer bank will not be cleared from the network. If there is more than one transformer bank in a single vault, the reduction in current after the first fuse blows, will not be great enough to prevent the remaining fuses blowing.

Figure 1. Typical low-voltage a-c network



Discussion of Calculations

The solution of the circuit for the conditions shown in figure 2 presents no unusual problem. The fault currents flowing in the circuit of figure 3 were first determined by

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E. L. Michelson and Noble G. Larson are both engineers for the Commonwealth Edison Company, Chicago, Ill. The authors wish to thank Professor Edward W. Kimbark, Polytechnic Institute of Brooklyn, for the development of the general equations for the solution of this problem by the method of symmetrical components.

For all numbered references, see list at end of paper.

solving the simultaneous circuit equations based on Kirchhoff's laws. However, it was recognized that this circuit was an interesting case of problems dealing with simulteneous faults on a balanced three-phase system, and could be solved using the method of symmetrical components. The two simultaneous faults in this case are the line-to-line fault on the high-voltage feeder and the open-circuit in the phase in which the low tension fuse is blown. The analysis of the problem using the method of symmetrical components is shown in the appendix. The results obtained using this method checked of course with those obtained using the first method, and the labor required for the solution was considerably shortened.

Table I gives the results of calculations made to determine the value of current flowing through the transformer windings for the case of a line-to-line fault on the high-voltage feeder, after the feeder breaker has opened. The equivalent system behind this transformer which was not cleared has been expressed in terms of an equivalent length of secondary cable between this transformer and the remainder of the system.

Description of Tests

In order to check the accuracy of the assumptions made in these calculations, it was decided to perform tests on a section of the actual a-c secondary network of the Commonwealth Edison Company, shown in figure 1. Tests were made at locations designated as vaults number 1 and number 2, which are supplied from the same high-voltage feeder. The fault was applied by closing two line disconnects to ground at the station, and was initiated and cleared by means of the station oil circuit breaker. At each location two tests were made: first with the *A* phase fuse removed and second with all fuses in. Table II gives the results of these tests.

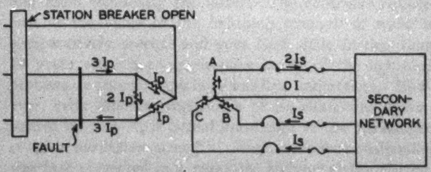

Figure 2. Flow of current in a network due to a line-to-line fault on the primary feeder

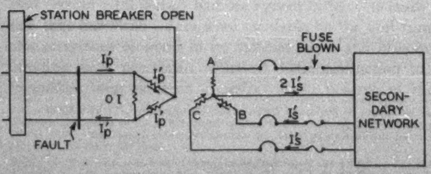

Figure 3. Flow of currents in a network, due to a line-to-line fault on the primary feeder, with the A-phase fuse blown

Discussion of Test Results

At both test locations, when the fault was applied with the *A*-phase fuse in, only this fuse was blown. The current was reduced to such a value in the remaining phases that these fuses did not blow and the fault was cleared by opening the oil circuit breakers at the station.

The test values checked essentially with the calculated values of current. The results verified the assumption that the effect of the stray ground current was unimportant. It will be noted that the current in the *B* and *C* phases is reduced to approximately 50 per cent of the initial value in both calculated and test results.

Table I. Transformer Secondary Currents for Line-to-Line Feeder Faults

Transformer Kva	Fuse Size (Amp.)	Equivalent Length of Secondary Mains to Banks Supplying Fault Current	Approximate Fault Current				
			Before first fuse blows			After first fuse blows	
			A	B	C	B	C
300	1,500	100 feet	9,900	4,950	4,950	3,150	3,150
300	1,500	200 feet	7,000	3,500	3,500	1,860	1,860
300	1,500	300 feet	5,600	2,800	2,800	1,320	1,360
300	1,500	400 feet	4,500	2,250	2,250	1,020	1,020
500	3,000	100 feet	13,100	6,550	6,550	3,600	3,600
500	3,000	200 feet	8,600	4,300	4,300	2,000	2,000
500	3,000	300 feet	6,300	3,150	3,150	1,400	1,400
500	3,000	400 feet	5,000	2,500	2,500	1,060	1,060

NOTE: Calculations based on 5 per cent impedance transformers, four single-conductor cables in a duct, 350,000-circular-mil phase cables, and 4/0 neutral cable, the arrangement of conductors assumed to be a square with the conductors as close together as possible. The effect of stray ground current was neglected.

Table II. Results of Tests

	A-Phase Fuse Out	*A*-Phase Fuse In
Vault 1		
A-phase amperes	0	5,680
B-phase amperes	1,500	2,840
C-phase amperes	1,500	2,840
Vault 2		
A-phase amperes	0	10,400
B-phase amperes	2,700	5,200
C-phase amperes	2,700	5,200

Appendix—Solution of Circuit of Figure 3, by Method of Symmetrical Components

The circuit of figure 3 can be solved on an a-c calculating board by setting up the sequence networks as shown in figure 4.[1] The open circuit in phase *A* is represented by connecting the three networks in parallel at points *y* and *z* as shown by the dashed lines. The line-to-line fault on phases *B* and *C* is represented by the connection between the positive and negative sequence networks. This connection must be made through one-to-one insulating transformers and because of the delta-wye transformation, the polarity of the

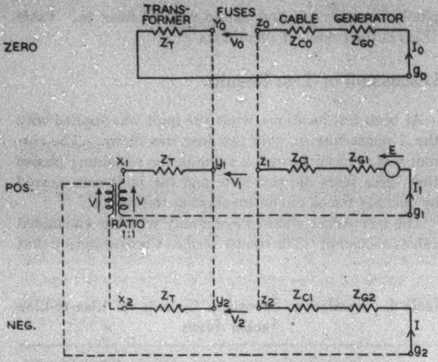

Figure 4. Calculating board set-up for the solution of figure 3 by the method of symmetrical components

insulating transformer must be reversed from the normal method of connection.

The algebraic solution of this circuit is as follows:

Relation between currents:

At open circuit: $I_0 + I_1 + I_2 = 0$ (1)

At short circuit: $I_2 = I_1$ (2)

Equating voltages between points y and z

$$v_0 = v_1 = v_2 \qquad (3)$$

or

$$I_0Z_0 = -E + I_1Z_1 + V = I_2Z_2 - V \qquad (4)$$

where

V = voltage across the one-to-one transformer.

Z_0, Z_1, Z_2 are the sum of the impedances in the respective networks.

Eliminating I_0, I_2, and V from (4), we find

$$E = I_1(Z_1 + Z_2 + 4Z_0) \qquad (5)$$

Let i be the actual value of current flowing through the transformer winding. (I_a' of figure 3.)

$$\begin{aligned} I_1 &= 1/3(I_a + aI_b + a^2I_c) \\ &= 1/3[0 + a(-i) + a^2(-i)] \\ &= 1/3i \qquad (6) \end{aligned}$$

Therefore

$$\begin{aligned} E &= i/3(Z_1 + Z_2 + 4Z_0) \\ &= i/3(Z_{g1} + Z_{g2} + 4Z_{g0} + 2Z_{c1} + 4Z_{c0} + 6Z_T) \end{aligned}$$

Where

E = Phase-to-neutral voltage on low-voltage network

and

I_1, I_2, I_0 are the sequence currents in the respective networks.
i = actual current in transformer winding. (See figure 3.)

References

1. Experimental Analysis of Double Unbalances, Edward Kimbark. AIEE Transactions, volume 54, 1935, pages 159–65.

2. Simultaneous Faults on Three-Phase Systems, Edith Clarke. AIEE Transactions, volume 50, 1931, pages 914–41.

Lightning Strength of Wood in Power Transmission Structures

By PHILIP SPORN
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I. Introduction

ALTHOUGH wood has been known to possess the excellent properties of mechanical strength in combination with insulation strength almost since the beginning of the electrical art, its real lightning strength was not generally appreciated until within comparatively recent years. It is true that it was rather extensively used in the past to supply power frequency insulation in treated form alone or with transil oil. For example, wood either solid or laminated has been employed for years in operating rods on oil circuit breakers and in terminal and spacing blocks in transformers. The 60-cycle strength of wood under those conditions has been studied, and although the results were not made available generally, the strength factors had been known with considerable preciseness for some time. In these applications, however, the 60-cycle (i.e., power frequency) strength of wood has been of paramount interest, and the lightning or impulse insulation value of secondary interest, either because the latter was considered comparatively unimportant or because the impulse strength furnished by other insulating members or mediums in the structure was considered sufficient.

The use of wood as supporting structures in transmission and distribution circuits goes back to the beginning of the power distribution art. Here, although it has not been generally realized, the lightning or impulse value of the wood has been utilized consciously or unconsciously and more frequently the latter. The conscious realization of the value of the lightning strength of wood is comparatively recent[1] and goes back little further than 12 years when a comprehensive study of the lightning problem on transmission lines was first undertaken. However, even this late realization of its value was accompanied by very little actual knowledge of the mechanism of such insulation strength. Such conscious realization was first presented in a paper by Melvin[2] giving the impulse insulation characteristics of wood and of wood in combination with insulators. A number of other investigations [3–8] have been described since then, each of which added to our store of knowledge. However, it was felt that certain vital factors had been given little or no study, and that insufficient data were available on the volt-time characteristics of wood in its various forms and in combination with porcelain. Further, little distinction has heretofore been made on the effect of the polarity of the impulse on wood flashover. Again, few test data have been given on actual assemblies of full-fledged transmission lines.

II. Purpose and Scope of Investigation

In the light of the above, the authors undertook a very comprehensive investigation of the lightning strength of wood alone, as used in transmission structures, and in combination with porcelain insulators (pin and suspension) to determine:

(*a*). The volt-time characteristics of the materials in question.

(*b*). Polarity effects, that is, the variation of the characteristics with positive and negative surges.

(*c*). The effect of moisture.

(*d*). The effect of age.

(*e*). The effect of the material employed, such as cedar, pine, and fir.

(*f*). The effect of treatment, particularly creosoting which is coming more and more into common use.

In the course of this investigation actual impulse tests were made on pole structures as used in the past few years on transmission lines, both for 33-kv and 66-kv service. Here tests were carried out on actual pole-line assemblies as they existed before and after structural changes were made to take advantage of the wood impulse insulation strength.

III. Test Equipment and Procedure

All of the flashover studies were carried out in the Ohio Brass High-Voltage Laboratory at Barberton with surges applied from the 3,000,000-volt impulse generator. The standard $1^1/_2$ by 40-microsecond wave was used, thereby allowing comparison with the impulse data of other insulation for the purpose of system co-ordination. Both positive and negative surges were applied in practically all cases, and oscillograph records were secured of the resultant voltage waves. From these oscillograms the volt and time data were taken for plotting the various curves here given.

Figure 1 illustrates some of the typical oscillograms, with impulses applied ranging from full waves to waves resulting in flashover of the structure under test in 1.8 microseconds. The nesting of the curves, as shown, is a very effective method of accurately correlating the records as the test work progresses, as well as greatly reducing the time otherwise required in changing film.

All flashover values were corrected to standard air density conditions. Inasmuch as no humidity correction factor for wood has been established as yet, no corrections were made for this atmospheric factor. To make possible

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1. For all numbered references, see list at end of paper.

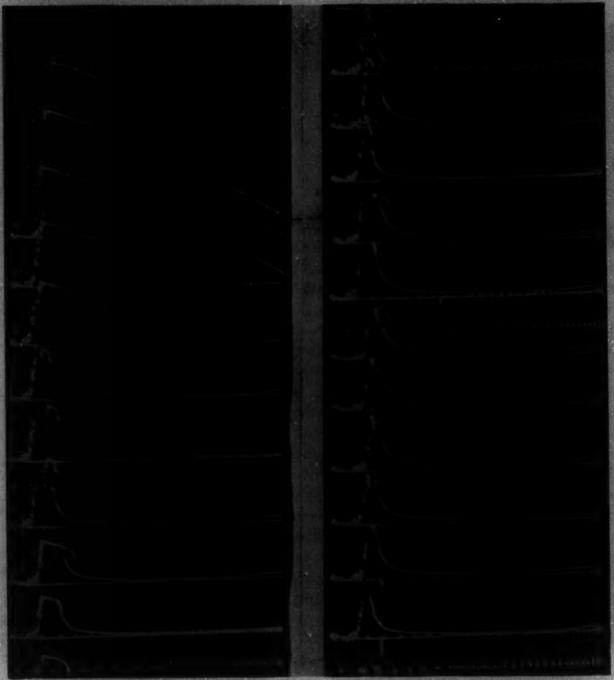


Figure 1. Typical oscillograms of impulse tests on wood and wood-porcelain structures. The above tests were made on three 5- by 10-inch suspension insulators plus four feet of creosoted pine crossarms with 1½x40 microsecond positive polarity

a proper comparison therefore of wood, porcelain, and air-flashover values, the latter two were corrected to the humidity at which the wood specimens were tested.

The wood crossarms studied consisted of typical specimens of untreated and creosote dipped Douglas fir, and creosote impregnated pine. Crossarm sizes varied from three and one-half by four and one-half inch to six by eight-inch cross section. In most cases new and old specimens were used, the latter having seen varying years of field service up to about ten years. For pole members, red cedar and creosoted pine were used, new and similar old specimens being chosen here as well.

Various lengths of individual wood specimens were flashed over in order to determine the impulse strength furnished by each. Braided copper bands which could be moved along the wood were employed as electrodes, and the wood lengths tested varied from a few inches up to eight feet. Voltage from the impulse generator was always applied to one band while the other band was grounded. In each case the wood specimen was suspended at a distance from ground equal to several times the length of gap being tested, in order to allow as little interference as possible with the field surrounding the wood gap. In securing flashover data upon successive lengths of the same specimen, care was taken to avoid wood surfaces which were too badly splintered from previous discharges. A certain amount of splintering had no appreciable effect upon the flashover voltages, although badly shattered members showed some reduction of insulation strength.

Tests were made on combinations of wood and porcelain to determine the total insulation strength afforded. In these studies varying sizes of pin-type insulators and lengths of suspension strings were used. These were mounted on the several types of crossarms noted above and flashover made with different lengths of wood in series with the insulator units. In this way the effect of various combinations of porcelain and wood insulation were investigated. An effort was made to confine the tests to combinations which were typical of field installations. Figure 2 illustrates such a typical specimen on test.

IV. Test Results—Structure Members

1. Crossarms

Eleven crossarm specimens of the three types of wood previously noted were given impulse flashover tests, and volt-time curves plotted from the oscillograph data. Figure 3 illustrates two sets of such curves for 12-inch and 36-inch wood gaps. Corresponding NEMA rod-gap curves are also given in order to show the comparative volt-time characteristics of air gaps. In figure 4 are given the minimum and two-microsecond flashover values for various lengths of wood arms.

These arms were of various cross sections. It being quite apparent from the test data that cross section is a vital factor in the impulse flashover value of an arm an effort was made to determine the effect of this factor upon impulse insulation strength. Accordingly, new curves were plotted of flashover voltage against cross-sectional area. Figure 5 (*A* and *B*) shows such curves for creosoted and untreated fir, and creosoted pine. It will be noted that these are practically straight lines with flashover

Figure 2. Typical impulse flashover in the laboratory of 33-kv pin insulator plus two feet of creosoted Douglas fir crossarm

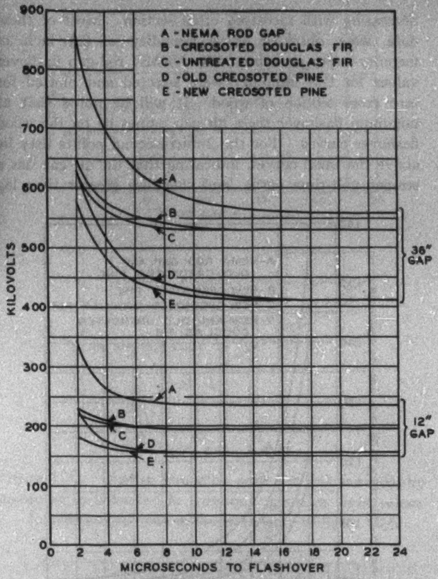


Figure 3. Impulse flashover of wood crossarms of 20-square-inch cross section; positive polarity

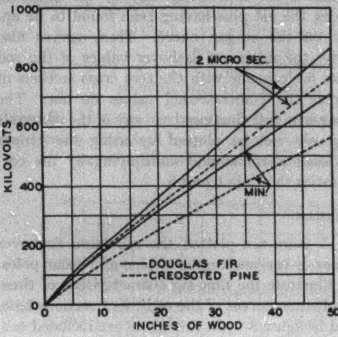
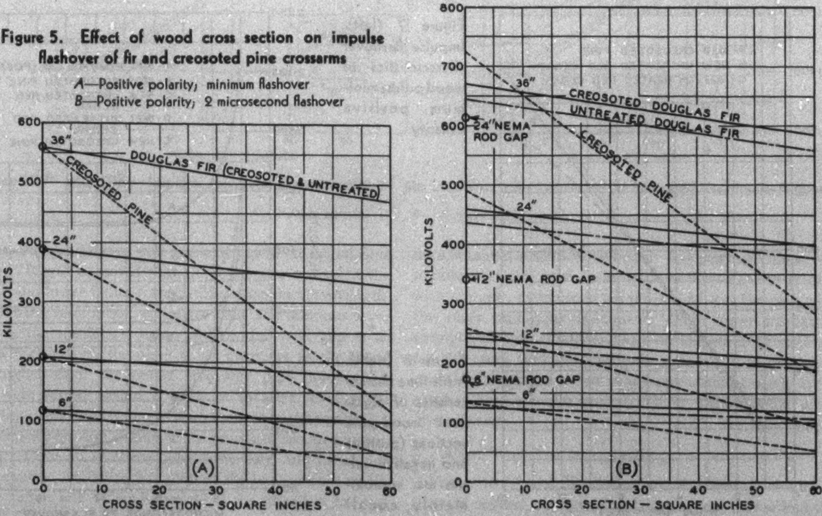


Figure 4 (left). Impulse flashover of Douglas fir and creosoted pine crossarms. Average of positive and negative average values. Two-microsecond and minimum values only are given

Figure 5. Effect of wood cross section on impulse flashover of fir and creosoted pine crossarms

A—Positive polarity; minimum flashover
B—Positive polarity; 2 microsecond flashover



decreasing with increased cross section. Most of these data were obtained at a humidity of 0.4 inch of mercury-vapor pressure. The NEMA rod-gap flashover values for this humidity were selected and plotted for zero cross section of wood. It will be noted that at minimum flashover these air-gap values lie on the wood flashover curves. For the 2-microsecond points they lie above the wood curves, indicating that the air gap has a steeper volt-time curve, and therefore greater time lag than the corresponding wood gap. This decrease of flashover strength with increase of cross section is probably due to the fact that there is a greater chance of weaker flashover paths, the larger the wood cross section. Obviously, too, the larger members have more chance of retaining internal moisture and other impurities which lessen the insulation strength. It will be noted, too, that the steepness of the curves is greatly influenced by the class of wood and treatment. The rapid decrease of strength in the case of the pine arms was unexpected and further work is to be done to check this.

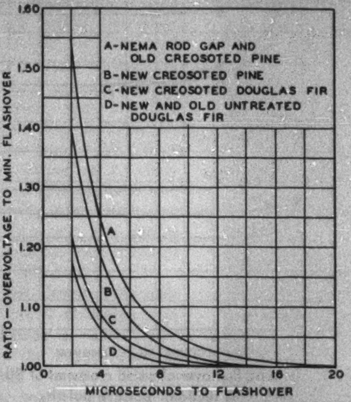


Figure 6. Volt-time characteristics of 36-inch crossarms; 20-square-inch cross section; of various woods; positive polarity

In figure 6 there is plotted a form of volt-time curve expressing ratios of overvoltage to minimum flashover in order to permit a direct comparison of the insulating properties of the various materials involved. It will be noted that the rod-gap curve is the highest one of the group. Its characteristics are approached only by the creosoted pine members, the old pine having been found to lie approximately on the air-gap curve. These curves also illustrate why the minimum flashover values of the rod gap on figure 5*A* coincide with the zero cross sections of wood, whereas the 2-microsecond values do not. The corresponding true volt-time curves of any of the materials shown obviously can be plotted by using the proper minimum flashover values and multiplying by the corresponding ratios shown in figure 6.

2. Wood Poles

In figure 7 there are plotted the minimum flashover voltage values of various lengths of pine and cedar poles. In order to illustrate the time-lag characteristics of these members, volt-time curves of the eight-foot sections have been plotted in figure 8. On this figure are included cor-

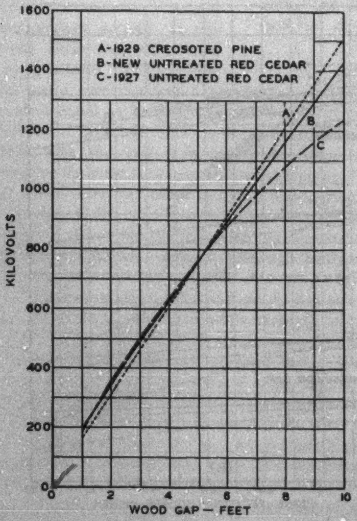


Figure 7 (left). Impulse flashover characteristics of wood poles; minimum positive polarity

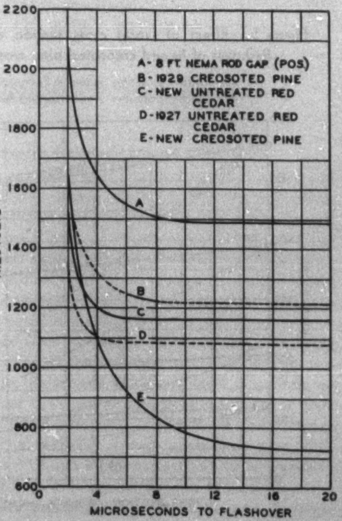


Figure 8 (right). Volt-time characteristics of eight-foot wood-pole sections (positive and negative values are approximately equal)

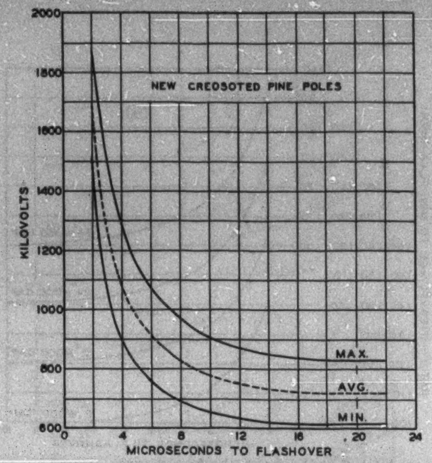


Figure 9. Variation in impulse flashover characteristics of wood poles; eight-foot length; positive wave (positive and negative values approximately equal)

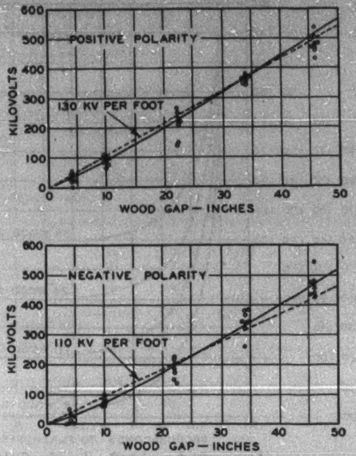
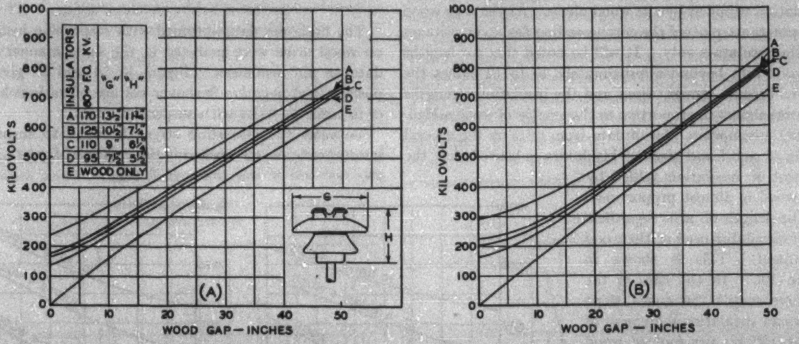


Figure 10C. Impulse insulation added to pin insulator by creosoted or untreated fir crossarm; minimum wave values. Insulators used same as shown in table of figure 10A



INSULATORS	60~ F.O. KV.	"G"	"H"
A	170	13½"	11 3/16"
B	125	10½"	7¼"
C	110	9"	6½"
D	95	7½"	5½"
E	WOOD ONLY		

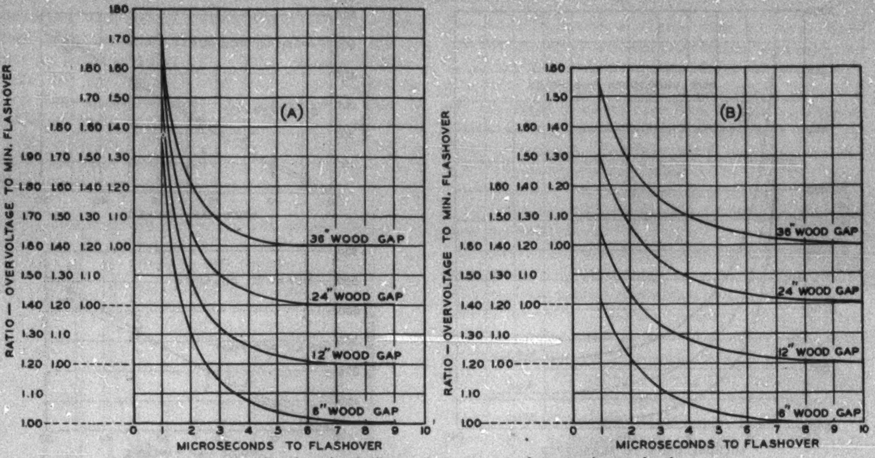
Figures 10A and 10B. Impulse flashover characteristics of pin insulators plus creosoted fir crossarm; minimum flashover

A—Positive polarity *B*—Negative polarity

responding spacings of rod gap and newly creosoted pine poles. It will be noted that the latter two represent the upper and lower limits respectively of insulation strength. A further examination will show that at minimum wave values, the insulation value of the plain rod gap is as much higher than the older wood members as the latter are higher than the newly creosoted pine poles. The last were tested within a week or so after leaving the treating plant, and it is felt that their reduced insulating properties were due largely to the moisture retained within them. Over 50 of these poles were used and numerous tests made. In figure 9 is plotted the band within which these values fell. This is typical of the spread in the test data obtained in some wood tests. It is pertinent, however, to note that this spread was found to be somewhat greater with the new, moisture-laden poles than was the case with the older and drier specimens. This lower insulation strength of new, undried poles would seem to account for the more frequent flashover and splitting often encountered during the first year's operation of newly creosoted poles, as compared to their performance after several years.

3. Wood Plus Pin Insulators

Figure 10 (*A* and *B*) shows data obtained on combinations of several pin-type insulators, and various lengths



Figures 11A and 11B. Impulse flashover characteristics of pin insulators plus fir crossarms

A—Positive polarity *B*—Negative polarity

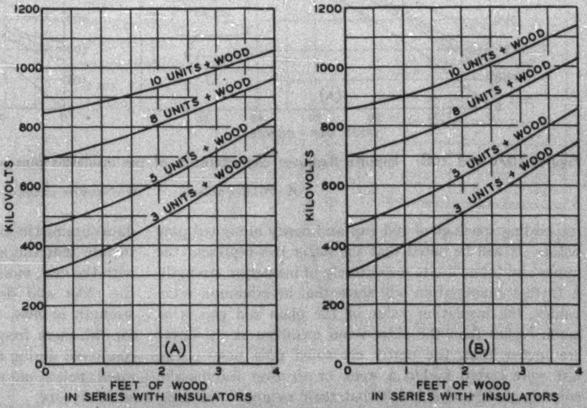
of wood arm. The curves for "wood only" show the insulation supplied by the wood alone. At the zero wood gap points are plotted the corresponding flashover voltages for the insulators only. It will be noted that for lengths of wood arm beyond approximately 10 to 12 inches the curves become straight lines and the insulation strengths increase almost in proportion to the lengths of wood added. In fact a composite plot of data from tests on the several forms of wood and sizes of insulators indicated that the amount of insulation added by the wood is almost proportional to the length of arm regardless of the insulator used or the wood treatment. This is shown in figure 10*C*. In the case of the positive surges the curve slope shows an insulation increment of about 130 kv per foot of wood and for the negative surges about 110 kv per foot, these values representing minimum flashover values.

In figure 11 (*A* and *B*) are given the overvoltage characteristics of pin insulators on wood arms. These curves are derived from data taken on both creosoted and untreated arms as it was found that the wood treatment had comparatively little effect upon the resultant insulation strengths of the arm and insulator combinations tested.

4. Wood Plus Suspension Insulators

The flashover data obtained with suspension insulators on wood arms were analyzed in the same manner as the data on pin insulators. Figure 12 (*A* and *B*) gives the positive and negative flashover voltage of various lengths of fir arms in series with suspension units.

To show the insulation strength added to suspension insulators by wood crossarms the curves of figure 13 were



Figures 12A and 12B. Impulse flashover characteristics of 5-inch by 10-inch suspension insulators plus fir crossarm; minimum flashover

A—Positive polarity *B*—Negative polarity

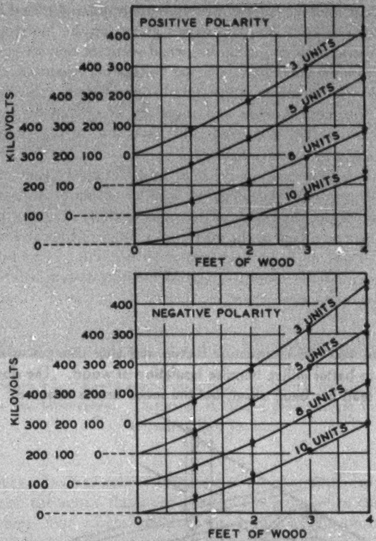


Figure 13. Impulse insulation added to suspension insulators by crossarms—minimum wave values

o—Creosoted pine
x—Untreated and creosoted fir

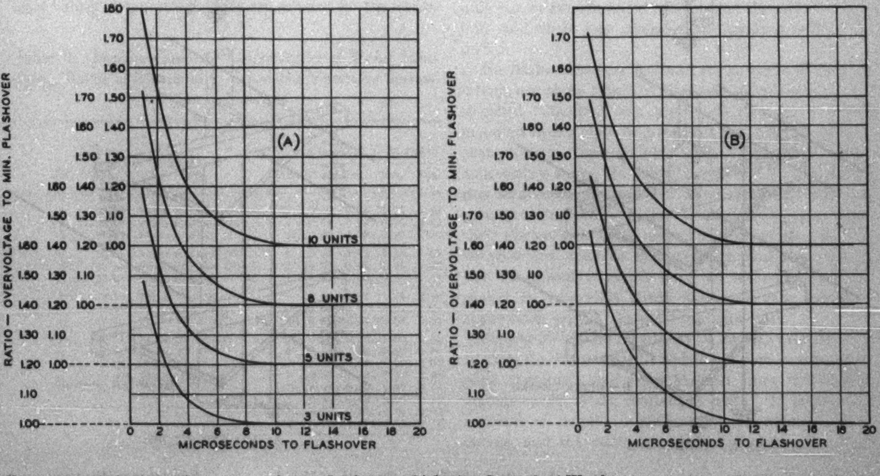
Table 1. Comparison of Calculated and Measured Impulse Flashover of 33-Kv Wood Pole (See Figure 15)

Member	Location	Min. Wave Pos.	Min. Wave Neg.	At 2 Ms. Pos.	At 2 Ms. Neg.
Case 1. Phase-to-ground flashover in kv (wood values from figures 4 and 6)					
Insulator	(*a*)	175	225	236	248
3½" Wood	(*b*)	65	65	85	85
Sum	(*a*)(*b*)	240	290	321	333
Measured	(*a*)(*b*)	214	256	300	276
Ratio—Measured/sum		0.89	0.88	0.93	0.83
Case 2. Phase-to-phase flashover—horizontal in kv (wood values from Figures 4 and 6)					
Insulator	(*a*)	175	225	236	248
3½" Wood	(*b*)	65	65	85	85
3½" Wood	(*c*)	65	65	85	85
Insulator	(*d*)	225	175	248	236
Sum	(*a*)(*b*)(*c*)(*d*)	530	530	654	654
Measured	(*a*)(*b*)(*c*)(*d*)	395	420	500	540
Ratio—Measured/sum		0.74	0.79	0.76	0.82
Case 2A. Same as (2) but calculated from combinations of insulation plus wood F.O. in Kv (wood values from figures 10*C*, 11*A*, and 11*B*)					
Insulation and wood	(*a*)(*b*)	205	245	265	295
Insulation and wood	(*c*)(*d*)	245	205	290	265
Sum	(*a*)(*b*)(*c*)(*d*)	450	450	560	560
Measured	(*a*)(*b*)(*c*)(*d*)	395	420	500	540
Ratio—Measured/sum		0.88	0.93	0.89	0.96

developed for both positive and negative impulses. It will be noted that the wood adds more insulation to the shorter string lengths. This is probably due to the voltage distribution between the porcelain and wood insulations in series, as affected by their relative capacities, a point which is being given further study. Another interesting factor developed was that although fir arms have higher flashover values than creosoted pine arms when alone, there seems to be little difference between the two when assembled with insulators. This is clearly indicated in figure 13. The wood values in these curves may be used with suspension units of any of the standard dimensions.

Figures 14A and 14B. Impulse flashover characteristics of suspension insulators plus fir crossarm, one to four feet long

A—Positive polarity
B—Negative polarity



The time-lag characteristics of typical suspension insulators on wood arms from one to four feet long are given in figure 14 (*A* and *B*).

V. Test Results—Complete Structures

Laboratory studies were made of typical transmission line structures in order to determine how well their insulating properties could be calculated from the flashover values of the component parts, these latter values being secured from data curves such as given in the previous figures. Figure 15 shows one of the 33-kv structures, (*A*) being the arrangement before, and (*B*) the arrangement after, construction changes had been made to gain lightning insulation.

Table I gives the data obtained from such an analysis and test. Separate data corresponding to the various parts or members of the structure analyzed and tested are given. In each case the values given for the individual members, such as insulators, wood sections, and air gaps, are from the particular flashover voltage curve corresponding to that member. The stated sums of these values are the arithmetical sums and these are compared by ratio with the actual measured values secured by test on the particular group of members.

In the first case analyzed in table I namely that of a phase-to-ground flashover on a 33-kv insulator and arm combination, the ratios of the sums of the voltages of the separate members to the measured voltage of the combination is seen to be around 0.90 or 90 per cent. This ratio was found to be about the order of magnitude generally obtained in the case where the sum of the flashover voltages

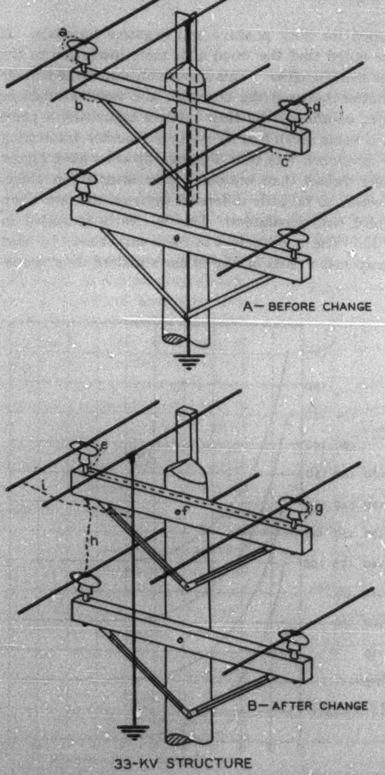


Figure 15. 33-kv structure before and after redesign to increase its impulse strength. Note the use of wood crossarm braces, and the down lead offset from the pole in B

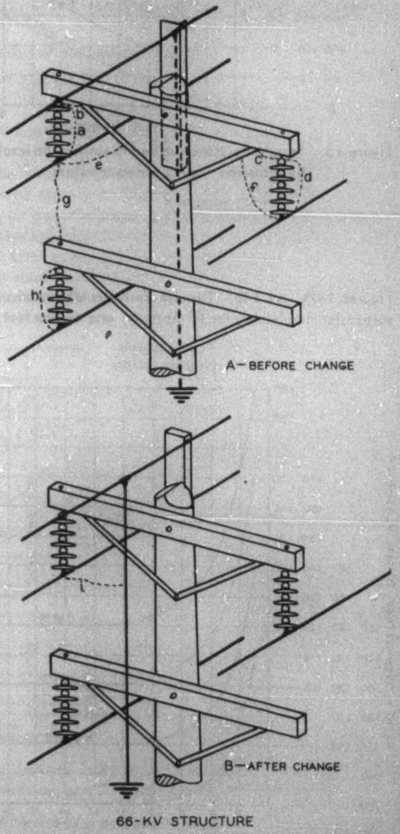


Figure 16. 66-kv structure before and after changes in design to better utilize impulse insulation of wood. The down lead has been offset, but the metal braces retained

Figure 17. 33-kv wood-pole structure being impulse tested in the laboratory. Flashover is taking place across the top crossarm, line to line

Figure 18. 66-kv wood-pole structure flashing over on laboratory test from conductor to down lead by way of the crossarm, brace, and wood pole

of two single members or groups of members are compared with the actual flashover value of the group. In the case where four single members or groups of members are involved, a ratio of about 0.75 or 75 per cent was found to hold. This is brought out in the second case of table I, where the sums of the individual flashover values of two insulators and two sections of wood are compared with the test values of the combination.

On the other hand if the four members involved, namely, two insulators and two wood sections, are grouped in pairs, each consisting of an insulator and a wood section, values may be chosen from figures 10*C*, 11*A*, and 11*B*. This results in a ratio of approximately 0.90 or 90 per cent as noted for case 2*A*.

In adding these phase-to-phase values, it will be noted that values of opposite polarity are selected for the insulators in each summation. The reason for this is that when one conductor is energized and the other grounded the insulator supporting the energized conductor experiences an electrostatic field of opposite polarity from that of the insulator supporting the grounded conductor. In all tests made phase-to-phase, the down lead was ungrounded.

Table II. Impulse Flashover Characteristics of Wood Structures; Ratios of Measured to Calculated Values by Summation of Elements

Ref.	Structure Type	Flashover Path	No. of Elements	Min. Wave Pos.	Min. Wave Neg.	At 2 Ms. Pos.	At 2 Ms. Neg.
1	33 kv	Phase to Ground Wire	2	0.89	0.88	0.93	0.83
2	33 kv	Phase to Phase (Hor.)	4	0.74	0.79	0.76	0.82
3	33 kv	Phase to Phase (Hor.)	2*	0.88	0.93	0.89	0.96
4	33 kv	Phase to Phase (Hor.)	4	0.74	0.72	0.84	0.68
5	33 kv	Phase to Phase (Hor.)	2*	0.95	0.93	1.06	0.85
6	33 kv	Phase to Phase (Vert.)	2	0.96	0.85	0.94	0.85
7	33 kv	Ph. to Grd. Wire	1	0.94	0.82	1.02	0.96
8	66 kv	Ph. to Grd. Wire	2	0.87	0.87	0.91	0.91
9	66 kv	Ph. to Ph. (Hor.)	4	0.72	0.72	0.82	0.82
10	66 kv	Ph. to Ph. (Hor.)	2*	0.92	0.92	1.02	1.02
11	66 kv	Ph. to Ph. (Hor.)	2	0.86	0.86	0.87	0.87
12	66 kv	Ph. to Ph. (Vert.)	2	0.89	0.84	1.00	0.91
13	66 kv	Ph. to Grd. Wire	1	0.93	0.88	0.95	0.90
		Average	1	0.94	0.85	0.99	0.93
			2	0.89	0.86	0.93	0.83
			2*	0.92	0.92	0.96	0.95
			4	0.75	0.75	0.81	0.77

* Each element consisted of porcelain and wood or air.

Figure 16 (*A* and *B*) illustrates the 66-kv structure where the down lead has been moved from the pole to secure increased insulation. Figure 17 shows the 33-kv structure flashing phase-to-phase on test. Figure 18 shows a similar flashover, phase-to-ground, on the 66-kv structure.

In table II the ratios of measured to calculated impulse flashovers for both 33-kv and 66-kv structures are given. It will be noted that where the flashover path is across only one or two elements of insulation the ratio is around 0.90, and where four elements are involved it is around 0.75.

The flashover values taken for the air paths in the calculations are those from the flashover data secured on the so-called "standard rod gap." As this gap is usually mounted rather close to ground, it has a distinct polarity characteristic, that is, the negative flashover value is appreciably above the positive. However, on transmission structures, air gaps are often rather far from large grounded structures, so that there is little to unbalance their electrostatic fields and cause appreciable differences between the positive and negative flashover voltages. For this reason it is felt that the ratio in these tables should be somewhat lower where air gaps involve an appreciable part of the total insulator path.

Figures 19 (*A* and *B*) and 20 (*A* and *B*) were developed to show the accuracy with which volt-time curves of complete structures can be calculated from the flashover voltage curves of the component parts. In these figures, two sets of curves are shown, one the actual measured curves, and the other the estimated curves secured by adding component voltages and multiplying by 90 per cent or 75 per cent dependent upon whether two or more figures were added, as discussed above. It will be noted that the two sets of curves agree rather well.

These curves also show the increased impulse strength obtained by the change in construction, that is, by using wood braces and offset down lead on the 33-kv structure, and by offsetting the down lead alone on the 66-kv structure.

A short series of wet tests on wood structures was also made. These tests are not reported here in detail. They apparently indicate, however, that the impulse flashover voltages would be little affected at short-time lags (less than 4 microseconds), but that at long-time lags and at minimum flashover, appreciable reductions might be experienced. In some cases the flashover voltages were decreased 50 per cent. Additional tests are being made on structures under wet conditions to obtain more complete data on their flashover characteristics.

VI. Conclusions

As a result of these tests comprising the application of some 4,000 to 5,000 individual impulse voltages on wood alone and in combination with porcelain, both on individual members and actual structures, the following conclusions seem to be justified:

1. Wood has a definite and sound field of application in supplying insulation against lightning voltages on wood structures such as are commonly used for transmission and distribution circuits.

2. The wood in crossarms can be taken advantage of to supply appreciable impulse insulation on lines using both pin insulators, where the impulse insulation is comparatively low, and on suspension insulators in the higher-voltage ranges where the insulator strings normally have a relatively high insulation strength.

Impulse insulation supplied by wood crossarms in combination with pin insulators is in the order of 110 to 130 kv per foot for minimum waves and somewhat higher for short-time lags.

Tests on individual members as well as complete structures indicate that the flashover path changes as the overvoltage increases. For example, in some cases, flashovers appearing through air for minimum waves will take a path across the wood as the overvoltage increases due to the faster rising volt-time characteristic of the air gap breakdown.

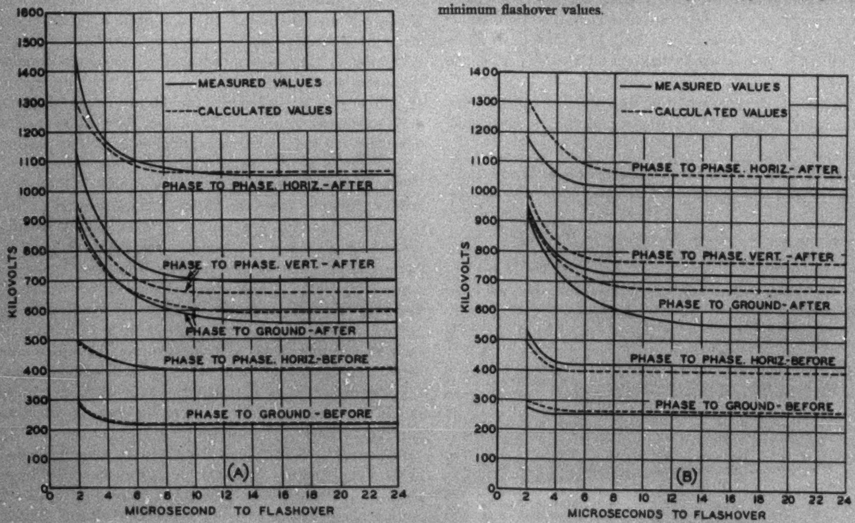
3. The flashover of wood insulating members is affected by the cross-sectional area of the wood, being in general lower the larger the wood section. The factors producing this result are not well understood and the subject should be further investigated.

4. The type of wood commonly used, for example, fir or pine, as used in crossarms, influences the flashover characteristics, fir having slightly higher flashover voltages than pine.

The volt-time characteristics also are different for the two woods, the characteristic for pine being steeper at short-time lags than for fir.

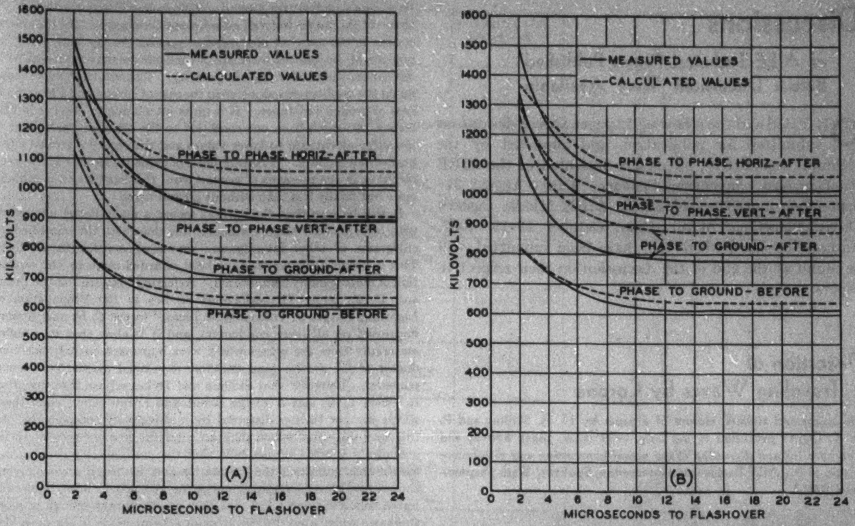
When used in combination with porcelain insulators, however, these different characteristics do not appear to be important.

5. The impulse flashover of cedar or aged pine wood poles appears to be very much the same and independent of treatment in lengths up to six feet. For greater lengths, a slight difference is indicated but of relative unimportance in selecting wood-pole material. New creosoted pine poles have considerably less impulse strength at minimum flashover values.



Figures 19A and 19B. Measured and calculated impulse strength of 33-kv wood structures of figure 15

A—Positive polarity *B*—Negative polarity



Figures 20A and 20B. Measured and calculated impulse strength of 66-kv wood structures of figure 16

A—Positive polarity
B—Negative polarity

6. The few tests made on wood under wet conditions indicate that its insulation strength is considerably decreased by the presence of moisture and rain. The preliminary work carried out indicates a reduction in impulse insulation of some 50 per cent under rain conditions. This is another point that needs further investigation.

7. The impulse insulation of wood in structures alone and in combination with porcelain can be calculated with a considerable degree of certainty. It is believed that the data presented here together with reduction factors obtained from tests on actual structures will be helpful in facilitating such calculations in the design of well-balanced wood transmission structures.

8. The data presented above, in the form of groups of curves giving the impulse characteristics of wood under various conditions, greatly amplifies our previous knowledge of the impulse insulating properties of wood, and should be useful in designing wood transmission structures to obtain greater benefit of the wood in supplying impulse insulation to the line. This is particularly important in the medium-voltage transmission lines where structures are inherently of wood, and until recent years, the insulation value of the wood has not been generally appreciated or made use of.

Considerable work still remains to be done in investigating further the impulse insulation strength of wood under various types of treatment, moisture content, and particularly under wet conditions simulating natural rain.

VII. Acknowledgments

The authors received material assistance in these studies from their associates and are particularly desirous of acknowledging the help received from Messrs. I. W. Gross of the American Gas and Electric Company, and C. J. Miller, Jr., of the Ohio Brass Company, in planning the tests and in analyzing the results. Most of the wood materials studied were furnished by The Ohio Power Company.

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Discussions

of AIEE Technical Papers Published Before Discussions Were Available

ON THIS and the following 12 pages appear discussions submitted for publication, and approved by the technical committees, on papers presented at the AIEE Pacific Coast convention, Spokane, Wash., August 31–September 3, 1937; and at the AIEE Middle Eastern District meeting, Akron, Ohio, October 13–15, 1937. Authors' closures, where they have been submitted, will be found at the end of the discussion on their respective papers.

Distortion of Traveling Waves by Corona

Discussion and authors' closure of a paper by H. H. Skilling and P. de K. Dykes published in the July 1937 issue, pages 850–7, and presented for oral discussion at the power transmission and distribution session of the AIEE Pacific Coast convention, Spokane, Wash., September 1, 1937.

L. V. Bewley (General Electric Company, Pittsfield, Mass.): Skilling and Dykes have made a contribution of fundamental importance toward the solution of wave distortion due to corona. In 1931 Skilling developed a formula for the attenuation of the wave, assuming the losses to vary as the excess voltage above the critical corona voltage, but no provision was made in his formula for the associated distortion. In the present paper he has succeeded in formulating an equation for the distortion which occurs on the front of the wave, and which appears to give surprisingly good agreement with the oscillographic record.

I wish to point out, however, that his equation does not take cognizance of the presence of mutually coupled parallel wires and is, in fact, based purely on a single conductor wave. Consequently, the authors' statement under "Other Theories" to the effect that there would be no sharp distinction between their analysis and multivelocity theory is hardly tentable. In the *Conclusions* of my paper "Attenuation and Distortion of Waves," ELECTRICAL ENGINEERING 1933, attenuation and distortion are attributed to three principal causes: (1) multivelocity components, (2) a slippage effect at high voltage levels caused by a reduction in velocity of propagation corresponding to the larger diameters of corona envelope (Boehne's hypothesis), and (3) energy losses. The distortion described by Skilling and Dykes is that due to (2) and (3), and *not* to (1). Three conditions must exist before multivelocity waves can develop: (*a*) There must be a mutually coupled *multiconductor* system, (*b*) the applied waves on this system must be unsymmetrical (for example, the main wave impressed on only one of the conductors), and (*c*) the geometric factors determining the electrostatic field must be *different* than those determining the electromagnetic field, which is possible with a resistive earth or corona. It is therefore manifestly impossible for multivelocity components to be called into existence if only one conductor is present, or if equal waves are simultaneously impressed on all conductors of a system; and yet the Skilling and Dykes distortion is active under both these conditions. I think, therefore, that the crucial test of their formula is to those situations where multivelocities do not exist to mask the results, and thus hinder the segregation of this particular effect.

Contrary to the view expressed by the authors I do not believe that multivelocity components are comparatively negligible at high corona voltages. As shown in my paper referred to, corona calls into existence components of an essentially different character than those due to a resistive ground. This is startlingly revealed by the shape of the waves induced on adjacent conductors. In the case of corona in the presence of a resistive ground the fast and slow components of the induced wave are of the *same* polarity; whereas without corona they are of *opposite* polarity. This vital difference shows up on the oscillograms as a step in the case of corona and a loop in the case of ground resistance. It is quite interesting to observe in field tests a transition from one condition to the other as the voltage of the surge attenuates to lower values and the resistive ground effect begins to dominate. A further piece of evidence lies in the observed fact that in the presence of ground wires the attenuation is initially more but finally less than without ground wires.

It will be seen that the authors secure a much better agreement with tests on their single-conductor line than with the published oscillograms of other investigators who used multiconductor lines. This is because the waves have been retarded more by the separation of multivelocity components. Referring to figure 1 of the Brune and Eaton paper "Experimental Studies in the Propagation of Lightning Surges in Transmission Lines," waves *C*, *D*, and *N* were impressed on all three conductors, and it is clear that they differ materially from the others which were impressed on only one conductor of the system (and therefore developed multivelocity components). I suggest that Skilling and Dykes adjust their equation to waves *C*, *D*, and *N* of the Brune and Eaton paper, since these waves are not further distorted by multivelocity components. In this way the effect which they are studying is more accurately segregated. While I do not hold that the multivelocity components are the chief culprits in the attenuation and distortion accompanying corona (see fourth paragraph under the *Conclusions* of my paper referred to above), yet I feel that they are important enough to mess things up when a worthwhile attempt is being made to segregate the corona *energy loss* factor. There are so many factors responsible for attenuation and distortion (corona, resistive grounds, skin effect, leakage conductance, series resistance) that an investigation of any one of them should endeavor to isolate that one alone as nearly as possible.

Concerning the filling in of the tail of a wave, the authors have repeated the explanation given by Brune and Eaton. They seem inclined to ignore it for "practical cases" which is questionable advice. The wave tail is important in station protection problems, because when the wave impinges on the station capacitance the resultant voltage is very much dependent on the length of the wave, and will be greater the more the wave is filled in. In order to estimate the amount of this filling in I have, on occasion, gone to the other extreme and assumed that all of the space charge returns to the wave as its tail decreases. Thus in an article "Protection of Stations Against Lightning" by Bewley and Rudge, *G. E. Review*, August 1937, a chart is given based on the assumption that the surge attenuates in accordance with the Foust and Menger formula, and that the charge of the wave remains constant. Actually, of course, a large amount of the space charge never returns to the conductor until after the passage of the bulk of the wave (particularly in the case of positive surges), but the assumption errs on the safe side in calculating station voltages.

H. H. Skilling: The authors find themselves in almost complete agreement with the excellent discussion by Mr. Bewley, and wish to offer a reply to the question that he has raised. This regards the relative importance of the effect of "multivelocity components" of a traveling wave and the effect of distortion due to corona, when the crest voltage of the wave is substantially greater than the voltage necessary to produce corona on the line.

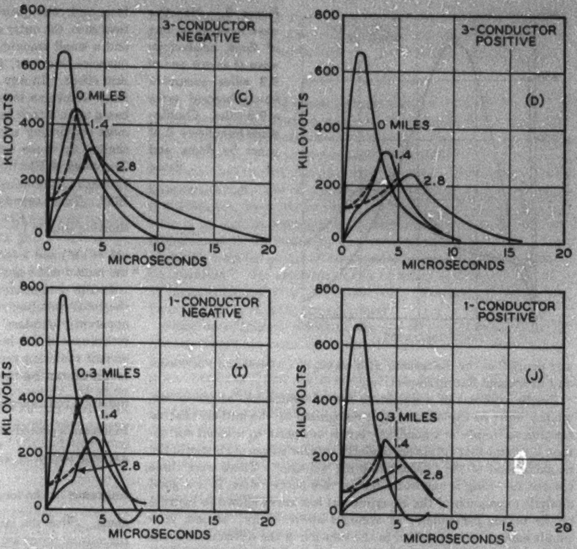
The question may be stated as follows: *Is the distortion of a traveling wave due to geometric factors, such as ground, ground wires, and near-by parallel conductors (as considered by Mr. Bewley in the theory of multi-velocity components) negligible when compared to the distortion of the wave due to corona loss, when the crest voltage of the wave is much greater than the corona-forming voltage of the line?*

It is certainly true that this question is not answered by the authors in their paper.

Mr. Bewley offers the following test: "I suggest that Skilling and

Figure 1. Computed waves compared with oscillograms by Brune and Eaton, waves C and D traveling on one of three conductors and waves I and J on all three

Dash lines computed



Dykes adjust their equations to waves *C*, *D*, and *N* of the Brune and Eaton paper, since these waves are not further distorted by multivelocity components." (Waves *C*, *D*, and *N* were impressed on all three conductors of the experimental transmission line simultaneously, and therefore would not develop multivelocity components.)

In carrying out Mr. Bewley's proposal, the authors went a step farther and compared computed values of waves with every one of the oscillograms reproduced by Brune and Eaton, if comparison was possible and appropriate. Brune and Eaton have sets of oscillograms for the following applied waves: their figures 1 and 2 show waves *A*, *B*, *C*, *D*, *I*, *J*, *K*, *L*, *M*, *N*; and their figure 8 shows a negative wave and a positive wave. Waves *K*, *L*, *M*, and *N* from figure 1 must be eliminated from this discussion because they are at all times below the corona-forming voltage of the line. The positive wave from figure 8 gives no opportunity for useful comparison because only one oscillogram of that wave is given with crest above the corona-forming voltage. There remain, then, waves *A*, *B*, *C*, *D*, *I*, and *J* of figure 1, and the negative wave of figure 8, for comparison with computed results. Of these, waves *C* and *D* were applied to all three conductors of the transmission line simultaneously, while *A*, *B*, *I*, and *J* and the negative wave of figure 8 were applied to only one of the three conductors, the other two conductors being present but not connected to the surge generator.

For comparison, curves from Brune and Eaton's paper are reproduced. Waves *A* and *B* from figures 1 and 2 of the paper by Brune and Eaton appear in figures 8*a* and 8*b*, respectively, of the paper by Skilling and Dykes (page 855). The other waves appear in figures 1 and 2 of the present discussion.

Distortion of all of these waves was computed with the following constants: $n^+ = 2^1/_4$, $n^- = 4$. No distinction was made between the case of waves on one conductor and the case of waves on three conductors. Of course, the corona-forming voltage and the capacitance were different in the two cases, and this was duly taken into account in the computation, but the factor n was unchanged. It will be remembered that the factor n is not merely an adjustable, empirical factor, as explained in the paper (pages 850 and 854). It is understood, however, that there is nothing in the authors' computations that takes into account the effect of near-by conductors on which no surge was applied. Consequently, if the distortion due to "multivelocity components" is a fairly large fraction of the distortion due to corona there should be decidedly better agreement between the computed results and the observed results when the surge was applied to all three conductors in parallel and there were no "multivelocity components."

Whether or not this is the case is purely a matter of judgment. Does it appear that there is better agreement between observed and computed results for waves *C* and *D* in figure 1 of this discussion than there is for waves *I* and *J* of figure 1, or for the wave of figure 2, or the waves of figure 8 of the paper? If the agreement seems about equally satisfactory in all cases (as it does to the authors) then it may be concluded that the distortion due to "multivelocity components" is, in these cases, very small compared to distortion due to simple corona loss, and that the formula (equation 14 of the paper) is reasonably correct despite the presence of near-by conductors.

One interesting detail of figure 2 of this discussion lies in the fact that Brune and Eaton give the form of the wave after it has traveled 2.8 miles, without giving the initial wave form. This figure, then, illustrates the statement in the paper that distortion can be predicted with *any* known form as data—it is not necessary to know the *initial* form. In fact, the initial form can be computed from the form of the wave at a distance of 2.8 miles along the line; this was done, and the result is shown in figure 2. Whether or not the computed initial wave is correct is not known, but it is quite consistent with other initial waves shown by Brune and Eaton. It must be noted, however, that the shape of the initial wave cannot be determined for voltages above the crest of the observed wave.

There is one point of interpretation on which the authors cannot agree with Mr. Bewley. He says, "It will be seen that the authors secure a much better agreement with tests on their single-conductor line than with the published oscillograms of other investigators who used multiconductor lines." The authors must confess that more trouble was encountered in fitting the distortion equation to their own results than to the published oscillograms of others. This is reflected in table I on page 856. Also it is felt that the poorest agreement appears in figure 3, page 852, where comparison is made to experimental results by Dykes; and the best in figure 9 of the paper

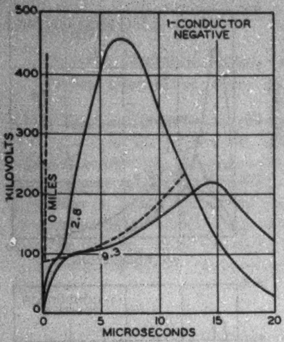


Figure 2. Negative wave traveling on one of three conductors; wave at source and at 9.3 miles computed from observed wave at 2.8 miles. Oscillograms from figure 8 of paper by Brune and Eaton

Dash lines computed

and figure 2 of the discussion, with waves of Conwell and Fortescue, and Brune and Eaton, respectively.

Finally, the use of a quadratic law of corona loss at power frequency must be explained. It is recognized by the authors that no relation so simple as a quadratic law is adequate to account for corona loss, and that this is particularly true for values of voltage in the neighborhood of the "corona-forming voltage." There were three reasons for using a quadratic law in this paper: first, it is a good analytic expression of the experimental loss curve shown in figure 4 of the paper, particularly at voltages above 75 kv; second, it is simple enough to be handled in the solution of the differential equation, equation 11; third, it is generally known (whether or not it is generally accepted) and it can readily be applied to a wide variety of conditions.

Several other expressions for corona loss were tried in the course of the study that led to this paper. At one time it was assumed that the loss was proportional to the first power of voltage above the corona-forming voltage; this described a traveling wave which was reduced at the crest but which maintained a constant slope of front—a result that is manifestly incorrect. Another attempt was based on loss proportional to the 1.6 power of the total voltage: this gave results that were good for high voltages, for it is a good approximation of the corona loss at high voltages, but it did not show a break in the front of the wave at the corona-forming voltage.

If it is possible experimentally to determine the actual corona loss on a line under investigation, that may be done. A true analytical expression can then be fitted to the loss curve, and equation 11 may be rewritten and solved for that special case.

Transmission Lines at Very High Radio Frequencies

Discussion and author's closure of a paper by Lester E. Reukema published in the August 1937 issue, pages 1002–11, and presented for oral discussion at the communication session of the AIEE Pacific Coast convention, Spokane, Wash., September 2, 1937.

S. A. Schelkunoff (Bell Telephone Laboratories, Inc., New York, N. Y.): In the course of his paper and in the concluding paragraph Professor Reukema states that a shorting disk used to short a coaxial line does not radiate. He further says: "Since such a disk does not radiate power itself, it cannot prevent the radiation of power through it from the current in the line." Thus, according to Professor Reukema, *power is radiated* from a coaxial pair shorted at both ends with conducting disks and energized by a generator somewhere in series with the inner conductor. But in reality, if the outer conductor and the shorting disks are perfect conductors, *there can be no radiation* since the electric field tangential to this outer surface, and consequently the Poynting flux, vanishes. In any practical case, the outer conductor and the shorting disks are not perfect and a small amount of power escapes through them. At high frequencies, however, this power is exceedingly small because of the skin effect. In any case, this is not the radiation with which Professor Reukema is concerned since his calculations of radiation are based upon a tacit assumption that the conductors are perfect. The most convincing argument consists, perhaps, of actual calculations similar to those contained in my paper on "Some Equivalence Theorems of Electromagnetics and Their Application to Radiation Problems," published in the *Bell System Technical Journal*, January 1936. These calculations show that the radiation resistance of the shorting disk is $\left(\frac{2\pi S}{\lambda^2}\right)^2$ ohms, where S is the area of the disk ($S = \pi b^2 - \pi a^2$) and λ is the wave length. It is assumed, of course, that the radii a and b are small compared with the wave length.

In the section on "Radiation Resistance of Parallel-Wire Lines" the total radiation resistance of a circuit comprised of two parts is apparently obtained by Professor Reukema by adding the radiation resistances of the individual parts without taking into account the *mutual* radiation resistances. That this procedure is dangerous is amply shown by the following example. It is well known that the radiation resistance of a short wire of length l is $80\pi^2\left(\frac{l}{\lambda}\right)^2$. Applying Reukema's procedure to two short wires, placed end-to-end to make a wire of length $2l$ we obtain $2 \times 80\pi^2\left(\frac{l}{\lambda}\right)^2$ ohms for the radiation resistance of the longer wire, instead of the correct value $80\pi^2\left(\frac{2l}{\lambda}\right)^2$ ohms. Since the latter figure is twice the former, it is evident that the mutual terms are equal to the radiation self-resistances. Another example is furnished by a coaxial transmission line one-half wave length long, open at both ends, and energized from the center. The radiation resistance of either conductor is about 73 ohms, barring small correction terms depending upon the radius (the radius is supposed to be small compared with λ). If the mutual radiation resistances are disregarded, the radiation resistance of the coaxial pair becomes 146 ohms. In reality the mutual radiation resistance is such that the principal terms in the total radiation resistance cancel, leaving only the small correction terms which depend upon the radii. In general, the complete expression for the flow of energy per unit area is

$$E \times H = (E_1 + E_2) \times (H_1 + H_2) = E_1 \times H_1 + E_1 \times H_2 + E_2 \times H_1 + E_2 \times H_2$$

where E_1, H_1 and E_2, H_2 are the fields produced by the currents in the separate parts of the complete radiating circuit. Evidently the middle terms are the mutual energy terms; they may be either positive or negative. In very special cases, of course, the middle terms may vanish.

C. W. Hansell and P. S. Carter (RCA Communications, Inc., Rocky Point, N. Y.): The section of this paper which treats the radiation from concentric lines is of particular interest to us since we have had considerable practical experience with numerous transmitters using concentric lines for frequency stability. We disagree with the author's conclusions concerning radiation from concentric lines. Contrary to the author's conclusion there would be no radiation from a concentric line of perfectly conducting material closed at both ends. In a practical arrangement wherein the transmitter is entirely shielded with copper of ordinary thickness and this shield is made continuous with the outside conductor of a concentric line the radiation is negligible. Both practice and correct theoretical analysis substantiate the truth of these statements.

Reukema cites as an example a quarter-wave line for 60,000,000 cycles per second for which we gave a Q of 20,000 whereas he has calculated the Q to be only 3,420. He states that radiation is 5.6 times the other power losses or 85 per cent of the total power input to the line. According to this statement resonant lines should radiate almost as much of the total input power as dipole antennas

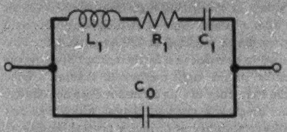


Figure 1

and there should be intense electromagnetic fields in the space surrounding them just as there would be in the space around an antenna. As a matter of fact completely shielded transmitters with resonant-line frequency control, such as those in commercial service on the radio relaying circuits between New York and Philadelphia, operated by RCA Communications, Inc., have so little radiation that a sensitive receiver is required to detect their radiation in the transmitter room. Even this radiation has been shown, by probe tests, to be leaking out over the power leads which are not quite perfectly filtered.

We have some resonant line installations in which power inputs to the resonant lines are on the order of kilowatts but still the electric fields surrounding them are negligible. If we actually radiated 85 per cent of the line input power, as Reukema says we do, we know from experience with unshielded circuits and indoor antennas that the transmitter room would be filled with intense electromagnetic energy by means of which lamps could be lighted without wires and operators would be subjected to great discomfort from high-frequency heating and artificial fever.

As a matter of fact Reukema's conclusions are contrary to common experience with high-frequency shielding. Most radio engineers are so well aware of the effectiveness of shielding that they take it for granted a water-tight metal enclosure completely surrounding any kind of high-frequency electrical circuit, regardless of shape or dimensions, will make the radiation negligible.

To connect up experience with theory let us consider a concentric line made up of perfect conductors which is closed at both ends and includes within it a radio transmitter. The energy generated by the transmitter is stored in the electromagnetic fields existing in the space between the conductors because a perfect conductor can sustain no electric field. In order for energy to be radiated from such a line an electromagnetic wave must penetrate the conductor. However, an electromagnetic wave cannot exist without storage of energy in the electric field and we have just stated that a perfect conductor can sustain no electric field. It therefore follows that passage of electromagnetic waves through the outer conductor and radiation from such a line could take place only by violating the fundamental laws of electromagnetic theory.

The author shows no derivation of the formulas given but, to one who is familiar with such problems, it is apparent that perfect conductors were assumed. Since these equations show very definite and comparatively large amounts of radiation the question naturally arises as to wherein lies the error. The well-known transmission line equations are only approximations which, while sufficient for most engineering purposes, are inadequate for treating the radiation from a concentric line. It is evident that the current distributions assumed in deriving the formulas given were taken from the common transmission line equations. A fundamental analysis of a concentric line indicates that such current distributions can exist only along the central portion of a line many wave lengths long, yet these current distributions have been assumed to hold for lines even shorter than a quarter wave length. Although an exact solution of the wave equation for a finite concentric line having closed ends is exceedingly difficult, if not mathematically impossible, we can state that no distribution of currents within such a system can exist which will produce other than zero electric or magnetic field strengths at *all* positions lying outside the conductors.

An electromagnetic wave will penetrate an imperfect conductor such as copper. However, at the very high radio frequencies under consideration here the attenuation of such a wave is so extremely high that the radiation passing through copper of any thickness ordinarily used would be of an infinitesimal amount in comparison with the quantities under consideration in the formulas of this paper.

W. P. Mason (Bell Telephone Laboratories, Inc., New York, N. Y.): In his interesting paper on radiation from transmission lines Professor Reukema has made some valuable calculations on the effect of radiation on the Q of balanced and concentric lines which should be a contribution to their use as circuit elements. He appears to feel, however, that the use of crystals in the ultrahigh-frequency region is not feasible, for he states that "The Q of a crystal (defined as the ratio of its inertial reactance to its frictional resistance) is inversely proportional to the frequency and assumes such a low value at the ultrahigh frequencies as to be practically worthless for frequency stabilization." He mentions values of the Q of a crystal as 170 at 30 megacycles and 17 at 300 megacycles. Fortunately these statements do not agree with our experience.

Since no published values of the Q's of crystals over wide frequency ranges are available it was thought worth while to measure a number of crystals covering a wide frequency range. Accordingly a number of *AT* and *BT* crystals whose fundamental frequencies varied from 800 kilocycles to 20 megacycles were measured. The Q of a crystal is defined with respect to the equivalent circuit of the crystal shown on figure 1. Here the static capacity of the crystal is designated by C_0, while the effect of the motional impedance of the crystal is represented by the series resonant circuit L_1, C_1, and R_1. The Q of the crystal is defined as the ratio $(2\pi f_R L_1)/R_1$ where f_R is the resonant frequency.

At the lower frequencies the method used to measure the Q was the one described in a former paper ("Electric Wave Filters Employing Crystal as Elements," *Bell System Technical Journal*, July 1934, page 431) and shown on figure 2. It consists of measuring the resonant frequency f_R, the antiresonant frequency f_A, the resistance at resonance R, and the static capacitance of the crystal C_0. From these the Q of the crystal can be calculated from the formula

$$Q = \frac{f_R^2/(f_A^2 - f_R^2)}{2\pi f_R C_0 R} \doteq \frac{1}{4\pi R C_0 \Delta} \text{ where } \Delta = f_A - f_R \qquad (1)$$

At higher frequencies the measurement of resistance becomes less reliable. At these frequencies, however, the reading of vacuum-tube voltmeters is satisfactory, so the circuit of figure 2 was modified to that of figure 3, and the resonant and antiresonant frequencies were measured as well as the voltages across the low terminal resistance at resonance and antiresonance. It is readily shown that the Q of the crystal will be given by the formula

$$Q = \frac{f_R}{2\Delta}\sqrt{\frac{V_R}{V_A}} \text{ where } \Delta = f_A - f_R \qquad (2)$$

and V_R and V_A are the voltages across the terminating resistance at the resonant and antiresonant frequencies, respectively. A third method tried was to measure the voltage across the terminating resistance at antiresonance and a few cycles from antiresonance. For this case the Q of the crystal is given by the formula

$$Q = \frac{f_A}{2\Delta}\left[\left(\frac{V_C}{V_A}\right)^2 - 1\right] \qquad (3)$$

where Δ is the difference in frequency between the antiresonance and the frequency for which the voltage V_c is measured. All three of these methods were compared on several crystals and checked within a few per cent.

The measurement of Q for the fundamental frequency for nine

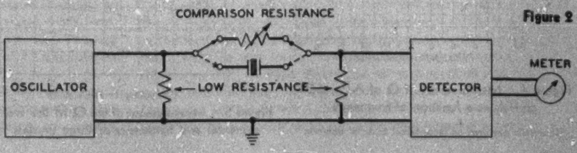


Figure 2

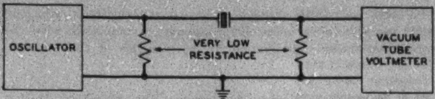


Figure 3

AT and *BT* crystals is shown on figure 4. These crystals were all measured in an air-gap holder in the open air. As can be seen from the figure, there is no significant trend with frequency. The individual difference between crystals is so large that it masks any frequency trend. This difference in Q between shear vibrating crystals is thought to be mainly due to differences in the surface caused by grinding. If the surface layer is taken off by etching or polishing, the Q in general is considerably higher. The shape or size of the crystal does not appear to affect the Q in any significant manner. In any case the Q of crystals at 20 megacycles—which is about as high in frequency as it is feasible to measure the characteristics of crystals with electric oscillators on account of the instability of the electric oscillator— is in the order of 30,000 to 70,000 which is much higher than the figure of 250 which would be obtained using Reukema's data.

Another method for attaining high frequencies with crystals whose thickness can be obtained commercially is to use a mechanical harmonic of a shear vibrating crystal. The electromechanical coupling of such a crystal decreases about inversely as the order of the harmonic, and the ratio of capacities inversely as the square of the harmonic. Nevertheless by using special oscillator circuits it is possible to drive such crystals and have them control the frequency of oscillators. Hence it is of some interest to find how the Q for the successive harmonics varies. For this purpose three crystals were measured having respectively fundamentals of one megacycle, two megacycles, and eight megacycles. The first 15 harmonics of the one-megacycle crystal were measured and the result is shown on curve 1 of figure 5. As can be seen the Q increased with frequency from 93,000 for the fundamental to nearly 400,000 for the fifteenth harmonic. This is what would be expected if most of the dissipation is located in the surface layers, for the harmonic crystal as shown on figure 6 is equivalent to a number of fundamental crystals in combination. If most of the dissipation is located on the two outside layers it is obvious that the effect will be inversely proportional to the number of dividing surfaces or loops and will be considerably smaller for a harmonic crystal than for a fundamental crystal. Crystals 2 and 3 also show its effect of increase in Q with harmonic order but they do not rise to as high limiting values as the first crystal. This is probably due to their lower initial value since the dissipation introduced by the outer faces is higher for them than for the first crystal.

It is concluded from these measurements that after eliminating known dissipative losses in crystals, that there is no evidence of a dependence of Q of a crystal upon frequency.

Professor Reukema, as a result of his opinion on the Q of crystals, seems to feel that crystals will not give frequency stability in the ultrahigh-frequency range as good as can be secured with resonant lines. This again is not in agreement with our experience. Evidence for the use of crystals in stabilizing high-frequency oscillator circuits is furnished by work recently done at Bell Laboratories on a direct crystal-controlled oscillator working at 120 megacycles. This oscillator, which it is hoped to describe in a forthcoming paper, works on the fifteenth harmonic of an eight-megacycle crystal. With plate voltage changes, the frequency of this oscillator does not change more than 0.05 cycles per megacycle per volt change which does not differ much from the voltage stability of ordinary crystal oscillators at low frequencies. Hence we can conclude that crystals in harmonic vibration can be used to control the frequency of ultrashort-wave oscillators and give good stability.

Lester E. Reukema: If my statement regarding the Q of crystals at very high frequencies is responsible for the initiation of the experimental work on the subject reported in Mr. Mason's discussion of my paper, it is fortunate that I made the statement even though it seems to be erroneous. For until this discussion of Mr Mason's appeared, it seemed to be universally accepted by all workers in the field of crystals, at least so far as the published literature is concerned, that the Q of a crystal was inversely proportional to frequency. As proof of this universal acceptance refer to any textbook which even mentions the subject or to the published literature dealing with the equivalent electrical circuit of the crystal. For instance, in Hund's "Phenomena in High-Frequency Systems," combine equation 51, page 311, namely

$$\omega_0 = \frac{540,000\,\pi}{b} \quad \text{or} \quad f_0 = \frac{270,000}{b}$$

where b is dimension of the crystal in the direction of vibration and f_0 is the resonant frequency of the crystal, with equation 57, page 313, namely

$$\Delta = \frac{R}{2f_0L} = \frac{0.00202}{b}$$

where Δ is the logarithmic decrement of the crystal, and R and L are the equivalent resistance and inductance of the crystal. This gives

$$Q = \frac{2\pi f_0 L}{R} = \frac{\pi}{\Delta} = \frac{\pi b}{0.00202} = \frac{270,000\,\pi}{0.00202 f_0} = \frac{420,000,000}{f_0}$$

According to this formula, at $f_0 = 30,000,000$ cycles, $Q = 14$, which is even lower than the value of 170 which I gave. However the decrement depends on the degree of polish of the crystal faces, the way the crystal is held in the holder, and on the air friction, and the constant involved in Hund's equation 57 may readily be decreased to considerably less than the value he gives. In the example given in my paper, I decreased the value of the constant to about one twelfth Hund's value, on the basis of values of Q given in the published literature, thus raising the value of $Q = 14$, as computed from Hund's formula, to my value of 170 at 30,000,000 cycles. The important part of Hund's formula is that it gives Q as inversely proportional to frequency.

I might also quote from Terman's "Radio Engineering," 1932 edition, top of page 265, "...and finally that the effective Q of the

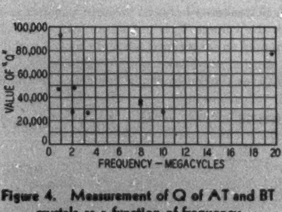


Figure 4. Measurement of Q of AT and BT crystals as a function of frequency

All crystals measured on fundamental mode of vibration

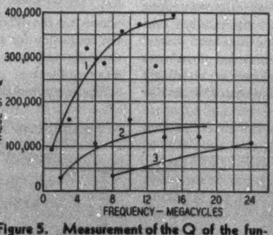


Figure 5. Measurement of the Q of the fundamental and harmonics of three crystals

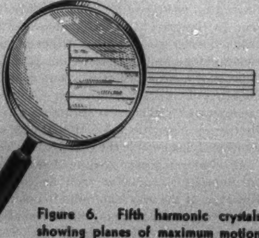


Figure 6. Fifth harmonic crystals showing planes of maximum motion

crystal vibrator varies *inversely* with frequency...." Terman's 1937 edition contains the same statement. This conclusion is reached also in computing Q from the equations for the equivalent R, L, and C of the crystal, as derived by K. S. Van Dyke in IRE *proceedings*, volume 16, 1928, page 743, and as accepted and repeated innumerably since then by the other workers in the field. If Mr. Mason's experimental results are correct, and I have no reason to believe they are not, then universally accepted mathematical analysis of the equivalent electric circuit of the crystal is sadly in need of revision.

Let us consider now the discussion of C. W. Hansell and P. S. Carter. They claim that according to my calculations resonant concentric lines should radiate almost as much of the total input power as dipole antennas. Actually my formulas show that the radiation resistance of their quarter-wave shorted concentric line at 60 megacycles is only 0.01455 ohms, as compared with 73.14 ohms for a dipole antenna. In other words, the concentric line, instead of radiating almost as much energy as a dipole, would radiate only one five-thousandth as much energy as a dipole. For a correctly designed short half-wave concentric line, the radiation resistance is only 1/33,200 that of a dipole. The reason the small amount of energy radiated from a concentric line, used to control the frequency of an oscillator, is not negligible, is that the other losses in the line *are so much smaller*. Of course I did not make the statement that 85 per cent of the power *input* to a concentric line is radiated. It is well known that the loss in concentric lines used to feed power to transmitting antennas is almost negligible, and no one would say that 85 per cent of the input power to such a line was radiated by the line. What I did state was that the loss due to radiation in a particular line used by Carter and Hansell to control the frequency of an ultrashort-wave oscillator, was 5.6 times as great *as the other losses in that line*, and the sum of all the losses is so small that the talk of generating artificial fever in the operators by means of the radiated energy is not valid.

They also state several times that "a perfect conductor can sustain no electric field." The outer surface of a concentric line is a perfect conductor only to the same extent that the surface of a radiating antenna of the same outer diameter is a perfect conductor. Yet everyone knows that voltages of many thousands of volts are set up between the ends of such radiating antennas, as evidenced by the necessity for insulators at the ends. What they should say is that a radiated field, striking such a perfect conductor, cannot set up an electric voltage across it because the resultant movement of electrons in the perfect conductor is such as to exactly neutralize the voltage induced by the field, just as the voltage induced in the secondary winding of a shorted transformer is neutralized by the movement of electrons through the winding itself, leaving zero voltage across the terminals. My computations of energy radiated do not presuppose that a field from *within the concentric line* penetrates the outer conductor, but rather that part of the current in the outer conductor flows on the outer surface, which allows it to radiate just as from any ordinary antenna.

To see that this is so, let us consider first a complete radiating system, entirely enclosed in a copper shield, the shield itself *forming no part of the enclosed circuit*. Any field striking such a shield from within induces in it a voltage which causes a current to flow on the *inner* surface of exactly the value to consume the induced voltage and no energy penetrates the shield (assuming perfect conductivity). Even for ordinary shields the energy which penetrates the shield is infinitesimally small. Since at every point of the shield the flow of electrons on the inner surface exactly neutralizes the voltage induced by the field the voltage difference between any two points on the inside of the shield is zero. Moreover, since no energy can penetrate the shield, the voltage between any two points on the outside of the shield is also zero, so of course no current flows on the outside. In other words, a shield *which forms no part of the enclosed circuit* is exceedingly effective in preventing the radiation of energy through it as amply proven by any number of accurate experimental measurements.

Consider now a concentric line, shorted at both ends and of such length as to be resonant, the length being assumed to extend from a circle halfway between inner and outer conductors on one shorting disk to the corresponding circle on the other shorting disk. Consider a source of high-frequency voltage to be impressed between inner and outer conductors of such frequency as to make the distance between inner and outer conductors an appreciable fraction of a quarter wave length. The field radiated from the inner conductor induces a voltage on the inner surface of the outer tube, causing a current to flow there lagging exactly 180 degrees behind the field which produced it and of such magnitude as to completely reflect the wave back into the inner space. But since the current induced in the outer tube by the field from the inner conductor lags 180° behind the field, it lags *more* than 180 degrees behind the current in the inner conductor, since it requires a finite time for the field to travel from inner conductor to outer tube.

However, the outer tube is acting as the return conductor for the inner tube, and therefore must carry a current *exactly* equal and opposite that carried by the inner tube, if there is to be no voltage set up between the ends of the concentric line. Since the current which flows on the inner surface of the outer tube, due to the field radiated from the inner conductor, is *not exactly equal and opposite* the current on the inner conductor, a slight voltage is set up across the ends of the concentric line, which causes a current to flow on the outer surface of the outer tube, equal to the vector difference between the currents already considered. Of course both radiated and inductive components of the field due to the current on the inner conductor must be considered, in computing the current which flows on the outer surface of the outer tube. When the spacing between inner and outer tubes is very small compared to a quarter wave length, as is the general case except at the ultrashort wave lengths, this outer current is practically infinitesimal. Even at the ultrashort waves, the outer current is so small compared to the current which flows on the inner surface of the tube, that the energy radiated is of the order of 0.01 of 1.0 per cent of that which would be radiated if all the current were on the outside. However, *small as it is, the energy radiated is by no means negligible, compared with the other losses in the concentric line.*

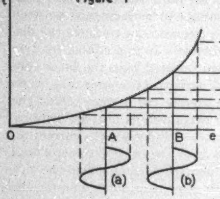
This point has been neglected both by Carter and Hansell and by Schelkunoff, which is the reason that the latter finds a finite radiation resistance from a perfectly conducting shorting disk, whereas my equations show this radiation resistance to be exactly zero. Mr. Schelkunoff's remarks concerning my direct addition of the radiation resistance of a shorted parallel-wire line to the radiation resistance of the shorting bar are perfectly correct. For high accuracy the *mutual* radiation resistance would have to be considered. However, the fact that the line and the shorting bar are perpendicular to each other makes the mutual radiation resistance quite small, compared with the total radiation resistance, and I neglected it because there seems no way to combine it with the other results *for all lengths of line* without making the equations unwieldy. In any case my conclusion that the shorted parallel-wire lines should be brought together at the far end gradually, thus eliminating the necessity for a shorting bar would be in no way changed.

A Review of Radio Interference Investigation

Discussion and authors' closure of a paper by F. E. Sanford and Willard Weise published in the October 1937 issue, pages 1248–52, and presented for oral discussion at the selected subjects session of the AIEE Middle Eastern District meeting, Akron, Ohio, October 13, 1937.

W. C. Osterbrock (University of Cincinnati, Cincinnati, Ohio): The location of sources of radio interference such as are described in this paper is in many cases quite difficult and this is largely due to the fact that faults capable of creating serious interference with radio reception may be entirely insignificant in their effects on the electric service. A good illustration of this possibility is reported in the editorial page of *Electronics*, September 1937, by R. L. Peterson. In this instance, the uncertain contact between turns of the armor on BX cable produced a variable impedance such as re-

Figure 1



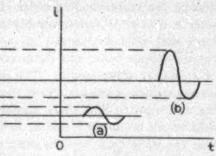


Figure 2

ferred to in the section on "External Cross-Modulation" of the paper, and this gave rise to the effect described, namely background tone from one or more interfering stations. It would seem that the location of such a fault might require, in addition to good measuring technique, a slight touch of genius.

The manner in which cross-modulation is produced by a nonlinear impedance may be considered qualitatively by reference to figure 1, which shows the current in such an impedance as a function of the voltage, and figure 2, which shows current as function of time for two different conditions.

In figure 1, (*a*) represents the desired signal voltage when the interfering signal has a value *OA*, and (*b*) represents the same voltage for an interfering voltage *OB*. The corresponding currents are shown at (*a*) and (*b*) of figure 2, and are seen to be quite different. In the course of modulation the interfering voltage varies from *OA* to *OB*, and the sensitivity of the system to the desired signal is caused to change correspondingly. As a result the modulation of the unwanted station is heard as a more or less distinct background.

The above analysis is not complete, as it ignores the rapid variation of interfering signal during the cycles (*a*) and (*b*). A more satisfactory procedure is to develop the current function into a Taylor's series, and if this is done, it turns out that the third-order term causes a current component to appear *having the same frequency as the desired signal*, but whose amplitude is a function of the interfering signal voltage. The receiver is of course tuned to this frequency in question, and responds as well to this particular component as to the original signal. Incidentally, the cross-modulation term is proportional to the *square* of the interfering signal voltage, and thus every increase in power of a local broadcast transmitter can be expected to produce a flood of new complaints.

As mentioned in the paper, the interference may be transmitted to the receiver by reradiation from the offending conductor, but it may also arrive there via the line cord. Whether it does or not will depend largely upon the effectiveness of the receiver grounding system.

The authors are to be commended for their thorough review of the situation, and their company for its enlightened policy in the investigation of complaints. Many cases have come to my attention of painstaking investigation by their radio squad, even though only a remote possibility existed that the trouble might be due to some fault in the lighting service.

J. J. Smith (General Electric Company, Schenectady, N. Y.): The authors deal with the problem of radio interference from the point of view of a utility whose territory is close to one of the broadcast centers. Under these conditions the radio set users would expect to get good reception. It is interesting to note in table I that 18 per cent of the complaints investigated were due to receiver trouble as compared with 11.5 per cent which were due to the company's distribution lines and equipment. The statement that the number of power cases is only about 25 per cent of what it was in the first years of the investigation work is encouraging. The trend toward greater co-operation between the power company investigations and the radio service men is also an indication that progress is being made.

The thing that is needed now is to carry out studies on a quantitative basis. The authors refer to the results that have been accomplished through inductive co-ordination of power and telephone systems. These results would not have been possible without the quantitative data obtained co-operatively through the joint Subcommittee on development and research of the EEI and Bell Telephone System. To obtain such data, wave-shape surveys have been made on a number of power systems in different parts of the country, coupling coefficients have been measured between typical power and telephone systems, data have been obtained on the characteristics of telephone circuits and apparatus, and many other studies have been made or are in progress.

A similar approach to the problem of radio interference which the paper suggests would involve the collection of data on signal strengths of broadcasting stations and noise levels in different localities, data on coupling between radio receivers and electric circuits in houses, data on characteristics of power apparatus, etc. Such a study is being carried on by the joint co-ordination committee on radio reception of EEI, NEMA, and RMA. The committee has recommended suitable instruments and methods, and issued a report entitled "Methods of Measuring Radio Noise" which is available through the headquarters of the EEI, NEMA, or RMA. This report gives specifications for the instrument recommended for measuring radio noise and the procedure to follow in the use of this instrument. Subcommittees have also been set up for collecting and correlating quantitative data and it would be of great assistance if those who are doing field work like the authors of the paper would obtain more data and report their results in the form suggested by this joint subcommittee.

P. L. Bellaschi (Westinghouse Electric & Manufacturing Company, Sharon, Pa.): Radio noise from power sources presents problems of a complex character but fortunately in practice many of these can be approached and disposed of satisfactorily through relatively simple procedure, as Messrs. Sanford and Weise have indicated in their paper. The fundamental aspect of radio noise remains, on which considerable progress already has been made.

An important contribution to this progress has been made possible by the joint co-ordination committee on radio reception of EEI, NEMA and RMA, in specifying suitable and adequate methods of measuring radio noise. Mention should be made of EEI publication No. *C*-9 or the corresponding NEMA (No. 102) or RMA (No. 13) publications.

The above mentioned technique and methods of measuring radio noise have been applied for a number of years at the East Pittsburgh laboratory of the Westinghouse Electric & Manufacturing Company ("Measuring Radio Interference from High Voltage Apparatus," C. V. Aggers and W. E. Pakala, *The Electrical Journal*, December 1933). Another laboratory, fulfilling the EEI-NEMA-RMA recommendations has recently been installed at the Sharon plant of the same company. Comparison of measurements have been made between these two laboratories to correlate results. For example, when a simple electrode arrangement as a rod to plate was employed the characteristic curves of voltage versus microvolts for the two laboratories are found to come within a few per cent of each other. Even wider variations would be considered good agreement at the present status of the art.

Fundamental investigations always should present room for possible improvement and refinements both in the technique and the methods of measurement. Yet experience has well demonstrated that in the long run much is to be gained through orderly standardization of methods and technique. It is for these reasons that the general application of the methods recommended by EEI-NEMA-RMA hold out promise toward a rapid rationalization of the radio noise problems. As Messrs. Sanford and Weise have clearly stated the complete elimination of all radio interference appears uneconomical, the aim being chiefly to establish permissible levels for electrical apparatus which are reasonable and economically feasible.

F. R. Benedict (nonmember; Westinghouse Electric & Manufacturing Company, East Pittsburgh, Pa.): Short-wave and broadcast program listeners are confronted at one time or another with radio interference which makes reception very unpleasant. Unfortunately, the general public has not been educated in the problems associated with interference and interference producing devices, but only know of it by the interference to their favorite program. Usually this interference is not under the control of the listener and his dissatisfaction is accompanied by a feeling of frustration, which is directed at the world in general. For the past seven years, a group of engineers have been working on the problem of interference and its relation to broadcast reception. This group has been studying the co-ordination of the newer uses of electricity with its older uses for power, light, heat, transportation, and communication, so that the growth of each of these will not be restricted by the requirements of the others. This program is being carried on by the joint committee on radio reception of the Edison Electric Institute, National Electrical Manufacturers Association, and Radio Manufacturers Association. Preferred methods of measurement have been developed and appear in the NEMA publication No. 102. Consideration is also being given to factors in the design of machines and power systems which may affect radio reception. The characteristics of radio receiving sets and the relation of shielding, sensitivity and selectivity to the elimination of radio noise, is also being carefully studied.

The source of radio interference can be determined by the audio characteristics produced in the radio loud speaker. The different types of interference may be classified in two groups, (*a*) uncontrollable interference and (*b*) controllable interference. Under class (*a*) are found atmospherics, i.e., lightning, background static, solar interference, snow and sleet static, etc. Under class (*b*) are found commutating machines, transmission line equipment (such as bushings, pin insulators, transformers, etc.), diathermy machines, sign flashers, auto ignition, etc.

The first class of interference is termed uncontrollable because of the engineer's inability to deal with the source of the noise. It is present at all times, although the amount of trouble it will produce varies considerably with time, weather conditions, and from one locality to another. Radio engineers have done much to improve this condition in modern receivers by increasing selectivity, or in short designing for a high signal to noise ratio. Another aid in eliminating trouble from this class of interference is obtained through increasing the power of broadcast stations and making use of transmitting antennas, which will increase the field strength within the normal coverage area. If the signal strength at the receiving antenna can be increased, the ratio of the signal voltage to the noise voltage appearing on the input of the receiver will be increased and the amount of annoyance caused the listener will be eliminated.

The greatest help for the distant listener would be an ideal receiving antenna system and an increase in broadcast station power to bring up the field strength at the listener's antenna. The problem is not so easily solved. Broadcast stations cannot use unlimited power since allotment of transmitting frequency necessitates more than one station on the same frequency with the possibility of interstation interference. Advances are being made in the use of directional antennas and improvement in broadcast station coverage may be expected in the future.

The progress in elimination of atmospherics has been slow as the static interference seems to have transmitting characteristics over the whole broadcast and short wave spectrum. Various schemes of frequency and amplitude modulation have been advanced for reducing atmospheric disturbances in radio receivers, but these schemes are still in the experimental stages and promise only a partial solution to the problem of uncontrollable interference.

That interference classed as controllable or man-made, has become a serious problem. With electric service increasing daily, the need for some consideration of radio interference problems and their relation to the rapidly growing electrical industry, is very essential. At the present time there are a great many devices in use which produce objectionable interference in nearby radio sets when operated on the supply mains. In the next few years many more will be sold and, in themselves, add measurably to the present problems. In many metropolitan areas, where field strengths are low, the problem is already acute. The interference in such areas, is mainly caused by appliances or other electrical devices common to all electrified households. Vacuum sweepers, food mixers, universal motors, electric razors, electric fans, electric toys and many other devices may come under the category of interfering devices. Of these, those devices classed as commutating machines cause the most radio interference troubles.

It is well known that scientifically designed and properly installed filters may be applied in many cases for the reduction of radio interference from electrical appliances. It may be asked—"Why not apply filters to all household appliances which may produce interference and thereby alleviate all future trouble from such appliances?" In answering such a question satisfactorily, consideration must be given to many factors, which are closely allied with the whole interference problem.

The problem begins in the radio owners' homes and simply stated is "For an appliance with known radio noise output voltage connected to the supply lines of the home, how much noise voltage will be induced in the receiver antenna and ground system?" This can only be determined by experimentally finding the ratio of interference noise measured in the factory to that which may be induced in the antenna when in operation on the home supply lines.

Tests made in homes in the Pittsburgh area indicated that about 50 per cent of the cases had less than 10 per cent of the noise as measured at the factory, induced in the antenna and 75 per cent of the cases had less than 18 per cent of the factory noise induced in the antenna. From actual tests, made with the radio receiver in operation, 5,500 microvolts on the supply lines caused no interference to programs in 50 per cent of the cases. Where the better antenna installations were found, the antenna pickup was less than 2 per cent of the factory noise. Such installations gave no interference to programs with 50,000 microvolts on the supply lines. It was proved conclusively that the coupling of the antenna lead in and ground, with the service wiring is a very important factor in noise reduction.

These tests indicated that a good antenna system is essential if a really successful fight is to be made against radio interference in the home. Unfortunately, the radio distributor is more interested in selling the set than in keeping it sold. Accordingly it has been left pretty much up to the consumer to either buy an antenna installation or put it in himself. The utility which supplies the power to run the set, is very much interested in keeping the set sold, as loss of revenue occurs when the set is not in operation. From the utility standpoint, it is more important that the radio dealer sell the consumer, first a good antenna system and, second, a radio set, for it is to the utility the consumer complains in case of interference. From experience it has been found that the best remedy for interfering household appliances is a modern, scientifically designed and properly installed antenna system.

Where interference from household appliances cannot be reduced by installation of the best antenna possible, filters are usually applied on the electrical circuit of the interfering machine. In applying filters, two types of apparatus installations must be considered; (*a*) apparatus operated with the frame grounded and (*b*) apparatus operated with the frame ungrounded. When the frame is grounded, or ungrounded, the noise can best be reduced by placing capacitors of the proper size and characteristics, from each line to frame. Little noise reduction is obtained by connecting capacitors line to line and may even increase the noise output.

In the grounded case, the line to frame capacitors may be any value as they are connected direct to ground. In the ungrounded case, however, large capacitors cannot be connected to the frame because of the shock hazard. Suitable noise reduction is most effectively obtained by connecting two capacitors in series, line to line, and a third capacitor from the midpoint to the frame. The value of this third capacitor has been a much discussed problem for the past few years. If the value of the frame capacitor is too high the shock is very objectionable. The human body is sensitive to about 500 microamperes, so this capacitor must be held to a very low value −0.005 microfarads or smaller. If the filter is used in damp or humid climates the shock hazard is increased. The most serious problem comes in the possibility of the frame capacitor shorting out or failing. Then the shock hazard may endanger life under moist or wet conditions of the skin. In the case of the grounded machine, which may become accidentally ungrounded, the shock hazard is also serious, as the capacitors may be 0.1 microfarad or more. If, for any reason, more than one capacitor fails full line voltage may appear on the frame. Then the condition is dangerous to life.

To be an effective remedy for all appliance ills, it is readily seen that all manufacturers would have to filter their devices. Immediately the question arises, "Who shall pay for making the appliances interference free?" It has been stated that with a good antenna system, considerable interference voltage is not objectionable in an appliance. If the filter is compulsory, on many of the cheaper household appliances the filter cost may be a large proportion of the total cost of the article, as, for instance, the 10¢ store variety of egg beater, which sells from 69¢ to $1.00. Addition of a filter to such a cheap article may make the price prohibitive for the quality of the article. Where the cost of the filter is only a very small part of the total cost of the appliance, the problem is simply one of absorbing the cost of the filter in the selling price.

Another complication to the problem is that not all of the appliances of a group produce serious interference. Out of 1,000 high-grade appliances only 1 per cent may cause objectionable interference. There is a probability that not all of this 1 per cent will find their way into locations where interference will cause trouble. The same may be said of cheaper appliances, but the percentage of interference producers per thousand may be much higher. In some cases, a simple change in design may eliminate a part or all of the intereference trouble of a group of appliances.

Mass production of a complete line of filtered appliances is a very involved manufacturing problem. It would require (*a*) redesign of the appliance to include space for a filter, (*b*) new dies, jigs, and tools for each class of appliance, (*c*) standardization of filters for production and renewal replacements, and (*d*) a special organization to design and test the filters and appliances. Such a program could only be reflected in the final cost of the appliance to the consumer.

Where the cost of the filter is a small part of the cost of the appliance, it may be economically feasible to apply filters. In fact, several large manufacturers do filter certain classes of their appliances. While the general interference problem is not helped much by such filtering, it is an asset to the manufacturer as it helps to keep the appliance sold. Such filtering reduces complaints on that manufacturer's equipment and promotes good will, which is the best intangible asset any company can have.

F. E. Sanford and W. R. Weise: Professor Osterbrock has offered a qualitative explanation of external cross modulation. This type of interference can well be made the subject of laboratory experiments and theoretical studies, to the end of obtaining an acceptable explanation, which will lead to solution and elimination. So far this type of interference is not common but it may be expected to increase with increasing signal strengths, which overcome some of the more common types of interference.

Mr. Smith suggests the need of more quantitative analyses of radio interference. Undoubtedly this will help, but we believe that exact measurements are of secondary importance compared to a general recognition of the problems and the value of a co-operative spirit among all of the groups concerned.

In the field of quantitative analyses, apparatus and levels of measurement are essential for the further alleviation of interference caused by appliances. Mr. Benedict has pointed out the present uncertainty in this field. The manufacturers of insulators and other equipment for transmission and distribution line use, have given a great deal of attention to this and as a result the electric utilities can purchase equipment which is inherently "radio interference free." As pointed out in the paper, a similar interest in this subject is in order, on the part of the numerous appliance manufacturers. Many are now following along these lines, but many others apparently are not. In time the public will surely recognize freedom from radio interference possibilities, as a desirable feature of any appliance purchased.

Determination of the permissible level rather than the complete elimination of all radio interference must be the aim in the work to be done. A standardization of methods and technique, as is being attempted by the joint EEI, NEMA, RMA committee, is an important step in the direction, as pointed out by Mr. Bellaschi. Manufacturers who have considered the interference possibilities of their products are to be complimented and encouraged.

In conclusion, we feel that this paper justifies itself to the extent that it serves to further focus attention on the radio interference problems yet to be solved. The discussion, both written and verbal indicated that there is wide interest in the subject.

Analysis of Series Capacitor Application Problems

Discussion and author's closure of a paper by J. W. Butler and C. Concordia published in the August 1937 issue, pages 975–88, and presented for oral discussion at the power transmission and distribution session of the AIEE Pacific Coast convention, Spokane, Wash., September 1, 1937.

E. C. Starr (Oregon State College, Corvallis): The paper by Messrs. Butler and Concordia is a valuable contribution to the field of power transmission. The authors are to be complimented upon the breadth and thoroughness of their analyses of the principal difficulties encountered in the application of series capacitors to transmission circuits.

The series capacitor offers an almost ideal means of improving the voltage regulation of transmission circuits. The action is instantaneous and line reactance, together with the transformer and generator reactances, can effectively be cancelled. The result should be not only that of eliminating the reactance component of system voltage drop but should also be that of improving the steady-state stability limit of the system.

A few series capacitor installations have at first been more or less unsuccessful. These have cast some doubt over the practicability of their use due to one or more of the characteristic troubles, analyzed by the authors, having arisen in their operation. In applications involving large induction- and synchronous-motor loads, where an attempt has been made to completely compensate, or in some cases overcompensate, for line and transformer reactances, serious surging difficulties have arisen. In other applications, where a large part of the load is of resistance character, and in cases where overcompensation of system reactance was not attempted, the success of operation has been complete.

From a very approximate analysis it can be shown that a simple criterion for the successful application of series capacitors to circuits involving large synchronous and induction machinery is to avoid any attempt at complete or overcompensation. That is, the series capacitive reactance in such circuits should not be made equal to or greater than the combined inductive reactance of the line, transformers and generators involved in the particular circuit in question. This is a very simple criterion and no attempt will be made at this time to carry it to a more exact solution. In some cases it will be objectionable because it does not permit of the complete cancellation of system reactance or in the effective compensation for both resistance and reactance as is possible under some limited conditions.

Reference to figure 1 of this discussion will bring out two interesting features. It will be noted that for any normal load current, I, the sending-end voltage, E_1, is ahead, in phase position, of the re-

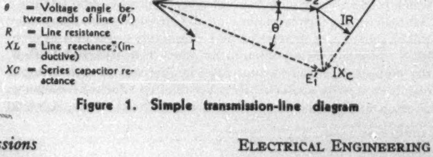

E_1 = Sending-end voltage (E_1')
E_2 = Receiving-end voltage
I = Line current
θ = Voltage angle between ends of line (θ')
R = Line resistance
X_L = Line reactance (inductive)
X_C = Series capacitor reactance
E_1
IX_L
θ
E_2
θ'
IR
I
E_1'
IX_C

Figure 1. Simple transmission-line diagram

ceiving-end voltage, E_2, by the angle θ. Any increase in the load current, I, is accompanied by an increase in the angle θ, and up to a certain limit the system is stable. If a series capacitor is introduced in the system with capacitive reactance greater than the system inductive reactance, then the dotted line diagram will apply. It will be observed that no change has taken place in the system resistance drop, IR, but that the sending-end voltage, E_1', is smaller in magnitude when compared to E_2 than previously. This indicates the desired improvement in system voltage regulation. It will be noted, however, that the sending-end voltage, E_1', now *lags* the receiving-end voltage, E_2, by the angle θ', and any increase in the load current, I, will cause a further lag of E_1' behind E_2. This means then that if we assume E_1' to be supplied by an infinite bus or by a generator of relatively large rotational inertia, the receiving-end voltage, E_2, will be governed in phase position by the line current, I_1, an increasing current meaning an *advancing* phase position.

This condition is inherently one of instability. If, for example, the load consists of a synchronous motor and a sudden increase in delivered torque should occur, then the load current, I, would increase correspondingly and the terminal voltage, E_2, would advance somewhat in phase. This momentary advance of E_2 would tend to accelerate the rotor of the motor and again increase the current drawn by the motor from the system. This further increase of current would tend, again, to increase the angle θ'. However, as soon as the rotor had accelerated to accommodate the new phase position of the terminal voltage, the current would drop off somewhat, allowing the phase of E_2 to drop back. The rotor would then drop back in phase position and the current momentarily would decrease. As soon as the rotor had reached its normal load angle, the current would have returned to normal and would again be changing in the upward direction, starting to repeat the cycle of oscillation just described. An unstable surging condition is thus established which involves the complete mechanical as well as electrical system.

The same general analysis can be applied to an induction-motor load. In that case, a suddenly increasing load produces an increased slip which in turn increases the line current and causes the phase of its terminal voltage to advance momentarily. This advance in terminal voltage is the equivalent of a momentary increase in frequency resulting in increased instantaneous slip and hence in a further increase of line current. The rotor then tends to accelerate under the influence of increased current, and thus again starts the oscillation described above.

A very crude analogy is the case of a differential-compound motor driving a cumulative-compound d-c generator. An increase in load on the generator tends to increase slightly the speed of the motor which immediately increases the voltage of the generator and automatically causes it to deliver more load. Such a system, when adjusted for a small degree of overcompounding, will tend to oscillate and surge rather badly. If, however, the overcompounding is excessive, particularly in the case of the motor, the system will surge beyond control.

It can be said in general that, so long as the receiving-end voltage of an electrical system tends to *fall back* in phase with respect to the sending end with increase of load, any oscillations set up externally will be subjected to a definite decrement due to system resistance and machine friction, and the operation will be stable up to the limit of the system. If, however, the receiving-end voltage tends to *advance* in phase with respect to the sending end with increase of load, the system is inherently unstable and a surging condition can develop when rotating equipment is involved.

In most practical systems, particularly those involving a fairly large percentage of resistive loading, a small negative angle will give rise to no difficulties. Hence it is possible to compensate for all, or nearly all, of the system reactance by means of series capacitors without introducing any abnormal operating conditions. However, if large rotating machines are involved in the load, it is not advisable to overcompensate for the system reactance in order to obtain an idealized voltage-regulation condition.

In designing a series-capacitor installation, the load should be carefully analyzed from the standpoint of size and types of rotating machines involved and the power factors encountered. Where large machines are involved in the load, the capacitive reactance should be limited to a value which will not cause the terminal voltage of the machines to advance with respect to the sending-end voltage more than a few per cent of the normal load angle of the machines. It is realized that the voltage-regulation improvement to be obtained by the application of series capacitors is limited by these conditions, but a very large improvement can be produced by their use and further applications should be greatly encouraged.

J. W. Butler and C. Concordia: We wish to thank Professor Starr for his remarks and explanation of the phenomena of self-excitation on the basis of a simple vector diagram. We feel however that in general it will not be feasible to compensate to as great an extent as is indicated by his criterion. As brought out in the paper, one cannot compensate for the total effective steady-state system reactance; even the short circuit or transient reactance can usually be only partly compensated, perhaps half-way compensated. Figures 10 and 12 illustrate this point and show the limits for a particular set of system constants.

Relay Operation During System Oscillations

Author's closing discussion of a paper published in the July 1937 issue, pages 823–32, and presented for oral discussion at the development of protective equipment session of the Pacific Coast convention, Spokane, Wash., August 31, 1937. Other discussion of this paper appeared in the December 1937 issue, pages 1513–14.

C. R. Mason: Mr. Neher has described an extremely valuable method for studying relay operations under abnormal system conditions. His suggestion for applying that method to the problems of this paper merits further consideration. By this method one can determine the operating tendency of various relays quickly, accurately, and with a minimum amount of calculation if he has the conception of the elements of the problem which the paper attempts to provide.

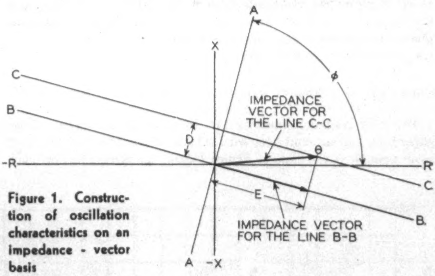

A
X
φ
C
B
IMPEDANCE VECTOR FOR THE LINE C-C
D
θ
-R
R
C
E
IMPEDANCE VECTOR FOR THE LINE B-B
B
A
-X

Figure 1. Construction of oscillation characteristics on an impedance - vector basis

CHARACTERISTICS OF A SYSTEM OSCILLATION

The relations of current, voltage, and the angle between them can be conveniently represented on a two-dimensional picture by showing the ratio of the voltage to the current vectorially. Consider figure 1. Lay out the resistance and reactance axes to the same scale and draw the line A-A through the origin at the system phase angle ϕ to the resistance axis. Draw the line B-B through the origin perpendicular to A-A. The line B-B applies to the impedance center of the system. At that point the current is always at the same angle to the voltage regardless of change in the displacement angle θ. The impedance vector will always lie on this line, terminating at the dis-

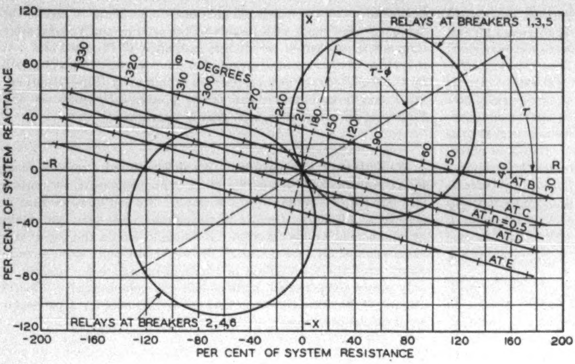


Figure 2. Voltage-restrained directional relay operating charactertistics combined with the oscillation characteristics to show relay operation

tance E to the right of the origin for any angle θ less than 180 degrees according to the relation $E = 50 \cot (\theta/2)$ where E is expressed as a percentage of the total system impedance. When θ exceeds 180 degrees, E will be measured to the left of the origin.

For some other point in the system toward the end, which in the paper was labeled "source A," the line C-C will apply. This line is parallel to the line B-B and at a distance D which is the per cent of the total system impedance that the point is from the impedance center of the system. For another point at the other side of the impedance center, the parallel line should be drawn on the other side of B-B.

Having determined θ for desired values of E or vice versa, according to the relation given for the impedance center line B-B, this same E and θ will apply for any other line if measured from the intersection of A-A and that line. In other words, as θ increases from zero, the head of the impedance vector for each location will move from right to left along its line, the heads of all of the vectors terminating on a line parallel to A-A as shown on figure 1. If source A starts in phase with source F and then goes lagging, the impedance vectors trace the lines from left to right.

Operation of a Directional Relay With Voltage Restraint

The relay diagram shown by Mr. Neher in his discussion of the paper is for a directional relay with different constants than that used as an example in the paper. Figure 2 shows the picture for the relay assumed in the paper having a three-phase minimum pick-up current of 7.0 amperes at 115 volts at its angle of maximum torque. The impedance pick-up of the relay at its maximum torque angle is therefore $V_A/7$. Since the system impedance is $2V_A/20$ according to the assumptions used, the relays' impedance pick-up is 10/7, or 143 per cent, of the system impedance at the angle of maximum torque. The relay circle is shown according to Mr. Neher's methods with its center on a line through the origin at an angle $(\tau - \phi)$ from the line A-A of figure 1, or at the angle τ from the R-axis. The diameter of the circle is 143 per cent of the total system impedance as previously determined.

Two circles are shown to account for the fact that the relays at breakers 2, 4, and 6 face in a direction opposite to the relays at breakers 1, 3, and 5.

For values of θ for which the oscillation line lies within the circle, the relay will operate to close its tripping contacts. If figure 2 is used to determine the point at which the relay changes its direction of torque, it will be noticed that slight errors in the curves of the paper are exposed. These curves were drawn from calculated data using the formulas of the paper, and during the process some error was introduced. A careful recheck of these calculations show that the errors are in the calculations and not in the formulas themselves. The fact that such a quick and easy check is possible using Mr. Neher's methods further emphasizes their value.

Figure 3 shows the circles for the voltage restrained directional unit of an actual distance relay which has a minimum pick-up ad-

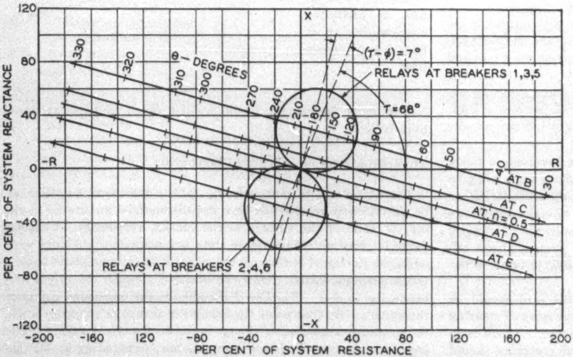
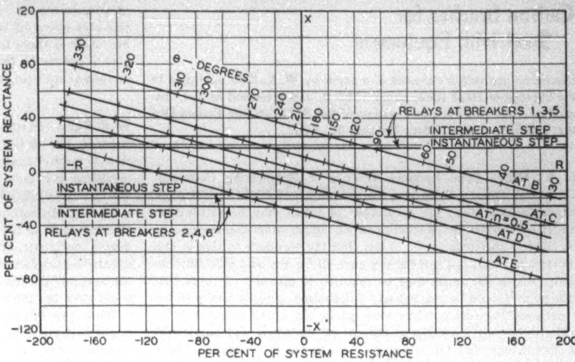


Figure 3. Combined oscillation and relay characteristics for an actual distance relay starting-element

Figure 4. Reactance element operating characteristics combined with the oscillation characteristics to show relay operation



justable to 16 amperes at rated voltage and at its angle of maximum torque (68 degrees). The diameter of this circle is thus seen to be 10/16 or 62.5 per cent of the system impedance; and $(\tau - \phi) = 7$ degrees.

Operation of a Reactance Relay

In figure 4 the operating characteristics of a reactance element adjusted as assumed in the paper are combined with the oscillation impedance characteristics. The operating characteristics of the reactance element are perpendicular to the X axis and intersect it at a distance from the origin which is the per cent of the total system impedance below which the element is adjusted to operate. The element operates to close its tripping contacts when the oscillation impedance vector terminates on the origin side of the relay characteristic. Here, as in the case of the voltage restrained directional relay, the characteristics for relays which face in opposite directions are drawn on opposite sides of the origin. These curves give a very accurate check of the curves in the paper.

Operation of an Impedance Relay

The characteristics of the impedance relay used in the paper are drawn in figure 5, together with the oscillation impedance characteristics. The radius of each curve is the per cent of the total system impedance below which the relay will operate. When the oscillation impedance vector falls inside of a circle the relay operates to close its tripping contacts.

Other Types of Relays

The characteristic of a directional relay without voltage restraint will be a straight line through the origin at right angles to the τ axis shown on figure 2. All oscillation impedance vectors to the right of this characteristic will cause the relays at breakers 1, 3, and 5 to close their tripping contacts; all impedance vectors to the left of the characteristic will cause the relays at breakers 2, 4, and 6 to close their tripping contacts.

The characteristics of single-quantity relays such as overcurrent, and undervoltage types cannot be shown on the diagrams, but their operation is easily determined otherwise.

Errata

In figure 12 *b* on page 828 the curves are incorrectly labeled. The curve labeled "At Breaker 4" applies to the relay at breaker 3 and the curve labeled "At Breaker 3" applies to the relay at breaker 4.

In the appendix under "Derivation of Equation 7" on page 832 there is an I missing in the denominator of the first expression. It should read:

$$Z = V/I$$

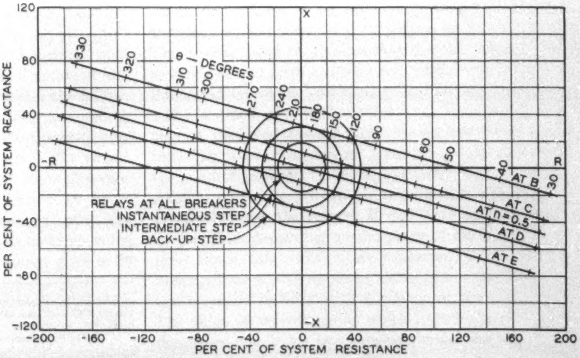


Figure 5. Impedance relay operating characteristics combined with the oscillation characteristics to show relay operation

Carbon Brushes for Steel-Mill Equipment

Discussion and author's closure of a paper by W. C. Kalb published in the September 1937 issue, pages 1165–8, and presented for oral discussion at the Iron and steel session of the AIEE Middle Eastern District meeting, Akron, Ohio, October 15, 1937.

L. E. Miller (The Reliance Electric & Engineering Company, Cleveland, Ohio): I am very glad to see Mr. Kalb bring out in his paper the fact that the same type of brush may be satisfactory under one certain set of conditions, but under other conditions may be entirely unsatisfactory. I feel that the corollary to this is that no type of brush is a satisfactory cure-all for any and all conditions under which the brush may be required to operate. I think that this point should be particularly emphasized, since it has been my experience that often times some particular brush is given to an operator to cure a certain specific set of conditions, and does its job so successfully that the operator desires to use this particular brush for every and all kinds of conditions. The result is many misapplications, causing endless difficulty, expense, and dissatisfaction.

Mr. Kalb, in speaking from notes, quoted from Doctor Glass's paper which occurred in an earlier issue of the *Iron and Steel Engineer*. While Mr. Kalb did not mention in his talk, I believe that Doctor Glass brings out the idea that the film on the commutator which gives the glossy chocolate-brown surface so desired, is a form of oxide in copper. This film, as a rule, is only produced after considerable operation of any particular machine. I would like to ask Mr. Kalb if there has been any experimental work done with the idea of producing this film quickly in any other way than the usual method of operating the machine.

W. C. Kalb: In reply to Mr. Miller's question, it is mentioned in the paper by Doctor Glass, to which reference is made, that the application of methyl alcohol as a reducing agent on a test commutator, operating with lampblack base brushes at an interface temperature of 125 degree centigrade caused the coefficient of friction to rise to 0.5. The application of hydrogen peroxide immediately brought the coefficient of friction down to 0.13, the normal friction, at 125 degree centigrade, of the grade being tested. In applying these reagents the brushes were lifted from the commutator and excess of the reagent was removed before replacing the brushes. Repeated trials gave the same results.

While this laboratory test indicates that a beneficial oxide film might be formed by the application of an oxidizing reagent to the commutator surface, I do not know of any instance where this plan has been tried in actual service.

Formation of oxide film can also be accelerated by the application of external heat to the commutator surface. It might be feasible for the electrical manufacturer to use this means for developing a commutator surface of the desired character before the machine leaves the factory, but the method does not seem well adapted to use in the field.

An Electronic Arc-Length Monitor

By WALTHER RICHTER
MEMBER AIEE

IN THE March 1935 issue of *Electronics*, a voltmeter for d-c arc welding was described. In this instrument the arc voltage was applied over a high resistance to the grid of a vacuum tube and a condenser was shunted across grid and cathode. The rapid fluctuations of the arc voltage are filtered out by this combination, and a meter placed in the plate circuit of the tube can therefore be calibrated in terms of average arc voltage. It was also possible to spread the scale of the meter so that it covered the range of welding voltage only, which leads to an increase of accuracy in determining the average arc voltage.

The application of this instrument in the shop not only showed that different welders using the same electrode and amperage would weld with voltages differing as much as four volts, but also that some kept the arc voltage constant within one to two volts, while others covered a range of five to six volts during welding. Some electrodes will operate well over a fairly wide range of values, but others were found to give decidedly inferior results when the welding voltage deviated as much as three volts from the value determined in the laboratory. The instrument described above permits a bystander to observe the performance of an operator, or it can be used to adjust the arc length held by an automatic weld head to the correct value, but it cannot be read by a hand welder since he must keep his eyes at all times on the arc. It therefore seemed desirable to have an instrument which would inform the welder about the voltage on his arc without requiring that he look away from the arc.

Acoustical means were considered first, but were found undesirable for several reasons. In the instrument described in this paper the problem is solved in the following manner: Two small flashlight bulbs are mounted in the welding shield or hood, one on each side of the window through which the welder looks at the arc. The circuit to which the bulbs, as well as the arc, are connected contains a potentiometer bearing a calibration in volts. This potentiometer can be set to any desired value, say 35 volts. If the average arc voltage is held to 35 volts, both bulbs will be extinguished; if the voltage rises to 36 volts the right bulb starts to glow faintly. increasing in brilliance with an increase in deviation; if the arc voltage falls below 35 volts, the left bulb acts in the same manner. The maximum brilliance is kept well below the point of hurting or blinding the welder's eyes. He can keep his conscious attention entirely on the arc, yet will be able to notice a surprisingly faint glow out of "the corner of his eyes" and can therefore immediately correct the deviation from the desired condition.

The principle of the circuit by means of which these results are obtained is the following: The arc voltage or a part thereof is connected in opposition to an adjustable d-c voltage. The resulting differential voltage is filtered by a resistance-capacity combination, leaving only the average difference. By means of an electron tube an alternating voltage is next produced with a magnitude which is a function of the above mentioned filtered differential voltage: it is zero, when the differential voltage is 2.5 or more, about ten volts, when the differential voltage is 1.5 volts, and about 20 volts, when the differential is 0.5 or less. This alternating voltage, varying between zero and 20 volts as just outlined, is now connected in series with a fixed alternating voltage of ten volts, but of opposite phase. The sum of these two voltages is obviously a new alternating voltage, which not only changes in magnitude but also reverses in phase; it will be zero, when the differential d-c voltage is 1.5 volts. This new alternating voltage is fed into the primary winding of a transformer, the secondary of which is center-tapped. The two sections of this winding and the bulbs in the welder's helmet form a bridge arrangement, and between the center tap of the transformer and the junction of the bulbs a fixed alternating voltage is introduced. The current produced by the latter in the two bulbs is adjusted by means of a variable resistor to a value just below the point of glowing. A voltage induced in the secondary winding of the center-tapped transformer will then obviously increase the current in one lamp and decrease it in the other, depending on the phase of the induced voltage. Since this voltage, as explained above, changes in magnitude as well as phase, either one or the other bulb can be made to burn with a change of the arc voltage. The value of arc voltage at which both bulbs will be extinguished can be set by changing the adjustable d-c voltage opposing it.

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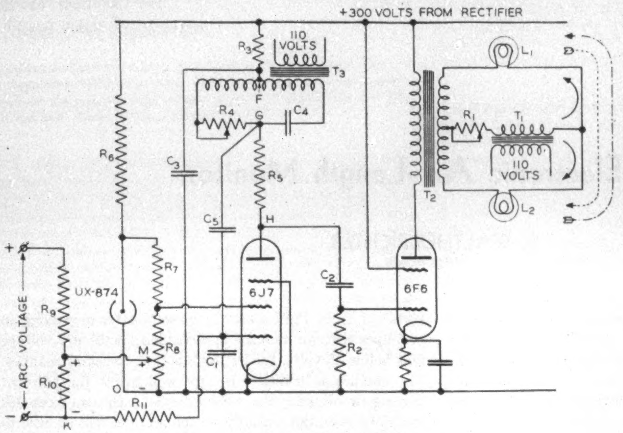

Figure 1. Diagram of arc-length monitor

The circuit putting the principle outlined above in practice is entirely electronic. It employs two amplifier tubes, a 6*J*7 and a 6*F*6, the first acting as voltage amplifier, the second as output tube. For simplicity the rectifier system, which is of the conventional type, as well as heater connections of the tubes are not shown in the diagram, figure 1. Voltage divider R_9 R_{10} is connected across the arc and the part of the arc voltage appearing across R_{10} is connected to the arm *M* of potentiometer R_8. This potentiometer is calibrated in terms of arc voltage, and in order to make this calibration as independent as possible from line voltage fluctuations, the current through R_8 is held constant by shunting voltage regulator tube *UX*874 across R_7 and R_8. The difference between the voltage *OM* over the lower part of potentiometer R_8 and the voltage *KM* across R_{10} is used as the d-c grid bias on the first tube. Resistance R_{11} which is 500,000 ohm and condenser C_1 of one microfarad capacity serve as a filtering element, so that the grid bias of the tube depends only on the average arc voltage. The production of an alternating voltage depending in magnitude on the d-c voltage is accomplished by applying a small alternating voltage to the grid of the first tube and changing the amount of amplification by changing the d-c bias voltage. The alternating voltage is obtained as follows: Point *G* has an alternating potential with respect to *O* of about ten volts. Condensers C_5 and C_1 act as a voltage divider; the capacity of C_5 is only about 0.025 microfarad, so that only a small alternating voltage appears on the grid. The amplified voltage will appear across the load resistor R_5 in the plate circuit of the tube, but the degree to which the tube will amplify the alternating voltage applied to the grid will depend on the d-c grid bias. In this particular circuit the 6*J*7 will cease to amplify when the negative grid bias exceeds 2.5 volts; with a negative grid bias of 1.5 volts, the alternating voltage appearing across the resistor R_5 will be ten volts, while with a negative grid bias of 0.5 volts or less the tube reaches full amplification, and the output voltage will be 20 volts across R_5. However, the alternating voltage drop across R_5 is only one part of the total alternating potential of point *H*, which is applied to the grid of the output tube by means of conventional capacity coupling C_2R_2. The other part is the alternating voltage *FG*, obtained from a center-tapped winding of transformer T_3. Resistor R_4 and condenser C_4 are connected across this winding, and since the voltages across R_4 and C_4 are always 90 degrees out of phase, a variation of R_4 will change the phase but not the magnitude of the voltage *FG*, which will always be one-half of the total voltage produced in the center-tapped winding at T_3. In this particular case the voltage *FG* is ten volts. The phase adjustment of this voltage is necessary to obtain proper balance in the final bridge circuit containing the bulbs. Due to the filtering effect of the resistance capacity combination R_3C_3, point *F* has no alternating voltage against O. As we had seen above, a small part of the alternating voltage *FG* was applied to the grid of the 6*J*7 by means of the voltage divider C_5C_1, and since any amplifying tube also produces a phase reversal, the output voltage *GH* will be 180 degrees out of phase with the voltage *FG*. This can be seen in another way: Consider the instant when the potential of point *G* has reached its negative maximum with respect to point *F*. Due to the voltage divider C_5C_1, the grid of the 6*J*7 will also reach its negative maximum at this instant.

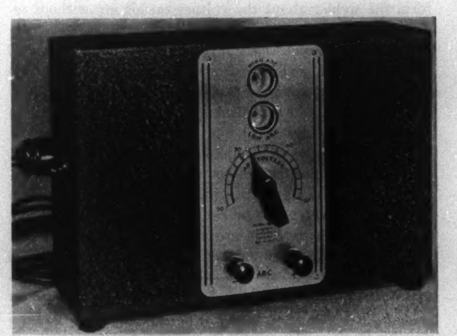

Figure 2. Front view of arc-length monitor

The alternating component of the plate current will then reach its minimum and with it the voltage drop across R_5. At this instant point H is therefore the least amount negative with respect to point G, and is seen to be 180 degrees out of phase with the potential of point G. When the alternating voltage across R_5 is just ten volts, the alternating potential applied to the grid of the 6*F*6 will be zero, since then the voltages *FG* and *GH* add up to zero. It is therefore seen, that as the arc voltage varies in such a manner, as to change the resulting negative grid bias on the first tube from 2.5 volts or more to 1.5 volts and finally to 0.5 volts or less, there will appear on the grid of the 6*F*6 an alternating voltage of 10 volts, which will diminish to zero and then build up again to 10 volts in opposite phase. Phase control of this voltage is obtainable by adjusting R_4. Incidentally, this system of converting a small d-c voltage change into a phase reversing alternating voltage can also be used where grid control of mercury vapor rectifiers (thyratrons) is desired from a small d-c voltage change. In the plate circuit of the output tube is found transformer T_2, the center-tapped secondary of which is connected in a bridge circuit containing lamps L_1 and L_2. The low-voltage winding of transformer T_1 is connected between the center tap of T_2 and the junction of the bulbs, and the current produced by it will flow at a given instant through the bulbs in the direction of the arrows drawn in full lines. The magnitude of it is adjusted by varying R_1 to a value just below the glowing point of L_1 and L_2. Grid excitation of the 6*F*6 will produce currents in the bridge circuit in the direction of the arrows in dotted or dash-dotted lines, depending on the phase of exciting alternating voltage on the grid. In the case of the dotted arrow the current through L_2 will be increased, through L_1 decreased, while the reverse takes place in the case of the dash-dotted arrow. The output of the tube is sufficient to operate more than one set of lights and on the finished instrument, figure 2, a second set of lights is provided which can be observed by the foreman or superintendent.

The results obtained with the instrument are very satisfactory. In training men, it was found that new men acquired the ability to hold a steady arc in about one-third of the time that it took without the instrument. The biggest advantage comes from the fact that all men could be trained to the technique best suited for the particular electrode. In one case the electrodes required for the work had to be welded within narrow limits of voltage. All work was X-rayed and any defects had to be chipped out and rewelded. After equipping the operators with these instruments the necessary repair work dropped to a fraction of what it was before. In this connection it was interesting to note that relatively new men produced good work sooner than old welders. It was obviously easier to acquire the correct method right from the start than it was to break away first from a faulty habit. Men that have been trained with bare-wire welding as a rule try to hold as short an arc as possible, which usually is undesirable for coated electrodes. With the aid of the instrument, men with this type of training could be switched over to coated electrode welding considerably faster than ordinarily.

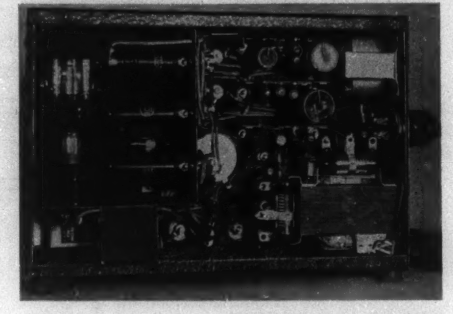

Figure 3. Rear view of arc-length monitor, back removed

The Application and Performance of Carrier-Current Relaying

By PHILIP SPORN
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MEMBER AIEE

I. Introduction

THE GENERAL idea of pilot relaying is quite old. It has been employed for several decades in the Mertz Price system where pilot wires are employed for balancing the currents at the two ends of the line and has met with considerable success in Europe, and particularly in England. A simpler pilot scheme in which the pilot wire is used merely to conduct an impulse, and the conditions of the line at the two ends of the section are indicated by the relative positions of a series of power directional relays, has been employed and has been described elsewhere.[1] Neither of the above schemes, however, has ever met with any great favor in the United States on account of the prohibitive cost of pilot wire circuits, and particularly on long lines, and because in the long run the scheme itself is no more reliable than the pilot wire. Experience has shown that the reliability of the latter is not quite at the 100 per cent level desired and mandatory for proper service.

One attempted solution for this difficulty was the substitution of carrier current for the pilot wire. A scheme embodying the idea was developed several years ago by Fitzgerald and is fully described in a paper before the AIEE.[2] The weakness of this scheme and the development of the first effective carrier relaying system as well as its performance in practical operation have been previously discussed.[3] This latter system was entirely successful but its operating speed was limited due to the racing of contacts. In practice this meant the maximum reliable speed was limited to a minimum time of approximately four cycles in terms of a 60-cycle system. However, it soon became evident that it would be necessary to reduce the relay time as close to the theoretical minimum (in power directional relay practice one-half of one cycle) as possible. A more detailed discussion of this requirement and the work that eventuated in the development of the one-cycle carrier relay system which is now the accepted system has been given elsewhere.[4]

Although developments of this system or slight modifications thereof have been described in papers presented or being presented before the Institute,[5,6] none of them however, have heretofore given a detailed discussion of the basis of application of carrier and particularly high-speed (one-cycle) relaying. In view of the fact that the authors have been engaged in the work of development of carrier relaying and its application over the past 11 years, they hope to be able to give here an intelligent and comprehensive discussion of the principles of relay practice as applied to carrier, to outline what can be accomplished by its use, and to show actually how it has been applied and the way in which it performs. To this end the authors believe it will be helpful to describe briefly the actual carrier current relaying equipments which have proved highly successful in actual operation since their installation on the transmission lines of the various power systems with which they are associated.

A diagram of the first effective carrier current arrangement is shown in figure 1 and will prove of interest. It consists basically of one three-phase power directional relay without voltage restraint, three instantaneous overcurrent starting relays, three definite time overcurrent tripping relays and one receiver blocking relay. In case of an external fault the starting relays operate to close their contacts instantaneously at both stations. At the station where the power flow is from the line into the bus the directional relay opens its tripping contacts preventing the tripping relays from opening up the breaker. In addition the transmitter plate contacts of the directional relay close, starting carrier transmission at this station. At the other station, the power flow is of necessity from the bus into the line and the tripping contacts of the directional relay remain closed, thus not permitting the transmitter to operate. Carrier will be received by the receiver relay which will open up the trip circuit at this other station. In case of an internal fault the power flow

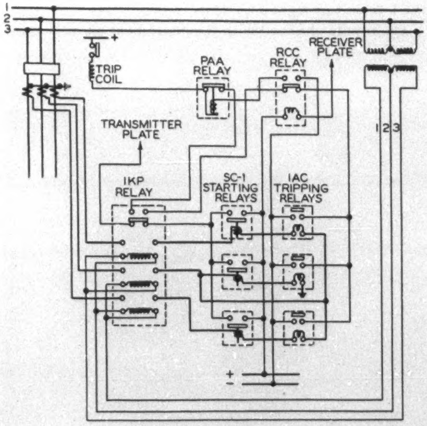

Figure 1. Four-cycle carrier-current relay arrangement

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[1] For all numbered references, see list at end of paper.

at both stations will be from the bus into the line, hence the tripping contacts of the directional relays at both stations will remain closed and the transmitters will be inoperative. Therefore the tripping relays at both stations will close their contacts and trip their respective circuit breakers.

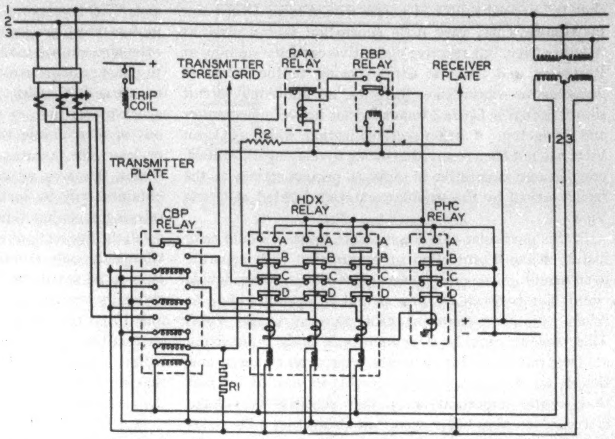


Figure 2. One-cycle carrier-current relay arrangement

As already mentioned, this scheme was entirely successful except for the comparatively slow operation speed. The one-cycle scheme of protection is shown in figure 2. It will be seen that the outstanding feature is the employment of three simple multicontact instantaneous impedance-type phase relays in one case and one multicontact instantaneous ground overcurrent relay in combination with one three-phase power directional relay with voltage restraint and one receiver relay. All contacts of the instantaneous fault-detector relays are circuit opening except the tripping contacts which are circuit closing. This combination gives a circuit having the equivalent of a conrinuous carrier circuit, with the battery taking the place of carrier under normal conditions.

Under normal conditions the *CBP* directional relay contacts are held closed by voltage restraint applying plate voltage to the transmitter, but the transmitter does not operate due to the normally closed contacts *C* of the fault detector relays applying a negative bias to the screen grid of the transmitter. Further, the *RBP* receiver relay contacts are held open by the closed contacts *B* of the fault-detector relays energizing receiver relay coil from the station battery. In case of an external fault, the instantaneous fault detectors operate at both stations to remove voltage restraint from the directional relays, bias from the screen grid of the transmitters and local battery supply from receiver relay coils. The instant that the bias is removed from the screen grids the transmitters at both stations operate, thereby holding receiver relay contacts open. At the station where the power flow is from the bus into the line the directional relay contacts open, stopping the local transmitter from operating, but at the other station the transmitter is operating causing carrier to be received, preventing the receiver relay contacts from closing. After three cycles, the *PAA* lockout relay contacts at both stations open, to prevent the breakers from tripping on sudden reversals of power flow after the fault is cleared. Therefore at both stations the receiver relay contacts remain open, thus preventing tripping of the circuit breakers. In case of an internal fault the operation is similar except that the directional relay contacts will open at both stations, stopping transmission of carrier and causing the receiver relay contacts to close at both stations and trip their respective breakers.

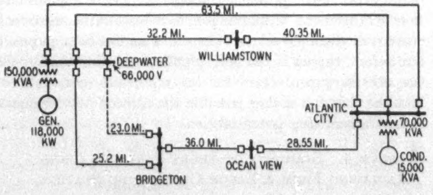


Figure 3. Illustrating problem 1: 66-kv system of Atlantic City Electric Company

II. Applications of High-Speed Carrier to a Power-Transmission System

In order to show the possibilities in the way of application of high-speed carrier relaying to the solution of either technically or economically difficult problems in the operation of high-tension power systems, a series of problems as they normally present themselves to the transmission designer or operator will be taken up and the use of carrier in their solution will be briefly discussed. Alternatives where such exist will be given and the advantages pro and con through the use of carrier will be pointed out.

PROBLEM 1. ADDITION OF ANOTHER STATION IN AN EXISTING TRANSMISSION LINE LOOP

Figure 3 is a typical existing transmission system loop which had been protected, adequately, until the change referred to below was carried through, by instantaneous overcurrent protection in conjunction with the standard time-delay reverse power protection. It is to be noted that the normal source of power flow is from Deepwater to Atlantic City, very little generation taking place at Atlantic City, but reactive corrective capacity running at Bridgeton and Atlantic City. Owing to the increased demands for transmission facilities, an additional circuit shown dotted in figure 3 was called for between Deepwater and Bridgeton. Further sectionalization, which at Ocean View had not been resorted to owing to relaying difficulties, now became imperative in order to protect service to the region served by the step-down station located at Ocean View.

In this particular case the standard solution would be to install balanced protection in conjunction with separate overcurrent or reverse power protection on the double circuit line between Deepwater and Bridgeton and go to reverse power overcurrent protection at Ocean View. This, however, would call for an increase in the time setting at Deepwater on the separate overcurrent relays and this, it was felt, would be intolerable in view of the fact that several important loads, very sensitive to voltage disturbances were being served at Bridgeton. The solution adopted, therefore, was the installation of one-cycle carrier protection between Bridgeton and Ocean View. Carrier here was the only practical solution.

It is to be noted in this case that no difficulties were taken into consideration from possibilities of double circuit faults between Deepwater and Bridgeton, in view of the fact that the two circuits between those two points run on different rights of way. But if this difficulty had to be considered, then the necessity of carrier on the section between Deepwater and Bridgeton would have had to be given serious consideration.

Problem 2. Prevention of Important Synchronous Motors Dropping Out of Step on Voltage Surges of Long Duration

Figure 4 shows the 44-kv transmission system of the Appalachian Electric Power Company radiating out from the Cabin Creek generating station. The Belle Substation supplies a large industrial chemical load of approximately 50,000 kw demand, which consists mainly of 2,000- and 2,700-horsepower low-speed synchronous compressor motors which are very sensitive to voltage surges. From extensive studies made, it was found that for any phase-to-phase fault occurring on this transmission system within an area of 20 miles of the Cabin Creek 44-kv bus, the synchronous motors at the chemical plant would drop out of step unless these faults were cleared within 10 cycles. The assurance that all phase-to-phase faults within this area would be cleared in this time could be obtained only by installing the one-cycle carrier current relaying in conjunction with high-speed oil circuit breakers on all the line sections indicated in figure 4. Again carrier offered the only practical remedy in a very difficult case. It is to be noted that no carrier current installation was necessary on the Cabin Creek-Charleston line. It was found that the instantaneous overcurrent relay protection on this line section would clear the phase-to-phase fault within a period of 10 cycles for all phase-to-phase faults that were likely to affect the stability of the motors at the chemical plant.

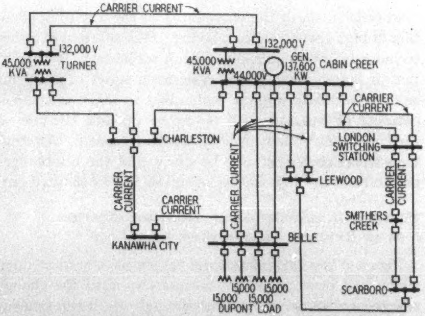


Figure 4. Illustrating problem 2: 44-kv system of Charleston division of Appalachian Electric Power Company

Problem 3. Stability of a High-Tension Transmission System

Figure 5 shows a 350-mile double-circuit tie line between the Philo station of The Ohio Power Company, the Twin Branch station of the Indiana & Michigan Electric Company, and the Michigan City station of the Northern Indiana Public Service Company. These three stations in turn tie in very extensive 132-kv networks totalling some 4,000,000 kw of generating capacity. On this tie line it was essential that all types of faults, such as, ground, phase-to-phase, and simultaneous faults on two circuits be cleared as rapidly as possible in order to keep within the stability limits of the tie line. In addition to high-speed oil circuit breaker installations, all the line sections with the exception of the short section between Twin Branch and South Bend stations were equipped with carrier current relay protection in addition to the standard schemes of balanced and instantaneous overcurrent protection. Only the carrier current system offered fast enough relaying in case of simultaneous trouble on two circuits and at the same time insured in case of either single-line or two-line faults that all breakers on the section in trouble would trip simultaneously. Cascading of oil circuit breakers would result in too slow clearing and would make the transmission system either inoperative or materially reduce its transmitting capacity.

This, of course, would be the case if an attempt were made to solve the problem through the use of a combination arrangement employing either instantaneous overcurrent or distance relays. In short, as has been pointed out before, carrier is the only practical solution that gives the necessary protection for an important sectionalized line like this and makes possible the utmost development of its transmitting potentialities.

Problem 4. Stability of Short Parallel Lines Radiating From a Large Generating Station

Figure 6 shows the short double-circuit lines radiating from the Philo generating station supplying Zanesville and

Crooksville stations. These 132-kv circuits are very short in length and when simultaneous faults occur on both circuits, the shock to the 132-kv system is extremely severe and results in opening the tie line between Philo and Twin Branch stations unless such faults are cleared very fast. For single-line faults occurring on either of these lines, high-speed clearing is obtained by the standard high-speed balanced schemes of protection. In case of simultaneous troubles on both circuits, the standard forms of relaying would not assure fast enough clearing under these conditions. Due to the infrequent occurrence of such faults, the cost of installing complete carrier current relaying equipment at all six terminals was not warranted. A scheme of carrier current relaying was therefore developed which would trip both oil circuit breakers at Philo station practically instantaneously only when a fault involving both 132-kv circuits occurred.

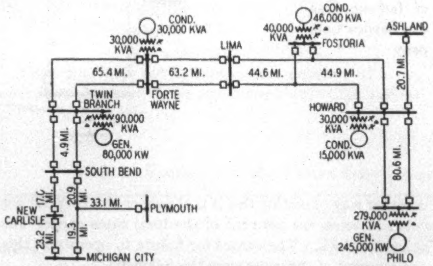


Figure 5. Illustrating problem 3: 132-kv double-circuit tie line of The Ohio Power Company and Indiana & Michigan Electric Company

PROBLEM 5. ULTRAHIGH-SPEED RECLOSING

The development of the idea of ultrahigh-speed reclosing and its application, including the performance of the equipment, has been presented in a previous paper.[7] That paper described the installation of an ultrahigh-speed reclosing setup on a line of the Indiana & Michigan Electric Company. Figure 7 shows the transmission line in question including a number of connecting lines all of which comprise a 132-kv transmission loop circuit radiating from the Fort Wayne station of the Indiana & Michigan Electric Company and supplying the Deer Creek and Delaware substations of the Indiana General Service Company. Although there is an interconnection with Kokomo, it has not enough capacity to supply the entire load of the Indiana General Service Company when separated from Fort Wayne station under double-line-fault conditions occurring on the two lines from Fort Wayne. The load here, therefore, really represents the general condition of a stub load fed from an interconnected system either by a single line or by a double-circuit line on a single tower line. The continuity problem is typical of the problem such a load presents of maintaining uninterrupted service either under a single-line or a double-line faulting, respectively. The only inexpensive solution for this problem is ultrahigh-speed reclosing. A trial installation made a year ago on the Fort Wayne-Deer Creek line showed effective results in 75 per cent of the cases. Specifically, out of eight cases of faulting of the line between Fort Wayne and Deer Creek on all of which ultrahigh-speed reclosing took place, six definitely held on first reclosure and the other two resulted in the restriking of the arc necessitating retripping of the line.

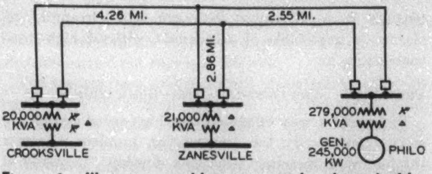


Figure 6. Illustrating problem 4: 132-kv short double-circuit line of The Ohio Power Company supplying Zanesville and Crooksville

It is obvious that this does not give a 100 per cent solution to the continuity problem but the fact that apparently 75 per cent of flashovers can be eliminated as far as the system effects are concerned is, the authors believe, without question a major contribution to the solution of the continuity problem on transmission-line operation.

In order to make high-speed reclosing practical, it was necessary to be able to re-energize the line not longer than 20 cycles after the initiation of the fault, and it was necessary to very definitely have a waiting time between arc outage and line re-energization of between 6 and 8 cycles. To accomplish this and to assure both breakers on the line operating simultaneously, it was necessary to have a one-cycle carrier-current system; any other arrangement would have given neither the speed of clearing nor the necessary interval between clearing and re-energization to get the beneficial effects of rapid reclosure.

The problem of the application of ultrarapid reclosing to high-tension networks to eliminate the effects of line outage and to improve stability limits of such networks is a very broad problem and one that will warrant further consideration and it is hoped to present a paper on that in the near future. It is interesting, however, to observe that in a case of a double-circuit tie between two systems such as the one represented by figure 5, studies have indicated that for single-circuit operation the amount of power that can be carried through a disturbance with rapid reclosure of the faulted circuit increases appreciably as the time of separation between the systems is decreased. This definitely points to the desirability of keeping the tripping and the reclosing time to a minimum even though the systems are large and the rate of decay of speed is slow. For double circuit operation with a clearing time of 12 cycles, it is possible to transmit 40 per cent more power with a successful rapid reclosure of the faulted circuit than if the circuit is not reclosed.

All of the above indicate a great many unexplored benefits from ultrarapid reclosure and ultrarapid reclosure is impossible of attainment without high-speed carrier relaying.

Problem 6. The General Case—Summation

In general it may be stated that high-speed carrier will solve economically the transmission problem wherever the following operating situations develop: necessity of speed and utmost reliability in the clearing of faults to accomplish minimum system disturbances; necessity of minimum disturbance to voltage sensitive equipment; need to bring up to a maximum load stability limits, either static or transient, of a transmission line; and the need of remedying the effect of line outage by ultrahigh-speed reclosure. In brief, whenever the necessity arises of counterbalancing the deleterious effects of system faults by the utmost speed in removing them from the faulted sections to leave the rest of the system undisturbed, or where high-speed reclosure of a faulted section is desired in a time so short that no ill effect of the initial fault results, the application of high-speed carrier relaying is imperative. As system concentrations increase and the injurious effects of system faulting correspondingly rise, the necessity of counterbalancing the deleterious effects of system faulting obviously increases.

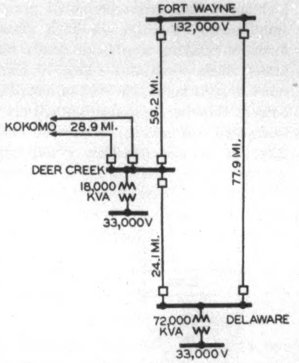
III. Performance of Carrier Equipment

The discussion of the application developments given above shows, the authors believe, the wide application open to high-speed carrier relaying but after all, the theoretical benefits are of far greater significance if accompanied by actual performance experience. In this connection the experience of the authors on their own system is, therefore, pertinent. At the time of writing (October 1937) there are 34 carrier current relay terminal installations in operation on the transmission systems of the various companies with which the authors are associated. The first ten of these were placed in operation in 1933. Thirty-one of these installations protect 14 line sections covering approximately 750 miles of single-circuit line. The other three installations are a solution of a special case and provide protection primarily for simultaneous trouble occurring on both circuits of a short double-circuit line.

A tabulated summary of the operation of these carrier sets since their installation and the performance both on internal and external faults is given in table I. It will be noted that a total of 203 internal faults were experienced on these sets during the period 1933–1937, all of which were cleared correctly by the carrier relays. In other words, there was no case where relaying was called for that the relays did not perform to clear the faulted section of line. This the authors believe, is unusually successful performance. During the same period 27 incorrect carrier relay operations occurred on external faults—that is, there were 27 cases where faults occurring on sections external to those on which the carrier relaying was installed, resulted in action—incorrect action—within the sections on which carrier relaying was installed.

An analysis of these 27 incorrect actions is given below. It is worth noting, however, that some possible 2,000 external faults occurred during the same periods in all of which the carrier relaying operated correctly, that is, the carrier operated to block within the healthy section.

Figure 7. Illustrating problem 5: 132-kv system of Indiana & Michigan Electric Company radiating from Fort Wayne and supplying Deer Creek and Delaware stations of Indiana General Service Company



Another way of stating this is that proper blocking did not occur in some one per cent of the total cases carrier was called to block. The causes for failure to operate in this one per cent of the cases were the following:

(*a*). Seven of these operations were due to rain splashing up onto the protective gaps on the coupling capacitors from the under side thus shorting the gaps out for short periods of time and preventing the transmission and reception of carrier. This difficulty was corrected by increasing the gap settings and by providing better gap shielding.

(*b*). Two of the incorrect operations were due to vacuum gaps used in the trap units becoming defective. This caused the gaps to break down during fault conditions, preventing the transmission and reception of carrier. This difficulty was remedied by testing and checking vacuum gaps periodically once a year.

(*c*). Four of the operations were due to the tripping relay on the four-cycle carrier scheme not opening its contacts on resetting before the receiver relay had closed its contacts in the reset position. This trouble was eliminated by the installation of an instantaneous pickup time delay dropout lockout relay which gave sufficient time for the tripping relays to reset, preventing false operations.

(*d*). Four were due to the power directional relay contacts getting out of adjustment so the contacts did not close positively enough when they should have closed to transmit carrier. All of these four operations occurred on the same day and happened before the relay man reached the station to determine the cause of trouble. This trouble was remedied by adjusting the directional contacts so as to give plenty of wipe to insure positive closing of contacts.

(*e*). Eight of the operations were due to the adjustments of contacts on the fault detector and power directional relays changing with time. This was remedied by installation of more positive fault detector and power directional relays which so far have shown that they do not alter their adjustments with time.

(*f*). One of these operations was due to the tripping relay at one terminal being set through an error lower in magnitude than the starting relay at the other terminal. This obviously was bound to cause an incorrect operation on an external fault. The remedy was simple.

(*g*). One of the operations was due to the voltage control relays employed with the four-cycle carrier scheme not being set properly at both terminals to take account of the difference in voltages at the two stations for a case of system instability. This condition has been remedied since then by employing proper settings on the undervoltage relays at the two terminals to take care of instability conditions.

It will be noticed in every case the definite cause for the failure to block properly was found and the necessary remedy determined. This should therefore eventually give perfect performance. That this is not an unreasonable expectation is evidenced by the record of only a single incorrect operation during 1937, the period when the maximum number of installations were in service.

IV. Maintenance Experience With Carrier Equipment

Those who have not had intimate experience with vacuum tube equipment have sometimes shied at the application of carrier because of the seemingly great complication of carrier circuits or the lack of sufficient confidence in the thorough reliability of carrier and vacuum-tube equipment in general. The experience running over some two decades with carrier and carrier tubes has been described elsewhere.[8,9] Subsequent to the time the above performance was described, further operating experience confirmed the utmost reliability of the carrier circuits and of the vacuum tubes and their concomitant equipment. Nevertheless it needs to be pointed out that reliability is not automatic and can be obtained only by a systematic, even though a comparatively simple, maintenance program. In this regard the experience on the system with which the authors are connected may be of interest. The program carried out here is as follows:

The transmission and reception of carrier are checked regularly by the station operators. At attended stations this check is made once each shift and at unattended stations it is made daily. This test is an over-all check of the carrier current operation and consists of each station transmitting in turn and reading the received signal from the distant station and the back-feed from the local transmitter, by means of a test plug and milliammeter provided for the purpose on the switchboard. Any abnormal variations in the readings obtained are reported immediately by phone and the trouble is located and corrected by the carrier current maintenance man.

More detailed tests and inspection of the carrier-current equipments are made regularly once every three months by the carrier current maintenance engineer. These detailed tests include checking the tubes, operating voltages and currents; checking the transmitter-receiver unit for proper frequency and output, margin of received signal available, and interference from other carrier channels; checking "*B*" batteries for output capacity; checking line trap units for proper operation, and a thorough visual inspection of all the equipment.

The following summarizes the troubles and experience with carrier current equipment over the past four and one-half years as determined from the routine tests.

Table I. Summary of Carrier-Current Relay Operations on 34 Installations for Period of 1933–1937

Company	Line	Kv of Line	Length of Line (Miles)	Date Carrier Relaying Put in Service	Operations on Internal Faults						Incorrect Operations on External Faults					
					1933	1934	1935	1936	1937	Total	1933	1934	1935	1936	1937	Total
Appalachian Electric Power Company	#1 Cabin Creek-Turner	132	23.6	Aug. 3, 1934		0	2	4	0	6		0	0	7	0	7
	#2 Cabin Creek-Turner	132	23.6	Aug. 3, 1934		0	5	1	0	6		1	4	0	0	5
	#1 & #2 Cabin Creek-Turner	132	47.2	Aug. 3, 1934		0	0	3	1	4		0	0	0	0	0
	Roanoke-Fieldale	132	37.9	Apr. 5, 1935			3	7	6	16			3	1	0	4
Indiana & Michigan Electric Company	Fort Wayne-Deer Creek	132	59.2	May 18, 1936				4	5	9				1	0	1
	South Bend-Michigan City	132	41.1	Apr. 16, 1937					2	2					0	0
	South Bend-Plymouth-Michigan City	132	77.6	Apr. 16, 1937					3	3					0	0
	South Bend-Michigan City & South Bend-Plymouth-Michigan City	132	118.7	Apr. 16, 1937					1	1					0	0
The Ohio Power Company	#1 Philo-Howard	132	80.6	June 22, 1934		4	2	7	5	18		0	0	0	0	0
	#2 Philo-Howard	132	80.6	June 22, 1934		1	5	0	3	9		0	0	0	0	0
	#1 & #2 Philo-Howard	132	161.2	June 22, 1934		1	2	4	3	10		0	0	0	0	0
	#1 Howard-Fostoria-Lima	132	89.5	Aug. 4, 1933	3	16	0	1	1	21	0	0	0	0	0	0
	#2 Howard-Fostoria-Lima	132	89.5	Aug. 4, 1933	3	9	3	5	5	25	0	0	0	0	0	0
	#1 & #2 Howard-Fostoria-Lima	132	179.0	Aug. 4, 1933	0	0	0	0	0	0	0	0	0	0	0	0
	#1 Fort Wayne-Lima	132	63.2	June 5, 1933	3	4	4	2	6	19	1	1	0	0	0	2
	#2 Fort Wayne-Lima	132	63.2	June 5, 1933	2	8	2	4	2	18	0	1	0	0	1	2
	#1 & #2 Fort Wayne-Lima	132	126.4	June 5, 1933	1	1	1	1	0	4	1	0	0	0	0	1
	#1 & #2 Philo-Zanesville-Crooksville	132	43.9	June 21, 1934		0	0	0	0	0		0	0	0	0	0
The Scranton Electric Company	#1 Stanton-Suburban	66	10.85	June 9, 1934		3	2	2	0	7		2	1	0	0	3
	#2 Stanton-Suburban	66	10.85	June 9, 1934		2	8	5	4	19		2	0	0	0	2
	#1 & #2 Stanton Suburban	66	21.7	June 9, 1934		0	1	4	1	6		0	0	0	0	0
	Total				12	49	40	54	48	203	2	7	8	9	1	27

1. Vacuum Tubes

Vacuum tubes have shown exceptionally long life and a high degree of reliability. Many of the type 210 tubes installed three to four years ago are still in service. The shortest life recorded on this type of tube was 3,600 hours and an average life of over three years is being obtained. In later equipments screen grid tubes were used for grid control of carrier transmission. Experience with the type 48 tube used in a few installations showed that it had insufficient insulation to withstand the lightning and switching surges which occur and a vacuum gap protector was added to the equipment to prevent the surge voltages from getting to the tubes. This corrected the trouble due to voltage breakdown of the tubes and an average life of approximately one year has since been obtained with these tubes. The latest sets use type 802 tubes and no failures have occurred during the six month service period to date.

2. "B" Batteries

At some stations heavy duty "B" batteries are used as plate supply for the transmitter and receiver tubes. These have a life of between one and two years in this service but are normally replaced once a year. In some cases where these batteries were not replaced yearly, the end of life was definitely indicated by the regular daily tests.

3. Trap Units

Line trap units developed the following faults:

(*a*). Paper condensers originally used varied their capacity with temperature, causing a change in tuning. This was corrected by using mica type condensers.

(*b*). It was found that the mica type condensers due to their low impulse strength broke down in several instances on lightning surges and protectors were added. In the one instance of trouble since the above change was made lightning destroyed the vacuum gap but did not harm the tuning condensers.

(*c*). Out of 37 traps in use there has been one case in which trap coils were deformed by a heavy fault current. This occurred on an internal line fault with an estimated short circuit current of 6,800 amperes and both line traps were affected.

4. Transmitter Receiver Units

Maintenance experience with the transmitter receiver units has been quite satisfactory after the few minor defects in early equipments were corrected. These included:

(*a*). The master oscillator was not sufficiently stable in frequency for satisfactory operation. This was corrected by redesigning the master oscillator circuit to provide more circulating energy in the oscillator tank and to reduce the coupling between the oscillator and power amplifier tubes.

(*b*). The air tuning condenser in the master oscillator circuit was originally connected between the tube plate and ground. It was found that any transient which broke this condenser down was followed by continuous arcing of the condenser fed from the d-c plate supply voltage. This trouble was corrected by using a fixed condenser in series with the variable condenser.

The relay circuit employed with carrier relaying is tested monthly to determine that the circuit breaker will trip from the carrier relays and in addition the blocking action for an external fault is checked. In the case of the one-cycle carrier system where more delicate adjustments of relays are necessary, their over-all adjustments are checked monthly by operating each fault detector relay manually with power flow into the bus and noting that the receiver relay contacts do not move and remain in the open position. Once every six months the relays are calibrated, inspected, and readjusted where necessary.

V. Conclusions

The authors believe as a result of their experience in developing and applying carrier current relaying, outlined in this paper that the following facts stand out particularly:

1. Carrier current relaying is definitely out of the experimental class and is now thoroughly practical, simple, reliable, and relatively low in cost.

2. This form of relaying is universal in its application.

3. Where conventional relay schemes have been applied up to their limit from the standpoint of getting proper selection, carrier relaying offers a solution for adding additional sectionalizing stations without disturbing the existing relay set-up or increasing sectionalizing time. At the same time, on the new protected sections sectionalizing time can be made a minimum.

4. Where a system is sensitive to fault initiated voltage disturbances, carrier relaying is the only practical form of protection which for every location and kind of fault will give short enough sectionalizing time to keep the duration of these voltage surges short enough.

5. The limit on loading of important tie lines without running into instability problems, is largely determined by the speed with which faults can be cleared. Carrier relay protection, which is the fastest practical universal scheme so far developed, materially aids in the operation of such tie lines to maximum advantage. Instability difficulties on short lines radiating from large generating stations can be effectively reduced by the application of this fast protection.

6. The development of ultrahigh-speed reclosing hinges directly on high-speed relaying both because total operating cycle time must be a minimum and because cascading of breakers cannot be tolerated. Carrier current relaying again is the only practical form of relay protection which will accomplish this.

7. The applications outlined above have been developed on the basis of experience. They have not been found difficult to carry out. It may be found possible to further simplify the present scheme with time, but even as it stands, it has been found to be comparatively simple. The equipment with which this work is being done has, on the other hand, during the period it has been in service proved to be most rugged. It has given quality and reliability of protection few dared to think possible only about a decade ago.

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The Rating of Resistance-Welding Transformers

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Synopsis: The present method of rating resistance-welding transformers varies with the different manufacturers, causing a certain amount of confusion among the users of this equipment. It is recommended that a standard method of rating, based on the thermally equivalent continuous output, be adopted and adhered to by the manufacturers. The general requirements of a welding transformer are discussed for the purpose of explaining the desirability of such a standard rating.

Certain of the recommendations made herein are already embodied in the present AIEE Standards for resistance welding apparatus, and in the standards adopted by the Resistance Welder Manufacturers Association, but it is further recommended that these standards be altered to require additional name plate data.

It is hoped that the transformer manufacturers and users will recognize the desirability of such standardization, and co-operate in its universal adoption.

FOR various reasons comparatively little technical information has been published concerning resistance welding, and today, only those who have had close contact and shop experience with this process are familiar with the problems involved, both as to the technique of welding as well as to the requirements of the equipment. It is the purpose of this paper to discuss one of the most important items of the equipment—the welding transformer—and offer further suggestions and recommendations as to the standardization of this part of the welding apparatus.

In the early days of resistance welding the transformer was considered not so much as a piece of electrical equipment but rather as incidental to the welding machine as a unit. The welding engineer himself knew comparatively little concerning the requirements of the transformer. He knew primarily that a relatively low voltage applied to a localized area of the pieces to be joined would cause a current to flow through these pieces, thus generating sufficient heat to fuse them together.

In stationary welding machines the transformer is connected to the welding electrodes through short conductors of rather large cross section. The secondary or load circuit is therefore of very low impedance, and the tendency is to consider it as effectively a short circuit on the transformer. This condition might be rather confusing to some engineers concerned with welding. How is it possible in the light of conventional transformer design knowledge, to design and build a transformer to stand up under the repeated short circuits demanded by the welding job? In fact I have heard it said by some that it was impossible to build a transformer which would stand up and that failures due to burn outs were to be expected. It is, I presume, partly because of this condition that welding transformers have the vagueness as to rating that they now have. It is fairly axiomatic in the welding profession that transformer ratings mean little if anything at all. I have found in the past that a transformer manufactured by "*A*" might be physically much larger than the unit of the same name-plate rating manufactured by "*B*," yet load tests and actual service have in certain instances, shown that the larger transformer may, under the same loading, fail before the smaller one. This condition is due primarily to the fact that although the name-plate kilovolt-ampere rating of the two transformers is the same, the method of determining this rating may vary considerably.

The welding manufacturers are aware of this condition and have taken certain steps toward standardization. Through mutual co-operation, under the sponsorship of the Resistance Welder Manufacturers Association, they have adopted transformer standards which call for a name-plate rating based on a 50 per cent duty cycle. In general, however, these standards pertain mostly to the method of manufacture, quality of materials, and limiting values of current and flux density to be used, and not so much to the over-all performance of the transformer.

Certain of these standards are desirable yet others, I feel, are superfluous. In order to allow the manufacturer latitude in his design he should not be limited, for example, as to the current density he uses. He should be allowed to increase the current density if he desires, thus reducing the weight of his unit, if he can at the same time provide sufficient cooling. On the other hand limitations as to flux density are desirable, as will be explained later. In general I feel that these particular standards do not necessarily insure uniformity as to performance, neither do they convey very definite information as to the capabilities of the transformer. A group of standards relating to performance is the logical way of relieving this condition.

The present AIEE Standard No. 39, pertaining to resistance welding apparatus require that the welding transformer rating shall be a continuous rating, the thermal effects of which shall be as nearly as possible those of the actual duty for which the transformer is intended. These standards pertain entirely to the performance of the unit. They furthermore require the following minimum name-plate data on each unit.

1. Manufacturers name, address, etc.
2. Input in kilovolt-amperes (continuous rating).
3. Frequency.
4. Primary current.
5. Primary voltage.

However, these standards do not entirely cover the subject. Also, my experience has been that comparatively

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1. For numbered reference, see end of paper.

few people active in the welding profession consider them very seriously. I therefore feel that some additional recommendations as to transformer standardization, based on the performance of the unit, are needed, and that a discussion might be helpful in bringing to the attention of manufacturer and user the reasons for these standards and the desirability of their universal adoption.

Before making recommendations, I should like first to discuss in general the requirement of a welding transformer in the hope that a better understanding of the operating conditions may be conveyed. This discussion will deal with the following:

1. Required current and voltage during the welding period.
2. Duty cycle.
3. Thermally equivalent continuous kilovolt-ampere rating.
4. Transformer impedance.
5. Flux density.
6. Secondary voltage range.
7. Transformer cooling.

The scope of this paper does not permit a discussion of the detail design features of the transformer.

Requirements of the Welding Transformer

In making a resistance weld, one requirement, among many, is a flow of electric current through the parts to be joined. It is the function of the welding transformer to supply this current. Its magnitude and the length of time during which it flows depends upon factors determined by the particular welding job and it is not the purpose of this article to discuss these factors. Let us start with the assumption that a certain current is required to flow for a certain length of time for each weld, and discuss the effect of these requirements upon the specification and rating of the transformer.

The function of the welding transformer being to supply current to the parts to be joined, implies that a certain voltage is necessary at the terminals of the transformer in order to force this current through the impedance of the circuit which conducts the current from the transformer to the welding electrodes. This impedance constitutes the load on the transformer and is composed of the resistance and reactance of the conductors and of the weld metal and joints. It is usually very low as compared with load impedances encountered in the average lighting or power application, yet it imposes a definite load on the transformer, and not a short circuit.

One of the first requirements of the welding transformer is therefore, that the magnitude of its secondary terminal voltage under load conditions shall be sufficient to force the required welding current through the impedance of the external welding circuit. This voltage will be equal to the numerical product of the circuit impedance in ohms, and the welding current in amperes, and in order to determine it we must of course know the numerical value of the load impedance. The size, spacing, and length of the conductors leading from transformer to welding electrode, determine this impedance, and these requirements are dictated by the practical requirements of the particular job to be welded. For example, if a large sheet is to be spot welded at its center by means of a stationary machine, the welding electrodes of the machine must be far enough away from the transformer to allow the edge of the sheet to clear it when the sheet is in the position for welding. This determines the length of the external circuit which is commonly referred to as the "throat depth" of the machine. If on the other hand the sheet has a vertical flange which must fit into the throat of the machine in order to place the sheet in position for welding, then the spacing of the conductors must be such as to clear this flange when in the welding position. This determines the "throat height" of the machine. Some commercial stationary machines are built with adjustable throat depth and height, thus presenting different load impedances depending on the particular adjustment being used.

In certain other applications, such as assembly operations where the welding machine must be carried to the work, the so-called portable welding equipment is employed. In such applications the welding tool is connected to the transformer by means of flexible copper conductors, the length, spacing, and size of which depend upon practical considerations. This in turn determines the impedance of the welding circuit, external to the transformer. In most instances the cables can be tied together throughout most of their length except at their ends where they must be separated in order to facilitate fastening to the transformer terminals and to the tool. This keeps the impedance to a minimum and prevents variations due to the cables swinging apart. In one such commercial applications these cables are five feet in length and the total external circuit impedance is about 0.001 ohm.

The load impedance required for any particular job can in most cases be calculated with sufficient accuracy. For practical purposes, however, if a sample set up can be conveniently made, the impedance can be measured by means of a voltmeter and ammeter.

One practical method of measuring this impedance is as follows. By means of any available transformer, force a certain current through the impedance. Then measure this current by means of an ammeter and current transformer and also measure the voltage across the impedance. The ohmic value of the impedance is then:

$$Z_{\text{load}} = E/I$$

Since the ratio of E to I will be constant for a given impedance, the numerical value of the current chosen will be immaterial in so far as the result is concerned. However if the impedance is exceptionally small a rather large current may be necessary in order to produce a voltage drop which can be read with any degree of accuracy with the average shop instrument. Under such conditions, a current transformer with ratio high enough to measure the current may not be available. In this case the turns ratio (A) may be determined by measuring the no-load primary and secondary voltage. Then with the load connected the primary current and load voltage are measured. The value of the load impedance is then:

$$Z_{\text{load}} = \frac{E_{\text{load}}}{A \times \text{primary current}}$$

Once this is determined, the numerical product of this impedance and the required welding current is equal to the voltage which must be maintained at the transformer terminals while it is delivering this current. In most practical welding applications this output is required for only a fractional part of a second to complete the weld. The power is then interrupted and the material progressed to the next weld location. For convenience I should like to refer to this product of root-mean-square current and volts during the welding period as the instantaneous kilovolt-ampere output of the transformer. The instantaneous kilovolt-ampere input will of course be somewhat higher due to the impedance voltage drop in the transformer.

Some manufacturers have in the past assigned kilovolt-ampere ratings to their transformers which they say is the maximum instantaneous kilovolt-ampere which it will demand from the line. In my opinion this rating means very little if anything in so far as the transformer itself is concerned.

Assuming a constant primary voltage, the instantaneous kilovolt-ampere input for a given ratio of transformation, will depend upon the load impedance, reaching a maximum when the load impedance is practically zero, under short circuit conditions. The internal impedance of the transformer will therefore establish the maximum instantaneous kilovolt-ampere that it can demand from the power supply. A rating stated in this manner is therefore nothing more than a statement of the internal impedance of the transformer. In cases where the transformer forms a part of the complete welding machine, the minimum load impedance is fixed by the minimum throat depth and height, and this impedance together with the internal impedance of the transformer will limit the maximum instantaneous kilovolt-ampere input. It is then perfectly legitimate to assign such a rating to the machine as a whole, but such a rating would mean very little if applied to the transformer alone. This rating when assigned to the machine informs the user as to the maximum instantaneous kilovolt-ampere that his lines and other power equipment must be capable of supplying, but it does not give him the complete story. It does not, for example, tell him whether the transformer is capable of performing a certain welding operation without burning out. This answer cannot as yet be given because we do not have sufficient information concerning the requirements of the transformer.

Duty Cycle

We have already assumed that in making a resistance weld, a certain current is required to flow for a definite length of time. This current flowing through the windings of the transformer will generate heat in the windings, which is proportional to the square of the current and to the length of time during which it flows. The average heating effect will therefore depend upon the time length of the current flow required to make one weld and upon the average number of welds per minute required of the machine. It follows therefore that a transformer which makes 20 ten-cycle welds per minute will develop twice as much heat in its windings as a transformer making only ten ten-cycle welds at the same current. As a result it must dissipate twice as much heat in the same length of time if the temperature of its insulation is to be kept below the danger point. It must therefore be designed accordingly, and although the two transformers deliver the same instantaneous output they must be quite different in design and construction.

The ability of any transformer to perform satisfactorily without overheating, depends upon its ability to dissipate the heat generated internally. If its thermal characteristics are such that it cannot dissipate this heat without exceeding the safe operating temperature of its insulation, the transformer will soon fail.

This immediately establishes another requirement of a satisfactory welding transformer, namely, that it shall have sufficient heat dissipating capacity to prevent overheating when delivering the required instantaneous kilovolt-ampere output, at a specified welding time and frequency of operation. The number of conducting cycles per weld, and the number of welds per minute establish the fractional part of a minute during which the transformer supplies current. This fraction is commonly referred to as the duty cycle of the transformer, and must be known by the designer in order that he may incorporate the above requirement into his design.

Thermally Equivalent Continuous Kilovolt-Amperes

A transformer fulfilling these two requirements as to secondary voltage and heat dissipating ability, will perform satisfactorily on any welding operation so long as the specified loading is not exceeded. The requirements so far have been specified with no reference whatever to kilovolt-ampere capacity. We have merely stated that the transformer shall be capable of delivering a specified secondary current at a specified duty cycle and maintaining under load conditions a specified voltage across its secondary terminals. There is no more simple way of stating the requirements, yet, it is desirable for various reasons to assign a kilovolt-ampere rating to the transformer. How then, from the information so far given, can we assign a logical and legitimate rating?

In all power and distribution transformers the kilovolt-ampere rating of each unit is equal to the kilovolt-ampere that it can deliver continuously without overheating, when operated under specified cooling conditions. Then why not, for the purpose of standardization, rate welding transformers accordingly? Some may say that this is not legitimate because the welding transformer is never called upon to operate continuously. The same may be said about power and distribution transformers, because during certain hours of the day they may operate under a considerable overload, while during the remaining hours they are very much underloaded. Yet they perform satisfactorily so long as the heating in the transformer, averaged over a period of time dependent upon the size and design, does not exceed the heating which would be caused if the transformer operated continuously at its rated kilovolt-amperes.

Since the heating effect of the current is proportional to the square of the current, it follows that a certain current flowing for 25 per cent of the time will produce the same average heating as a current of one-half that value flowing continuously. Therefore from a heating standpoint, this continuous current is thermally equivalent to the instantaneous current multiplied by the square root of the duty cycle. This, I should like to call the thermally equivalent continuous current, which multiplied by the secondary load voltage becomes the thermally equivalent continuous kilovolt-ampere output of the transformer. This we may call the kilovolt-ampere output capacity of the transformer, but since in the case of power and distribution transformers the rating is determined by the product of load current and open circuit secondary volts, I recommend that the same method be employed in rating welding transformers.

Impedance

In discussing the welding transformer, my remarks have so far been confined to the secondary load voltage. To determine the open circuit secondary voltage we must know the impedance of the transformer. In other types of transformers this is usually expressed as impedance voltage drop in per cent of open-circuit secondary volts, at a specified loading which is usually the thermally equivalent continuous kilovolt-ampere rating of the transformer. It is my further recommendation that the same custom be adopted for welding transformers. However, since the instantaneous kilovolt-ampere output of the welding transformer will be equal to its thermally equivalent continuous output divided by the square root of the duty cycle, then the per cent impedance voltage drop in the transformer at its instantaneous output will be equal to the per cent impedance drop at the equivalent continuous output divided by the square root of the duty cycle.

Therefore the impedance voltage drop in the transformer during the welding period will be

$$Z/\sqrt{D}$$

where

Z = per cent impedance voltage drop at the thermally equivalent continuous output

D = duty cycle

The open-circuit voltage of the transformer will then be the vector sum of the load voltage and the internal impedance drop. The phase angle of these two voltages will vary, depending on the phase angles of the transformer impedance and of the external welding circuit. Their vector sum will therefore reach a maximum when both phase angles are the same. In other words, if we assume a certain required secondary load voltage, the required open circuit secondary voltage will be a maximum when the load voltage and the transformer impedance drop are exactly in phase. I therefore recommend that, in calculating the required open circuit secondary voltages or in specifying the transformer impedance voltage drop, these two voltages be assumed to be in phase.

Some users may be concerned with the magnitude of the internal impedance of the transformer at continuous rating. If this is high, a higher open-circuit voltage will be necessary in order to maintain a given secondary load voltage. This means a lower ratio of transformation and hence a larger primary current. In most cases this means very little, in so far as power consumption is concerned, but in any case it causes more heating and more voltage drop in the primary line feeding the transformer. On the other hand if the specified impedance is too low the additional transformer cost may not be justified. This impedance should therefore be specified so as to strike an economic balance between the two extreme conditions. After consulting various manufacturers and experimenting with various designs, it appears now that the most economical impedance is from 4 per cent to 6 per cent. I, therefore, recommend that a transformer impedance of 6 per cent be adopted as standard unless otherwise specified.

It is usually not considered good practice to design or specify a transformer to operate at an instantaneous output of more than about three times the thermally equivalent continuous output. If the required instantaneous output is higher than this, the instantaneous impedance voltage drop in the transformer becomes excessive, and it is advisable to employ a transformer with a higher continuous rating. This means that although the actual welding duty cycle may be only 1 per cent on a certain job, the duty cycle used in calculating the thermally equivalent continuous output should be not less than 9 per cent. As an illustration let us assume the following conditions as determined by the welding job.

Required secondary current during weld = 10,000 amperes
Welding circuit impedance = 0.001 ohms
Length of current flow per weld = 7 cycles
Number of welds per minute = 10

From this we see that the number of conducting cycles per minute is 7 × 10 = 70 and the actual duty cycle 70/3600 = 0.0195 or 1.95 per cent. Also that the required load voltage is 10,000 × 0.001 = 10 volts. If then we assume a transformer impedance of 4 per cent at its continuous rating, its impedance voltage drop during the weld will be:

$$Z/\sqrt{D} = \frac{0.04}{\sqrt{0.0195}} = \frac{0.04}{0.14} = 0.285$$

or 28.5 per cent.

The required open-circuit secondary voltage will then be 10/(1–0.285) = 14 volts.

The thermally equivalent continuous kilovolt-ampere output will be

$$\frac{10{,}000 \times 14 \times \sqrt{0.0195}}{1{,}000} = 19.6 \text{ kva}$$

but due to the fact that the transformer loading during the welding period is (10,000 × 14)/1,000 = 140 kilovolt-amperes or 7.15 times the continuous rating, the instantaneous impedance drop in the transformer is 7.15 × 4 per cent = 28.5 per cent.

Assuming a 220-volt primary the instantaneous flow of current from the line (neglecting exciting current) will be

$$\frac{14}{220} \times 10{,}000 = 638 \text{ amperes}$$

If, on the other hand, we use a duty cycle of nine per cent in calculating the thermally equivalent continuous output, the instantaneous impedance voltage drop in the transformer will be

$$\frac{0.04}{\sqrt{0.09}} = \frac{0.04}{0.3} = 0.133 \text{ or } 13.3 \text{ per cent}$$

The open circuit secondary voltage will be 10/(1–0.133) = 11.6 volts and the instantaneous flow of current from the line will be

$$\frac{11.6 \times 10{,}000}{220} = 525 \text{ amperes}$$

The thermally equivalent continuous kilovolt-ampere rating of the transformer will in this case be

$$\frac{10{,}000 \times 11.6 \times \sqrt{0.09}}{1{,}000} = 34.7 \text{ kva}$$

This rating is almost twice that required from a thermal standpoint, yet from a standpoint of voltage regulation and primary current flow, this larger rating is usually justified.

Flux Density

It is desirable to limit the maximum flux density in a welding transformer in order to limit the transient exciting current during starting. In energizing any transformer, unless the circuit is closed at the proper point with respect to the voltage wave, the exciting current will contain a transient component which may be many times the normal exciting current.[1] The magnitude of this transient component will depend upon the point of closure of the circuit with respect to the voltage wave, and upon the normal flux density in the core. In normal welding service the transformer is started, or energized, many times per minute, and since many of the commercial welding transformer controls do not provide for repetitively closing the circuit at the proper point, rather high transient exciting currents might result, unless the maximum flux density is kept within reason. The existence of these high transient currents, places an additional burden upon the control apparatus and also produces additional heating in the primary of the welding transformer. With this type of control therefore it is desirable to limit the maximum flux density. The limiting value will depend upon the characteristics of the iron used in the core. The maximum flux density should be kept below the knee of the saturation curve. For the usual commercial transformer iron, I have found that a maximum flux density of 90,000 lines per square inch gives satisfactory operation but should not be exceeded. I believe that most manufacturers are conforming to this limit at the present time.

It might further be said that the full electronic type of control usually has provision for repetitively closing the circuit at the proper point to eliminate transients irrespective of the value of the flux density.

In such cases it is not so important to limit the flux density, except as it may be desirable to limit the normal exciting current. I feel, however, that irrespective of the type of control anticipated the maximum flux density, at maximum secondary voltage, should be limited so as to keep it below the knee of the saturation curve of the particular iron being used.

Secondary-Voltage Range

In practically all resistance-welding applications, it is desirable for various reasons, to provide some means of varying or adjusting the welding current. For a given welding condition, the external circuit impedance will be substantially constant. The only means therefore of varying the secondary or welding current is to vary the secondary voltage of the transformer. This can be accomplished either by varying the primary voltage impressed on the transformer or, by varying, the number of primary turns in the transformer. Since the first method requires a separate voltage-adjusting transformer in addition to the welding transformer, this is, in a number of instances, uneconomical. The most commonly used method of obtaining this welding current variation is to provide the welding transformer with a tapped primary winding. The previous discussion has dealt entirely with the maximum secondary voltage at which the rating of the transformer should be taken. Since the maximum secondary voltage occurs with minimum primary turns, in order to provide a lower secondary voltage a greater number of primary turns must be provided. For example if a maximum secondary voltage of 10 volts is obtained when using 22 primary turns, in order to obtain a minimum of 5 secondary volts with the same transformer, 44 primary turns must be provided. Since these additional primary turns take up space and add weight to the transformer, it is therefore desirable to specify the minimum as well as the maximum secondary voltage, so as to limit the number of primary turns. In order to insure uniform adjustment of voltage in between these maximum and minimum values it is furthermore desirable to specify the number of voltage adjustments available. For the average portable welding application, I have found that eight voltage steps ranging from 50 per cent to 100 per cent secondary voltage gives satisfactory operation. Applications may vary widely however, so that I do not recommend this as standard. I do recommend that the standards require the manufacturer to give this information, whatever it may be, on the name plate of the transformer.

Cooling

The above requirements can be met with an air-cooled transformer, however, this is considered uneconomical, particularly on the larger sizes. Most of the shops which use welding equipment are equipped with running water, and by using this water internally to cool the secondary winding of the transformer, the size, weight, and cost can be considerably reduced. In order to know the amount of cooling he can depend on from this source, the manufacturer must know or assume values for:

1. The temperature of the intake water.
2. The pressure available to force the water through the winding.

It must of course be remembered that the part of the internal heating attributable to core losses is directly proportional to the duty cycle, and that when tested at its continuous rating the transformer will develop slightly more internal heat than when operating at its equivalent intermittent capacity. This difference however is rather small due to the present tendency in design to make the core loss low and the copper loss high. This is done because most of the cooling is derived from the circulating water in the secondary winding and therefore the heat generated here can be dissipated more readily. In a typical design the ratio of copper loss to core loss was 4.5 to one measured at the continuous rating of the transformer. This gives a total loss of 5.5. The intermittent rating was at 50 per cent duty cycle so that under this condition the total loss would be 4.5 plus 0.5 or about 91 per cent of the total losses which would occur at the continuous rating. This allows a five per cent permissible increase in intermittent loading over the specified value, which gives a desirable safety factor.

Recommendations

In the foregoing, I have attempted to outline the electrical requirements of a welding transformer, placing them on a sound and logical basis. I have also attempted to outline my ideas and recommendations as to a logical basis of rating for such transformers. If the transformer is to give satisfactory and uninterrupted service, its thermal capacity must certainly be adequate for the particular job.

Due to ever-changing manufacturing methods, the user constantly finds it necessary to shift his welding transformers from one job to another or from one machine frame to another. He must therefore have knowledge of the capabilities of each of his units in order intelligently to apply them to his jobs, with assurance that they will perform satisfactorily, without overheating. This knowledge can be conveyed only when the transformer name plate bears the following information:

1. Manufacturer's name and address.
2. Rated primary voltage.
3. Frequency.
4. Open-circuit secondary-voltage range.
5. Thermally equivalent continuous kilovolt-ampere capacity (at maximum secondary voltage).
6. Per cent impedance at rated kilovolt-amperes (6 per cent unless otherwise requested).
7. Temperature rise.
8. Type of cooling required (if water cooled, state required gallons per minute at specified intake temperature).

It is therefore recommended that such name-plate data be considered for adoption as standard on all welding transformers whether they be individual units or part of a complete welding machine. In the latter case it is desirable that the maximum instantaneous kilovolt-ampere demand, or the maximum primary and secondary current of the machine as a whole be stated, in addition to the above data pertaining to the transformer.

With this data available the user, knowing the requirements of his job, does not need to go through the painful and expensive experience of a transformer failure to discover that it is too small for the job. Such experiences can be entirely eliminated to the decided benefit of user and manufacturer as well.

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Protection Features for the Joint Use of Wood Poles

Carrying Communication Circuits and Power-Distribution Circuits Above 5,000 Volts

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Synopsis: The paper reviews the historical development of joint use and the general results to date of studies of protective problems of lower and higher voltage joint use. The safety features are reviewed from the standpoint of (1) subscribers' premises, (2) employees, and (3) telephone plant. Characteristics of equipment of power and telephone plant as far as they relate to this problem are given. The various factors which determine magnitude and duration of the current and voltage in the telephone plant resulting from a contact with power conductors are discussed. Improved methods for obtaining safety under various conditions, where higher voltage joint use is found to be the best over-all solution, are described.

IN THE EARLY days it was the opinion of both the power and telephone industries that their respective lines should be kept apart except at unavoidable crossings. As the plants grew, this became more difficult and much overbuilding, conflicting construction, and some joint use resulted. By 1906 it was recognized that properly engineered joint use was generally preferable to overbuilding one line with another. Also, even at this early date it was recognized that, from the public point of view, joint use was more desirable than the construction of paralleling pole lines on the same street. However, no generally recognized construction specifications providing for adequate clearances, strength of construction, and suitable material were available and each situation had to be studied individually.

The first general joint use agreement was negotiated between a power company and a telephone company about 1906. This included joint use construction specifications worked out by the companies concerned. This formed the basis for many other subsequent agreements. Joint use construction methods have been under development ever since that time and the successful use during the last 25 years of joint poles for power circuits and telephone circuits has shown that such lines when constructed and maintained in accordance with recognized practices are the best engineering solution in many situations. There are today over 5,000,000 such joint poles in this country.

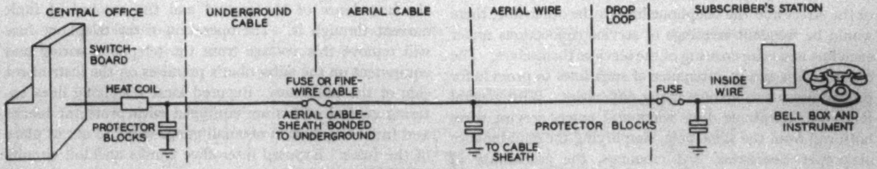

Figure 1

In 1926 the joint general committee of the National Electric Light Association and the Bell Telephone System recommended that the following general principles be adhered to as a guide in establishing joint use arrangements:

"Each party should:

a. Be the judge of the quality and requirements of its own service, including the character and design of its own facilities.

b. Provide and maintain facilities adequate to meet the service requirements including such future modifications in these facilities as changing conditions indicate to be necessary and proper.

c. Determine the character of its own circuits and structures to be placed or continued in joint use, and determine the character of the circuits and structures of others with which it will enter into or continue in joint use.

d. Co-operate with the other party so that in carrying out the foregoing duties, proper consideration will be given to the mutual problems which may arise and so that the parties can jointly determine the best engineering solution in situations where the facilities of both are involved."

In the earlier agreements for general joint use, a limitation of 5,000 volts was placed on the power circuits. This limit did not prove to be, in general, unduly restrictive in the earlier days, as distribution circuits were largely below this voltage and transmission circuits were above it. However, the engineers who made the studies upon which the early agreements were based recognized that there would be need for joint use above 5,000 volts in certain specific situations and that joint studies should be made with a view to providing standards for such cases. Shortly thereafter, specifications were adopted which provided for "higher grades" of construction for joint use involving the

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higher voltage circuits and for many years in specific situations where joint use could not practicably be avoided, such construction was adopted.

Later on, however, there was a marked increase in the use of higher voltage distribution circuits to meet growing loads and expanding service areas, and in recent years there have been an ever increasing number of instances where it has been necessary to consider general joint use in specific areas with voltages up to about 13 kv. In view of this, the joint subcommittee on development and research of the Edison Electric Institute and the Bell system undertook to find a less burdensome and more satisfactory solution to this problem. This paper outlines the progress, to date, which has been made in this connection.

Their study has included a detailed investigation of many of the factors in both the power and telephone systems which affect safety in the event of accidental contacts. Co-ordinative measures have now been developed which, where applicable, will provide safety for joint use with the higher voltages comparable to that enjoyed with joint use with the ordinary distribution voltages. In fact, it has been established that the degree of hazard in any proposed situation cannot be measured solely by the voltage of the power system and that numerous other factors must be taken into account.

Separate Lines Versus Joint Lines

Power transmission lines and telephone toll lines are of such a nature and so located that in general it has not been desirable to have them joint, and it has been generally feasible to find separate nonconflicting routes for such lines.

On the other hand, distribution circuits of both plants in urban and suburban and other built-up areas present an entirely different problem. In order to render power and telephone service to the individual customers, it is necessary to make provision for service connections of each utility to all houses on both sides of the street. In such a situation, if the power line were constructed on one side of the street and the telephone line on the other side, there would be frequent crossings of service connections under each line and criss-crossing of the services themselves. The construction and maintenance of such lines to provide for proper clearances involve many difficulties. With general joint use, telephone drop wires and power service wires both run from the same pole, simplifying the maintenance of proper clearances. At crossings, the probability of contact, in case of line wire failure, is materially less if the crossing occurs at a common pole rather than in mid-span. In view of these considerations, joint distribution lines are in general use in urban areas.

In rural or thinly settled areas the conditions are, of course, substantially different and it has been generally practicable and desirable to construct separate lines.

Telephone Circuits

Figure 1 shows a schematic layout of a Bell system local telephone circuit such as might be involved in joint use. Telephone plant between the central office and the subscriber includes underground cable, aerial cable, and aerial wire, any one or two of which, of course, may be absent in particular cases.

Table I. Causes of Contacts Between Power-Supply Conductors and the Telephone Plant

	250 Volt-5 Kv	5-15 Kv
Conductor Failure	1,545	224
Pole Failure	96	54
Other Causes	355	85
Total	1,996	360
Conductor Failure	1,545	224
Cause Unknown	391	75
Storm	214	9
Trees	481	34
Lightning	41	28
Acts of Public	99	34
Glaze, Wind, Cold	189	15
Conductor Hits	93	10
Fires Near Line	11	..
Defects	9	4
Insulator Failure	...	9
Splices	...	2
Miscellaneous	17	4
Pole Failure	96	54
Cause unknown	20	3
Storm	10	4
Acts of Public	23	17
Wind	15	14
Rotten	13	4
Falling Trees	4	..
Fires Near Line	4	2
Electrical Burns	3	4
Guy Failure	3	4
Lightning	1	..
Pole Inadeq. Reinf	...	1
Soft Ground	...	1
Other Causes	355	85
Increased Sag	133	38
Trees	42	7
Constr. & Maint	125	16
Indirect	39	6
Miscellaneous	16	18

Exposed telephone lines entering subscribers' premises are protected by a fuse in series with each line conductor and protector blocks which provide a spark gap between each line conductor and ground. When an overvoltage occurs, the protector blocks operate and ground the circuit. The value of the voltage on the subscriber's station wiring after operation of the protector blocks depends upon the impedance of the ground and the amount of fault current through it. The operation of the telephone fuse will remove this voltage from the telephone wiring and equipment on the subscriber's premises on the instrument side of the protector. Exposed local telephone lines entering central offices are equipped with protector blocks and fuses or a section of small gauge cable to act in place of the fuses. Exposed interoffice trunks and toll circuits are similarly protected.

Power Circuits

Figure 2 shows the elements of a power distribution circuit between the substation bus and the consumers' premises. In this diagram are included such elements as a reactor, voltage regulator, and residual current relay which might or might not be employed in specific cases. For these circuits a ground is provided on the wiring which is extended into the customers' premises in order to prevent a rise in potential of the interior wiring in case of contacts between primary and secondary or breakdown of the service transformer. This ground also drains off any static potential on these wires. In case of failure the operation of the transformer fuse or the substation circuit breaker interrupts the primary voltage. In certain cases branch circuit fuses are also included so that the whole primary circuit is not interrupted in case of a fault.

Factors Affecting Safety

In addition to the generally recognized construction features and protection practices, safety in joint use is largely affected by the four following factors:

1. Frequency of occurrence of accidental contacts between power and telephone circuits.
2. Voltage on the telephone plant when contact occurs.
3. Duration of excessive voltage on the telephone plant.
4. Current in the telephone plant when contact occurs.

Safety is a relative condition, there being no such thing, of course, as absolute safety. However, the highest degree of safety reasonably obtainable under the circumstances should, of course, be provided. In the studies upon which this paper is based, judgment from the safety standpoint was based on long experience with joint use with lower voltage distribution circuits.

Frequency of Occurrence of Contacts

Over the past 15 years Bell Telephone System companies have been collecting information on accidental contacts and their causes, the data being reasonably complete for the past seven years. These data consist mostly of cases involving joint use for the lower voltages and crossings for the higher voltages. The available data were analyzed and a summary of the analysis is given in table I.

This tabulation shows that for both voltage classifications a large majority of the failures resulting in contacts were caused by factors independent of the circuit voltage, such as storms, falling trees or branches, acts of the public, etc.

This study, combined with observations made during field inspections and general studies of causes of line failure, indicates, assuming proper construction and maintenance for the various voltages involved, that the frequency of occurrences of contacts will not be materially different at the higher distribution voltages from that experienced at the lower voltages.

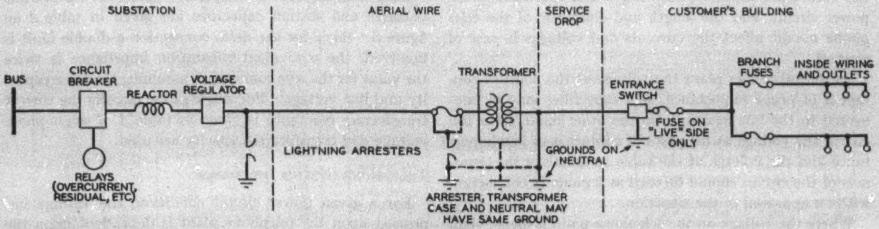


Figure 2

Type of Plant

For joint construction following recognized practices, the probability of contact varies with the types of plant involved as indicated by the following:

1. Power circuits and telephone circuits, both of aerial wire construction. A failure of a power conductor will generally result in contact with one or more of the telephone conductors.
2. Aerial wire power circuits and lead-sheathed telephone cable. The probability of contact due to a power conductor failure is less than in (1) above.
3. Aerial wire power circuits and one or more telephone paired wires attached directly to the pole. Here the probability of contacts is less than (2) above, due to the protection afforded by the paired wire insulation.
4. Power conductors in cable. Regardless of the type of telephone plant, the probability of a fault which would result in a contact with the telephone plant is remote.

The type of plant also has an important bearing on the effects when a contact does occur as is discussed later, and should be given due consideration in any study of the safety of joint use situations. This is particularly important in situations of limited extent where the application of general co-ordinated measures would not be justified, as for example, where a few spans of paired wire are carried on power poles used for the higher voltage distribution circuits.

Trees

The presence of trees along streets may have a material effect upon the frequency of occurrence of contacts. If trees grow into the line, arcing may occur between the trees and the wires, weakening the wire and causing failure. To obtain satisfactory operation, higher voltage circuits generally require more definite clearance from trees.

The effect of the trees can be overcome to some extent by the use of tree wire where the lower-voltage primaries are involved, but such wire has not been extensively used for the higher voltages. Where the power circuits are located above the trees, the branches afford some protection to telephone plant below them and in addition the

Table II. The Effects of Various Elements of the Circuit Involved in a Contact With the Telephone Plant

		4-Kv Wye	4.6-Kv Delta*	6.6-Kv Wye	6.6-Kv Delta*	13.2-Kv Wye	13.2-Kv Delta*
Kva of Substation and Power System Voltage. In general the substation may be represented by an impedance in series between the line and the power source which increases as the square of the voltage and decreases inversely as the kva. To illustrate the effect of substation kva and power-system voltage, a contact between the power circuit at a point one mile from the substation and a 1.5-inch diameter telephone cable grounded four miles away is assumed. The tabulation shows the contact voltage and current for a 200-kva and a 5,000-kva substation.	200 kva	I = 170	I = 220	I = 181	I = 195	I = 127	I = 118
		V = 1,340	V = 1,730	V = 1,420	V = 1,530	V = 1,000	V = 928
	5,000 kva	I = 235	I = 460	I = 381	I = 632	I = 675	I = 1,012
		V = 1,860	V = 3,620	V = 3,000	V = 4,960	V = 5,300	V = 7,950
Length of Power Line. The greater the length of power circuit between the substation and the point of contact, the smaller will be the voltage impressed on the telephone plant and also the smaller will be the currents in the fault circuit. To illustrate this, a power circuit three miles and 15 miles long, respectively, is assumed in contact with a telephone aerial cable of 1.5-inch diameter which is four miles from an underground section. This situation would result in voltages and currents as shown.	3 miles	I = 175	I = 348	I = 288	I = 500	I = 577	I = 1,000
		V = 1,380	V = 2,740	V = 2,280	V = 3,940	V = 4,550	V = 7,860
	15 miles	I = 67	I = 132	I = 110	I = 190	I = 219	I = 380
		V = 525	V = 1,044	V = 865	V = 1,500	V = 1,730	V = 3,000
Length of Aerial Cable in a Cable Contact. A 5,000 kva substation and five miles of power conductor is assumed in contact with an aerial cable of length one-quarter mile and four miles, respectively, the cable being one inch in diameter.	1/4-mile cable	I = 241	I = 474	I = 383	I = 635	I = 681	I = 1,010
		V = 137	V = 270	V = 220	V = 362	V = 388	V = 575
	4-mile cable	I = 135	I = 266	I = 219	I = 370	I = 410	I = 650
		V = 1,210	V = 2,400	V = 1,970	V = 3,340	V = 3,700	V = 5,850
Open-Wire Contact. To illustrate the effect of contact with open-wire circuit, assume a contact in middle of a four-mile telephone open wire run (0.109 iron), one station at end. Also assume a 5,000-kva substation and five miles of number 2 power conductor. Resistance of station ground and impedance of cable sheath assumed negligible.	Current at contact	I = 40	I = 80	I = 66	I = 114	I = 131	I = 225
	Voltage at contact	V = 1,990	V = 3,980	V = 3,280	V = 5,690	V = 6,550	V = 11,250
	Fuse duty	V = 2,160	V = 4,280	V = 3,540	V = 6,150	V = 7,090	V = 12,220
	Current through subscriber's fuse	I = 20	I = 40	I = 33	I = 57	I = 66	I = 113
Open-Wire Contact. Same as above except telephone wire parted and power conductor remaining in contact with subscriber end.	Current at contact	I = 22	I = 43	I = 35	I = 62	I = 71	I = 122
	Voltage at contact	V = 2,160	V = 4,280	V = 3,540	V = 6,150	V = 7,090	V = 12,220
	Subscriber fuse duty	V = 2,310	V = 4,600	V = 3,810	V = 6,600	V = 7,620	V = 13,200

* For delta circuits a double fault is assumed, one fault being located at the substation.

probability of faults resulting from broken branches, falling trees, etc., is reduced.

Voltage on Telephone Plant

The voltage on the telephone plant in case of contact is dependent upon the power circuit voltage and the ratio of the impedance in the telephone plant between the point of contact and ground to the total impedance of the fault circuit. Table II shows how the power circuit voltage, the substation capacity, the length and character of the power circuit, and the length and character of the telephone circuit affect the currents and voltages in case of contact.

In the subscriber plant the voltage at the point of contact is of prime importance, as a subscriber may be connected to the line nearby. On the other hand, for a toll circuit the voltage at the point of contact is of less importance and the voltage at the cable terminals or the terminals of the circuit should be used as a guide in connection with an appraisal of the situation.

Where the voltage on the telephone plant is higher than the capacity of the telephone fuse, the fuse may fail to clear the circuit and an arc be maintained across the fuse. Such an arc, if continued, may do considerable damage or start a fire. If the telephone fuse operates and opens the circuit the potential will remain on the drop wire. If this potential is above the dielectric strength between the conductors of the drop wire, failure of the insulation may also result.

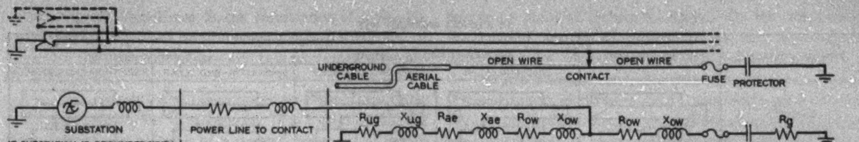
POWER-SYSTEM IMPEDANCE

The major elements involved in the power-system impedance are the substation and the power line up to the point of contact. The line impedance is dependent mainly on the length of the line and the wire size, as indicated in table *C* of figure 3. Line impedance is independent of the circuit voltage. The substation transformer may be considered as an impedance in series with the line. Transformer impedance (assuming the same percentage reactance) varies directly with the square of the voltage and inversely with the kilovolt-amperes. Values for various line voltages and station capacities are given in table *A* on figure 3. Since for the delta connection a double fault is involved, the equivalent substation impedance is twice the value for the wye connection, assuming the same capacity and line voltage. For single-phase circuits the correct transformer reactance is given in table *A*, if single-phase voltage and transformer capacity are used.

TELEPHONE SYSTEM IMPEDANCE

For a given power circuit condition, the voltage impressed upon the telephone plant is dependent upon the telephone circuit impedance from the point of contact to ground.

Contacts with telephone cable result as a rule in lower voltages on the telephone plant than do contacts with open wire, because cable sheaths usually present a lower impedance to ground. When a contact with a telephone cable occurs, the impedance to ground of the telephone plant is the impedance of the aerial cable sheath between



IF SUBSTATION IS GROUNDED WYE:
E = VOLTAGE FROM LINE TO NEUTRAL
SUBSTATION REACTANCE = X_t FROM TABLE A
IF STATION IS ISOLATED DELTA:
E = VOLTAGE FROM LINE TO LINE
X_t = 2 TIMES VALUE SHOWN IN TABLE A
NOTE: — FOR DELTA CIRCUITS A DOUBLE FAULT IS ASSUMED

POWER LINE IMPEDANCE = $(R_L + jX_L)$ FROM TABLE C

TELEPHONE LINE. CONTACT TO CENTRAL OFFICE

TELEPHONE PLANT IMPEDANCE

$= [R_{ow}$ (TABLE B) $+ R_{ae}$ (TABLE D) $+ R_{ug}$ (TABLE E)$]$
$+ j[X_{ow}$ (TABLE B) $+ X_{ae}$ (TABLE D) $+ X_{ug}$ (TABLE E)$]$
$= (R_{co} + jX_{co})$
$= \overline{Z}_{co} \angle \theta_{co}$ MAGNITUDE AND ANGLE FROM CHART

TELEPHONE LINE. CONTACT TO SUBSCRIBER

TELEPHONE PLANT IMPEDANCE

$= [R_{ow}$ (TABLE B) $+ R_g$ (TABLE E)$]$
$+ jX_{ow}$ (TABLE B)
$= (R_{sub} + jX_{sub})$
$= \overline{Z}_{sub} \angle \theta_{sub}$ MAGNITUDE AND ANGLE FROM CHART

IMPEDANCE OF THE 2 PATHS TO GROUND IN THE TELEPHONE PLANT

SERIES $= (R_{co} + R_{sub}) + j(X_{co} + X_{sub}) = (R_{ser} + jX_{ser}) = \overline{Z}_{ser} \angle \theta_{ser}$ MAGNITUDE & ANGLE FROM CHART

PARALLEL $= \dfrac{\overline{(Z_{co} \times Z_{sub})}}{Z_{ser}} \angle \theta_{co} + \theta_{sub} - \theta_{ser} = \overline{Z}_{tel} \angle \theta_{tel} = (R_{tel} + jX_{tel})$ COMPONENTS FROM CHART

NATURE OF CONTACT		TOTAL FAULT CIRCUIT IMPEDANCE	CONTACT AMPERES (I_c)	CONTACT VOLTS (V_c)	FUSE AMPERES (I_{fu})	FUSE DUTY (V_{fu})	SUB. VOLTS (V_{sub})	DROP-WIRE VOLTS (V_d)
OPEN-WIRE CONTACT	NO FUSE OPERATION OR PLANT BURN OFF	$(R_L + R_{tel}) + j(X_t + X_L + X_{tel}) = R_1 + jX_1 = Z_1$ FROM CHART	E/Z_1	$I_c Z_{tel}$	V_c/Z_{sub}	—	$I_{fu} R_g$	$I_{fu} R_g$
	NO PLANT BURN OFF — FUSE OPERATED	$(R_L + R_{co}) + j(X_t + X_L + X_{co}) = R_2 + jX_2 = Z_2$ FROM CHART	E/Z_2	$I_c Z_{co}$	ZERO	$I_c Z_{co}$	ZERO	$I_c Z_{co}$
	PLANT TO C.O. BURNED OFF — NO FUSE OPER.	$(R_L + R_{sub}) + j(X_t + X_L + X_{sub}) = R_3 + jX_3 = Z_3$ FROM CHART	E/Z_3	$I_c Z_{sub}$	V_c/Z_{sub}	—	$I_{fu} R_g$	$I_{fu} R_g$
	PLANT TO C.O. BURNED OFF — FUSE OPERATED	INFINITE	ZERO	E	ZERO	E	ZERO	E
AERIAL CABLE CONTACT		$(R_L + R_{co}) + j(X_t + X_L + X_{co}) = R_4 + jX_4 = Z_4$ FROM CHART	E/Z_4	$I_c Z_{co}$	—	—	—	—
CONTACT WITH DROP WIRE FROM CABLE		SAME AS OPEN-WIRE CONTACTS WITH OMISSION OF OPEN WIRE FROM TELEPHONE PLANT IMPEDANCES						

— ASSUMPTIONS —

1 — EARTH RESISTIVITY = 100 METER-OHMS
2 — SPACING OF PHASE WIRE AND NEUTRAL = 6 FEET
3 — INFINITE SUPPLY CAPACITY TO THE SUBSTATION, WITH NO PATH FOR EARTH CURRENTS

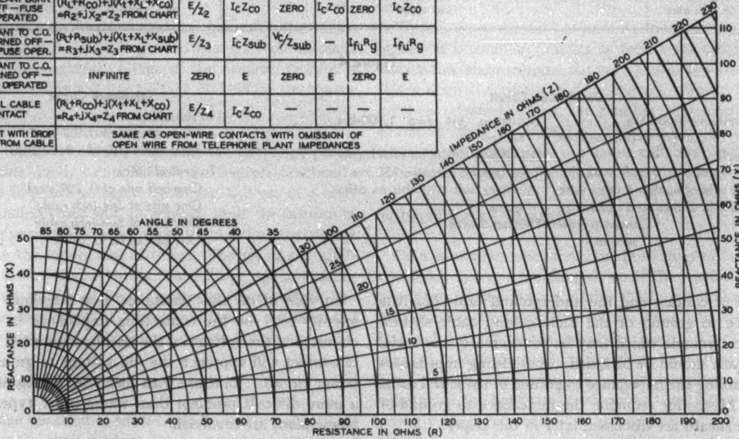


TABLE A
SUBSTATION REACTANCE IN OHMS
(TRANSFORMERS — 6 PER CENT REACTANCE)

KILOVOLT-AMPERES	X			
	4 KV	6.9 KV	11 KV	13.2 KV
100	9.60	28.57	72.6	104.54
200	4.80	14.29	36.3	52.27
300	3.20	9.52	24.2	34.85
500	1.92	5.71	14.52	20.91
750	1.28	3.81	9.68	13.94
1000	0.96	2.86	7.26	10.45
1500	0.64	1.90	4.84	6.96
2000	0.48	1.43	3.63	5.23
3000	0.32	0.95	2.42	3.49
5000	0.19	0.57	1.45	2.09
7500	0.13	0.38	0.99	1.39
10,000	0.10	0.29	0.72	1.05

TABLE B
TELEPHONE OPEN-WIRE IMPEDANCE
IN OHMS PER 1000 FEET

WIRE	R	X
0.080 CU	1.63	0.32
0.104 CU	0.97	0.32
0.128 CU	0.64	0.30
0.165 CU	0.40	0.30
0.083 STEEL	16.3	1.52
0.109 STEEL	9.5	1.52
0.134 STEEL	6.3	1.52
DROP WIRE	14.2	0.38

TABLE C
POWER-LINE IMPEDANCES FOR PHASE-TO-GROUND
FAULTS IN OHMS PER 1000 FEET

WIRE	3-WIRE CIRCUIT		4-WIRE CIRCUIT	
	R	X	R	X
4/0	0.070	0.275	0.083	0.201
3/0	0.083	0.275	0.102	0.203
2/0	0.100	0.277	0.117	0.210
0	0.123	0.278	0.144	0.218
1	0.150	0.280	0.176	0.225
2	0.167	0.280	0.214	0.235
3	0.227	0.282	0.259	0.244
4	0.284	0.284	0.312	0.256
5	0.352	0.286	0.381	0.258
6	0.439	0.288	0.467	0.259

TABLE D
AERIAL CABLE-SHEATH IMPEDANCES IN OHMS
PER 1000 FEET (SEE NOTE)

APPROX. NUMBER OF PAIRS IN CABLE				CABLE DIAMETER IN INCHES	R	X
GAUGE						
19	22	24	26			
450	800	1200	—	2.6	0.110	0.235
300	600	900	—	2.2	0.140	0.239
—	450	600	—	1.9	0.172	0.240
200	400	—	—	1.75	0.193	0.242
150	300	400	600	1.55	0.222	0.244
100	200	300	400	1.30	0.267	0.248
50	100	150	200	1.0	0.343	0.254
25	50	75	100	0.75	0.439	0.263

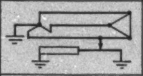
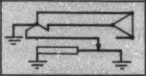
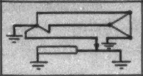
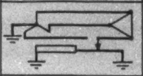
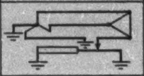
TABLE E
IMPEDANCE-TO-GROUND OF TELEPHONE
LINE TERMINATIONS IN OHMS

UNDERGROUND CABLE	$(0.6 + j0.4)$
WATER PIPE GROUND	(0.5 TO 3 OHMS RESISTANCE)
DRIVEN GROUND	(25 TO 150 OHMS RESISTANCE)

NOTE: — (TABLE D)
IMPEDANCE OF SHEATH ONLY. FOR 2 OR MORE CABLES ON SAME POLE, USE THE CABLE RESISTANCES IN PARALLEL, 85 PER CENT OF THE REACTANCE OF THE LARGER CABLE FOR 2 CABLES, AND RESPECTIVELY 81 PER CENT AND 79 PER CENT OF THE REACTANCE OF THE LARGEST CABLE FOR 3 AND 4 CABLES

Figure 3

6.6 KILOVOLT GROUNDED WYE

TYPE OF CONTACT	NO BROKEN WIRE	BROKEN WIRE			
		SUBSTATION-END IN CONTACT		LOAD-END IN CONTACT	
		LOAD-END CLEAR	LOAD-END GROUNDED	SUBSTATION-END CLEAR	SUBSTATION-END GROUNDED
CIRCUIT					
CONTACT CURRENT	209 AMPERES			11.8 AMPERES	
CONTACT POTENTIAL	2980 VOLTS			215 VOLTS	
FUSE CURRENT	106 AMPERES			7.8 AMPERES	
POTENTIAL FUSE MUST OPEN	3350 VOLTS			215 VOLTS	
POTENTIAL OF SUB. WIRE	318 VOLTS			35 VOLTS	

6.6 KILOVOLT ISOLATED DELTA

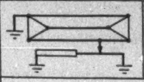
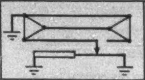
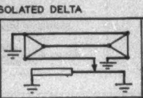
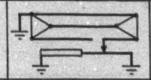
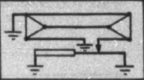
CORRESPONDING DELTA CIRCUIT					
CONTACT CURRENT	357 AMPERES			14.9 AMPERES	16.4 AMPERES
CONTACT POTENTIAL	5090 VOLTS			271 VOLTS	298 VOLTS
FUSE CURRENT	180 AMPERES			9.8 AMPERES	10.6 AMPERES
POTENTIAL FUSE MUST OPEN	5800 VOLTS			271 VOLTS	298 VOLTS
POTENTIAL OF SUB. WIRE	540 VOLTS			29 VOLTS	32 VOLTS

Figure 4

Distribution Circuit

Substation—3,000 kva
0.20 mile of 4/0 three-conductor cable
1.30 miles of 2/0 open wire
0.80 mile of number 2 open wire
0.90 mile of number 6 open wire

Substation has two 1,500-kva feeders
15 miles of circuit on each feeder
Load beyond contact—150 kva
Auxiliary fault of 0 ohm on delta system

Contact in center of number 6 wire section

Telephone Circuit

To subscriber
One-half mile of 0.109 steel
Subscribers ground—three ohms
To central office
One-half mile of 0.109 steel
One mile of one-inch cable
One mile of two-inch cable
One mile of U. G. cable

the point of contact and the underground cable plus the impedance to ground of the underground cable sheath. Any other grounds which may be connected to the aerial cable sheath should be included as paralleling impedances in determining the impedance to ground of the aerial cable sheath. Means for reducing the impedance to ground of the cable sheath are discussed later in this paper.

There is also a possibility that the suspension strand and the cable may burn off at the point of contact or at some other location depending upon the magnitude of the current and the duration of the contact. Where the severing of the cable results in the elimination of the low-impedance path to ground toward the central office, the relatively higher impedance between the contact and the subscriber may materially raise the voltage on the telephone plant.

When a contact occurs with telephone open wire, the impedance of the telephone plant is determined by the two parallel paths; one toward the central office, including the open wire to the aerial cable junction, and the aerial and underground cable sheaths; the other from the point of contact to the subscriber station, including the open wire, the drop wire, and the protector ground. If the telephone open wire parts, the impedance will be determined by one of these paths alone, depending upon which end of the telephone wire remains in contact with the power wire.

In view of its high resistance, iron telephone wire frequently may be the controlling impedance in the fault circuit, and the voltage on the telephone plant at the point of contact will approach the full voltage to ground of the power circuit. Joint use with such wire therefore presents a more difficult problem than where other types of telephone plant are involved.

TYPE OF CONTACT

A contact may involve any one of five different physical situations, depending upon whether or not the power wire parts and, if it parts, which end remains in contact with the telephone plant, and whether or not the other end becomes grounded. Examples of these five situations are shown diagrammatically on figure 4 for a wye and a delta system.

It will be noticed that single faults are assumed on wye circuits, and double faults, one of which is located at the substation bus, are assumed on delta circuits. A double fault is chosen on delta circuits for two reasons: (1) a single fault ordinarily produces no appreciable voltage or current on the telephone plant and can exist without making the power circuit inoperative; (2) routine contact reports analyzed by the committee show that with delta circuits an appreciable number of double faults involving the telephone plant have occurred. A single-phase fault on a delta circuit sometimes is the forerunner of a double fault, because a ground on one phase raises the voltage to ground of the other two phases and this may cause a breakdown to ground.

From the currents and the voltages resulting from these contacts as shown on this diagram, it is evident that the fault current and voltages at a contact are greatest where the power wire remains continuous from the contact to the supply point. Hereafter in this paper when a contact is referred to, the most severe type of contact is assumed, although it is recognized that such conditions do not always obtain.

COMPUTATION OF VOLTAGE IN TELEPHONE PLANT

To estimate the voltages and currents in the telephone plant resulting from a contact, it is necessary to have impedance data on the various elements involved in the fault circuit. These data are given on figure 3, which also includes formulas and curves to facilitate the computations. These formulas are for steady-state conditions and do not include transient conditions.

In the grounded-neutral systems generally the maximum voltage that can be impressed on the telephone plant is the phase-to-neutral voltage of the system, i.e., for a three-phase system the line voltage divided by the square root of three.

In a delta system, as stated above, a single fault impresses a relatively small voltage on the telephone plant. On the other hand, a double fault can impress practically full phase-to-phase voltage. However, the use of a suitable grounding bank will, in general, limit the maximum voltage impressed on the telephone plant to the line voltage divided by the square root of three. The use of a grounding bank means that a substantial voltage, up to a maximum equal to the phase-to-neutral voltage will be impressed for every contact, whereas without it a substantial voltage is impressed only when a double fault occurs.

Duration of Contact

While the duration of some contacts is short as a result of the nature of the contact (such as "side swiping" and other types of momentary contacts), in general, the duration is determined by the magnitude of the fault current and the type and setting of the interrupting devices in the power circuit. Either fuses or circuit breakers are used to de-energize the power circuits, depending on the service requirements and the general system arrangement. Frequently fuses are used to isolate faulty branches without causing an interruption to the entire circuit. From the safety standpoint it is immaterial how the fault is de-energized as long as this is accomplished promptly.

Power-Circuit Interrupting Devices

Phase or overcurrent relays or fuses must be set sufficiently high to avoid tripping the circuit under maximum load conditions. If the capacity of the circuit is relatively large, this setting may be higher than the fault currents which would result from a fault involving contact with telephone plant at certain locations. On the other hand, the current setting of ground relays can generally be lower than for phase relays, which makes it possible to de-energize a circuit for faults which would not be cleared if phase relays alone were used. Generally ground relaying has not been applied to an isolated delta system except by the addition of a grounding transformer. Under certain conditions phase relays can be so arranged and set as to provide positive de-energization.

The current value to which the ground relays can be set without false operation depends, among other things, upon the load and upon the type of circuit used, and upon the ratings of transformer and branch fuses where definite selectivity between the fuses and the relays is planned.

Ground detectors are used to some extent in connection with the operation of delta distribution circuits. These devices, which indicate a single fault on a delta system, will reduce the probability of double faults. However, in general, the same reliance cannot be placed in them as in automatic devices that are associated with a particular circuit.

When a power circuit becomes de-energized by a circuit breaker it is common practice to reclose the circuit one or more times, since tripping out may be due to a fault which clears when the circuit is de-energized. Where the fault involves telephone plant, the number of reclosures may adversely affect the situation. Holding the number of reclosures to a reasonable minimum will tend to limit the damage to the telephone plant.

Subscriber's Premises

On a subscriber's premises, the possibility of fire or electric shock due to a breaking down of the insulation on the telephone wiring can ordinarily exist only following the introduction of an excessive voltage, from whatever cause, imposed directly on the telephone plant. Practically all cases of fires or personal injury which have occurred were due to potentials causing failure of telephone plant insulation or the holdover of telephone fuses along with the power voltage remaining on the telephone circuit for some time.

In some cases, excessive voltage due to accidental contacts caused arcing, such as holdover of fuses, but the short duration of the fault did not permit the arcing to spread to other objects and start a fire. Where injury to persons has occurred it has resulted mostly from the fact that when arcing or some other disturbance attracted attention, the person attempted to use the telephone, cut wires, or extinguish fires while the telephone circuit was still energized. There are no records of such injuries having occurred where the duration of the fault was short.

Other things being equal, breakdown of insulation on telephone wiring and fuse holdovers at the subscribers' premises are more likely to occur in the event of accidental contacts with the higher voltage power distribution circuits than with the lower voltage distribution circuits. However, as pointed out in later paragraphs, means have been developed such as to avoid such occurrences in connection with higher voltage joint use.

This objective is obtained by: (1) limiting the duration of foreign voltage, (2) co-ordinating the duty imposed on the fuse and its capacity, (3) co-ordinating the voltage which may be impressed and the drop wire insulation, and (4) minimizing the potential to ground and also the differences of potential on the premises.

Central Offices

Telephone central offices are generally located in areas where extensive underground metallic structures of low impedance are available and these are used for the grounds. It is Bell System practice to bond underground cable sheaths to the central office ground, thus further assuring a low impedance.

When a contact occurs between a power circuit and telephone plant, the protector blocks at the central office will limit the potential difference within the office to the breakdown voltage of the protector block. This is true regardless of the voltage of the power circuit involved in the contact or the impedance of the central office ground. The resulting current to ground depends upon the voltage and impedance of the power circuit, and the impedance of the telephone plant from the contact to ground. The potential, to which the central office ground can be raised, depends upon its resistance and the portion of the fault current that passes through it. If the ground potential in the office, that is, the potential of the ground bus with respect to remote ground, should exceed approximately 300 volts, many of the protector blocks connected to subscriber lines not otherwise involved in the fault will break down.

Where extensive underground structures are used as the central office grounds, the probability of raising the potential of a central office to 300 volts is remote, and this is particularly true where underground cable is also present. Also, where telephone cable is involved, dielectric breakdown from sheath to conductors and between conductors will provide paths to other grounds as well as to the central office and these will further limit the voltage that may develop. However, where such low impedance grounds are not available, the situation from the safety standpoint at the central office becomes very similar to that encountered at a subscriber's premises, and the factors previously discussed in connection with such premises should be taken into account.

Safety to Telephone Employees

Considering employees working on telephone circuits on joint poles carrying higher voltage distribution circuits, attention must be given to four types of situations: (1) contact with power conductors or equipment in normal operating condition, (2) contact with a broken power conductor, (3) contact with telephone wires energized at some other location by a broken power conductor, and (4) energized poles.

With adequate clearances between power and telephone attachments, the probability of a properly trained telephone workman making contact with a power conductor is remote. Assuming approved construction, therefore, it can be said that the hazard to linemen existing under normal power circuit operating conditions is slight.

Unless the broken power wire is so located that it is difficult to see, the probability of a telephone outside plant employee making contact with it is small. These employees have definite instructions not to handle or approach a broken power conductor unless it is absolutely necessary, and then only with the proper equipment (rubber gloves, etc.). However, if one should make contact with such a wire, the probability of being seriously injured is great, but this is true for any primary power circuit voltage.

When a contact occurs between a power conductor and telephone plant, there is generally some visible evidence of this condition at the central office which gives a warning that the line has been energized and it is the usual practice to caution any lineman detailed to investigate such trouble to be on the lookout for energized wires. If, however, he actually comes into contact with such an energized telephone wire and also with some grounded object or another wire, the effect would be serious for either the lower or the higher primary voltages. The degree of safety here is more directly influenced by the duration of the excessive voltage on the telephone plant than by the magnitude of the voltage.

Current leakage along poles may result either from an insulator failure or from a loose wire resting on the crossarm or pole. A potential is then impressed upon the pole, the magnitude and gradient of which depend upon many factors, such as the contact resistance, moisture content of the pole, the resistance of the crossarm wood compared with the resistance of the pole wood, and the presence of various attachments on the pole, such as a grounded telephone cable, lamp bracket, ground wire, etc.

Where there is a grounded cable on the pole the potential to ground around the telephone space is generally quite small; of the order of 50 volts or less. Where there are no grounded cables, the potential-to-ground from the pole surface in the telephone space may be materially greater, ranging up to several hundred volts.

Tests on pole leakage were conducted by the committee on two poles under dry and wet conditions. It is recognized that test results based on measurements on two poles are not conclusive. They, however, provide sufficient qualitative information to indicate that:

(*a*). Under similar conditions the potential gradient on the pole surface increases with the increase of voltage. However, with the higher voltages, there is a marked tendency for the top of the pole to smoke or burn with the result that the fault is more likely to be detected and remedied.

(*b*). The presence of a grounded telephone cable materially reduces the voltage to ground on all parts of the pole below the location of the cable.

(*c*). The likelihood that the voltage to ground at points on the pole would reach substantial magnitudes under the condition mentioned in (*b*) without some visible indication either by smoke or flame at the top of the pole seems small, since currents of the order of one-quarter ampere give visible effects in the nature of smoke or flame.

It would seem from this that the use of higher distribution voltages under the conditions that have been discussed does not introduce any elements which would make the hazard to linemen materially different from that which exists at the lower voltages.

Damage to Telephone Plant

The costs of repairs to telephone plant as a result of contacts vary over a wide range, depending upon conditions such as type of plant, duration of contact, magnitude of current, etc. Since duration of contact has such an important bearing on the extent of damage done, it appears reasonable to expect that higher voltage joint use with provision for prompt de-energizing of the circuit will not increase and may even decrease the total cost of telephone plant damage as compared to the total cost of such damage with lower voltage circuits, where such prompt disconnection is not generally obtained.

Protective Measures

It is evident, in view of the foregoing paragraphs, that joint use with the higher-voltage distribution circuits entails consideration of a number of factors which are not involved in joint use with the ordinary distribution voltages. However, co-ordinated measures have now been developed such that the degree of safety which can be attained in many situations involving joint use with the higher distribution voltages is comparable to that involved in joint use with the ordinary distribution voltages. The protective measures which have been developed in this connection are discussed in the following paragraphs.

It is not intended to imply that all of the protective measures should be employed in every situation where higher voltage joint use is contemplated, since the solution in any specific situation will depend upon many local factors. Some of the suggestions given below may already be incorporated in the existing plants for operating reasons. The methods cover only those which have been carefully considered and applied in one or more situations. There are no doubt other means which can be employed in certain situations. Studies are of course being continued, looking forward to improvements and developments in the protective measures of both systems.

The methods and apparatus used for limiting the magnitude and duration of the fault current and voltage should be of a high degree of reliability, whether applied to the power or the telephone plant. They should also be of such a nature as not materially to affect the service of either utility. Where such current, voltage, and duration limitations are obtained as a result of the inherent nature of the circuits, or are due to devices installed for operating reasons, the degree of reliability obtained is generally greater than where such devices or methods are used primarily for co-ordination purposes. Also, the reliability of protective devices is greater for those giving positive indication when they become inoperative.

Drop Wire and Fuses

The use of telephone drop wire with heavier insulation and telephone fuses capable of operating properly under higher voltages and currents would provide increased protection. This added protection would, of course, have to be applied at all exposed subscribers' stations and would not reduce the voltages on the telephone plant or reduce the duration of contacts. Satisfactory drop wire with heavier insulation or fuses with a higher interrupting capacity are not economically available for general use at the present time. The use of these measures is not further considered in this paper for a general solution, as other measures appear more promising, although they may offer a solution for isolated cases.

Reducing Impedances of Cables

Aerial telephone cable sheaths are generally effectively grounded by bonding to the underground cable sheath. Where the aerial and underground cables are not bonded together, a substantial bond at their junction will give the aerial cable the benefit of the low impedance of the underground cable sheath. For long, small-size aerial cables, the telephone plant impedance to ground may become relatively large. In situations of this kind, it is generally practicable to reduce the cable sheath impedance by placing auxiliary grounds on the aerial cable sheath.

Common Ground for Power and Telephone

The interconnection of power and telephone grounds on subscribers' premises limits differences of potential between the power and telephone wiring. In addition, it will in general provide a lower resistance ground for protection than can be practicably obtained otherwise. This lower resistance ground will, of course, tend to limit the potential on the telephone plant and to ensure sufficient current to operate protective devices in cases of contact. Such common grounding, of course, has been usual in areas where extensive piping systems have been available and both have been connected to them.

A common ground will under some conditions result in some increase in telephone circuit noise where low impedance grounded ringers are used. Where, on specific lines, noise is introduced it may be necessary to apply remedial measures.

Gaps on Open Wire

The use of protective gaps, having a breakdown voltage of about 3 kv on open-wire telephone lines provides a means for limiting the voltage impressed on the drop wire and across the station fuse, and also ensures a path to ground for fault current to operate the power system protective devices. These gaps to be effective require relatively low resistance grounds, which under certain conditions may be obtained in connection with the use of well grounded power circuit neutral grid.

Bare Power Wires

The use of bare, as compared with covered power conductors, where joint use with telephone open wire is involved, lessens the probability of the telephone wire parting in case of contacts due to the tendency of the wires to weld. The parting of the telephone wire would cause an increase in impedance in the telephone plant. It is desirable to maintain the low impedance to ground of the telephone plant for reclosures of the power circuit as well as for the initial contact.

Grounding Banks on 6.9-Kv Delta System

The use of a suitable grounding bank on a 6.9-kv delta system inherently limits the magnitude of fault currents and also limits to 4 kv the maximum voltage to which the telephone plant can ordinarily be subjected. The use of ground relays or low capacity fuses will ensure prompt de-energization of the circuit in case of contact with the telephone plant.

13.2-Kv Grounded-Neutral System

In a 13.2-kv grounded-neutral system the phase-to-ground voltage is 7.6 kv. For relatively low-capacity substations there is appreciable impedance inherent in the power system to limit the magnitude of fault current and to aid in obtaining a favorable impedance condition for cases of contact. For a large capacity station, feeder reactors or an impedance in the neutral will aid in obtaining favorable impedance conditions. Ground relays of a relatively low setting can be applied. For situations where the fault current may be relatively large the duration of fault can frequently be shortened by the addition of an instantaneous ground relay set to operate on a current several times larger than the setting of the inverse time ground relay.

Grounding Banks on 13.2-Kv Delta System

The use of a grounding bank makes it possible to apply ground relays to such a system which will ensure prompt de-energizing in case of a contact with the telephone plant. The voltages to ground of such a system are in general the same as the voltages to ground of a 13.2-kv grounded-neutral system. However, the magnitude of the fault current will generally be less due to the impedance of the grounding bank.

Summary

To summarize, for situations where joint use with higher voltage distribution circuits is the best engineering solution, safety conditions comparable to those existing at the lower voltages can, in general, be obtained by co-ordinating the protection of the two systems. Various combinations of the protective measures suggested in this paper, depending on the characteristics of the power and telephone plant involved, have been successfully applied to actual field cases.

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Mechanical Uniformity of Paper-Insulated Cables

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Synopsis: Application of a recently developed method whereby it is possible virtually to see the voids in the layer structure of high voltage cable insulation shows most such insulation to be mechanically irregular. Lack of mechanical uniformity in the cable as manufactured is responsible for much of the ionization after the load cycle, and probably to a great extent for ionization breakdown. Application of the method and of supplementary methods should lead to better load cycle stability, improved installation practice, and ultimately to reduced insulation thicknesses.

Introduction

THE SEARCH for ways of improving solid-type, high-voltage underground cable, so as to raise its reliability in operation to the same high level as that of other major links in power systems, has gone on intensively for the last 10 or 15 years in this country and in Europe. The characteristics of the materials of which they are made, particularly paper, oil, and lead; their design; the thoroughness of drying, degassing, and impregnating of the insulation; the deterioration in service through ionization and oxidation; the electrical characteristics before and after the load-cycle: all these have been the subject of intense research. In consequence, progress has been made in raising quality of solid cables, as is indicated by the ETL index[1] and by reduction in failure rate.[2]

Indeed, this progress has been great enough to raise the question in some quarters as to whether solid-type cable had not reached the practical limit of its development, and that therefore from now on only minor improvements might be expected. There are a number of observations which throw very serious doubt on such a point of view. One of these is the great difference in insulation thickness used respectively with solid-type cable and oil-filled or pressure-type cable. Solid cable has twice, and sometimes more than twice, the wall thickness of the pressure type for the same voltage. Is this marked difference due solely to effects of differential expansion of the oil in the solid-type cable during operation, as is usually represented? Or is it possible that, in this long-continued research, some important factor has been overlooked?

On looking back over the cable researches and the literature of dielectrics for the past 10 or 15 years, one finds that little or no information has been published concerning the compactness and uniformity of the layer structure of cable insulation. A great deal has been written about electrical uniformity, both longitudinally and radially, but nothing about mechanical uniformity. Yet it is the common everyday experience of those who dissect cable which has failed in service to find that the insulation of some cables is much looser than that of others, that wrinkles are often present, and that in three-conductor cable heavy wax deposits occur sometimes on only one shoulder of the insulation of one conductor. Poor mechanical assembly of a similar nature is noticed in dissecting cables in the laboratory after load-cycle aging. Earlier studies in which dyed oil was forced through short lengths of cable showed that there were preferential channels in the insulation wall itself.[3] Experimental work with cables in which voids were kept from forming by the application of fluid pressure has revealed the presence in the insulation of tight and loose annular zones. There is thus a great deal of field and laboratory observation which indicates cable insulation to be frequently nonuniform in its layer structure.

How general are mechanical irregularities in cable insulation, and what is their significance in terms of operation and life? Is mechanical nonuniformity a factor of importance in solid-type cable which heretofore has been overlooked? To answer these questions an investigation was undertaken. As a first step, quantitative methods of measuring irregularities in the insulation wall were developed. One of these is a particularly powerful tool because it makes possible the quantitative visual examination of the cable structure at any selected point. When these methods were applied, a great many defects of design and of mechanical assembly in practically all modern cable became apparent. Further study showed these irregularities to be related quite closely to electrical characteristics. Much effort was expended in interpreting mechanical nonuniformity in terms of manufacturing technique. It is the purpose of this paper to present these new test methods and the results of their application to a large number of new and used cables.

Methods

In the attempt to find some quantitative way of evaluating the mechanical characteristics of cable insulation, early efforts were centered on the development of simple mechanical tests on the cross section for measuring average compactness and for detecting tight and loose annular zones in the laminated dielectric. At this stage the difficulties involved in obtaining reliable measurements on spongy oil and paper insulation seemed insuperable. Later, microscopic studies gave a clue which, when followed up, led to a new technique which makes it possible to prepare thin translucent cross sections of wafers. These show the internal structure of a cable at a glance. The insight into cable design and construction afforded by the wafer method was so great that it was decided to spend most of the available time on its perfection. As a

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[1] For all numbered references, see list at end of paper.

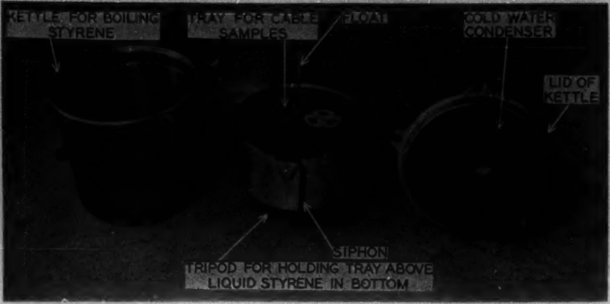
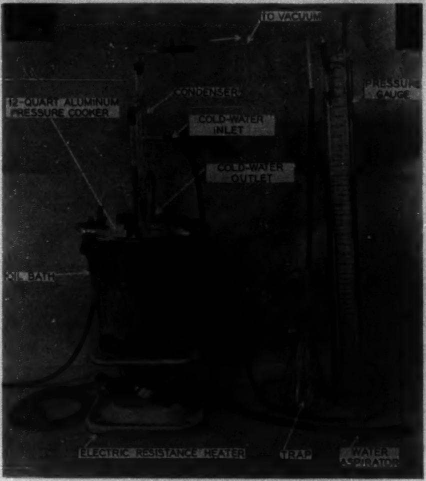

KETTLE FOR BOILING STYRENE
TRAY FOR CABLE SAMPLES
FLOAT
COLD WATER CONDENSER
LID OF KETTLE
SIPHON
TRIPOD FOR HOLDING TRAY ABOVE LIQUID STYRENE IN BOTTOM

Figure 1. Oil extraction and styrene impregnation apparatus for preparing translucent cable cross sections; apparatus disassembled

Figure 2 (below). Oil extraction and styrene impregnation apparatus for preparing translucent cable cross sections; general view



consequence the mechanical test methods referred to above were not carried to the same degree of refinement; nevertheless, each of the tests to be described is capable of yielding reliable and useful information.

PENETROMETER METHOD

A simple method of determining general compactness is to drop a weighted needle into the cable cross section at chosen points, measuring the depth of penetration. For this purpose the special sharp-pointed five-degree needle of a chemical penetrometer is adjusted so as just to touch the insulation, and then, released by means of a mechanical trip, a 150-gram weight is dropped through about one centimeter, driving the needle into the insulation. Although the method is useful in making rough comparisons of the compactness of different insulations, it is most valuable for evaluating the density of the filler spaces, this being an important factor in compound migration in cable laid on slopes and in the behavior of cable during load-cycle aging and use.

It is important, in this and the two succeeding mechanical tests, to prepare the sample carefully. In cutting the sample, the length of which has been standardized at two and one-half inches, a sharp bandsaw is used so as not to disturb the layer structure. Since in some cables the sheath constricts the insulation, it seems desirable to make a longitudinal saw cut so as to just sever the lead but not the binder tape. Exposure of the cable end to air for more than 30 minutes results in swelling of the layer structure due to moisture absorption, and consequently to erroneous values of compactness; hence samples should be stored in jars away from the air.

THE PUSH-OUT METHOD

A second method of measuring insulation compactness is to push out the conductor of a short length, recording the pressure required and observing the shape of the insulation cone so produced. A uniform insulation should yield a smoothly stepped cone, whereas the presence of loose zones should be indicated by wide steps in the cone. The test is carried out by a simple instrument consisting of a worm-operated push-out shaft into which has been built a dynamometer and which is equipped with an end-piece of the same shape and cross section as the cable conductor. A table, having at its center a hole sufficient to pass the pushed-out insulation, is provided for holding the sample, and three adjustable fingers support the shielding tape and outer layer of insulation of the cable sample. In use, the shaft is moved downward for a distance of three centimeters at a uniform speed (about ten centimeters per minute), forcing out the conductor and coning the insulation, the maximum pressure required being observed. The test is useful only as a quick means of detecting relatively large variations in compactness of the insulation, i.e., for revealing exceptionally loose annular zones, the pressure readings being a more valuable criterion than the shape of the cone.

THE TORSION METHOD

A more delicate method of measuring compactness of the layer structure depends on the fact that, when a small flat probe is inserted between the paper layers, the force required to twist it through a small angle is much greater for compact than for loose insulation. Since the twisting forces are small, a torsion wire dynamometer was employed in the construction of the apparatus. The latter consists of a torsion head (from a Cenco de Nouy tensiometer) connected by a number 24 AWG piano wire or drill rod to a pin-vise coupling carrying the probe, and a fixed scale from which the angle of rotation of the coupling may be read. The probe, made from the end of a jeweler's hacksaw blade, has an active length of one-half inch and is 9 by 15 mils in cross section, one end being ground to a diamond shape. In use, the pointed end of the probe is inserted in the insulation to a depth of one-half inch and the force required to twist it through a 30-degree angle is determined from the torsion head scale reading. If readings are made at intervals across the insulation wall corresponding to about five layers thickness (the distance being measured with a finely graduated scale from a marked conductor strand), a radial curve from conductor to sheath for insulation compactness is obtained. The method is sufficiently sensitive to detect minor variations in tightness of layer structure, and hence is extremely valuable in studying the mechanical structure of cables, particularly after the general structure has been inspected by the wafer method to be described below; it yields important information which the wafer method does not give.

THE WAFER METHOD

An effective way to detect mechanical irregularities in cable insulation is to prepare cross-sectional cable wafers sufficiently thin to be translucent. Until recently there has been no way of accomplishing this because long before the wafer could be made sufficiently thin the insulation would fall apart. Even had it been possible to prepare thin wafers by ordinary methods the layer structure would have been so distorted that interpretation of the irregularities would have been meaningless.

A satisfactory solution has been developed. A short length of cable is first "petrified" so as to hold the layer structure in place; and ordinary mechanical methods are then used to cut out cross sections 5 to 50 mils in thickness. "Petrification" is accomplished by the use of a synthetic resin, styrene, which has recently become commercially available in liquid form.[4] Styrene has three characteristics which fit it for the preparation of cable wafers: it is highly soluble in ordinary insulating oils so that it can be used to extract and replace the oil in cable insulation; it changes on heating from a liquid to a clear water-white solid without the evolution of gas, moisture or other products, thus making it possible to "freeze" the layer structure in place; and it has no harmful effect on cable paper so that it does not distort the cable structure.

Study of wafers from a wide variety of cables shows that the method makes clearly visible tight and loose zones in the insulation, voids in which ionization may occur, wrinkling, poor cabling in three-conductor cables, errors in design, and many other important characteristics. Wafers may be used to reveal the effects on the insulation structure produced by service aging, load cycling, and bending. From a study of the wafer it is possible to reason back to the manufacturing processes and machinery used in the construction of the cable.

Preparation of Sample. A three-inch length is cut with a sharp bandsaw, care being taken not to disturb the layer structure. As before, the open ends must be protected against moisture from the air to prevent swelling which would increase the time of extraction and possibly distort the insulation structure. This is most easily accomplished by submerging the sample in liquid styrene immediately after cutting.

Extraction. Extraction is carried out in a modified Soxhlet extractor consisting of a 12-quart boiler in which styrene, vaporized in the bottom, condenses at the top and drips onto the cable samples which are supported on a tray. The latter is provided with a syphon which peri-

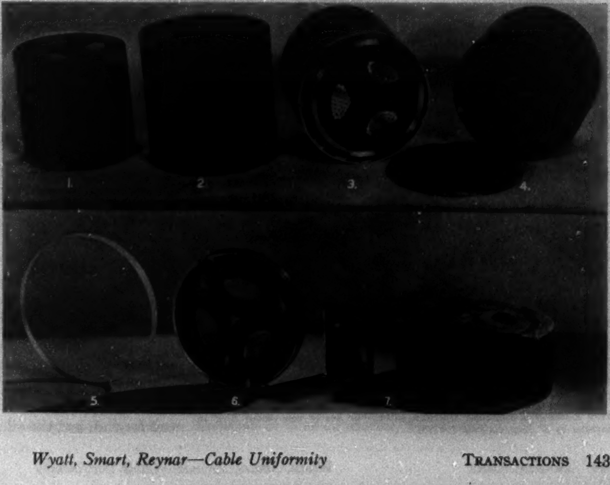


Figure 3. Preparation of a cable wafer by steps

1—Sample cut from cable and ready for extraction

2—Sample after polymerization

3—Sample with one face machined in lathe

4—One-sixteenth inch thick cross section with machined face cut from sample with band saw

5—Lucite disk to which machined face of one-sixteenth inch cross section is cemented

6—Finished cross-sectional wafer after other face has been machined and wafer reduced to proper thickness

7—Longitudinal wafer in preparation

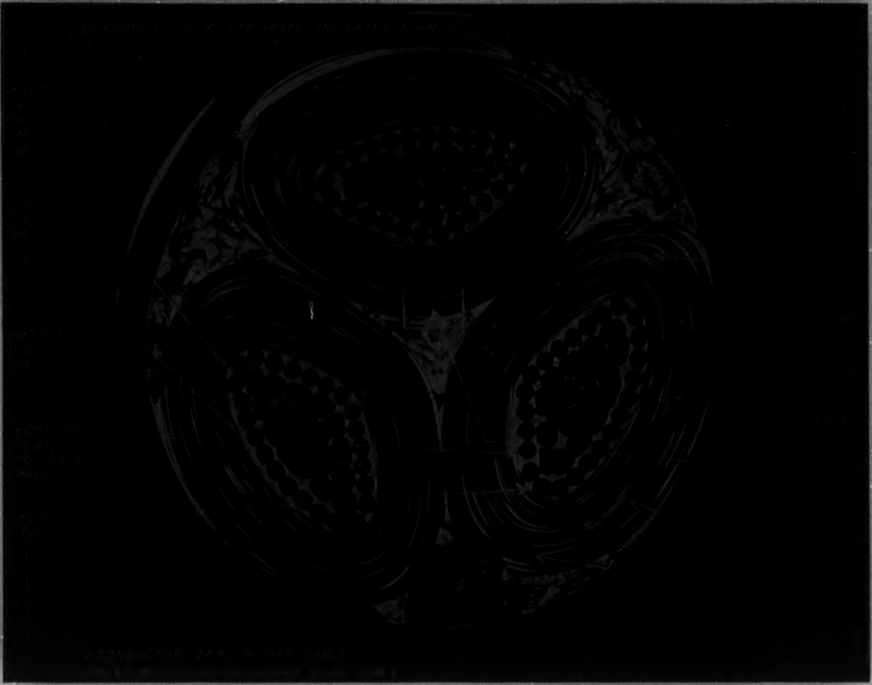


Figure 4. Wafer showing typically nonuniform insulation of three-conductor, 24-kv cable

odically returns the styrene and extracted oil to the bottom of the vessel (figures 1 and 2).*

The extraction must be carried out at a pressure of about one inch of mercury, since styrene polymerizes below its atmospheric boiling temperature (146 degrees centigrade). Vacuum is obtained by connecting the system to a water aspirator, a trap and a manometer being included in the line. In order to insure complete condensation of styrene vapors, a glass condenser is inserted in the vacuum line immediately above the boiler, the cooling water circulating first through the glass condenser outside the boiler and then through the metal condenser inside. The hole in the center of the radiator core should be blocked off by soldering in a circular piece of sheet metal so as to reduce styrene evaporation losses.

The sample tray is made by cutting down to a depth of four inches, a tin or aluminum kettle whose diameter is one-half inch less than the inside diameter of the boiler. An inverted U-shaped five-sixteenths-inch copper syphon tube is soldered into a hole near the top of the side wall of the tray with one leg extending to within one-eighth inch of the bottom of the tray on the inside, and the other leg extending to one-half inch below the bottom of the tray on the outside. The tripod which supports the tray above the styrene has a height of three and one-half inches. In order to determine during operation when the tray is full of styrene, and whether the syphon is working, it is necessary to install a float. This may consist simply of a cork impaled on a rigid wire which extends into the middle of the glass condenser tube. The boiler is heated in a glycerine or oil bath so that overheating, and consequent slow polymerization of the styrene, does not occur.

In operation, styrene is poured into the bottom of the kettle in an amount 50 to 100 per cent in excess of that required to fill the tray when filled with cable samples (about 1,500 cubic centimeters total); to this about one per cent by weight of powdered sulphur is added to reduce polymerization of the styrene. The cable samples are placed in the tray, a coarse wire grid being first laid on the bottom to make extraction more easy. Dead spaces in the tray may be filled with glass rods to reduce the amount of liquid. Vacuum is applied until the internal pressure is about one inch of mercury and heating of the kettle is delayed for a few minutes in order to prevent foaming. After 8 hours, the samples should be reversed end for end, and the oil and styrene solution in the bottom of the kettle should be replaced by fresh styrene. Oil-extraction and styrene impregnation will be complete in 48 to 72 hours depending on the nature of the samples. The boiler is then opened up at a time when the tray is nearly full of styrene so as to prevent contact of air with the samples. The tray is lifted out and the samples transferred to the polymerizing vessels.

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* *Boiler* may be obtained from National Pressure Cooker Co., Eau Claire, Wis. *Metal condenser* is a circular Harrison automobile heater core about 8½" in diameter by 2" deep, core no. 3107381.

The syphon operates in cycles of about 10 to 15 minutes' duration when the temperature of the outside bath is 130 degrees centigrade, that of the styrene and oil in the bottom of the boiler about 65 degrees centigrade and that of the liquid in the tray about 55 degrees centigrade. The extraction temperature of 55 degrees centigrade may not be sufficient to remove all the rosin from cables containing large amounts. This difficulty may be overcome by raising the pressure and thus increasing the extraction temperature. Cycling may be interrupted by boiling of the styrene in the tray if the boiler is submerged more than one and one-half to two inches in the oil bath.

It is important to use 100 per cent pure styrene free from inhibitor other than the added sulphur. The setting qualities of each shipment should be tested by placing ten cubic centimeters in a closed test tube for 16 hours at 120 degrees centigrade, when it should have become solid. If pure, the refractive index should be between 1.5437 and 1.5440. The sytrene should be freed from moisture by passing it through a column of anhydrous sodium sulphate. Styrene may be recovered from the styrene-oil mixture by vacuum distillation so that it may be used over again. However, care should be taken not to let air contact the styrene more than necessary; otherwise oxidation products may be formed which will interfere with polymerization.

Polymerization. Each cable sample is transferred from the extraction tray to a tin can of suitable size. It is elevated above the bottom by means of three small pieces of lead cable sheath or other non-absorbent material and carefully centered by inserting three U-shaped strips of lead between the sample and the walls of the can. Styrene is then added until the cable sample is covered and the can is nearly but not quite full. The styrene is then boiled for 15 minutes by placing the can in an oil bath at about

Figure 5. Wafer showing typically uniform insulation of three-conductor, 24-kv cable

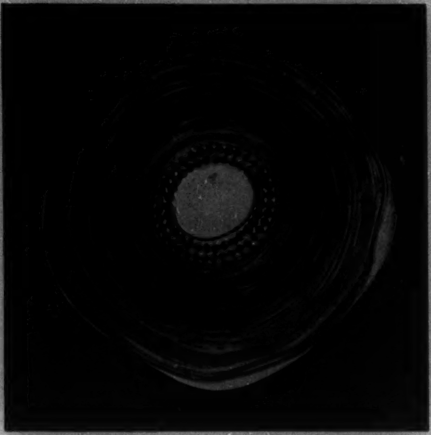


Figure 6. Wafer of 132-kv oil-filled cable of European manufacture

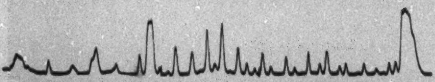


Figure 7. Radial densitometer curve taken across upper shoulder of right-hand core of insulation shown in figure 4

155 degrees centigrade. Immediately after removing the can from the bath it is made vacuum-tight by means of a lid pressed on with a standard can sealer. The styrene is then polymerized by immersing the can for 48 to 72 hours in a well-stirred glycerine or oil bath maintained at 120 degrees centigrade. After polymerization the can and its contents must be cooled slowly over a period of several hours, preferably by cooling the oil bath. After cooling is complete the can is removed from the bath and stripped from the hardened block of clear styrene containing the cable sample.

If the styrene is not completely polymerized, it will "craze," i.e., the surface will take on a whitish appearance after a few weeks' exposure. If cooling is too rapid, small cracks will appear at the surface. Continuing the polymerization for longer than 72 hours will not impair its mechanical properties. The temperature of the polymerization may be reduced but the time required will be roughly doubled for each 10 degrees centigrade decrease.

Mechanical Preparation. To prepare a wafer, the styrenated cable is cut in the middle with a metal-cutting bandsaw. To obtain a square cut it is necessary to use a carriage. First a cut one-half inch deep is made all round by turning the sample to several positions. Then a deeper cut is made all round, after which the final cut is made.

Conditions must be right for making satisfactory cuts on the bandsaw, because the styrene gums on heating. A sharp saw with hardened edge, having eight to ten teeth to the inch and traveling 1,200 feet per minute, will make about eight satisfactory cuts of a three-conductor, 350,000-circular-mil, 24-kv, styrenated cable.

The face of one-half of the sample is then turned on a lathe. The speed may be as high as 600 rpm if the sample, particularly the conductors, are cooled before each cut with a stream of carbon dioxide from a pressure cylinder of the syphon type. For roughing, cuts up to 10 mils may be made; and for finishing, cuts of one-half mil with one mil a maximum. Stellited tools are the most satisfactory. The best tool for roughing is one that is V-pointed; and for finishing, one that is sharp-edged, round-pointed, with 35- to 40- degree top rake, 35- to 40- degree side rake and 6- to 10- degree clearance.

A cut is then made with the bandsaw one-twentieth to one-tenth inch away from the finished face, the latter being pressed against a supporting face on the saw carriage. Before making this cut it is advisable to cool the conductors with a blast of compressed carbon dioxide. The lathe-finished face is cemented to a one-fourth inch disk of transparent synthetic resin such as Lucite, using syrupy styrene (prepared by boiling styrene away from air for an hour or more or until it becomes viscous). To set the cement it is necessary to place the wafer in an oven maintained at 60 to 65 degrees centigrade and to heat it for not less than 12 hours. A weight is placed on top of the wafer. To prevent warping of the Lucite, it is essential that it be placed on the plane face of a sheet of metal or glass.

The disk with the styrene wafer cemented to it is cooled and then placed in the lathe. The rough sawn face is turned down until the wafer is sufficiently translucent to

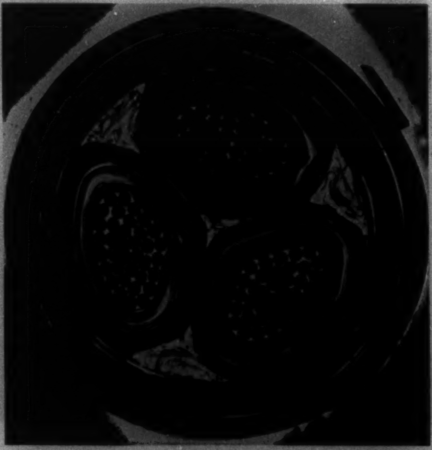


Figure 8. Wafer of three-conductor cable of European manufacture

show up all defects in its structure. It is usually necessary to turn down only to 30 or 40 mils thickness, although it is possible to produce wafers as thin as five to ten mils. Carbon dioxide cooling is resorted to between each cut as before. It should be noted that it is possible to cool the wafer too much with the result that the cement is cracked.

As a final step, a coating of transparent moisture-resistant lacquer should be given the wafer directly after it is finished to prevent swelling of the paper through moisture absorption. A coating of syrupy styrene heated over night at 60 degrees centigrade serves the purpose satisfactorily.

The various steps in making a cable wafer are shown in figure 3.

Application of Cross-Sectional Wafer Method

In analyzing wafer structure it is sufficient for most purposes to view the wafer against a strong light. Detailed study is rendered easier by the use of an enlarged photograph made by transmitted light, using antihalation film. However, for the manufacturing engineer who is familiar with the fine details of taping machinery and general fabrication processes, much additional information may be gained by more quantitative methods. Precise measurements of the layer structure may be made by transmitted light with a traveling microscope employing a magnification of 20 to 30 diameters. A binocular microscope with calibrated eyepiece is ideal.

Analysis of Typically Nonuniform and Uniform Cable Wafers

After waferizing about 100 cables it became apparent that the presence of mechanical imperfections was the rule rather than the exception and that in most cases the irregularities were sufficiently prominent to be seen immediately on casual inspection. This was true even in the case of new cable of recent manufacture. A wafer of a typically nonuniform cable photographed by transmitted light is shown in figure 4. A number of irregularities are immediately apparent. Starting at the conductor, it is seen that the paper makes line contact with the outer strands instead of surface contact, thereby increasing the electrical stress. The gaps between insulation and conductor at the flats on either side of the conductor apices indicate that the conductor is not properly shaped to conform to the natural bulge in the paper. A gap between the insulation and conductor is visible at the base of the sector of one conductor. The insulation is, in general, very loose, particularly at shoulders of the conductors. A group of five or ten tapes next each conductor is unusually slack. Many groups or bunches of tapes, which are themselves fairly compact, are separated from each other by gaps or by a number of loose tapes, giving the insulation the appearance of a series of tight and loose annular zones. The metal shielding tapes are loose, failing adequately to protect the insulation. The lateral filler spaces are unsymmetrical.

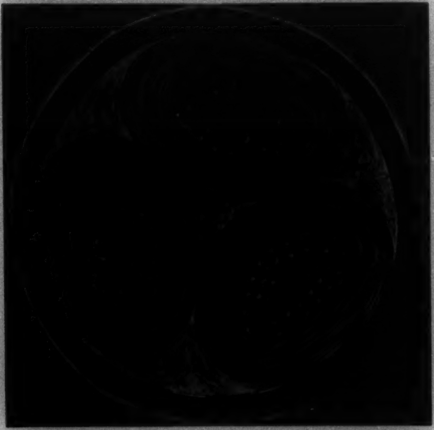


Figure 9. Wafer of three-conductor, 24-kv cable showing effect of pretwisted conductor on insulation characteristics

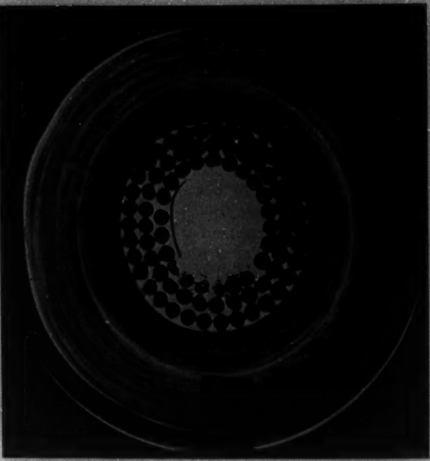


Figure 10. Wafer of 132-kv oil-filled cable showing gap between papers of different density

A wafer of a typically uniform cable is shown in figure 5. Although many mechanical imperfections are in evidence, the gross irregularities which were pointed out in connection with figure 4 are absent.

Before going into the causes of the irregularities in cable insulation as revealed by waferizing, it should be pointed out that experience in making over 200 wafers shows that they are not the result of the styrenation process, but were actually present in the original sample. Although styrene in polymerizing from the liquid to the solid form shrinks about 13 per cent by volume, most of the change occurs in the liquid stage, and even as polymerization nears completion in the 120 degrees centigrade bath, the styrene can flow longitudinally between the layers of the three-inch sample, filling up voids which would otherwise form. Since impregnation temperatures of 120 degrees centigrade are reached during manufacture, there should be no distortion as a result of the polymerization temperature; and the preliminary boiling treatment at 146 degrees centigrade is so short that its effect is believed negligible. No difference was observable in the wafer structure when the short boiling treatment was omitted and polymerization carried out at temperatures below 120 degrees centigrade. Wafers from different lengths of a uniform cable, made several months apart, all show the same uniform structure; similarly, wafers from nonuniform cable always show characteristic nonuniformities. The visible irregularities in the wafer, therefore, are not due to the process of preparation, but are characteristics of the insulation.

Certain precautions in the preparation and interpretation of wafers must be emphasized. As in any other method of analysis, reasonable care must be exercised. It is possible to make a good cable appear bad by the wafer method if the cable from which the sample is taken has been badly treated mechanically, if the wafer is sawn from the end instead of from the middle of the styrenated sample, or if excessive heat is generated in sawing off the wafer. Likewise a bad cable might appear good if the rosin were not completely extracted or if the wafer were cut too thick. It is most important to emphasize once again the necessity for keeping the finished wafer protected against moisture absorption with a film of nonabsorbent varnish.

Interpretation of Wafer Structure in Terms of Cable Design and of Manufacturing Processes

The mechanical irregularities visible in the wafer structure may be interpreted in terms of both manufacturing design and processes and of installation and operation. Among the former the effects of design, taping, and cabling machinery, and the shrinkage characteristics of the paper should be emphasized.[5]

Considering first design, the shape of the conductor is important in obtaining a uniformly compact insulation. While a round conductor offers little difficulty, the extreme sector of figure 4 is a source of trouble; had the "flats" on either side of the apex been rounded to meet the bulge in the insulation the gaps at this location would have been eliminated, and had the shoulders been less sharp there would have been less tendency to form interlayer spaces in the adjoining insulation wall. Tangential contact of the paper with the strands may be avoided by the use of crushed compact conductors (see figure 9), or perhaps by using softer papers and applying them with greater tension.

The taping machinery is responsible for many irregularities. The use of too low a tension on the individual tapes or of too high a speed of the machines results in loose insulation with visible spaces between layers. An example of loose insulation may be seen in the wafer (figure 6) of a new foreign cable submitted for test. Irregular values of tension of the different tapes may result in tight and loose zones (as in figure 4). The sudden stoppage of improperly designed machinery may loosen some of the tapes for a short distance producing wrinkles or other irregularities. The effect of difference in taping action at adjacent taping heads, and in European cable the effect of reversal of direction of taping, manifests itself in periodic variations in compactness across the insulation wall. The periodic effect was brought out clearly in the case of the wafer of figure 4 by passing a positive transparency of the photograph through a densitometer across the insulation wall at *A*. The resultant graph is shown in figure 7.

Cabling or spiralling of the three insulated cores of three-conductor sector type cable may distort the insulation walls. In order to twist the copper conductors, considerable torsional forces are applied to the insulation wall by metal guide shoes as the three insulated conductors are slowly rotated and enter the closing-in die which forces the three cores, together with the fillers, into a compact construction. It is during this rotation that any slack in the insulation is pushed away from the leading shoulder toward the lagging or trailing shoulder, the compressed leading shoulder and the extended lagging shoulder giving an effect which may be likened in extreme cases to the compressed head and extended tail of a comet. According to this explanation, all three tails should be in phase. Actually in figure 4 only two are in phase, the third, that of the lower left-hand conductor, being reversed. This may be explained by the "over-twist" which is given to one conductor by the guide shoe or die in order to make it lock in place with the other two conductors. The slack in the insulation in this case results in a 30 per cent increase in insulation wall thickness at the "comet's tail."

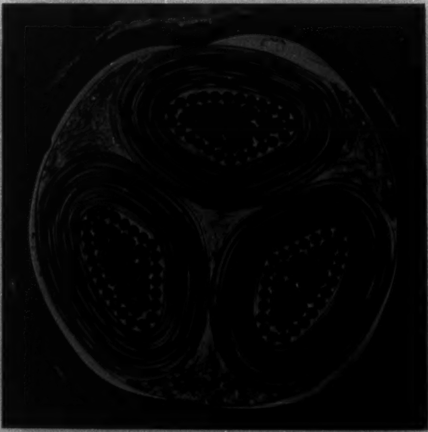


Figure 11. Wafer of cable after load-cycle aging at maximum sheath temperature of 89 degrees centigrade

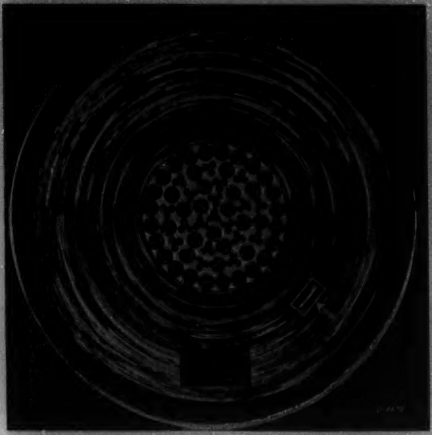


Figure 12. Wafer of single conductor 66-kv cable after load-cycle aging

In a similar cable as much as 40 per cent greater thickness has been found. A striking example of the "comet's tail" effect was found in a piece of new foreign cable submitted for test (figure 8). Pretwisting the copper conductors eliminates the rather considerable twisting forces on the insulation with the result that "comets' tails" are less likely to be found (figure 9).

Occasionally the three cores are not spaced symmetrically as a result of improper adjustment of the guide shoes or dies in the cabling machine. This trouble may usually be detected by unsymmetrical lateral filler spaces as in figure 4. The consequence is increased diameter of the cable and perhaps distortion of some parts of the insulation wall. Cabling is also frequently responsible for wrinkling.

An important cause of mechanical irregularity of insulation lies in the shrinkage characteristics of kraft paper when dried. When two different papers which have been conditioned at the same relative humidity are dried together, one may contract more than the other; for example, in "graded" insulation, if the inner denser paper shrinks more than the outer more porous paper, a gap appears such as is indicated in plate 4. Sometimes due to lack of preconditioning the paper tapes have different moisture contents at the time they are applied to the cable, and as a result differential shrinkage occurs on drying, and interlayer spaces, such as are visible in the wafers, are formed. Since paper conditioned at high humidities shrinks more than paper conditioned at lower humidities, cable made on days of high humidity, such as might occur during the summer, differs with respect to compactness of layer structure from cable made under low humidity conditions. In building insulation walls of more than 100 tapes' thickness it is often necessary to pass the conductor through the taping machine twice; if the first tapes are applied on a day of high humidity and the final tapes on a day of low humidity, a gap may be formed between the two groups of tapes during drying, because the inner tapes being more moist would show greater shrinkage. Such an explanation may account for the plainly visible gap of figure 10, although the same effect might be produced by using graded insulation of different shrinkage characteristics, or by making the second pass through a different taping machine.

INTERPRETATION OF WAFER STRUCTURE IN TERMS OF INSTALLATION AND SERVICE CONDITIONS

When cable which has been withdrawn from service is waferized, interlayer voids are usually visible; these may have been formed, however, not during manufacture, but during installation and use. Installation involves reeling and unreeling, bending and pulling; and operation involves load cycles. The effects of bending and of the load cycle have been evaluated by the wafer method.

Bending. Two makes of 1937 cable were subjected to the standard cold bending test specified by the AEIC. Examination of wafers made before and after the bend test revealed that, although slight changes in structure were apparent under the microscope, the additional irregularities due to bending were so small in comparison with normal imperfections as to be negligible.

Load Cycle. The effects of service can perhaps best be determined by studying wafers of cables which have been subjected to severe load cycles in the course of accelerated aging. The effect on the cable structure of load cycles having a maximum sheath temperature of about 90 degrees centigrade is shown by the wafer of figure 11. When this was compared with the wafer of the new cable it was seen that the layer structure had been opened up, and that the lead sheath which initially fitted the insulation closely had been greatly expanded. In addition, the shielding tape, which tends to resist the distention of the insulation on load cycling, had been slightly stretched. A similar cable, subjected to sheath temperatures of 70 degrees centigrade during load-cycle aging, showed about the same degree of loosening of the layer structure, but the distortion of the lead sheath was less pronounced. On the

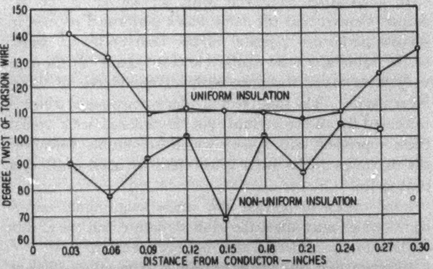


Figure 13. Curves showing radial variation in compactness across insulation wall of cables similar to those shown in figures 4 and 5

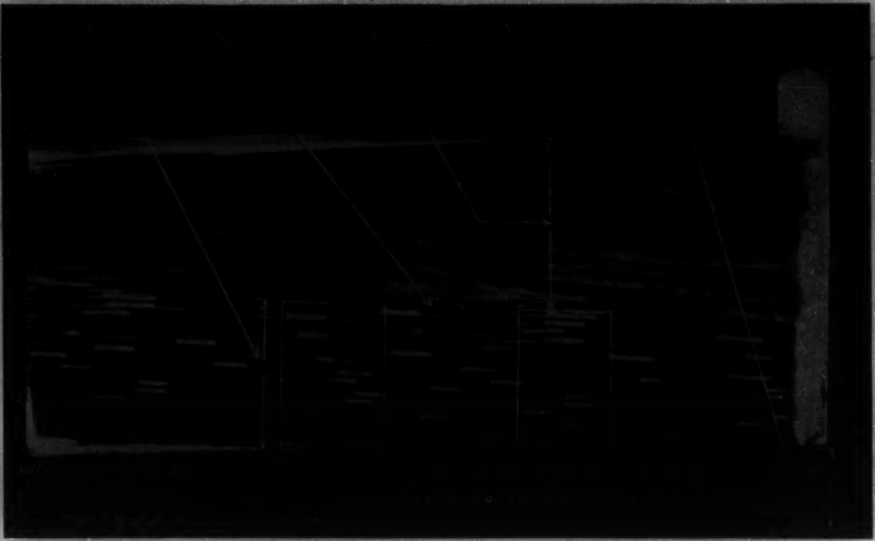


Figure 14. Longitudinal wafer taken from plane perpendicular to axis X-X shown in figure 4

other hand, another make of cable which had been subjected to a copper temperature of 45 degrees centigrade during load-cycle aging showed no structural changes whatever in the insulation or sheath, and even 55 degrees centigrade load cycles appeared to have only a slight effect, if any, on the layer structure. The tentative conclusion from waferizing over 15 cables which had received load-cycle treatment is that up to 55 degrees centigrade or over, either the oil is so fluid that it flows outward and inward with the heating and cooling cycles causing no distortion, or what is more probable, that up to some critical temperature above 55 degrees centigrade the oil and paper insulation expand as a unit, and that above this temperature differential expansion of the oil occurs.[6]

In correlating load-cycle with service aging, D. W Roper[7] showed that the cable which performed poorest in service performed poorest on the load cycle. A wafer of the cable after the load-cycle test (figure 12) shows a very loose layer structure with an abundance of interlayer spaces. The small rectangular spaces, one of which is indicated by an arrow, must not be confused with voids; these represent butt spaces which are visible whenever the thickness of the wafer is less than the axial width of a butt space.

Load cycles in service also cause longitudinal movements which may affect the cable structure, but the effects of these on the structure have not been studied.[8]

In waferizing cable from service it is usually sufficient to prepare only one wafer from a cable length, since this is generally representative. For example, wafers of one make of cable over the past seven years reveal characteristic nonuniformities. On the other hand, where it is known that local hot-spots are present, it is necessary to waferize at more frequent intervals.

CORRELATION WITH MECHANICAL TESTS

The results of the mechanical tests support and supplement the results of waferizing. The torsion method is particularly valuable. When applied to the sector shoulders of the insulation of cables similar to those shown in figures 4 and 5, the torsion test gave results which, when plotted against distance from conductor, gave the radial curves shown in figure 13. The test is sufficiently sensitive to show the decrease in compactness of insulation after severe load cycles.

The penetrometer test is chiefly useful for evaluating compactness of filler spaces, but it also differentiates between compact and loose insulation. The results of tests on the cables mentioned above were as shown in table I.

The push-out test gave pressure readings of 13 and 4 pounds, respectively, for the compact and loose insulations, indicating the sensitiveness of this criterion for compactness. The types of cones which were formed during push-out did not appear to be a reliable measure of compactness.

Longitudinal Wafers

An entirely different set of insulation characteristics is brought out by waferizing longitudinally. Such a wafer for the axis X-X of the wafer shown in figure 4 is shown in figure 14. It is immediately obvious that the

Table I. Depth of Penetration (in Millimeters)

	Insulation	Lateral Fillers	Central Fillers
Nonuniform cable	3.9	5.6	11.5
Uniform cable	3.3	4.5	Too soft to measure

Table II

Percentage of Overlay	Amount of Overlay (Inch)	Length of Shortest Leakage Path (Inch)	
		"Zig-Zag"	"Step"
50	0.450	20.2	22.7
40	0.360	20.2	18.2
30	0.270	20.2	13.7
25	0.225	20.2	11.4
20	0.180	20.2	9.2
10	0.090	20.2	4.7

butt spaces, instead of being symmetrically staggered diagonally from conductor to sheath, are arranged somewhat at random, and that in consequence there are two locations where several butt spaces are superimposed along a radius with a corresponding reduction in effective solid insulation thickness. In one case this thickness is reduced by 33 per cent. Moreover, the length of the shortest leakage path from conductor to sheath, believed to be an important factor in breakdown, is greatly reduced at these locations. In addition to revealing flaws and faults in the layer structure the longitudinal wafers therefore provide an ideal method for making measurements for statistical studies of regularity of taping.

Analysis of Taping Theory

A simple and effective method of quantitatively evaluating uniformity of butt-space arrangement has been devised by Mildner and Scott, who were using it in studies of taping machine control at the time one of the authors was visiting their laboratory.[9] It consists of making measurements of the overlay of each tape in the insulation wall and of plotting these values, expressed to the nearest five per cent in percentage of lay, i.e., pitch or axial width of a tape and butt space, against frequency of occurrence in the form of a distribution curve. These values may be obtained with a traveling microscope from a longitudinal wafer. The method when applied to two samples of American cable yielded the distribution curves of figure 15. In the upper curve it is evident that, although a 67–33 overlay was aimed at, the butt-space distribution is not far from random. In the cable of the lower curve, in which a 60–40 overlay was evidently the goal, fairly close control is indicated.

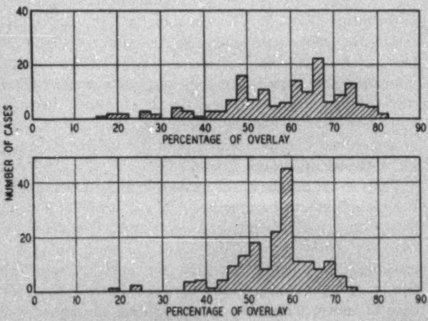
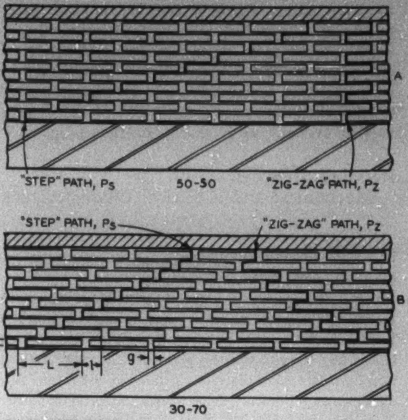


Figure 15. Mildner diagram of butt-space distribution



N = NUMBER OF LAYERS

P_Z (LENGTH OF SHORTEST "ZIG-ZAG" PATH) $= tN+(N-1)\left(\frac{l}{2}-g\right)$

P_S (LENGTH OF SHORTEST "STEP" PATH $= tN+(N-1)l-g$, WHEN $l \geqq g$

Figure 16. Diagram showing effect of various percentages of overlay on minimum insulation thickness and leakage paths

Tape "wander" may also be plotted in the form of Mildner distribution curves.

Influence of Overlay on Effective Thickness and Shortest Leakage Path

The proper choice of overlay is important in producing cable of good quality because it determines the minimum number of butt spaces which may line up across a radius, thus reducing the effective solid insulation thickness, and rendering the cable particularly susceptible to breakdown as a result of transient voltages.[10] It also influences the length of the shortest leakage path, which according to the tracking theory of Robinson[11] may be more important than effective insulation thickness. There are two general types of path from conductor to sheath over the surface of the paper tapes, either of which may be the shortest, depending on the overlay. One is the "zig-zag" type, and the other the "step" type, both of which are shown in diagrams *A* and *B* of figure 16. The length of the "zig-zag" path is independent of overlay, whereas that of the "step" path is almost a direct function thereof. The equations are given on figure 16.

In analyzing cable structure, the length of the step path is of importance, the length of the zig-zag path being fixed for a given width of tape and of butt space. It may be greatly reduced either by tape "wander" and consequent registration, or by poor choice of overlay. For example, in a cable having 51 layers of tape 0.850 inches wide and 0.005 inches thick, and a butt space width of 0.050 inches, the lengths of shortest leakage path for various overlays vary widely as shown by table II.

Relation Between Mechanical and Electrical Characteristics

While it would be unreasonable to seek a relationship between mechanical uniformity and dielectric loss resulting from poorly dried insulation or the use of high-loss materials, yet it is entirely logical to expect a relationship between mechanical uniformity and ionization characteristics. The latter should be true in view of the numerous researches described in the literature showing that ionization in cables takes place in gas films of not inconsiderable dimensions. Further, since ionization is held to be the chief cause of inherent failure, it is reasonable to expect a relationship between mechanical uniformity and life.

IONIZATION

The search for a relationship between ionization factor and imperfections in the cable insulation structure, as made visible by the cross-sectional wafer method, was made comparatively easy by the availability of a number of samples of three-conductor 24-kv cable of known ionization characteristics. These had been saved in a sealed condition from load-cycle aging tests run over a period of years. In addition six samples of single conductor 66-kv cable, also of known electrical characteristics, were very kindly supplied by Mr. H. Halperin of the Commonwealth Edison Company, Chicago, Illinois. Upon waferizing the 66-kv samples, it was found that the best with respect to visible defects in the wafer structure had a maximum ionization factor of minus 0.01 per cent; this cable gave satisfactory results on load cycle and in service. The worst (figure 12) had an ionization factor of 1.15 per cent, the highest of any of the samples; it had a poor load-cycle and service record.[7] In the remaining four samples, with one exception, mechanical characteristics and ionization factor show good agreement.

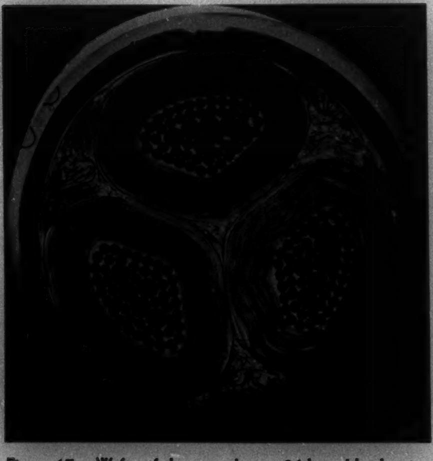


Figure 17. Wafer of three-conductor, 24-kv cable showing wrinkled insulation of conductor which failed on load cycle

Out of 18 three-conductor 24-kv cables 17 showed good agreement between ionization factor and number and distribution of voids as determined by visual inspection of the wafers. The relation was especially good for cables of the same make and age. The wafer showing the most uniform insulation had an ionization factor of only 0.08 per cent, whereas in a cable with pronounced imperfections (figure 11) the ionization factor was 3.1 per cent.

The results obtained in this study confirm the belief that there is a relationship between mechanical gaps and imperfections in the insulation, and ionization factor. However, when these gaps are filled with oil, as when the cable is new, the ionization factor may be low. It is only when the oil drains out of these gaps as a result of laying the cable on slopes, or is forced out by the effect of the load cycle, that ionization can occur in them and that the relationship stated above holds true.

In the light of this relationship, the explanation of the observation[12] that two conductors of the three-conductor sector-type cable show a different ionization factor than the third, becomes simple, because as has already been mentioned, two insulated conductors of such a cable are crushed in the cabling process to a different degree from the third.

LIFE

Since ionization is probably the chief cause of inherent failures, and in view of the relation between ionization and visible voids, it is inevitable that a relation between mechanical characteristics and life should exist. Failures in service in which sufficient evidence remains to permit definite assignment of the cause are not easy to obtain, and some time must necessarily elapse before a complete case can be made. Some evidence is already at hand, however, in the form of life on load-cycle aging and on high-voltage test. The cable represented by the wafer of figure 17 failed on test after only three load-cycles on the conductor on which the wrinkle appears so prominently. When three-conductor sector-type cable is subjected to high-voltage tests at the factory, seven out of ten failures occur in some makes at the insulation of the lagging shoulder, i.e., in the vicinity of the "comet's tail."

Voids in the middle of the insulation wall are thought by Robinson[11] not to be a serious cause of trouble, it being his contention that breakdown starts at the conductor and proceeds slowly through the insulation by a coring and tracking mechanism. Granting the correctness of his theory of breakdown, it must be conceded that the presence of severe ionization in the middle insulation wall would aid in initiating coring at the conductor, since ioniza-

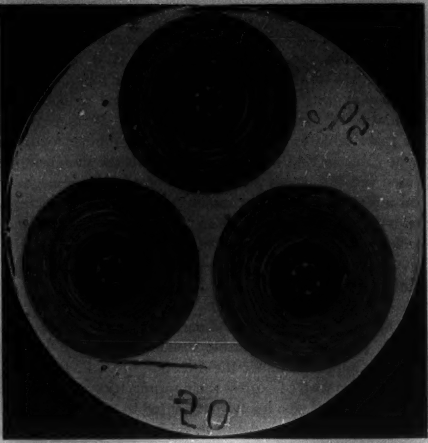


Figure 18a. Wafer of three-conductor, 24-kv joint made by experienced splicer

tion in a void reduces the potential drop across it to very low values, thus increasing the gradient in the rest of the insulation and, in turn, at the conductor. Where voids in different layers are interconnected and form chains of ionized layers, the gradient at the conductor might be very seriously increased. Superimposed on the effect of increased stress resulting from ionization in the middle insulation wall is a high frequency component; Robinson points out the greater ease with which breakdown occurs as the frequency increases.

Tracking along paper surfaces, which is now coming to be accepted as the mechanism of ionization failure after a core has once formed, is believed to be dependent not only on the type of oil used and the stress, but on the radial thickness of the interlayer space and on the extent to which the adjacent layers are pressed together. In other words, tracking is thought to be more difficult in a compact insulation. For these reasons it seems unwise to rule out the effect on life of large gaps and voids at any point throughout the insulation wall.

From the evidence obtained so far there appears to be a good correlation between mechanical uniformity of cable insulation and ionization factor, and, in those cases where ionization is responsible for failure between uniformity and time-to-breakdown in service.

Suggested Uses of Tests for Mechanical Uniformity

Because so little attention appears to have been given to mechanical uniformity of high voltage insulation up to the present time, a wide range of useful applications of the test methods which have been described immediately suggest themselves.[13] These methods should be equally useful to both divisions of the industry.

Among the many uses to which the methods might be put by the utilities, perhaps the most important is that of helping evaluate more precisely the comparative quality of different makes of cable. For this the occasional use of the wafer method is indicated, supplemented by the more frequent use of the torsion and other tests. By waferizing before and after subjecting the cable to severe load-cycles, it should be possible to judge the heat-cycle stability of cables; and indeed, when more data has been accumulated, it may be possible to substitute this and other tests for the tedious and costly load-cycle test. In a similar manner, it may be possible to determine the critical temperature at which the cable structure becomes permanently damaged, that is, to determine maximum safe operating temperatures. Another important use for the test methods, particularly the longitudinal wafer method, should be to analyze the cause of failures in service more satisfactorily. By careful study it will be possible to learn whether present methods of installation of cable cause serious damage to the insulation. In the jointing school the wafer may be used effectively to demonstrate the result of tightly and loosely wound taping on the final structure. (See figure 18.) By pulling from the ground cables of different types and vintages and then waferizing them, a useful educational illuminated display of the history of improvement of cable construction could be produced.

To the manufacturer, the test methods should find their most important use in factory control. Through their continued use, it should be possible to improve both design and the mechanical assembly of cables. The load-cycle stability should now be capable of improvement through selection of oils and papers and adjustment of processes in the light of the results obtainable by the wafer method. Another important problem which should be capable of

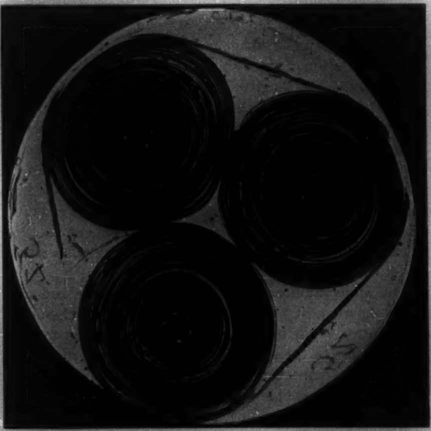


Figure 18b. Wafer of three-conductor, 24-kv joint made by less experienced splicer than the one making the joint shown in figure 18a

solution with these methods is that of the best manner of varying the tape tensions across the radius so as to make an insulation which is compact and which will at the same time satisfactorily pass the specification bending test.

When these improvements have been effected and methods developed of applying paper tapes to the conductor with such precision that the overlay cannot be other than perfect, so that the shortest leakage path must always be at a maximum, and so that it is impossible to have registration and resulting weak spots periodically through portions of the insulation wall, it should be possible radically to reduce insulation thicknesses. In cutting down the insulation wall, once mechanical perfection is possible, the task of the manufacturer is made easier, not harder, because the volume of oil causing expansion troubles is reduced, the radial resistance to oil flow is reduced making return of the oil after the heat cycle more easy, and the thermal resistance is diminished. Reduction of insulation thickness should mean, on the one hand, ability to transmit more power through a congested duct system, and on the other, reduction in cost and a further closing of the gap between economics of underground and overhead construction.

Carried to its logical end, improvement in mechanical uniformity should at some time in the future make possible the use of solid cable at 100 kv or over, without resort to pressure devices to suppress ionization.

Conclusions

The test methods which have been described make it possible to evaluate the mechanical characteristics of cable insulation. They are reliable and suitable for general use. The wafer method is most important because by its use it is possible virtually to see into the insulation structure. The cross-sectional wafer brings out irregularities resulting from taping with variable tension, the use of paper with nonuniform shrinkage characteristics, and distortion of the insulation of three-conductor sector cable as a result of cabling. The longitudinal wafer reveals the percentage of overlay used in the taping, and the precision with which the tapes were applied. The torsion method is simple, rapid, and sufficiently sensitive to make possible comparisons of radial variations in compactness across the insulation wall of new cable, and the evaluation of changes in compactness as a result of load-cycles or of bending. The penetrometer test is useful in measuring compactness of filler spaces. The push-out test furnishes a quick means of detecting especially loose insulation.

The application of the test methods to new and used cable shows most modern cable still to be mechanically imperfect. Wide differences may be found in average compactness of the insulation wall, and in the manner of varying the compactness across it.

Granted the use of low-loss paper and oil, and reasonably thorough drying, degassing, and impregnation, mechanical uniformity of cable as manufactured is the foundation of cable quality for it determines in large degree the ionization characteristics, and probably accounts for a large proportion of inherent failures.

The study of mechanical uniformity throws open the door to improvement of high voltage cable in several directions. Improved load-cycle stability may now be expected, and ultimately, when ways can be found of applying taping with absolute precision, radical reductions in insulation thickness should be possible. Perhaps it will be practicable to raise the upper voltage limits for solid-type cable so that it may compete with the more complicated pressure types. Possibly improvement of mechanical uniformity is the last major step to be taken to bring oil-impregnated paper cable to the same level of efficiency and reliability as other types of electrical equipment.

Acknowledgment

The work described forms part of a comprehensive research on high-voltage cable insulation by The Detroit Edison Company, under the direction of C. F. Hirshfeld. The authors wish to express their appreciation to the entire laboratory staff who freely contributed to this work in suggestions and help; and to the cable manufacturers, who through throwing open their plants and through frank discussion, made possible interpretation of cable wafers in terms of design and manufacturing processes. They are also indebted to Professor C. S. Schoepfle of the University of Michigan for preparing the densitometer curve, and for his advice and encouragement.

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The Current-Carrying Capacity of Rubber-Insulated Conductors

By S. J. ROSCH
ASSOCIATE AIEE

Introduction

THE INVESTIGATION described in this paper was initiated in 1931 by The Rubber Insulated Wire and Cable Section of The National Electrical Manufacturers' Association (NEMA), for the purpose of determining the current-carrying capacities of rubber-insulated wires and cables installed in buildings. The reasons for this investigation were as follows:

(*a*). Rubber-covered wires and cables had undergone considerable changes in design and chemical composition during the preceding ten years, without any advantage being reflected in increased current-carrying capacity. On the other hand, many changes in building materials and methods during a number of years may have had an adverse effect on the heat-dissipating condition of wires and cables.

(*b*). The table of permissible current ratings for rubber-insulated wires and cables, as printed in the National Electrical Code, was presumably based on a maximum safe operating temperature of 49 degrees centigrade (120.2 degrees Fahrenheit), although it refers to this temperature as the "ambient" temperature. The standardization rules of the AIEE specify a maximum safe operating temperature for rubber-insulated conductors of ($60-E/4$) degrees centigrade, which in the case of 600 volts operation would correspond approximately to 60 degrees centigrade (140 degrees Fahrenheit). Since the AIEE is represented in the formation of the National Electrical Code, it would appear as though the rules promulgated by these respective organizations were in conflict.

(*c*). If the AIEE interpretation of ambient temperature were applied literally to the Code ratings, the ultimate temperature of some of the conductors when loaded according to these ratings, would approach close to 70 degrees centigrade (158 degrees Fahrenheit).

(*d*). Although it had been known for some time that grouping of the conductors results in the lowering of the allowable current carrying capacity, there was no indication in the Code of the reduction to be expected, for example, in a three-conductor as contrasted with a single-conductor cable.

(*e*). As a result of misinterpretation of the current-carrying capacity table in the Code, frequent cases of trouble had been experienced in the operation of rubber-covered cables, without any apparent violation of the Code having been committed in the wiring layout or installation. A typical example of one of these cases may be seen in figure 1.

This represents two samples of rubber insulation taken from a 400-foot length of single-conductor 500,000-circular-mil, taped and weatherproof braided cable, which had been installed in the basement of a public building. Of the entire run, about 17 feet passed through the boiler room where the ambient temperature averaged between 102 and 110 degrees Fahrenheit (38.9 and 43.3 degrees centigrade), while the balance of the run outside of the boiler room had an average ambient temperature of 70 degrees Fahrenheit (21.1 degrees centigrade). The installation consisted of three single conductors in conduit with 380 amperes per phase and had been in operation for about two years before being withdrawn due to alterations in the wiring layout. It was then that the conditions as indicated in figure 1 were noted. The upper portion of figure 1 shows the appearance of the insulation taken from the cooler section of the cable, while the lower indicates the appearance of a section taken from the run installed in the boiler room.

Obviously, the hot-spot condition should have been provided with a larger conductor size so as to reduce the rise in temperature above the ambient, so that the maximum temperature rise would not exceed the maximum permissible insulation temperature of 122 degrees Fahrenheit (50 degrees centigrade), as provided by the Code. This installation was carried out in accordance with the then existing Code rules.

(*f*). Although the table of allowable current-carrying capacity appearing in the Code had been reproduced in various engineering handbooks, trade publications, catalogs, textbooks and other sources of authoritative engineering information for more than 30 years, none of these publications ever gave any indication of the basis used in the formation of this table as in the case of current-carrying capacity tables given for impregnated paper insulated cables.[1]

(*g*). Periodically, various engineering publications would contain articles dealing with the subject of current-carrying capacity of rubber insulated wires and cables, but giving permissible current loadings at variance with those found in the code. These would not only be confined to technical articles, but would also appear in the form of standards to be observed in a given territory.[2]

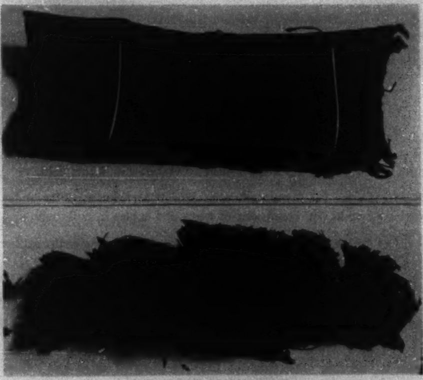

Figure 1. Effect of hot-spot temperature on rubber insulation

Plan of Investigation

After a thorough study, the committee consisting of Messrs. E. D. Youmans, G. W. Zink, and the author as chairman, formulated the following program.

Paper number 37-155 recommended by the AIEE committees on power transmission and distribution and research, and presented at the AIEE winter convention, New York, N. Y., January 24-28, 1938. Manuscript submitted November 3, 1937; made available for preprinting January 6, 1938.

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1. For all numbered references, see list at end of paper.

Table I. Summary of Cable Ratings

Conductor Size (Awg or Circular Mils)	Current-Carrying Capacity (Amperes)																				
	(a)	(b)	(c)	(d)	(e)	(f)		(g)	(h)	(i)				(j)						(k)	(l)
	1889	1890	1894	1894	1896	1903		1906	1912	1913				1934						1931–37	1895
						IEE Wiring Rules Ambient				Dushman				IEE Wiring Rules							
										Temperature Rise °C				2–1/Cond Cables							
	Kennelly	Fisher	Insurance Rules	National Electrical Code	AIEE & National Elec. Code	High	Normal	Fisher	R. H. Rice	30	20	10	G-E Bulletin	D-C	A-C	4–1/C or 1–3/C	6–1/C or 1–4/C	8–1/C or 2–3/C	10–1/C or 2–4/C	National Electrical Code	Puffer:Amperes to Produce Smoking
18		4.5			3			4.5												3	
16		6.4			6			6.4								5		5		6	
14	25	9.5	10	13	12			9.1								10		8		15	48
12	33	13.5	15	17	17			12.9								15		12		20	65
10	46	19.1	20	23	24			18.3		41	34	24	20	24	24	19	17		14	25	87
8	58	27.0	25	29	33			31.3						37	37	30	26		22	35	127
6	78	38.2	35	39	46			44.3		76	63	45	50	46	46	37	32		28	50	155
5	90	45.4	45	45	54			52.5													178
4	104	54.1	50	52	65			62.7		105	88	63	75	53	53	42	37		32	70	207
3	120	64.4	60	60	76			74.4												80	237
2	144	76.6	70	72	90	42	64	86.6												90	273
1	172	91.1	85	86	107			105.4						83	83	66	58		50	100	319
0	206	108.5	100	103	127			127.8		199	166	119	165							125	372
00	246	129.0	120	123	150	71	113	151.7						118	118	94	83		71	150	438
000	298	153.5	145	149	177	84	136	180.8												175	519
0000	360	182.7	175	180	210	96	158	215.2												225	630
200,000					200									152	152	122	106		91	200	
300,000		237.0			270	145	241	281						214	214	171	150			275	
400,000		294.0			330	166	279	349												325	
500,000		348.0			390	208	354	413	500	572	476	342	550	288	288	230				400	
600,000					450	248	425	474												450	
700,000					500			532												500	
800,000					550			587						384	366					550	
900,000					600			641												600	
1,000,000					650			694	800	925	769	554	900							650	
1,100,000					690	390	688	746						512	463					690	
1,200,000					730			797												730	
1,300,000					770	424	750	846												770	
1,400,000					810			894						595	520					810	
1,500,000					850			942												850	
1,600,000					890			989												890	
1,700,000					930			1,035												930	
1,800,000					970			1,080												970	
1,900,000					1,010			1,125												1,010	
2,000,000					1,050			1,169		1,501	1,247	898	1,400							1,050	

1. Investigate the historical data in connection with the Code values of permissible current carrying capacities for rubber insulated conductors, as well as the work of other investigators in this field.

2. Determine the factors affecting current carrying capacity under various installation conditions of wires and cables in buildings.

3. Determine the maximum continuous safe operating temperature of Code-grade rubber insulation.

4. Determine the thermal surface resistivity of the customary outer coverings of building wires and cables, also similar factors for metal conduit.

5. Set up the necessary formulae for the calculation of allowable current carrying capacities for building sizes of wires and cables under the various conditions of installation.

Historical Data

Space does not permit the inclusion of all of the historical data, but the following references will serve as an indication of the development of the present Code values of current carrying capacity, as well as some of the independent ratings that appeared during various periods. To facilitate the study of this development the various ratings have been abstracted from their respective sources and set up in comparative form in table I.

In 1849, John Müller[3] reached the conclusion that for a bare conductor, the temperature rise varies as the 1.5 power of the current flowing through the conductor.

In 1881, Lord Kelvin, speaking before the British Association for the Advancement of Science,[4] first enunciated the famous law associated with his name "that the most economical size of conductor is found by placing the cost of the annual conductor loss equal to the annual charge for maintenance, depreciation and interest on the money invested."

In 1883, Thomas Gray[5] found that a current density of 134 amperes per square centimeter was a representative figure for copper conductors from an economic point of view. However, he went further than Lord Kelvin by recognizing the need of voltage regulation and limiting the current so as not to cause overheating.

In 1884, J. T. Bottomley[6] presented results of an investigation, the primary purpose of which was to determine the ultimate steady state temperature of electric light conductors under a given loading, as well as their thermal surface emissivity, basing his results on conductors of comparatively small diameter and various insulating coverings. His was probably the first effort at recording a table of materials in the order of their heat emissivities. He also showed that under a given loading, insulated conductors did not run hotter than bare conductors, but on the contrary, with higher current densities had a tendency to run cooler. This had previously been shown to be theoretically possible according to an editorial by Sir William Thomson (Lord Kelvin),[7] and was based on the fact that the thermal resistance of the insulating coverings was offset by the increased dissipating surface provided by the increase in diameter.

In 1884, Prof. George Forbes[8] presented a paper which was destined to have a very important bearing on the formation of our present Code values. In this paper he concerned himself with the heating effects produced by the current ratings then in effect, severely criticizing the British Fire Risks Committee for their arbitrary practice in 1882 in setting up a limit of 1,000 amperes per square inch of conductor cross section, pointing out that whereas, this may be a safe limit for the smaller wires, it might produce dangerous overheating in the case of larger conductors. He also claimed that the square of the current is proportional to the cube of the conductor diameter, adding however, that in the case of small conductors, convection is of greater consequence than radiation, his law is only applicable to the larger conductors where radiation is the major method of heat dissipation.

In 1889, Dr. A. E. Kennelly[9] published a series of articles based on an original investigation conducted at the Edison Laboratory in Orange, New Jersey. This investigation was undertaken on behalf of some fire insurance companies to determine the heating produced by certain current loadings and this afforded him an opportunity to verify Professor Forbes' findings. In his published data, Doctor Kennelly gave a table of current ratings (see table I, column *a*) based on an ambient temperature of 75 degrees Fahrenheit (23.9 degrees centigrade) and an allowable temperature rise of 75 degrees Fahrenheit (41.6 degrees centigrade), or an ultimate temperature of 150 degrees Fahrenheit (65.6 degrees centigrade). The reason for setting the upper temperature limit at 150 degrees Fahrenheit (65.6 degrees centigrade) was that Doctor Kennelly felt this to be the danger point for rubber insulation. We shall see later how this table became the basis of our present Code ratings.

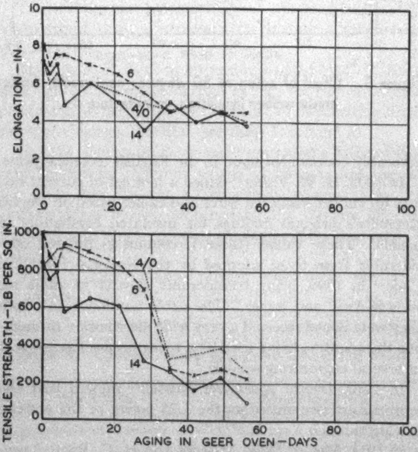

Figure 2. Effect of aging at 60 degrees centigrade, Code-grade rubber insulation, manufacture A

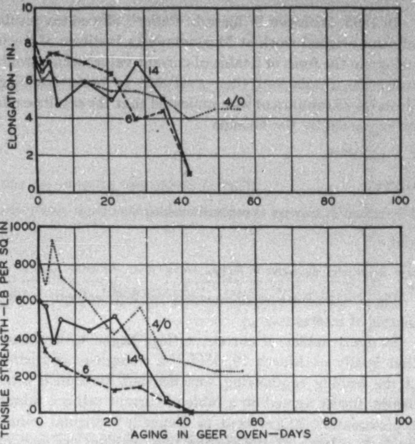

Figure 3. Effect of aging at 60 degrees centigrade, Code-grade rubber insulation, manufacture B

Doctor Kennelly's investigation included three different installation conditions.

Insulated copper conductors installed in wooden paneling to represent house wiring.

Bare and insulated copper conductors installed outdoors on poles.

Bare copper conductors suspended indoors to represent an installation in a central station.

In 1890, H. W. Fisher[10] published a table of current ratings which were very widely used (see table I, column *b*). This table was based on the results of a series of original researches.

In 1894, the insurance rules[11] prevalent in the country prescribed a table of maximum current ratings (see table I, column *c*). These were apparently based on 50 per cent of the values proposed by Kennelly, but rounded off to the nearest 5 amperes.

In 1894, the National Electrical Code[11] (then known as the Underwriters' National Electrical Association) also published a table of similar ratings, but based on an exact 50 per cent of Kennelly's values (see table I, column *d*).

On January 10, 1895[11] this same body proposed that their 1894 values be increased 25 per cent for concealed installations, such as in paneling, and 75 per cent for open wiring, such as cleating or knob and tube wiring. This was strongly protested at a meeting of the AIEE as being arbitrary and tending to be unsafe.

In 1895, Professor William L. Puffer[11] offered the results of some original work at Massachusetts Institute of Technology in the form of a table of current values which would cause the insulations then available to begin smoking. Tests on 75 samples of wire indicated that the results could be expressed by the formula

$$I = 1{,}610d^{1.38}$$

where

I = current in amperes to produce smoking

and

d = conductor diameter in inches

These values are reproduced in table I, column 1, as a matter of interest.

So many tables of current ratings began to be issued that finally on March 19, 1896,[12] a committee consisting of the leading engineering societies and insurance companies, finally agreed on a table of current ratings, taking approximately 60 per cent of Kennelly's original values (table I, column *a*) and stating that with an ambient of 75 degrees Fahrenheit (23.9 degrees centigrade), these new current ratings would yield a temperature rise of 29 degrees Fahrenheit (16.1 degrees centigrade) above the temperature of the surrounding air. The value of 75 degrees Fahrenheit (23.9 degrees centigrade) was chosen as the average indoor temperature. The values adopted are given in table I, column *e*, and it is to be noted that "these currents are specified for rubber-covered wires to prevent gradual deterioration of the insulation by the heat of the wires, but not from fear of igniting the insulation. The question of drop is not taken into consideration in the above table."

The reason assigned for adopting only 60 per cent of Kennelly's original values, was that the margin of 40 per cent was to allow for the inevitable increase in current, such as that produced by the changing from one size lamp to those of a larger candlepower, the adding of more lamps to a circuit, the overloading of a motor, etc.

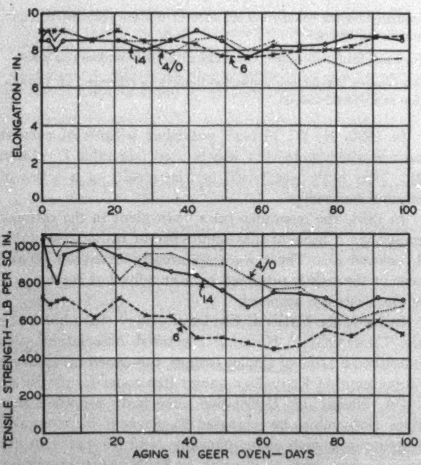

Figure 4. Effect of aging at 50 degrees centigrade, Code-grade rubber insulation, manufacture A

In 1903, the Institution of Electrical Engineers (British),[13] issued a set of wiring rules in which they recognized the maximum operating temperature of rubber-insulated conductors as 130 degrees Fahrenheit (54.5 degrees centigrade). They also set up two ratings, one for locations of high ambients and the other for ordinary conditions. In view of the difference in the British wire gauge, table I, column *f*, gives excerpts from these rules in terms of the nearest American wire gauge (Awg.). It is interesting to note that these ratings were established even though Doctor Kennelly's findings had already been reported and favorably commented upon in the English technical press.

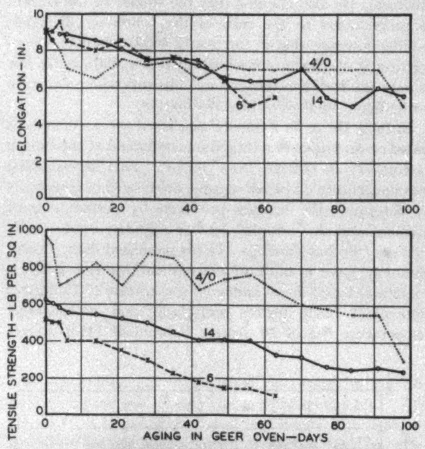

Figure 5. Effect of aging at 50 degrees centigrade, Code-grade rubber insulation, manufacture B

In 1906, H. W. Fisher[14] issued a new set of current ratings for rubber-insulated wires and cables based on Doctor Kennelly's original findings for insulated conductors in panels. These values (table I, column *g*) differed considerably from those adopted by the National Electrical Code[12] in 1896, being considerably greater in value for sizes 8 Awg. and larger. The cable handbook in which they were issued received a very wide distribution throughout the country, being used as a recognized reference text in several engineering schools.

In 1910, Doctor Alexander Russell[15] showed that the temperature rise varied as the 1.25 power of the current flowing through a conductor.

In 1911, Messrs. S. W. Melsom and H. C. Booth[16] presented their classical paper before the IEE, giving the results of a two-year investigation conducted at the National Physical Laboratory at the request of the IEE wiring rules committee. The object of the investigation was to determine the temperature rise as well as the current density for a given temperature rise in rubber- and paper-insulated cables of various sizes and with different types of coverings. In the case of the rubber-insulated conductors, the sizes were confined to those ordinarily encountered in interior wiring. This was the first scientifically conducted experiment, the study actually taking into account observations made under different conditions of installation.

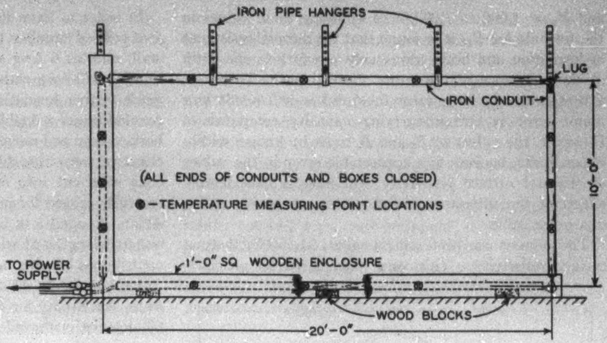
IRON PIPE HANGERS
LUG
IRON CONDUIT
(ALL ENDS OF CONDUITS AND BOXES CLOSED)
● — TEMPERATURE MEASURING POINT LOCATIONS
10'-0"
TO POWER SUPPLY
1'-0" SQ WOODEN ENCLOSURE
WOOD BLOCKS
20'-0"

Figure 6. Test setup

The authors were interested in the current densities necessary to produce two different values of temperature rise, namely, 20 and 30 degrees Fahrenheit (11.1 and 16.7 degrees centigrade), respectively. Their findings indicated that this data could be expressed by the general formula

$$i = K\left(\frac{D}{S}\right)^n$$

where

i = current density in amperes per square inch
K = a constant varying with each condition
D = outer diameter of completed cable in inches
S = area of copper conductor in square inches
n = an exponent varying with installation conditions

the actual values being found in table II.

The authors also investigated the effects of higher, but intermittent ratings, necessary to produce a given temperature rise.

In 1912, R. H. Rice in discussing a paper by C. T. Mosman[2] gave some of the current ratings for rubber-covered conductors as permitted by the Board of Supervising Engineers of Chicago Traction (see table I, column *h*).

In 1913, Doctor S. Dushman[17] presented a table of current ratings for different values of temperature rise, at the same time drawing a comparison between these and corresponding values published by the General Electric Company for the respective conductor sizes (see table I, column *i*).

Table I, also contains ratings taken from the latest standards of the IEE, as well as the 1931 (same as 1937) ratings given in the National Electrical Code. A study of this tabulation not only reveals the inconsistencies between the various ratings, but that the present Code values are practically the same as adopted in 1896, the difference being that in the present table the values have been expressed to the nearest five amperes.

Table II. Current-Density Factors—Melsom and Booth

Type of Cable	Temperature Rise of 20°F(11.1°C)	Temperature Rise of 30°F(16.7°C)
Rubber-insulated and braided in air..	$i = 364\left(\frac{D}{S}\right)^{0.816}$	$i = 457\left(\frac{D}{S}\right)^{0.816}$
Rubber-insulated and braided in moulding....................	$i = 309\left(\frac{D}{S}\right)^{0.86}$	$i = 358\left(\frac{D}{S}\right)^{0.86}$

Factors Affecting Current-Carrying Capacity

The fundamental formula for the current-carrying capacity of a single-conductor rubber-insulated braided cable in conduit is

$$I = \sqrt{\frac{T_0 - T_a}{R_{cu}R_{th}}} \text{ amperes}$$

wherein

T_0 = maximum safe operating temperature of insulation, in degrees centigrade
T_a = ambient temperature, in degrees centigrade
R_{cu} = effective conductor resistance, in ohms per foot
R_{th} = thermal resistance of path from conductor to ambient air, in thermal ohms per foot of cable.

$$R_{th} = 0.00522\left(\rho \log_e \frac{d_1}{d} + \rho_1 \log_e \frac{D}{d_1}\right) + \frac{0.00411B_b}{D} + \frac{0.00411B_c}{D_c}$$

wherein

d = diameter over copper conductor, in inches
d_1 = diameter over insulation, in inches
D = over-all diameter of cable, in inches
D_c = over-all diameter of conduit, in inches
ρ = thermal resistivity of insulation, degrees centigrade per watt per centimeter cube
ρ_1 = thermal resistivity of braid, degrees centigrade per watt per centimeter cube
B_b = thermal surface resistivity of braid, degrees centigrade per watt per centimeter square
B_c = thermal surface resistivity of conduit, degrees centigrade per watt per centimeter square

Examination of the various factors which make up I, shows that the values of T_0, R_{cu}, and R_{th} are the unknowns. Analyzing R_{th}, we find that the industry has been using the following values: $\rho = 500$[18]; $\rho_1 = 300$[19]

and B_b = 1,000 to 1,200.[1] In applying these values in the formula for R_{th}, it is found that the thermal resistance of insulation and braid forms only one-fifth to one-tenth of the resistance of the entire thermal path. Therefore, a comparatively large error in ρ and ρ_1 will result in a minor error in current-carrying capacity computation. However, the values of B_b and B_c must be known within closer limits, because any appreciable error in the values of thermal surface resistivity will have a considerable effect on the ultimate current-carrying capacity rating of a given cable.

The industry has been using a value of 1,200 for thermal surface resistivity. This value was based on measurements conducted on cables having an over-all diameter of two inches and larger and was assumed to cover the entire range of diameters. Calculations, as well as experimental determination of the current necessary to produce a given permissible temperature rise on some small conductors, definitely indicated that the value of 1,200 was in error and, consequently, it became necessary to determine the actual thermal surface-resistivity values.

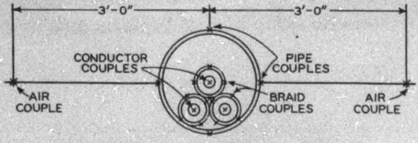

Figure 7. Location of Thermocouples

Number 12 Awg—2 couples on braid
Number 4/0 & 2 Awg—3 couples on braid
500 MCM—4 couples on braid

In addition to determining the values of thermal surface resistivity of cable coverings and conduits, it was also felt necessary to determine the effect of the following variables:

(*a*). Effect of number of conductors in conduit.

(*b*). Nature of outer covering of cable, whether braided or lead covered.

(*c*). The effect of color of outer coverings and conduits, whether black or white.

(*d*). The effect of installation in air or in conduit, concealed or exposed, vertically or horizontally.

All of these factors were taken into consideration as will be shown elsewhere in the paper.

Determination of Maximum Operating Temperature of Code-Grade Rubber Insulation

It has already been pointed out that an apparent conflict exists between the maximum permissible temperatures specified by the Code and those contained in the AIEE–ASA standards, the former calling for a maximum of 49 degrees centigrade, while the latter specifies (60-E/4) degrees centigrade. It was therefore decided to first investigate the behavior of this type of insulation at 60 degrees centigrade.

In order to have the results truly representative, 100-foot coils of number 14 Awg with three-sixty-fourth-inch wall, number 6 Awg with four-sixty-fourth-inch wall, and number 4/0 Awg with five-sixty-fourth-inch wall of Code-grade rubber insulation, finished with a single-moistureproofed braid, a double-moistureproofed braid, and a rubberized tape and moistureproofed braid, respectively, were obtained from nine different cable manufacturers. These coils were cut into one-foot lengths, each length being carefully tagged for proper identification and in that condition, grouped into bundles. Each bundle contained 27 one-foot lengths of wire representing three different sizes contributed by nine manufacturers.

Approximately 40 of these bundles were placed in an oven, containing air in circulation, at a temperature of 60 degrees centigrade ± 0.5 degrees centigrade. The outer coverings were not removed from any of the samples during their stay in the oven. One group of samples which was not subjected to heating, was tested for physical properties and the values obtained were considered the initial values for both tensile strength and elongation.

Table III. Thermal Surface Resistivity B_b (Braided Cables in Air)

Conductor Size (Awg or Mcm)	Over-All Diameter of Cable (Inches)	Thermal Surface Resistivity Bb (Degrees Centigrade per Watt per Centimeter Square)
12	0.216	530
2	0.518	611
4/0	0.784	712
500	1.100	750
1,000	1.500	818

During the first week, tests were conducted after two, four, and six days, respectively, of heating in the oven. Thereafter these tests were conducted at the end of each week. When the time for testing would arrive, an entire bundle would be removed from the oven and allowed to cool for 24 hours to room temperature, after which the samples were tested for both tensile and elongation.

Figures 2 and 3 are typical curves of the behavior of the different insulations. In these curves the actual values of tensile and elongation were plotted against time of exposure to 60 degrees centigrade and the trend in the deterioration noted. It was expected that some deterioration would be experienced during the progress of the investigation, but that if the insulation was capable of operating continuously at 60 degrees centigrade, the physical properties would gradually level off to a constant value with time of exposure. Reference to these curves, as well as to the actual results obtained, indicates that at the end of the ninth week of exposure to 60 degrees centigrade, the insulation had deteriorated to such a point that it was impossible for it to be removed from the conductor. Some insulations had become plastic and very soft, while others had hardened and become brittle. It was, therefore concluded that 60 degrees centigrade was too high for the permissible operating temperature of Code-grade insulation.

It was next decided to carry on a similar investigation at 50 degrees centigrade. The procedure was carried out identically as in the previous investigation except that this time two additional cable manufacturers contributed similar samples. Therefore, each bundle now contained 33 different wire samples.

Figures 4 and 5 are representative of the typical behavior of the insulation at 50 degrees centigrade. The materials for figures 2 and 4 were supplied by manufacturer *A* while those in figures 3 and 5 were supplied by manufacturer *B*. It is interesting to note that after approximately 50 days of heating at 50 degrees centigrade, the physical properties began to level off and remained fairly constant thereafter. After the data shown for an elapsed period of 98 days had been secured, as a matter of interest, tests were conducted for 133 days with the values showing a practically constant trend. As a result of this investigation, it was decided that Code-grade insulation could be operated continuously at 50 degrees centigrade.

Before concluding this section of the paper, it might be of interest to note that, other than the experiments conducted by Professor Puffer[11] to determine the values of current at which insulation begins to smoke, there is no published account of any work having been performed in this country to determine the maximum permissible operating temperature of Code-grade insulation.

Attention should be called, however, to some tests conducted by the Union des Syndicats de L'Eléctricité for the Paris City Council.[20] In the latter investigation, five French cable manufacturers contributed a total of 68 samples of rubber covered wire and cable and tests similar in nature to those described in our investigation were carried on to determine the maximum permissible safe operating temperature of rubber insulation. As a result of their findings a decision was reached that the maximum permissible operating temperature at the surface of the conductor in a rubber insulated cable should not exceed 40 degrees centigrade (104 degrees Fahrenheit).

The current ratings contained in the IEE wiring rules[21] are based on a maximum permissible operating temperature of 120 degrees Fahrenheit (48.9 degrees centigrade) for rubber-insulated wires and cables. This is based on an ambient of 100 degrees Fahrenheit (37.7 degrees centigrade) and a maximum temperature rise of 20 degrees Fahrenheit (11.1 degrees centigrade).

The maximum permissible operating temperature for rubber covered wires used in Japan[22] is 55 degrees centigrade (131 degrees Fahrenheit) and is based on an ambient of 40 degrees centigrade (104 degrees Fahrenheit) with a maximum temperature rise of 15 degrees centigrade (27 degrees Fahrenheit).

Determination of Thermal Surface Resistivity of Cable Coverings and Conduits

PREPARATION OF TEST SPECIMENS AND EQUIPMENT

The cables used in this part of the investigation consisted of the following:

Number 12 Awg—three-sixty-fourths-inch wall of rubber, black finished braid

Number 12 Awg—three-sixty-fourths-inch wall of rubber, white finished braid

Number 2 Awg—fouri-sxty-fourths-inch wall of rubber, rubberized tape and black finished braid

Number 2 Awg—four-sixty-fourths-inch wall of rubber, rubberized tape and white finished braid

Number 2 Awg—four-sixty-fourths-inch wall of rubber, rubberized tape and four-sixty-fourths-inch wall of lead sheathing

Number 4/0 Awg—five-sixty-fourths-inch wall of rubber, rubberized tape and black finished braid

Number 4/0 Awg—five-sixty-fourths-inch wall of rubber, rubberized tape and white finished braid

500,000 circular mils—six-sixty-fourths-inch wall of rubber, rubberized tape and black finished braid

Table IV. Thermal Surface Resistivities B_b and B_c Under Various Conditions (Braided Cables in Conduit)

Conductor Size (Awg or Mcm)	Number of Cables per Conduit	Nominal Conduit Size (Inches)	Color of Conduit	Loading Current		Thermal Surface Resistivity of Braid Bb (Degrees Centigrade per Watt per Centimeter Square)					Thermal Surface Resistivity of Conduit B_c (Degrees Centigrade per Watt per Centimeter Square)			
						Horizontal		Vertical			Horizontal		Vertical	
				Amps.	Type	Enclosed	Open	Enclosed	Open	Average	Enclosed	Open	Enclosed	Open
12	1	0.50	Black	31	d-c	402	378	351	376	377	948	898	1,061	904
	2	0.50	Black	26	d-c	531	526	477	483	504	897	809	927	858
	3	0.50	Black	20	d-c	581	574	583	535	560	922	824	993	817
	3	0.50	Black	20	a-c	537	551	558	553		931	837	996	852
	3	0.50	White	20	d-c	538	575	595	538		940	804	994	838
2	1	0.75	Black	134	d-c		411			411		840		
	2	1.25	Black	112	d-c		776			776		905		
	3	1.50	Black	100	d-c		1,001			1,001		955		
4/0	1	1.25	Black	260	d-c		622			622		915		
	2	2.00	Black	223	d-c		952			952		1,000		
	3	2.50	Black	200	d-c		1,115			1,115		1,028		
500	1	1.50	Black	450	d-c	716	651	704	701	693	1,081	931	1,256	926
	2	3.00	Black	390	d-c	950	1,022	1,035	1,182	1,047	1,501	1,125	1,604	1,037
	3	3.00	Black	336	d-c	1,030	988	1,063	1,289	1,106	1,307	1,081	1,442	967
	3	3.00	Black	339	a-c	1,025	991	1,083	1,264		1,265	1,083	1,383	962
	3	3.00	White	336	d-c	1,049	1,090	1,137	1,265		1,468	1,050	1,528	899
1,000	2	4.00	Black	638	d-c		1,017			980		1,121		
	2	4.00	Black	548	a-c		940					1,101		

500,000 circular mils—six-sixty-fourths-inch wall of rubber, rubberized tape and white finished braid

500,000 circular mils—six-sixty-fourths-inch wall of rubber, rubberized tape and five-sixty-fourths-inch wall of lead sheathing

1,000,000 circular mils—seven-sixty-fourths-inch wall of rubber, rubberized tape and black finished braid

All of the rubber insulation was of the Code-grade and the saturating and finishing compounds used on the cable were of the flame-retarding, moisture-resisting type per the requirements of The Underwriters' Laboratories. Each type of braided cable was furnished by a different cable manufacturer so as to make the results truly representative.

All tests were conducted in a specially enclosed, draft-free room where suitable sources of a-c and d-c power were available. The 500,000-circular-mil cable and number 12 Awg wire, when measured in air or in conduit, were set up as shown in figure 6. For all other tests the cables were supported horizontally in air or in conduit, three and one-half feet from the floor.

On all horizontal sections, thermocouples were located at approximately 4, 9, and 14 feet from one end of the conduit. On vertical sections, the couples were located at the center and two feet above and below the center. In any one section, thermocouples were placed on the copper, braid, conduit, and in the air, as shown in figure 7.

All thermocouples were made from number 30 Awg enameled, single-cotton-covered, copper and constantan wires. All junctions were soldered and cut to the same length. Cold junctions were immersed in a cup of oil kept in a bath of melting ice.

TEST PROCEDURE

For each test setup, a current, estimated to give 50 degrees centigrade copper temperature, was applied at 8 a.m. This current was maintained for six hours and the first set of readings taken. Readings were repeated hourly until thermal equilibrium had been reached. This procedure was repeated on each setup for three days.

The temperatures at any measuring point were invariably read in the following order, before proceeding to the next measuring point.

1. Conductor temperature (loading current—direct and reversed).
2. Braid temperatures.
3. Conduit to air temperature difference.
4. Air temperature.

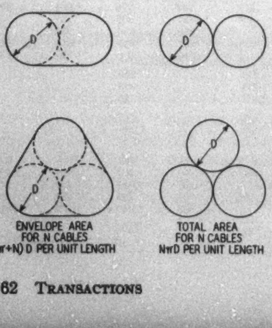
D
D
D
D
ENVELOPE AREA FOR N CABLES $(\pi+N)D$ PER UNIT LENGTH
TOTAL AREA FOR N CABLES $N\pi D$ PER UNIT LENGTH

Figure 8. Dissipating surface

The effective resistance for cables in air, was calculated by the use of Ewan's formulae.[24] The effective resistance of cables in conduit was determined by actual measurement. The power input was computed from the current and the effective resistance.

TEST RESULTS

We have already given the test results for the determination of the maximum permissible operating temperature for Code-grade rubber insulation, in figures 2, 3, 4, and 5, respectively.

Tables III, IV, V, and VI, contained the results of all measurements for the determination of thermal surface resistivity for braided cables in air and in conduit, some additional measurements on lead covered cables in conduit, and the a-c resistance of braided and lead covered cables in conduit.

DISCUSSION OF TEST RESULTS

1. The data in table III shows that the value of B_b (thermal surface resistivity of braided cables) in air is not a constant, as previously assumed. For the range of cable sizes investigated, it varies with the over-all diameter of the respective cables. Due to space limitations, only a summary of the data is given, but it can be stated that, although the values of B_b for white finished braids were higher than those for the corresponding sizes with black finished braid, the difference was insufficient to make it necessary to complicate current-carrying capacity calculations by introducing two sets of values. No appreciable difference could be detected in the values of B_b for vertical or horizontal installation.

Table V. Thermal Surface Resistivity B_b (Lead-Covered Cables in Conduit)

Conductor Size (Awg or Mcm)	Over-All Diameter of Cable (Inches)	Number of Cables per Conduit	Nominal Conduit Size (Inches)	Type of Loading Current	Thermal Surface Resistivity B_b (Degrees Centigrade per Watt per Centimeter Square)
2	0.582	2	1.25	a-c	747
2	0.582	2	1.25	d-c	711
500	1.240	2	3.00	a-c	934
500	1.240	2	3.00	d-c	982

2. The data in table IV indicates the following:

(*a*). The value of B_c (thermal surface resistivity of conduit) is independent of the source of power, the respective values for a given combination of conduit and cable sizes being practically the same whether determined with alternating current or direct current. It should be noted that when computing B_c, care should be taken to use the effective value of R_{cu} for the operating temperature.

(*b*). The value of B_c is independent of the color of the conduit surface, the values being practically the same for black and white conduit. These findings are substantiated by McAdams[25] who, in a table of radiation emissivities for sixteen different colors of oil paint, gives values of 0.92 to 0.96 and also assigns the same value of emissivity for black and white lacquer. It appears that as far as heat dissipation by convection is concerned, the color of the conduit has no significance, but in the case of heat dissipation by radiation, the texture of the surface and the chemical composition of the surface treatment have a far greater effect on heat emissivity than color.

(*c*). The value of B_c increases with increasing conduit diameter.

(*d*). The value of B_c varies with the conduit location, whether it be exposed or concealed, vertical or horizontal. According to McAdams,[15] the air currents passing vertical surfaces are complicated and irregular enough so that the experimental data of several authorities on this subject cannot be successfully correlated. However, he further indicates that for pipes about seven inches in diameter, the convection for a vertical pipe is nearly the same as for a horizontal pipe. This is in substantial agreement with the results as shown in table IV.

To facilitate the use of B_c in the formula for current-carrying capacity, advantage is taken of a correction factor Q, which is discussed elsewhere in the paper.

(*e*). The values of B_b in conduit have been averaged for the respective conduit sizes and cable groupings, so that the resulting values will be truly representative of all of the installation conditions. It should be noted that it is difficult to know the relative position which a group of conductors, when drawn into a conduit, will assume with respect to each other. It should also be pointed out that the respective conduit sizes specified in the Code for certain arrangement of conductors, are based on ease of pulling the conductors into the conduit, and not on a proportional ratio between conductor and conduit diameters. In the light of the foregoing, the data obtained shows excellent agreement with the trend exhibited by the values of B_b in air.

Table VI. A-C/D-C Resistance Ratio (Measured Values for Cables in Iron Conduit)

Conductor Size (Awg or Mcm)	Type of Covering	Number of Cables per Conduit	Nominal Conduit Size (Inches)	Average Ratio A-C/D-C
12	Braid	2	1/2	1.003
12	Braid	3	1/2	1.007
2	Braid	2	1 1/4	1.017
2	Lead	2	1 1/4	1.016
2	Braid	3	1 1/2	1.007
4/0	Braid	2	2	1.034
4/0	Braid	3	2 1/2	1.024
500	Braid	2	3	1.124
500	Lead	2	3	1.224
500	Braid	3	3	1.143
1000	Braid	2	4	1.333

3. Since lead sheathing is used occasionally instead of braid, especially under conditions of moisture, a check was made on the value of B_b in conduit, but using lead covered instead of braided cables. The results are given in table V and show excellent agreement with the values of B_b as shown in table IV.

4. The data in table VI represents the results of a-c resistance determinations for various cable sizes, finishes, and groupings, in the iron conduit sizes prescribed by the code. Each value is the average of two readings, taken with the current direct and reversed, in order to eliminate the effect of stray fields. Among the various factors which have an important bearing on the a-c resistance measurements, are:

(*a*). "Skin effect" in copper conductor, especially larger sizes.

(*b*). "Proximity effect" of one cable upon another, particularly due to the close spacings existing between several cables in conduit.

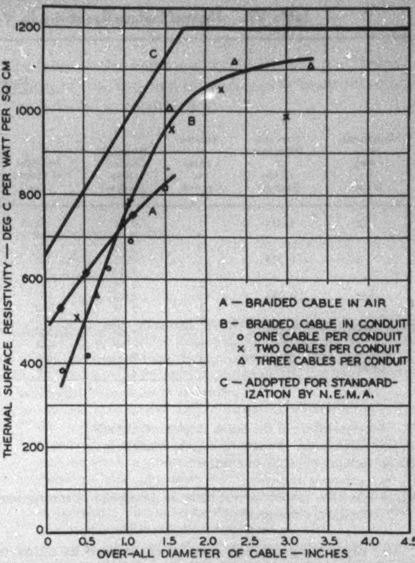

Figure 9. Thermal surface resistivity of cable coverings

(*c*). Hysteresis and eddy currents in the iron conduits causing power losses and increasing the effective resistance of the conductors.

(*d*). In the case of lead-covered cables, there are also eddy-current losses in the lead sheathing, which in turn produce a magnetic field opposite in polarity to that produced by the current in the conductor. Although this reduces the flux in the iron conduit and, consequently, the iron losses, there are heating losses in the lead sheath due to stray currents. If the shielding effect is greater than the sheath loss, the a-c resistance of the lead covered cables will be less than the braided cables, but if the shielding effect is less than the sheath loss, the a-c resistance of the lead-covered cables will be greater than that for the braided cables.

Although table VI contains a considerable number of measurements, it was felt that they were not enough to form the basis for a table of a-c/d-c resistance ratios to be used for computation purposes. Consultation with cable engineers of other cable companies indicated the availability of some additional measurements which enabled the drawing up of a fairly complete set of data. This will be found in table IX.

5. In attempting to calculate the values of B_b in conduit, the question arose as to whether these values should be calculated on the basis of "total" or "envelope area." In order to clarify this point, let us examine the formula for calculating B_b in conduit.

$$B_b = \frac{T_b - T_c}{NI^2R_{cu}} \times \text{dissipating area}$$

wherein

B_b = thermal surface resistivity of braided cable in conduit, degrees centigrade per watt per centimeter square

Table VII. Thermal Surface Resistivity B_b Versus Dissipating Area (Braided Cables in Conduit)

				Dissipating Surface				Thermal Surface Resistivity of Braid B_b Calculated From	
				Envelope Area		Total Area			
Conductor Size (Awg or Mcm)	Over-All Diameter of Cable (Inches)	Number of Cables per Conduit	Nominal Conduit Size (Inches)	Per Foot of Circuit (Cm^2)	Diameter of Equivalent Cylinder (Inches)	Per Foot of Circuit (Cm^2)	Diameter of Equivalent Cylinder (Inches)	Envelope Area (Degrees Centigrade per Watt per Centimeter Square)	Total Area (Degrees Centigrade per Watt per Centimeter Square)
12	0.216	1	0.50	52.6	0.216	52.6	0.216	378	378
		2	0.50	86.3	0.354	105	0.432	431	526
		3	0.50	103	0.421	158	0.648	373	574
2	0.518	1	0.75	126	0.52	126	0.518	411	411
		2	1.25	207	0.85	252	1.04	636	776
		3	1.50	246	1.01	378	1.55	652	1,001
4/0	0.784	1	1.25	191	0.79	191	0.785	622	622
		2	2.00	313	1.29	382	1.57	781	952
		3	2.50	373	1.54	573	2.35	726	1,115
500	1.100	1	1.50	267	1.10	267	1.10	651	651
		2	3.00	438	1.81	534	2.20	840	1,022
		3	3.00	522	2.15	801	3.30	644	988
1,000	1.50	2	4.00	596	2.45	730	3.00	830	1,017

T_b = temperature of the braid, degrees centigrade
T_c = temperature of conduit, degrees centigrade
N = number of cables in conduit
I = current in amperes
R_{cu} = effective conductor resistance at maximum operating temperature, ohms per ft.

The dissipating surface may be considered as either of the areas shown in figure 8 and since there was no precedent available in the literature as to which should be used calculations were made with both the "envelope" as well as the "total area." The resulting values of B_b have been incorporated in table VII and, as can be seen, the values are more consistent when computed on the basis of "total area." This can be more readily seen from a study of curve *B* in figure 9. It will be noted that almost all of the points lie on this curve, even though the points represent values for one, two, and three cables per conduit. Therefore, all of our values for B_b in conduit are based on a dissipating surface defined as the "total area."

Table VIII. Conduit Correction Factor Q

Nominal Conduit Size Inches	Values of Q		
	No. of Single or Multi-Conductor Cables per Conduit		
	One	Two	Three
0.50	3.9	9.5	15.4
0.75	3.6	8.6	13.9
1.00	3.2	7.6	12.4
1.25	2.8	6.7	11.0
1.50	2.6	6.2	10.2
2.00	2.3	5.5	8.9
2.50	2.1	4.8	7.6
3.00	1.9	4.2	6.6
3.50	1.8	3.8	6.0
4.00	1.7	3.5	5.5
4.50	1.6	3.3	5.2

NOTE: *a.* The above values refer to number of cables in a conduit, not to number of conductors in a conduit.
Example: The correction factor for one three-conductor cable in a two-inch conduit is 2.3, while for three single-conductor cables in same size conduit, it is 8.9.
b. Tables do not extend beyond 4.5 inch nominal conduit size because of unknown losses when very large conductor sizes are used.

6. After curves *A* and *B* in figure 9 had been plotted, it was felt that some factor of safety should be introduced. After a thorough review of the subject by the technical representatives of the respective Wire and Cable Sections of NEMA, curve *C* in figure 9 was chosen for standardization as representative of the values of B_b for braided or lead covered cables, in air or in conduit. This curve is a straight line from a value of 650 at 0 diameter to a value of 1,200 degrees centigrade per watt per centimeter square, for a diameter of 1.75 inches and continuing in a horizontal straight line thereafter at a constant value of 1,200 degrees centigrade per watt per centimeter square.

DERIVATION OF CONDUIT CORRECTION FACTOR *Q*

To understand the significance of the conduit correction factor *Q* and its function in the calculation of current-carrying capacity, let us consider the case of a single conductor cable, suspended in free air. In an installation of this type the cable creates a "uniform" heat field and its current-carrying capacity may be expressed by

$$I = \sqrt{\frac{T_0 - T_a}{R_{th}R_{cu}}} \text{ amperes} \qquad (1)$$

$$R_{th} = 0.00522\,\rho \log_e \frac{D}{d} + 0.00411\,\frac{B}{D} \text{ thermal ohms per foot} \qquad (2)$$

wherein

R_{th} = thermal resistance of path from conductor to ambient air, thermal ohms per foot
ρ = 500 degrees centigrade per watt per centimeter cube
D = over-all diameter of cable, inches
d = diameter over copper conductor, inches
B = thermal surface resistivity of the cable in still air, degrees centigrade per watt per centimeter square
T_0 = maximum permissible operating temperature of conductor, degrees centigrade
T_a = ambient temperature, degrees centigrade
R_{cu} = effective conductor resistance at operating temperature, ohms per foot

If we now introduce a conduit into this "uniform" heat field, the latter is distorted and the thermal resistance is increased. The amount of this increase, we define as Q, or the correction factor made necessary by the introduction of the cable in a conduit. As a result of the new thermal resistance, equation 2 now becomes:

$$R_{th}' = 0.00522\,\rho \log_e \frac{D}{d} + 0.00411\,\frac{B}{D} + Q \text{ thermal ohms per foot} \quad (3)$$

or

$$R_{th}' = R_{th} + Q \qquad \text{thermal ohms per foot}$$

The derivation of the values of Q for any given set of installation conditions, may be experimentally determined by measurements made on an actual setup of cables in conduit, as follows:

From equation 1

$$I^2 R_{cu} R_{th} = T_0 - T_a$$

substituting the new value of R_{th} from equation 3, we have

$$I^2 R_{cu}\left(0.00522\,\rho \log_e \frac{D}{d} + 0.00411\,\frac{B}{D} + Q\right) = T_0 - T_a$$

and

$$Q = \frac{T_0 - T_a}{I^2 R_{cu}} - \left(0.00522\,\rho \log_e \frac{D}{d} + 0.00411\,\frac{B}{D}\right)$$

The values of T_0, T_a, I, R_{cu}, D, and d are all obtained from actual measurements and B is taken from the standard curve. Experimental determinations conducted in this manner, indicate that for a given size of conduit containing a given number of cables, the conduit correction factor Q is independent of cable diameter. Values of Q based on the data obtained during the NEMA investigation, will be found in table VIII.

The conduit correction factor Q makes it unnecessary to use the value of B_c (thermal surface resistivity of the conduit) in the formula for R_{th}, since Q contains the elements from which B_c would ordinarily be calculated.

Table IX. A-C/D-C Resistance Ratio

Conductor Size (Awg or Mcm)	A-C/D-C Resistance Ratio	
	Single Conductor Cable in Air or Separate Nonmetallic Conduit	Multiconductor Cable or Two or Three Single-Conductor Cables in Same Conduit
Up to 3	1.00	1.00
2 and 1	1.00	1.01
1/0	1.00	1.02
2/0	1.00	1.03
3/0	1.00	1.04
4/0	1.00	1.05
250	1.005	1.06
300	1.006	1.07
350	1.009	1.08
400	1.011	1.10
500	1.018	1.13
600	1.025	1.16
700	1.034	1.19
750	1.039	1.21
800	1.044	
1,000	1.067	
1,250	1.102	
1,500	1.142	
1,750	1.185	
2,000	1.233	

Formulae for Current-Carrying Capacity Calculation

The formulae for current-carrying capacity and thermal resistance of one, two, and three, single- or multiconductor cables installed in conduit, are:

$$I = \sqrt{\frac{T_0 - T_a}{n\,R_{cu} R_{th}}} \text{ amperes}$$

and

$$R_{th} = \frac{0.00522\rho\,G_1}{n} + \frac{0.00411B}{D} + Q \text{ thermal ohms per foot of cable}$$

wherein

T_0 = maximum safe operating temperature, degrees centigrade. For Code-grade rubber insulation, T = 50 degrees centigrade.

T_a = ambient temperature, degrees centigrade. For building installations T_a = 30 degrees centigrade

n = number of conductors in a cable.

R_{cu} = effective resistance of copper conductor, ohms per foot. For d-c resistance use values given in National Bureau of Standards Circular 31, for 100 per cent IACS conductivity and corrected to proper operating temperature. For a-c/d-c resistance ratio, use values in table IX.

ρ = thermal resistivity, degrees centigrade per watt per centimeter cube. For rubber insulation = 500. For tape or braid over rubber insulation use same value.

G_1 = geometric factor to be taken from Simmons[13] or Underground Systems Reference Book,[1] page 297. In computing the geometric factor consider the tape and/or braid on single conductor cables, and tape of individual conductors of multiconductor cables, as part of the insulation. Tape and braid over assembled conductors are to be considered as belt insulation.

B = thermal surface resistivity, degrees centigrade per watt per centimeter square. For braid, lead and other coverings, use values from curve C in figure 9. The value of B can also be determined as follows:

For diameters below 1.75 inches: $B = 314D + 650$
For diameters 1.75 inches and larger: $B = 1{,}200$

D = over-all diameter of cable, inches.

Q = conduit correction factor. Values to be taken from table VIII. The respective conduit sizes to be in accordance with the National Electrical Code.

AMBIENT TEMPERATURE

Under the formula for current-carrying capacity, we listed T_a = 30 degrees centigrade. Our investigation indicates that 30 degrees centigrade (86 degrees Fahrenheit) is a fair average ambient temperature for indoor installations in general and, accordingly, this value has been chosen by NEMA as standard for all future calculations of interior wiring problems. There may, of course, be conditions of installation where it is definitely known that the temperature of the surroundings is other than 30 degrees centigrade, in which case the correction factors given in table X should be applied to the standard values computed on the basis of a 30 degrees centigrade ambient.

CABLE RATINGS BASED ON NEMA INVESTIGATION

In table I given in the earlier part of the paper, an attempt was made to bring together under one tabulation, all of the principal tables of current carrying capacity for rubber insulated conductors from 1889 to 1937. Included in that tabulation, were excerpts from the present IEE wiring rules[21] issued in 1934, and attention is directed to the fact that the latter have recognized the distinction between a-c and d-c current ratings, especially in the case of the larger conductors. It is also of interest to note that in contrast to our own code ratings, the IEE wiring rules provide ratings for various conductor groupings or conductor assemblies, and that these ratings are based on a definite ambient temperature and a definite maximum permissible temperature rise, a practice which conforms to the best universal engineering thought on the subject. Even in 1903, the IEE[13] made a distinction between high and normal ambients.

In table XI, we have tabulated some cable ratings, based on the factors established by the NEMA investigation, for a single cable in air, and three cables in iron conduit. The calculations were made both for a-c, as well as d-c, loadings. For comparative purposes, we have also included in this tabulation, the corresponding ratings taken from the 1937 edition of The National Electrical Code.[23] It may be of interest to compare the respective ratings for number 4/0 Awg cable. For three 4/0 Awg single cables in iron conduit, carrying alternating current, the data indicates approximately 160 amperes as compared to 225 amperes permitted by the Code. Under these conditions of installation, three 4/0 Awg cables, carrying 225 amperes per cable, would actually achieve a maximum temperature at the surface of the conductor of approximately 66 degrees centigrade.

Conclusions

The factors established in this investigation are equally applicable to varnished cambric, and impregnated paper-insulated cables, as well as to rubber-insulated cables. The formulae are applicable, not only to Code-grade rubber insulations, but to all other grades as well, the important point being to have definite knowledge concerning the maximum permissible operating temperature of the particular insulation in question.

It is the author's hope that the results of this investigation will not only be reflected in a revision of future Code ratings for rubber-insulated cables, but that the factors established by this investigation will be used by others for the revision of existing current-carrying capacity tables for other types of insulated cables as well.

Table X. Correction Factors for Change in Ambient Temperature Based on Maximum Operating Temperature of 50 Degrees Centigrade

Ambient Temperature (Degrees Centigrade)	Correction Factor
10	1.42
20	1.23
30	1.00
40	0.71

Note: For ambients higher than 40 degrees centigrade, insulations other than Code-grade rubber, having a higher permissible operating temperature, should be used.

Acknowledgments

To Anaconda Wire and Cable Company and its engineering staff for the use of laboratory and other facilities: to the Rubber Covered Building Wire Section of NEMA for sponsoring this investigation and for permission to publish these results: to Mr. E. D. Youmans, chairman of the NEMA building wire technical committee: to the members of the technical committee of the Rubber Covered Building Wire Section and the Rubber and Varnished Cambric Power Cable Sections of NEMA, for their very kind criticisms and many helpful suggestions: to all of these and many others who may have contributed to the completion of this investigation, the author wishes to convey his sincere gratitude.

Table XI. Cable Ratings Based on NEMA Investigation

Conductor Size (Awg or Mcm)	Current-Carrying Capacity in Amperes				
	Based on Constants From NEMA Investigation				1937 National Electrical Code
	Alternating Current		Direct Current		
	One Cable in Air	Three Cables in Conduit	One Cable in Air	Three Cables in Conduit	
14	20	15	20	15	15
12	26	19	26	19	20
10	34	26	34	26	25
8	48	35	48	35	35
6	66	47	66	47	50
4	88	61	88	61	70
2	118	78	118	79	90
1/0	160	105	160	106	125
2/0	185	119	185	121	150
3/0	215	141	215	144	175
4/0	249	160	249	164	225
500	430	267	433	284	400
800	571	345	584	384	550
1,000	653		675		650

Note: The above calculations are based upon conduit sizes recommended by the National Electrical Code.

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Discussions

of AIEE Technical Papers Published Before Discussions Were Available

ON THIS and the following 2 pages appear all remaining discussions submitted for publication, and approved by the technical committees, on papers presented at the AIEE Pacific Coast convention, Spokane, Wash., August 31–September 3, 1937; and at the AIEE Middle Eastern District meeting, Akron, Ohio, October 13–15, 1937. Authors' closures, where they have been submitted, will be found at the end of the discussion on their respective papers.

Regulation of Grid-Controlled Rectifier

Discussion and authors' closure of a paper by L. A. Kilgore and J. H. Cox published in the September 1937 issue, pages 1134–40, and presented for oral discussion at the electronics session of the AIEE Middle Eastern District meeting, Akron, Ohio, October 14, 1937.

E. H. Reid (General Electric Company, Schenectady, N. Y.): Messrs. Kilgore and Cox have given a very interesting paper on regulation of grid controlled rectifiers. In regard to the effect of grid characteristics on regulation, I would like to say that it is our experience that most practical commercial designs have what we call a positive characteristic throughout the guaranteed load range. That is, the potential applied to the grid must be positive with respect to cathode before the anode will fire. This being the case, the effect of the grid characteristic on regulation we believe would be small.

I would like to add that by a simple manipulation of a few watts of grid control power the regulation of a grid controlled rectifier may be corrected or changed almost at will.

I would like to suggest that some indication of magnitudes on figure 6 would be of interest. I assume that cathode potential is referred to as 0 potential in this diagram.

W. M. Goodhue (Harvard University, Cambridge, Mass): The authors should be commended for quantitative study of abnormal regulation in grid-controlled rectifiers, particularly where system reactance, or smoothing deficiency at light load are involved.

In figure 2 are shown rectifier characteristics for the increase in regulation due to system reactance, and most of these characteristics show a high degree of curvature. Usually, rectifier regulation characteristics are essentially straight lines, such as those in figure 1. The authors do not give any physical explanation of this curvature. Such an explanation would be very helpful to the writer particularly in his understanding of other factors along these lines.

Is it possible that the assumption on page 1135 of "fundamental frequency voltage E, at the primary terminals may be taken equal to the root-mean-square value" is the cause of the curvature? It would be expected that the effect of system reactance is to increase the "*source*" voltage *linearly* at the same time that the regulation is linearly increased. This is because the effect of reactance is to introduce large harmonic voltage drops, in addition to ordinary phase lag and sine wave drops. Without taking the time to analyze thoroughly the problem, I would expect such a method, when employed with only two anodes involved in commutation, to be useful only in the straight line portions of the curves.

The discussion of light load voltage rise on pages 1136–7, taking grid delay and interphase into account, is very clear and provides a useful graphical method.

If grid delay is used in circuits having interphase transformers there arises the question as to the proper distribution of average current between the two halves of the interphase winding at loads greater than critical, say full load. The grid control is responsible in large measure for the average voltage output of each of the anode groups (double-three-phase circuit). Unlike conditions without grids, any unbalance of these voltages caused by grid control will greatly affect the average current distribution in the interphase, since inductance is powerless to limit the flow of average current. Since the paper implies the use of interphase transformers with grid control, it would be very important to know the average current distribution in the two halves of the interphase, at various loads, particularly from half load to full load and overload, for various delay angles (α). Have the authors any information on the interphase current distribution, at least at full load, and at one or more typical grid delay angles? The distribution is important commercially in that input current harmonics, output voltage harmonics, and limit of load by heating of transformer and rectifier may be affected. For example, a three-phase rectifier has a *second harmonic* of input current which will be introduced by the unbalance of a double-three-phase circuit.

L. A. Kilgore: Mr. Goodhue asks the cause of the curvature of the curves for the correction to be applied for system reactance. It is true that the drop in voltage due to commutating reactance is proportional to the total reactance. However, the increase in the sine-wave voltage for a given primary voltage is not proportional, since the reactive drop adds vectorially. This causes the source voltage to increase more than proportionally, which is offset by a proportional reduction; hence, the net correction to regulation increases less than proportionally.

Mr. Reid has asked concerning the scale of figure 6. This is about 20 volts per division and the voltages are measured relative to the cathode. It is true that if the grid characteristics are positive throughout the range, they will make very little difference on the pick-up provided several hundred volt impulses are used on the grids. However, the authors do not consider that this is essential if the characteristics do not go too much negative throughout the operating range.

Empirical Method of Calculating Corona Loss From High-Voltage Transmission Lines

Discussion and authors' closure of a paper by Joseph H. Carroll and Mabel Macferran Rockwell published in the May 1937 issue, pages 558–65, and presented for oral discussion at the power transmission and distribution session of the AIEE Pacific Coast convention, Spokane, Wash., September 1, 1937.

E. C. Starr (Oregon State College, Corvallis): The paper by Mrs. Rockwell and Doctor Carroll is interesting and valuable in that it summarizes a great deal of corona loss information, which has been obtained over a long period of time, and makes it available in a general way for future application.

The problem is an old one and has been attacked by a great many investigators. Formulas which give reasonably accurate data for losses of the order of five to ten kw per mile of three-phase line were developed by F. W. Peek, Jr., and his associates over 20 years ago. The important thing today, however, is to be able to design a line which will be known to have only a very small amount of corona loss under normal operating conditions. The data contained in this paper for the one-kw-per-mile condition are very valuable and are not obtainable accurately by calculation from any existing formula. It will become increasingly desirable that we reduce corona losses from transmission lines to values even less than one kw per mile, and there is need for an accumulation of data showing the practical starting point of corona on the various sizes and types of conductors used in modern transmission line practice. I realize that the starting point of corona is a very indefinite thing and that we should specify some definite loss as indicating the starting point. The figure of 100 watts per mile of three-phase line might be sufficiently close to the actual starting point to be satisfactory for most applications.

From the standpoint of the power saved, the reduction of losses below a few kw per mile is not especially justifiable, but the problem of interference with radio reception, particularly on the short-wave bands, is becoming increasingly acute. Information is not available on the magnitude and coverage of interference created by corona loss of different power values from transmission circuits, but observation has shown that a degree of loss which is economically satisfactory produces a great deal of interference which blankets a fairly wide area in the short-wave radio communication bands.

An interesting case is that of the Pitt River lines of the Pacific Gas and Electric Company in Northern California. The Seattle-San Diego Airway crosses this line several miles south of Mt. Shasta and parallels it at some distance for a number of miles. At altitudes as great as 14,000 feet a considerable amount of interference is created by this line in the short-wave radio receivers of the transport planes. This interference is not ordinarily so great as to render normal communication ineffective, but it is nevertheless disturbing. This line is, of course, a rather old one and was designed to operate in a small amount of corona. It would be interesting to know just how much loss per mile is occurring from that particular transmission line in the vicinity of the airway crossing.

Present-day conductor design has advanced to the point where it is economically feasible to reduce corona losses to zero, or nearly so, for all normal operating conditions and it is felt that all future lines should be designed with the objective in view of reducing corona loss to an absolute minimum. The industry would be greatly benefited by the publication of a carefully compiled set of data showing the initial corona points for practical transmission line conductors under different conditions of altitude and atmospheric temperature.

M. M. Rockwell: Professor Starr's discussion hits the nail squarely on the head in stating that economic considerations are not the only ones to be taken into account in determining how much corona loss is permissible on a line. The matter of radio interference is undoubtedly of importance too, especially where the line traverses populous territory.

I should like to speak a word of caution, however, against taking even the radio angle too seriously. To begin with, the Pitt River line which Professor Starr cites as disturbing airways communication, is made up of rope-lay cable and is known to be operating with much heavier corona than is permitted on any lines of more recent design. In the second place, much or all of the disturbance commonly attributed to corona on so-called noisy lines is actually caused by corona on sharp-edged hardware and fittings, not on the conductor itself. This has been verified by examinations made in the dead of night on various Southern California high-voltage lines operating at high altitudes. These lines were very noisy and caused appreciable interference on car radios, yet examination showed no corona whatever on the conductors themselves, but copious brushes from the suspension clamps, vibration dampers, and occasionally the splices. Hence it would be well to eliminate these sources of trouble before turning too much suspicion on the conductor itself. In fact, I have heard my colleague, Doctor Carroll, state that he once made radio interference studies in the high-voltage laboratory at Stanford, in which the conductors were allowed to go into copious corona, yet little or no interference could be detected in the radio set. At the same time, if brushes were allowed to form on the hardware or insulators, interference resulted. Apparently the inductive-capacitive circuit conditions necessary for setting up radio waves were not present in simple corona from line conductors, and only occurred where small gaps between fittings, etc., were broken down. Of course, I do not know whether this analysis extended to the ultra-short waves now used in radio work. In any event I believe that a good deal of careful experimentation on radio interference due to corona from the conductor itself should be carried on before jumping to the conclusion that conductors should be operated below the corona starting point to avoid interference. If this rule were adopted it would cause the cost of lines to jump appreciably, as the economic balance-point appears to be such as to permit a loss amounting to several kilowatts a mile. The pronounced flattening of the corona curves at the lower end indicates that the size conductor for a given voltage would have to be very markedly increased to avoid the chance of one or two brushes appearing per span, especially in foggy weather. Perhaps it will be found that if brushes from hardware, etc., are eliminated, these one or two brushes on the conductor itself will do little or no harm.

In designing the 220-kv lines for transmitting power from Boulder Dam to the pumping plants along the aqueduct of the Metropolitan Water District of Southern California, with which the writer was associated, it was found economic to permit quite a bit of corona on the lines. Since these lines passed through desert territory the radio interference problem did not seem of importance. Nevertheless, the conductor diameter was selected large enough to limit the corona to quite low values, just to be on the safe side. The matter of airways communication was not thought of at the time, but perhaps from this standpoint, the conservative design in the matter of corona will prove a good thing.

Present-Day and Probable Future Electrical Applications in Aircraft

Discussion and author's closure of a paper by W. V. Boughton published in the August 1937 issue, pages 959–63, and presented for oral discussion at the selected subjects session of the AIEE Pacific Coast Convention, Spokane, Wash., August 31, 1937.

E. C. Starr (Oregon State College, Corvallis): It is interesting to note how completely the modern aircraft are becoming electrified. In the past it has been possible to employ storage-battery and low-voltage systems quite satisfactorily in most transport work but, as indicated by Mr. Boughton, we will soon be compelled to go to higher voltages and much greater power capacities. He indicates that alternating current of a frequency probably between 360 and 800 cycles will be employed. There is some question as to whether single- or three-phase supply will be chosen. It is true that polyphase motors have an advantage over single phase but for frequencies as great as 800 cycles the single-phase capacitor motor should be almost ideal. It would be light in weight since it would require only a small amount of iron and copper, and modern high-capacitance capacitors are available in very small sizes. The simplicity of the single-phase system, together with the efficiency of capacitor motors, should be given careful consideration in the final choice.

The higher power frequencies employed in connection with the radio equipment will make necessary more careful filtering and shielding than is employed at present to prevent undesirable speech interference. This problem, however, should not be at all difficult.

It is interesting to note here that in addition to the regular electrical equipment which one might expect to find on all large transport planes, we may soon have rather complex rain- and snow-static discharging devices. Intense radio interference produced by certain storm conditions makes the development of such equipment desirable. Storm-static research during the past few months has clarified the problem and indicated the necessary developments.

W. V. Boughton: Motor applications in aircraft are rapidly increasing, and the use of motors as large as ten horsepower is quite probable. It is rather doubtful whether the use of single-phase capacitor motors would be desirable for these sizes. Three-phase supply presents no serious distribution problems and does present advantages when synchronization of two or more generating units is considered.

The saving in weight of motors at 800 cycles over 400 cycles is extremely doubtful when the capacitor weight and reduction gearing is considered. In many cases mechanical limitations in manufacturing may prevent taking full advantage of the theoretical weight saving at 800 cycles.

The problem of radio interference from the higher power frequencies has been studied by the radio manufacturers and can be readily solved.

Extension of 2-Reaction Theory to Multiphase Synchronous Machines

Discussion and author's closure of a paper by Y. H. Ku published in the September 1937 issue, pages 1197–1201.

R. H. Park (The Calco Chemical Company, Inc., Bound Brook, N. J.): It may be desirable, in connection with this paper, to point out that the assumption that all space harmonics are zero and further that the mutual slot leakage reactance is space fundamentally distributed departs from the assumptions of reference 11 of the paper wherein it was merely assumed that "as far as concerns effects depending on the position of the rotor, each armature winding is sinusoidally distributed.

On the basis of the latter assumption β in equation 3 of appendix I would be regarded as merely the space fundamental component of air gap flux and the total linkages in phase a would be computed as $\psi_a(\beta) + \psi_a(l)$ where by symmetry

$$\psi_a(l) = -x_{l\sigma}i_a + x_{map}(i_b + i_n) + x_{mac}\,(i_c + i_{n-1}) + x_{mad}\,(i_d + i_{n-2}) + \ldots\ldots$$

and the x_m's are suitable mutual coefficients which may be either positive or negative, but which take into account the detail nature of the slot mutual and harmonic air gap fluxes, which are merely assumed to be unaffected by rotor position.

But for a three-phase machine we have in general, with balanced terminal connections, under transient as well as steady conditions, $e_a = \boldsymbol{e}\cdot\boldsymbol{a}$, $\psi_a = \boldsymbol{\psi}\cdot\boldsymbol{a}$, $i_a = \boldsymbol{i}\cdot\boldsymbol{a}$ where $\boldsymbol{e}$, $\boldsymbol{\psi}$, and $\boldsymbol{i}$ are instantaneous space vectors and $\boldsymbol{a}$ is a unit space vector representing the position of any phase a.

Evidently we may tentatively assume that the same type of relationship holds generally for a multiphase machine. On this assumption and if the number of phases n is equal to or greater than three, it would follow from equation 6 of reference 11, after making the necessary changes in notation that if ϕ is the angle between $\boldsymbol{i}$ and $\boldsymbol{a}$ and $\theta = \phi + \theta_0$, $\theta_0 = 360/n$ that $i_a = \cos\phi$, $i_b = i\,(\cos\phi - \theta_0)$, etc.,

$$\psi_a = I_d\cos\theta - I_q\sin\theta - \left(x_{l\sigma} + x_m + \frac{p_d + p_q}{2}\right) i\cos\phi - \frac{p_d - p_q}{2}\cos\,(\phi + 2\phi_0)$$

where

$$x_m = -2(x_{mab}\cos\theta_0 + x_{mac}\cos 2\theta_0 + \ldots\ldots)$$

a result which may be seen to be independent of n.

But as this is the same form as the corresponding equation for a three-phase machine it will be apparent that the validity of the tentative assumption stated, namely, that the space vectors $\boldsymbol{e}$, $\boldsymbol{\psi}$, and $\boldsymbol{i}$ which correspond to the behavior of a three-phase machine do in fact represent the vector form of the solution for any machine of three or more phases under any balanced terminal conditions whether transient or not, has been demonstrated.

But under balanced load conditions a two-phase machine would behave in the same way as a four-phase machine, in view of symmetry conditions.

Hence we may conclude that any solution of the equations of a three-phase machine under balanced load conditions may properly be thought of as automatically constituting a solution for a machine of any number of phases in excess of one.

Y. H. Ku: The author appreciates very much the discussion of Mr. R. H. Park regarding the assumptions in appendix I. Though the armature-reaction linkages are free from space harmonics as given by equation 4, the simplification that the mutual slot leakage reactance is space fundamentally distributed is not necessary, as rightly pointed out by Mr. Park. So, in general, equation 5 may be written as follows:

$$\psi_A = \psi_{A(\beta)} + \psi_{A(l)}$$

where $\psi_A(\beta)$ is given by equation 4 in appendix I and

$$\psi_{A(l)} = -x_{l\sigma}i_A - [x_{mAB}i_B + x_{mAC}i_C + \ldots\ldots + x_{mAN}i_N]$$

From symmetry, $x_{mAB} = x_{mAN}$, and similarly for others. These x_m's are mutual coefficients which take into account the detail nature of the slot mutual and harmonic air gap fluxes.

Under balanced conditions, there are

$i_A\cos t$, $i_B\cos(t-\theta_0)$, etc.

where $\theta_0 = 2\pi/n$ and n is the number of phases.

Substituting these current expressions, we get

$$\psi_{A(l)} = -[x_{e\sigma} + x_{mAB}\cos\theta_0 + x_{mAC}\cos 2\theta_0 + \ldots\ldots + x_{mAN}\cos\theta_0]\cos t$$

Notice that the above expression holds for any number of phases, even or odd. For n odd, it can be easily seen that the sin t terms cancel each other on account of symmetry. For n even, however, we have a mutual term involving $\cos\,[t-(n/2)\,\theta_0]$ which can be expanded into $[\cos t\cos\,(n/2)\,\theta_0 + \sin t\sin\,(n/2)\,\theta_0]$. Now since $\theta_0 = 2\pi/n$, $\sin\,(n/2)$, θ_0 is zero, and hence there will be no sin t terms.

Equation 6 of appendix I can then be rewritten as

$$\psi_A = -\left[x_{l\sigma} + x_m + \frac{p_d + p_q}{2}\right] - \cos t\,\frac{p_d - p_q}{2}\cos\,(t + 2t_0)$$

where

$$x_m = x_{mAB}\cos\theta_0 + x_{mAC}\cos 2\theta_0 + \ldots\ldots\ldots + x_{mAN}\cos\theta_0$$

With this new expression of x_m, equations 7 and 8 are also extended to the more general form. However, x_0 in equation 9 should be changed to the following:

$$x_0 = x_{l\sigma} + x_{mAB} + x_{mAC} + \ldots\ldots\ldots + x_{mAN}$$

Sharp Cutoff in Vacuum Tubes, With Applications to the Slide-Back Voltmeter

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Synopsis: That the current-voltage characteristics of vacuum tubes obey an exponential law near cutoff is pointed out. The larger the coefficient k appearing in the exponent, the steeper will be the current-voltage curve, and the better suited the tube will be for use in a slide-back voltmeter. The problem of obtaining high values of k is considered, and a simple theory of rectification by an exponential conductor is given. The error inherent in slide-back voltmeters of the ordinary type is discussed, and a correction factor applicable to sine waves is derived. The circuit of a slide-back vacuum-tube voltmeter which includes means for measuring the k of the tube is shown.

WHEN a high-vacuum tube is used as a small-voltage rectifier, or as a threshold indicator, it is usually desirable that the current-voltage curve have as abrupt a cutoff as possible. In many cases a sharp angle at the foot of the characteristic would be valuable, but, since this cannot be realized in a high-vacuum device, we must be content with obtaining a rapid rise in current as the control voltage is increased in the positive direction.

Despite the fact that many very satisfactory forms of direct-reading vacuum-tube voltmeters have been developed during the last few years, the slide-back[1,2] type is still an extremely useful instrument. In this type, a triode, or multielectrode tube, is biased almost to cutoff, and the reading of a d-c voltmeter which indicates the bias is noted. The signal to be measured is then applied to the grid in series with the bias voltage, and the bias is increased until the cutoff point is again reached. The difference in the two readings of the d-c voltmeter equals the peak value of the alternating voltage under measurement. The simplicity of this instrument, together with the fact that an accurate d-c voltmeter may be used to measure a-c peak voltages, makes it the most suitable device for many applications. Moreover, it is easily built and manipulated by men who have had no experience with vacuum tube circuits.

However, the slide-back voltmeter will give accurate results only if the voltage under measurement is reasonably large. When taking a reading with it, we must allow the peak of the impressed alternating voltage to swing past cutoff by an amount sufficient to produce a detectable current in the plate circuit of the tube. Just how much past cutoff this is is not ordinarily known, and consequently an uncertainty is introduced into the reading. If the potential under measurement is only a few volts, a considerable error may result.

In this paper, a study is made of the conditions which lead to the most rapid rise of the plate current versus grid voltage characteristic of a vacuum tube, and the results are applied to the operation of a slide-back voltmeter. The error involved in the measurement of small voltages is discussed, and a correction factor is derived which makes it possible to obtain accurate measurements of sinusoidal waves having peak values of as little as half a volt.

Form of Current-Voltage Curves

At very low currents, the theoretical equation of a diode is given by[3]

$$I = I_s e^{\frac{-eV}{KT}} \qquad (1)$$

in which V is the voltage between the electrodes, I_s is the saturation current, T is the absolute temperature, e is the charge of an electron $= -1.59 \times 10^{-19}$ coulombs, and K is Boltzmann's constant $= 1.37 \times 10^{-23}$ joules per degree. Actually, the grid current-grid voltage characteristic of commercial high-vacuum tubes is found to be exponential in form over a limited range.

Let the coefficient of V be represented by k. Then, putting in the numerical values quoted, there results

$$k = \frac{-e}{KT} = \frac{11{,}600}{T} \qquad (2)$$

Then

$$I = I_s e^{kV} \qquad (3)$$

We may evaluate k experimentally at any given temperature. Its value for the grid voltage-grid current curve of average receiving tubes lies between 5 and 11, and the exponential law holds over a range of about 0.6 volts.

The quantity I_s is of no great interest to us here, since the factor which multiplies the exponential term in the current equation can be varied at will by adjusting the bias

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1. For all numbered references, see list at end of paper.

voltage. Thus, if we apply a bias $-V_0$ and a variable voltage v, (3) gives

$$I = I_s\epsilon^{kV_0}\epsilon^{kv} = G\epsilon^{kv} \tag{4}$$

G being a new constant. Evidently, it is k, and not I_s, which, from a circuit-theory standpoint, is an important parameter of a given tube.

In the case of a multielectrode tube, there is, of course, no necessary relation between the plate current-grid voltage characteristic and the curve defined by (1), since the degree of control which the grid exercises upon the electron stream is dependent upon a number of different factors. However, it is not unreasonable to suppose that, with an efficient grid structure allowing very little leakage of lines of force around it, the equation for low currents might be similar to (1). The most efficient grid structure encountered in ordinary tubes is that of the screen in a tetrode or pentode. We may expect that when the screen is used as the control element, a more rapid variation of plate current with grid voltage will be obtained near cutoff than when the ordinary grid is used. Whether or not the current-voltage curve is exponential is best determined by experiment.

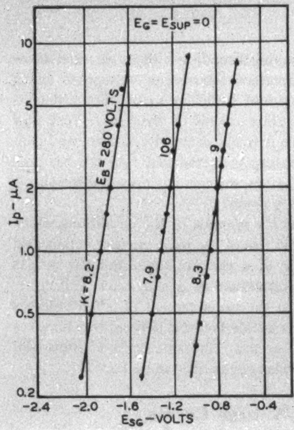


Figure 1. Curves of plate current versus screen-grid voltage for a type 77 tube

It is to be noted that plate voltage has only a small effect on the slope of these curves

Figure 1 shows semilogarithmic plots which were taken of the plate current-screen voltage characteristics of a type 77 vacuum tube, the suppressor and control grid being connected to the cathode. The three curves are for widely different values of plate voltage, and it is to be seen that an exponential law is obeyed in all three cases. Furthermore, the slope of all the curves is practically the same.

Figure 2 shows $I_p - E_{sg}$ characteristics of two other type 77 tubes and one 6*D*6. The former are exponential up to about ten microamperes, and the latter to about five microamperes.

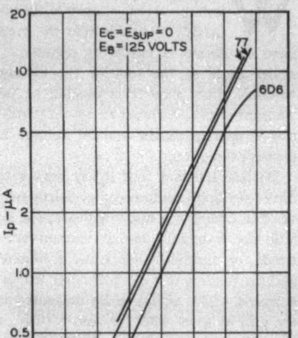
Strictly speaking, an exponential characteristic has no cutoff point, since the curve approaches the voltage axis asymptotically. In practice, however, the plate current is regarded as being cut off when it is reduced to so small a value that it can no longer be read on the plate-circuit microammeter. As the control element is made less negative, the meter will finally show a reading, which will then increase, as the control voltage is further changed, with a rapidity that is determined by the constant k of (3). The larger k is, the more abrupt is the rise of current, and hence the more satisfactory the tube will be for use in a slide-back voltmeter or similar device.

The values of k determined graphically from the slopes of the curves of figure 1 are marked on them, and it will be seen that k is not greatly dependent upon the plate voltage. With such steep slopes, the graphical determination may easily be in error by several units in the second digit, and consequently we may say that k is roughly constant with respect to plate voltage.

Effect of Operating Conditions on k

The curves of figure 1 have shown the small effect of changes in plate voltage on k. Figure 3 shows a group of curves, taken on another type 77 tube, in which the suppressor voltage is the parameter. Here again an exponential law is obeyed over the range which was measured. The graphically determined values of k are again shown on these curves, and it is evident that suppressor voltage has no marked effect upon this quantity, although positive voltages tend to reduce it somewhat.

Figure 2. Curves of plate current versus screen-grid voltage for two different model 77 tubes and a 6D6



Having established the fact that the current-voltage characteristics with which we are dealing are substantially exponential at low currents, a better method of measuring the quantity k was devised. This was desirable, since the graphical determination is both slow and inaccurate. The new method involved adjusting the voltage of the control element until the plate current read one microampere, and then applying a sine-wave alternating voltage from an adjustable potential divider and varying its magnitude until the plate current was increased to exactly two microamperes. If E_a is the peak value of the applied alternating voltage, k is given by

$$k = 1.81/E_a \qquad (5)$$

The justification for this relation will be given later on.

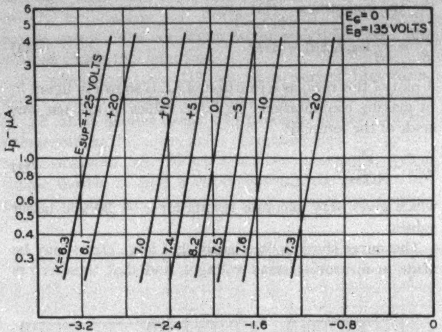


Figure 3. Effect of suppressor grid voltage on the plate current versus screen-grid voltage curves. Large positive voltages tend to reduce the slope

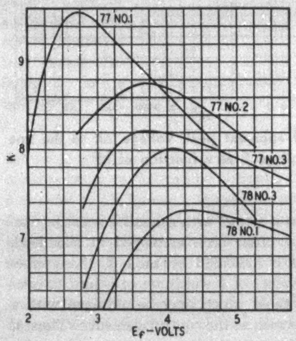


Figure 4. Variation of k with heater voltage for five different screen-grid tubes

The effect of heater voltage on the k of these same tubes was next studied, with results as shown in figure 4 for five different tubes. As the filament temperature is reduced, k increases, and this is as predicted by (2). However, if the filament temperature is dropped far enough, k must of course decrease, and would become zero when the emission stopped altogether.

If k is calculated from (2) for 1,120 degrees absolute, which is a typical operating temperature for oxide-coated cathodes,[3] a value of 10.37 is obtained. This is a little greater than the largest observed value of k, even though the latter occurred at a temperature below that of normal operation.

The values of k calculated from (2) for thoriated (2,000 degrees) and pure tungsten (2,550 degrees) cathodes are 5.80 and 4.55, respectively. Evidently oxide-cathode tubes are to be preferred for use in a slide-back voltmeter.

It was found that the voltage of number one grid had only a small effect on k over the range of +1.0 to −0.2 volts. However, negative biases of half a volt or more caused a large reduction in this parameter.

Table I

Type of tube	k
79—number 1	3.4
79—number 2	3.6
76—number 1	2.6
76—number 2	1.9
77*—number 1	2.7

* Number 1 grid and screen grid connected together as control element.

Table II

Type of tube	k
24—number 1	8.1
24—number 3	7.7
35—number 1	4.6
57—number 1	7.6
58—number 1	8.6

Values of k for Other Electrode Arrangements and for Other Tubes

As might be expected, a screen-grid tube showed a smaller value of k when the number one grid was used as the control element. This was true even with type 77 tubes, which cut off very much more quickly than do those having variable-mu structures. Thus, values of from 2.0 to 2.8 were found for 77's used in the normal manner.

A number of other tubes were also measured, with the results shown in table I. In all cases the plate voltage was 135 volts and the bias of the control element was adjusted to give an initial current of one microampere. Table II shows the values of k for several screen grid tubes of older types. The screen was used as the control element in each case, and the number one grid and suppressor, if any, were connected to the cathode. It is evident from these tables that tubes used with screen control give very much better values of k than do triodes, or pentodes used in the normal manner.

From the data which have been presented, it is clear that the operating voltages, with the exception of the heater voltage, of a pentode used with screen-grid control do not greatly affect the value of k. If the heater voltage is kept strictly constant, k is reasonably constant. Measurements taken at four different times during a period of a week, on three different type 77 tubes, showed a maximum variation of k with time, for any tube, of less than four per cent.

Rectification by an Exponential Conductor

If a sinusoidal voltage is applied to a device whose current-voltage curve is defined by (4), we have

$$I = G\epsilon^{-kE\cos\omega t} \tag{6}$$

The d-c component of this expression can be shown to be[4]

$$I = GI_0(kE) \tag{7}$$

in which $I_0(kE) = J_0(\sqrt{-1}\,kE)$. I_0 is the zero-order Bessel function of a pure imaginary argument. Values of I_0 are to be found tabulated on pages 277–82 of "Tables of Functions," by Jahnke and Emde, second edition, 1933.

As has already been explained, G can be adjusted to any desired value by means of an appropriate bias voltage. When the impressed alternating voltage is zero, $I_0(kE) = I_0(0) =$ unity. Suppose that the bias is so adjusted that $G = 1$ microampere when there is no impressed voltage wave. If, then, an alternating voltage is placed on the grid, the current in microamperes will be equal to $I_0(kE)$. If this impressed wave is adjusted to a value E_a, such that the d-c plate current is two microamperes, we have

$$I_0(kE_a) = 2 \tag{8}$$

From the tables we find that $I_0(x) = 2$ when $x = 1.81$. Hence, (5) follows immediately. It is to be remembered that E_a is the peak value of the impressed sinusoidal wave of voltage which will change the d-c plate current from an initial value of one microampere to twice that value.

When a sinusoidal voltage is to be measured with a slide-back voltmeter, the bias is first adjusted, in the absence of impressed voltage, until the plate current has a low, but readable value. If the grid bias corresponding to this condition is E_1, we have

$$I_1 = G\epsilon^{-kE_1} \tag{9}$$

The alternating voltage is then applied and the bias readjusted until the original current reading is restored. Then

$$I_1 = G\epsilon^{-kE_2}I_0(kE) \tag{10}$$

Table III

Peak Alternating Current	f(kE)	ΔE Calculated	E Measured
0.565	0.631	0.356	0.35
0.848	0.724	0.613	0.60
1.132	0.776	0.88	0.87
1.414	0.812	1.15	1.14
2.83	0.890	2.52	2.51
4.24	0.920	3.90	3.90
5.66	0.938	5.31	5.28
7.07	0.950	6.72	6.71
8.49	0.957	8.13	8.11
9.90	0.966	9.56	9.51
14.14	0.970	13.72	13.77
16.98	0.973	16.53	16.53
21.21	0.980	20.8	20.8

Let this difference between the two bias voltages $(E_2 - E_1)$ be ΔE. Then, equating (9) and (10) we get

$$\epsilon^{k\Delta E} = I_0(kE) \tag{11}$$

In the ordinary use of this type of voltmeter, it is assumed that $\Delta E = E$. Actually, this is not the case, and, in order to find the magnitude of the error, let us take the ratio of the two quantities. (11) gives

$$\frac{\Delta E}{E} = \frac{1}{kE}\log_\epsilon I_0(kE) = f(kE) \tag{12}$$

A plot of this ratio as a function of kE is shown in figure 5. In making computations for large values of kE, use was made of the formula[5]

$$I_0(x) = \frac{\epsilon^x}{\sqrt{2\pi x}} \tag{13}$$

which gives very accurate results for $x = 20$ and larger values.

This curve shows quite clearly that large errors may be made in measuring small voltages, and that these errors are the more serious if a tube with a small k is used. Actually, triodes have often been employed in slide-back voltmeters, and, as is indicated by table I, these tubes are likely to have values of k which are one-third to one-fourth as great as those which may be obtained with a pentode, using the screen as the control element. Thus, if we have a k of eight and a peak voltage of seven volts, the error in measurement will be five per cent (corresponding to $kE = 56$), whereas if k is only two, the error for the same applied peak voltage will be about 16 per cent.

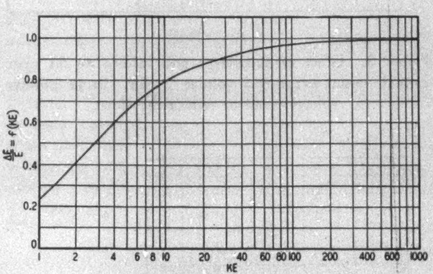


Figure 5. Ratio of apparent peak voltage ΔE to the true peak voltage E as a function of kE

A check of the validity of (12) is given by table III. In taking data for this table, an a-c source was adjusted to 15.0 volts root-mean-square by means of an accurate voltmeter, and a potential divider was used to obtain the smaller voltages. The value of $f(kE)$ corresponding to each peak voltage was determined from the curve of figure 5, and this, multiplied by the peak voltage, gives the value of ΔE which should result. The measured and calculated values are shown in table III.

The per cent correction is small for the higher voltages, but is quite large for peak values of a few volts or less. For the lowest voltage recorded, the error which would be made by omitting the correction factor would be 23 per cent of the impressed voltage. There is no reason why still smaller voltages could not be measured if sufficiently sensitive meters were used. When a slide-back voltmeter has been set up to measure large voltages, it often turns out that small ones must also be dealt with, and it is usually inconvenient to provide another voltmeter to handle them. The correction factor which has been developed makes this unnecessary.

In calculating table III, we have worked backwards from a known alternating voltage, and obtained the value of ΔE which should result. In practice, the alternating voltage will be unknown, and a correction must be applied to the observed ΔE. When k has been measured, it is a simple matter to plot a curve showing the relation between the observed ΔE and the peak value of the sinusoidal wave which is being measured, since the value of E corresponding to ΔE can be obtained directly from figure 5 as soon as k is known. A typical curve of E versus ΔE for $k = 8.2$ is shown in figure 6. Of course, this correction is accurate only if the voltage being measured is reasonably sinusoidal. It cannot be applied to nonsinusoidal waves, nor can a simple correction factor for measuring such waves be derived, for the ratio of ΔE to the true peak voltage would depend upon both the amplitudes and the phases of the harmonics, and these are usually unknown.

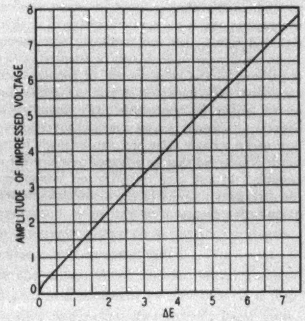


Figure 6. Calibration curve of a slide-back voltmeter derived from the curve of figure 5 for a value of k = 8.2

Circuit of the Slide-Back Voltmeter

Figure 7 shows the circuit of a slide-back voltmeter which is arranged for making measurements of k, as well as of unknown voltages. In order to measure k, the switch S is thrown into position 2, and resistance R_1 is adjusted until the voltage E_0 is equal to some reference value, say 50 volts peak. This value is determined by measurement with the slide-back voltmeter in the following manner. With the 60-cycle supply cut off, potentiometer P_3 is adjusted until the plate-current meter reads some small value, such as one or two microamperes, and the reading of voltmeter E_c is noted. P_3 is then increased by exactly 50 volts, the 60-cycle supply is switched on, and R_1 is adjusted until the plate current again reads the same as before. The voltage across R_2 and P_2 will then be 50 volts peak, except for the error inherent in a slide-back voltmeter, which we have been discussing in this paper. However, if a type-77 pentode tube is used in the voltmeter, it will have a value of k not far from 8, and hence kE will be about 400, and reference to figure 5 shows that the error will not be greater than one per cent. This can be eliminated if we use an increment in screen grid bias of 49.5 volts instead of 50 volts. Having adjusted the voltage across R_2 and P_2 to the proper value, the switch S is next thrown into position 3, and P_2 is adjusted until the plate current reads twice the no-signal value. Very small currents should, of course, be used. The value of k will be inversely proportional to the voltage across P_2, as shown by (5), and consequently the dial of P_2 may be calibrated to read k directly.

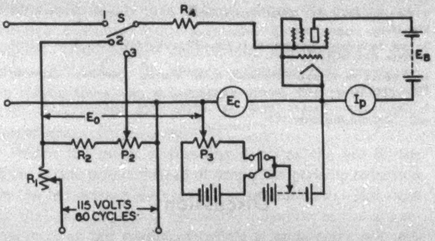


Figure 7. Circuit of a slide-back voltmeter with arrangements for measuring the value of k

When k has been measured, a curve such as that shown in figure 6 can be plotted with the help of figure 5, thereby calibrating the instrument. An unknown voltage is then measured by throwing the switch to position 1, short-circuiting the input terminals, and adjusting P_3 until the plate current is about one microampere. E_c is then increased to a value larger than the peak voltage to be measured, and the latter is applied to the input terminals. It is important to make E_c large enough in order to avoid damage to the plate-current meter. Next, P_3 is adjusted until the plate current is restored to its original value, and the increment in E_c is noted. This increment gives the peak of the voltage under measurement, by reference to the calibration curve.

R_4 is a high resistance of the order of 0.25 to 0.5 megohm, which will prevent the grid from going positive if the input voltage is accidentally allowed to exceed the bias voltage by any large amount. This furnishes some protection to the plate-circuit meter, but reasonable care will still be required to prevent damage to a sensitive instrument.

A plate voltage of 150 to 170 volts is satisfactory with type-77 tubes. If lower voltages are used, there is danger that the tube will draw grid current when a reading of one microampere plate-current is obtained, and this would lower the input impedance of the device. Moreover, if R_4 is in circuit, this would also introduce large errors in the measurements. If higher plate voltages are employed the maximum plate current which can occur during overload is increased, thereby adding unnecessarily to the ease with which the plate meter may be injured.

This voltmeter is simple and accurate, and greatly extends the range of usefulness of the slide-back type of instrument.

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1. HIGH-FREQUENCY MEASUREMENTS, August Hund. McGraw-Hill, section 67.

2. MEASUREMENTS IN RADIO ENGINEERING, F. E. Terman. McGraw-Hill, page 28.

3. See, for instance, PHENOMENA IN HIGH-FREQUENCY SYSTEMS, August Hund. McGraw-Hill, page 7.

4. THEORY OF BESSEL FUNCTIONS, G. N. Watson. Cambridge University Press, page 22.

5. See ELECTRIC CIRCUIT THEORY AND OPERATIONAL CALCULUS, J R. Carson. McGraw-Hill, page 129.

Discussion

C. F. Harding (Purdue University, Lafayette, Ind.): The unique method, described in this paper, in making possible the measurement of crest potentials with the slide-back voltmeter principle not only marks another important step in the varied application of standard electronic tubes but also provides a simple and fairly accurate device for determining these rather evasive values. It is readily used, in connection with a resistance or capacitance potentiometer, for determining the important crest values of sustained wave forms in the extra-high-potential laboratory.

However, this combination with the slide-back voltmeter, requiring as it does two readings for every potential determination, is not applicable to steep-front surges nor to nonrepetitive wave shapes. It is hoped that, as the result of this study, the electronic tube, which seems to be well adapted to such control, may be soon adapted to the measurement of the individual crest values of surges of the order of one microsecond duration or less with a single reading and that ultimately such may be reduced down to cgs units to provide the much-desired primary standards for extra-high-potential measurements. Unfortunately, in spite of the long and careful experimentation and consideration of the committee on high-voltage standards, nothing better than the sphere gap has been developed for surge potential measurement. This gap has many disadvantages and, furthermore, is not a primary standard. It is hoped that this paper may act as a possible solution and a valuable progress report looking toward that important selection of such a primary standard.

C. B. Aiken and L. C. Birdsall: As Doctor Harding points out, the slide-back voltmeter is not, of course, applicable to nonrepetitive wave forms, nor is the correction derived in this paper applicable to repeated waves that are markedly nonsinusoidal. However, the diode rectifier can undoubtedly be adapted, with the help of auxiliary apparatus, to the measurement of transient voltages in which the maximum value of dE/dt is not too great, the limit being determined largely by irreducible stray capacitances in the measuring circuits. It would require a special investigation to determine just how useful a diode voltmeter can be made for surge measurements, but it is believed that an advance over present equipment would be possible. In any such investigation, advantage could be taken of the exponential nature of the diode characteristic in working out a quantitative theory of operation.

Interpretation of Oscillograms of Arc-Welding Generators in Terms of Welding Performance

By K. L. HANSEN
FELLOW AIEE

IT HAS long been recognized that the transient performance of an arc-welding generator greatly exceeds its static performance in importance. By transient performance is meant the behavior of the machine during the period of readjustment after a disturbance has taken place in the external circuit. Unlike most generators, an arc-welding generator supplies energy to a load in which conditions are continually changing, and the importance of the machine's ability to respond quickly to these rapid variations is obvious.

In the literature on arc-welding generators can be found many discussions on the static, or steady-state volt-ampere curve. The static volt-ampere curve is obtained by taking readings of voltage and current when the external load resistance is varied in suitable steps from infinite value on open circuit to approximately zero value on short circuit. Sufficient time is taken on each step of resistance to allow the current and voltage to settle to steady values before readings are taken; in other words, it is a characteristic volt-ampere curve for a slowly varying resistance only. Many arguments have been advanced for a certain shape or a particular degree of steepness of the static volt-ampere curve.

The volt-ampere characteristic of the arc when plotted with volts as ordinates and amperes as abscissas is a line approximately parallel with the horizontal base line. From this it is inferred that a steep volt-ampere characteristic of the welding system, which crosses the arc characteristic in such a manner as to make a large angle at the intersection, will be conducive to arc stability.

However, with the possible exception of the constant potential welding generator with negligible inductance in the arc circuit, the relation between voltage and current indicated by the static volt-ampere curve never prevails under actual welding conditions. For that reason it is not the steepness of the static volt-ampere curve which is the principal factor in determining arc stability, but the steepness of the transient or dynamic volt-ampere curves. Yet the writer has never observed any discussion on the shape or steepness of the much more important transient or dynamic volt-ampere curves.

Figure 1 shows an oscillogram of the arc voltage and current which may be considered typical, the generator and the covered electrode used both being standard commercial articles. An automatic head was employed to feed the electrode. The voltage drops to zero as the drops of molten metal short-circuit the arc at more or less regular intervals. Whenever the drops short-circuit the arc the familiar momentary rise of current can be observed. However, even between drops the current and voltage fluctuate continually and never reach steady values. The voltage fluctuations are especially pronounced. Unlike the steady resistance load employed in taking the static volt-ampere curve, the resistance of the welding arc constantly undergoes rapid and wide variations.

When the metal is transferred across the arc in the form of fine pellets instead of drops, the periodic reduction in the arc voltage may not be so pronounced. But even in that case there is a noticeable variation in the arc resistance, as the stream of pellets is apparently not uniform but takes place in gushes. It has been generally assumed that with a coated weld rod the metal is transferred in a more finely divided state than when a bare rod is used.

However, the degree of comminution of the molten metal appears to depend on the magnitude of the current rather than the coating. When using a current which is large for the size of the rod the fluctuations of voltage and current are greatly minimized with either a coated or a bare weld rod. With the current densities usually employed, the voltage fluctuations are substantially as shown in figure 1 and they may be considered typical.

Figure 1. Oscillogram of arc current and voltage; approximately 200 amperes, 30 volts

The current fluctuations attendant upon the short circuiting of the arc have been subject to considerable discussion. Some maintain that they are detrimental in that they increase spatter and lower the deposition efficiency, and may even cause porosity. Others claim that they improve the penetration and are therefore de-

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1. For all numbered references, see list at end of paper.

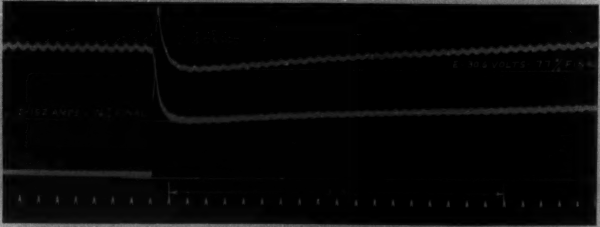

Figure 2. Oscillogram of current and voltage on short to load test on 200-ampere welding generator set at 200 amperes, 40 volts

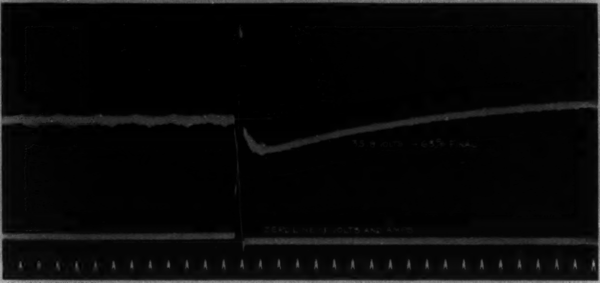

Figure 3 (below). Oscillogram of voltage on short to open circuit test on 200-ampere welding generator

sirable. Although it is improbable that the penetration is improved by these fluctuations, strange as it may seem, there is something to be said for each of these contradictory claims.

It should be understood that there are two sources of current fluctuation when the arc is short circuited. The current may increase because the permanent short-circuit current on the volt-ampere characteristic is considerably higher than the current at welding voltage. The current increase may also be produced by a transient overshoot, which will be large if the transient characteristic of the machine is poor. It is the former type of fluctuations which is the subject of this discussion, as no one will claim any advantage for the latter type.

It is not likely that any great benefit is derived from these fluctuations in producing better penetration. Since the first draft of this manuscript was submitted, an article has appeared which completely verifies this intuitive statement.[1] However, they do serve a useful function, which will be discussed later, but they must be kept within certain limits or they become harmful.

It is generally agreed that oscillograms of arc current and voltage when welding are useful in comparing the performance of two machines if they can be tested under identical conditions. However, they are not well adapted to serve as a basis for specifications to cover the transient performance of a welding generator. To answer this purpose the test must be of such nature that it can be duplicated anywhere under certain prescribed conditions.

Inadequacy of Present Specifications

It is natural that specifications which have so far been formulated to cover tests for transient performance of welding generators should simulate as far as possible the conditions prevailing in the arc. That is, oscillograms are taken of the current while the external resistance undergoes a prescribed variation. For example, the AIEE test of a welding generator for momentary current fluctuations and arc recovery is as follows.

Adjust generator on resistance load to obtain normal current at 25 volts. Short-circuit one half the resistance and allow current to settle. Then suddenly open the short circuit and reinsert the resistance in the circuit. The current momentarily drops below normal and the time of recovery is the time required for the current to return to within five per cent of its original value. The resistance variation prescribed by this specification is so narrow as to preclude its being of any use.

Another specification for arc recovery states that the machine should be loaded on a resistance to give 40 volts across the resistance and a current corresponding to the setting at which the test is made. The entire resistance is then short-circuited and the conditions allowed to become stable. The short circuit is then suddenly opened, reinserting the resistance. The momentary current drop should not fall below 70 per cent of normal and the current should be back to within five per cent of normal in not more than 0.3 second.

Figure 2 shows an oscillogram made according to this test on a 200-ampere machine at full load.

This test is an improvement on the AIEE specification in so far as the prescribed resistance variation is wide enough to produce an appreciable current fluctuation. However, the usual interpretation of this test leaves much to be desired. Experience has shown that a machine which meets this specification may be definitely inferior to one which does not, and vice versa. The requirement that the current should return to within five per cent of normal in not more than 0.3 second is a particularly valueless stipulation, inasmuch as the disturbances in the arc occur at much shorter intervals than 0.3 second. What happens on a test of this kind after, say 0.1 second, is of no interest. This test unquestionably has some merit, but it is felt that it may be interpreted in a manner to give it greater usefulness than has so far been the case.

In addition to specifications for current recovery, there is at present in existence a specification for voltage recovery. It states that when a machine is suddenly open circuited from short circuit the time of voltage recovery is the time required for the voltage to reach 50 per cent of normal open-circuit voltage. Figure 3 shows an oscillogram of voltage recovery from short circuit. The test itself is valuable and gives more information about the transient performance of the machine than the other tests discussed, but the definition of voltage recovery given above is absolutely valueless.

It will be observed that the voltage rises quickly to a high value, then drops to a comparatively low value, from which it gradually recovers to normal. The rapidity with which the first rise of voltage takes place depends entirely upon the rapidity of the break of the current. This is the voltage induced by the rapid change of current in an inductance and it will be instantaneous if no arc is formed at the break, or if means are employed to blow out the arc if one is formed. In general, this voltage rise reaches a maximum when the current reaches zero, and the time required to reach this maximum is of no value as a criterion of performance of the machine, but its magnitude is of great importance. The peak value of this first voltage rise should always be considerably higher than the normal open-circuit voltage, and if the time required to reach this peak is of no importance, the time required to reach 50 per cent of normal can certainly have no significance.

The minimum value to which the voltage drops after the first sudden rise is an indication of the building-up characteristic of the fields. The higher this value is maintained, the faster the fields build up to produce a sustained voltage. As before, the gradual increase in voltage after this minimum voltage is passed is of little significance. It is the magnitudes of the maximum and minimum voltages on this test that are of importance. They are indications of the machine's ability to react quickly to sudden current changes by virtue of an appreciable amount of inductance in the circuit, and also the ability of the field to build up rapidly enough to sustain the voltage after the quick change has taken place. It will be shown later how this test, taken in conjunction with the previously discussed load test, can be interpreted to give a clear picture of the entire transient performance.

A Prerequisite for Arc Stability

Experience gained from many years of operation of constant-potential welding generators furnishes a clue to the reason for failure of the current-recovery test described above to tell the whole story. On a test of this kind the constant potential machine, where the drooping volt-ampere curve is produced by inserting resistance in series with the arc, will show 100 per cent current recovery. It shows no overshoots or under-rides on the recovery test.

Likewise, on the voltage-recovery test, the constant potential machine will show instant recovery to full open-circuit voltage. This, of course, is evident. Yet, experience showed that a variable voltage machine in which the drooping volt-ampere characteristic is produced by magnetic means frequently performed better on actual welding, even though it has considerable overshoots and under-rides on the current-

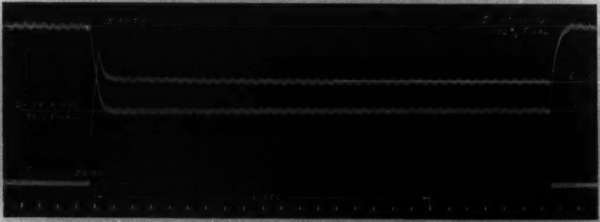

Figure 4 (above). Oscillogram of current and voltage on short to load test on 200-ampere welding generator set; 60 amperes, 40 volts

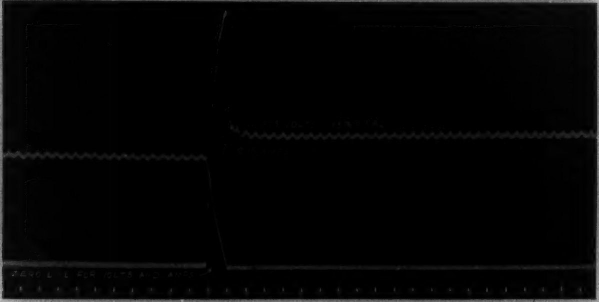

Figure 5. Oscillogram of voltage on short to open test on 200-ampere welding generator

recovery test. Furthermore, experience also showed that the operation of the constant potential machine was markedly improved by insertion of a certain amount of inductance in the circuit, especially if the volt-ampere characteristic is fairly flat.

Figure 4 shows an oscillogram of a current-recovery test on a 200-ampere machine with sufficient resistance in the circuit to give it the constant potential characteristic. However, it is also supplied with an inductive stabilizer. It will be observed that the current drops from the short-circuit current to normal with practically no dip. Without the inductive stabilizer in the circuit, substantially the same current-recovery curve would be obtained. However, the insertion of the inductance modifies the voltage curve very materially. Without the inductance the voltage would rise quickly to normal and stay there. With the inductance in the circuit the voltage rises rapidly to a value considerably above normal before settling down. This overshoot of voltage when there is a sudden reduction in current is apparently of great benefit in stabilizing the arc.

Similar phenomena will also be observed on the voltage recovery test on the same machine shown in figure 5. Without the inductance the voltage would rise to the normal open-circuit voltage without overshoot. With the inductance, a higher voltage peak is obtained.

These observations merely verify the familiar fact that the more electromagnetic energy there is stored in an electric circuit, the greater is the tendency to establish an arc when the circuit is broken. When a drop of metal forms a short circuit the arc is extinguished and the high instantaneous voltage induced when there is a sudden reduction in current is a direct measure of the tendency to re-establish the arc upon the reopening of the arc gap. The minimum sustained voltage is a measure of the tendency to maintain the arc once it has been established. The importance of the magnitudes of these voltages can, therefore, readily be appreciated.

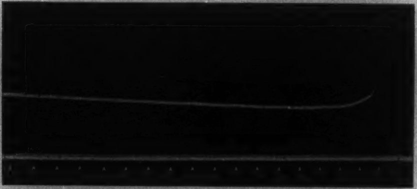

Figure 6. Oscillogram of voltage on short to load test on 400-ampere welding generator set at 400 amperes, 40 volts

Long experience with another type of equipment points in the same direction. In endeavoring to apply a-c current to arc welding it was soon discovered that a drooping volt-ampere characteristic produced by resistance in series with the arc, similar to the constant-potential d-c system, was of practically no value in bringing about arc stability. On the other hand, when the drooping volt-ampere characteristic was produced by reactance in either primary or secondary circuit, the arc stability was greatly enhanced.

With resistance in the circuit, the power factor is high, and the secondary voltage and current rise and fall practically in unison. When the current passes through the zero point the arc goes out and the voltage rise following an approximate sine wave is too slow to re-establish the arc, and it stays out. With reactance in the circuit the power factor on the input side is low and on the output side the voltage wave is distorted into an almost rectangular shape, frequently with a sharp peak on the leading side of the rectangle. This abrupt rise of voltage immediately after the current passes through the zero point is an essential requirement to quickly and surely reignite the arc. The inference from experience, therefore, is that whenever the arc has gone out for any reason, such as being short-circuited by a drop of metal in d-c welding or the current passing through zero in a-c welding, a rapid rise of voltage to a relatively high value, even if of short duration, is a necessary requisite for the reignition of the arc, and therefore highly desirable.

The theory of the arc, as it has been developed so far, appears to agree with experience. According to Doctor C. G. Suits,[2] the resistance of the arc depends on the state of ionization of the gaseous medium, which in turn depends on the ionizing voltage and the temperature. Furthermore, the ionization follows a variation in the temperature almost without time lag, the period of readjustment being of the order of 0.001 second. The short circuiting of the arc by a drop of metal and the consequent momentary extinction of the arc is unquestionably followed by a drop in temperature of the surrounding gases.

The initial resistance of the arc upon the reopening of the arc gap as the drop leaves the electrode is, therefore, considerably higher than the average arc resistance. A rapid and relatively high voltage rise at that instant is then effective in reducing the initial resistance by rapidly increasing the state of ionization of the arc gases. The failure of the current-recovery test, as interpreted at present, to give a correct picture of the transient performance, is undoubtedly due to the fact that it does not take into account the variation in arc resistance with temperature and applied voltage. To give this test the maximum of usefulness, the voltage fluctuations which take place when the test is made should also be taken in consideration.

Transient or Dynamic Volt-Ampere Curves

As the maximum and minimum voltages induced when there is a sudden interruption of the circuit, or a sudden insertion of a resistance in the circuit, are determining factors in producing arc stability during welding, these values plotted in the form of curves similar to the static volt-ampere curves should prove of interest. The open-circuit points for these curves are obtained from the voltage-recovery test already described. The load points are obtained by tests similar to the current-recovery tests, except that oscillograph records are made of the voltage only. The machine is loaded on resistance, which gives a point on the static volt-ampere curve. The resistance is then short-circuited and conditions allowed to become stable. The resistance is then suddenly opened and a record made of the maximum and minimum voltages, as shown in figure 6. In making this test the film should travel fast to obtain a distinct maximum voltage point, also the short should be interrupted quickly. If necessary, a blowout of the arc should be provided. The resistance is then varied in suitable steps and the test repeated.

Figure 7 shows the static and transient volt-ampere curves for approximately full load setting of a 400-ampere welder. The curves give a clear picture of what happens when a resistance is suddenly inserted from a short circuit. While the current drops quickly from the steady short-circuit value, the voltage rises quickly along curve 2, reaching point *A* approximately at the time the current has dropped back to normal. From this point on the current drops slowly. The voltage at first drops quickly along *ab* until the current and voltage simultaneously reach their minimum values at *B*. From there on they increase rather slowly along *bc* to *C*. As pointed out, this gradual approach back to normal is of no significance. The point *B* can readily be determined from the fact that the percentage reduction in current is substantially equal to the percentage in voltage below normal. This eliminates the necessity for recording the current with the oscillograph and simplifies the operation considerably.

Curve 1—Transient volt-ampere curve of momentary short-circuit current values

Curve 2—Volt-ampere curve of transient peak voltages obtained during change from short-circuit condition to a resistance load condition

Curve 3—Volt-ampere curve of static or steady-state conditions obtained by various resistance loads

Curve 4—Volt-ampere curve of minimum voltage obtained when changing from short-circuit condition to a given resistance load condition

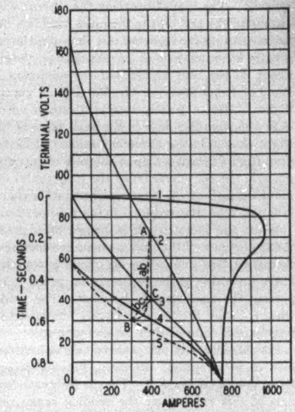

Figure 7. Transient and static (steady-state) volt-ampere characteristics of 400-ampere welding generator

It can readily be seen that the steady short-circuit current must be appreciably higher than the welding current in order to induce a high substantially instantaneous voltage when a resistance is suddenly inserted in the circuit. On the other hand, it is desirable to keep the current fluctuation from becoming excessive when a sudden short takes place. Recent tests have shown that large short-circuit currents tend to produce weld metal of large grain size with poor impact value and greatly increased spatter loss. For convenience, the record of the current when the machine is suddenly shorted from open circuit may be plotted on the same sheet as shown by curve 1. The overshoot is in this case higher than it would be if the short was made from a resistance load at lower voltage, but it represents the possible maximum current fluctuation and is, therefore sufficient.

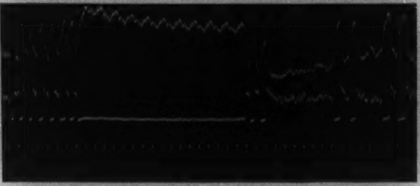

Figure 8. Oscillogram of arc current and voltage when welding with three-sixteenths-inch bare electrode. Approximately 160 amperes, 20 volts

The degrees of steepness of curves 2 and 5 are the important factors in determining arc stability. Of course, the degree of steepness has no meaning unless the scales of co-ordinates are agreed upon. However, the ratio of volts to excess of short-circuit current over normal current may be taken as the measure of steepness. For example, the curves in figure 7 were taken at 400 amperes, 40-volt setting. At 400 amperes curve 2 shows a voltage of 72 volts; 72 divided by (760 − 400 = 360 amperes) = 0.2. In the same manner the steepness factor of curve 5 is 0.0695.

A moderately steep static volt-ampere curve together with a much steeper curve 2 produces a very desirable characteristic. The steepness of curve 2 is very effective in producing arc stability provided the arc length is kept within certain limits. At the same time, the moderate steepness of curve 3 precludes an excessively long arc. In many applications the operators have to break the arc frequently and they prefer a clean break when the arc is drawn beyond a certain length, but they also desire the arc to hang on tenaciously within that length. A wide spread between curves 2 and 3 is conducive to this desirable feature.

It is interesting to observe the voltage peaks resulting from a rapidly decreasing current when welding with a machine with good transient characteristics. Figure 8 shows these peaks distinctly and is especially interesting as the operator happened to short-circuit the arc when the film was taken. The sharp voltage rise after each short circuit as the drops go across is indeed the most important criterion in judging the transient performance of a welding generator from an oscillogram taken while welding.

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Discussion

R. C. Freeman (nonmember; General Electric Company, Schenectady, N. Y.): The article shows two important features of the voltage-recovery characteristics of a d-c power source for arc welding, graphically represented on the basis of the same voltage ordinates and current abscissas as are used for the static volt-ampere curves.

The prime importance of various recovery characteristics has long been recognized by those acquainted with the design and operation of arc-welding generators. There has been much discussion as to the relative importance of the different features of voltage and current recovery under variously specified conditions. From the extensive experience of many engineers there has developed the consensus that peak-voltage recovery, or certainly the constants indicated by the presence of such a voltage surge, and minimum voltage recovery are two of the most significant phases of the transient phenomena.

The transient characteristics of a welding generator as graphically represented in figure 7 of the article, would become more meaningful if the time element were evaluated. This is particularly true in curve 2 of figure 7 which is a plot of transient peak-voltage values. Since this peak voltage is an electromotive force induced by the rapid change of current when a short circuit of the generator is completely opened or opened across some predetermined resistance, this value then is a function not only of the constants of the welding generator but also of the speed of opening of the short circuit. Significant though such peak values are, they do not lend themselves to a simple and accurate quantitative determination of the welding-generator constants which are necessary for the maintenance of a good welding arc.

The author minimizes the significance of the time element in the ultimate recovery and current to steady-state values. Although the magnitude of minimum voltage recovery is of much more importance in the maintenance of an arc, yet it is also true that the time required for the current in the arc to recover to a normal value (that is, the time required for the transition from point *B* on curve 5 to point *C* on curve 3 of figure 7 of the article) is worthy of consideration as a means of obtaining more uniform welding performance. It is noteworthy that in the ultimate recovery of current and voltage (to point *C*, figure 7) from the minimum value (point *B*, figure 7) there is an increase of 75 per cent in the watts of electrical energy delivered to the arc. Obviously it is advantageous to complete a transition of such proportion in a minimum period of time.

In a time where there are so many conclusions drawn and claims made regarding welding performances which are based upon only a superficial consideration of static volt-ampere curves, this article stands for a true analysis of arc welding performance based upon the all-important consideration of transient characteristics.

K. L. Hansen: The maximum and minimum voltages following the sudden insertion of a resistance in the circuit are apparently recognized as the most important criteria in judging the welding performance of a generator. However, Mr. Freeman is skeptical about the ability to determine the peak voltages readily and with a reasonable degree of accuracy. He is of the opinion that the magnitude of the voltage peak is materially influenced by the time element involved in breaking the circuit.

It is true that the time required for the voltage to reach its maximum value on the short-circuit-to-open-circuit test is directly dependent on the rapidity of the break. In general, the voltage reaches the peak when the current has dropped to zero, as stated in the paper. But, strange as it may seem, the magnitude of the voltage peak is by no means inversely proportional to the time required for the current to reach zero, although it is considered to be induced principally by this rate of current change. This point was brought out rather forcibly by a test made some years ago.

In discussing my paper ("Recent Developments in Design of Arc Welding Generators," AIEE Transactions, June 1932, page 576), Mr. Ver Planck contended that a "slow" break would assist the generator in maintaining a high minimum sustained voltage in so far as it gave the generator fields more time to build up. He compared the oscillogram of a recovery test shown in my paper with one taken on a Bergman machine and found that the short circuit was interrupted six times more slowly on the Hansen machine. This, Mr. Ver Planck claimed, assisted the Hansen machine in showing a high "minimum recovery" voltage.

This point was carefully checked by repeating the test on the Hansen machine and taking special precaution to break the arc quickly. The rate of breaking the current was speeded up to compare with the test on the Bergman machine, that is, about six times faster than on the recovery test shown in the paper. In spite of the greatly increased rate of current change, the peak voltage was only about 30 per cent higher and the minimum recovery voltage, instead of being lower as Mr. Ver Planck expected, was a few volts higher. Here a greatly increased rate of current change produced only a moderate change in the peak value of the voltage and a very slight change in the minimum value. It is their comparative freedom from dependence upon the rate of interruption that led me to propose the consideration of the magnitudes of the peak and minimum voltages on the recovery tests rather than the ridiculous stipulation of "the time required to reach 50 per cent of normal voltage."

When the short circuit is opened on a predetermined resistance the importance of the time element in affecting the magnitude of the voltages is less than when the circuit is completely opened, and the lower the value of the resistance on which it is opened the less important the time element becomes. The tendency to form an arc when a circuit is opened is greatly minimized when a circuit of low resistance is connected in parallel with it.

Possible errors in the peak-voltage values due to variation in the rate of interruption, therefore, become progressively smaller for lower points on curve 2. That is fortunate because it is the steepness of this curve over the welding range that is of importance. What happens as the curve approaches the open-circuit point is not of much consequence. The fact that the peak values come out on a smooth curve is an indication that they cannot be very erratic. It is certainly easier to determine the voltage peaks than the gradual approach to within five per cent of normal current on the present current recovery test.

Mr. Freeman's second criticism that little consideration is given in the paper to the time of current recovery deserves to be commented on. Mr. Freeman states that there is an increase of 75 per cent in the energy delivered to the arc as the current and voltage pass from their minimum values to normal.

This statement is evidently based on the assumption that the behavior of the current and voltage is the same when a fixed resistance is inserted in the circuit as it is when the arc gap is opened due to a drop leaving the electrode. This, however, is far from being the case, as pointed out in the paper. A close study of many oscillograms taken during actual welding operations has not disclosed any tendency for the current and voltage to start at low values and gradually approach normal after recovering from a short circuit caused by a drop. Even when there is no drop passing across for as many as 10 to 12 cycles there is no evidence of such a tendency. The reason probably is that the conductance of the arc is itself a function of the applied voltage. A high peak value of voltage following immediately upon the opening of the short circuit appears to establish normal current flow quicker in the arc than it does in a fixed resistance. This is the reason that so little consideration is given in the paper to the time of current recovery when it is taken on a resistance.

Co-ordination of Power Transformers for Steep-Front Impulse Waves

By V. M. MONTSINGER
FELLOW AIEE

Introduction

THE REQUIREMENT that transformers withstand certain specified impulse tests has been accepted by the industry as desirable, and the Institute has recommended a standard method of testing transformers with a 1.5 x 40 microsecond wave.

These tests, however, do not demonstrate that transformers can withstand steep-front waves when the voltage is limited by flashover of their bushings or of rod gaps that give ample protection for long waves. The reason, of course, for this is that the flashover voltage of an air gap increases faster than the breakdown voltage of insulation as the time (to flashover or breakdown) gets shorter. This means that the margin of safety of the transformer, when protected by an air gap, decreases as the impulse wave front becomes steeper and for very steep fronts the margin may disappear. It is now generally recognized that to protect a transformer by an air gap against steep front waves, the spacing of the gap must be much lower than that necessary to protect against long waves.

Furthermore, in cases where transformer windings have an inherently poor initial impulse voltage distribution at the line end, the steep-front waves produce steeper gradients (in per cent of the applied wave) than are produced by long waves with the result that breakdown may occur between the turns and coils, before the major insulation fails. This is particularly true of the higher-voltage unshielded windings. In such cases shorter air gaps may be required to protect the line end coils than are required to protect the major insulation.

Though greatly improved in impulse strength, within the past few years, it has to be admitted that modern high voltage transformers cannot withstand the kilovolts associated with excessive steep-front lightning waves resulting from direct strokes on or near its terminals. The major insulation would fail. Transformers, of course, could be built to withstand direct strokes but this cannot be economically justified. Extremely steep-front waves should, of course, be kept out of the station by suitable means and suitable protective devices used for protection against dangerous incoming traveling waves.

Finally, to co-ordinate or protect transformers intelligently for all kinds of impulse waves requires an accurate knowledge of the volt-time characteristics of transformer insulations. Data of this kind can be determined only by careful laboratory investigations of typical transformer insulations. Such investigations have been under way during the past few years.

Purposes of Paper

The purpose of this paper is to report the results of these laboratory tests made to determine the impulse volt-time curve characteristics of transformer insulations, based upon the effect of both single and repeated applications of impulse voltages on both the corona injury level and the breakdown. Front-of-wave testing of transformers is also discussed.

A. Volt-Time Characteristics of Transformer Insulations

Considerable data have already been published[1-4] on volt-time curves of solid insulation, on transformer oil and on solid insulation in series with oil (representing the major insulation in a transformer). Most of these curves were based upon breakdown caused by a single application of voltage for the various "times to breakdown." Unlike an air gap, insulation has the characteristic of being weakened by repeated applications of voltage above the injury level. Therefore, in comparing the volt-time curve of transformer windings with air gaps, it is quite necessary to use insulation curves based upon repeated voltage applications for the reason that we have no assurance that a transformer will not be subjected to more than one lightning stroke. In fact, it has been shown[5] that in a (supposedly) single stroke a large number of impulses may

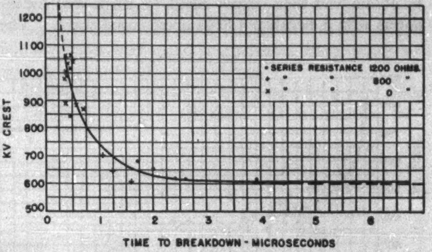

Figure 1. Single-shot volt-time curve of barrier A (36 inches by 48 inches). One-quarter inch oil plus one-eighth inch P. B. plus one-quarter inch oil plus one-eighth inch P. B. plus one-quarter inch oil—total thickness one inch, four-inch-diameter square-edged electrodes, 1.5 x 40 negative wave

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1. For all numbered references, see list at end of paper.

follow each other in rapid succession. Another reason why co-ordination and protection should be based upon the "repeated impulse wave" volt-time curves of insulation is that the application of a single wave having an amplitude higher than the repeated wave strength will cause injury even though failure does not occur.

Insulation Barriers

Tests were made on four different barriers designated as *A*, *B*, *C*, and *D*, designed to simulate the major insulation in transformers. Barriers A and B in all cases had bare electrodes. Barriers *C* and *D* had insulated line (generator-side) electrodes and a flat metal sheet for ground electrodes.

Tests on Barriers *A* and *B*

Barrier *A* consisted of one-fourth inch oil, one-eighth inch pressboard, one-fourth inch oil, one-eighth inch pressboard, and one-fourth inch oil—total thickness of barrier of one inch. Barrier *B* was the same as *A* except that it consisted of one-eighth inch oil ducts and one-sixteenth inch pressboard sheets—total thickness one-half inch. In all cases the oil ducts were maintained by two spacing strips one-half inch in width under the electrodes.

(*1*). *Single-Shot Breakdown Volt-Time Curves.* Figures 1 and 2 give the results of tests made on barriers *A* and *B* to obtain volt-time curves based on single-shot breakdowns with a negative 1.5x40 microsecond wave. The line and ground electrodes were four inch diameter disks with square edges.

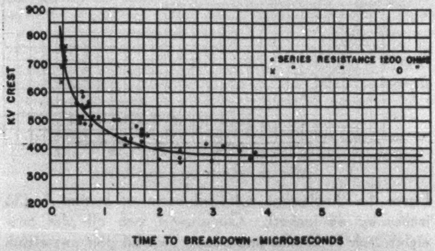


Figure 2. Single-shot volt-time curve of barrier B (36 inches by 48 inches). One-eighth inch oil plus one-sixteenth inch P. B. plus one-eighth inch oil plus one-sixteenth inch P. B. plus one-eighth inch oil—total thickness one-half inch, four inch diameter square-edged elecrodes, 1.5x40 negative wave

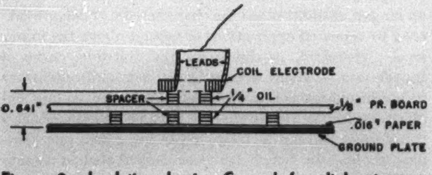
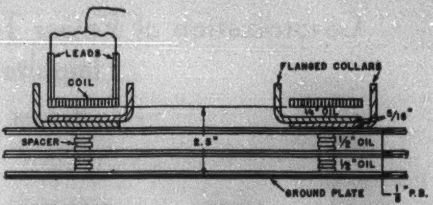


Figure 3. Insulation barrier C used for dielectric tests



BARRIER PARTS	BARRIER C	BARRIER D
COIL CONDUCTOR	.300 X .120"	.300 X .100"
COIL INS. (ONE SIDE)	.006" PAPER .003" COTTON	.138" PAPER .003" COTTON
COIL DIAMETER (INSIDE)	4.0"	9.25"
COIL RADIAL BUILD	1.875"	1.875"
SIZE OF PRESSBOARD	36 X 48"	36 X 48"
SIZE OF GROUND PLATE	36 X 48"	36 X 48"

Figure 4. Insulation barrier D used for dielectric tests

Expressed in percentages of kilovolts versus time to breakdown the curves in figures 1 and 2 give the values shown in table I.

(*2*). *Life Tests With Full Waves.* To obtain the effect of repeated shots on the breakdown strength as compared with the single-shot strength with a 1.5 x 40 wave, several thousand shots (covering a period of about one year) were made on barriers of type *A*. Three shapes of line electrodes were tried out, these being (1) four-inch-diameter square edge; (2) rectangular U-shaped rods; and (3) one inch round U-shaped rods. The rectangular and round rods were well rounded at the bends. The ground electrode was a metal sheet 36 inches by 48 inches.

The only definite conclusions that could be drawn from these life tests were, first, that on repeated shots the "wait" between shots apparently had no effect on the final breakdown value. In other words, there appears to be no heating effect with impulse voltages as occurs with low frequency voltage.[6]

Second, the breakdown value was reduced (below the single-shot value) quite rapidly on the first few shots, approximately 50 per cent of the reduction occurring in the first ten shots and then decreasing to the final or minimum breakdown value in the next 25 or 50 shots.

No dependable curve of breakdown versus number of shots could be obtained for the reason that injurious corona in some cases started as low as 60 to 65 per cent of the single-shot breakdown value, but failure did not usually occur (under many shots) until approximately 90 per cent of the single-shot breakdown value was reached. Obviously, when the insulation was tracked near the line electrode at 65 to 75 per cent voltage the final breakdown value of approximately 90 per cent voltage did not mean anything. Apparently dependable life tests cannot be obtained with bare line electrodes. This, of course, is not a practical condition as all transformer conductors are insulated.

Tests on Barriers *C* and *D*

At this point it was decided to use insulated line electrodes or small coils with barriers, designated previously as *C* and *D*.

Table I. Impulse Volt-Time Values by Test

Time to Breakdown (Microseconds)	Kilovolts in Per Cent of Full-Wave Kilovolts
0.25	200
0.5	150
1.0	125
1.5	113
2.0	105
Full wave	100

Barrier *C* consisted of a small coil as the line electrode and a plane as the ground electrode, the two electrodes being separated 0.641 inch consisting of one-fourth inch oil, one-eighth inch pressboard, one-fourth inch oil, and a 0.016 inch sheet of paper adjacent to the ground plate. See figure 3. The turns in the coil consisted of 0.300-inch by 0.120-inch copper strands insulated with 0.006 inch paper (one side thickness) and 0.003 inch cotton thread, the cotton serving merely as a mechanical protection. This barrier represents the type of major insulation of approximately a 15-kv power transformer.

Barrier *D* consisted of a small coil (more heavily insulated than the one in barrier *C*), a one-fourth inch oil duct adjacent to the coil, two five-sixteenth inch flanged collars, (5) one-eighth inch pressboard sheets, and two one-half inch oil ducts—total thickness two and one-half inch—as shown in figure 4. The turns in the coil consisted of 0.300 inch by 0.100 inch copper strands insulated with 0.138 inch paper (one side thickness) and 0.003 inch cotton thread. This barrier represents the type of major insulation of a power transformer of approximately 92-kv rating.

Figure 5 shows the impulse generator and circuit connections used.

Twenty or more *C* barriers were tested—some on life tests of several hundred shots—before consistent results could be obtained. The difficulty was found to be due to small particles of air entrapped in the coil insulation. The presence of air was very misleading both in the beginning of corona and in the breakdown strength. Quite often, on repeated shots the first appearance of corona gradually disappeared. This process might occur several times as the voltage was increased. During this time the air was eliminated.

As it was not practicable to eliminate the air in the manner usually employed in transformers (by short-circuit heat runs, or evacuation) before starting the tests, it was necessary to assemble the barrier under oil. The coils were dried, oil impregnated, and kept under oil at least 24 hours previous to use and not allowed to come in contact with air during the assembly process. The pressboard, paper sheets, flanged collars, and spacers were all vacuum dried and oil impregnated, and assembled under oil. Approximately 6,000 shots were made on barrier *C* alone after the entrapped air problem was solved. Approximately 4,600 shots were made on barrier *D*. Altogether during the entire investigation some 15,000 to 20,000 impulse tests were made, ranging over a period of approximately two years' time. The investigation proved to be one of the most difficult ones ever followed by the author.

The results of the tests made on barriers *C* and *D* are shown in curve form in figures 6, 7, and 8. The full-wave points are shown at from four to five microseconds' time, regardless of the exact point on the wave at which either corona, injury, or breakdown occurred. The corona no doubt occurred near the crest of the wave (approximately 1.5 microseconds). As has been pointed out, breakdown of insulation barriers also usually occurs near the crest of the wave, though in some cases it may be somewhat beyond the crest—perhaps up to two or three microseconds for a 1.5-microsecond front. Each point represents anywhere from 20 to 100 shots and in some cases several hundred shots were made in graduated steps before corona, injury, or breakdown occurred.

Table II. Impulse Ratios

Negative Full-Wave Impulses	Impulse Ratios for Barriers	
	C	D
1. Single-shot breakdown to one-minute 60-cycle breakdown	$\left(\frac{320}{155}\right) = 2.07$	
2. Repeated shot breakdown to one-minute 60-cycle breakdown	$\left(\frac{280}{155}\right) = 1.8$	$\left(\frac{780}{300}\right) = 2.6$
3. Injurious corona impulse to injurious corona 60 cycle	$\left(\frac{240}{137}\right) = 1.75$	$\left(\frac{680}{275}\right) = 2.48$
4. Injurious corona impulse to one-minute 60-cycle breakdown	$\left(\frac{240}{155}\right) = 1.55$	$\left(\frac{680}{300}\right) = 2.27$

All front-of-wave tests, with one exception, were made by chopping the wave with either 100-centimeter or 200-centimeter spheres. The front of the wave chopped with spheres had no bend-over, as often occurs when chopped with a rod gap. No gap was used when obtaining the single-shot breakdown volt-time curve shown in figure 6, the applied wave being raised on a new barrier after each breakdown. The time and kilovolt values at breakdown were obtained by cathode-ray oscillograms.

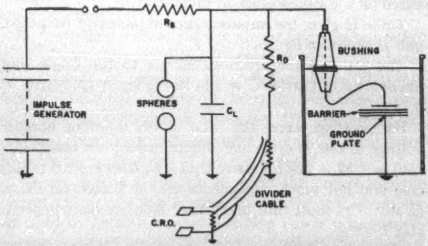


Figure 5. Impulse generator and circuit connections for testing insulation barriers

The curves shown by dashes in figures 6, 7, and 8 and

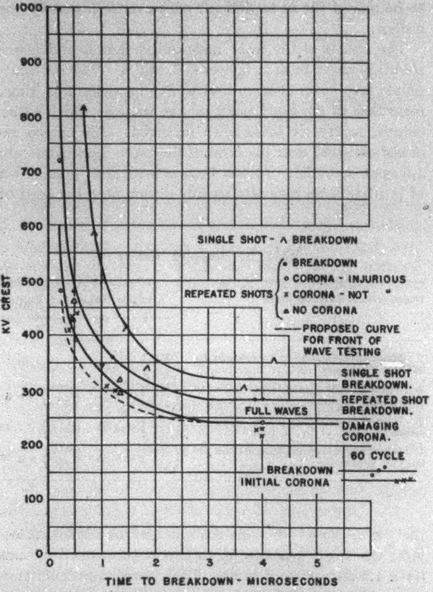


Figure 6. Volt-time curves of transformer insulations; impulse tests on 0.64 inch barrier C; negative waves; 1.5 x 40 used on full-wave tests

marked "Proposed curve for front-of-wave testing" are based on the values given in table I.

Table II gives the impulse ratios obtained on barriers *C* and *D* shown in figures 6 and 7.

The ratio of the injurious corona to the single shot breakdown of barrier *C* was in the order of (240/320) = 0.75.

For positive waves the ratio of the injurious impulse voltage to the 60-cycle one-minute breakdown was (235/155) = 1.52. The full-wave data are rather limited in this case and not much dependence can be placed on them. (Note: The short-time breakdown data are more dependable.)

The results of this investigation indicate that there is no definite impulse ratio that applies to all voltage ratings of transformers.

B. Testing Transformers on Front of Wave

If front-of-wave testing is to be standardized, careful consideration must be given to the rate of voltage rise to be used. Just how steep a wave front should be for front-of-wave testing is an open question. It is not felt that enough is known at the present time of the steepness of wave fronts that occur under service conditions and especially whether or not the same steepness of fronts occur on all rated voltage circuits to make any definite recommendations at this time. The suggestion, however, has been made that a wave whose front rises at the rate of 1,000 kv per microsecond be used for front-of-wave testing of transformers of all voltage ratings. In fact, several high-voltage transformers have recently been tested with this wave by flashing over a rod gap having a spacing in the order of 70 per cent of the (previously standard) test gap.[7]

The present knowledge of direct strokes indicates that the rate of voltage rise is not constant over the whole front of the wave, being lower at the initiation of the wave. Since the same kind of lightning waves will strike both low- and high-voltage circuits, the rate of rise to flashover of the line or station insulation would be less for low-voltage circuits. If 1,000 kv per microsecond is correct for high-voltage circuits it may be only 500 kv per microsecond for low-voltage circuits.

Service records on old transformers protected only by gaps coupled with experience in making factory impulse tests (using 1,000 kv per microsecond wave fronts) on transformers of relatively high-voltage ratings and of similar construction and insulation levels, indicate that seldom are wave fronts as steep as 1,000 kv per microsecond imposed on transformers in service. In other words, based on laboratory tests many of these old transformers would fail if subjected to 1,000 kv per microsecond wave fronts. This absence of failures might be accounted for by the fact that the wave fronts are usually sloped off to less than 1,000 kv per microsecond by attenuation or by the station capacitance—of bus bars, bushings (for both circuit breakers and transformers) and of the

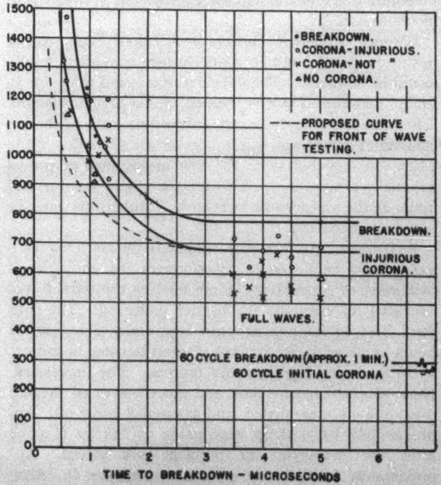


Figure 7. Volt-time curves of transformer insulations; impulse life tests on 2.5 inch barrier D; negative waves; 1.5x40 used on full-wave tests

transformer windings. This indicates that 1,000 kv per microsecond rate of voltage rise might be too high for front-of-wave testing, especially for transformers of the lower voltage ratings.

Front-of-wave testing can be specified in either of two ways; first, by kilovolts and time; second, by inches of rod gap and rate of voltage rise. In either case the kilovolts imposed on transformers in front-of-wave testing should be governed by the volt-time curve of transformer windings if the same factor of safety is to be maintained for full waves, chopped waves, and front of waves. Until the EEI-NEMA subcommittee on correlation of laboratory data establishes volt-time curves for gaps, insulators, etc., it appears that some front-of-wave tests will be specified by gap spacings. The preferable method, however, is to specify the tests by kilovolts and time to flashover of a gap.

Specifying front-of-wave testing by inches of rod gaps is more involved than specifying the test by kilovolt values for the reason that the proper gap spacing depends upon the way the rate of rise is defined. There are two ways of defining the rate of rise; namely,

First, the rate of rise is determined by the slope of a straight line drawn through the 10 and 90 per cent voltage points. This is usually termed the "effective" rate of rise.

Second, the rate of rise corresponds to the slope of a straight line drawn from virtual zero to the time of actual flashover. This is termed the "average" rate of rise.

NOTE: In either case "time" starts from "virtual time zero" which is determined by drawing a straight line through the 10 and 90 per cent voltage points. The intersection of this line with the time axis (zero voltage line) is the virtual time zero.

If there is no bend-over just before flashover, both the effective and average rates of rise are, of course, the same.

However, when using a rod gap to chop the wave front, there may be an appreciable bend-over just before flashover takes place (due to heavy corona current drawn by rod gap), in which case the effective rate of rise will be greater than the average rate of rise. This is illustrated in figure 9. Merely applying a higher voltage will not entirely eliminate the bend-over.

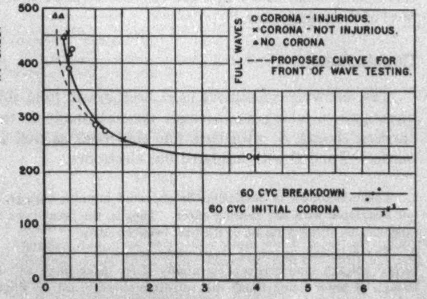


Figure 8. Volt-time curves of transformer insulations; impulse life tests on 0.64 inch barrier C; positive waves; 1.5x40 for full-wave tests

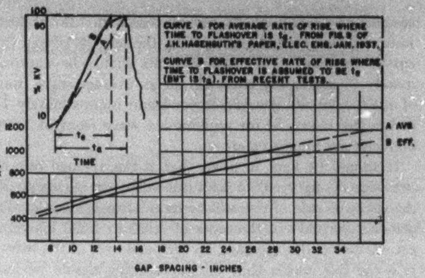


Figure 9. Impulse flashover of rod gaps, 1,000 kilovolts per microsecond; negative wave fronts

The remainder of the discussion of this subject will be based, merely for illustrative purposes, upon testing transformers with a 1,000 kilovolts per microsecond wave front. It should be understood, however, that this does not mean that a 1,000 kilovolts per microsecond wave is advocated for front-of-wave testing for the reasons previously mentioned. If some other rate is used where the test is specified by gaps, the same method should be used to determine the proper gap spacing.

The impulse flashover of rod gaps at 1,000 kilovolts per microsecond wave front is shown in figure 9, where curve *A* is based on the average front and curve *B* on the effective front.

Curve *A* is based on three points for 10-, 20-, and 30-inch gap spacings taken from figure 9 of reference 8. Curve *B* is based on two points for 10-inch and 24-inch rod gaps recently taken in the Pittsfield works high-voltage laboratory.

As previously pointed out the proper kilovolts to use for front-of-wave testing of transformers with standard insulation should be based on the volt-time curve of transformer insulation. Figure 10 shows the volt-time curves of transformer insulation based on (1) breakdown by single application of voltage, curve *A*, and (2) breakdown by repeated applications of voltage, curve *B*.

We are now in a position to select the proper kilovolt values and proper gap spacings (when the test levels are specified by gaps) for impulse testing power transformers having standard insulation levels and where the specified wave front is 1,000 kilovolts per microsecond and of negative polarity.

If the time of gap flashover is 0.6-microsecond (determined by cut-and-try method) the impulse kilovolts by curve *A*, figure 10, should be 1.44 times the long (chopped) wave (chopped) test kilovolts.

Table II of article "Insulation Strength of Transformers"[9] gives the chopped wave test as 405 kv for a transformer of 69-kv rating.

Therefore, the front-of-wave test voltage for a 69-kv transformer should be 405 × 1.44 = 583 kv which flashes over in 0.583 microsecond (or approximately 0.6 microsecond) at 1,000 kilovolts per microsecond. If the tests were specified by rod-gap spacing, the lengths of gap (by curves in figure 9) for 583 kilovolts should be approximately 11 inches (by curve *A*) for an average wave front of 1,000 kilovolts per microsecond and approximately 13 inches (by curve *B*) for an effective wave front of 1,000 kilovolts per microsecond.

It should be kept in mind that if the average rate of rise (line *A*, figure 9) is 1,000 kilovolts per microsecond, the effective rate of rise will be more than 1,000 kilovolts per microsecond—in the order of 1,200 to 1,500 kilovolts per microsecond, depending on the bend-over of the wave. It is for this reason that the gap spacing should be less when using average rate of rise.

In a similar manner the test-gap spacings were determined for 92-, 115-, and 138-kv transformers having standard insulation levels. The results are given in table III.

The above shows that the test-gap spacing for front-of-wave testing will depend not only upon the insulation level but also upon the method of defining the rate of voltage rise. It should be kept in mind that while the gap spacings are different in the last two columns, they represent the same flashover kv values. Further laboratory data are needed on 1,000 kilovolts per microsecond flashover of rod gaps below ten inch and above 30 inch spacings before working out gap spacings for other circuit kilovolt ratings, especially ratings above 138 kv.

Table III. Rod Gap Spacings for Front-of-Wave Tests of Power Transformers

Rated Circuit Kv	AIEE Recom. Chopped Wave Kv	Multi-plying Factor	Kv	Front-of-Negative-Wave Tests: Approx. Time to F. O. μ sec	Gap Spacing—Inches for 1,000 Kv per μ sec: Eff. Front	Gap Spacing—Inches for 1,000 Kv per μ sec: Avg. Front
69	405 ×	1.44 =	583	0.6	13″	11″
92	515 ×	1.38 =	710	0.7	18	15.5
115	625 ×	1.33 =	830	0.83	23.5	20
138	750 ×	1.27 =	950	0.95	30.0	25.5

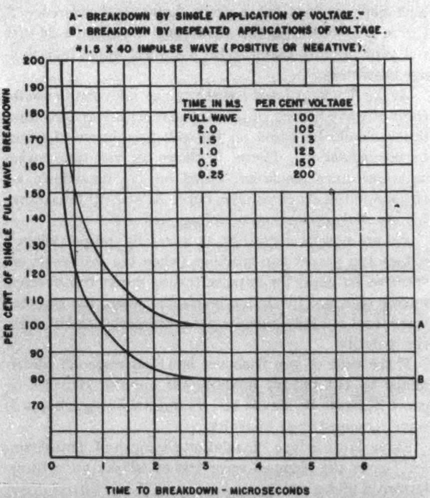


Figure 10. Volt-time curves of insulation, consisting of solid insulation and oil in series (simulating major insulation in a power transformer)

Furthermore, as the curves in figure 9 are based on limited data from only one laboratory, the spacings shown in table III are subject to change when final volt-time curves for rod gaps are agreed upon by the EEI-NEMA subcommittee on correlation of laboratory data. The main purpose in presenting this data is to show the proper method of deriving gap spacings for front-of-wave testing.

Protecting Old Power Transformers Against Steep-Front Waves by Rod Gaps

It should be realized that the bend-up of volt-time curves of insulation shown in figure 10 does not necessarily apply to old unshielded transformers where the impulse stresses between turns and between coils are much higher for steep-front waves than for long waves of the same applied kilovolts. This is true mostly for the higher-voltage ratings where the windings were not shielded to prevent excessive voltage stresses between turns and coils when subjected to steep waves.

In other words, while figure 10 shows the shape of volt-time curves for the major insulation for both old and new transformers it does not necessarily represent the shape of the volt-time curve for turn to turn and coil to coil insulation for all transformers in service. In some cases of old transformers it may not be possible to permit any turn-up in the volt-time curve. It should be understood, however, that this reduced turn-up is not due in any way to the characteristics of the insulation itself but rather to the increased turn and coil voltage stresses (as expressed by percentage of the applied wave) resulting from steep waves. In all cases the flat part of the curve, figure 10, should correspond to the safe long wave (1.5 × 40) impulse test level of the transformer.

Conclusions

The following conclusions have been drawn from this investigation, taking into account the tests made on the barriers, *A* and *B*, with bare line electrodes as well as barriers *C* and *D* with insulated line electrodes.

1. The breakdown of insulation by repeated impulse voltage is not associated with any heating effect. That is, the breakdown is independent of the duration of the wait between shots.

2. The volt-time curve of transformer insulations based on repeated voltage applications is essentially of the same shape as (although of lower value than) the volt-time curve based on single voltage applications.

3. There is no polarity effect on either the beginning of corona or the breakdown of insulation. (Note: If any polarity effect exists for insulation it should have shown up in these tests made with a small coil as one electrode and a plane 36 inches by 46 inches as the other electrode.)

4. Streamer corona damages insulation, while glow or brush discharge corona under conditions within the limits of this investigation (impulse of short time duration compared to 60 cycles) may not cause any measurable damage. There is no definite voltage at which corona of the brush discharge type first occurs on a given barrier.

5. The corona band on the edges (either inside or outside) of the coils with turn insulation was generally wider on full waves than on steep-front waves. In some cases on life tests there was no visible corona on the coil with steep waves until puncture occurred. The kv at which corona appeared on different coils, however, varied to some extent for steep waves.

6. With bare line (generator-side) electrodes the ratio between the repeated shot injurious corona and the repeated shot breakdown (reached by graduated voltage steps of 100 shots at each step) varies over quite a range. In some cases the repeated shot injurious corona was as low as 65 per cent of the single shot breakdown while the breakdown with 100 or more shots was as high as 90 per cent of the single-shot breakdown.

7. With an insulated line electrode the injurious corona on full waves generally ranges from approximately 75 per cent to 85 per cent of the single shot breakdown although in some few cases it was as high as 90 per cent. While the repeated-shot strength (100 shots or more) may be higher than the injurious corona point, the volt-time curves of repeated shot strengths of insulation should be based on the injurious corona level since failure would eventually take place if enough shots were applied. A safe over-all average value to use appears to be about 80 per cent.

8. When front-of-wave testing of transformers is desired it is preferable that the test be specified by kilovolts and time—the values being based on the volt-time curve of transformer insulation, figure 10. If the larger gap spacings (last two columns of Table III) are specified the effective rate of rise should be used.

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Discussion

J. E. Clem (General Electric Company, Schenectady, N. Y.): Mr. Montsinger has given us another paper which should be useful in improving the design of transformers in regard to their lightning-strength characteristics. This discussion will suggest an application of the material in the paper.

It is interesting to note that the data in table I can be closely approximated by a simple empirical expression as follows:

$$R = 0.75 \times \frac{0.5}{t^{\frac{2}{3}}}$$

Table I. Impulse Volt-Time Values by Test and Equation

Time t to Breakdown (Microseconds)	Ratio R of Breakdown Voltage at Time t to Full-Wave Breakdown Voltage	
	Paper	Equation
0.25	2.00	2.010
0.5	1.50	1.544
1.0	1.25	1.250
1.5	1.13	1.132
2.0	1.05	1.065
3.0	1.00	0.990

R = ratio of voltage at breakdown to full-wave breakdown voltage.

t = time to breakdown in microseconds.

See table I of this discussion for comparison of test and calculated values.

The average steepness to the point of cut-off of the wave front of lightning as it appears on transmission lines is less for the lower-voltage circuits than for the higher-voltage circuits. Measured values of kilovolts per microsecond range from 40 to 2,000 with only a few exceeding 500 volts per microsecond. These facts should be considered in any proposals for wave-front testing of transformers.

When the present Institute Impulse Test Code was first prepared it was recognized that changes or additions probably would be necessary, as the knowledge of the nature of lightning increased and as experience in testing developed. The basic reason for setting up impulse tests is to establish definite insulation values which may be demonstrated by test and which may serve as a measure of the protection which must be provided when the transformer is in service.

It has long been recognized that the main source of damaging overvoltages is lightning and accordingly the impulse test should be of such a nature that the results can be correlated with operating conditions. The present impulse test consists of two applications of a chopped wave followed by one application of a full wave. This test simulates to a certain degree traveling waves and waves chopped some distance from the station. However, the Institute should consider some test which better demonstrates the insulation level of transformers for conditions corresponding to a nearby direct stroke which imposes a very rapidly rising voltage.

There has been a demand for tests of this latter nature, in which a wave of a specified rate of rise and magnitude is applied to the transformer. In order that there be a common basis for specifying the test requirements and making the tests I am proposing that the Institute immediately begin the preparation of standards for front-of-wave testing of transformers.

K. B. McEachron (General Electric Company, Pittsfield, Mass.): Mr. Montsinger has presented a paper setting forth the results of

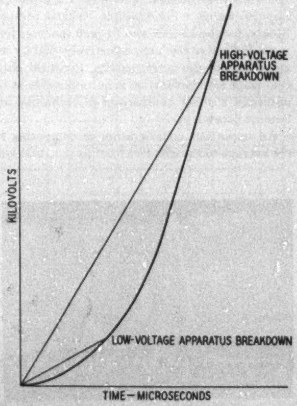

Figure 1. Effect of wave shape on rate of rise from zero volts

an investigation in the high-voltage engineering laboratory on transformer insulation under front-of-the-wave conditions. It must not be inferred, however, that information is available as yet indicating how steep the waves may be expected to be in practice. A figure of 1,000 kilovolts per microsecond has been mentioned and used at times, but it is not known how often such a wave front would be measured in service. Most of the data on wave front comes from the work of Norinder,[1] but these results must be used with caution on account of the possibility of error.

Furthermore, although the author is discussing only power-transformer insulation, yet I think it is wise to point out that rates of rise, to be expected for low-voltage circuits, may well be appreciably less than for high-voltage equipment. This situation is illustrated in figure 1, which shows the probable shape of a direct stroke of lightning initiated by a downward leader. McEachron[2] and McMorris have shown that this wave shape would be expected as the result of the motion of propagation of the downward leader carrying a charge. The only oscillogram[3] measuring the potential of a direct stroke of lightning close by also indicates the same kind of front. I have no doubt but that many kinds of fronts are possible, but figure 1 indicates that the same rate of rise should not be applied to low-voltage transformers as is applied to the high-voltage transformer, on account of the lower insulation level.

Attention is being directed toward the securing of wave-shape data, but it cannot be obtained quickly, even after means and methods are devised for making a sufficient number of measurements to be significant. This all states that much care must be taken in undertaking any program of front-of-wave testing, and low-voltage transformers will require in all probability lower rates of voltage rise than high voltage transformers.

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P. L. Bellaschi (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): Mr. Montsinger has presented a valuable contribution on the co-ordination of transformer insulation to steep-front impulse waves. Similar extensive investigations at the Sharon high-voltage laboratory have established that what has been accomplished for the distribution transformer in the way of self-protection against lightning strokes[1] appears feasible and economical also for the modern power transformer.

One of the factors which has contributed to this development is the ability to produce and measure in the laboratory steep-front impulse waves. For example, typical rod-gap values obtained at Sharon for the 15-inch and 24-inch spacings for a 1,000 kv per microsecond effective rise are respectively, 620 kv and 830 kv. The good agreement of the corresponding Pittsfield data in figure 9 suggests that some standardization is quite possible at this date. In so doing naturally a closer comparison of technique and test methods may become necessary.

An important consideration in comparing results is the ratio of the average to the effective front as this ratio is a measure of the shape of the wave and of the rounding at the chopped crest. The experience at Sharon in testing many barriers and a number of transformers has shown that this ratio for the various conditions hereto encountered has ranged from 0.70 to practically 1.00. On the average the ratio is in the order of 0.85 which indicates that the wave in general is quite sharp at the crest. Locating the rod gap right at the bushing of the apparatus tested and operating the impulse generator as high up in the voltage setting as possible with respect to the chopped test voltage should contribute to a front in which the ratio of average to effective is high, such as the oscillograms of the transformer model tests given in figures 2 and 3 show.

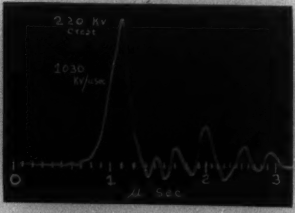

Figure 2. Commercial impulse test on 13.8-kv transformer. This oscillogram also illustrates the impulse voltage applied repeatedly to a 13.8-kv transformer model

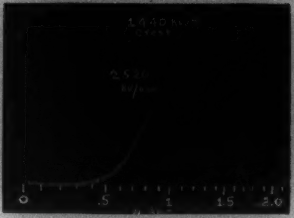

Figure 3. Impulse voltage applied to 138-kv transformer model

The conclusion Mr. Montsinger arrives at in table III where quite different rod gap values are set up for each of the two methods of defining the front, is difficult to understand. The insulation characteristic in figure 10 is a volt-time curve, therefore the rod-gap schedule under the average front column appears as the logical one. The appreciable difference in the gap values for the average and the effective front columns is hardly to be expected for the reason that the insulation characteristic and the rod gap characteristic each vary in the same direction and about the same relative amount. For example, increasing or decreasing the basis of the rate of rise in table III either to 1,500 kv per microsecond or 750 kv per microsecond (according to the Sharon rod-gap data) would indicate substantially the same schedule of rod gaps. In other words by virtue of the close similarity in the insulation and rod-gap characteristics over a certain range of the time once the correct schedule of gaps has been established co-ordination for the shorter or longer impulses should be expected. It is clear, too, on this score that the method of testing would be rather immaterial. On the other hand there are certain advantages inherent in the effective method, as better control in testing. The effective front responds more directly to generator setting and is accordingly subject to somewhat better control.

Mr. Montsinger indicates that reason for a 1,000 kv per microsecond rate of rise as a basis for testing transformers. From the extensive experience of apparatus in the field that has given adequate service and judging the relative ability of this old apparatus in withstanding the steep-impulse tests that have been applied to modern transformers, the 1,000 kv per microsecond appears in general quite justified. The measurements taken at the end of the 220 kv Wallenpaupack-Siegfried line several years ago[2] indicated front steepnesses lower than 1,000 kv per microsecond. On the other hand an oscillograph record[3] measured within a few hundred feet of a stroke to this same line shows a rate of rise in the order of 1,500 to 2,000 kv per microsecond. Had there been a substation close by, the front at the apparatus quite likely would have suffered some reduction due to the station capacitance. There have been indications of steeper fronts appearing on wood-pole lines with no ground wire as the result of direct strokes to the line conductors and the high insulation level of the line. A rate of rise apparently as high as 5,000 kv per microsecond seems to have been measured.[4]

While abroad this past summer, the writer had the occasion to discuss with various investigators the problem of the probable rate of rise of lightning. One of the more important field studies in this connection has been the investigation in Switzerland under the direction of Doctor K. Berger.[5] Oscillograph records taken at one important substation which connects to a 150-kv line and where an 80-kv line terminates, have not to this day indicated fronts up to 1,000 kv per microsecond, though direct strokes on the lines have rather frequently occurred. In an interesting analysis prepared for

IEC,[6] Doctor Berger calls attention to the possibility of impulse voltages attaining a rate of rise as high as 5,000 kv per microsecond—figure representing the upper limit, such as in the case of a severe direct stroke to the line conductor. These impulses attenuate rapidly and as stated above the station capacitance would contribute also in reducing the steepness. For lines properly shielded with ground wires and having reasonably low tower-footing resistance, even in the case of a back flash onto the line a rate of rise steeper than 1,000 kv per microsecond does not appear probable.[6] With due weight to the investigations and experiences on lightning-stroke phenomena both here and abroad, in my opinion the 1,000 kv per microsecond rate of rise for testing transformers is as sound a figure as our present knowledge will permit.

The tests on distribution and power transformers with steep impulses such as shown in figures 2 and 3 mark quite a step in the progress of insulation co-ordination. In view of the greater difficulty steep-front impulse testing should however, be considered more in the nature of a type test. Somewhat greater tolerance should, for the present at least, be recognized in making such tests as compared to the standard AIEE tests on full wave and waves chopped on the tail.[7]

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L. H. Hill (Allis-Chalmers Manufacturing Company, Milwaukee, Wis.): Mr. Montsinger's paper is worthy of considerable interest. When it is realized that the data in his paper represent approximately 10,600 impulse tests, the difficulty of obtaining reliable data relating to impulse phenomena may be better appreciated.

After considering the data submitted by Mr. Montsinger, a few questions arise. These refer to figures 6, 7, and 8 in the advance copy of the paper. It will be noted the ordinates of the curves are labeled "kv crest." From this, one assumes the 60-cycle values are also in crest kilovolts.

Consider, first, figure 6, which represents an insulating barrier for a 15-kv class transformer. The AIEE low-frequency dielectric test for this class is 34 kv effective, or 48 kv crest. Assuming the plotted 60-cycle corona and breakdown values as crest kilovolts, it is found the factor of safety or ratio of 60-cycle initial corona to low frequency dielectric test is 137/48 = 2.85. This appears to be a reasonable factor of safety.

Upon going to figure 7 and the 92-kv class of insulating barrier, a different picture develops. For this class the low frequency dielectric test is 185 kv effective, equivalent to 261.2 kv crest. Mr. Montsinger's data give the 60-cycle initial corona point as 275 kv, which gives a factor of safety of 275/261.2 = 1.05, a value which seems too close for 100 per cent assurance against damaging the insulation even before the transformer has left the manufacturer's shops.

Because of the wide spread between impulse injurious corona and one-shot breakdown, it is of course essential to design transformers using the injurious corona as the impulse strength of the transformer. If encroachment is made into the region above the injurious corona level, damage may be done without knowing it. As the transformer must withstand the crest value of the chopped wave when subjected to an impulse test, it is essential to regard this figure rather than the lower full-wave impulse shot as the one determining the impulse ratio.

Referring to figure 6, the impulse injurious corona level is found to be 240 kv. In accordance with the latest recommendations of the transformer subcommittee, the chopped-wave crest for this class is 130 kv, resulting in a factor of safety of 240/130 = 1.85. For the 92-kv class the impulse injurious corona level is 680 kv, and the chopped-wave crest value is 515 kv, giving a factor of safety of 680/515 = 1.32.

No mention is made of the method used to determine the point of initial corona under impulse test. Injurious corona can be detected by markings, but initial corona supposedly is non-injurious to the material and would leave no mark; hence an outline of the method used would be of interest.

J. H. Hagenguth (General Electric Company, Pittsfield, Mass.): Mr. Montsinger clearly points out the necessity of differentiation between an impulse test made with a 1.5 x 40 wave and a wave having a rate of rise of say, 1,000 kv per microsecond. Whether or not steep-front tests are desirable and what rate of rise should be used for transformers of different voltage classes has to be decided on consideration of actual lightning fronts (information of which is meager) and on tests such as described in the paper. If practical, these tests should be supplemented by steep-front tests to establish volt-time curves for turn, coil, and major insulation.

Several front-of-wave tests have been made on a commercial basis by the laboratory with which I am connected. The results of these tests were satisfactory from the point of view that failure did not occur. The kilovolt ratings of the windings tested, and the impulse voltages applied are given in table I of this discussion. At present, no method of definitely detecting small failures, such as turn-to-turn failure or coil-to-coil failure is available. My experience has shown that normal-frequency voltage applied to experimental transformers has never resulted in follow currents although failure due to impulse was detected by other methods. This type of failure, however, is progressive and finally will cause a normal frequency short circuit, when the failure path is sufficiently carbonized. In transformers with uniform voltage distribution the front-of-wave test limits of figure 10 of the paper apply probably for the transformer as a unit

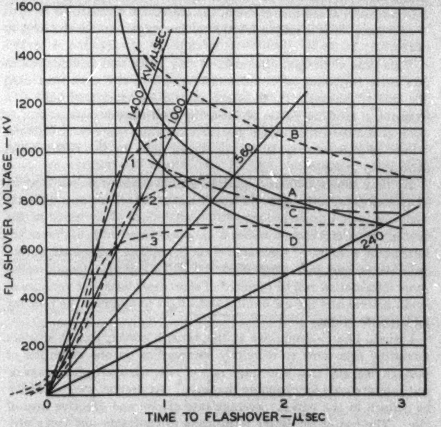

Figure 4. Flashover voltages of a 30-inch rod gap on the front of the wave, negative polarity, for 1,000 kv per microsecond rate of rise

Wave 1—On the basis of average rate of rise

Waves 2 and 3—On the basis of effective rate of rise

Curve *A*—Volt-time curve for 30-inch rod gap obtained with 1.5 × 40 wave

Curve *B*—Volt-time curve for 30-inch rod gap obtained with practically straight-line voltage rise, flashover on the rising front

Curve *C*—Volt-time curve of transformer insulation, 138-kv voltage class

Curve *D*—Volt-time curve of 25.5-inch rod gap obtained with 1.5 × 40 wave

Straight lines indicate rate of voltage rise in kilovolts per microsecond since turn insulation exhibits characteristics similar to the sample barriers tested. Transformers with poor initial voltage distribution would be subjected to considerably higher internal stresses during the steep front test than during the present standard AIEE. A point of some concern is the method of testing with steep-fronted waves.

1. Voltage Measurement

At short times to flashover the flashover voltage measured by different laboratories for a given spacing and time to flashover differs by 20 per cent and more. This is mostly due to the use of different types of voltage dividers. It appears, therefore, doubtful, that a test standard could be devised on the basis of flashover voltage of a rod gap, at present, because this would definitely mean that transformers of different manufacturers were receiving tests of different severity. Similarly, of course, it seems quite impossible to use the rate of rise of the wave as a standard measure, since this in turn would imply a more exact knowledge of the actual flashover voltage.

2. Rod-Gap Method (Temporary Expedient)

For the reasons stated above, rate of rise does not appear as a satisfactory yardstick for impulse testing at present. However, rod-gap spacings can be specified for the various voltage classes of transformers which have been found by experience to be capable of limiting the applied voltage to a maximum voltage value. The spacing alone would not suffice to assure application of similar voltage waves to a transformer by different manufacturers (as will be shown later) and it is necessary to specify the maximum time to flashover to be measured by a cathode-ray oscillograph. Irrespective of the type of divider used, the time to flashover will be measured with almost equal accuracy and a test specification of this type should, therefore, yield uniform test results by all manufacturers. The principal objection against the use of rod gaps in the present standard was that its flashover voltage varies greatly with humidity. The humidity effect is less objectionable for front-of-wave tests since at short times to flashover the humidity correction is greatly reduced. For instance, at 0.5 microsecond, the correction seems to be zero, at 2 microseconds it is only a fraction of the correction for the 1.5×40 wave.

This type of testing, of course, is very crude and reminds one of the early beginning of impulse testing, where rod gaps were used for similar reasons. It, therefore, should be considered only as a transitory measure to be replaced by volt-time measurements as soon as standardized voltage measurements and short-time flashover values of gaps and insulators are agreed upon by the EEI-NEMA laboratory group.

At that time sphere-gap spacings should perhaps be used rather than rod gaps, although this would mean the use of two different types of gaps during a complete impulse test. For waves of the present standard test requiring a chopped wave with flashover occurring at a specified minimum time the sphere gap would be unsuitable, at least for voltage classes of 25 kv and up. Considerably more information will be required of short-time flashover voltages of small spheres and the use of ultraviolet light to diminish the time lag and erratic arcing.

I would like to emphasize at this time, however, that it appears advisable even now to definitely standardize on one definition of voltage rise, and that is average rate of rise, whether sphere gaps or rod gaps are used for chopping the wave. At present, two are in use as defined in the paper: average rate of rise and effective rate of rise. The use of the latter term leads to confusion and lacks precision as will be explained by the use of figure 4 of the discussion. For illustrative purposes this figure has been drawn for a 30-inch rod gap. Curve *A* represents the volt-time curve of a 30-inch rod (as recommended for front-of-wave testing for 138 kv rated circuit voltage) obtained with a 1.5×40 wave. Curve *B* represents the volt-time curve of a 30-inch rod gap obtained with waves rising at substantially a uniform rate of rise. Curve *C* represents the safe voltage limits for power transformers of the 138-kv voltage class. Curves *A*, *B*, and *C* were obtained using the same methods of measurement and are, therefore, directly comparable. Superimposed on these curves are straight lines representing different rates of rise. The question of average and effective rate of rise would be irrelevant if the wave fronts applied to the test piece were straight lines, because then the voltage and time to flashover would be the same using either definition. However, waves usually obtained are of the form shown by curves 1, 2, and 3. Curves 2 and 3 represent waves, both of which have an effective rate of rise of 1,000 kv per microsecond. The straight line represents another front at 1,000 kv per microsecond and, therefore, the following values of flashover voltage and time to flashover could be obtained for waves of the same rate of rise on the front.

This table, I believe, shows clearly that the effective rate of rise is too loose a term and cannot be used as a standard of measurement, especially since it can lead to a condition where steep-front waves of the required rate of rise are applied, but the amplitude is decreased by making the time to flashover long. For instance, the flashover voltage of curve 3 of figure 4 (this discussion) would actually be lower than the specified chopped wave voltage of the present standard test with 1.5x40 wave.

Table II

Wave	Flashover Voltage	Time	Per Cent Kilovolts
1	1,070	1.07	100
2	900	1.6	84
3	700	2.87	65.0

On the other hand, the use of average rate of rise establishes definite limits. For instance, curve 1, figure 4 (this discussion) represents a typical wave which would be applied during a test specified by average rate of rise of 1,000 kv per microsecond. The flashover voltage with this definition cannot be made to be materially lower than, 1,070 kv but rather can be increased by straightening out the wave front to reach a possible maximum of 1,240 kv. In this case, therefore, the bend over of wave front is entirely beneficial to the transformer as far as the amplitude of the voltage is concerned and yet the flashover voltage cannot be lowered materially without obtaining too low a rate of rise, i.e., if the kilovolt value were reduced to 1,000 kv (flashover at 1.26 microseconds) the rate of rise would be reduced to 800 kv per microsecond which would be too low to conform with specifications.

A further advantage of the use of average rate of rise is that it can be obtained from any volt-time curve by simply dividing the flashover voltage by the time to flashover, while the use of effective rate of rise requires entirely separate curves which will be a function of the bend over of the curve as illustrated by curves 2 and 3, figure 4 (this discussion). For instance, curve *B* of figure 9 of the paper applies only to one particular wave front used. Any number of such curves could be drawn, the highest one would coincide with curve *A* while curves with flashover voltages lower than *B* would be entirely possible.

It appears, therefore, necessary that in the final standard for front-of-wave testing, crest voltage and time to flashover or crest voltage and average rate of rise be specified and that the use of effective rate of rise be discontinued.

Concerning the rod-gap spacings to be used at present for front-of-wave tests, figure 4 (this discussion) shows that if average rate of rise is used a rod gap spacing of 25.5 inches (curve *D*, figure 4, this discussion) is required to limit the flashover voltage to 950 kv. This checks values given in table III of the paper. If a rod-gap spacing of 30 inches is used as shown in the paper for effective rate of rise, the flashover voltage should be limited to about 755 kv in order not to exceed the safe limits of the transformer. The time to flashover in this case would be approximately 2.4 microseconds. This test, therefore, would, practically speaking, be exactly the same as the chopped-wave test of the present Impulse Test Standards. Therefore, if the gap spacing of 30 inches is used, either the safe insulation limits are exceeded by up to 25 per cent or if the voltage is kept below the safe limits of the transformer, the value of the test is doubtful, since the only difference from the present chopped wave is the steeper front.

F. J. Vogel (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): Mr. Montsinger has made a real contribution in furnishing data on the strength of insulation under oil. Such tests as those described are indeed difficult and expensive to make. By increasing our knowledge of the behavior of materials, we are provided on the one hand with information necessary to determine what is possible, and on the other hand how to accomplish these possibilities with reasonable factors of safety in actual designs.

When data are furnished, it is usually possible to draw many diverse conclusions from it. It is often dangerous to draw conclusions from another's data but nevertheless there are certain interpretations of Mr. Montsinger's tests which are of great interest and it seems to me that they should be noted.

Barrier *C* is indeed an approximation of 15-kv transformer major insulation, except for the extension of the barrier beyond the coil. It is my impression that in actual transformers, the extension is of the order of one or two inches for the 15-kv class; it is believed that this extension is not generally sufficient to result in puncture, but that the insulation would fail by creeping around the barrier. In Mr. Montsinger's paper this extension was 13 or 14 inches, sufficient to result in a puncture of the barrier *C* in the tests. However, taking the full-wave value of 240 kv as correct for 15-kv insulation, it is possible to show certain relationships. My figure 5 shows time-voltage values for the five and three-fourths inch gap as determined in Sharon, compared directly with Mr. Montsinger's curve figure 6. There is a margin of 30 per cent between these curves, even at extremely short times. This curve indicates to the writer that it might be possible economically to make transformers of this type of construction completely co-ordinated. Barrier *C* is typical, depending upon the manufacture, of transformers insulations up to 46 kv, except for the amount of extension. The barrier thickness will, of course, vary up to one or one and one-half inches or more.

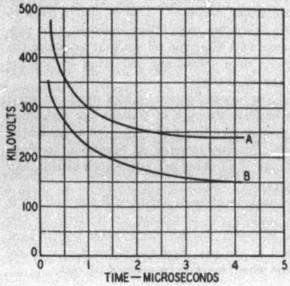

Figure 5

A—Data for barrier *C*, 13.8-kv insulation, Mr. Montsinger's paper "Co-ordination of Power Transformers for Steep-Front Impulse Waves," figure 6

B—Data for five-and-three-quarters-inch rod gap, negative waves

Barrier *D* is described as typical of approximately 92-kv class insulation, tested at 185 kv root-mean-square. It is to be noted that 275 kv crest is 194 kv root-mean-square; 275 kv crest at 60 cycles is the maximum voltage which barrier *D* will withstand without injurious corona. If the insulation in an actual transformer, is not to be injured on test at 185 kv root-mean-square, there must be a margin in the design. According to table II, row 3, the insulation would withstand without corona, full waves of 680 times 185 divided by 194 or 650 kv, with the designer's margin. One might reduce this value somewhat further by taking the lowest value for injurious corona instead of the average value which would reduce the figure to 590 kv. The data on figure 7 may then be redrawn as shown in figure 6 to be representative of 92-kv class insulation. For comparison, a curve given by Mr. Hagenguth for a 20-inch gap, published in the January issue of Electrical Engineering, is also shown. This would indicate that a 20-inch gap could protect a 92-kv transformer against extremely high voltages for extremely short times, even for direct strokes of lightning. It is to be noted that barrier *D* is similar to the insulation generally used in the 69-kv class and above and similar relationships should hold. This would tend to confirm Mr. Montsinger's opinion that it is impractical to build completely co-ordinated transformers when a gap alone is to be relied upon.

The results of tests on barrier *D* only confirm my opinion further that higher impulse strengths should be obtained than those at present standardized for the higher voltage classes. Data published by the writer has been chiefly on the basis of interleaved insulation similar to barrier *D* and typical of actual transformer insulation. Mr. Montsinger's data also show 2.2 to be a fairly average value for the ratio between impulse and 60-cycle breakdown and corona voltages for interleaved insulation. Under these circumstances, it is difficult to see why a 570 kv is not a more reasonable value for impulse-testing 92 kv insulation than the present AIEE subcommittee recommendations of 515 kv. This is particularly true when the higher value is desirable from the standpoint of obtaining protection under the worst conditions. Whether Mr. Montsinger is correct or not in stating that there is no definite impulse ratio that applies to all voltage ratings of transformers, he has at least furnished confirming evidence that ratios of 2.2 or higher can be obtained with some constructions. Further it would seem that the impulse strength is really the important one from the service standpoint and that the 60-cycle test is relatively unimportant. If present impulse test values are retained, it would seem that the 60-cycle test values might be abandoned or at least reduced in order to provide a greater degree of flexibility for the designer.

It is difficult to see how it is possible to change the gap values as much as Mr. Montsinger states when using effective values as compared with average values. Referring to figure 7 in my discussion, a sample oscillogram is shown and the two methods of interpretation described by Mr. Montsinger are illustrated. These data can be plotted on the curve using either average or effective times. This curve shows how a higher voltage will be obtained for a given rate of voltage rise using the average value. Assuming that these curves apply for a rod gap, is there any reason to believe that similar curves do not apply to transformer insulation? If so, figure 8 shows what might be obtained for transformer insulation. Mr. Montsinger, in his method of deriving gap lengths, is really comparing volt-time curves for gaps with similar curves for transformer insulation. If average values are used to compare results, it is seen that we obtain approximately the same length as obtained by Mr. Montsinger. If we compare the effective values for the rod gap with the average rate-of-rise data for the insulation, we find that the gap can apparently be increased. The writer does not believe this to be actually the case. It is suggested that to prove this, it might be desirable to test insulation, protected by a rod gap, with different shapes of waves to see if there is really a difference, as Mr. Montsinger believes. It would be desirable for another reason also. Those customers desiring tests with gaps and steep-wave

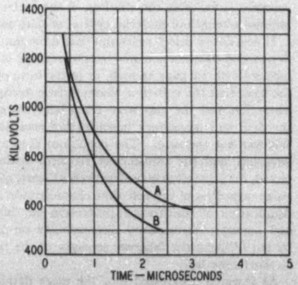

Figure 6

A—Data for barrier reduced to 92-kv class, Mr. Montsinger's paper "Co-ordination of Power Transformers for Steep-Front Impulse Waves," figure 7

B—Data from maximum values for 20-inch rod gap (positive or negative), Mr. J. H. Hagenguth's paper "Short-Time Sparkover of Gaps," figure 14, ELECTRICAL ENGINEERING January 193

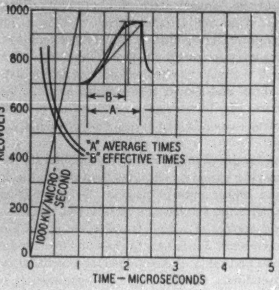

Figure 7. Data for 11-inch rod gap, showing effect of method of scaling oscillogram on time-voltage curve

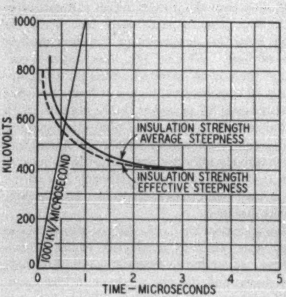

Figure 8. Data for 69-kv transformer insulation, from proposed method given in Mr. Montsinger's paper

fronts intend to use the gap as a protective device. They are not therefore, interested in voltage and time values but only in proof that the co-ordination actually works. Therefore, it is believed that it is desirable to use the gap directly and avoid the use of volt-time curves for this purpose.

There is still another suggestion, that is, that tests on sample transformers under these conditions are desirable. When the test values of 860 and 1,430 kv were proposed for 138- and 230-kv transformers respectively, transformers had been given endurance tests to see that these values could be withstood, and likewise tests with steep-wave-front waves on sample transformers are desirable before values actually withstood can be stated without question.

I. W. Gross (American Gas and Electric Service Corporation, New York, N. Y.) [**Editor's Note:** This discussion covers also "Corona" Voltages of Typical Transformers for Steep-Front Impulse Waves, F. J. Vogel. *EE, Jan. '38, Trans. p. 34–8.*]: These two papers comprise a valuable contribution to the better understanding of the impulse strength of material typical of that used in transformers.

It was not so many years ago that those familiar with transformer design and construction were recommending the protection of transformers with air gaps as such, or in the form of insulator strings, on the basis that the volt-time characteristic strength of the transformer insulation and the gap were essentially the same. Experience in the field and laboratory investigation soon showed definitely that this was not the case. The pendulum then swung in the opposite direction, and the transformer strength against lightning was depicted, for protection considerations at least, as being a fixed voltage value regardless of the volt-time characteristic of the lightning wave. Application of transformer protection on this basis placed heavy and perhaps unwarranted responsibility on the protective device by not utilizing the inherent strength of the transformer insulation at short time lags.

As research progressed into the more detailed characteristics of transformer insulation under impulse stresses, information which formerly had been qualitative became quantitative. We now find in Mr. Montsinger's paper definite suggestions for applying front-of-wave impulse tests at voltages considerably higher than the present full or chopped-wave tests. Such tests should demonstrate in a practical way that the short-time insulation strength which is now supposed to be built into the transformer is actually there. These suggestions, I am sure, will prove extremely valuable in setting up a test procedure for standardizing front-of-wave testing on transformers, a subject the transformer subcommittee has been actively working on for more than a year. Further, I thoroughly agree with Mr. Montsinger that front-of-wave testing be set up on a voltage and time basis which are definite units of insulation measurement rather than by specifying inches of gap, thus leaving the gap as a useful test device to chop the wave on the front, as may be specified.

Both Messrs. Montsinger and Vogel have encountered the same difficulty in carrying on their test work, namely, entrapped air in the insulation which has not received special treatment. The special treatment used by the authors, assembly under oil, vacuum treatment, tapping of the assembled sample to remove this air and thus increase the impulse strength leads directly to the question, why should it not be possible to increase the impulse strength of commercial oil-insulated transformers by admitting oil to the tank while the winding is still under vacuum? This of course leads to the next question: if this procedure were followed would it be possible in the field to change and filter the oil without breaking the vacuum, or without more than temporary reduction of impulse strength resulting from the oil change?

Could the authors give us any indication of how much, or what percentage, increase in impulse strength of transformer insulation it is possible to obtain by this oil filling under vacuum as compared with nonvacuum filling? Possible increase of insulation strength of transformers by a change of oil-filling procedure such as this may well be worth considering.

It is to be hoped the authors of both papers will continue their valuable researches on transformer insulation particularly in the short time region of the volt-time characteristic where a great deal of speculation has existed in the past, but altogether too little actually known to the point where it could be verified by test.

V. M. Montsinger: I believe the reason that Mr. Bellaschi gets a smaller difference in gap spacing for different rates of voltage rise (of, say, 750 to 1,500 kv per microsecond) than the data in my paper indicates, is because the Pittsfield gap volt-time curves probably have higher kilovolt values for short times than his volt-time curves have for the same gap spacing. In other words, the turn-up may be somewhat more. Figure 9 of this discussion shows volt-time curves of a 25-inch rod gap and a transformer winding (same shape as curve in figure 10) having an insulation level for a standard 138-kv circuit. It will be seen that there is quite a difference in their slopes which does not seem to bear out Mr. Bellaschi's expectations that once the correct schedule of gaps has been established, co-ordination for the shorter or longer waves would be established. This question is best answered by citing the results of tests made about five years ago (table III, reference 1) when a one-half inch pad of insulation

Table III

Applied Voltage (Per Cent of Full Wave)	Gap Spacing (Inches)	Kilovolts*	Time (Microseconds)
100	20.5	355	5.5
135	16.5	371	2.0
208	13.7	447	0.9
350	10.5	447	0.8
500**	6.25		0.3 (approx.)

* By sphere gap.
** Zero series resistance in generator circuit.

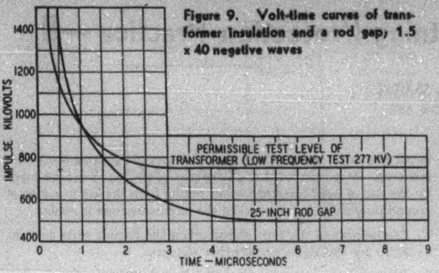

Figure 9. Volt-time curves of transformer insulation and a rod gap; 1.5 x 40 negative waves

(same as barrier *B*) was impulse tested in parallel with a rod gap.

The following tabulation shows to what extent the rod gaps had to be shortened as the wave fronts became steep by applying more and more overvoltages. In each case the gap was adjusted such that its flashover on the wave front was equal to the single-shot breakdown of the pad of insulation.

A study of the oscillograms taken at the time these tests were made shows the following results given in table III.

The above tests show conclusively that to protect even a low-voltage transformer, the gap spacing must be rapidly reduced as the wave fronts become steeper. In other words, any one gap spacing will have an impulse strength equal to the insulation over a very narrow range of time.

Mr. Gross' question about the increase in impulse strength of transformers by admitting oil to the tank with the windings under vacuum, is a difficult question to answer except in a general way. It is the air pockets entrapped in or under the insulation structures and not the air in solution (of the oil) that cause trouble. The seriousness of air pockets in series with oil and solid insulation in a highly stressed electric field can be appreciated when it is remembered that the voltage stresses are in the inverse ratio of their permittivities; i.e., the stress in volts per mil on the air, is approximately 2.2 times the stress in volts per mil on the oil and approximately 4.4 times the stress on the solid, while the dielectric strength of air is in the order of one-fourth to one-sixth that of oil and solid. Any air pockets in series with oil will, therefore, reduce the breakdown of the oil in the order of eight to ten times because when the air becomes ionized and breaks down, the oil breakdown follows. It is, therefore, out of the question to economically design transformers to withstand high impulse voltages with air pockets in the insulation under high stress.

Since the air in solution is not harmful to the dielectric strength of oil (and if the air pockets have been previously removed from the insulation) very little would be gained in the field by changing and filtering the oil under vacuum. Of course, if the oil were drained out of the transformer, air pockets most likely would be introduced when refilling. If the air pockets are adjacent to the oil in circulation under load conditions, they are absorbed in a few hours after the transformer is put in service. Their temporary presence is not harmful if the transformer has been in operation a few hours before being subjected to a high impulse voltage.

Mr. Hagenguth's discussion of average versus effective rate of rise emphasizes the importance of eliminating, if possible, the bend-over of the voltage chopped by the gap. If the bend over is eliminated, and there is reason to believe it can be reduced to a negligible value by using sphere gaps to chop the wave and proper circuit constants, the question of "average rate of rise" versus "effective rate of rise" is done away with, and the testing is simplified. The reason so much space was devoted to this question in the paper was because we are at present being asked to use rod gaps to chop off the wave in front of wave testing of transformers.

Regarding Mr. Hill's discussion, the 60-cycle kilovolt values shown on figures 6, 7, and 8 are the crest values.

As to barrier *C* representing a 92-kv (185-kv low-frequency test) transformer, this barrier represents merely the type of insulation used in a 92-kv transformer. No importance should be attached to its actual kilovolt strength.

The method used in determining the initial corona was by observation—in a dark room. It is true that initial corona may not be injurious. It is more difficult to determine when corona becomes injurious than to detect initial corona. The most satisfactory method of detecting damaging corona, particularly on the coil insulation, was to lower the voltage to the highest value that previously had not shown any corona. If corona appeared it was evident that the coil insulation was damaged. Damage to the pad itself did not occur until after the insulation on the small coil was injured. Injury to the pad (if the test was continued up to this point) did not mean anything. When using bare electrodes injury to the pad had to be determined by removing from the oil and examining it for tracking.

Distribution Transformer Lightning-Protection Practice—II

By L. G. SMITH
FELLOW AIEE

TO DETERMINE the relative effectiveness of various methods of protecting distribution transformers from lightning an annual survey of operating companies was initiated in 1934 by the transmission and distribution committee of the Edison Electric Institute and continued through 1935 and 1936. In 1934 operating data were obtained from 38 companies, in 1935 from 43 companies, and in 1936 from 41 companies. The results for 1934 were presented in a previous paper. This paper presents a summary of the data for the three years and draws conclusions therefrom. It is only by collecting such data from a number of companies for numerous installations for a number of years will conclusive data be obtained so that annual variations in storm frequency and severity will be averaged. It is planned to continue the collection of these data for several years. In the future it is hoped that data for a sufficient number of years will be obtained from each company so that the troubles may be weighted by the isokeraunic level of each territory served.

Conclusions

Based upon the data collected, the following conclusions appear to be justified:

1. In so far as the rate of primary fuses blown the various protection schemes fall into the following relative order of effectiveness with not much choice among the last three named:

   First: The various methods of solid interconnection of which the solid interconnection with tie to case seems the best followed by the conventional solid interconnection and the common primary and secondary neutral schemes.

   Second: The type *SP* transformer.

   Third: The three-point connection.

   Fourth: The standard connection.

   Fifth: The gapped interconnection.

2. In protecting against transformer-winding failures the relative order of merit is not clear due to inconsistencies in the data. However, it is believed that the newer methods of lightning protection are better than the standard connection with the exception that the three-point connection and the gapped interconnection are practically no better.

3. The data indicate that lightning-arrester failure rates are lower for the types of solid interconnection and type *CSP* transformer than for the standard connection.

4. Solid interconnection does not seem to increase the number of meters burned out or troubles on customers' premises.

5. The ground resistance of lightning-arrester grounds is the most important single factor influencing the degree of lightning protection rendered. It is believed that a large part of the improvement noted by the use of solid interconnection results from the lower values of ground resistance that exist when the interconnection is made.

6. Irrespective of the type of protection the trouble rates seem to be higher for the higher distribution voltages.

7. There is no apparent trend of the effect of the type of protection upon the location of transformer-winding failures. On the average 35.2 per cent of the failures involved the primary only, 11.3 per cent involved the secondary only, and 53.5 per cent involved both the primary and secondary.

8. There seems to be a growing tendency to install secondary arresters on customers' premises.

9. It is believed that the higher the fusing schedule, the lower will be rate of primary fuses blown.

10. In the case of all types of protection a higher rate of primary fuses blown exists when the arrester is installed on the transformer side of the fuse.

11. The rates of primary fuses blown per 100 transformers are practically the same irrespective of whether one or two fuses per transformer are used.

12. The practice of grounding transformer tanks is growing.

13. The amount of attention paid to obtaining low resistance of arrester grounds is not as great as the data warrants except for solid interconnection.

Various Methods of Lightning Protection

The various protective schemes considered in this survey may be classified as follows:

1. *Standard connection* (figure 1) in which the lightning-arrester ground is not interconnected with the secondary neutral. In the case of three-phase four-wire systems the ground lead is usually tied

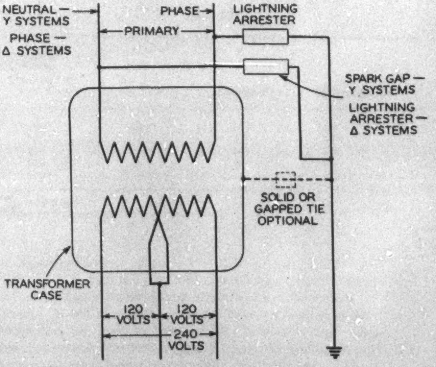

Figure 1. Standard connection

Paper number 37-111, recommended by the AIEE committee on protective devices and presented at the AIEE winter convention, New York, N. Y., January 24–28, 1938. Manuscript submitted October 20, 1937; made available for preprinting December 21, 1937.

L. G. Smith is assistant to general superintendent, Consolidated Gas, Electric Light and Power Company of Baltimore, Md.

The author wishes to acknowledge the assistance of R. D. Bader, H. C. Miller, and W. J. Witte in summarizing data for the tables, and the co-operation of the various operating companies who supplied operating data.

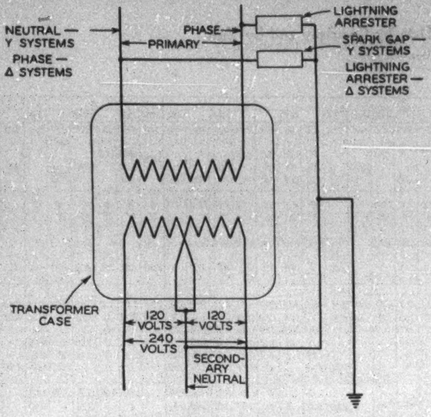
NEUTRAL — Y SYSTEMS
PHASE — Δ SYSTEMS
PHASE
PRIMARY
LIGHTNING ARRESTER
SPARK GAP — Y SYSTEMS
LIGHTNING ARRESTER — Δ SYSTEMS
TRANSFORMER CASE
120 VOLTS
120 VOLTS
240 VOLTS
SECONDARY NEUTRAL

Figure 2. Solid interconnection

with the primary neutral through a gap. In some cases the transformer tank is grounded.

2. *Various types of interconnection* classified as follows:

(*a*) Solid interconnection (figure 2) in which the arrester ground lead is interconnected with the secondary neutral. The following sub-classification exists:

(1) No tie to the transformer tank (conventional type).

(2) The transformer tank tied solidly to the ground lead as in figure 3.

(3) The transformer tank tied through a gap to the ground lead as shown in figure 3.

(*b*) *Common primary and secondary neutral* (figure 6) which is obviously similar to the solid interconnection. There may be three types of this scheme as follows:

(1) Transformer tank ungrounded.

(2) Transformer tank solidly grounded.

(3) Transformer tank grounded through a gap.

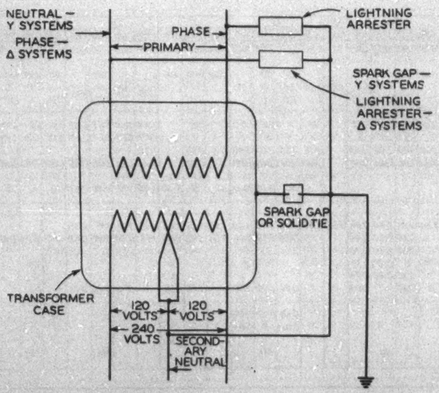
NEUTRAL — Y SYSTEMS
PHASE — Δ SYSTEMS
PHASE
PRIMARY
LIGHTNING ARRESTER
SPARK GAP — Y SYSTEMS
LIGHTNING ARRESTER — Δ SYSTEMS
SPARK GAP OR SOLID TIE
TRANSFORMER CASE
120 VOLTS
120 VOLTS
240 VOLTS
SECONDARY NEUTRAL

Figure 3. Solid interconnection with tie to transformer case

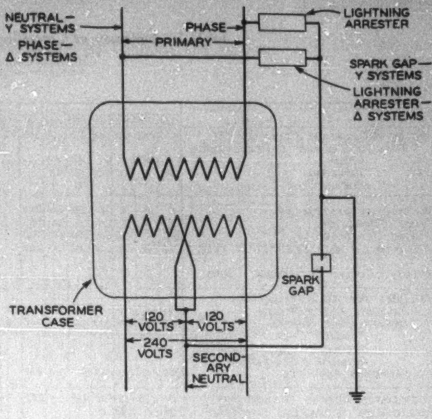
NEUTRAL — Y SYSTEMS
PHASE — Δ SYSTEMS
PHASE
PRIMARY
LIGHTNING ARRESTER
SPARK GAP — Y SYSTEMS
LIGHTNING ARRESTER — Δ SYSTEMS
SPARK GAP
TRANSFORMER CASE
120 VOLTS
120 VOLTS
240 VOLTS
SECONDARY NEUTRAL

Figure 4. Gapped interconnection

(*c*) *Gapped interconnection* in which the arrester ground is interconnected with the secondary neutral through a gap. This scheme may be divided into the following subclasses:

(1) No tie to the transformer case (figure 4). The conventional type.

(2) The transformer tank tied solidly to the ground lead (figure 5).

(3) The transformer tank tied through a gap to the ground lead (figure 5).

(4) The three point connection (figure 8) in which the ground lead is connected to the tank through a gap and the arrester lead connected solidly to the tank the secondary neutral being connected to the tank through a gap.

3. *Type SP transformer* (figure 7) in which self-clearing spark gaps (deion gaps) are used instead of lightning arresters. It is essentially a gapped interconnection with the tank grounded through a gap and the connections are similar to the three-point scheme.

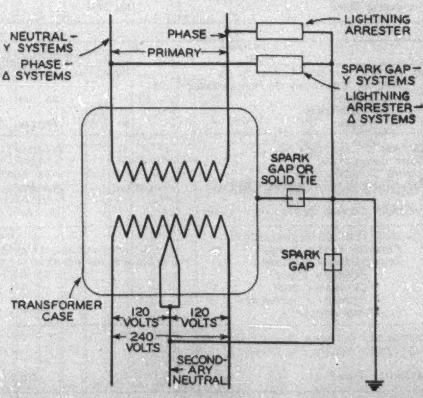
NEUTRAL — Y SYSTEMS
PHASE — Δ SYSTEMS
PHASE
PRIMARY
LIGHTNING ARRESTER
SPARK GAP — Y SYSTEMS
LIGHTNING ARRESTER — Δ SYSTEMS
SPARK GAP OR SOLID TIE
SPARK GAP
TRANSFORMER CASE
120 VOLTS
120 VOLTS
240 VOLTS
SECONDARY NEUTRAL

Figure 5. Gapped interconnection with tie to transformer case

Table I. Summary of Operating Data by Company and by Year

Standard Connection—Figure 1

Company	1			2				3					4								6							
Year	1934	1935	Avg.	1934	1935	1936	Avg.	1934	1935	1935	1936	Avg.	1934	1935	1935	1935	1936	1936	1936	Avg.	1934	1935	1936	1936	1936	Avg.	1934	1934
No of Installations Reported On	4150	4300	4225	3623	3221	3221	3355	5607	1177	949	6237	4657	5222	450	26	29	2976	1677	1457	3946	516[a]	567[a]	155[a]	392[a]	37[b]	556	18182	50
Type of Territory By Percentages																												
Urban	100	85	100	100	40	44	100	100			100	100		50	0	52	100				100	21.9	100				100	
Suburban		15			60	48								30	46	3		100				67.2		100				
Rural		0			0	8				100				20	54	45			100			7.9			100			100
Voltage of System - KV.	4	4	4	2.4-4	2.4-4	2.4-4	2.4-4		2.3	13	2.3/13			4	6.6	11	2.3 To 13.2				4	4	4	4	4	4		
Phase Connection	Y	Y	Y	Δ	Δ	Δ	Δ		Δ	Y	Δ			Y	Δ	Δ	Δ	&	Y		Y	Y	Y	Y	Y	Y		
Fusing Schedule - %	240	240	240	200	200	200	200																					
Position L.A. With Respect To Fuse	Line	Line	Line	Line	Line	Line	Line		Line	Line	Trans			Line	Line	Line	60% Trans 40% Line				All on Trans. Side							
Average Ground Resistance - Ohms	15	15	15	50									25-200	3	40	45	20[c]	40[c]	50[c]		250	250	250	250	250	250	130	90
Failures Per 100 Installations																												
Primary Fuses Blown	13.45	3.95	8.61	0.94	3.72	5.37	3.25	2.14	6.54	17.3	6.42	5.45	3.16	48.9	57.7	89.7	3.53	7.83	30.9	12.02	1.94	1.41	5.16	2.80	5.40	2.34	10.65	10.00
Transformer Winding	0.29	1.05	0.66	0.19	0.68	0.56	0.47	0.96	0.17	0.95	1.28	1.04	0.92	6.22	15.4	17.2	0.54	0.95	2.61	1.31	0.20	0.18	0	1.02	0	0.26	0.52	0
Primary Only		0.88												3.33	3.85	3.45	0.34	0.36	0.96	0.71		0	0	0	0	0		0
Secondary Only		0.17												1.11	0	3.45	0.07	0.12	0.69	0.30		0	0	0.76	0	0.26		0
Primary & Secondary		0												1.78	11.5	10.3	0.13	0.48	0.96	0.60		0.18	0	0.26	0	0.17		0
Lightning Arrester	0.60	2.32	1.48	0.14	0.12	0.03	0.10						0.06	7.78		0	0.27	0.66	2.75	0.82		0	0	0	0	0	3.30	
Customers' Meters Burned Out	0	2.00	1.03	0.39	0.68		0.52						0.11	1.78	0	0	1.21	3.34	11.8	2.34		1.06	4.50	0.76	2.70	1.48		
Customers' Wiring, Etc Damaged	7.08	0	3.48	0									0.09	3.33	0	0	0.47	0.48	0.48	0.41		0		0.26		0.10		
Grounded Tanks						No					Yes						Above 6.6 KV.						No	No	No			

Company	16					17						18			19					20			21					
Year	1934	1935	1935	1936	Avg.	1934	1935	1935	1936	1936	Avg.	1936	1936	Avg.	1934	1935	1936	1936	Avg.	1934	1935	Avg.	1934	1935	Avg.	1934	1935	1935
No. of Installations Reported On	13255	8980	1700	11095	11677	6795	5331	435	415	371	4449	1817	13	1830	569	3995	1020	2000	2528	4700	2734	3747	124	22	73	1400	974	573
Type of Territory By Percentages																												
Urban		40	40	30	35	100	100		100		100	100		100	100	29.2	100		100	100	100	100	100	36	100	100	35	0
Suburban		30	30	40	35			21					100			0								0			5	40
Rural		30	30	30	30			79		100						70.8		100						64			0	60
Voltage of System - KV.		2.3/4	6.6	2.3-6.9	2.3-6.9		4	6.6	4	11.95	4-11.95	4	13.2	4 & 13.2		2.3-13	2.3-13		2.3-13		2.4/4			13			2.4	6.9
Phase Connection		Δ&Y	Δ	Δ&Y	Δ&Y		Y	Y	Y	Y	Y	Y	Y	Y		Δ	Δ		Δ		Δ&Y			Δ			Δ	Δ
Fusing Schedule - %				300					200	200														1540			240	240
Position L.A. With Respect To Fuse	All on Line Side						Line	Both	Both	Both	Both	Line	Line	Line		Line	Line	Line	Line		Line			Line			Both	Both
Average Ground Resistance - Ohms	25	20	20		20-25	19.45	13	15	22	23	17	3	3	3	30		25	50 To 500		35			12.2			10	20	15
Failures Per 100 Installations																												
Primary Fuses Blown						3.42	0.92	8.28	0.72	4.85	2.53	0.83	46.1	1.15	8.80	5.48	2.35	10.75	6.70	2.75	1.54	2.30	4.03	4.54	4.10			
Transformer Winding	1.10	0.78	1.35	1.00	1.00	0.12			1.68	0.27	0.21	0.61	15.4	0.71	1.05	0.33	0.10	0.70	0.45	0.13	0.25	0.17	0	4.54	0.68	0.71	0.41	0.52
Primary Only									0.48	0		0.05	15.4	0.16									0	0	0			
Secondary Only									0.24	0		0.05	0	0.05									0	0	0			
Primary & Secondary									0.96	0.27		0.50	0	0.49									0	4.34	0.68			
Lightning Arrester						0.34	0.17	2.95	0	1.89	0.39				0.88	0.08	0	0.40	0.36	0		0	4.03	0	3.43			
Customers' Meters Burned Out	0			0	0	0.015			0.48	0	0.04				0	0	2.45	1.35	1.45	0		0	0	0	0			
Customers' Wiring, Etc Damaged				0		0.47					0.47				0.70	0	0	0	0.11	0		0	0	0	0	1.85		
Grounded Tanks			Yes	Yes					Some	Some		Some	Some	Some			No	No										

Company	31				33				34				35	36				37			38				39			40
Year	1934	1936	1936	Avg.	1934	1935	1936	Avg.	1934	1935	1936	Avg.	1934	1934	1935	1936	Avg.	1935	1936	Avg.	1934	1935	1936	Avg.	1935	1936	Avg.	1935
No. of Installations Reported On	3380	551	3946	4939	4366	476	1143	1995	3362	3343	3272	3325	51	4033	5314	2594	3920	3601	1896	2729	473[a]	16883	16592	11296	12951	12960	12950	2109
Type of Territory By Percentages																												
Urban	100	90		100		90	75.5		100	100	100	100		100	60	75	100	100	100	100	100	50	50	100	80	80	80	66
Suburban						0	0						100		20	15						50	50		10	10	10	0
Rural			95			10	24.5								20	10						0	0		10	10	10	34
Voltage of System - KV.	4	4	11			2.4-6.6	2.3-6.6		2.3	2.3	2.3	2.3		4	4	4	4		4		2.3/4	2.3/4	2.3/4	2.3/4	2.3-11	2.3-11	2.3-11	4
Phase Connection	Y	Y	Y			Δ Y	Δ&Y		Δ	Δ	Δ	Δ		Y	Y	Y	Y		Y		Δ&Y	Δ&Y	Δ&Y	Δ&Y	Δ&Y	Δ&Y	Δ&Y	Y
Fusing Schedule - %									240	240	240	240		240	240	240	240		250+									
Position L.A. With Respect To Fuse		Line	Line			Both	Both		All on Line Side					All on Line Side					Line		Line	Line	Line	Line	Line	Line	Line	Line
Average Ground Resistance - Ohms	30					30	30								50	30-200	30-200	10	5	5-10	15-100	100	15-100	15-100	10	10	10	2
Failures Per 100 Installations																												
Primary Fuses Blown	2.23	0	5.50	3.41	13.52	6.10	6.55	11.61	3.90	0.39	7.40	3.86	56.0	6.99	1.96	5.47	4.40	1.90	1.29	1.68	5.07	3.21	3.71	3.49				2.37
Transformer Winding	0.08	0.36	0.76	0.36	0.80	0.84	0.53	0.75	0.64	0.24	0.62	0.46			0.34	0.79	0.48	0.51	0.38	0.46	1.06	0.30	0.47	0.40		0.52	0.52	0.57
Primary Only										0.12	0.09	0.11																
Secondary Only										0	0.03	0.15																
Primary & Secondary										0.12	0.40	0.26																
Lightning Arrester									0	0.03		0.02			0.04	0.21	0.09		1.78	1.78		0.16	0.26	0.21				0.33
Customers' Meters Burned Out															0.04	0.13	0.07								0		0	0.05
Customers' Wiring, Etc Damaged															0.04	0	0.03								0		0	0.05
Grounded Tanks			No				Yes				No					No							No			No		

Table I (Continued). Summary of Operating Data by Company and by Year

Standard Connection—Figure 1

7									8	9						10[c]				13						14				15							
	1935			1936				Avg.	1934	1934		1935	1936		Avg.	1934	1935	1936	Avg.	1934	1935	1936			Avg.	1934	1935	1936	Avg.	1934	1935			1936			Avg.
30	1930	7404	7336	7118	4490	3723	130	16831	4000	3575	7405	1125	811	872	4596	13000	6500	3900	7600	1932	5300	1565	1313	1932	4107	97	100	100	99	9287	518	409	545	500	400	550[d]	4071
	7.5		100	100						100		100				100	100	100	100	100	37	100									74	0	0	74	0	0	
								100	100		100				100						27		100		100					100	26	100	100	26	100	100	100
00	92.5	100			100	100	100														36			100		100	100	100	100		0			0			
	2.3 To 6.6	4	4	4	4	4	4			4	2.3 To 13.8	4	4	2.3		4	4	4	4	4	4	4	4	4	4	4.8	4.8	4.8	4.8		2.3	2.3	4.0	2.3	2.3	4.0	
	Δ	Y	Y	Y	Y	Y	Y			Y	Δ	Y	Y	Δ		Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Δ	Δ	Δ	Δ		2Φ3W	Δ	Y	2Φ3W	Δ	Y	
										240	240	240	240	240	240											300	300	300	300								
	ALL	ON	LINE		SIDE						ALL	ON	LINE	SIDE			LINE	SIDE		ALL	ON	LINE		SIDE			LINE	SIDE			ALL	ON	LINE		SIDE		
46	200	200	200				<30		70	50	60	50	1 To	200		14	14	14	14	30	25	25	25	25	25	93.5	68	68	68 To 93.5								
27.7	13.3	11.3	2.33	3.88	15.3	18.98	26.9	9.74	2.75	27.4	50.3				42.7	1.11	1.07	2.82	1.35	0.35	1.60	0.58	0.68	3.95	1.53	19.6	14.0	32.0	21.9	7.80	2.90	6.60	2.75	3.60	9.25	7.27	7.08
0.77	0.52	0.43	0.18	0.27	0.69	0.89	0	0.46	0.88	2.97	2.71	1.69	6.17	4.47	3.01	0.14	0.16	0.23	0.16	0.10	0.25	0.06	0.13	0.36	0.21	1.09	0	0	0.34	346[e]			0				
	0.10	0.09	0.07				0											0.03			0.11	0.06	0	0.15	0.10		0	0					0				
	0.26	0.12	0.04				0											0.03			0.02	0	0	0.05	0.02		0	0					0				
	0.16	0.22	0.07				0											0.18			0.13	0	0.13	0.15	0.10		0	0					0				
	4.10	4.10	0.82	0.91	4.42	3.74	6.92	2.89	0.38			3.82	3.82	6.30	4.60	0.09	0.13	0.20	0.11	0.15	0.15	0.26	0.19	0.31	0.20	0	7.0	0	2.96	0.33	0	0	0	0	0.50	0	0.28
	0.26	0.14	0.03	0	0.58	0	0	0.13	2.50								0.035	0.097	0.05	0						0	1.0	0	0.34		0	0	0				
																	0.004	0.002	0.003	0						0	0	0	0		0	0	0				
				No	No	No	No						No	No				No				No	No	No				No						No	No	No	

22[f]								23						24				25	26					27			28			29				30			
1936							Avg.	1934	1935	1936			Avg.	1936			Avg.	1934	1934	1936			Avg.	1934	1935	Avg.	1935	1936	Avg.	1934	1935	1936	Avg.	1934	1935	1936	Avg.
965	25	210	65	248	13	75	1516	3481	1623	260	256	71	1897	2515	2780	350	5645	1500	4500	2050	2050	180	4390	14500	8010	11255	8	19[g]	14	5200	3300	8277	5592	16172	13850	7850	12624
100		100							55.0	50	0	0		100						100								80	100			75		100	100	100	100
			100		100		100	100	14.4	20	10	0	100		100		100	100	100		100		100	100			100	20		100		15	100				
	100			100		100			30.6	30	90	100				100						100						0				10					
2.3	2.3	6.9	6.9	6.9	12	12	24-12		2.3&4	4	2.3	13.8		2.3	4	6.3	2.3&6.3		4	4	4	4	4		2.4		4	4	4		4&6.6	4&6.9	4&6.9	4.8	4.8	4.8	4.8
Δ	Δ	Δ	Δ	Δ	Y	Y	Δ&Y		Δ&Y	Y	Δ	Δ		Y	Y	Δ	Δ&Y		Y	Y	Y	Y	Y		Δ		Y	Y	Y		Y&Δ	Y&Δ	Y&Δ	Δ	Δ	Δ	Δ
240	240	240	240	240	240	240	240		200	200	200	200	200	200	200	200	200														200	250	250		300	250	250
LINE	LINE	LINE	LINE	LINE			BOTH		LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE		LINE	LINE	LINE	LINE	LINE		TRANS		LINE	SIDE			LINE	LINE	LINE	LINE	LINE	LINE	LINE
10	10	7	7	7	7	7		38	38					40	40	40	40	12	80	75	75	75	75				75	70	70-75	30	20	20	20-30	15	10	0+	10
13.	75	2	0.65		23.05	26.65	16.68	1.15	3.21	1.54	0.39	8.45	1.81	1.87	0.90	18.0	1.45			1.90	5.76	7.23	3.97				0	0	0	5.76	7.15	5.15	5.74	2.72	1.01		1.93
0.10	4.00	0.48	0	0.40	0	0	0.46	0	0.25	0	0	1.41	0.11	1.07	0.65	2.86	0.98		2.44	0.83	3.66	4.44	2.39	0.25	0.19	0.23	0	0	0	0.39	0.64	2.46	1.46	0.16	0.12	0.36	0.19
0	0	0	0	0.40	0	0		0		0	0			0.80	0.34	2.00	0.75			0.05	0.20	0.56	0.14				0	0	0		0.39				0.05	0.04	0.03
0	0	0	0	0	0	0		0		0	0			0.06	0.07	0.26	0.09			0	0.20	0.56	0.12				0	0	0		0.12	0.28			0	0	0
0.10	4.00	0.48	0	0	0	0		0		0	0			0.20	0.04	0.37	0.14			0.78	3.26	3.33	2.08				0	0	0		0.12				0.09	0.32	0.17
								0.03	0.12				0.06	0.68	0.29	3.43	1.56			0.16	0.34	0	0.23				0	0	0			0.02	0.02	2.17	0.95		1.61
								0	0.06	1.15			0.07							0.05	0.58	2.78	0.42							0.25			0.25				
								0	0				0																	0.29	0		0.18				
No	No	No	No	No	No	No	No			No	No	No		No	No	No	No			No	No	No					SOME	YES				No				SOME	

42	43	44			45	47	50					55	67					68	69			70					
1935	1935	1935	1936	Avg.	1936	1935	1935		1936		Avg.	1936	1935	1936			Avg.	1935	1935	1936	Avg.	1936					Avg.
13638	6000	300	1217	758	4071	689	1651	1318	1542	1220	2866	315	363	1284	985	172	1404	10000	15	12	14	84	10	45	94	98	331
	67	0	100		80	5	100	55.5						100								100					
	0	0		100			0	0							100		100		100	100	100		100				100
	33	100			20	35	0	44.5				100				100								100	100	100	
24-12	24-66	11.5	4	4&11.5	23-69	24-11	2.4	6.6	4	12	24-12	23-69	2.4	2.3	2.3	2.3	2.3		4.8	4.8	4.8	2.3	2.3	2.3	6.9	6.9&13.8	2.3-13.8
Δ&Y	Δ&Y	Y	Y	Y	Δ&Y	Δ&Y	Y	Y	Y	Y	Y	Δ&Y	Δ	Δ	Δ	Δ	Δ		2Φ4W	2Φ4W	2Φ4W	Δ	Δ	Δ	Δ	Δ	Δ
	200	450	450	450					250	250	250		400					250								230	
LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	TRANS	TRANS	TRANS	TRANS					LINE	
		13	20-25				17	17	16.5	16.5	17	28		668	668	668	668	25-30									
4.01	3.55	10.70	2.22	3.89	8.85	15.1	3.51	21.7	3.05		8.65	14.29	4.68	0	8.19	18.02	4.60		0	8.33	3.70	15.49	10.00	6.67	21.3	6.13	13.0
0.24	0.62	0.67	0.16	0.26	0.81	0.58	1.57	0.46	1.36	0.41	1.01	1.27	1.10	0.08	0.51	0.58	0.39	0.59	0	0	0						
	0.05					0.58	1.09	0.38	0.71	0.16	0.63		0.55	0.08	0	0.58	0.14	0.07	0	0	0						
	0.02					0	0.12	0	0.13	0	0.07		0	0	0.20	0	0.07	0.03	0	0	0						
	0.55					0	0.36	0.08	0.52	0.25	0.31		0.55	0	0.30	0	0.18	0.49	0	0	0						
	0.44				0.20								0.28	0	0	0.58	0.07		0	0	0			2.22			
		0			3.64	0	0	0	0	0	0	1.90	0	0	0.10	0	0.04		0		0	2.38			1.06		1.65
		0				0	0	0			0		0						0		0						
			No		No				No	No				No	No	No				No		No	No	No	No	No	No

a. Covers 45% of Installations.
b. 3Φ Transformers.
c. Failures per 100 Installations taken from Company rates
d. 550 3Φ Transformers.
e. Includes both bushing and winding.
f. Installation total includes a few interconnected transformers. However, all failures were on Standard Connection
g. No Spark gap used.
h. 3 Years

Table II. Summary of Operating Data by Company and by Year

Solid Interconnection—Figure 2

COMPANY	2				4									6*						7				8		
YEAR	1934	1935	1936	AVG.	1934	1935				1936			AVG.	1934	1935	1936			AVG.	1934	1935	1936	AVG.	1934	1934	
NO. OF INSTALLATIONS REPORTED ON	3708	3051	13371	3577	502	30	546	25	25	693	253	215	783	4652	5031	1154	3404	612	4951	28	853	1321	801	154	3536	9291
TYPE OF TERRITORY BY PERCENTAGES																										
URBAN		80	73			70		0	0	100					21.8	100										60
SUBURBAN	100	20	17	100	100	0		50	50		100		100	100	63.7		100		100					100	100	40
RURAL		0	10			30		50	50			100			18.5			100		100	100	100	100			0
VOLTAGE OF SYSTEM - KV.	2&4	2&4	2&4	2&4	2.3	2.3	2.3	11	13	2.3&4	2.3&4	2.3&4	2.3-13	4	4	4	4	4	4	4	4	4	4			2.4
PHASE CONNECTION	Δ	Δ	Δ	Δ	Δ	Δ	Δ	Δ	Δ	Δ&Y	Δ&Y	Δ&Y	Δ&Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y		Δ	Δ
FUSING SCHEDULE - %	300	200	200	200						250	250	250	250													
POSITION L.A. WITH RESPECT TO FUSE	TRANS	TRANS	TRANS	TRANS		TRANS	BOTH	LINE	LINE	80%	TRANS	[illegible] LINE	BOTH	TRANS	TRANS	TRANS	TRANS	TRANS	TRANS	LINE	LINE	LINE	LINE			
AVERAGE GROUND RESISTANCE OHMS	3				25	23.7	4.5	15	15	4	6	6		5	6.7	6.7	6.7	6.7	6	8.7	2.5	2		70	120	20
FAILURE DATA PER 100 INSTALLATIONS																										
PRIMARY FUSES BLOWN	0.94	1.67	2.68	2.13	19.5	1.11	4.58	0	0	3.03	10.69	4.19	7.70	1.38	0.86	0.26	1.97	2.46	1.92	0	3.64	5.05	4.50		32.5	
RATIO-CHANGE OVER STD. CONNECTION-%	100.0	44.9	48.9	65.5	213	2.27	9.57	0	0	85.8	136.3	13.5	64.0	81.5	61	5.03	70.3	45.4	36.4	0	32.2	29.5	46.2		64.2	
TRANSFORMER WINDING	0.14	0.34	0.36	0.52	0.40	0	0.18	0	0	0.29	0.40	0	0.25	0.16	0.22	0.36	0.18	0	0.19	0	0.47	0.86	0.71		1.73	0.68
RATIO CHANGE OVER STD. CONNECTION-%	63.9	50.0		68.0	43.5	0	28.9	0	0	53.8	42.1	0	19.1	80.5	122	∞	17.6	*	73.0	0	103	112	157		63.9	15.2
PRIMARY ONLY						0	0.18	0	0	0.29	0	0	0.16		0	0.09	0.09	0	0.04	0	0.12					
RATIO CHANGE OVER STD. CONNECTION-%						0	5.40	0	0	85.3	0	0	22.5		*	∞	∞	*	∞		133					
SECONDARY ONLY						0	0	0	0	0	0	0	0		0.04	0	0.03	0	0.03	0	0.12					
RATIO CHANGE OVER STD CONNECTION %						0	0	0	0	0	0	0	0		∞	*	3.94	*	12.5		100					
PRIMARY & SECONDARY						0	0	0	0	0	0.40	0	0.05		0.18	0.26	0.06	0	0.14	0	0.25					
RATIO CHANGE OVER STD CONNECTION %						0	0	0	0	0	83.5	0	8.35		100	∞	23.1	*	80.4		104					
LIGHTNING ARRESTER	0	0.55	0.04	0.20	0	0	1.46	0		0.28	1.19	3.72	0.90		0.39	0.17	0.21	0.49	0.30	3.87	0.94	0.86	0.92			
RATIO CHANGE OVER STD. CONNECTION %	0	438	133	195	0	0	18.15	*		104.0	180.5	135.2	110.0		∞	∞	∞	∞	∞	102.1	22.9	20.7	31.8			
CUSTOMERS' METERS BURNED OUT	0.32	0.57		0.50	0.40	0	0	0		0.43	0	0.47	0.36			1.38	2.46	0.65	2.01							
RATIO CHANGE OVER STD CONNECTION %	83.7	83.9		96.1	363	0	0	*		36.0	0	4.06	11.10			306	324	24.1	136							
CUSTOMERS' WIRING, ETC. DAMAGED	0				0.40	0	0	0		1.01	0	0	0.39		0.68	3.29	0.35	0.16	0.83		0.23	0.66	0.51			
RATIO CHANGE OVER STD. CONNECTION %	*				444	0	0	*		215	0	0	0.95		∞		135		830		164	212	381			

COMPANY	20	21				23				24	28			29			30				31	32	33			
YEAR	1934	1934	1936		AVG.	1934	1935	1936	AVG.	1936	1935	1936	AVG.	1934	1935	AVG.	1934	1935	1936	AVG.	1934	1934	1934	1935	1936	AVG.
NO. OF INSTALLATIONS REPORTED ON	1300	4382	134	18	2367	1919	2175	2757	2284	10	38	33	36	20	20	20	29980	36300	34689	33556	50	6831	866	813	411	697
TYPE OF TERRITORY BY PERCENTAGES																										
URBAN			100			100	100	98		100		80			55			77	0		100		100	100	93.9	98.8
SUBURBAN	100	100			100			2	100		100	20	100	100	45	100	100	32.3	100	100		100			0	0
RURAL				100				0							0										6.1	1.2
VOLTAGES OF SYSTEM-KV			13	13		4	4	4	4	2.3&4	4	4	4	4-6.6	4-6.6	4-6.6	4.8	4.8	4.8	4.8				2.4-4.4	[illegible]	
PHASE CONNECTION			Y	Y		Y	Y	Y	Y	Δ&Y	Y	Y	Y	Y-Δ	Y-Δ	Y-Δ	Δ	Δ	Δ	Δ				Δ	Δ&Y	
FUSING SCHEDULE - %			250	250		200	200	200	200	200				200	200	200	300	300	250	300						
POSITION L.A. WITH RESPECT TO FUSE			LINE	LINE		TRANS	TRANS	TRANS	TRANS	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE	LINE				BOTH	BOTH	BOTH
AVERAGE GROUND RESISTANCE-OHMS	0.25	3.8				1	1	2	1-2		10	10	10	19	2	2	10	10	12	10-12		1	30		30	30
FAILURE DATA PER 100 INSTALLATIONS																										
PRIMARY FUSES BLOWN	1.46	0.15	1.49	0	0.18	0.10	1.20	0.22	0.30		0	0	0	0	0	0	2.78	1.89		2.02		0.51	1.50	4.68	2.92	3.01
RATIO CHANGE OVER STD CONNECTION-%	53.1	3.72	†	†	4.63	3.04	37.4	14.5	27.6		*	*	*	0	0	0	102.5	138		105		†	11.1	76.8	445	25.5
TRANSFORMER WINDING	0.08	0	0	0	0	0.05	0.05	0.04	0.04		0	0	0	0	0	0	0.47	0.15	0.46	0.35		0.09	0.12	0.12	0.34	0.14
RATIO CHANGE OVER STD CONNECTION-%	62.5	*	†	†	0	∞	20.0	∞	40.0		*	*	*	0	0	0	308	125	128	189		†	14.5	14.3	453	19.1
PRIMARY ONLY		0	0	0	0						0	0	0	0	0	0		0.06	0.13	0.10						
RATIO CHANGE OVER STD CONNECTION-%		*	†	†	*						*	*	*	0	0	0		200	325	297						
SECONDARY ONLY		0	0	0	0						0	0	0	0	0	0		0.03	0.05	0.03						
RATIO CHANGE OVER STD CONNECTION-%		*	†	†	*						*	*	*	0	0	0		∞	∞	∞						
PRIMARY & SECONDARY		0	0	0	0						0	0	0	0	0	0		0.07	0.30	0.180						
RATIO CHANGE OVER STD CONNECTION %		*	†	†	0						*	*	*	0	0	0		77.8	59.7	107						
LIGHTNING ARRESTER	0	0.07	0	0	0.06	0	0.14	0.07	0.07		0	0	0	0	0	0	2.39	1.38		1.83		0.61				
RATIO CHANGE OVER STD. CONNECTION %	*	1.64	†	†		0	117		122		*	*	*		0	0	110	145		114		†				
CUSTOMERS' METERS BURNED OUT	0	0	0	0	0	0	0.09	0.11	0.07					0	0	0		0		0	0	0.15				
RATIO CHANGE OVER STD CONNECTION-%	*	*	†	†	*	*	150	9.55	98.7					0	0	0						†				
CUSTOMERS' WIRING, ETC. DAMAGED	0	0	0	0	0	0								0	0	0		0		0	0	0				
RATIO CHANGE OVER STD CONNECTION-%	*	*	†	†	*	*								0	*	0						†				

4. *Type CSP transformer* (figure 9) which is similar to the type *SP* transformer except a secondary breaker is used and a high-rating fuse link is installed in the transformer. The intention is that no primary fuse be used.

In each of the protection schemes described, except the type *SP* and *CSP* transformers, the primary fuse or fuses may be installed on either the line or transformer side of the lightning arresters. On three-phase four-wire systems except for type *SP* and *CSP* transformers the lightning arrester ground is interconnected with the primary neutral usually through a gap whereas on three-wire systems two arresters and primary fuses are used. These alternatives are indicated on figures 1 to 5 and 8.

Operating Results 1934–1935–1936

DETAIL DATA BY COMPANIES

While the data collected have been summarized to draw various comparisons in the tables of this paper it is believed the principal value of a paper of this type is to include the detail data submitted by each operating company so that the information can be set up to draw

Table II (Continued). Summary of Operating Data by Company and by Year

Solid Interconnection—Figure 2

9					10				12	13						14				15				16				17				18	19				
1936				Avg.	1934	1935	1936	Avg.	1935	1934	1935	1936			Avg.	1934	1935	1936	Avg.	1934	1935	1936	Avg.	1934	1935	1936	Avg.	1934	1936		Avg.	1934	1934	1935	1936		Avg.
7618	559	2344	857	12085	21000	26167	30614	25927	2847	1834	4380	838	2846	309	3436	37	100	99	99	4143	2165	2150	2819	900	2340	3668 c	2303	9	73	2	45	427	144	235	300	83	244
85 18 0	0 20 80	10 20 70	10 20 70	100	100	100	100	100	100	100	15 70 15	100	100	100	100	100	100	100	100	100	100	100	100		60 20 20	30 40 30	100	100	100	100	100	100	100	81.2 0 18.8	100	100	100
4 Y	4816 Δ&Y	6.9 Y	11.5-115 Δ&Y	24-115 Δ&Y	4 Y Line	4 Y Line	4 Y Line	4 Y Line	4 Y Line	4 Y Line	4 Y Line	4 Y Line	4 Y Line	4 Y Line	4 Y Line	4.8 Δ 300 Line	4.8 Δ 300 Line	4.8 Δ 300 Line	4.8 Δ 300 Line	2.4 2413W Line	2.4 2413W Line	2.4 2413W Line	2.4 2413W Line		2.4-4.16 Δ&Y Line	2.4-4.16 4.8 Δ&Y 300 Line	2.4-4.16 Δ&Y		4 Y 200 Town	12 Y 200 Town		4-13.2 Y Line		2.4 Δ Line	2.3-13.8 Δ&Y Line	2.3-13.8 Δ&Y Line	
20	50	30	50		0.5	0.5	0.2	<0.5		2	3	2	2	2	2	22.8	17	17	17-23					25	20			10.5	1-5	1-5		3	30				
					0.38 26.5	0.39 36.4	0.76 26.9	0.31 37.9	0.49 †	0.13 44.1	0.41 25.6	0.22 37.9	0.35 51.5	0.13 3.19	0.36 23.5	10.00 52.5	10.00 71.4	18.20 56.9	12.82 38.5	0.34 12.0	0.60 20.7	0.56 15.6	0.76 10.7						1.27 176		1.27 30.1	0.47 40.3	5.6 62.6	2.55 46.5	1.67 71.0	18.85 17.5	3.83 37.1
0.91 148	1.43	2.51	7.47	1.34 44.5	0.07 47.9	0.09 38.1	0.13 36.1	0.10 63.5	0.11 †	0.109 103	0.11 44	0 0	0 0	0 0	0.07 32.9	1.03 100	0 *	0 *	0.34 101	0.60 17.3	0.42	0.05	0.41		0.21 36.9	1.09 109	0.75 75		1.27 75.6	0 0	0.12 57.0	0.7 98.5	2.78 264	0 0	0 0	0 0	0.55 81.8
							0.02 80.0	0.02 80.0	0.07 †		0.02 1.82	0 0	0 *	0 0	0.01 10.3		0 *	0 *	0 *										1.27 265	0 *	0.12			0 0	0	0	0
							0.02 96.4	0.02 96.4	0.04 †		0.05 250	0 *	0 *	0 0	0.02 105		0 *	0 *	0 *										0 0	0 *	0			0 0	0	0	0
							0.08 46.5	0.08 46.5	0 †		0.05 38.4	0 *	0 0	0 0	0.02 20.6		0 *	0 *	0 *										0 0	0 0	0			0 0	0	0	0
	0 0				0.141 164.1	0.13 101.0	0.25 124	0.18 159	0.17 †	0.15 102	0.16 107	0.11 35.0	0.14 73.7	0 0	0.14 71.7	1.03 ∞	6.00 85.7	0 *	2.36 100	0.05 14.5	0.65 ∞	0.05 ∞	0.20 96.5						3.8 ∞	0 0	0.37 34.9	0	1.39 158	0.43 598	0 *	1.89 472	0.85 183
7.34						0.06 171.5	0.09 94.3	0.08 151.0	0.36 †	0 *		0	0	0	0	0 *	0 0	0 *	0 0	0.57			0.57	0 *		0 *	0 *	0 0	0 0	0 *	0 0	0	0 0	0 0			0 0
						0.005 62.5	0.001 50.0	0.001 46.7	0 †	0 *		0	0	0	0	0 *	0 *	0 *	0 *					0		0 *	0	0 0				0	0 *	0 0	0 *	0 *	0 0

24				36				37			40	42	45	50					21			25	69		
1934	1935	1936	Avg.	1934	1935	1936	Avg.	1934	1935	Avg.	1935	1935	1936	1935		1936		Avg.	1936		Avg.	1936	1935	1936	Avg.
41	40	38	40	11122 a	11488	15227	12536	2063	2772	2418	1900	1885	1298	360	5	487	535 e	709	2170	2057	4216	38	392	403	398
100	100	100	100	100	40 60 0	40 60 0	100	100	100	100	100		100	100	100				100	100	100	100	100	100	100
2.3 Δ 230 Town	2.3 Δ 230 Town	2.3 Δ 230 Town	2.3 Δ 230 Town	4 Y 230 Line	4 Y 230 Line	4 Y 230 Line	4 Y 230 Line	4 Y 250 Line	4 Y 250 Line	4 Y 250 Line	4 Y Town	2.4-12 Δ&Y Line	2.3 Δ Line	6.6 Y Line	2.4 Y Town	12 Y Line	12 Y		4 Y 240 Town	12 Y 240 Town	240 Town	2.3-4.16 Δ&Y Line	4.8 2400W Town	4.8 2400W Town	4.8 2400W Town
0	0	0	0		0.5	0.2		10.0	5.0	5-10	1.0			17	17				4	10		28	14.1	14.1	14.1
4.88 125	5.00 1280	7.30 107	5.86 152	2.57 339	0.56 28.6	1.11 20.3	1.32 30.0	1.02 58.7	0.35 18.4	0.58 34.5	1.00 42.1	4.30 107	1.88 21.2		0 0		0		1.14 †	6.30 †	3.62 †	1.70 11.9	13.5 ∞	3.22 38.6	8.30 224
0 0	0 0	0 0	0 0		0.11 32.4	0.13 16.4	0.12 25.0	0.30 57.7	0.14 36.8	0.21 45.0	0.16 28.1	0.27 112	0.16 19.7	0.88 180	0 0	1.01 246	0.18 44.0	0.64 62.9	0.03 †	0.20 †	0.14 †	0 0	0.31 ∞	0.25 ∞	0.38 ∞
0	0 0	0 0	0 0								0.11			0.28 73.8	0 0	0.40 250	0 0	0.21 32.7	0 †	0.10 †	0.05 †	0	0 *	0 *	0 *
0	0 *	0 0	0 0								0 0			0.55 ∞	0 0	0.60 ∞	0 *	0.333 304	0 †	0 †	0 †	0	0.25 ∞	0.25 ∞	0.25 ∞
0	0 0	0 0	0 0								0.05			0 0	0 0	0 0	0.18 72.0	0.07 22.6	0.09 †	0.10 †	0.10 †	0	0.25 ∞	0 *	0.13 ∞
0 *	0 0		0 0		0.10 250	0.12 37.1	0.11 121		0.50 28.1	0.50 28.1	0.16 48.6		0.31 148		0 0				0.64 †	1.08 †	0.85 †		1.73 ∞	0.50 ∞	1.13 ∞
					0.01 250	0.03 28.1	0.02 33.8				0 0		5.87 161	0 *	0 *	0 *		0 *	0.05 †	0.05 †	0.05 †	0 0	1.28 ∞		1.28 ∞
					0 0	0 *	0 0				0 0			0 *	0 *				0 †	0 †	0 †	0	0.51 ∞		0.51 ∞

* No failures either connection.
† Standard Connection not used.
a Not directly comparable with Fig. 1.
b Lightning arrester used on each wire.
c Includes some Gapped Interconnection
d Includes some Gapped Interconnection.
e Spill gap used instead of lightning arr.

any comparison desired. The data submitted by each company for the three-year period have been included in tables I to IX. Each table covers a type of protection. All failures and primary fuses blown are those attributed to lightning. All rates of trouble are the number per 100 transformer installations. Primary fuses blown are exclusive of those blown by arrester or transformer failures.

Comparison of Various Schemes of Protection

In tables X, XI, and XII the data have been summarized for the three years. Since in 1935 and 1936 the data was obtained in more detail, tables XIII, XIV and XV have been prepared covering average data for these two years. Since the reports for many companies did not segregate operating data by types of territory tables X and XIII represent a greater number of installations than the sum of those represented in the component tables covering rural and more thickly populated territories. In the summary tables, i.e., X to XXIV, the average number of transformer installations in each case is the average of the total installations for the years covered in each table. However, the trouble rate for

Table III. Summary of Operating Data by Company and by Year

Solid Interconnection With Tie to Case—Figure 3

Company	3	4						11			13	16			18	33		
Year	1935	1934	1935			1936	Avg.	1935	1936	Avg.	1936	1935	1936	Avg.	1936	1935	1936	Avg.
No. of Installations Reported On	103	35	7	514	31	58	215	1477	1500	1489	103	570	316[a]	743	28	45	79	62
Type of Territory By Percentages																		
Urban	100			48	0		100	100	100	100		30	20	100		100	82.3	88.7
Suburban		100		17	39							50	30		100		0	0
Rural			100	35	61	100					100	20	50				17.7	11.3
Voltage of System - KV	4	11&13.2	24	24	11	23-112		2.4	2.4	2.4	4	6.9	6.9	6.9	13.2	2.4-6.6	2.4-12	
Phase Connection	Y	Δ	Δ	Δ	Y	Δ&Y		2&4W	2&4W	2&4W	Y	Δ	Δ	Δ	Y	Δ&Y	Δ&Y	
Fusing Schedule - %						300							300					
Position L.A. With Respect To Fuse	Line		Trans	Line	Trans	Both		Line	Line	Line	Line	Line	Line	Line	Line		Both	
Average Ground Resistance - Ohms		20	56		133	10		0.10	0+	0+	2	20			0.5		30	
Failure Data Per 100 Installations																		
Primary Fuses Blown	4.85	22.8	0	6.03	0	24.15	8.22	0.07	0	0.03	0.97				50.0		5.06	5.06
Ratio - Change over Std. Connection - %	74.2		0	12.3	0	78.0	68.3	†	†	†	24.6				108		77.3	43.5
Transformer Winding	0.97	14.3	0	1.36	0	0	1.86	0	0	0	0	0.53	0.11	0.37	7.15	0	1.27	0.81
Ratio Change over Std. Connection - %	5.70		0	21.8	0	0	1.42	†	†	†	0	39.3	11.0	37.1	46.5	0	240	107
Primary Only			0	0.39	0	0	0.33	0	0	0	0					0		
Ratio Change over Std. Connection - %			0	11.7	0	0	46.2	†	†	†	0							
Secondary Only			0	0.39	0	0	0.33	0	0	0	0					0		
Ratio Change over Std. Connection - %			0	35.2	0	0	109.2	†	†	†	0							
Primary & Secondary			0	0.58	0	0	0.49	0	0	0	0					0		
Ratio Change over Std. Connection %			0	32.6	0	0	82.0	†	†	†	0							
Lightning Arrester		0	0	0.19	0	0	0.16	0	0	0	0							
Ratio Change over Std. Connection - %			0	2.44	*	0	18.9	†	†	†	0							
Customers' Meters Burned Out		0	0	0	0	0	0	0	0	0	0		0	0				
Ratio Change over Std Connection - %			0	0	*	0	0	†	†	†			*	*				
Customers' Wiring, Etc. Damaged		5.72	0	0	0	0	0.31	0	0	0	0		0	0				
Ratio Change over Std Connection - %			0	0	*	0	75.6	†	†	†			*					
Tie to Case - Solid(S) or Gapped(G)	S	S	S	G	G	Both	Both	S	S	S	S	S	Both	Both	S	G	G	G

* No failures either connection.
† Standard Connection not used.

a. Includes some Gapped Interconnection with Tie to Case.

Table IV. Summary of Operating Data by Company and by Year

Gapped Interconnection—Figure 4

Company	1	4							5[a]						7	13	14				19					
Year	1935	1935			1936			Avg.	1934	1935	1936			Avg.	1936	1936	1934	1935	1936	Avg.	1934	1935	1936		Avg.	
No. of Installations Reported On	1000	573	23	89	191	150	690	858	646	707	17[b]	434[b]	291[b]	698	883	260	98	101	101	100	72	998	50	1220	780	139
Type of Territory by Percentages																										
Urban	0	7		3	100			100		2.0	100			100									100		21.3	15
Suburban	30	5		10		100			100	50.2		100														60
Rural	70	88	100	87			100			47.8			100		100	100	100	100	100	100	100	100		100	78.7	25
Voltage of System - KV	4-12	2.4	7.6	13	23-132	23-132	23-132	23-132	4	4	4	4	4	4	4	4	4.8	4.8	4.8	4.8	24-138	24-138	24-138	24-138	24-138	4
Phase Connection	Y	Δ	Δ	Δ	Δ	Δ	Δ	Δ	Y	Y	Y	Y	Y	Y	Y	Y	Δ	Δ	Δ	Δ	Δ&Y	Δ&Y	Δ&Y	Δ&Y	Δ&Y	Y
Fusing Schedule - %					250	250	250										300	300	300	300						200
Position L.A. With Respect To Fuse	Line	Both	Both	Trans	Both	Both	Both	Both	Trans	Trans	Trans	Trans	Trans	Trans	Line	Line	Line	Line	Line	Line	Line	Line	Line	Line	Line	Trans
Average Ground Resistance - Ohms	15	30	60	20	25	50	50	58	5	250	250	250	250	174	2	25	101	75	75	83	30					10
Failure Data Per 100 Installations																										
Primary Fuses Blown	3.50	10.1	4.35	38.5	10.5	28.0	10.0	14.10	3.13	1.27	5.89	3.69	3.44	2.68	3.96	1.92	3.2	139	228	15.33	7.20	3.91	8.00	7.37	5.90	3.02
Ratio Change over Std Connection - %	88.6	20.7	7.34	65.2	298	338	324	117	161	90.0	114	132	63.7	114	23.2	48.5	47.0	99.2	72.2	70.0	81.8	71.4	340	68.5	86.1	196
Transformer Winding	0.70	0.35	4.35	0	0.52	1.33	1.01	0.76	0.16	0.14	0	0.92	0	0.29	0.45	0	0	1.98	1.98	1.33	0	0.10	0	0.74	0.43	0
Ratio Change over Std Connection - %	66.6	5.62	28.2	0	97.0	140	387	58.0	78.0	77.8	*	90.1	*	110	58.5	0	0	∞	∞	396	0	30.3	0	106	93.2	*
Primary Only	0.60	0.35	4.35	0	0	0.67	0.58	0.47		0	0	0.46	0	0.14		0	0	0.99	0.99	0.66	0					0
Ratio Change over Std Connection - %	68.2	10.5	113	0	0	186	60.5	66.2		*	*	∞	*	∞		0		∞	∞							*
Secondary Only	0.10	0	0	0	0	0	0.15	0.06		0.14	0	0	0	0.07		0	0	0	0	0	0					0
Ratio Change over Std Connection - %	62.5	0	*	0	0	0	21.0	20.0		∞	*	0	*	26.6		0		*	*							*
Primary & Secondary	0	0	0	0	0.52	0.67	0.29	0.23		0	0	0.46	0	0.14		0	0	0.99	0.99	0.66	0					0
Ratio Change over Std Connection - %	*	0	0	0	388	139.0	30.2	38.4		0	*	177	*	78.3		0		∞	∞							*
Lightning Arrester	4.00	2.10	0	13.5	2.62	1.33	0.72	2.10		0.42	0	0.69	0.34	0.48	0.29	0.38	0	3.96	0	1.33	1.39	0.30	2.00	0.37	0.31	0
Ratio Change over Std Connection - %	172	270	0	∞	970	198	26.2	286		∞	*	∞	∞	∞	7.80	123	*	86.6	*	36.3	138	375	∞	143	143	
Customers' Meters Burned Out	0.20	2.62	0	13.5	4.19	4.66	2.09	3.26								0	0	0	0	0	0	0				0
Ratio Change over Std Connection - %	10.0	147	*	∞	346	140	172	27.6									*	0	*	0	*	*				0
Customers' Wiring, Etc. Damaged	0	0	0	0	0	0	0.15	0.06		0.28	0	0.46	0	0.29	0.11	0	0	0	0	0	0	0				
Ratio Change over Std Connection - %	*	0	*	*	0	0	30.2	12.5		∞		177		276	33.5		*	*	*	*	0	*				

* No failures either connection
† Standard Connection not used.

a Not directly comparable with Fig. 1

b Lightning Arrester used on neutral.

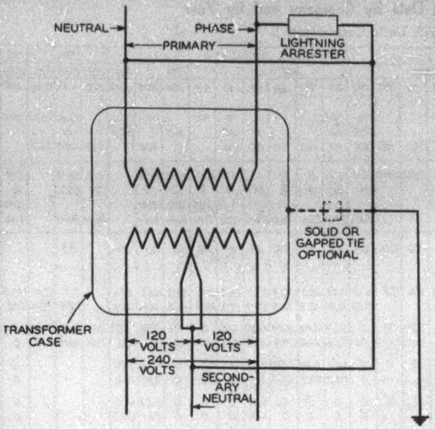

Figure 6 (left). Common primary and secondary neutral

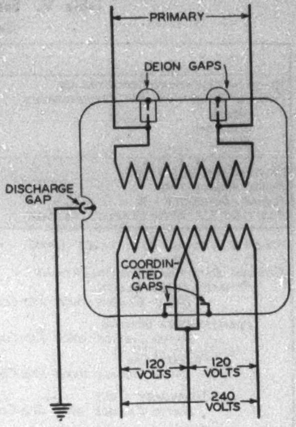

Figure 7 (right). Surge-proof transformer

each item of trouble is determined by taking the sum of troubles for each item for the period under consideration as a percentage of the sum of the installations on which data were reported for that item. Since very few companies failed to supply data for primary fuses blown, transformer winding failures, and lightning arrester failures, the trouble rate in each of these cases may be considered as based upon the average of the total number of installations shown in the tables. However in the case of the segregation of the location of transformer winding failures, a number of companies did not supply segregated data. Therefore, the sum of the segregated rates will not equal the total rates of transformer winding failures.

In comparing the relative *rates of primary fuses blown* using the various protective schemes, a study of tables X to XV shows that the schemes using solid interconnection show the lowest rate of primary fuses blown using any of the six bases of comparison. Where sufficient data is available to compare the conventional solid interconnection, the interconnection with tie to transformer tank; and the common primary and secondary neutral,

Table IV (Continued). Summary of Operating Data by Company and by Year

Gapped Interconnection—Figure 4

23			26			33			34				35
1936		Avg.	1936		Avg.	1935	1936	Avg.	1934	1935	1936	Avg.	1934
76	9	289	660	350	1010	2543	2419	2481	57	58	53	56	193
0							18.3	8.9	100	100	100	100	
53		}100	100		}100		0						
47	100			100		100	81.7	91.1					100
2.3	13.8		4	4	4	24-6.6	23-12	23-12	2.4	2.4	2.4	2.4	
Δ	Δ		Y	Y	Y	Δ	Δ+Y	Δ+Y	Δ	Δ	Δ	Δ	
200	200	200							240	240	240	240	
Town	Town	Town	Line	Line	Line	Both	Both	Both	Line	Line	Line	Line	
10	10	10	73	73	75		30		0				
0	11.1	2.46	1.06	8.86	3.76	8.26	7.86	8.06	0	3.45	7.55	2.80	30.0
0	131	136	18.4	123	94.7	135	120	63.5	0	88.4	102	72.5	53.5
0	0	0	1.06	8.00	3.46	0.04	0.41	0.22	0	0	0	0	
*	0	0	29.0	180	145	480	77.3	29.6	0	0	0	0	
0	0	0	0	0	0				0	0	0	0	
*			0	0						0	0	0	
0	0	0	0.15	0	0.10				0	0	0	0	
*			75.0	0						*	0	0	
0	0	0	0.31	8.00	3.37				0	0	0	0	
*			27.9	240						0	0	0	
0	0	0	0.30	1.43	0.69				0				
			88.2	∞					*				
0	0	0	2.25[c]	13.42[c]	6.14[c]								
		0	393	482									

36			38				39			40	42	44		
1935	1936	Avg.	1934	1935	1936	Avg.	1935	1936	Avg.	1935	1935	1935	1936	Avg.
4634	5083	4839	202	151	151	168	527	357	442	400	112	50	50	50
0	0	0					75	80						
10	10	10	100	100	100	100	15	10	}100					
90	90	90					10	10		100		100	100	100
4	4	4	4	4	4	4	24-12	24-12	24-12	4	4	11.5	11.0	11.0
Y	Y	Y	Y	Y	Y	Y	Δ+Y	Δ+Y	Δ+Y	Y	Y	Y	Y	Y
240	240	240					200	200	200			450	450	450
Town	Town	Town	Line	Line	Line	Line	Town	Town	Town	Town		Line	Line	Line
20	20	20	15 to 100	15 to 100	15 to 100	15 to 100	10	10	10	25		13	20-25	
10.0	18.40	14.41	0.97	0.66	2.65	1.39				3.00	3.57	10.0	10.0	10.0
510	3.36	328	19.1	20.6	70.8	39.8				126	89.0	93.4	450	267
0.63	0.36	0.81	0.33	0.66	0	0.40	0	0	0	0.50	0	0		0
185	122	168	31.1	220	0	99.0		0	0	87.7	0	0		
					0		0	0	0		0	0		0
											0	0		
					0		0	0	0		0	0		0
											0	0		
					0		0	0	0		0	0		0
											0	0		
0.54	1.38	0.98		0.66	0.66	0.66	0	0	0	0.75		0		0
1350	657	1090		412	254	314				227		0		
0.02	0.26	0.15						0	0	0		0		0
50	200	223	39.0[d]							0		*		
0.09	0.14	0.11						0	0	0		0		0
225	∞	456								0		*		

45	55	69			70							
1936	1936	1935	1936	Avg.	1936							Avg.
2380	447	122	136	129	153	10	8	12	121	26	20	350
					100	100			100			
		100	100	100			100			100		}100
100	100							100			100	
6.9	23-6.9	4.8	4.8	4.8	2.3	13	13.2	2.3	2.4	12	13.8	
Y	Δ+Y	2φ4W	2φ4W	2φ4W	Δ	Y	Δ	Δ	Δ	Δ	Δ	
					200	200	200	200				
Line	Line	Town	Town	Town	Line	Line	Line	Line	Town	Town	Town	
	28					15	15	15	6	5.5	6.5	
6.30	4.32	13.9	2.94	8.15	0.65				0	0	0	0.31
70.1	34.4	∞	353	220	4.20				0	0	0	2.40
0.43	0.22	0	0.74	0.39								
53.1	17.3	*	∞	∞								
		0	0.74	0.39								
		*	∞	∞								
		0	0	0								
		*	*	*								
		0	0	0								
		*	*	*								
0.12		0.82	0	0.39				16.67	0	0	0	1.12
57.1		∞	*	∞				750	0			90.5
7.14	0.22	0.82		0.82				8.33	3.31			
196	11.6	∞		∞					139			
		0		0					8.27			
		*		*								

c. Company estimate

d. As compared with surrounding area.

Table V. Summary of Operating Data by Company and by Year

Gapped Interconnection With Tie to Case—Figure 5

Company	4														33			34
Year	1934				1935						1936			Avg.	1935	1936	Avg.	1936
No. of Installations Reported On	120	354	20	27	193	60	70	66	196[a]	13	62	124	367	557	312	777	545	10[a]
Type of Territory by Percentages																		
Urban	45	90			6	0			2	0	100					4.6	3.30	100
Suburban	34	10			25	22			0	38		100		100		0	0	
Rural	41	0	100	100	69	78	100	100	98	62			100		100	95.4	96.7	
Voltage of System - KV.	2.3	2.3	11-13.2	11-13.2	2.4	6.6	11	11	13	13	2.3-13.2	2.3-13.2	2.3-13.2		6.6	2.3-12		2.3
Phase Connection	Δ	Δ			Δ	Δ	Δ	Δ	Δ&Y	Δ	Δ	Δ	Δ		Δ	Δ&Y		Δ
Fusing Schedule - %											250	250	250					250
Position L.A. With Respect To Fuse					Both	Trans	Line	Trans	Both	Line	Both	Both	Both		Both	Both		Line
Average Ground Resistance - Ohms	75	25	25	60	60	54	50	23	30	6	20	25	25	38		30		
Failure Data Per 100 Installations																		
Primary Fuses Blown	0.83	2.26	20	111	4.15	1.67	14.3	30.3	75.0	15.4	37.1	35.5	44.6	27.6		8.11	8.11	10.0
Ratio - Change over Std. Connection - %					8.50	2.30	15.9	33.6	83.5	17.2	1051	454	145	230		124	69.9	135
Transformer Winding	0.83	0	0	0	0	1.67	2.86	3.03	4.08	7.70	0	0	3.54	1.67	0.96	0.26	0.46	0
Ratio Change over Std Connection - %					0	10.8	16.6	17.6	23.7	44.7	0	0	135	128	114	49.0	61.3	0
Primary Only		0	0	0	0	1.67	2.86	1.32	3.57	0	0	0	2.45	1.29				0
Ratio Change over Std. Connection - %					0	43.3	82.9	44.0	103	0	0	0	256	182				0
Secondary Only		0	0	0	0	0	0	0	0.51	0	0	0	0.27	0.13				0
Ratio Change over Std. Connection - %					0	*	0	0	14.8	0	0	0	39.1	43.0				0
Primary & Secondary		0	0	0	0	0	0	1.52	0	7.70	0	0	0.82	0.32				0
Ratio Change over Std. Connection - %					0	0	0	14.8	0	74.6	0	0	85.5	53.7				0
Lightning Arrester	0		0		2.59	0	0	0	5.10	0	0	0	2.45	1.86				
Ratio Change over Std. Connection - %					33.3	0	*	*	∞	*	0	0	89.1	227				
Customers' Meters Burned Out	0	0	0	0	0	0	0	0	0	0	0	0	1.36	0.30				
Ratio Change over Std. Connection - %					0	*	*	*	*	*	0	0	11.5	12.8				
Customers' Wiring, Etc. Damaged	0	0	0	0	0	0		0	0	0	0	0	0	0				
Ratio Change over Std. Connection - %					0	*		*	*	*	0	0	0	0				
Tie to Case - Solid (S) or Gapped (G)	G	G	S	S	S	S	S	S	S	S	Both	Both	Both		G	S		G

* No Failures either connection.

a. Slight modification of Fig. 5.

Table VI. Summary of Operating Data by Company and by Year

Solid Interconnection Obtained by Common Primary and Secondary Neutral—Figure 6

Company	1	3			4						9			11			12			13		
Year	1935	1935	1936	Avg.	1935		1936			Avg.	1934	1935	Avg.	1934	1936	Avg.	1934	1936	Avg.	1935		1936
No. of Installations Reported On	10	52	5500	2771	7	7	29	42	572	329	3375	6364	4970	4709	757	2733	3175	2490	2833	1050	301	1142
Type of Territory by Percentages																						
Urban			100			100	100				100			100	100	100		100		10	100	
Suburban								100		100		100	100				100		100	40		100
Rural	100	70-30			100				100											50		
Voltage of System - KV.	2.4	13	4&12.5		2.4	4	4, 11, and 13.2				4	4	4	4	4	4	4	4	4	4	4	4
Phase Connection		Y	Y				Y				Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Fusing Schedule - %	240						250	250	250		240	240	240									
Position L.A. With Respect To Fuse	Line	Line	Trans		Trans	Trans	Trans	Trans	Trans	Trans		Both	Both	Line	Line	Line	Line	Line	Line	Line	Line	Line
Average Ground Resistance - Ohms	2				18	3	3	3	3	30	10	50	35	0.6	0+		1.0	0+	0-1		20-	20-
Failure Data Per 100 Installations																						
Primary Fuses Blown	0	7.69	2.95	2.99	0	0	0	11.9	8.92	8.53	6.10	0	2.20	0.06	1.32	0.24	0.72	0.40	0.58	0.76	1.33	1.05
Ratio - Change over Std Connection - %	0	44.4	46.0	54.9	0	0	0	152.0	28.9	70.3	22.2		5.15	†	†	†	†	†	†	47.5	229	154
Transformer Winding	0	0	0.58	0.58	0	0	0	0	1.57	1.37	0.78	0.19	0.40	0.04	0.40	0.09	0.22	0.36	0.28	0	0	0
Ratio Change over Std Connection - %	0	0	45.4	35.4	0	0	0	0	60.1	104	26.2	11.2	13.4	†	†	†	†	†	†	0	0	0
Primary Only	0	0			0	0	0	0	0.35	0.30					0.40			0.20		0	0	0
Ratio Change over Std Connection - %	0	0			0	0	0	0	36.5	42.2					†			†		0	0	*
Secondary Only	0	0			0	0	0	0	0.35	0.31					0					0	0	0
Ratio Change over Std. Connection - %	0	0			0	0	0	0	50.7	102					†					0	*	*
Primary & Secondary	0	0			0	0	0	0	0.87	0.76					0					0	0	0
Ratio Change over Std Connection - %	*	0			0	0	0	0	90.6	127					†					0	*	0
Lightning Arrester					0	0	0	0	0.87	0.76		0.49	0.49	0.02	0.53	0.09	0.22	0.24	0.23	0	0	0.09
Ratio Change over Std. Connection - %					0	0	0	0	31.6	929		12.8	10.7	†	†	†	†	†	†	0	0	47.4
Customers' Meters Burned Out	0				0	0	0	0	0.17	0.15				0.02	4.10	0.59	0.22	1.65	0.86			
Ratio Change over Std. Connection - %	0				0	0	0	0	1.44	6.48				†	†	†	†	†	†			
Customers' Wiring, Etc. Damaged	0				0	0	0	0	0	0				0	0	0	0	0.04	0.02			
Ratio Change over Std Connection - %	*				0	0	0	0	0	0				†	†	†	†	†	†			
Grounded Tanks	No	No	Yes		No	No	Yes	Yes	Yes			No			Some			Yes		No	No	No

* No failures either connection.

† Standard Connection not used.

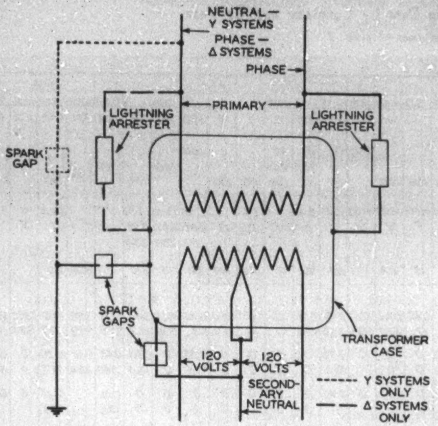

Figure 8 (left). Three-point protection

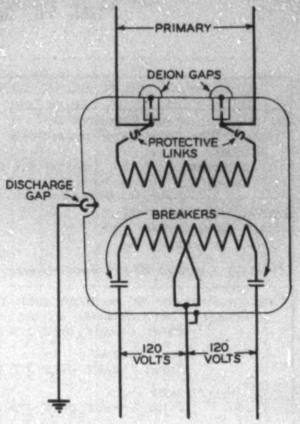

Figure 9 (right). Type CSP transformer

the data indicates some benefit of the tie with the tank. The common primary and secondary neutral seem to be about equally effective as the conventional interconnection. In rural areas the *SP* transformer ranks next to the solid interconnection schemes. In general it is believed that three-point protection ranks next. The gapped interconnection and the standard connection rank last. The gapped interconnection seems no better if as good as the standard connection. A possible explanation is that the impulse flashover of the gaps used may be as high as the bushing flashover voltages. If the secondary bushings flashover on a transformer protected

Table VI (Continued). Summary of Operating Data by Company and by Year

Solid Interconnection Obtained by Common Primary and Secondary Neutral—Figure 6

		15			18			20	21			
	Avg.	1935	1936	Avg.	1935	1936	Avg.	1935	1935	1936		Avg.
754	1624	10000	10000	10000	793	1539	1166	2020	4600	4835	5ᵃ	4719
100	100	100	100	100	100	100	100	100	85 15 0	100	100	100
4	4	4	4	4	4	4	4	24	4	4	4	4
Y	Y	Y	Y	Y	Y	Y	Y	Δ	Y	Y	Y	Y
									240	240	240	240
Line	Line	Line	Line	Line	Line	Line	Line	Line	Line	Line	Line	Line
20-	20-				3	0.5-			21	0.2±1		
2.12 53.6	1.23 80.3	1.78 64.7	3.00 41.2	2.39 33.8	1.01 †	0.52 62.7	0.69 59.7	1.14 74.0	0.41 9.0	0.41 †	0 †	0.41 10.1
0.13 36.2	0.03 14.5	0.98 ∞	1.17	1.08	0.25 †	0.33 54.0	0.30 42.3	0.05 20.0	0.07 1.5	0.02 †	0 †	0.04 6.17
0 0	0 0				0.25 †	0.33 66.0	0.30 18.7		0.04 ∞	0.02 †	0 †	0.03 ∞
0 0	0 0				0 †	0 0	0 0		0.02 ∞	0 †	0 †	0.01 ∞
0.13 86.6	0.03 32.0				0 †	0 0	0 0		0 0	0 †	0 †	0 0
0.27 87.0	0.09 47.2	0 *	0.06 ∞	0.03 10.7					0.04 ∞	0.04 †	0 †	0.04 1.25
									0 *	0.02 †	0 †	0.11 ∞
									0 *	0 †	0 †	0 *
No	No	No	No	No	No	Some		No	No	Yes	Yes	

28					31	32					33
1934	1935	1936		Avg.	1936	1935	1936			Avg.	1936
434	453	444ᵇ	111ᵇ	481	2847	7250	5120	1378	1182	7365	494
100	100	100	100	100	98.0	70 20 10	100	100	100	100	12.4 0 87.6
4	4	4	4	4	4	4	4	4	4	4	23-12
Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Δ4Y
						240	240	240	240	240	
Line	Line	Line	Line	Line	Line	Line	Line	Line	Line	Line	
10	10	10	10	10		1.0	0.7	1.0	1.5	1.0	30
0.23 †	0 *	0 0	0 0	0.07 1.74	0.84 ∞	0.32 †	0.10 †	0.25 †	0.42 †	0.24 †	3.04 46.4
	0 *	0 0	0 0	0 0	0.46 128	0.14 †	0.04 †	0.13 †	0.17 †	0.11 †	0.41 77.3
	0 *	0 0	0 0	0		0.04 †	0.02 †	0 †	0.08 †	0.03 †	
	0 *	0 *	0 0	0		0 †	0 †	0 †	0 †	0 †	
	0 *	0 0	0 0	0		0.10 †	0.02 †	0.13 †	0.08 †	0.07 †	
0.46 †	0 *	0.45 300	0 0	0.28		0.70 †	0.33 †	0.95 †	1.89 †	0.69 †	
1.62 †						0.29 †					
0 †						0.33 †					
	Some	Yes	Yes		No	No	No	No	No	No	Yes

50				51			56	70		
	1936		Avg.	1935		Avg.	1936	1936		Avg.
2ᶜ	80ᶜ	503ᶜ	585	2078	3638	5716	2830	210	53	263
100	100	100	100	100	100	36.3 0 63.7	100	100	100	79.8 20.2
12	12	12	12	4	12		4	4	4	4
Y	Y	Y	Y	Y	Y		Y	Y	Y	Y
				240	240	240		240	240	240
				Trans	Trans	Trans	Line	Line	Line	Line
17	17	17	17		20	20	0+			
100	6.25	2.78	3.59 41.5	2.89 †	9.12 †	6.85 †	0 †	0.95 6.14	3.77 35.5	1.52 11.7
0 0	0 0	0.40 97.7	0.34 29.6	0.05 †	0.27 †	0.19 †	0 †			
0 0	0 0	0.20 12.5	0.17 27.2	0.05 †	0.14 †	0.11 †	0 †			
0 *	0 *	0 *	0 0	0 †	0 †	0 †	0 †			
0 0	0 0	0.20 80.0	0.17 54.5	0 †	0.14 †	0.09 †	0 †			
				0.24 †	0.41 †	0.35 †	0 †			
0 *	0 *	0 *	0 *	0 †	0.16 †	0.11 †	0 †			
				0 †	0 †	0 †	0 †			
Yes	Yes	Yes	Yes	No	No	No	Yes	No	No	No

a. Rural type transformers using spill gap   b Includes Fig. 3   c. Horn gap instead of L.A.

Table VII. Summary of Operating Data by Company and by Year

Surge-Proof Transformer—Figure 7

Company	1			3	4									7			
Year	1934	1935	Avg.	1936	1935					1936			Avg.	1934	1935	1936	Avg.
No. of Installations Reported On	60	150	105	32	27	255	16	1	4	120	56	418	449	551	672	672	652
Type of Territory by Percentages																	
Urban				100	13	31	12			100					}10		
Suburban					0	18	50				100		}100	}100			}100
Rural	100	100	100		87	51	38	100	100			100			90		
Voltage of System - KV.	44125	44125	44125	44132	2.4	2.4	4	7.6	13	2.3-132	2.3-132	2.3-132			2414	4	
Phase Connection	Y	Y	Y	Y	Δ	Δ	Y	Δ	Δ	Δ&Y	Δ&Y	Δ&Y		✓	Y	Y	
Fusing Schedule - %										250	250	250					
Average Ground Resistance - Ohms	20	15	16.4		83	120	3	150	160	50	75	100		35.6	150		
Failure Data per 100 Installations																	
Primary Fuses Blown	0	0.67	0.49	37.5	3.70	3.14	0	0	25.0	5.83	46.5	9.80	9.36	9.78	11.0	15.92	18.10
Ratio - Change over Std. Connection - %	0	17	5.65	688	7.60	6.42	0	0	27.9	16.5	594	31.7	77.9	91.9	82.7	93.0	106
Transformer Winding	0	0	0	0	3.70	0	0	0	0	0.83	0	0.48	0.45	0.18	0.30	0	0.32
Ratio Change over Std. Connection - %	0	0	0		59.5	0	0	0	0	134	0	18.4	34.0	35.4	57.7	0	41.2
Primary Only	0	0	0	0	3.70	0	0	0	0	0	0	0	0.11		0.15	0	0.07
Ratio Change over Std. Connection - %		0			111	0	0	0	0	0	0	0	15.6		150		
Secondary Only	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0
Ratio Change over Std. Connection - %		0			0	0	0	*	0	0	0	0	0		0		
Primary & Secondary	0	0	0	0	0	0	0	0	0	0.83	0	0.48	0.33		0.15	0	0.30
Ratio Change over Std. Connection - %		*			0	0	0	0	0	615	0	50.0	36.6		93.7		
Deion Gap	0	0	0	0	0	0.78	0	0	0	0	0	0.72	0.56	1.99	1.48	3.28	2.96
Ratio Change over Std. Connection - %	0	0	0		0	10.0	0	0	*	0	0	26.2	68.0	60.2	36.2	111	102
Customers' Meters Burned Out	0	0	0		0	0	0	0	0	0	1.79	0	0.11				
Ratio Change over Std. Connection - %	0	0	0		0	0	0	*	*	0	271	.0	4.70				
Customers' Wiring, Etc. Damaged	0	0	0		0	0	0	0	0	0.83	0	0	0.11		0.15	0.30	0.22
Ratio Change over Std. Connection - %	*	*			0	0	0	*	*	177	0	0	27.1		107	∞	167
Cover Removal after Fuse Failure to Inspect Gaps	No	Yes			Yes	Some	No		No	50%	50%	50%		Yes	Yes	Yes	Yes

Company	23				24			25	29	33				34			
Year	1934	1935	1936	Avg.	1934	1936	Avg.	1934	1934	1934	1935	1936	Avg.	1934	1935	1936	Avg.
No. of Installations Reported On	5	5	5	5	5	528	267	8	18	96	138	141	125	14	11	11	12
Type of Territory by Percentages																	
Urban		0	60			}97					14	13.5	10.1	100	100	100	100
Suburban	}100	60	0	}100			}100	}100	}100		0	0	0				
Rural		40	40		100	3				100	86	86.5	89.9				
Voltages of System - KV.		2414	2314			2369				24-66	24-66	2.3-12		2.4	2.4	2.4	2.4
Phase Connection		1&Y	Δ&Y			1&Δ				Δ&Y	Δ&Y	Δ&Y		Δ	Δ	Δ	Δ
Fusing Schedule - %	200	200	200	200		240								240	240	240	240
Average Ground Resistance - Ohms	20				20	25	25	12				30					
Failure Data per 100 Installations																	
Primary Fuses Blown	20	0	0	6.67		2.27	2.27		0	2.09	0	7.10	3.12	0	9.09	9.09	5.55
Ratio Change over Std. Connection - %	1739	0	0	368		121	157		0	15.4	0	108	26.8	0	2330	123	144
Transformer Winding	0	0	0	0	0	2.08	2.06	0	0	0	0	0.71	0.27	0	0	9.09	2.78
Ratio Change over Std. Connection - %	*	0	*	0	†	195	211		0	0	0	134	41.0	0	0	1750	602
Primary Only	0	0	0	0	0					0	0		0	0	0	0	0
Ratio Change over Std. Connection - %	*	0	*		†										0	0	0
Secondary Only	0	0	0	0	0					0	0		0	0	0	0	0
Ratio Change over Std. Connection - %	*	0	*		†										*	0	0
Primary & Secondary	0	0	0	0	0					0	0		0	0	0	9.09	2.78
Ratio Change over Std. Connection %	*	0	*		†										0	2270	1080
Deion Gap	0	0	0	0	0			0	0	0				0	0	9.09	2.78
Ratio Change over Std. Connection %	0	0		0	†				0					*	0		10880
Customers' Meters Burned Out	0	0	0	0	0	0	0	0	0	0							
Ratio Change over Std. Connection - %	*	0		0	†				0								
Customers' Wiring, Etc. Damaged	0	0	0	0	0	0	0	0	0	0							
Ratio Change over Std. Connection - %	*	*		*	†				0								
Cover Removal after Fuse Failure to Inspect Gaps	Yes	Yes	Yes	Yes	No	Yes		No	No	No	No	No	No	No	No	No	No

Table VII (Continued). Summary of Operating Data by Company and by Year

Surge-Proof Transformer—Figure 7

9^a						14				15				16	17				19				21			22		
1934	1936				Avg.	1934	1935	1936	Avg.	1934	1935	1936	Avg.	1934	1934	1935	1936	Avg.	1934	1935	1936	Avg.	1935	1936	Avg.	1934	1936	Avg.
99	534	230	511	36	745	11	11	16^b	13	591	1051	1262	968	30	12	13	16	14	29	30	30	30	3	3	3	50	45	48
	70	0	10	10								5											100	100	100		33.3	
	30	20	20	20	}100																						11.1	}100
	0	80	70	70		100	100	100	100	}100	}100	}95			100	100	100	100	100	100	100	100				100	55.6	
	2.3	3.8&4.8	5.9&11.5	11.5&13.8		4.8	4.8	4.8	4.8	4	4	4	4		12	12	12	12		2.4	2.3&6.9		4	4	4		6.9	
	Δ	Δ&Y	Δ&Y	Δ		Δ	Δ	Δ	Δ	Y	Y	Y	Y		Y	Y	Y	Y		Δ	Δ		Y	Y	Y		Δ	
	230	230	230	230	230	300	300	300	300						200	200	200	200					240	240	240		240	
200						66.5	72	64	68					25	10.5	15			43							10	7	8.5
1.01	0	0	0	0	0.07	9.10	27.3	31.2	23.7	0.34		2.30	1.62	0	16.67	13.30	12.50	13.95	13.70	13.30	6.67	11.25	0	0	0	0	20.0	9.48
2.00					0.16	46.4	195	97.5	108	4.45		31.6	22.9		487	161	258	551	156	243	62.0	168	0	†	0		97.0	56.9
0	0		0.39	2.08	0.32	0	0	0	0	1.01	0.38	0.40	0.52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0					10.5	0	*	*	0	29.2	∞			0	0	0	0	0	0	0	0	0	0	†	0	0	0	0
0	0					0	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							*	*			*					0	*						*	†	*		0	
0	0					0	0	0	0		0.19	0.08	0.13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							*	*			∞					0	*						*	†	*		*	
0	0					0	0	0	0		0.19	0.32	0.26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							*	*			∞					0	0						0	†	0		0	
16.16	0.18	1.30			2.29	0	0	0	0	3.05	0.38	0.47	0.97		16.67	0	0	4.65	3.45	0	0	1.13	0	0	0	0	0	0
	2.86				49.8		0	*		925	∞	∞	345		4900	0	0	1191	392	0	0	314	*	†	0			
0						0	0	0	0					0	0		0	0	0	0		0	0	0	0			
							0	*						*	0		*	0	*	*		0	*	†	*			
0						0	0	0	0						0		0	0	0	0		0	0	0	0	0		
							*	*							0			0	0	*		0	*	†	*	0		
Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No		No	No	No	No		Yes	Yes	No		No	No	No	No		Yes		No	No	No

36				42	50							33	70		
1934	1935	1936	Avg.	1935	1935		1936				Avg.	1936	1936		Avg.
71	140	140	117	76	79	18	4	4	9	64	89	8	10	13	23
					100	33	100			100					
}100	100	100}	}100					100			}100				
						67			100					100	
	4	4		4-12	2.4	6.6	12	12	12	4		6.9	12	6.9	
	Y	Y		Y	Y	Y	Y	Y	Y	Y		Δ	Y	Δ	
	240	240					240	240	240	240			200		
2.0	25	0.2	10.4		17	17						2.8		15	
0	0	0	0	18.4	0	0	25.0	25.0	0	9.38	4.50	0	10.00	0	4.35
0	0	0	0	469	0	0				308	52.0	0	64.5	0	33.4
0	0	0	0	0	1.27	0	0	0	0	0	0.56	0	0	0	0
	0	0	0	0	80.9	0	0	0	0	0	56.5	0			
0	0	0	0	0	1.27	0	0	0	0	0	0.56	0	0	0	0
				0	116	0	0	0	0	0	83.8				
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
				0	0	*	0	0	0	0	0				
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
				0	0	0	0	0	0	0	0				
0	0	0	0		0	0				0	0	0	0	0	0
	0	0	0		0	0									0
0	0	0	0		0	0				0	0	0	0	0	0
	0	0	0		*	*				*	*	0			0
0	0	0	0		0	0				0	0	0	0	0	0
	0	*	0		*	*					*				
	Yes			Yes			No	No	No	No	No	No	No	No	No

* No failures either connection.

† Standard Connection not used.

a. Includes Type C.S.P. Transformers.

b. Data on units subject to most lightning surges.

Table VIII. Summary of Operating Data by Company and by Year

Three-Point Protection—Figure 8

Company	4					7						10				22	24	25	39				42	50		
Year	1935	1936			Avg	1934		1935	1936		Avg	1934	1935	1936	Avg	1934	1936	1936	1934	1935	1936	Avg	1935	1935	1936	Avg
No. of Installations Reported On	723	234	58	130	373	62	50	110	60	50	111	20	19	19	19	100	350	20	22	69	19	33	14	213	314	264
Type of Territory by Percentages																										
Urban	47	100															100							100	100	100
Suburban	20		100		100								100	100	100	100		100								
Rural	33			100		100	100	100	100	100	100	100							100	100	100	100				
Voltage of System - KV	2.4	2.4&13.2	2.4&13.2	2.4&13.2	2.4&13.2	4	4	4	4	4	4	13.2	13.2	13.2	13.2		2.4&4.8 11.0&13.2			6.6	6.6&12		4&12	4	4	4
Phase Connection	Δ	Δ	Δ	Δ	Δ	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y					Δ	Δ&Y		Y	Y	Y	Y
Fusing Schedule - %		250	250	250													200									
Position L A With Respect To Fuse	Both	Both	Both	Both	Both	Line	Line	Line	Line	Line	Line	Line	Line	Line	Line		Line		Both	Both	Both	Both				
Average Ground Resistance - Ohms	175	7	10	20		80	21	70				5	3	3	3	10		12			50			17		
Failure Data Per 100 Installations																										
Primary Fuses Blown	3.60	1.28	20.7	1.54	3.76	11.30	6.00	15.50	16.67	28.0	15.35	5	5.27	15.80	8.62				0	0	0	0	0		1.27	1.27
Ratio Change over Std. Connection-%	740	36.2	264	4.98	30.1	106	56.3	137	97.4	164	138	†	†	34.2	750				0	0	0	0	0		41.6	14.7
Transformer Winding	0.97	0.43	3.45	1.54	1.05	0	0	0	0	0	0	5	0	0	1.79	0		0	0	0	0	0		0	0	0
Ratio Change over Std Connection-%	15.6	79.6	363	39.0	80.1	0	0	0	0	0	0	†	†	0	243	0			0	0	0	0		0	0	0
Primary Only	0.69	0.43	0	0.77	0.61	0	0	0	0	0	0		0	0	0	0		0	0	0	0	0		0	0	0
Ratio Change over Std Connection-%	20.7	127	0	80.2	86.2			0					†	0	0					0				0	0	0
Secondary Only	0.14	0	1.72	0	0.17	0	0	0	0	0	0		0	0	0	0		0	0	0	0	0		0	0	0
Ratio Change over Std Connection-%	11.7	0	1432	0	57.9			0					†	*	0					0				0	0	0
Primary & Secondary	0.14	0	1.72	0.77	0.26	0	0	0	0	0	0		0	0	0	0		0	0	0	0	0		0	0	0
Ratio Change over Std Connection-%	7.30	0	338	80.2	43.6			0					†	*	0					0				0	0	0
Lightning Arrester	1.25	0.43	6.90	0	1.22	12.8	6.00	19.10	8.33	20.0	14.15	0						0	0		0					
Ratio Change over Std Connection-%	16.1	189	1046	0	149	388	182	465	282	677	490	†														
Customers' Meters Burned Out	0	0.43	0	0	0.09				0	0		0						0	0		0			0	0	0
Ratio Change over Std. Connection-%	0	35.6	0	0	3.72							†												*	*	*
Customers' Wiring, Etc. Damaged	0	1.71	0	0.77	0.35			0.31				0				0		0	0		0			0		
Ratio Change over Std Connection-%	0	3.64	0	460	85.4			650				†												*		

* No failures either connection
† Standard Connection not used

by the standard connection, the effect is similar to the action of a gapped interconnection on three-point connection. This may explain why the three-point connection, the gapped interconnection, and the standard connection show similar operating results. It is also interesting to note that the relative performance of the various schemes of protection do not align in their expected relative order of merit when data on urban and suburban installations are compared. Of the 19 companies reporting data on type *CSP* transformers three state that they install primary fuses with them. Of 11 companies reporting data on type *CSP* transformers only one states that an increase in branch fuses blown was noted.

Table IX. Summary of Operating Data by Company and by Year

Type CSP Transformer—Figure 9

Company	1	3	4												5	6^b				7			12			16		
Year	1935	1936	1934	1935							1936			Avg.	1934	1935	1936		Avg.	1935	1936	Avg.	1935	1936	Avg.	1935	1936	Avg.
No. of Installations Reported On	25	41	176	11	133	18	5	24	11	34	56	31	188	236	34^a	17	26	1	22	361	590	480	330	421^c	386	262	758	508
Type of Territory by Percentages																												
Urban		100		10	23	17			14	3	100									25						30	20	
Suburban			100	0	26	28			21	9		100		100		94	100		100				100	100	100	40	30	100
Rural	100			90	51	55	100	100	65	88			100		100	6		100		75						30	50	
Voltage of System - KV	2.4&12	4.4&13.2		2.4	2.4	4	6.6		11	13	2.4-13.2	2.4-13.2	2.4-13.2			4	4	4	4	2.4&4	4	2.4&4	4	4	4	4.0-6.6	2.3-6.3	
Phase Connection	Y	Y		Δ	Δ	Y	Δ	Δ	Δ	Δ&Y	Δ&Y	Δ&Y	Δ&Y			Y	Y	Y	Y	Δ&Y	Y	Δ&Y	Y	Y	Y	Δ	Δ&Y	
Average Ground Resistance - Ohms	8		125		65	5		250	35	25	4	20	50		5	250	250	250	250							20		
Failure Data Per 100 Installations																												
Primary Fuses Blown	0	0	4.55	0	0	0	0	0	9.09	0	0	0	0.53	1.41	0	0	0	0	0	0	0	0		0	0			
Ratio Change over Std Connection-%	0	0	48.7	0	0	0	0	0	10.1	0	0	0	1.72	10.7		0	0	0	0	0	0	0		0	0			
Transformer Winding	0	0	0	0	0	5.55	0	0	9.09	0	0	0	0	0.25	0	0	0	0	0	0	0	0	0	0.71	0.39	0.76	0.33	0.69
Ratio Change over Std Connection-%	0	0	0	0	0	89.2	0	0	52.8	0	0	0	0	21.6		0	0	*	0	0	0	0	*			56.2	93.0	89.0
Primary Only	0	0	0	0	0	5.55	0	0	0	0	0	0	0	0.141	0	0	0	0	0	0	0	0	0	0	0	0		
Ratio Change over Std Connection %	0				0	167	0	0	0	0	0	0	0	19.9		*	*	*	*	0			*			0		
Secondary Only	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.24	0.15	0		
Ratio Change over Std Connection %	0				0	0	*	0	0	0	0	0	0	0		*	0	*	0	0			*			0		
Primary & Secondary	0	0	0	0	0	0	0	0	9.09	0	0	0	0	0.14	0	0	0	0	0	0	0	0	0	0.47	0.26	0.76		
Ratio Change over Std. Connection %	*				0	0	0	0	98.2	0	0	0	0	23.8		0	0	*	0	0			*					
Deion Gap	0	0	0	0	0	5.55	0	0	9.09	0	0	0	0	0.28		0	0	0	0	0	0	0	0	0	0	0		
Ratio Change over Std Connection %	0		0	0	0	71.4	0	*	∞	*	0	0	0	34.4		*	*	*	*	0	0	0	*	*	0	0		
Customers' Meters Burned Out	0		1.14		0	0	0	0	0	0	0	0	0.53	0.42	0	0				0							0	0
Ratio Change over Std Connection %	0		1036		0	0	*	*	*	*	0	0	4.49	18.4		0				0							*	*
Customers' Wiring, Etc. Damaged	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0	3.85	0	2.27	0.29	0.17	0.21					0	0
Ratio Change over Std Connection %	*		0	0	0	0	*	*	*	*	0	0	0	0		*	1480		2235	200	64.8	156					*	
Increase in Tap or Branch Fuses Blown	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No								

* No failures either connection
† Standard Connection not used

a. Includes approx 10 Surge-Proof Transf's. b. Not directly comparable with Fig 1. c. Transf's equipped with one bushing and one gap.

In comparing the relative merit of the various schemes of protection in reducing *transformer winding failures* a study of tables X to XV indicates conflicting trends which make it difficult to draw definite conclusions. If rural installations only are considered the *CSP* transformer shows the best performance with the common primary and secondary neutral system next with the *SP* transformers, the three-point connection, the standard connection, the gapped interconnection, and the solid interconnection following in the order listed. However, when urban and suburban installations are considered almost the reverse trend is noted in that the various types of solid interconnection show the best performance with the *CSP* transformer, the *SP* transformer, the three-point connection, the gapped interconnection, and the standard connection following in that order. This lack of consistency may be due to the fact that in all cases the rate of transformer-winding failures is so small that there is less probability of distinct trends being established without collecting data over a number of years. Moreover the age and design of the transformer, the number of lightning surges to which it has been previously exposed, the degree of exposure, and isokeraunic level all affect the probability of transformer failure. The fact that *SP* and *CSP* transformers are both relatively new and of more modern design is undoubtedly at least in part a contributing factor in their good performance as well as the protection scheme used on them. It is interesting to note that in both methods of comparison that the three-point connection, the standard connection, and the gapped interconnection are all grouped together possibly for the same reason that was discussed under rates of primary fuses blown. In comparing the performance of rural installations it is difficult to explain the difference in the performance of the common neutral system and the solid interconnection as they are essentially the same in so far as lightning protection is concerned. It is believed that it is safe to conclude that the newer schemes are better than the standard connection but with little to choose between the three-point connection, the gapped interconnection, and the standard connection.

In considering the effect of the various methods of protection on the *rate of arrester failures* the data are somewhat conflicting but show that the *CSP* transformer and the several methods of solid interconnection have the lowest failure rates. The three-point connection consistently shows the maximum failure rate. However, this may be due to the small number of installations and the fact that most of them were protected by a type of arrester with a known high failure rate. Since the type of arrester has a considerable bearing upon the failure rate, definite conclusions cannot be drawn relative to the effect of the type of protection scheme used. However, since the failure rates for solid interconnections are not higher than those for the standard connection, it can be concluded that the fear that has been expressed that with solid interconnection a heavier duty may be placed on the arrester causing more failures is without foundation. Since in case of type *SP* transformers a deion-gap failure cannot be noted unless the top of the transformer tank is removed and an examination made, the practice of the companies as to this point was determined. Of 17 companies reporting this point on type *SP* transformers, seven reported that the tank covers are removed after primary fuses are blown. Unless this is done gap failures may not be observed.

Table IX (Continued). Summary of Operating Data by Company and by Year

Type CSP Transformer—Figure 9

17				18				21			22			33			34	36	38			40	42	44	45	50			55	66			69			70		
1935	1936		Avg.	1935	1936		Avg.	1935	1936	Avg.	1935	1936	Avg.	1935	1936	Avg.	1935	1935	1936	1936	Avg.	1935	1935	1935	1936	1935	1936	Avg.	1936	1935	1936	Avg.	1935	1936	Avg.	1936		Avg.
4	4	4	6	14	44	7	50	4	4	4	133	98	115	145	654	400	3	5	39	39	39	83	3	10	1338	74	132	103	43	50	50	50	1	1	1	8	38	46
100	100		66.7	100				100	100	100	0	29.6			1.8	1.5	100					59			20	42	22.8									37.5		
					100	100	100				40	15.9	100		0			100	100	100	100	17			0	0	24.2	100					100	100	100	0		100
		100	33.3								60	55.1		100	98.2	98.5						24		100	60	58	53.0		100							62.5	100	
4	4	12	4&12	4	4	132		4	4	4	6.6	6.9	6.9	24-66	23-12	23-12	2.4	4	2.4	2.4	2.4	4	4-12	4	23&69	6.6	12	6.6&12	6.9	23-13	4-13.8	23-13.8	4.8	4.8	4.8	23&12	6.9	
Y	Y	Y	Y	Y	Y	Y		Y	Y	Y	Δ	Δ	Δ	Δ&Y	Δ&Y	Δ&Y	Δ	Y	Δ	Δ	Δ	Y	Y	Y	Δ	Y	Y	Y	Δ	Y	Δ&Y	Δ&Y	2&4W	2&4W	2&4W	Δ&Y	Δ	
15				3	3	3	3	1.5			15				30			25		15-20		2		13		17			28	10	10	10				15		
0	0	0	0	0	0	1.43	1.54	0	0	0		0		0	0.76	0.63	0	0	0	0	0	0	0	0	0				0	0			0	0	0		0	0
0	0	0	0	f	0	31.0	1.34	0	f	0		0		0	11.6	5.38	0	0	0	0	0	0	0	0	0				0	f			•	0	0		0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.07	0	0	0	0	0	0	0	0	0	0		0	0
0	0	0	0	f	0	0	0	0	f	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8.65	0	0	0	0	f	f	f	•	•	•			
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0
0	0	•		f	0	0	0	•	f	•	0	0		0			0	0	0	0		0	0	0		0	0	0		f	f	f	•	•	•			
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0
0	0	•		f	0	•	0	•	f	•	0	•		0			•	0	0	0		0	0	0		•	•	0		f	f	f	•	•	•			
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.07	0	0	0	0	0	0	0	0	0	0		0	0
0	0	0		f	0	•	0	0	f	0	0	0		0			0	0	0	0		0	0	0		0	0	0		f	f	f	•	•	•			
25				0	0	0	0	0	0	0	0	0	0		0.46	0.46	0	0	0	0	0	0		0	0.07	0	0	0	0	0	0	0	0	0	0		0	0
14,700				f				•	f	0	0						0	0	0	0	0	0		0	33.3	0				f	f	f	•	•	•			
	0	0	0					0	0	0								0		0		0		0		0	0	0	0	0	0	0	0				0	0
	0	•	0					•	f	•								0		0		0		•		•	•	•	0	f	f	f	•				0	0
	0	0	0					0	0	0								0		0		0		0		0				0	0	0	0				0	0
			0					•	f	•								0		0		0		•		•				f	f	f	•					
	No	No	No								No				No					No		No		No		No	No	No	d	No	No	No	No	No	No	•	No	

d. Decrease noted. e. Increase noted on urban installations

Table X. Combined Operating Data on Various Methods of Protecting Distribution Transformers From Lightning for 1934, 1935, and 1936

All Types of Territory

Type of Protection	Average Number Installations	Average Ground Resistance	Failures per 100 Installations				
						Customers'	
			Primary Fuses Blown	Transformer Winding Failures	Lightning Arrester Failures	Meters Burned Out	Wiring, Etc., Damaged
Standard connection	159,138	60.0	6.35	0.717	1.17	1.10	0.103
Solid interconnection**	146,738	13.2	1.85	0.350	0.551	1.01	0.079
Gapped interconnection	11,029	48.8	10.10	0.676	0.963	1.97	0.141
Three-point protection	769	34.8	5.22	0.590	3.92	0.054	0.364
Surge-proof transformer	3,164	58.2	5.38	0.43	1.21*	0.036	0.102

* Deion gaps damaged per 100 installations. ** Includes common-neutral type.

Table XI. Combined Operating Data on Rural Installations for Various Methods of Protecting Distribution Transformers From Lightning for 1934, 1935, and 1936

Type of Protection	Average Number Installations	Average Ground Resistance	Failures per 100 Installations				
						Customers'	
			Primary Fuses Blown	Transformer Winding Failures	Lightning Arrester Failures	Meters Burned Out	Wiring, Etc., Damaged
Standard connection	15,645	93.0	9.98	0.565	2.24	1.205	0.132
Solid interconnection*	5,738	19.1	6.26	1.04	0.680	0.352	0.137
Gapped interconnection	9,020	45.3	10.95	0.730	0.965	2.04	0.080
Three-point protection	217	42.7	8.28	0.460	8.06	0	0.554
Surge-proof transformers	890	85.5	7.72	0.272	1.66**	0	0.056

* Includes common primary and secondary neutral. ** Deion gap failures.

Table XII. Combined Operating Data Upon Various Methods of Protecting Distribution Transformers From Lightning for 1934, 1935, and 1936

Urban and Suburban Installations

Type of Protection	Average Number Installations	Average Ground Resistance (Ohms)	Failures per 100 Installations				
						Customers'	
			Primary Fuses Blown	Transformer Winding Failures	Lightning Arrester Failures	Meters Burned Out	Wiring, Etc., Damaged
Standard connection	84,991	77.7	3.65	0.660	0.593	0.736	0.066
Solid interconnection*	80,235	9.1	0.989	0.204	0.192	1.45	0.099
Gapped interconnection	1,270	56.7	6.62	0.455	0.451	1.16	1.19
Three-point protection			No urban or suburban installations in 1934				
Surge-proof transformers	221	33.4	9.25	0.453	0.153	0.167	0.168

* Includes common primary and secondary neutral. ** Deion gap failures per 100 installations.

While the data for *meters burned out* and *troubles on customers' premises* are not conclusive, it can be concluded that solid interconnection does not increase the probability of trouble on the customers' premises due to lightning.

Effect of Ground Resistance

Since the resistance of lightning-arrester grounds is believed to have a material effect upon the degree of protection rendered, the data have been analyzed to determine such effects in tables XVI and XVII. In the analyses it will be noted while no consistent trend is evident that in general the trend of primary fuses blown and transformer-winding failures are upward as the ground resistance increases. When it is considered that the data are analyzed by the average ground resistance of each company reporting and that this average ground resistance is estimated, it is evident that these two tables cannot be considered entirely conclusive. However, it is believed that they are indicative of general trends.

Table XVI in most cases, except for the three-point connection, has sufficient number of installations in each range of ground resistances. However, this table includes all types of territory. When the data for rural installations only is segregated as in table XVII, too few installations exist in many ranges of ground resistances to give usable averages. In table XVII it should be noted that the rate of primary fuses blown and transformer winding failures for ground resistances up to 30 ohms are slightly higher for interconnection than for the standard connection. This may be due in part to the inclusion of gapped interconnection in the classification

Table XIII. Combined Operating Data on Various Methods of Protecting Distribution Transformers From Lightning for 1935 and 1936

All Types of Territory

			Failures per 100 Installations										
			Primary Fuses Blown			Transformer Windings Damaged					Customers'	
Type of Protection	Average Number Installations	Average Ground Resistance	Total	Lightning Arrester on Line Side of Fuse	Lightning Arrester on Transformer Side of Fuse	Total	Primary Only	Secondary Only	Primary and Secondary	Lightning Arrester Failures	Meters Burned Out	Wiring, Etc., Damaged
Standard connection	153,698	59.0	5.08	4.55	6.29	0.655	0.214	0.081	0.317	0.878	1.04	0.100
Solid interconnection	123,102	10.7	1.36	1.01	2.12	0.333	0.068	0.030	0.130	0.455	1.37	0.092
Solid interconnection with tie to case	2,715	27.9	1.80	1.40	N.D.	0.276	0.096	0.096	0.145	0.049	0	0
Gapped interconnection	14,627	56.1	9.20	5.23	12.40	0.634	0.254	0.053	0.561	0.875	2.12	0.156
Gapped interconnection with tie to case	1,125	36.9	24.90	13.99	16.67	1.42	1.72	0.172	0.432	2.08	0.435	0
Common primary and secondary neutral	40,734	10.1	1.74	1.10	5.15	0.413	0.063	0.007	0.053	0.300	0.343	0.085
Surge proof transformer	3,850	48.1	5.81			0.442	0.048	0.048	0.178	0.840*	0.046	0.115
Three-point protection	1,016	44.0	5.05	17.45	N.D.	0.595	0.347	0.099	0.149	3.61	0.115	0.405
CSP transformer	3,215	64.2	0.172			0.234	0.018	0.018	0.106	0.127*	0.074	0.112

* Deion gap failures per 100 installations.

Table XIV. Combined Operating Data on Various Methods of Protecting Distribution Transformers From Lightning for 1935 and 1936

Rural Installations

			Failures per 100 Installations										
			Primary Fuses Blown			Transformer Windings Damaged					Customers'	
Type of Protection	Average Number Installations	Average Ground Resistance (Ohms)	Total	Lightning Arrester on Line Side of Fuse	Lightning Arrester on Transformer Side of Fuse	Total	Primary Only	Secondary Only	Primary and Secondary	Lightning Arrester Failures	Meters Burned Out	Wiring, Etc., Damaged
Standard connection	17,756	106	12.17	11.45	N.D.	0.691	0.259	0.175	0.278	2.90	2.25	0.301
Solid interconnection with and without tie to case	4,903	18.2	5.26	4.88	5.39	1.55	0.066	0.022	0.088	0.932	0.572	0.215
Gapped interconnection with and without tie to case	13,408	46.3	10.63	5.83	13.70	0.714	0.691	0.067	0.780	1.07	2.06	0.075
Common primary and secondary neutral	3,383	13.7	6.28	1.32	8.98	0.358	0.129	0.032	0.193	0.392	0.166	0
Surge-proof transformer	832	52.2	8.53			0.361	0.131	0	0.197	0.945*	0	0.075
Three-point protection	219	47.5	9.81	18.60	N.D.	0.456	0.228	0	0.228	9.75	0	0.772
CSP transformer	834	89.5	0.376			0	0	0	0	0.198*	0.123	0.135

* Deion gap failures per 100 installations.

Table XV. Combined Operating Data Upon Various Methods of Protecting Distribution Transformers From Lightning for 1935 and 1936

Urban and Suburban Installations

			Failures per 100 Installations										
			Primary Fuses Blown			Transformer Windings Damaged					Customers'	
Type of Protection	Average Number Installations	Average Ground Resistance (Ohms)	Total	Lightning Arrester on Line Side of Fuse	Lightning Arrester on Transformer Side of Fuse	Total	Primary Only	Secondary Only	Primary and Secondary	Lightning Arrester Failures	Meters Burned Out	Wiring, Etc., Damaged
Standard connection	105,029	76.6	3.34	3.11	5.80	0.687	0.228	0.056	0.290	0.465	0.731	0.063
Solid interconnection with and without tie to case	74,637	10.2	1.05	0.552	1.90	0.185	0.031	0.028	0.073	0.174	1.91	0.104
Gapped interconnection with and without tie to case	1,876	61.3	6.75	1.87	3.89	0.464	0.121	0.030	0.303	0.460	1.16	1.19
Common primary and secondary neutral	26,670	8.3	0.749	0.438	3.96	0.180	0.053	0.003	0.030	0.337	0.378	0.100
Surge-proof transformers	325	33.4	9.47			0.462	0.154	0	0.308	0.156*	0.167	0.168
Three-point protection	429	8.0	3.57	N.D.	N.D.	0.350	0.117	0.117	0.117	1.71	0.122	0.792
CSP transformers	396	49.5	0.214			0.253	0	0	0.253	0.127	0	0.370

* Deion gap failures per 100 installations.

Table XVI. Effect of Ground Resistance Upon Operating Data for Various Methods of Protecting Distribution Transformers From Lightning—1934, 1935, and 1936 Lightning Seasons

All Types of Territory

Type of Protection	Range of Average Ground Resistance (Ohms)	Average Number of Installations	Number of Companies			Failures per 100 Installations				
									Customers'	
			1934	1935	1936	Primary Fuses Blown	Transformer Windings Damaged	Lightning Arrester Failures	Meters Damaged	Wiring, Etc., Damaged
Standard connection (figure 1)	1– 10	17,166	3	4	4	3.50	0.425	0.930	0.653	0.625
	11– 20	25,542	5	8	4	3.82	0.743	0.432	2.00	0.084
	21– 30	7,376	3	2	5	3.34	0.344	0.262	0.316	0.168
	31– 40	9,051	2	3	2	2.92	0.558	0.188	0.495	0.069
	45– 75	5,477	1	5	4	5.91	1.31	0.711	1.92	0.129
	80–125	28,181	4	0	0	10.4	0.92	2.67	0.996	0.883
	200–250	9,772	2	2	1	20.5	1.26	3.84	1.47	0.101
Solid interconnection (figures 2 and 3) and common primary and secondary neutral (figure 6)	0– 1	40,761	7	6	10	0.545	0.112	0.256	1.43	0.050
	2– 5	13,291	5	8	6	1.35	0.216	0.199	0.272	0.100
	6– 10	29,585	4	3	6	1.91	0.260	1.49	0.211	0.152
	11– 20	22,375	0	6	4	6.65	0.410	0.480	0.236	0.043
	21– 30	4,077	5	3	4	1.84	0.163	0.143	0.019	0.039
	50–133	6,858	2	2	0	18.52	0.74	0.484	0	0
Gapped interconnection (figures 4 and 5)	0– 10	921	2	2	3	3.40	0.217	0.094	0	0.080
	11– 20	5,439	0	4	2	13.90	0.792	1.35	0.81	0.101
	21– 30	1,724	1	2	8	10.70	0.600	1.28	0.726	0
	31– 50	542	1	1	1	11.50	1.23	1.54	1.29	0.064
	51–100	1,030	0	2	2	5.96	2.09	1.36	3.74	0
	101–250	516	1	1	1	2.91	0.324	0.453	0	.259
Surge-proof transformers (figures 7 and 9)	1– 25	761	7	8	6	1.75	0.657	0.228*	0	0
	26–100	857	5	3	6	6.45	0.272	0.823*	0.185	0.063
	101–400	378	2	4	1	7.40	0.176	2.03*	0	0.176
Three-point protection (figure 8)	1– 10	150	2	1	2	5.70	0.89	1.60	0.320	0.970
	11– 20	121	1	1	1	1.54	0.55	0	0	0.276
	21– 30	35	1	0	1	4.35	0	4.35	0	0
	70– 80	86	1	1	0	13.95	0	16.87	N.D.	0.91
	175	723	0	1	0	3.60	0.969	1.24	0	0

* Deion gaps damaged per 100 installations.

Table XVII. Effect of Ground Resistance Upon Operating Data for Various Methods of Protecting Distribution Transformers From Lightning—1934, 1935, and 1936 Lightning Seasons

Rural Installations

Type of Protection	Range of Average Ground Resistance (Ohms)	Average Number of Installations	Number of Companies			Failures per 100 Installations				
			1934	1935	1936	Primary Fuses Blown	Transformer Windings Damaged	Lightning Arrester Failures	Customers' Meters	Customers' Wiring, Etc., Damaged
Standard connection (figure 1)	1– 30	4,976	2	2	6	3.09	0.173	0.45	0.52	0.01
	40– 70	712	1	1	3	27.9	2.30	2.94	10.43	0.423
	71–100	164	2	0	1	11.30	2.75	N.D.	1.81	N.D.
	200–250	4,686	0	2	1	11.70	0.45	4.07	2.70	0.160
All types of interconnection (figures 2, 3, 4, 5, and 6)	0– 10	7,533	0	0	13	4.46	0.44	0.73	0.18	0.19
	11– 20	7,207	0	6	3	12.10	0.62	1.10	0.64	0.08
	21– 30	2,130	3	3	7	8.49	0.58	1.12	0.34	.09
	31– 50	1,588	1	2	4	20.9	2.98	0.73	0.80	0
	51– 60	283	0	1	0	3.53	0.71	1.77	0	0
	75– 90	563	0	2	2	11.20	3.02	1.87	5.50	0
	101–250	195	1	0	1	4.89	0	0.34	N.D.	0
Surge-proof and *CSP* transformers (figures 7 and 9)	1– 25	131	4	6	1	1.35	0	0.51	0	0
	26–100	696	3	2	6	5.79	0.24	0.98	0.13	0
	101–400	418	2	5	0	9.10	0.272	2.62	0	0.12
Three-point protection (figure 8)	1– 10	20	1	0	0	5.00	5.00	0	0	0
	11– 20	95	1	0	1	1.05	1.05	0	0	0.53
	21– 30	35	1	0	1	8.57	0	8.57	0	0
	70– 80	94	1	1	1	17.00	0	12.10	N.D.	N.D.

of interconnection. The data in table XVI show the reverse trend when the standard connection and solid interconnection only are compared. This leads to the conclusion that a large part of the improvement noted by the use of solid interconnection is the result of the lower values of ground resistance that almost invariably exists when the latter is used.

EFFECT OF OPERATING VOLTAGE

An analysis of the data by operating voltage has been made in tables XVIII, XIX, and XX. In drawing this comparison allowance has been made for type of territory, ground resistance, and location of primary fuse. Due to lack of data no comparison could be drawn for urban and

Table XVIII. Effect of Operating Voltage and Average Ground Resistance on Operating Data for Various Methods of Lightning Protection—1935 and 1936 Lightning Seasons

All Types of Territory

Type of Protection	Range of Average Ground Resistance (Ohms)	Average Number of Installations			Failures per 100 Installations																	
					Primary Fuses Blown																	
					Lightning Arrester on Line Side of Fuse			Lightning Arrester on Transformer Side of Fuse			Transformer Windings			Lightning Arresters			Customers' Meters Burned Out			Customers' Wiring, Etc., Damaged		
		1–5 Kv	6–10 Kv	11–15 Kv	1–5 Kv	6–10 Kv	11–15 Kv	1–5 Kv	6–10 Kv	11–15 Kv	1–5 Kv	6–10 Kv	11–15 Kv	1–5 Kv	6–10 Kv	11–15 Kv	1–5 Kv	6–10 Kv	11–15 Kv	1–5 Kv	6–10 Kv	11–15 Kv
Standard connection (figure 1)	1– 10	18,392	523*	101*	2.58	20.6	28.8	N.D.	N.D.	N.D.	0.283	0.382	1.98	1.13	N.D.	N.D.	0.352	N.D.	N.D.	0.625	N.D.	N.D.
	11– 25	22,408	4,026	945	2.00	10.2	7.45	N.D.	N.D.	N.D.	0.577	0.475	0.423	0.485	2.99	1.89	1.70	N.D.	0	0.162	N.D.	N.D.
	26– 50	7,875			2.46			N.D.			0.665			0.489			0.064			0.021		
	51–100	10,695			3.54			N.D.			0.705			0.206			0.424			N.D.		
	200–250	7,945			6.86			2.52			0.315			2.28			1.47			0.082		
Solid interconnection with and without tie to case (figures 2 and 3)	0– 1	39,754			0.725			1.17			0.114			0.167			1.67			0.036		
	1– 5	9,541			0.926			0.875			0.152			0.352			0.067			0.187		
	6– 10	23,520			1.39			1.20			0.047			0.089			0.25			0.122		
	11– 20	27,398	930	1,049	14.1	N.D.	0	8.25	N.D.	6.30	0.545	0.645	0.191	1.50	N.D.	1.07	6.86	0	0.048	0.336	0	0
	21– 30	501			3.51			1.11			0.40			0.85			N.D.			0.200		
Gapped interconnection with and without tie to case (figures 4 and 5)	0– 10	1,515	None	11*	3.96		15.40	2.18		11.1	0.278		4.55	0.132		0	N.D.		0	0.080		0
	11– 20	None	None	139			10.00			58.5			0			0			0			0
	21– 30	5,168	None	58*	1.92		10.00	1.40		30.3	0.773		3.03	0.958		N.D.	0.135		N.D.	0.106		N.D.
	50– 75	702	83*	266	6.19	N.D.	1.43	N.D.	1.67	N.D.	2.78	2.41	3.76	1.14	0	3.76	1.07	0	0	N.D.	0	0
	75–100	573	None	None	N.D.			N.D.			0.349			2.10			2.62			0		
	250	725	None	None	N.D.			2.48			0.345			0.483			N.D.			0.276		
Common primary and secondary neutral (figure 6)	0– 5	14,195	None	None	0.328			N.D.			0.127			0.440			0.517			0.138		
	6– 10	504*	None	None	0			N.D.			0			0.20			N.D.			N.D.		
	11– 20	None	None	2,111			N.D.			9.13			0.284			0.412			0.412			0
	21	4,600*	None	None	0.413			N.D.			0.065			0.043			0			0		
Westinghouse surge-proof transformers (figure 7)	0– 25	187	38	None	0	15.5					0.266	0		0	0		0	0		0	0	
	26–100	38	24	None	10.52	20.8					2.63	0		0	0		0	0		0	0	
	100–150	927	None	None	20.5						0.54			2.48			N.D.			0.108		
Westinghouse *CSP* transformers (figure 9)	0– 10	106	None	7*	0		14.28				0.47		0	0.5		0	0		N.D.	0		N.D.
	11– 25	19	207	None	0	N.D.					0	0		5.26	0		0	0		0	0	
	26– 50	None	43*	45*		0	2.22					0	2.22		0	2.22		0	0		N.D.	0
	51– 75	155	None	None	0						0			0			0			0		
	250	22	None	None	0						0			0			0			0		

* Based on report of only one company for either one or two years.

Insufficient data for tabulation on three-point protection—figure 8.

suburban installations for voltages above 5 kv (see table XX). It is thought that the data indicates that in spite of the paucity of data the trouble rates are higher for the higher distribution voltages.

Location of Transformer Winding Failures

Where possible data were obtained relative to the location of transformer-winding failures, i.e., whether the primary only, the secondary only, or both the primary and secondary were involved in the failure. While with some types of protection a sufficient number of failures to indicate trends have not been reported, a sufficient number have been reported for several types of protection. These data have been summarized in table XXI. Except for the gapped interconnection, the *SP* and *CSP* transformers in which cases relatively few failures were reported, the various types of protection show consistent trends.

Secondary Protection

Since the average data in table XXI show that in 64.8 per cent of the transformer winding failures the secondaries are involved, the question arises as to whether or not this high percentage may be due to the lightning surges entered from the secondary. A secondary-winding failure does not necessarily mean a secondary entrance of the surge as a surge entering through the primary in its attempt to reach the

Table XIX. Effect of Ground Resistance Upon Operating Data for Various Methods of Protecting Distribution Transformers From Lightning—1935 and 1936 Lightning Seasons

Rural Installations

Type of Protection	Range of Average Ground Resistance (Ohms)	Average Number of Installations			Failures per 100 Installations																	
					Primary Fuses Blown																	
					Lightning Arrester on Line Side of Fuse			Lightning Arrester on Transformer Side of Fuse			Transformer Windings Damaged			Lightning Arrester Failures			Customers' Meters Burned Out			Customers' Wiring, Etc., Damaged		
		1–5 Kv	6–10 Kv	11–15 Kv	1–5 Kv	6–10 Kv	11–15 Kv	1–5 Kv	6–10 Kv	11–15 Kv	1–5 Kv	6–10 Kv	11–15 Kv	1–5 Kv	6–10 Kv	11–15 Kv	1–5 Kv	6–10 Kv	11–15 Kv	1–5 Kv	6–10 Kv	11–15 Kv
Standard connection (figure 1)	7– 15	None	342	188		N.D.	13.89		N.D.	N.D.		5.42	0.534		2.99	N.D.		N.D.	0		N.D.	0
	21– 30	2,082	None	371*	5.38		N.D.	N.D.		N.D.	0.337		0.270	0.720		1.89	N.D.		0	0		N.D.
	31– 40	None	350*	None		18.00			N.D.			2.86			3.43			N.D.			N.D.	
	68– 75	160	None	None	15.53			N.D.			2.10			1.84			1.58			0		
	200–250	3,721	None	None	11.35			5.40			0.43			4.07			2.70			0.134		
All types of interconnection	0– 10	2,418	None	2,068	3.04		N.D.	2.45		6.24	0.394		0.195	0.580		1.06	0.400		0.048	0.329		0
	11– 20	5,319	None	3,777	4.61		10.0	14.4		10.30	0.743		0.265	0.969		0.715	0.142		0.476	0.111		0
	21– 30	786	None	116	3.16		10.0	3.00		30.30	0.382		3.03	0.792		0	0		0	0.127		0
	50– 60	200*	1,214	563	N.D.	N.D.	14.3	N.D.	1.6	N.D.	0	2.51	6.59	2.50	0	3.76	0	0	0	0	0	0
	75– 90	563	None	None	12.31			N.D.			3.02			1.87			5.50			0		
	250	291*	None	None	N.D.			3.44			0			0.344			N.D.			0		

Insufficient data for tabulation on *SP* and *CSP* transformers and three-point protection.
* Based upon single company for either one or two years.

ground may flash over a secondary bushing and raise the potential of the secondary winding sufficiently to cause it to fail. On the other hand the rate of meter failures per hundred transformers shown in tables X to XV indicate the possibility of surges on the secondary.

Of the companies reporting six report the use of secondary protection. In most of these cases secondary protection is used in a few installations apparently as an experiment. However, one company makes it a practice to install secondary arresters on polyphase meters and another company installs secondary arresters at customers where electric ranges are installed.

Effect of Fusing Schedule

Since the primary fusing schedule used by any company cannot be expressed in a single numerical statement, any comparison must be only approximate. In making the comparison in table XXII an attempt has been made to reduce all the fusing schedules to a percentage basis. In spite of the fact that no definite trend is established it is believed that irrespective of the type of protection used the rate of primary fuses blown should decrease with the use of higher rated fuses. This has been borne out by studies made by several individual companies.

Effect of Location of Primary Fuse

As a result of the question raised regarding the effect of the surge current causing the primary fuse to blow, the data have been analyzed in tables XIII to XV showing the rates of fuse blowing when the primary fuse is installed on the line side of the arrester separate from those when the fuse is installed on the transformer side of the arrester. From a study of these tables it is apparent that with all types of protection the rates of primary fuses blown are higher when the arrester is on the transformer side of the fuse and the surge current passes entirely through the fuse.

Effect of One or Two Fuses per Transformer

In three-phase four-wire systems it is customary to connect single-phase transformers between phase and neutral using a single fuse; whereas in three-phase three-wire systems single-phase transformers must be connected between phases necessitating two fuses per transformer. In table XXIII a comparison is drawn to determine whether or not there is any difference in the number of fuses blown per hundred transformers when one or two fuses per transformer are used. The data show that the rates of primary fuses blown are practically the same in both cases.

Grounding of Transformer Tanks

Since theory, laboratory tests, and certain field data indicate that if the transformer tank is left free to assume any potential a greater strain may be imposed upon the insulation by lightning surges than if the tank were held at ground potential, a growing tendency to ground the tanks of distribution transformers is noted. Fourteen of the companies reporting state that they ground the transformer tanks on some 40,437 transformers. Seven

Table XX. Effect of Ground Resistance Upon Operating Data* for Various Methods of Protecting Distribution Transformers From Lightning—1935 and 1936 Lightning Seasons

Urban and Suburban Installations

Type of Protection	Range of Average Ground Resistance (Ohms)	Average Number of Installations	Failures per 100 Installations: Primary Fuses Blown: Lightning Arrester on Line Side	Lightning Arrester on Transformer Side	Transformer Windings Damaged	Lightning Arrester	Customers' Meters	Customers' Wiring, Etc.
Standard connection (figure 1)	1– 10	19,657	1.70	N.D.	0.333	1.04	N.D.	N.D.
	11– 15	9,716	2.18	N.D.	0.710	0.653	1.89	0.085
	16– 20	11,605	3.51	N.D.	0.860	N.D.	N.D.	N.D.
	21– 30	4,510	1.10	N.D.	0.266	0.213	0.482	N.D.
	31– 50	5,867	1.66	N.D.	0.698	0.595	0.038	0.038
	70–100	10,501	3.32	N.D.	0.680	0.176	0.317	N.D.
	200	7,336	2.34	N.D.	0.177	0.819	N.D.	0.028
	250	567	N.D.	1.41	0.176	0	1.06	0
All types of interconnection (figures 2, 3, 4, 5, and 6)	0– 1	47,291	0.638	7.90	0.115	0.184	0.570	0.021
	1– 3	11,542	0.378	0.751	0.100	0.332	0.145	0.186
	4– 10	8,282	1.08	1.20	0.179	0.376	1.32	0.715
	11– 30	12,959	0.580	1.05	0.555	0.049	0	0
	50	6,364	N.D.	N.D.	0.188	0.487	N.D.	N.D.

* This data for 0–5 kv only; insufficient data for tabulation for 6–15 kv.
Insufficient data for tabulation on three-point protection, surge-proof and *CSP* transformers.

Table XXI. Location of Transformer Winding Failures Expressed as Per Cent of Total Failures—1935 and 1936 Lightning Seasons Combined

All Types of Territory

Type of Protection	Number of Segregated Transformer Failures	Per Cent Primary Only	Per Cent Secondary Only	Per Cent Primary and Secondary
Standard	685	35.8	11.1	53.1
Solid interconnection	307	29.6	13.4	57.0
Solid interconnection with tie to case	7	28.6	28.6	42.8
Gapped interconnection	65	29.2	6.2	64.6
Gapped interconnection with tie to case	27	74.1	7.4	18.5
Common primary and secondary neutral	53	51.0	5.6	43.4
Surge proof	17	17.6	17.6	64.8
Three-point protection	12	58.3	16.7	25.0
CSP	8	12.5	12.5	75.0
Total	1,181	35.2	11.3	53.5

Table XXII. Effect of Fusing Schedule on Number of Primary Fuses Blown—1935 and 1936 Data Combined

Fuses on Transformer Side of Lightning Arrester

Schedule Range (Per Cent)	Standard Connection: Number Installations Involved	Standard Connection: Per Cent Primary Fuses Blown	Interconnection (All Types): Number Installations Involved	Interconnection (All Types): Per Cent Primary Fuses Blown
150–200	26,134	3.78	Insufficient data	
201–250	20,324	4.60	57,221	1.43
251–300	31,835	4.30	3,173	2.27
301–400	Insufficient data		36,300	1.39
401–500	1,517	3.89	Insufficient data	

companies ground the tanks on 18,478 transformers protected with the standard connection. Six companies do the same on 2,684 transformers using solid interconnection. Three companies ground the tanks on 1,340 transformers using gapped interconnection. Nine companies using the common primary and secondary neutral ground the tanks on 17,935 transformers.

LIMITATIONS PLACED ON GROUND RESISTANCE

Since the data indicate that the value of resistance of the arrester ground is a factor of major importance in rendering effective lightning protection, the practice of the various companies in reducing or placing limitations on ground resistance has been analyzed in table XXIV. Of the 44 groups of installations of the standard connection only 15 make any specific attempt to lower or to limit the ground resistance. On the other hand only seven groups of installations of solid interconnection have no limitation on the ground resistance of which five have grounds salted. Of the 33 groups of installations of solid interconnection 13 have definite limitations

Table XXIII. Difference Upon Rate of Primary Fuses Blown Due to One or Two Primary Fuses per Transformer—1935 and 1936 Data Combined

Type of Protection	One Fuse per Transformer Installation: Average Number Installations	One Fuse per Transformer Installation: Fuses Blown per 100 Transformers	Two Fuses per Transformer Installation: Installations	Two Fuses per Transformer Installation: Fuses Blown per 100 Transformers
Standard	43,080	5.60	27,574	5.22
Solid interconnection	55,200	0.945	34,630	1.84
Solid interconnection with tie to case	103	2.91	1,808	1.66
Gapped interconnection	7,516	10.4	3,950	7.80
Gapped interconnection with tie to case	0	N.D.	483	28.60*
Type *SP* transformer	1,302	6.72	504	4.66
Three-point protection	293	7.85	617	3.72
Type *CSP* transformer	640	0.08	261	0.19
Total—all types	108,134	3.54	69,827	3.74

* Based on one company which had much damage done by a severe storm during 1936.

Table XXIV. Ground Resistance Limitations Placed Upon Various Schemes of Lightning Protection—1935 and 1936 Data

	Standard Connection	Solid Interconnection	Solid Interconnection Tie to Case	Gapped Interconnection	Gapped Interconnection Tie to Case	Common Primary and Secondary Neutral	Surge-Proof Transformers	Three-Point Protection	CSP Transformers
Installation Groups Reported Upon	44	33	6	24	4	20	21	7	25
Number reducing ground resistance to specified value:	15	13	1	7	2	6	5	2	6
200 ohm					1				
100 ohm	1					1			
50		2	1	2					
30	2	1		1				1	1
25	6	3		2		1	3	1	2
20					1				
15	4	3		1		1	2		2
10		3		1					
5						2			1
Less than 5	2	1				1			
By salting only	1			1					1
Water pipe grounds	1	3							
Driven grounds	4	3		1		3	3	1	2
Salting and water pipes		2							
Salting and driven grounds	5		1	2	2	1	1	1	1
Water pipe and driven grounds		1				1			
No method	4	4		3		1	1		2
Number specifying water pipes only:	1	5	2	2	1	1			
1	1	2							
2		1	1	1		1			
3		2		1	1				
4									
5									
Number specifying driven grounds only:		5	1	8			3		1
1		1	1	6			1		
2		1		2			1		1
3		3					1		
Number specifying either water pipe or driven grounds:									
One water pipe, two driven		3							
Two water pipe, one driven		2							
Two water pipe, two driven		1							
Number salting grounds without limitation as to resistance		3		1	1		1		3
Number salting grounds and driving rods without limitation as to resistance	1	2		2		1	1	1	1
No limitation	27	2	2	4		12	11	4	14
Number placing limitation because of:									
Experience		8	1	4	1				
National Electrical Code		3		5					
Both		1		3					

N.B. Installation group does not necessarily mean company. Some companies impose limitations upon certain territories or certain voltages.

of the maximum value of ground resistance, eight require water pipe grounds, and four require more than one driven ground.

Discussion

Herman Halperin (Consolidated Edison Company, Chicago, Ill.): While analysis of extensive data involving numerous variables generally proves difficult, Smith has done a commendable job. The large number of tables, which at first seems confusing, presents the data in such a way as to make further analysis readily possible.

From the standpoint of protection to equipment, it is of interest to examine the combined rates of transformer winding failures and fuse blowings. The combined figures for 1935 and 1936, expressed as the ratio of transformer and fuse troubles to similar troubles with the solid interconnection, were: common neutral, 1.3; standard connection, 3.4; gapped interconnection, 5.8. For the remaining protective schemes, including the self-protecting types of transformers, the numbers of installations involved were statistically insufficient, that is, only a few per cent of the number with solid interconnection, and definite conclusions for these types appear unwarranted as yet.

In view of the fairly extensive application of the gapped interconnection, it must be disturbing to a number of companies to learn of the generally poor performance of this protective scheme. With this connection, the surge voltage impressed on the transformer is that permitted by the phase arrester plus the drop in connecting leads and plus the breakdown of the spark gap. This latter value may be about the same as the voltage across the lightning arrester. Two alternative improvements may be suggested for such installations: (*a*) reduction in breakdown voltage of the interconnection gap, by the use of lower rated or special gaps; or (*b*) a change to the solid interconnection. The objection to this latter connection was based mainly upon fear for the customers' safety, but, according to our Chicago studies ("Lightning Investigations on a Distribution System," Halperin and Grosser, ELECTRICAL ENGINEERING, January 1936, pages 63–70) and the data in this paper, this objection to solid interconnection is of no consequence and should gradually disappear.

Conclusion 3 of the paper, indicating that the rate of arrester failures is lower with solid interconnection than with the standard connection, is based on a general averaging of the data from all companies. This conclusion does not seem to be entirely justified by the data, when the operating records of individual companies are considered. Of 12 companies, which report on a sufficiently large number of installations of both types to be suitable for comparison, seven, including Chicago, showed higher rates of arrester failure with the solid interconnection than with the standard connection. Theoretically, some increase in failure rate should be expected with solid interconnection, because of the greater duty on the arrester in discharging surge current to a low resistance ground. The best way to determine the effect of one variable is to have just one variable in the study, and that was the case in Chicago when we had both solid interconnection and standard connection of arresters for a few years over various parts of the city and found that arrester failures were relatively high with the solid interconnection.

The installation of the solid interconnection in Chicago was very much cheaper than trying to improve the ground resistance with the standard connection. The solid interconnection was obtained simply by the installation of a short piece of Number 6 wire between the ground side of the arresters and the secondary neutral. The effect of the solid interconnection in Chicago was to reduce the total rate of transformer failures and fuse blowings by over 50 per cent as compared to the rate that obtained with the standard connection of arrester.

Regarding conclusion 8, while it is recognized that the majority of the reported installations of secondary arresters were of an experimental nature, it would be of interest to learn something of the reasons for providing these protective measures.

The compilation of a large amount of data such as this may make it possible to analyze the effect of thunderstorm frequency on lightning troubles. It is suggested that the next questionnaire circulated should be modified to determine, for the four years beginning with 1934, the number of thunderstorms per year, both from weather bureau and company records, and an attempt made to correlate these data with lightning trouble rates.

W. A. McMorris (General Electric Company, Pittsfield, Mass.): Mr. Smith and those who have co-operated with him in the accumulation and analysis of operating data on distribution circuits, are to be congratulated on what they have accomplished. Differences in the types and ages of equipment in use, and in the conditions under which it must operate, require a study of broad scope in order that definite trends in performance may be established.

The data obtained have made it possible to draw a number of conclusions with regard to best operating practices. A few inconsistencies still remain, but it is to be expected that these will decrease in number as the investigation is continued.

On page 5 of the preprint it is stated that "the fear that has been expressed that with solid interconnection a heavier duty may be placed on the arrester causing more failures is without foundation." This conclusion is well supported, without contradictions, by data in tables X to XV, inclusive. Table X which summarizes the results, shows an arrester failure rate of 1.17 per hundred for 159,138 installations with standard connection, and only 0.551 per hundred for 146,738 installations with solid interconnection. Rather than increasing the arrester failure rate, solid interconnection has reduced it to 47 per cent of the value for standard connection, and to 57.5 per cent of the value for gapped interconnection, the nearest competitor in this table.

Information given in recent AIEE papers by Mr. Flanigen ("Lightning Protection of Distribution Transformers," AIEE TRANSACTIONS, volume 54, pages 1400–05, December 1935) and by Messrs. Halperin and Grosser ("Lightning Investigations on a Distribution System," AIEE TRANSACTIONS, volume 55, pages 63–70, January 1936) suggests that many arrester failures result from other causes than high-current lightning discharges. This was borne out in the investigation reported in the paper by Mr. McEachron and myself, presented at this convention, in which measurements were made of the discharge currents associated with arrester failures. Figure 11 of that paper shows a very small percentage of arrester failures to be due to high discharge currents.

The obvious conclusion is that solid interconnection probably does result in some increase in the magnitude of currents passed by arresters, but that high current discharges are not a major cause of arrester failure, so the predicted increase in arrester failure rate does not occur. Interconnection may result in an increase in arrester discharge currents by lowering ground resistances, but this tends to localize the disturbance so that fewer arresters operate.

This possibly helps to explain the lower arrester failure rate with solid interconnection. Failures from power current resulting from sixty-cycle overvoltage, gap corrosion, or similar causes, would decrease as the number of arrester operations is decreased by localizing the disturbances, more or less independently of any change in the magnitude of lightning discharge currents.

In general, the performance of the various gapped interconnections, including three-point protection and surgeproof transformers, has been considerably less satisfactory, even though the surgeproof transformers and their deion gaps were all new equipment. The protective scheme of the surgeproof transformer is essentially a gapped interconnection, and the over-all service performance from table X does not appear to be greatly out of line with that of other gapped interconnections.

Since the paper reports field data for 1934, 1935, and 1936, the surgeproof transformers referred to are presumably of the type with the deion gap inside the transformer tank. With such a protective device inherently a part of the transformer, either winding failure or gap failure would probably require transformer removal and maintenance.

Table X, summarizing all data, shows 1.21 gap failures and 0.43 winding failures per hundred installations of surgeproof transformers, compared to values of 0.551 and 0.350, respectively, for conventional transformers protected by valve-type arresters with solid interconnection. In some cases, both gap and winding failures may have occurred on the same transformers, so the number of surgeproof transformers requiring maintenance, per hundred installed, would be between 1.21 and 1.64. For solid interconnection with valve-type arresters mounted outside the transformer tank, arrester failure alone does not make transformer replacement or maintenance necessary, and only 0.350 transformers per hundred installed would have to go to the shop. Thus for the surgeproof transformers, all of which were relatively new, maintenance was required from 3.5 to 4.7 times as often as for the older standard transformers with solidly interconnected arresters. Both table XI showing data from rural circuits, and table XII showing data on urban and suburban circuits, but which are based on many fewer installations than table X, indicate that more transformer maintenance is required for surgeproof transformers than for standard transformers with solid interconnection.

Such an analysis based on the data reported in the paper, shows that distinct disadvantages result from combining the protective device with the transformer in such a way that the failure of the protective device is likely to interrupt service of the transformer or require its removal for maintenance. Similarly it is shown that the full benefits of interconnection are not realized by any of the schemes in which the interconnection is made through a gap.

F. E. Andrews and **R. O. Askey** (Public Service Company of Northern Illinois, Chicago, Ill.): As a result of a brief study of the data in this paper made within the short time during which a printed copy has been available, it appears that there are several factors which may influence the various failure rates and which may be of greater importance in the results shown by the different types of protection than the inherent differences in the protective scheme.

It is noted in table X summarizing results of all types of territory that the rate of primary fuses blown is in general higher with the gapped interconnection than either the solid interconnection or standard connection. It is also noted by reference to table XI that 82 per cent of the total number of gapped interconnection installations are classified as rural and that for the rural installations there is less difference in performance of the gapped installation in comparison with the others. As noted in the text of the paper, the failure rate for the rural installations for all types of protection is apparently considerably higher than for urban, which suggests that differences in exposure, arrester density, etc., which are inherent in the nature of the territory probably have a considerable influence.

We have noted in studies of our operating data with respect to the interconnection, much the same relationship with respect to the gapped interconnection as is indicated by the data in the paper. We have also noted in attempting to explain the apparent failure of the gapped interconnection to perform as satisfactorily as the solid interconnection that the gapped interconnection in general has been used in a much larger proportion of situations where grounding conditions are less satisfactory and where specific trouble due to fuse blowing has been previously experienced. This trouble due to the larger amount of fuse blowing has been found in many instances to have been due to inadequate lead and bushing insulation such that even with the interconnection satisfactory co-ordination with the lightning arrester could not be obtained. Under this condition it has been apparent that material improvement by the use of the interconnection could not be expected.

It would accordingly seem that if data on the various types of protection are to be compared purely on the merits of the various protective schemes it is necessary that the transformers involved also be classified in such a manner as to factor out those of known susceptibility to trouble of this nature. The principal items affecting this susceptibility are probably transformer size, age, and manner in which maintenance and repair work have been done. In line with this idea it would seem advisable to set up a separate grouping to include only transformers built according to present day standards which in general would probably include those bought since about 1934.

In furtherance of this idea we have made a partial study of performance of these newer transformers and while at the present time we cannot give specific data on them with reference to the different types of protection, it is obvious that rates of failure substantially different from those applying to the older transformers on the system will be found. One apparently significant point has been developed thus far from this investigation. It was believed that with new transformers of adequate design with respect to leads and bushings, it should be relatively easy to determine the explanation for each case of fuse blowing. We accordingly investigated the record of 41 cases of fuse blowing in a group of 672 modern transformers in the size range from one and one-half to seven and one-half kva, inclusive. The results of this investigation are shown as follows:

Number of transformers in group	672
Total number of fuse operations	41
Number of installations at which fuses were blown	39
Number of installations inspected in detail. (The arresters were connected to the line side of the fuse on all of these installations.)	13
Installations showing evidence of lead or bushing flashover	5
Installations at which no evidence was found to explain the fuse operations	8

From this data it is evident that there were a few cases in which transformer fuses were blown due to bushing flashovers in spite of the lightning arrester protection. There were, however, eight cases in which no ordinary explanation of the fuse blowing was evident. The transformers at these eight locations at which fuse blowing occurred were given the standard AIEE high-potential tests to determine if fuse blowing might have been caused by a coil puncture which was sealed off, perhaps by the transformer oil. All of these transformers stood the high-potential test successfully, indicating that the fuse failure occurred without passage of fault currents through the fuse.

An investigation, now in progress, of the performance of a similar group of modern transformers has disclosed that a number of instances of fuse blowing without leaving visible evidence of the path of the current responsible have occurred. In general these have been at rural installations using three- and five-ampere fuses and with the lightning arresters connected to the line side of the cutout. For the most part, these have been confined to gapped interconnected installations in rural territory.

These data point to the likelihood of the fuse blowing being due to some other type of current through the primary fuse not heretofore recognized in connection with distribution transformers such as perhaps transients due to capacity coupling from primary winding to secondary or to core and tank. There is also the possibility of high current in the fuse due to resonance between the inductance of the transformer and the capacity of the interconnection gap. It is believed that with modern transformers, at least in the smaller sizes, a relatively large proportion of fuse blowings is due to the effect described above and that it is important to direct any future investigation toward an explanation of this effect.

W. G. Roman (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): This report has tabulated a large mass of data on distribution transformer lightning-protection practice. Since these data were taken from a large number of operating companys they represent an interesting cross section or average practice.

An examination of table X shows the solid interconnection gives the best all around lightning protection although the meter burnouts are relatively high. These data are somewhat conflicting since the damage to customer's wiring is the lowest for the solid interconnection. The reasons for the low arrester failure rate for the solid interconnection are not obvious. Because of the lower effective ground resistance of the interconnection the surge currents discharged by the arrester should be higher and should result in a higher arrester failure rate. The lower rate shown in the table may be because installations having the solid interconnection also on an average have more modern arresters.

Table XVI shows that for the standard connection the low ground resistance is definitely desirable in reducing trouble. For the various forms of interconnection this is not apparent which checks with what is to be expected since in the interconnection schemes the ground drop is not added to the voltage appearing across the protective device. The reason for the high meter-failure rate for the solid interconnection for ground resistances of zero to one ohm is not apparent unless these low ground resistances are usually obtained by tying the secondary neutral to the water system on the customer's premises. If this were true, the major portion of the surge would be discharged through the customer's ground and might result in a higher meter-failure rate. This is borne out by table XVII which shows a low meter-failure rate for rural installations having a low ground resistance.

Frank Sanford (Cincinnati Gas and Electric Company, Cincinnati, Ohio): Perhaps the most important conclusion of this paper is that interconnection of grounding on distribution transformer installations is an established practice. The general data confirm the data that individual utilities have that this form of protection has accomplished a great deal and that its continuation and extension is justified. Three detailed comments may be noted:

1. The increase that is noted in the paper in grounding of the transformer tanks may be due more to the use of rural-type transformers rather than any change in policy on urban type installations.

2. The relationship between fusing schedule and per cent blown fuses caused by lightning is not shown in the paper. Its importance may justify a more detailed study and the economics of failure and service outages may justify a higher fuse schedule, especially on the smaller size transformers.

3. It would seem better not to group together all systems using one to five kv distribution, in comparing standard connection and interconnection. Of twenty companies, only four failed to show a large reduction in both transformer failures and fuses blown, with the interconnection method. Of these, one showed an increase in fuses blown but they operate with the arrester on transformer side of fuse and they may not fuse high enough for this condition. The other three operate a 4.8-kv delta system. Other records indicate a higher trouble experience with 4,150 volts or 4,800 volts than with the 2,400/4,150-volt system, other conditions being equivalent, and this would seem to justify the break-down of the records in comparison of this kind.

J. K. Hodnette (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): Mr. L. G. Smith is to be complimented on the fine job he has done collecting and compiling the data on distribution transformer lightning protection. Data of this type should prove very valuable to distribution engineers in working out the problem of lightning protection to distribution transformers.

It is possible that some of the inconsistent results obtained to date could be explained if more were known of the operating conditions surrounding the transformers. It would be desirable to tabulate the storm days per year experienced by each company reporting. In this manner the results obtained by different operating companies could be more accurately compared as well as the year to year results of individual companies.

It would be desirable to establish a more accurate nomenclature for the different methods of protection. Referring in particular to figure 8 in the paper which is designated as "three point protection," this phrase was originated by Mr. Fred Hanker to designate a type of protection in which the protective devices joined the high-voltage winding and low-voltage winding and the tank together, such as is illustrated in the author's figures 3, 5, 6, 7, 8, and 9. Since this interpretation is still in common use, a designation of a specific method of obtaining this result as three-point protection is somewhat confusing.

The results to date indicate that primary fuses may be expected to blow even though the protective device is connected ahead of the fuse link and presumably shunts the surge current around it. In my opinion this is due to (1) mechanical damage of the link; (2) short circuits attending storms; (3) surge currents flowing through the link together with associated secondary short circuits. Even though a protective device successfully shunts the major portion of the surge current around the fuse link, the charging current of the winding flows through the link. For steep front lightning surges this is by no means a negligible current and considering repetitive strokes the accumulative effect may be sufficient to blow the fuse link, particularly if the rating of the link is small. In addition to this effect, short circuits occur in the house wiring due to induced surges on the ungrounded secondary wires flashing over to grounded neutral. It is apparent that this condition cannot be eliminated and improvement in service can only be expected either by eliminating primary fuses or using large sizes.

C. F. Harding (Purdue University, Lafayette, Ind.): The principle and method of attack of this whole problem of protection of distribution systems may well be emphasized again. When first conceived the length of time and expense involved in its solution seemed prohibitive. One executive in the Middle West said it would take $50,000 and many years to get an answer to the interconnection problem if undertaken by statistical analysis of lightning discharges in natural thunderstorms in representative areas of several large cities.

As a predictive and foreshortened study, therefore, an experimental yet full-sized, four-span standard installation was made at Purdue University under the auspices of the Utilities Research Commission of Chicago whereupon thousands of induced surges were superimposed by means of a high-voltage laboratory surge generator. The results of these two years of testing of various types of interconnection and protection were reported at length in the AIEE TRANSACTIONS, March 1932, and in Bulletin No. 42 of the Engineering Experiment Station of Purdue University.

Since the solid interconnection was definitely proved by these tests to be most effective and since no dangerous excessive voltages appeared on the consumers' premises *if adequately low-resistance grounds were provided thereon*, many companies were encouraged to install such interconnections at a very much earlier date than would have been possible by the statistical field-study method. These predictions of the best methods of protection are well substantiated by the conclusions of this paper. Reports, during recent years, of large reductions in transformer burnouts, and of no trouble from excess potentials at the consumers' premises, after the interconnection was made, are confirmed by this paper and the resulting discussion thereon.

This general procedure, therefore, seems to justify the practical results and warrant the further consideration of laboratory methods of attack as co-operative projects between university research staffs and public utility and manufacturing corporations in order that the probable best solutions to such problems may be secured more promptly and with minimum of expense.

It is encouraging to note the very large increase in such interconnections during the last few years as well as the tendency to ground transformer cases and to reduce ground resistances on rural consumer circuits. The success that these changes have attained is a further confirmation of the worth of such full-sized laboratory tests. However, the limited number of meter burnouts may not necessarily be a criterion for indicating the correspondingly small number of cases of high potential on the consumers' premises as some types of meters, at least, will withstand relatively high surge potentials without burnout or serious change of calibration, the potential coils having sufficiently high inductance and such design as to permit flashover and the current coils having sufficiently low inductance and high carrying capacity to avoid damage due to heavy surge currents therein. The laboratory tests previously mentioned indicated that other consumer apparatus would flashover or burn out before meters would be damaged and that with house circuits closed and lamps lighted the attenuation of such surges was a worth-while safety factor.

J. R. North (The Commonwealth and Southern Corporation, Jackson, Mich.): Mr. Smith's paper presents a very considerable number of operating records covering the experience of different companies with distribution transformer protection from lightning. The conclusions reached should be very interesting to distribution engineers and others concerned with the application of protective equipment.

It would appear that the results of this study could be made more conclusive if it were possible to evaluate further the effects of:

(*a*). Number and severity of storms for the various locations.

(*b*). Age and relative insulation strength of the transformers involved.

(*c*). Relative protective characteristics of the lightning arresters.

The various methods of lightning protection of distribution transformers and the conclusions given are classified in the paper according to current terminology and by means of nine connection diagrams. As mentioned by Mr. Smith, it is difficult to compare the operating performance and the relative merits of the various schemes because of the lack of uniformity in nomenclature used throughout

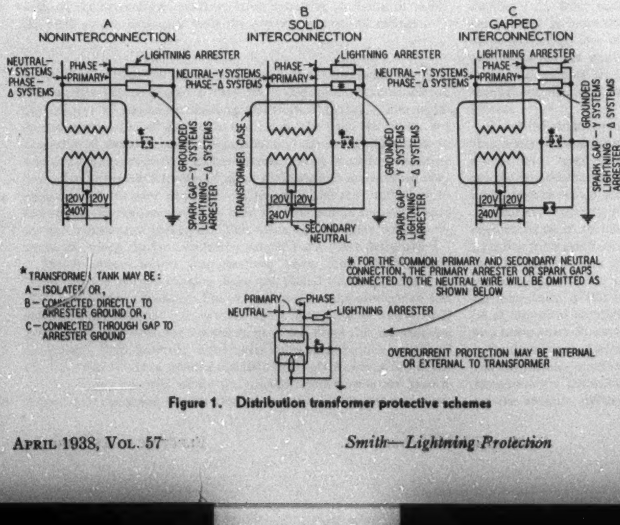
A
NONINTERCONNECTION
B
SOLID
INTERCONNECTION
C
GAPPED
INTERCONNECTION
LIGHTNING ARRESTER
NEUTRAL-Y SYSTEMS
PHASE-Δ SYSTEMS
PHASE
PRIMARY
GROUNDED
SPARK GAP – Y SYSTEMS
LIGHTNING ARRESTER – Δ SYSTEMS
TRANSFORMER CASE
SECONDARY NEUTRAL
120V
120V
240V
* TRANSFORMER TANK MAY BE:
A – ISOLATED OR,
B – CONNECTED DIRECTLY TO ARRESTER GROUND OR,
C – CONNECTED THROUGH GAP TO ARRESTER GROUND
# FOR THE COMMON PRIMARY AND SECONDARY NEUTRAL CONNECTION, THE PRIMARY ARRESTER OR SPARK GAPS CONNECTED TO THE NEUTRAL WIRE WILL BE OMITTED AS SHOWN BELOW
PRIMARY
PHASE
NEUTRAL
LIGHTNING ARRESTER
OVERCURRENT PROTECTION MAY BE INTERNAL OR EXTERNAL TO TRANSFORMER

Figure 1. Distribution transformer protective schemes

the industry. Therefore, it would appear to be highly desirable to adopt a uniform and simple system of classification of the protective methods.

The lightning arrester subcommittee of the Institute's protective devices committee has given considerable thought to this, and figure 1 shows three diagrams illustrating all the connections commonly used for protecting distribution transformers. The three basic schemes of connections may be designated as (*A*) noninterconnection, (*B*) solid interconnection or (*C*) gapped interconnection, according to the connections of the ground leads of the primary protective devices and the secondary neutral. These leads may not be connected to the secondary neutral. They may be solidly connected or connected through a gap.

It will be seen from the notes on figure 1 that these three diagrams are applicable to all types of systems and schemes of connection. It is urged that some such uniform scheme of diagrams and terminology be adopted to avoid confusion and to eliminate lengthly descriptions of the various different protective schemes.

L. G. Smith: The suggestions made by Halperin, North, and Hodnette that the storm frequency for each year in each locality be weighted in determining the totalized data in the summary tables, might be misleading in that there is still another factor which cannot be determined, namely, storm severity. For example, on our own company system we have had years in which as many thunder storm days were experienced as in other years but the percentage troubles were materially lower due to variations in storm frequency. It is planned after five or six years' data have been obtained to weight the data supplied by each company by isoceraunic level for the territory for which the data was reported. Likewise the age and condition of the transformers involved should be considered as suggested by Andrews and North. However, in collecting data of this kind it is necessary not only to obtain from each company the numbers of transformers failing and number of primary fuses blown for each type of transformer but it is also necessary to know the number of transformers of each type on the system. If it was decided to obtain data in this amount of detail it is believed complete operating data would be obtained from only a few companies, as most companies do not keep their records in as much detail as would be required. It is believed that the obtaining of operating data on a large number of transformer installations will tend to average the effect of age and condition of transformers.

As Halperin suggests, the use of the solid interconnection is probably the best and cheapest means of lowering the ground resistance of the lightning arrester ground. Both Halperin and McMorris have discussed the effect of interconnection upon lightning arrester failures from two different points of view. While it is true as Halperin stated the effect of lowered ground resistance is to permit more surge current to pass through the arrester and hence probably result in more arrester failures, attention is called to the data in the McEachron and McMorris paper, which indicates that a very large percentage of the surges experienced on distribution systems are of lower current values than those which would cause arrester failures. It is also true that many failures of arresters even during storms may be due to other causes than high surge currents.

In reference to the poor performance of gapped interconnections as discussed by Halperin and Andrews it is quite possible that the voltage at which the gap breaks down is not materially different from the bushing flashover of the transformers protected. This is particularly true of the older types of transformers with smaller bushings. If such is the case it would be anticipated that the performance with the gapped interconnection would not be materially different from that with the standard connection in view of the fact that the flashover of the bushings results in effecting an interconnection with the secondary neutral. It is true as Andrews has mentioned that the rates of trouble are materially higher in rural districts. It is possible that the unexplained blowing of fuses mentioned by Andrews may be due to bushing flashovers or flashovers from the leads to the case that do not leave marks sufficiently to be distinguished by a subsequent inspection.

In reference to the relationship between the fusing schedule and the rate of primary fuses blown as discussed by Sanford, an attempt was made to correlate these two points in obtaining data. However, it is quite difficult to reduce a fusing schedule to a single statement. In summarizing the data an attempt was made to reduce all fusing schedules to a percentage basis. As Hodnette has suggested, from a theoretical standpoint at least the higher the fusing schedule the lower the probable rate of primary fuses blown.

Discussions

of AIEE Technical Papers Published Before Discussions Were Available

ON THIS and the following five pages appear discussions submitted for publication, and approved by the technical committees, on papers presented at the AIEE winter convention, New York, N. Y., January 24–28, 1938. Authors' closures, where they have been submitted, will be found at the end of the discussion on their respective papers.

The Rating of Resistance-Welding Transformers

Discussion and author's closure of a paper by C. E. Heitman published on pages 125–30 of this volume (March section) and presented for oral discussion at the electric welding session of the winter convention, New York, N. Y., January 26, 1938.

D. I. Bohn (Aluminum Company of America, Pittsburgh, Pa.): I am in hopes that Mr. Heitman's excellent paper on this subject will provoke quite vigorous comments, not only from those in the welding-machine industry, but from users and power companies.

In my opinion a welding machine should have a dual electrical rating. Part of this rating should be that of the transformer only. It is not so important as to just what method is used to rate this transformer as long as it is agreed on. The second part of the welding machine rating should advise the electrical conditions of the machine as a whole. A spot welder for instance may have a transformer of a 100-kva rating defined in whatever method is agreed on. The machine itself however may be of such a design as to require a demand of 225 kva. The same transformer in another machine of different design may require a maximum demand of only 150 kva. It is quite obvious therefore that to rate the machine on the basis of the transformer alone is misleading to all concerned.

Inasmuch as the thing that makes the weld is amperage, the maximum available amperes at the point of weld should be given as part of the machine rating.

If all machines have ratings of this nature, the question as to whether a machine will do a certain job could be answered directly without the necessity of making rather extensive electrical or welding tests as are necessary with the present systems of rating.

L. G. Levoy (General Electric Company, Schenectady, N. Y.): The author has presented a very lucid and timely exposition on the subject of resistance welding transformer rating. The widespread application of electronic control in the past few years has necessitated more dependable information to enable the user to select the proper control. Either overestimating or underestimating control requirements from the name-plate ratings may cause considerable unnecessary expense. In the past the only safe way to determine the requirements has been to make actual measurements on the transformer involved. This often entails delay and inconvenience, which can be avoided by proper name-plate designation.

The resistance welder manufacturers know these requirements. For a variety of reasons, they have not placed this information on the name plate. Probably the chief reason for the absence of this information is that the user, in many cases, does not know the secondary current requirements for welds, in which cases this information is useless. Now, however, many users know these requirements and yet from the data on a welding machine name plate cannot determine the suitability for jobs where the current requirements are known. Many manufacturers furnish this information on request and probably will welcome a new sane standard for name-plate designations to be followed by all manufacturers.

In order to eliminate all ambiguity and enable the user to determine whether or not his requirements are within name-plate rating of the transformer, it is necessary to specify, in addition to the items listed by the author, the maximum allowable time over which the duty cycle may be averaged with the understanding that this average is to be taken to include the most severe duty cycle imposed. This time is intimately associated with the thermal time constant of the transformer. The question arises whenever the duty cycle is irregular and the transformer is working near its rated capacity. Since this characteristic will vary with each size and design of transformer, the maximum allowable averaging time for the most severe duty cycle should be stated on the name plate to protect both the manufacturer and user against unintentional overloading.

The author discusses the use of an autotransformer or a tapped primary winding for varying the secondary current. There is another practical method known as phase control, which eliminates the necessity and expense of supplying either an autotransformer or a tapped-primary winding. It is accomplished, with electronic control, by delaying the start of the period of conduction of each voltage wave beyond the power-factor angle of the welder. This permits a smooth, stepless variation of heat over the entire range. Ease and speed of adjustment has made this method popular with operators where frequent changes in heat are required on work of varying nature, or where the steps of current adjustment by the available taps are too large.

L. W. Clark (The Detroit Edison Company, Detroit, Mich.): Mr. Heitman deserves the thanks of all concerned with the welding industry for his comprehensive discussion of the fundamentals involved in properly rating resistance welding transformers. There is bound to be some disagreement as to the details of standardization and recommended name-plate data, but the author's logical presentation should be a great help in crystallizing the ideas of various interested groups with a resulting standardization valuable to everyone.

From the standpoint of those interested in providing an adequate power supply for resistance welders, both in the plant itself where the welders are located and also from the power system providing the service, there are two factors mentioned by the author which may be further emphasized. He clearly shows the value of reduced impedance drop in the transformer but only mentions the effect of the impedance of the secondary circuit and states that it is this impedance that constitutes the load on the transformer. Only a very small percentage of the total secondary impedance is actually accounted for in the weld itself, the bulk of it being found in the conducting path from the transformer to the weld. I believe that, particularly in large machines, further emphasis should be placed upon the importance of keeping the secondary circuit as compact and of as low an impedance as possible thereby showing definite savings in welding transformer size, required distribution facilities throughout the plant, and power demands drawn from the power system.

It hardly seems necessary to discuss this point but it is a fact that many of the large present-day flash and projection welders have much higher reactive losses than necessary, resulting in oversized supply equipment all along the line. Money spent in improved design will be repaid many times over in savings in expenditures for transformer capacity and distribution facilities. The amount of current required to make a good weld is pretty much of a fixed quantity for a given job, but the type of path provided for conducting the current from the welding transformer to the work can be varied. Some of the welder manufacturers may not have realized the importance of this factor, and it is hoped that new machines will be of improved electrical design, resulting in lower power demands and lower costs of service for the user.

On the subject of name-plate rating, it is highly desirable that when the transformer is part of a complete welding machine, the maximum instantaneous kilovolt-ampere demand be given, as recommended by the author. With the present practice of a name-plate rating based on a 50-per-cent-duty cycle, there is no logical basis upon which to lay out an adequate supply system. It becomes necessary to "guess" that the instantaneous demand will be two to five times the name-plate rating which is certainly poor basic information upon which to authorize plant expenditures for serving new equipment. The maximum instantaneous demand with the transformer operating on the highest voltage tap should be incorporated as part of the standard name-plate data.

H. S. Hubbard (nonmember; General Electric Company, Pittsfield, Mass.): Mr. Heitman's paper is of timely interest to both the user and manufacturer of welding transformers. Certainly it is apparent that the subject of transformer ratings for this class of service should be given serious consideration and it is logical that this should follow other standard lines of equipment.

The 50-per-cent-duty cycle, so commonly used as a rating basis, has little bearing on the capability of a transformer to perform satisfactorily on duty cycles for other than which it was originally designed. While the theoretical equivalent rating for any other period of operation may be easily determined from this value, there are real practical limitations dependent upon the transformer characteristics, which may restrict the use of a particular unit for a specific application.

Above all, the manufacturer should know the maximum instantaneous load requirement and the maximum duty cycle at that load. This permits him to so proportion his design that reasonable values of resistance and reactance are obtained and to determine the thermal rating on an equivalent continuous basis.

It is desirable that reactance and resistance be considered separately rather than as combined impedance for the reason that with the low power factor in the usual type of welding circuit, the reactance component is more nearly in phase with the load current and consequently influences the regulation much more than the resistance component. Thus two similarly rated transformers having the same impedance value might vary considerably in the matter of regulation.

In specifying a definite impedance in per cent, it is assumed that the designer has full control over the governing factors. This is not always so and cases will arise where it is economically impractical to meet such requirements. Perhaps it would be best to state the impedance or reactance rather than limit it to a fixed value.

Two transformers of the same kilovolt-ampere rating, but with different secondary voltages might have the same impedance drop in volts, but in per cent would vary inversely with the actual voltage output. This it would seem, might penalize an ordinary satisfactory design, if limits are too closely restricted.

While the true ohmic value of the loop impedance is numerically low as stated in the paper, the per cent impedance is usually extremely high, approaching 100 per cent of the transformer voltage. The power component across the weld is a very small factor and the voltage rating of the transformer is consequently influenced largely by the inductance of welding circuit or loop. Thus it is of utmost importance that the voltage drop in any welding circuit be carefully determined before the kilovolt-ampere rating can be specified.

In the matter of temperature rise, would it not be consistent to limit this to the AIEE standards for whichever class of insulation is used in the transformer design, that is, for class *A*, 55 degrees centigrade; and for class *B*, 75 degrees centigrade.

In connection with the chairman's request for items which might profitably be studied by the committee, I would like to suggest that an effort be made to correlate existing data on "loop" inductance and the methods used to calculate this circuit. There appears to be a divergence of opinion in this regard and it is difficult to obtain consistent data which might be applied generally. This is particularly important for as mentioned earlier in this discussion, the voltage drop in the secondary circuit is the major determining factor in the selection of the proper transformer voltage and hence the kilovolt-ampere rating for a particular job.

Some time ago in co-operation with one of the equipment manufacturers, we made a brief study of the kilovolt-ampere requirements for various size loops and based on the data submitted, developed a simple family of graphs. Although largely empirical, we have found these curves quite useful as a reference and I am sure that a more thorough study of this particular problem would be generally helpful to the industry.

S. M. Humphrey (nonmember; Taylor-Winfield Corporation, Warren, Ohio): I do not believe that anything will be accomplished unless the AIEE can agree to adopt a standard method of rating which has a reasonable chance of being accepted by other organizations such as the manufacturers of resistance-welding equipment, and their customers. Fundamentally it does not make any particular difference what the system of rating is as long as it is based on sound engineering principles and is universally acceptable.

Since a very large percentage of purchasers and users of resistance welding equipment are not electrical engineers, in fact they are not conversant with electrical terms, but have been brought up to consider that a transformer with certain kilovolt-ampere name plate will stand about so much, it would not be at all practical to attempt at this time to suddenly change this name-plate figure and then attempt to explain that a 70-kva transformer is now the same as a 100-kva used to be.

For this reason I strongly recommend that the present system of rating based on a 50-per cent duty cycle be adopted as standard by the AIEE. This would be as equally understandable by electrical engineers as the 100-per cent duty cycle and in the eyes of many users would not constitute any change at all.

In order to completely specify the transformer, it seems to me it would also be worth while to specify what its rating should be on the low tap as well as the high tap.

In regards to per cent impedance, I believe the best system would be to actually specify the impedance on the name plate in all cases. I am not in favor of specifying both resistance and reactance since in most cases the impedance is sufficiently close for all practical calculations and, furthermore, the actual resistance is a small part of the impedance and also very difficult to measure without machining special short-circuiting members for that purpose.

In regard to the temperature rise, I believe it best to adopt the present standard of 55 degrees for class-*A* insulation, and 75 degrees to class-*B* insulation.

Regarding the cooling water required, it might also be worth while to specify what pressure is needed to obtain the required gallons per minute for cooling.

In regard to the machines, it is very difficult to specify on the machine name plate the maximum primary current which will be drawn. If this were done it would have to be specified with the lower knee in its raised position and with the shortest horns which the machine could use, and if this figure were then used as a basis for power installation and purchase of control equipment, it might burden the purchaser with a much more expensive installation than was required for the work to be done.

At the present moment the Taylor-Winfield Corporation are supplying with a portion of their standard machines complete curves covering both the primary and secondary currents for all possible geometrical combinations of the secondary circuit and will extend this service to cover their entire line of machines as soon as possible.

The use of electronic control has been mentioned here by one of the representatives of the General Electric Company. There is one fact regarding its use which I believe should be pointed out since it will have some bearing on the power company's problem, and that is that heat reduction by means of electronic control results in a higher root-mean-square primary current than is obtained if the current reduction is made by means of a tap transformer or autotransformer. The increase in root-mean-square value is accounted for entirely by harmonics due to the wave form obtained from electronic heat control. The fundamental component of the current is the same with either system on secondary current reduction.

The above comments should not be construed to mean that I am opposed to electronic control. I believe it to be a very valuable addition to resistance welding equipment and from actual use have found it to be as fine a means of welding-current control as has yet been devised.

G. S. Bernard (nonmember; Aluminum Company of America, Pittsburgh, Pa.): I certainly agree with Mr. Heitman in that the name-plate data now furnished on resistance-welding transformers is of little value to the user of this equipment. The use of a thermal equivalent continuous kilovolt-ampere rating based on the maximum continuous secondary current which can be drawn from the transformer without exceeding the safe operating temperature of the insulation materials employed and the maximum rated open-circuit voltage is a logical method of rating any transformer whether for continuous or intermittent duty.

In making use of this kilovolt-ampere rating as proposed by Mr. Heitman, we must not lose sight of the fact that the current delivered by the transformer and not the kilovolt-amperes delivered should be used as a basis to determine if any given loading is within the safe limits for a given transformer.

Thus, regardless of the secondary voltage which may be used to obtain the welding current, the thermal equivalent continuous kilovolt-amperes is obtained by multiplying the secondary current by the maximum rated open circuit secondary voltage and the square root of the duty cycle.

As an example, consider the 34.7-kilovolt-ampere transformer proposed by Mr. Heitman.

Suppose that we wish to find the maximum duty cycle at which each of the following welding loads could be supplied.

CASE I

Welding current = 10,000 amperes
Load impedance = 0.001 ohm

CASE II

Welding current = 10,000 amperes
Load impedance = 0.0005 ohm

CASE III

Welding current = 5,000 amperes
Load impedance = 0.001 ohm

CASE I

Open-circuit voltage required is:

$$V_1 = 10{,}000 \times 0.001 + 0.04 \times \frac{10{,}000}{3{,}000} \times 11.6 = 11.6 \text{ volts}$$

$$\frac{10{,}000 \times 11.6}{1{,}000} \times \sqrt{D_1} = 34.7 \text{ kva}$$

$$\sqrt{D_1} = 0.30,\ D_1 = 0.09 = 9.0 \text{ per cent}$$

CASE II

Open-circuit voltage required is:
$V_2 = 10{,}000 \times 0.0005 + 0.04 \times 10{,}000 \times 11.6 = 6.6$ volts.
But in calculating thermal equivalent continuous kilovolt-amperes we use the rated voltage

$$\frac{10{,}000 \times 11.6}{1{,}000} \times \sqrt{D_2} = 34.7 \text{ kva}$$

$$\sqrt{D_2} = 0.30,\ D_2 = 0.09 = 9.0 \text{ per cent}$$

CASE III

Open-circuit voltage required is:

$$V_3 = 5{,}000 \times 0.001 + 0.04 \times \frac{5{,}000}{3{,}000} \times 11.6 = 5.8 \text{ volts}$$

Again we use the rated secondary voltage

$$\frac{5{,}000 \times 11.6}{1{,}000} \times \sqrt{D_3} = 34.7 \text{ kva}$$

$$\sqrt{D_3} = 0.60,\ D_3 = 0.36 = 36 \text{ per cent}$$

The above calculations show the method of using the T.E.C. [thermally equivalent continuous] kilovolt-amperes in calculating safe loadings for welding transformers. Had we used the actual secondary voltage instead of the rated maximum voltage, we would have obtained duty cycles of 27 per cent for case II and 143 per cent for case III which, considering the currents used, are obviously incorrect.

In selecting the proper maximum open circuit secondary voltage to obtain a given secondary current, an allowance for voltage drop in the lines supplying the welder transformer must be made. Unless the welder happens to be located favorably with relation to a large source of power, it will not be economical to provide a voltage regulation better than 2 per cent based on the T.E.C. kilovolt-ampere rating or six per cent based on the maximum current demand during the weld. As this supply line impedance is comparable to the transformer impedance, it should be included in the calculations of open circuit secondary volts required.

The inclusion of this factor will indicate the use of open circuit voltages some six to ten per cent higher than those which would be used if only transformer and load impedance were considered. The transformer kilovolt-amperes required will also have to be increased in proportion to the increased secondary voltage.

F. H. Roby (Square D Company, Milwaukee, Wis.): Manufacturers and users of resistance welding control equipment welcome any standardization of the kind suggested in Mr. Heitman's paper entitled, "The Rating of Resistance Welding Transformers."

Although welding transformers presumably have heretofore been rated on a 50-per-cent-duty-cycle basis, it has been almost impossible to calculate the magnetic contactor load in amperes, because of widely varying transformer design. Contactor ratings are properly based upon the load current (primary of transformer), primary voltage, interruptions per unit of time and length of welding period. Of the four factors involved, the load current is the most important and least often determinable.

It is true that nearly all manufacturers of control equipment have conducted tests to fix values of currents that can be safely interrupted under given operating conditions but this data is useful only to the individual in possession of the transformer or welding machine performance characteristics. If sufficient information is not included as name-plate data on both transformers and machines, there is little hope that contactors will be successfully applied by any one except the manufacturer of the equipment.

If the standardization under discussion becomes effective and all concerned use it, then manufacturers of control equipment can publish contactor-rating data that will be helpful to all in the industry.

R. L. Briggs (nonmember; Thomson-Gibb Electric Welding Company, Lynn, Mass.): I agree in principle with the recommendations suggested in this paper. It should be possible for a user of resistance-welding equipment to determine the exact capabilities of his machine. He should know how much it is capable of doing and how rapidly it can be used. I should like, however, to take certain exceptions to his recommendations and statements.

In a great majority of cases finished machines never experience

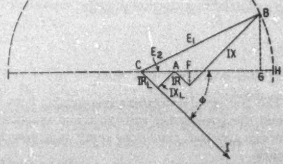

Figure 1

any changing of their transformers, portable spot welding equipment being practically the only exception. I therefore recommend that the complete welder be rated with the exception that the short-circuit impedance of the complete welder be stated rather than the per cent impedance at rated kilovolt-amperes. I also recommend that the method of rating suggested in this paper be applied to all transformers installed or to be used with portable welding equipment, with the exception that the six-per-cent value assigned as a standard for the percentage impedance be altered.

The value of six per cent suggested as a standard for the impedance drop does not seem consistent with the actual conditions involved in a resistance welding transformer. Consider figure 1 which shows the load vector diagram of a transformer referred to the secondary. Now

E_1 = open-circuit secondary voltage
E_2 = secondary voltage across load
I = load current
R_L = load resistance
X_L = load reactance
R = transformer resistance
X = transformer reactance
ϕ = current phase angle

By the definition of the impedance drop

$$Z_d = \frac{E_1 - E_2}{E_1} = \frac{\overline{AH}}{E_1} \tag{1}$$

But

$$\overline{AH} = \overline{AF} + \overline{FG} + \overline{GH} \tag{2}$$

$$\overline{AF} = IR \cos \phi \tag{3}$$

$$\overline{FG} = IX \sin \phi \tag{4}$$

The value of $\overline{GH}$ in a transformer is small in proportion to $\overline{AH}$ and this is true for all loads except at or near no load and for any power factor. Let us assume that $\overline{GH} = 0.2\overline{AH}$ as a maximum and use this value in our analysis.
Therefore

$$\overline{AH} - 0.2\overline{AH} = IR \cos \phi + IX \sin \phi \tag{5}$$

$$\overline{AH} = 1.25\ (IR \cos \phi + IX \sin \phi) \tag{6}$$

Now

$$Z_d = \frac{\overline{AH}}{E_1} = \frac{1.25}{E_1} (IR \cos \phi + IX \sin \phi)$$

$$= 1.25 \frac{I}{E_1} (R \cos \phi + X \sin \phi) \tag{7}$$

But

$$\frac{E_1}{I} = Z_t \text{ the total impedance} \tag{8}$$

Hence

$$Z_d = \frac{1.25}{Z_t} (R \cos \phi + X \sin \phi) \tag{9}$$

Equation 9 should be substantially accurate for the thermally equivalent kilovolt-ampere load condition, and at this loading it has been suggested that $Z_d = 0.06$. Therefore

$$0.06 = \frac{1.25}{Z_t} (R \cos \phi + X \sin \phi) \tag{10}$$

or

$$0.048Z_t = R \cos \phi + X \sin \phi \tag{11}$$

This last equation states a relationship between the total impedance and the transformer resistance and reactance. If the transformer design and load conditions result in a very high power factor load, then

$$0.048Z_t = R \text{ approximately} \tag{12}$$

This is a condition closely met in actual practice and demonstrates that the $Z_d = 6$ per cent is a proper value for high-power-factor loads used with power transformers. If the transformer is a resistance-welding transformer and the loads are those met in resistance welding the power factor is low, and

$$0.048Z_t = 0.7R + 0.7X \text{ as an example} \tag{13}$$

This is a condition almost impossible to meet in actual practice. Actually the sum of the fractions of the transformer resistance and reactance in the case of butt welders may be $0.5Z_t$ which indicates that Z_d should approximately be 65 per cent for these types of welders. This value of Z_d must necessarily be different for the transformers used in the other types of welding machinery.

I therefore recommend that a value of Z_d be established for the welding transformer which is commensurate with the type of welder in which the transformer is to be installed. It is still my belief, however, that excepting the portable welding equipment it is better to use the short-circuit impedance data for the purpose of rating the welder complete rather than to rate the transformer proper in terms of its impedance drop at a specified loading.

C. E. Heitman: It is gratifying to the author to note the number of very pertinent discussions of this paper which indicate that there is considerable interest in, and desire for, standardization of welding transformer ratings. It is sincerely hoped that this paper and these discussions will stimulate further activity in this direction which will bear fruitful results.

The discussion by Mr. Bernard brings out two important points which I failed to stress in the paper. First, that in calculating the maximum permissible duty cycle, the maximum open-circuit secondary voltage should be used in conjunction with the thermally equivalent continuous kilovolt-amperes. However in this connection I should like to bring out one more point which some of you may have considered already. Referring to case II of Mr. Bernard's discussion, with an open-circuit voltage of 6.6 volts, the member of primary turns will be roughly twice that used in obtaining the maximum secondary voltage. The primary resistance will therefore be approximately twice, but for the same secondary current, 10,000 amperes, the primary current will be one-half. The primary heating will therefore be $(I/2)^2 \times 2R = {}^1/_2\, I^2R$ or one-half its previous value.

The secondary heating will of course be the same. The determination as to whether or not the transformer can stand a duty cycle higher than nine per cent will depend upon what percentage of the total copper loss is contributed by the primary winding. If the primary could contribute all the copper loss it is obvious that the duty cycle could be doubled in this case, since the core loss, per weld, is approximately one-fourth its previous value. However, since the present design tendency is to make the primary copper loss low, a reduction of 50 per cent in this factor will make very little difference in the total copper loss. However for most present-day designs, the duty cycle could be increased above nine per cent in case II, but for safety it is not desirable to do so. If the actual values of primary and secondary copper losses are known, an accurate determination of the maximum permissible duty cycle (somewhat above nine per cent) could be made.

Second, Mr. Bernard brings out the point that the impedance of the primary line feeding the transformer should be considered in calculating the open-circuit secondary voltage of the transformers.

Both of these considerations were omitted from my paper since I felt that they might be confusing in a discussion of transformer ratings. I furthermore felt that although the question of primary feeder impedance is very pertinent to resistance welding, it was not within the scope of this paper. It is a subject within itself and one which needs further discussion.

Regarding the question raised as to the maximum allowable time over which the duty cycle may be averaged, I feel that this can best be answered by the manufacturers. I am therefore suggesting that they offer recommendations in this connection, with the idea in mind that this information will not be included on the name plate, but that values of time, depending upon the rating, or group of ratings, be given in the standards. For example, in transformers rated from 25 to 75 kva the duty cycle may be averaged over a certain period, whereas transformers rated from 75 to 150 kva may be averaged over a different time period. I recommend that the ratings be divided into such groups and a definite averaging time be assigned to each group and included in the standards, rather than adding this information to the name plate.

Mr. Hubbard's suggestion that the reactance and resistance be considered separately rather than as a combined impedance, is very pertinent. Such information would undoubtedly be desirable, if the manufacturers would agree to supply it. However, I feel that it is not essential. That a standard based on the assumption that the transformer impedance voltage is in phase with the load voltage, will take care of the worst condition, from a standpoint of regulation, and will therefore serve the purpose. If certain manufacturers desire to include this information on the name plate, or submit it to the customer in the form of a test report, I do not have any objections. However, in outlining standards such as these, I feel that the required name-plate information should be held to a minimum.

I am in hearty agreement with Mr. Hubbard's suggestion that the resistance welding committee make a study of existing data on loop impedances and methods used for calculating this data. I have certain data on this subject and I am sure many others connected with the art have likewise. A correlation of this data would prove most useful and beneficial to all concerned.

I am in partial agreement with Mr. Humphrey in that it does not make any appreciable difference what standard of rating is adopted, so long as it is technically sound and rigidly adhered to by the various manufacturers. From a technical standpoint, a rating based on a 50-per cent duty cycle is just as sound as one based on the continuous or 100-per cent duty cycle. I feel that the 100-per cent duty cycle rating is the most logical and I have recommended this for adoption as standard. However, if it is felt by all concerned that a standard rating based on a 50-per-cent-duty cycle will be more universally accepted and adhered to, I feel that this should be adopted.

In this connection I might add that in the discussion which followed the presentation of this paper, the suggestion was made that a standard be adopted which would keep the name-plate kilovolt-amperes as low as possible. Such a standard would obviously be based on a 100-per cent duty cycle. Rated on this basis a certain transformer might have a name-plate rating of 100 kva, whereas the same transformer if rated on the basis of a 50-per-cent duty cycle would have a name-plate rating of 141 kva.

I would like to express my further approval of Mr. Humphrey's suggestion that the required cooling be expressed in terms of water pressure required rather than in gallons per minute.

The thought expressed in Mr. Brigg's discussion has been mentioned before in certain verbal discussions which I have had with various interested parties. Mr. Briggs feels that it is not practical to obtain a transformer impedance as low as six per cent in units having a very low open-circuit secondary voltage. In the example he cites of a butt-welding transformer, the maximum open-circuit secondary voltage would be in the neighborhood of five volts. A normal impedance of six per cent would mean an impedance voltage drop of only 0.3 volts in the transformer. This, Mr. Briggs feels, is impractical to obtain. I think it best to abide by the advice and opinions of the various manufacturers in this matter, since through their experience, they are in a better position to state what they can obtain economically. I do know however that an impedance of six per cent can be obtained with an open-circuit secondary voltage of 15 volts in ratings up to 200 kva. To outline a group of standard impedances based on a kilovolt-ampere rating and open-circuit secondary voltage ranges might be too complicated for the purpose of standardization. It therefore seems desirable to omit from the standards any specification as to the quantitative value of the transformer impedance, but rather to require that the manufacturer state on the name plate, the value of the impedance, whatever it might be, at rated load. It will therefore be left to the judgment of the user to select the transformer having the lowest impedances, all other factors being equal.

Most of these discussions have been confined to items 5, 6, and 8 of my recommendations, and from them I am prompted to make the following changes in the recommendations:

Item 6 to read "Per cent impedance at rated kilovolt-amperes"

Item 8 to read "Type of cooling required (if water-cooled state required pressure at specified intake temperature)"

I will not revise item 5 at present, but if it is mutually agreed that a rating based on a 50-per-cent-duty cycle will be more satisfactory to all concerned, I will co-operate toward its adoption as standard.

In conclusion I should like to emphasize the fact that these standards, in order to be effective, must first of all be technically sound, and secondly universally accepted by all concerned. It is sincerely hoped that the manufacturers and the users of welding equipment will co-operate toward arriving at a standard that will meet these two conditions.

Recent Advances in Resistance Welding

Discussion and closure of a report of the subcommittee on resistance welding of the AIEE committee on electric welding published on pages 37–8 of this volume (January section) and presented for oral discussion at the electric welding session of the winter convention, New York, N. Y., January 26, 1938.

L. W. Clark (The Detroit Edison Company, Detroit, Mich.): The report mentions the increased use of large-size welders, with transformer ratings in excess of 1,000 kva not being uncommon. Where there are many such large welders in a single plant as is true in most automobile and accessory plants, the problem of providing an adequate plant distribution system for serving the welders becomes important. The system must allow only a minimum of voltage drop which means a low-reactance design capable of serving the extremely high-current, low-power-factor loads.

A low reactance bus system designed and installed during the past year by one such user of welders is a distinct advance along such lines and a short description would seem appropriate. The main section of bus consists of a four-inch standard copper tube surrounding an extra-heavy three-inch copper tube with "micarta" tubing insulation between. The outer conductor is grounded, doing away with the necessity for any bus supports or outer insulation. Taps are brought out from the inner conducting tube at six-foot intervals, at which point a maximum of four welder control contactors can be located. Branch busses, each consisting of a two-inch standard tube surrounding a one-and-one-quarter-inch extra-heavy tube, radiate from the main bus at intervals throughout the plant.

The main bus can transmit 1,000 amperes of 40-per-cent-power-factor 440-volt welding current, a distance of 800 feet with only one-per-cent voltage drop, and the branches can transmit the same current 300 feet with one-per-cent drop. Large welders with current swings of 3,000 and 4,000 amperes can thus be served at considerable distances from the supply transformers and not cause over three- or four-per cent voltage drop in the bus system.

One such bus system, fed by 1,500 kva of transformers, serves four floors of factory area with a maximum transmission distance to the furthest machine of about 300 feet. Another is laid out in the form of a loop with over 1,000 feet of bus for serving a large ground floor manufacturing space. There are a total of about 18,000 kva of various types and sizes of resistance welders operating in regular production from these two systems.

C. L. Pfeiffer: I wish to thank Mr. Clark for his description of a low-reactance bus system for large-capacity resistance-welding machines. The question of power supply has always been not only an important but a vexing one in resistance welding for both the user and power company and improvements along these lines are of vital interest to the industry.

An Electronic Arc-Length Monitor

Discussion and author's closure of a paper by Walther Richter published on pages 115–17 of this volume (March section) and presented for oral discussion at the electronics session of the winter convention, New York, N. Y., January 25, 1938.

J. E. Waugh (nonmember; General Electric Company, Schenectady, N. Y.): This discussion covers merely the scope of an "Electronic Arc-Length Monitor" and not the equipment involved.

A layman is very apt to infer that, if the monitor is set within proper limits for a given electrode and the arc length is maintained within these limits by the welding operator, the result will be a perfect weld. This would mean that at last the boiler code authorities had found an instrument to check the quality of the welds.

However, it is not necessarily true that good welds will be produced if the operator maintains an arc length within the monitor limits. It is possible to obtain correct arc-length indications from the monitor and still obtain a very unsatisfactory weld. Conversely, it is possible to obtain very erratic results with an arc length monitor and still produce a satisfactory weld-metal deposit.

Under the first condition, one of the chief causes of weld defects is the failure on the part of the operator or automatic welding equipment to obtain proper fusion to the side walls of the joint being welded. The arc-length monitor will not indicate such a condition, since the operator can hold the correct arc length and still not direct the arc sufficiently against the side walls to obtain proper fusion.

In the second case, in making the first pass of a vertical fillet or butt weld, the operator must "whip" his electrode in and out of the molten pool in order to properly locate the weld metal. This condition will show very erratic indications on the monitor, yet the weld is perfectly sound.

Both of these conditions are very clearly shown on a recording voltmeter which we have used with the same purpose in mind as the monitor, namely, to predict the quality of the weld. An examination of many charts and the result of the physical tests forced us to reach the conclusion that we could not predict the quality of the weld from the chart readings.

From our point of view a monitor of the type described in Mr. Richter's paper would prove useful in training new men as welders, indicating to the man the correct voltage he should be maintaining. It should also prove valuable as an occasional check on production welders, but we feel that its general use, while acting as an aid toward improving the quality of the weld, could not represent a "cure-all" for weld defects.

Walther Richter: The instrument was never intended to predict the quality of the weld, since we know of course, that the arc voltage is only one factor of many affecting the quality of the weld. We therefore fully agree with Mr. Waugh that the use of the instrument cannot guarantee a perfect weld. But on the other hand, it is well known that various electrodes require different voltages for best results and it is clear that if the welder fails to use the proper voltage, an inferior weld will certainly be obtained. The instrument will in this case eliminate at least the possibility of a bad weld due to using the improper voltage, and therefore seems to render a worthwhile service. We feel that the instrument has great value in training men, checking up on their performance and in all cases where it is desirable or necessary to hold the arc voltage within close limits.

A Carrier Telephone System for Toll Cables

By C. W. GREEN
MEMBER AIEE

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Synopsis: A new 12-channel carrier telephone system for existing cables is described. This system, which incorporates a number of interesting departures from the previous carrier art, is now being manufactured in considerable quantities to meet increased traffic requirements.

AN IMPORTANT advance in the art of carrier telephony has been made by the development of a new 12-channel system, known as the type *K*, for toll telephone cables of existing type. It is applicable both to cables installed underground, and also to aerial cables, for which the wide range of temperature variation introduces quite difficult transmission problems. Field trials on cables previously installed between Toledo and South Bend have been successful, and the system is now being manufactured to meet field demands.

This new development is an outgrowth of the experiments at Morristown, N. J., described by Messrs. Clark and Kendall before this Institute in 1933,[1] and the essential principles of the new system were included in those experiments. The earlier work dealt, however, with cable specially designed for carrier operation, and only underground cable was experimented with. As that work drew to a close, it became clearer that because of general economic conditions several years would elapse before the Bell System would require any substantial increase in toll facilities. Hence this early system was not put into commercial form, but work was continued to determine the extent to which carrier could be applied to existing cables, of which more than 15,000 miles were available for such use. Serious problems of crosstalk at high frequencies had to be reckoned with. A more serious problem, however, was that of maintaining stability of transmission, since with aerial cable, which comprises about two-thirds of the existing cable mileage, the total variation in attenuation, due to temperature variation, is about three times that for underground cable, and the rate of variation not infrequently is several hundred times as great.

In spite of these and other difficulties, the capabilities of the present system go far beyond those of previous systems. As a development objective the maximum length was taken as 4,000 miles, with as many as five separate systems linked together. On the basis of results thus far obtained it is expected that for these exacting conditions the performance with respect to crosstalk, noise, transmission stability, width of voice band, and other characteristics will equal or exceed that of previous facilities for much shorter distances.

Superior performance has been achieved without material effect on the cost of the system. For distances of a few hundred miles, on moderately heavy traffic routes, it will provide telephone circuits at a much lower cost than previous facilities. The minimum distance for which the system will be useful may be less than 100 miles.

Interesting features of the new system are:

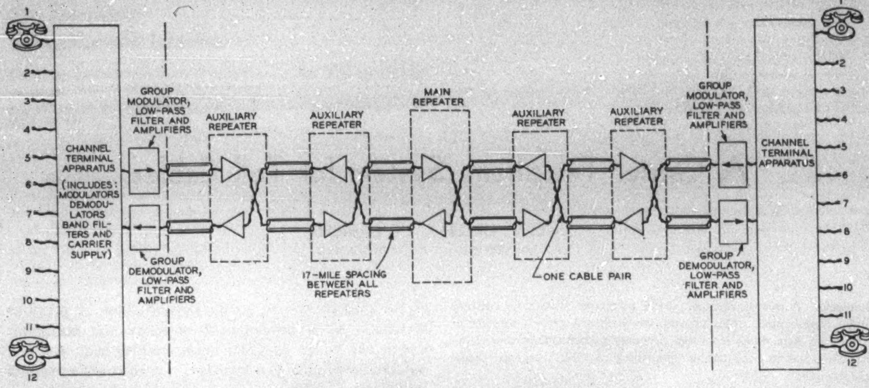
1. A line of very high attenuation, requiring high-gain repeaters spaced at approximately 17-mile intervals. This would mean, for the maximum distance for which the system is designed, more than 200 repeaters in tandem.
2. The use in the repeaters of the negative feedback principle of amplification to obtain the requisite stability and freedom from modulation.
3. Small auxiliary repeater stations, established between existing voice-frequency repeater stations, housing equipment which can be left for considerable periods of time without attention.
4. A system of transmission regulation whereby huge variations of attenuation, differing at each frequency, are automatically equalized to a high degree of accuracy.
5. New methods of crosstalk and noise reduction. Small adjustable mutual inductance coils are connected between carrier pairs to balance out the crosstalk. The noise is kept at an extremely low level to permit the high gains.
6. Channel terminal equipment designed so that it may be used in other types of carrier systems, thus simplifying development and manufacture, and facilitating the interconnection of different types of systems.
7. Speech bands considerably wider than those of existing facilities. The increase is obtained by spacing the channels at uniform 4,000-cycle intervals, and employing channel band filters containing quartz-crystal elements.
8. High speed transmission, which is of considerable value from the standpoint of minimizing delays and echoes.

A general description of the system is presented herein, and the different parts are taken up in greater detail in other papers.

Paper number 37-136, recommended by the AIEE committee on communication, and presented at the AIEE winter convention, New York, N. Y., January 24–28, 1938. Manuscript submitted October 29, 1937; made available for preprinting December 18, 1937.

C. W. GREEN is carrier telephone engineer with the Bell Telephone Laboratories, Inc., New York, N. Y., and E. I. GREEN is in the transmission development department of the same company.

1. For all numbered references, see list at end of paper.



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CHANNEL TERMINAL APPARATUS (INCLUDES: MODULATORS DEMODULATORS BAND FILTERS AND CARRIER SUPPLY)
GROUP MODULATOR, LOW-PASS FILTER AND AMPLIFIERS
GROUP DEMODULATOR, LOW-PASS FILTER AND AMPLIFIERS
AUXILIARY REPEATER
AUXILIARY REPEATER
MAIN REPEATER
AUXILIARY REPEATER
AUXILIARY REPEATER
17-MILE SPACING BETWEEN ALL REPEATERS
ONE CABLE PAIR
GROUP MODULATOR, LOW-PASS FILTER AND AMPLIFIERS
GROUP DEMODULATOR, LOW-PASS FILTER AND AMPLIFIERS
CHANNEL TERMINAL APPARATUS

Figure 1. Schematic of type K system

General Considerations

The type *K* system, whose elements are illustrated schematically in figure 1, operates on a "four-wire" basis, using the same frequency range, but different electrical paths, for opposite directions of transmission. Thus it differs from open-wire carrier systems, for which the line is not suitable for four-wire operation, and which therefore require complicated and expensive filters to separate the different frequency bands used for transmission in opposite directions. A high degree of shielding between the two cable paths is necessary to avoid the effects of near-end crosstalk, which would be serious because of the large level differences existing at the repeaters and the terminals. On routes where two or more cables exist, such shielding is obtained by employing two separate cables, with transmission in one direction only in each section of cable. On single cable routes, a similar arrangement is obtained by adding a small cable. Where there is no cable, two small cables may be provided. Also satisfactory shielding between the carrier pairs used for opposite directions of transmission has been obtained in short experimental lengths of cable by the use of a layer shield.

FREQUENCY ALLOCATION

In contrast to the original Morristown system which gave nine one-way channels per pair in the range from 4 to 40 kilocycles, the type *K* system has 12 channels in the range from 12 to 60 kilocycles. As shown in figure 2, the frequency range of the type *K* system is roughly

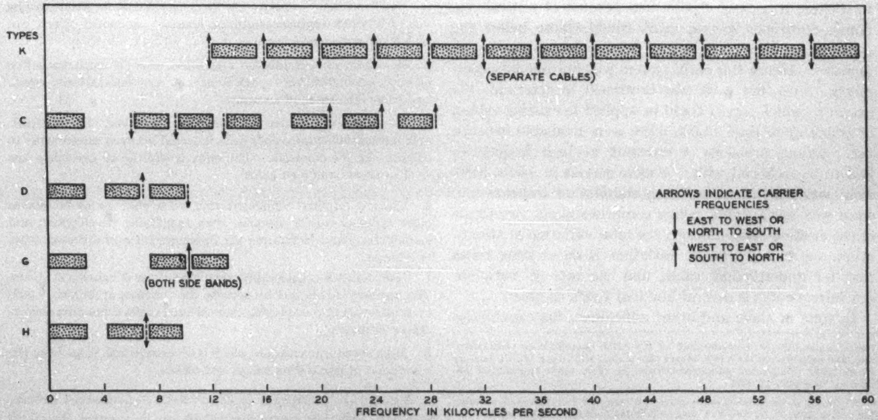

Figure 2. Frequency allocation of carrier telephone systems

double that of preceding open-wire carrier systems.[2] The choice of 12 and 60 kilocycles, respectively, as the lower and upper frequency limits was governed by economic considerations, and there is nothing technically insurmountable either in going to considerably higher frequencies or in utilizing the lower frequency range, which is now idle except for the use of the d-c path for purposes of transmission regulation and fault location. Important factors influencing the selection of the upper frequency are the crosstalk, which depends on the number of pairs utilized for carrier in one cable and the extent to which special crosstalk balancing means are used, and the attenuation, which largely controls the spacing between repeaters. Factors affecting the lower limit include the difficulty of maintaining accurate transmission regulation over the whole frequency range, and the design of the repeater, which becomes harder as the ratio of maximum to minimum transmitted frequency is increased.

The frequency range between 12 and 60 kilocycles accommodates 12 speech channels, each occupying a gross band of four kilocycles. The single-sideband method of transmission is employed, with carrier frequencies suppressed. The choice of a group comprising 12 channels was influenced not alone by the requirements of the type *K* system itself but also by those of other broad-band systems. From the earliest stages of the broad-band development it was recognized that there would be considerable advantage from the standpoints of flexibility of interconnection, of minimum development effort, and of large scale production of equipment units, if the designs of different broad-band systems could be so co-ordinated as to enable the same design of channel terminal equipment to be employed for each. A common 12-channel terminal unit developed for this purpose is used in the type *K* system.

The spacing of the channels in broad-band systems is important from the standpoint of the channel selecting circuits and the width of the derived voice circuit. As discussed in a recent article, a uniform 4,000-cycle interval has been adopted for the different channels of all broad-band systems.[3] The speech band width obtained with this spacing is in keeping with recent improvements in telephone instruments and other parts of the telephone plant. Over-all transmission-frequency characteristics for a single link and a five-link connection are shown in figure 3.

Cable Attenuation

The type *K* system is designed to be applied to the number 19 American Wire Gauge (0.9 millimeter) pairs commonly found in existing cables. (The Morristown system used 16-gauge pairs.) Because the conductors are small and closely spaced, with paper and air dielectric, the attenuation of a nonloaded 19-gauge pair at the frequencies involved is inherently high, as will be seen from figure 4. Because of the high attenuation, the repeaters must be placed much closer together than is necessary for voice-frequency cable circuits. Fortunately this effect is partly offset by the fact that it is possible, as discussed later, to use higher gains in the carrier repeaters.

The cable pairs exhibit the rise in attenuation with frequency which is familiar in most transmission circuits. This effect is brought about largely by the increase in conductor resistance, due to skin effect, and the increasing dielectric losses. More important than this, however, is the fact that the resistance of the wires and the other "constants" of the cable pair undergo variations with temperature, which in turn affect the attenuation. The magnitude of the result for a representative nonloaded 19-gauge cable pair is illustrated by the curves of figure 4, which show, respectively, the attenuation for an average temperature, assumed to be 55 degrees Fahrenheit, and for 0 and 110 degrees Fahrenheit. The latter values, often taken as the extremes of annual variation for an aerial cable, are in fact frequently exceeded. One reason for this is that when the sun is shining directly on an aerial cable, it may assume a temperature from 15 degrees to 25 degrees above that of the ambient air. The range of temperature variation (and attenuation variation) for an aerial cable may be half as much in one day as in an entire year. For an underground cable, changes of temperature occur quite gradually and the total annual variation is about one-third of that for an aerial cable.

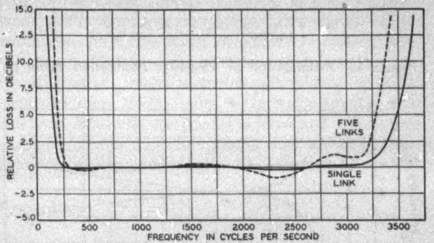

Figure 3. Transmission-frequency characteristics of over-all circuit

These relations between the attenuation of a cable pair and the frequency and temperature are of fundamental importance in the design of the type *K* system. First of all, since the attenuation at 60 kilocycles is about four decibels per mile, the total attenuation for a cable circuit of the length used in designing the type *K* system, i. e., 4,000 miles, would be approximately 16,000 decibels. This must be offset by a corresponding gain.

In the next place, differences in the attenuation at the different frequencies would, if uncorrected, become so great that signals of the less attenuated channels would overload the repeaters, while those of the more attenuated channels would drop down into the noise region. Hence, each repeater must be given a gain-frequency slope which is complementary to the attenuation slope of the line.

Finally, the changes of transmission due to temperature variations and other causes must be compensated so precisely that the net variation in each channel is held within very narrow limits. The method of doing this is explained later. Here it is interesting merely to consider the magni-

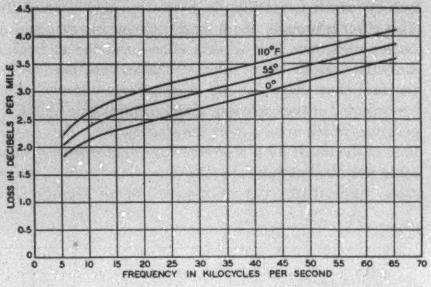

Figure 4. Attenuation of 19-gauge nonloaded cable pair

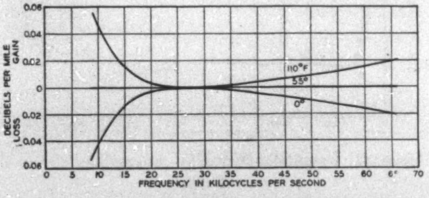

Figure 5. Twist characteristics of 19-gauge nonloaded cable pair

tude of the problem. For the top channel, assuming a 4,000-mile circuit, the annual variation in attenuation of an aerial cable pair might be approximately 2,000 decibels. The systems thus far installed have, of course, been limited to much shorter distances than this.

Even if the change of attenuation with temperature were related to frequency by a simple law, correct compensation over the frequency range would be far from easy. To a casual inspection the differential between any two curves of figure 4, for example those for 55 and 110 degrees Fahrenheit, will not appear serious. This differential, which becomes very large for a long circuit, is, however, a complicated function of the frequency.

The attenuation differential with temperature can be considered as made up of two components, one which is independent of frequency and another which varies with frequency. The former component, which is much the larger, requires a gain adjustment which is uniform or flat over the frequency range of the system. The latter component is frequently referred to as the "twist." For the range from 12 to 60 kilocycles, the maximum change of attenuation with temperature occurs near 28 kilocycles. Hence this frequency has been used as a datum point in determining the twist. The shape of the twist component is apparent from figure 5, which shows the net loss per mile at temperatures of 0 and 110 degrees Fahrenheit, assuming that the attenuation has been equalized so as to obtain a flat characteristic at 55 degrees Fahrenheit, and that the gain is then adjusted so as to hold the transmission constant at 28 kilocycles as the temperature varies. Although the twist is small enough so that it need not be corrected at each repeater, it is too large to be allowed to accumulate over a very long distance.

CROSSTALK

As noted above, crosstalk between opposite directions of transmission is avoided by using two separate cables (or shielded compartments in the same cable). To prevent crosstalk in offices, special measures are employed. There remains the problem of "far-end" crosstalk between pairs in the same cable which transmit in the same direction. The pairs are packed closely together, and substantial crosstalk occurs between them because of small departures from symmetry and slight imperfections of twisting. However, by abandoning the use of phantoms and by connecting small adjustable mutual inductance coils between each carrier pair and every other carrier pair, sufficient crosstalk reduction is obtained to permit transmission up to 60 kilocycles on a substantial number of pairs.[4] The scheme is illustrated in figure 6.

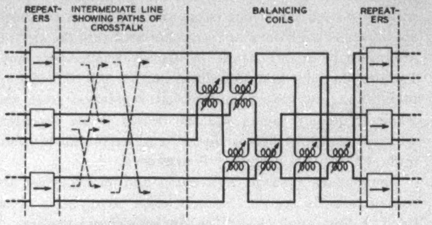

Figure 6. Method of balancing out crosstalk

In the original Morristown cable, the crosstalk was reduced in part by separating the 16-gauge carrier pairs from one another by 19-gauge quads which served as spacers. With existing cables, however, the use of spacers would be impracticable since this would require resplicing the cable at every joint, and therefore reliance must be placed largely on balancing. Since the number of combinations to be balanced increases approximately as the square of the number of pairs employed for carrier, the number of balancing coils required for even a moderate complement of carrier pairs becomes quite large. With 40 carrier pairs, for example, the number of coils required is 780. The balancing coils are mounted on panels as shown in figure 7 and are connected together in a crisscross arrangement. Each repeater section is balanced separately, the balancing panels being located in the repeater station.

Other measures are necessary to supplement this balancing technique. To reduce the crosstalk coupling, and also to average the transmission characteristics of different pairs, the carrier pairs in different quads of an existing cable are respliced about every mile so as to approach random splicing. The crosstalk coupling between the two sides of a quad is reduced by test splicing at the middle of a repeater section and the quads are split at repeater points.

In a new cable, of course, the desired splicing arrangements are introduced at the time of installation. The carrier pairs are also transposed from one cable to the other as indicated in figure 1. This avoids interaction crosstalk that would take place, through the medium of the voice-frequency pairs, between the high level carrier outputs on one side of a repeater station and the low level inputs on the other side. There is, of course, a similar effect between carrier pairs at any point in a repeater section. This is much less serious since no level difference is involved, but it does tend to limit the effectiveness of the balancing over a range of frequencies.

Reflections resulting from impedance irregularities reverse the direction of propagation and therefore produce far-end crosstalk from near-end crosstalk. This crosstalk cannot readily be balanced out over a range of frequencies. To avoid it, it is necessary that the impedances of successive lengths of cable pair be substantially uniform and also that the impedance of the equipment be closely matched to the characteristic impedance of the cable pair.

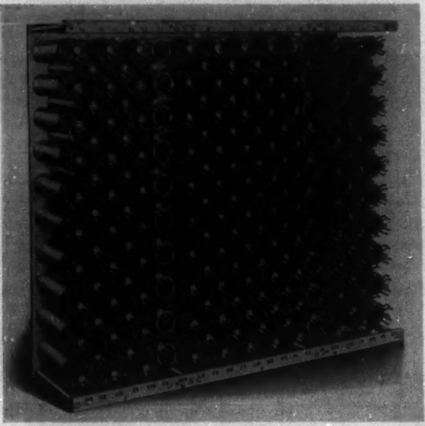
Figure 7. Crosstalk balancing panel

NOISE

The cable pairs are fairly well protected by the lead sheath from external electrical disturbances. However, high-frequency noise originating in the voice-frequency repeater stations due to relay operation, etc., would, unless prevented, enter the cable over the voice-frequency pairs and thence would be induced in the carrier pairs. Such noise would be excessive at the low-level carrier inputs. It is avoided by connecting, in each voice-frequency pair in the "low-level" cable on each side of a voice-frequency repeater station, a coil which supresses longitudinal noise currents. Similarly, it is necessary to keep high-frequency noise from entering the cable where open-wire pairs tap into it and frequently also where branch cables are connected. For this purpose simple noise-suppression filters are employed. With these and other measures the noise on the carrier pairs can, at the highest frequencies where the amplification is greatest, be brought within a few decibels of the basic noise due to thermal agitation of electricity in the conductors themselves.

VELOCITY OF TRANSMISSION

Voice waves travel through loaded cable circuits at from 10,000 to 20,000 miles per second, the higher speed being used for the longer circuits. On very long connections, even if echo suppressors are employed this velocity results in transmission delays which introduce difficulties in conversation.[5] The use of nonloaded conductors for the type K systems results in an over-all velocity of transmission of about 100,000 miles per second, a speed so high that such difficulties are greatly reduced and satisfactory telephone conversations are possible over the longest distances for which connections may be required.

Repeaters

Since the noise level in the cable circuits can be made quite low, the carrier currents may be permitted to drop to levels below those used on voice-frequency circuits or on open-wire carrier circuits, and the repeaters may have higher gains. In the cable carrier system the noise has been so reduced that the level of the top channel at the repeater input may on the average be dropped about 60 decibels below the voice level at the transmitting switchboard. The amplifier gains at the top frequency range from about 50 to 75 decibels and the output level of each of

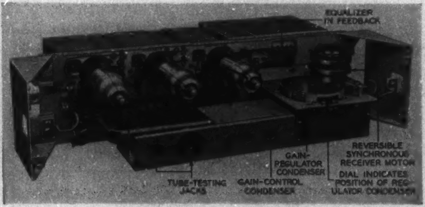

Figure 8. Line amplifier

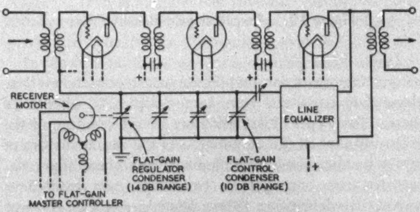
RECEIVER MOTOR
LINE EQUALIZER
FLAT-GAIN REGULATOR CONDENSER (14 DB RANGE)
FLAT-GAIN CONTROL CONDENSER (10 DB RANGE)
TO FLAT-GAIN MASTER CONTROLLER
+
+
+

Figure 9. Schematic of line amplifier

the 12 channels is about ten decibels above that at the switchboard. The average repeater spacing is about 17 miles.

The tube which was developed for the gain stage of the amplifier is a pentode with indirect heater. The heater requires a potential of ten volts and a current of 0.32 ampere and the plate, 150 volts. The tube in the power stage is similar in type but requires a heater current of 0.64 ampere at ten volts. With this power tube a feedback of about 40 decibels has been found to provide a satisfactory reduction of inter-channel modulation.[6] Both tubes were designed to have long life with very reliable performance.

Description of Amplifier

Each repeater comprises two amplifiers of the type illustrated in figure 8. A schematic diagram of the amplifier circuit is shown in figure 9. Three stages with impedance coupling are used and the feedback circuit is connected between the plate circuit of the last tube and the grid circuit of the first. The amount of gain and the slope of the gain-frequency characteristic are controlled by the condensers and line equalizer in the circuit.

The line equalizer has the same attenuation slope as the line. In the introduction of this equalizer in the feedback circuit careful attention to phase-shift requirements was required. Four types of equalizers are available, for different repeater spacings, to compensate for the cable distortion which occurs at a temperature of 55 degrees Fahrenheit, additional means being provided to compensate for variations which occur as the cable temperature swings away from this value. The solid curve of figure 10 shows a repeater gain characteristic with one of these equalizers in the feedback circuit.

The correction introduced by the line equalizers is subject to errors which, although small at each repeater point, become important for a moderate length of system. Supplementary equalizers have been designed to correct for these. Two types of deviation are considered, that of the cable and that of the amplifier. As the characteristics of cables manufactured at different times show slight departures from one another, two shapes are required to correct their deviations. There is one corrector for concave deviations and another for convex. The amplifier requires but one type of correction. The characteristics of these equalizers are shown in figures 11 and 12. The equalizers for amplifier deviations are used about every tenth repeater and those for the cable deviations at distances of 300 to 400 miles. At normal temperature (55 degrees Fahrenheit), the correction applied by these networks will, for a 500-mile system, result in a frequency characteristic which is flat to within less than two decibels over the range from 12 to 60 kilocycles. For a longer circuit further corrective measures will be provided.

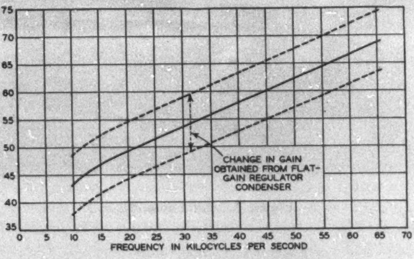

Figure 10. Amplifier gain characteristics

Regulation for Temperature Effects

The method adopted for controlling the repeater gain to compensate for temperature changes is similar to that which has been found satisfactory for voice-frequency cable circuits. This is the pilot-wire method in which a pair of cable conductors extending over the section to be regulated forms one arm of a Wheatstone bridge. This bridge is designed for automatic self-balancing and the mechanical motion required for establishing the balance has been made to adjust the gain of the amplifiers. The d-c resistance of the pilot wire gives an accurate indication of the temperature of the carrier pairs, which determines their attenuation to a close approximation. The motion of the bridge mechanism is communicated to the repeater amplifiers by means of self-synchronizing motors, a master motor being associated with the bridge and an individual motor with each amplifier.

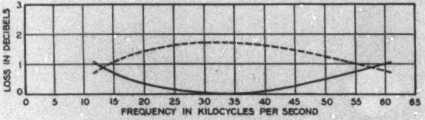

Figure 11. Characteristics of cable deviation equalizers

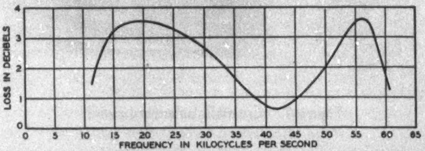

Figure 12. Characteristics of amplifier deviation equalizer

With aerial cable a flat gain correction must be made at every repeater. With underground cable the flat gain correction may be omitted at some repeater points. Figure 13 shows a master controller with its galvanometer, driving motor, and self-synchronizing motor. The air condenser in the feedback circuit of figure 8 makes this correction. The small self-synchronizing motor which may be seen in this figure is geared to the condenser and it moves the air condenser in accordance with the motion of the master motor. The resulting change in repeater gain is virtually the same for all frequencies in the transmitted band. In figure 10 the repeater gain is plotted against frequency for three angular positions of the condenser.

As was mentioned earlier, an additional correction for the residual effect or "twist" is required about every six repeaters to supplement the flat gain adjustment. This distortion is a function of frequency and has been found to vary from cable to cable. A network whose characteristics are shown in figure 14 has been developed to meet this condition. Certain fixed resistances in the network are selected to correspond to the length and twist characteristic of the cable section considered. A variable resistance in the network is adjusted automatically using a control similar to the flat gain regulator. Figure 15 gives the transmission characteristics of a 150-mile regulator-controlled circuit under two temperature conditions.

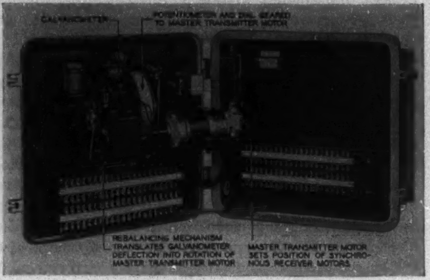
GALVANOMETER
POTENTIOMETER AND DIAL GEARED TO MASTER TRANSMITTER MOTOR
REBALANCING MECHANISM TRANSLATES GALVANOMETER DEFLECTION INTO ROTATION OF MASTER TRANSMITTER MOTOR
MASTER TRANSMITTER MOTOR SETS POSITION OF SYNCHRONOUS RECEIVER MOTORS

Figure 13. Flat gain master controller

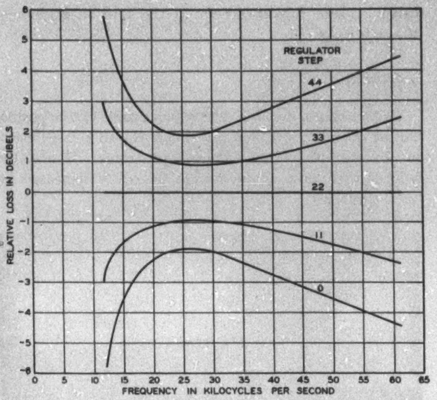

Figure 14. Characteristics of twist regulator networks

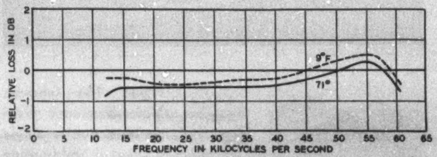

Figure 15. Over-all transmission-frequency characteristic of 150-mile line

AUXILIARY REPEATER STATIONS

Cable carrier systems are expected to be used largely on existing toll-cable routes which now carry voice-frequency circuits. The average spacing of the stations housing the voice-frequency repeaters on these routes is about 50 miles. The same buildings with their power plants will also care for the cable carrier repeaters. Since the maximum spacing for the carrier repeaters is about 19 miles, additional carrier repeaters must be provided at intermediate stations (two is the usual number). The various design features of the equipment to be located in these stations have been made the subject of extensive development work and field tests. These stations are designed to function with a minimum of attention and are visited at intervals for routine testing work or as required by some emergency, but resident maintenance forces are not planned for them. The present equipment is expected to be suitable not only for auxiliary stations on existing cable routes but also for cases where a greater spacing than 50 miles between the attended stations may be desired on new routes.

A voice-frequency repeater station for a single cable and a cable carrier auxiliary station are shown to approximately the same scale in figures 16*a* and 16*b*, respectively. Many of the existing voice-frequency stations are even larger than that shown in figure 16*a*. The auxiliary building shown in figure 16*b* has about 600 feet of floor space with a ceiling height sufficient to take care of 11-foot six-inch relay racks. This building will house 100 repeaters with necessary auxiliary equipment, thus providing ultimately for a total of 1,200 carrier circuits. The interior of a typical auxiliary station is shown in figure 17.

The main power plant for the repeaters consists of a 152-volt storage battery, which is continuously floated across a grid-controlled rectifier fed from the 60-cycle power mains. The voltage of the entire battery applies the plate voltage for the tubes. Each amplifier requires about 22 volts for the tube heaters and this is obtained by dividing the battery into seven sections, each section supplying several amplifiers in parallel. Additional power supplies of 55 volts alternating current and 140 volts direct current are required for the regulator system.

In the station, there are alarm circuits, which signal the nearest attended office if trouble develops. There are alarms for blown fuses, high or low battery voltages, power failures, etc. A telephone order wire to the nearest attended station is provided for the maintenance force.

In addition to the line amplifiers with their regulating equipment, there are racks mounting the crosstalk-balancing coils. There are also sealed terminal units between the outside cable or the balancing units and the office cable. These furnish access to the line or equipment through jacks.

Terminals

The minimum distance over which a cable carrier system can be operated economically is determined in large measure by the cost of the terminal apparatus. Hence, the field of usefulness of the system is greatly increased by keeping the terminal cost as low as is consistent with satisfactory performance. Numerous developments during the past few years in connection with modulation, filtering, and methods of carrier supply have all contributed materially toward this end. At the same time, the standards of performance have not only been maintained, but in many respects substantially improved.

Figure 16a (above). Voice-frequency repeater station on single cable route

Figure 16b. Auxiliary cable carrier repeater station

Channel and Group Modulation

In the design of the terminals for the type *K* system, a number of circuit arrangements were considered, the final choice being influenced to a considerable extent by the conditions imposed upon the filters. As noted above, the desirability of using the channel terminal equipment in other broad-band systems such as those for open wire or coaxial cable was also an important factor. The circuit arrangements selected have a first stage of modulation which raises the voice frequencies of the 12 channels up to a range of 60 to 108 kilocycles. This range is favorable to the use of crystal type band filters,[7] which have transmission characteristics superior to the coil and condenser type and seem to be no more costly. For the type *K* system, a single stage of group modulation shifts the frequencies to the range required on the line, 12 to 60 kilocycles and a similar stage at the receiving end returns them to the 60 to 108-kilocycle range. Other carrier systems will also use the 60 to 108-kilocycle channels and by group modulation shift them to the desired position in the frequency spectrum.

The band filter occupies a space on the relay rack equal to one-eighth of that required by the coil and condenser type which was used in the earlier model of this system. Its attenuation characteristic in the transmitting region is flat to within one decibel over a range of about 3,100 cycles. Immediately outside of this range the attenuation rises very rapidly, thus permitting very efficient use of the frequency spectrum.

Another new device on the terminal is the copper-oxide unit used in the modulating process. These units are expected to show a stability of the same order as that of coils and condensers, and require practically no maintenance as compared to vacuum tubes.

The translation of the channels from the 60 to 108-kilocycle range to the position required for cable carrier, 12 to 60 kilocycles, is made by a stage of group modulation. A copper-oxide group modulator is used and a carrier frequency of 120 kilocycles. The reverse of this process in a similar group demodulator at the receiving end steps the frequency back to its original range, 60 to 108 kilocycles. These processes of modulation take place at points of low energy level in the circuit with a comparatively high level of carrier, so that the interchannel crosstalk which results from unwanted products of modulation is unobjectionable. Low-pass filters are inserted after the group modulator and demodulator, and amplifiers with

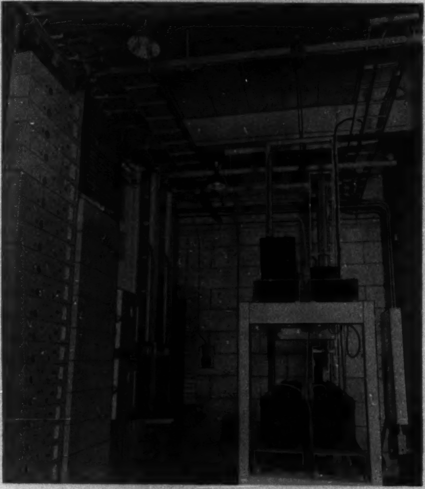

Figure 17. Interior of auxiliary repeater station

flat gain characteristics are supplied to raise the levels of the output currents of the group modulators or demodulators.

CARRIER SUPPLY

The carrier frequencies which are required at a terminal are obtained from the harmonics of a base frequency. The carrier supply system is common to as many as ten systems in one office. This simplification was made possible by the selection of the channel frequencies as multiples of a base frequency, four kilocycles being chosen for this system. This base frequency is produced by an oscillator in which the control element is a tuning fork, the whole unit being designed to have the necessary output and frequency stabilities. The output of the oscillator is amplified and fed to a circuit which produces the desired harmonics. All of the carrier frequencies which are required for the different channels as well as for group modulation and demodulation are obtained from these harmonics. A small coil with a permalloy core is the important agent in this process.[8]

Failure of the four-kilocycle supply, or failure of the 120-kilocycle supply used for group modulation, would cause failure in the channels of all systems operating from this supply. Provision is made for such a contingency by an emergency carrier supply which is automatically switched into service when the regular supply fails. This reserve source duplicates all of the parts of the regular supply, four-kilocycle fork, amplifier, harmonic producer, and amplifier for the 120-kilocycle carrier.

ASSEMBLY

The different panel units which make up the terminal of a type *K* system are assembled on a functional basis with similar panels of other *K* systems, the channel modulator-demodulator panels in one bay, the carrier supply in a second, the group modulator and demodulator in a third, etc. The compactness of the equipment makes it possible to mount the modulators and demodulators for 18 channels on one 11-foot six-inch bay 19 inches wide.

SIGNALING

The same type of ringdown signaling equipment is used with the channels of this system as with the voice-frequency toll circuits. A 1,000-cycle tone, interrupted 20 times per second, is impressed on a channel terminal, modulated, and transmitted over the carrier system in the allotted channel band. At the far end, it is demodulated to operate the receiving end of the standard voice-frequency receiving circuit, or to be transmitted along an extended voice-frequency circuit to its terminal.

TELEGRAPH AND PROGRAM APPLICATIONS

Voice-frequency telegraph can be superposed on any of the carrier channels as is now done on the three-channel open-wire systems. Equipment is being developed to superpose a program channel on the cable carrier system. This will be done by devoting to the program circuit the frequency space occupied by two of the four-kilocycle speech bands.

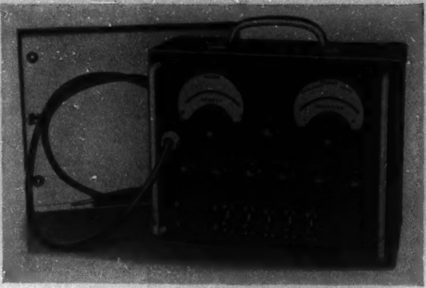

Figure 18. Vacuum tube test set

System Maintenance

Arrangements are provided whereby the tubes may be tested on a routine basis as has been done in voice-frequency practice. The amplifier panels, however, are provided with test jacks which are connected to resistances in the plate circuits. A reading of the voltage across the resistance gives a measure of the plate current for the associated tube without disturbing the amplifier performance while the amplifier is in service. A portable tube testing set, figure 18, has been designed for this measurement.

Provision is being made for the removal of an amplifier from an active circuit without interruption of service. A spare amplifier at each repeater station can be substituted for the active one by connecting it to jacks at the sealed terminal and operating associated relays to make a quick transfer.

Apparatus is also furnished which permits the substitution of a new link between attended points for one which develops trouble. A complete high-frequency circuit for each direction of transmission will generally be reserved as a spare. It can be substituted for any working high-frequency circuit without interfering with service by paralleling the transmitting ends of the spare and working circuits and patching the receiving ends through relays. The operation of a key controlling these relays substitutes the spare circuit for the working one with a transient disturbance of but one or two milliseconds.

Three pilot frequencies, 15.9, 27.9, and 55.9 kilocycles, which are produced at the transmitting terminals, may be used to check the levels at the main repeater points and the receiving terminals. This is done by means of a special testing circuit which can be bridged across a pair to detect the level of the pilots without interference to service.

A heterodyne oscillator having a frequency range from 60 cycles to 150 kilocycles has been developed for use in testing this and other carrier systems. Its frequency is calibrated at 60 cycles against the power mains and at 100 kilocycles against a quartz crystal. This oscillator is shown in figure 19. A portable test set, developed for measuring transmission gains and losses with high precision, is shown with the cover removed in figure 20.

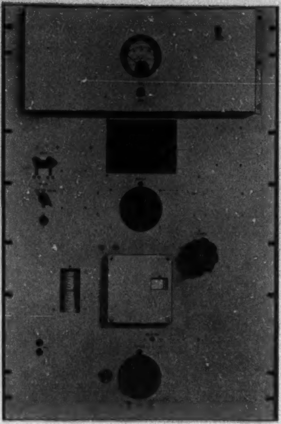

Figure 19. Testing oscillator

Conclusion

The type *K* system makes possible the application of carrier to toll cables of existing type, whether installed underground or aerially. The blocks of 12 circuits each, which it furnishes, seem to be a convenient size for routes where large numbers of circuits are concentrated. It is to be expected, of course, that substantial modifications and improvements will be made in this system through further development effort. In its present form, however, it constitutes an important stage in the history of carrier development. Plans already under way call for the application of large numbers of such systems to meet rapid growth in long-distance traffic.

This new system forms merely one phase of a concerted development effort on broad-band carrier transmission systems.[9,10] There is every indication that, taken collectively, these broad-band systems will have far reaching effects upon the toll telephone plant of the Bell System. A transition is already under way from the time when carrier was used only on open wire, and comprised only a small part of the toll plant, to a time when carrier systems will furnish a major part of the toll circuit mileage of the Bell System. The type *K* system is clearly destined to play an outstanding part in this evolution of the toll plant along carrier lines.

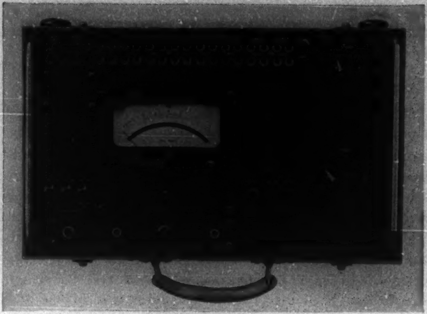

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Figure 20. Transmission measuring set

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Discussion

For discussion of this paper see page 259.

Cable Carrier-Telephone Terminals

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Synopsis: This paper describes the circuits, performance, and equipment features of the terminals of a new 12-channel carrier system for application to existing toll cables. The 12-channel group of terminal apparatus has been designed also to form a basic part of the terminals of other carrier systems now under development, such as the type-*J* system for open wire and the coaxial system.

Introduction

ABOUT 20 years ago the first commercial carrier telephone system was installed between Baltimore and Pittsburgh. Until recently, telephone circuits were obtained by carrier methods largely on open-wire lines. The notable exceptions were on short deep sea submarine cables.[1,2] Ten years ago, experiments were initiated which now have resulted in the design of a carrier system which can be applied with substantial economy to existing long distance toll cables on land. Its general features are described in another paper.[3] The present paper describes in detail the circuits and performance of the carrier terminals of this system.

General Features

The carrier system for existing cables, designated type *K*, is designed to provide 12 telephone channels in the frequency range between 12 and 60 kilocycles, using one nonloaded 19-gauge paper insulated cable pair in each direction. Previous carrier systems employed for open-wire lines used vacuum tubes for the modulating or translating circuits and electrical filters composed of coil and condenser networks for separating the frequency bands associated with the respective channels. The terminals of the new type-*K* system are simpler and yet provide improved performance by using copper-oxide bridges for the modulation function and quartz-crystal filters[4] for the separation of the individual channel bands.

The quartz-crystal filter is economical only in a comparatively high frequency range, necessitating the use of high intermediate frequencies. The high intermediate frequencies are reduced by a second stage of modulation to the desired range of frequencies for transmission over the line. Copper-oxide bridge circuits again are used for this group-modulation stage. In all cases they are connected to suppress the carrier. To provide the various carriers required for modulation and demodulation, a carrier supply system has been designed somewhat along the lines of an office power-distribution system using bus bars and protective arrangements for the various carriers. Each carrier supply system is capable of supplying as many as ten carrier terminals, or a total of 120 two-way channels.

Because of the large number of circuits involved, every effort has been made to provide reliable operation of the carrier supply and common terminal equipment. The terminal and carrier supply equipment is designed to permit maintenance tests for checking the performance of amplifier tubes and to permit switching between regular and spare equipment without interruption of the large number of circuits involved.

The emphasis placed upon ease of maintenance and the necessity for more careful handling of higher frequency circuits have resulted in new equipment design features. These include new cable terminals, new shielded office cabling, and panels arranged for front wiring and maintenance which are mounted on racks having wiring ducts at both edges of the bays. In the following sections a more detailed description is given of the circuits, their performance, equipment, and maintenance features.

Circuits

The frequency allocation for one direction of transmission and a block schematic of one terminal are shown in figures 1 and 2 which supplement each other and need little explanation. The 12 voice bands shown at the left in figure 1 are modulated individually in the channel modems. (The term "modem" has been coined to mean a panel or equipment unit in which there is both a modulator and a demodulator to take care of both the outgoing and incoming signal.) This forms a 12-channel block lying between 60 and 108 kilocycles which is then modulated in the group modulator by a 120-kilocycle carrier to move the block down in the range from 12 to 60 kilocycles for transmission to the distant terminal. On the receiving side the processes are reversed. One of the channels as well as the group modem of figure 2 is presented in more circuit detail in figure 3. This shows the circuit from the point where the voice comes into the carrier system to the point where the 12 carrier sidebands go out onto the cable and vice versa.

At the left the four-wire terminating circuit serves, not only as a device to transform from a two-wire to a four-wire circuit, but also as a high-pass filter to eliminate, from the input to the carrier system, noises below about 200 cycles, such as telegraph harmonics, 20-cycle ringing, 60-cycle power, etc., which may be present on connected voice-frequency circuits. Otherwise these noise frequencies which are below the voice range would modulate and pass through the terminal to load unnecessarily the carrier repeaters along the line, as well as to interfere with the level indications of the pilot channels.

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1. For all numbered references, see list at end of paper.

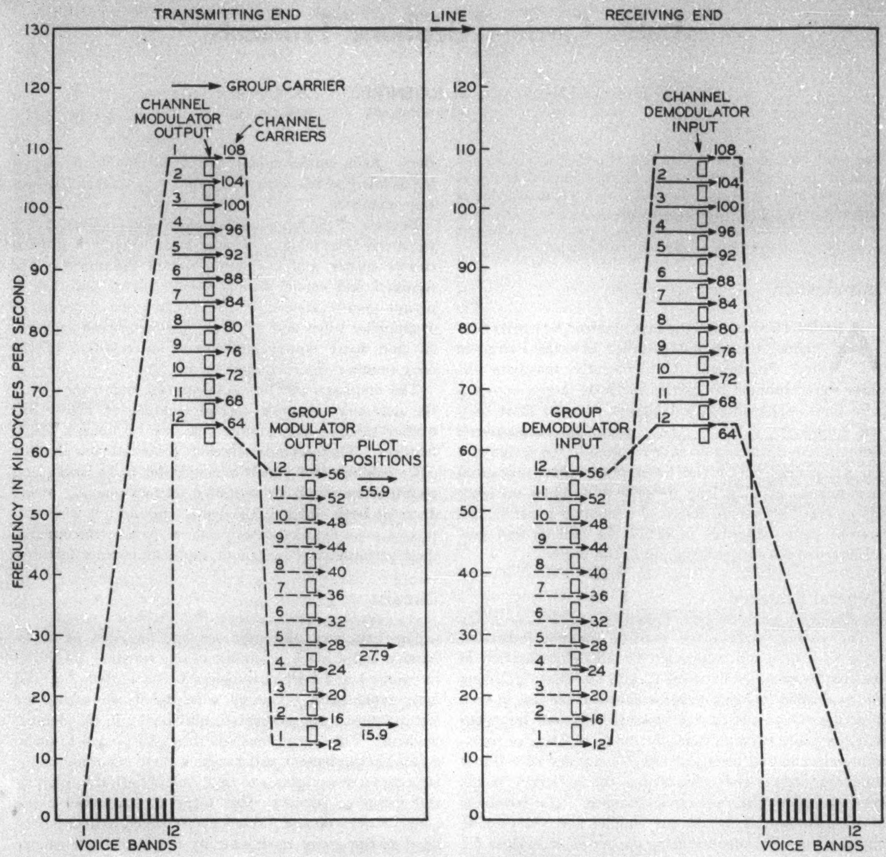
TRANSMITTING END
LINE
RECEIVING END
FREQUENCY IN KILOCYCLES PER SECOND
130
120
110
100
90
80
70
60
50
40
30
20
10
0
GROUP CARRIER
CHANNEL MODULATOR OUTPUT
CHANNEL CARRIERS
108
104
100
96
92
88
84
80
76
72
68
64
GROUP MODULATOR OUTPUT
PILOT POSITIONS
55.9
27.9
15.9
56
52
48
44
40
36
32
28
24
20
16
12
VOICE BANDS
CHANNEL DEMODULATOR INPUT
GROUP DEMODULATOR INPUT
1
2
3
4
5
6
7
8
9
10
11
12

Figure 1. Frequency allocation

From the terminating equipment the circuit loops through jacks which have paralleled contacts for reliability. The level at this point is −13 decibels compared with the transmitting toll switchboard, which level is expected to be generally used in the Bell System for all multichannel carrier-telephone systems. Then comes the channel modulator which consists of four copper-oxide disks, each three-sixteenths of an inch in diameter, potted in a small can. This makes a very simple and inexpensive modulator which is much more satisfactory than tubes. It seems to have an indefinite life (some have been on life tests as modulators for about five years). The carrier power required is about one-half milliwatt to modulate satisfactorily a single telephone circuit level of −13 decibels.

The modulator produces the usual two sidebands and the lower one is selected by the quartz crystal channel filter described in another paper.[4] This sideband, joined by eleven others, is stepped down to about the iterative impedance of the shielded office cabling. In the office cabling the 12 channels pass through the high frequency patching bay to the double balanced group modulator of copper oxide where they are joined by three pilot-channel frequencies.

The group modulator uses the same copper oxide as that in the channel modulator described above, but the carrier power is about 50 times greater (about 25 milliwatts) in order to keep down unwanted modulation produced between the 12 sidebands. To that same end the level of each sideband is made low (−46 decibels), and the double balanced type of circuit is used to balance out some of the undesired products. It also balances out the

12 incoming bands in the range 60 to 108 kilocycles from the output and so simplifies the following group modulator filter.

From a level of −57 decibels the 12 channels, now in the range from 12 to 60 kilocycles, are amplified to +9 decibels for delivery to the 19-gauge pair in the lead covered toll cable. The amplifier is a three-tube negative-feedback type, using pentodes and operating with 154 volts plate battery which is composed of the usual 24-volt filament battery and 130-volt plate battery in series. The last tube is a power tube and does not overload until a single-frequency output of about one watt is reached.

On the receiving side in figure 3, the 12 incoming channels, in the range from 12 to 60 kilocycles, pass from the amplifying and regulating equipment,[3] to the group demodulator. This is identical with the group modulator described above and transfers the 12 channels to the range 60 to 108 kilocycles. The channels are then amplified to a −5 decibel level by an amplifier of the negative-feedback type using two low-power pentodes with 154-volt plate battery as described above for the transmitting amplifier.

From there the 12 channels are separated by the filters which are identical with those on the transmitting side, and are then demodulated and amplified to a +4 decibel level as shown for one channel in figure 3. The demodulator is identical with the modulator but it is poled oppositely on the carrier supply so that the d-c components of modulation in the modulator and demodulator neutralize each other and thereby avoid developing an undesirable voltage bias. The poling also reduces somewhat the amount by which stray frequencies have to be suppressed in the carrier supply. The demodulator amplifier has a slide-wire gain-control rheostat to equalize channel levels which functions by changing both the grid bias on the tube and the amount of negative feedback which is introduced by the rheostat. The sliding contact in the slide wire is made practically free from contact trouble by the space current of the tube flowing through it. As the rheostats are only about 1,000 ohms and small in size, they can easily be mounted at a distance from the amplifier in the voice-frequency jack field.

The carrier supply for the 12 channels from 64 to 108 kilocycles, and for the group modems of 120 kilocycles, is derived in the circuit shown in figure 4. A regular generator is shown at the top in solid lines and an emergency generator at the bottom is shown in dotted lines. Between the two is an automatic transfer circuit (in dotted lines) which transfers to the emergency whenever the regular generator fails to supply the proper amount of 120 kilocycles to the 120-kilocycle bus.

At the upper left-hand corner is shown a four-kilocycle tuning fork, of an alloy having a low temperature coefficient driven by the tube to its right to operate as an oscillator of very stable frequency. The next, or control tube, amplifies the four kilocycles to drive the push-pull power stage where a power of about four watts is developed. This passes through the four-kilocycle filter to the nonlinear coil where odd harmonics of four kilocycles are produced. The underlying principles of operation of this coil have been published.[5] To derive even harmonics of four kilocycles, the copper-oxide bridge is used which rectifies about half the energy of the complex wave of odd harmonics but, by balance, greatly reduces the amount of the odd harmonics present in its output. Odd harmonics are obtained at one point and even at the other. This separation into odd and even harmonics by the balance of the copper-oxide bridge provides effective loss of about 30 to 40 decibels and reduces the requirements on the carrier-supply filters which follow.

The two branches pass through hybrid coils to the banks of channel-carrier supply filters. These separate the frequencies and feed them to 12 carrier-supply bus bars, one for each channel frequency. From these the individual modems are fed through protective resistances so that an accidental short circuit on one of the modems will not cut off the carrier supply to the others.

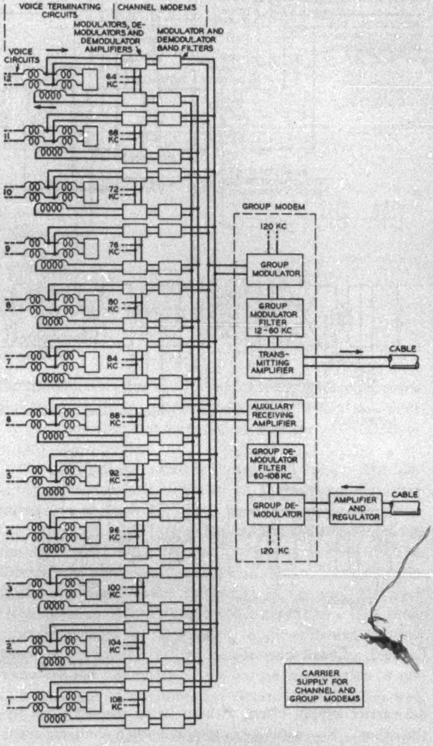
VOICE TERMINATING CIRCUITS
CHANNEL MODEMS
MODULATORS, DEMODULATORS AND DEMODULATOR AMPLIFIERS
MODULATOR AND DEMODULATOR BAND FILTERS
VOICE CIRCUITS
12
64 KC
11
68 KC
10
72 KC
9
76 KC
8
80 KC
7
84 KC
6
88 KC
5
92 KC
4
96 KC
3
100 KC
2
104 KC
1
108 KC
GROUP MODEM
120 KC
GROUP MODULATOR
GROUP MODULATOR FILTER 12-60 KC
TRANSMITTING AMPLIFIER
CABLE
AUXILIARY RECEIVING AMPLIFIER
GROUP DEMODULATOR FILTER 60-108 KC
GROUP DEMODULATOR
AMPLIFIER AND REGULATOR
CABLE
120 KC
CARRIER SUPPLY FOR CHANNEL AND GROUP MODEMS

Figure 2. Block schematic

The hybrid coils permit the two generators to be connected so that either can feed into the same bank of chan-

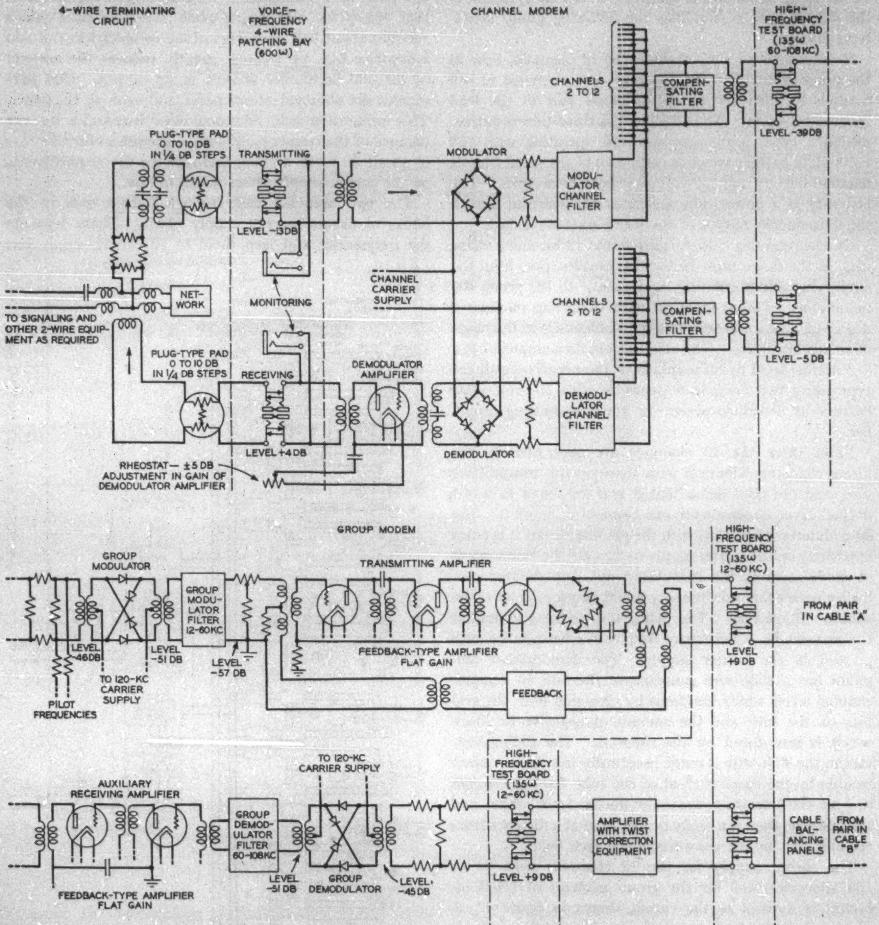
4-WIRE TERMINATING CIRCUIT
VOICE-FREQUENCY 4-WIRE PATCHING BAY (600ω)
CHANNEL MODEM
HIGH-FREQUENCY TEST BOARD (135ω 60-108 KC)
CHANNELS 2 TO 12
COMPEN-SATING FILTER
LEVEL -39 DB
PLUG-TYPE PAD 0 TO 10 DB IN 1/4 DB STEPS
TRANSMITTING
MODULATOR
MODU-LATOR CHANNEL FILTER
LEVEL -13 DB
NET-WORK
TO SIGNALING AND OTHER 2-WIRE EQUIPMENT AS REQUIRED
MONITORING
CHANNEL CARRIER SUPPLY
CHANNELS 2 TO 12
COMPEN-SATING FILTER
LEVEL -5 DB
PLUG-TYPE PAD 0 TO 10 DB IN 1/4 DB STEPS
RECEIVING
DEMODULATOR AMPLIFIER
DEMODU-LATOR CHANNEL FILTER
LEVEL +4 DB
DEMODULATOR
RHEOSTAT— ±5 DB ADJUSTMENT IN GAIN OF DEMODULATOR AMPLIFIER
GROUP MODEM
HIGH-FREQUENCY TEST BOARD (135ω 12-60 KC)
GROUP MODULATOR
GROUP MODU-LATOR FILTER 12-60KC
TRANSMITTING AMPLIFIER
FROM PAIR IN CABLE "A"
LEVEL -46 DB
LEVEL -51 DB
LEVEL -57 DB
FEEDBACK-TYPE AMPLIFIER FLAT GAIN
LEVEL +9 DB
TO 120-KC CARRIER SUPPLY
PILOT FREQUENCIES
FEEDBACK
TO 120-KC CARRIER SUPPLY
HIGH-FREQUENCY TEST BOARD (135ω 12-60 KC)
AUXILIARY RECEIVING AMPLIFIER
GROUP DEMOD-ULATOR FILTER 60-108KC
AMPLIFIER WITH TWIST CORRECTION EQUIPMENT
CABLE BAL-ANCING PANELS
FROM PAIR IN CABLE "B"
LEVEL -51 DB
GROUP DEMODULATOR
LEVEL -45 DB
LEVEL +9 DB
FEEDBACK-TYPE AMPLIFIER FLAT GAIN

Figure 3. Circuit schematic

—NOTE—
LEVELS ARE SHOWN WITH RESPECT
TO THE TRANSMITTING TOLL SWITCHBOARD

nel carrier supply filters without being reacted upon by the other. No switching is required when changing from regular to emergency supply.

The 120-kilocycle carrier which feeds the group modulators of ten systems or a total of 120 talking channels must be very dependable. Therefore separate filters are used for the regular and emergency supply and separate amplifiers for the large power required by group modulators. Regular and emergency distributing busses are provided. Each group modulator and each group demodulator is wired through protective resistances to the regular bus and through another set of protective resistances to the emergency bus. With this arrangement an accidental short circuit even across one of the busses or across one of the output coils of one of the 120-kilocycle amplifiers will not stop the whole supply of 120 kilocycles.

The four-kilocycle oscillator of the emergency generator is in constant operation so that when it is needed no time is required to start it, but the grid bias on the second tube is held above its cutoff value by the automatic transfer circuit. This prevents the four kilocycles from going further until called for in an emergency.

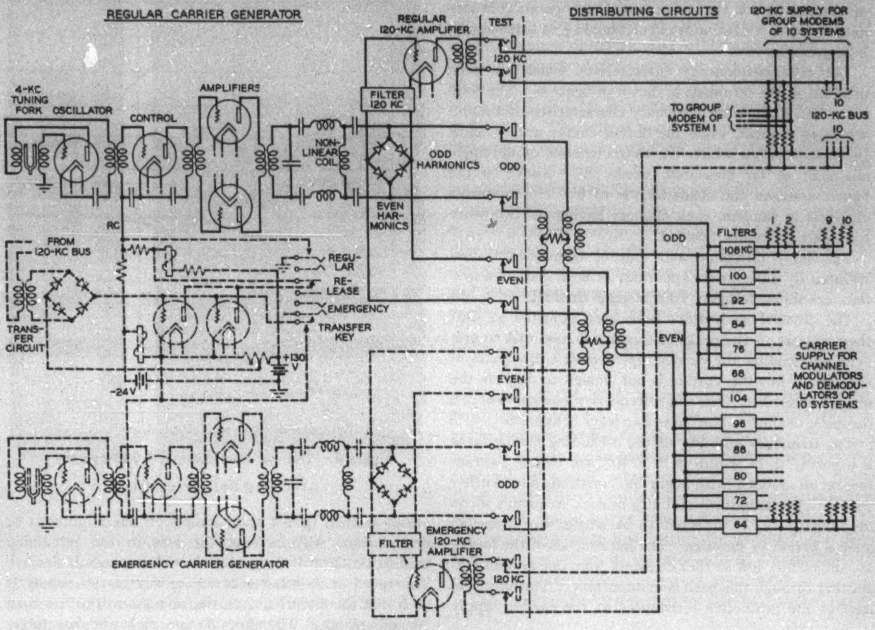
REGULAR CARRIER GENERATOR
REGULAR 120-KC AMPLIFIER
TEST
DISTRIBUTING CIRCUITS
120-KC SUPPLY FOR GROUP MODEMS OF 10 SYSTEMS
120 KC
4-KC TUNING FORK OSCILLATOR
CONTROL
AMPLIFIERS
FILTER 120 KC
NON-LINEAR COIL
ODD HARMONICS
ODD
EVEN HAR-MONICS
TO GROUP MODEM OF SYSTEM I
10
120-KC BUS
10
RC
FROM 120-KC BUS
REGU-LAR
RE-LEASE
EMERGENCY
TRANSFER KEY
TRANS-FER CIRCUIT
+130 V
-24 V
EVEN
ODD
FILTERS
1 2
9 10
108 KC
100
92
84
76
68
104
96
88
80
72
64
EVEN
CARRIER SUPPLY FOR CHANNEL MODULATORS AND DEMODU-LATORS OF 10 SYSTEMS
EVEN
ODD
FILTER
EMERGENCY 120-KC AMPLIFIER
120 KC
EMERGENCY CARRIER GENERATOR

Figure 4. Carrier-supply circuit

An emergency is indicated when there is no 120-kilocycle supply on either the regular or emergency bus. When this happens, the copper-oxide rectifier in the transfer circuit gets no 120 kilocycles and so loses its rectified voltage. This triggers off one or both of the two gas-filled tubes (multipled for safety) which increases the grid bias on the control tube of the regular generator to stop its four-kilocycle supply and at the same instant restores the bias to normal on the control tube of the emergency to let its four kilocycles pass through and put the whole emergency circuit into operation. The keys in the transfer circuit are provided for maintenance purposes and to return from emergency to regular operation, since the gas tube circuit is arranged to transfer automatically in only one direction.

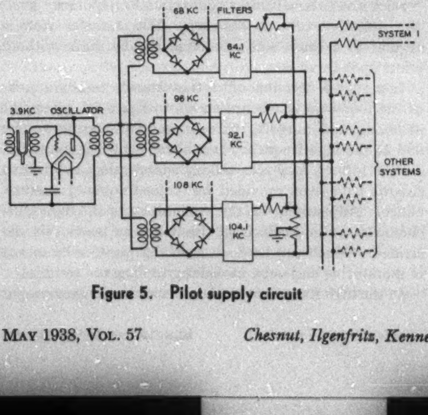
68 KC
FILTERS
64.1 KC
SYSTEM I
3.9KC OSCILLATOR
96 KC
92.1 KC
OTHER SYSTEMS
108 KC
104.1 KC

Figure 5. Pilot supply circuit

The pilot supply circuit is shown in figure 5. The 3.9-kilocycle tuning-fork oscillator at the left supplies that frequency, through the three transformers, to the three copper-oxide modulators whose carriers are obtained from the regular channel-carrier supply busses as shown. The three filters, which are identical with channel-carrier supply filters, select the lower sidebands to be used for pilot frequencies at 64.1, 92.1, and 104.1 kilocycles. The three pilot frequencies are distributed to the different systems through protective resistances from a bus as shown. They are set 100 cycles off the carrier frequencies to obtain locations of minimum interference from carrier leak and other sources.

Signaling circuits do not form an integral part of the carrier terminal equipment. Signaling equipment of a type already widely used in the Bell System for toll circuits, is connected between the toll switchboard and the four-wire terminating set of the individual channel.

Transmission Performance

In general, the performance requirements set down as objectives in the development of this system were based on the assumption that five carrier links operating in tandem and over a 4,000-mile circuit should give satisfactory, high-grade service.

The channel-frequency characteristic which has been attained in the terminals is shown in figure 6. The solid curve below shows the frequency characteristic of a representative channel, while the dotted curves near it show the limits within which the characteristics of all single channels, so far measured, would fall. Above in the figure is shown the characteristic of five representative channels in tandem, each channel having its two voice terminating circuits included.

The delay distortion and time of transmission, contributed by all terminal apparatus at both ends of a system, are shown in figure 7 for a single channel.

The channel modulators have been adjusted so that they will cut off the peaks of excessively loud talk to prevent overloading the carrier repeaters or other parts of the circuit, but this cutting is not enough to degrade the quality of speech. The single frequency load curve of one complete channel is plotted in two ways in figure 8.

The frequency stability of the oscillating tuning forks is expected to be within ± 1 × 10⁻⁶ per degree Fahrenheit on all systems, with negligible variations due to other causes. The amplitude stability of each frequency at its distributing bus is expected to be within ± $^1/_4$ decibel over a period of months. The impedances of the busses are sufficiently low so that crosstalk from one system into another through this path is unimportant. The effectiveness of the protective resistances at the carrier supply busses is such that a short circuit on one modulator or demodulator will increase the loss in the remaining modulators and demodulators less than one-half decibel. The speed of switchover to emergency carrier supply is such that the disturbance to transmission will be less than 10 milliseconds. The effect on speech is not detectable.

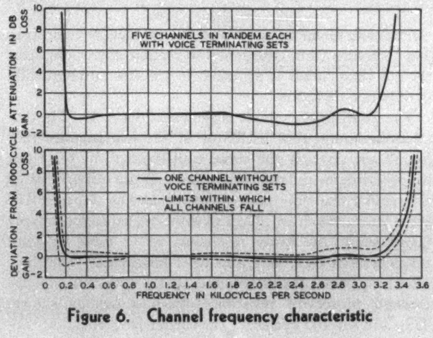

Figure 6. Channel frequency characteristic

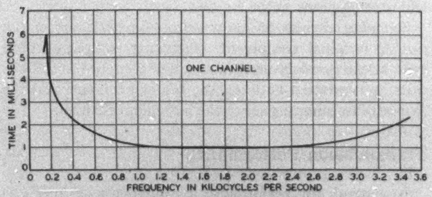

Figure 7. Delay characteristic

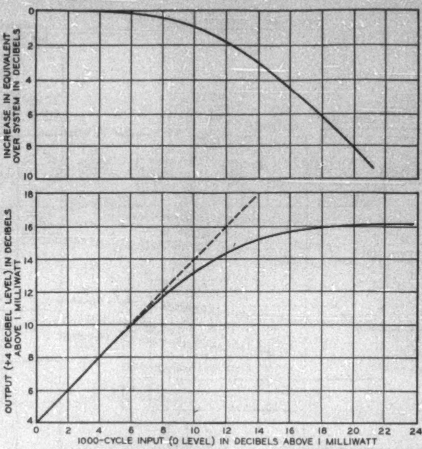

Figure 8. Channel load curve at 1,000 cycles

Maintenance Features

Since the type-*K* system provides more circuits in a group than ever before, it is essential that appropriately better maintenance facilities be furnished. Wherever vacuum tubes are used, jacks have been included to permit testing the condition of the tube by plugging in a new type of test set. The testing of a working tube with this set will not produce an appreciable reaction on performance of the circuits involved. When it has been determined that a tube in the common equipment is nearing the end of its useful life, a special transfer cord circuit is used to remove the circuit involving the tube from service and to substitute a spare circuit temporarily while the defective tube is replaced. This transfer from a regular to a spare and vice versa can be made without effect upon service.

In a type-*K* terminal office transmission tests are made at the four-wire test board shown in figure 9*A* where the incoming and outgoing voice-frequency circuits appear and at the high frequency test board shown in figure 9*B*. At the former, four-wire talking, monitoring and testing circuits have been provided for voice-frequency maintenance. Adjustment of the equivalents of the individual channels can be made from this point as previously described. Much can be done from this position by means of monitoring and noise measuring to diagnose troubles.

At the high-frequency testboard the circuits are brought

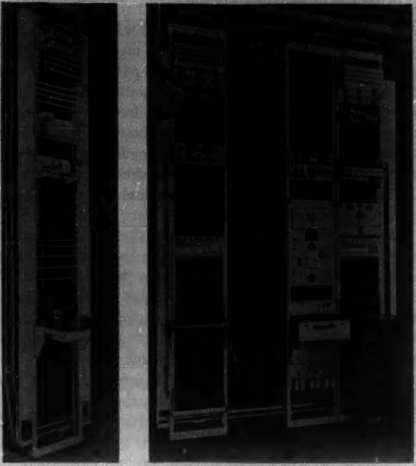

Figure 9. Test positions

A—Voice frequency B—High frequency

through jacks and high-frequency measuring apparatus is provided. Measurements can be made on operating systems to determine the performance of the intermediate repeaters and regulators with respect to level and equalization over the frequency range from 12 to 60 kilocycles. Loss and gain measurements also can be made either between this point and the voice-frequency four-wire test board, through the carrier terminal equipment or through the next adjacent repeater or terminal office at high frequencies. It is possible to test the high-frequency portion of the terminal and to substitute a spare, by patching or rapid transfer, for a defective or potentially defective group modulator, transmitting amplifier, group demodulator or receiving amplifier.

Some of the high-frequency testing equipment is shown mounted on the middle bay of figure 9*B*, of which one of the most important units is the 1- to 150-kilocycle test oscillator located at the center of this bay. It is a heterodyne type of oscillator which covers the frequency range with a continuous film strip scale about 300 inches long. Its maximum output is about one watt and this varies less than one decibel over the entire range. It is provided with built-in calibrating features and can be set to any frequency with an absolute accuracy of about 25 cycles. It is used as the tuning control of the pilot level measuring circuit. An auxiliary scale on the oscillator permits tuning the measuring circuit directly in terms of frequency.

The pilot-level measuring circuit is of the double heterodyne type and includes a copper-oxide modulator which is supplied with carrier from the heterodyne oscillator, an intermediate frequency 130-kilocycle crystal filter of 10 cycles band width, a high frequency amplifier for this frequency, a copper-oxide demodulator supplied with carrier from a 129-kilocycle fixed frequency oscillator, a voice-frequency amplifier, and calibrating circuit. The input impedance of the measuring circuit is high so that when it is bridged across a line pair at the high-frequency test-board jack fields it does not produce appreciable loss to the line. The circuit permits measuring each of the three pilot frequencies to check levels and equalization of operating systems. The panels comprising the circuit are shown below the oscillator in figure 9*B*.

Mounted on a shelf just below the oscillator is the transmission-measuring set which contains a highly accurate thermocouple and meter-combination with calibrating circuits, wide-range repeating coils, a test-key circuit, and attenuators, one of which can be set in steps of one decibel up to a total of 90 decibels.

Equipment Features

Because of the large number of systems likely to be terminated in an office, the jacks are concentrated in a group of bays located together for ease in patching and testing. There are in general five major divisions of the terminal equipment consisting of channel-modem bays, group-modem bays, carrier-supply bays, high-frequency test board, and four-wire voice-frequency patching board and associated voice terminating equipment. The general arrangement in an office is such as to simplify the cabling between various groups of equipment.

The cabling of a high-frequency jack bay, shown in fig-

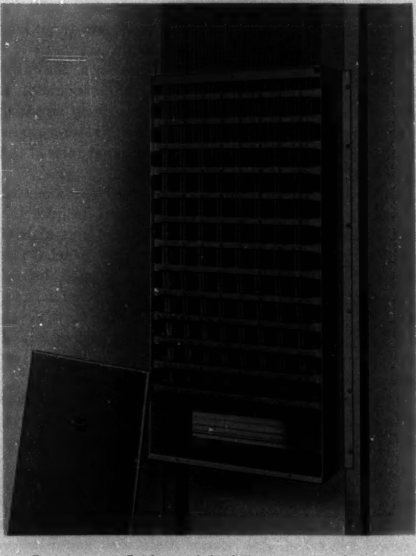

Figure 10. Cabling of high-frequency jack bay

Figure 11. Carrier-equipment bays

A—Channel *B*—Group
C—Carrier and pilot supply

ure 10, illustrates the congested wiring condition occurring when a large number of heavy shielded wires is run to one location. Because of this congestion the jacks in this bay are mounted or removed from the front.

The concentration of equipment in the modem unit is made possible by the small size of the copper-oxide bridges and the filters. Figure 11*A* shows 12 modem units for two systems on adjacent bays with space left at the bottom for the six modems of a third system.

The group modems are about the same size as the channel modems and include a modulator, a demodulator, a transmitting amplifier, and auxiliary receiving amplifier with associated filters. Figure 11*B* shows three units for three systems with space for six additional units at the bottom of the bay.

The carrier supply equipment for ten systems is mounted in one bay as shown in figure 11*C*, which includes the regular and emergency generators, transfer unit distributing equipment and pilot channel supply panel. The bays of this type are located near their associated channel equipment because the supply is chiefly for channel modems. One carrier distributing unit provides for the even and another for the odd harmonics. All terminals and bus bars of these units which are common to the ten systems are protected by insulating covers.

The four-wire voice-frequency jacks for all the systems in an office will ordinarily be grouped in associated bays, one of which is shown on figure 9*A*. A bay will accommodate five systems as an average, that is, 60 voice circuits including the necessary pads and telephone set.

The high-frequency test board is an arrangement of sealed test terminals, high-frequency patching jacks and high-frequency testing equipment mounted on bays as shown in figure 9*B*. Only a few high-frequency patching jacks were required initially and these were therefore mounted above the sealed terminals. This arrangement of bays with the addition of a high-frequency patching bay at the right of each sealed terminal bay will accommodate 100 systems.

The carrier pairs are split off from the main toll cables at splices in the cable vault. The input circuits are carried thence in lead covered cable to the cable crosstalk balancing bays and thence to the input sealed terminal. The output pairs run directly from the output sealed test terminal to the splice in the cable vault. The remaining high frequency wiring from rack to rack is shielded wire.

Conclusion

The carrier-telephone terminals for the type-*K* system which have been described are simpler, occupy less space, and provide better transmission performance than multichannel carrier terminals used previously in the Bell System. As part of a general development of broad-band transmission systems, it is very desirable to employ equipment which can be used in common with several systems. The 12-channel bay, much of the carrier supply, and all of the voice-frequency terminating equipment of this type-*K* system terminal will be used to form corresponding parts of the terminals for the 12-channel open-wire system and the coaxial system, both of which are under development. This not only has simplified the development work, but also will result in greater mass production of these common parts and provide desired uniformity of voice-frequency terminating levels and maintenance arrangements.

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2. NEW KEY WEST-HAVANA CARRIER TELEPHONE CABLE, **H. A. Affel, W. S. Gorton, and R. W. Chesnut,** *BSTJ*, April 1932.

3. CARRIER TELEPHONE SYSTEMS FOR TOLL CABLES, **C. W. Green and E. I. Green.** Presented at the 1938 winter convention of the AIEE.

4. CRYSTAL CHANNEL FILTERS FOR THE CABLE CARRIER SYSTEM. **C. E. Lane.** Presented at the 1938 winter convention of the AIEE.

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Discussion

For discussion of this paper see page 259.

Crystal Channel Filters for the Cable Carrier System

By C. E. LANE
ASSOCIATE AIEE

SINCE the channel selecting filters used at the terminals of the 12-channel cable carrier system are the principal filters in the system this paper is concerned primarily with these filters. Their importance is evident from the fact that they represent over one-third of the cost of the system terminals.

Many new features appear in these channel filters. The most outstanding is the use as filter elements, along with inductance coils and condensers, of plates cut from crystalline quartz. It is for this reason they are called "crystal filters." In addition, however, the inductance coils, some of the condensers, and also the filter assemblies have in them new features. Only after a number of years of laboratory experimentation with filters using crystal elements, studying their advantages and limitations, was the cable carrier system planned to use such filters.

There are 12 channel filters which transmit the lower side bands derived from the modulation of the speech signals with carrier frequencies spaced four kilocycles apart from 64 to 108 kilocycles. An insertion loss frequency characteristic which applies for each of the 12 filters is shown in figure 1. Regarding a ten decibel loss increase as the cutoff as compared with transmission at 1,000 cycles, the voice-frequency band for a single carrier link, largely determined by the characteristics of the channel filters, extends from approximately 150 to 3,600 cycles. For five links the band extends from about 200 to 3,300 cycles. This is a 600 or 700 cycles wider frequency band than the present three-channel open-wire carrier system. The maximum delay distortion in the transmission band of each of the filters is about 0.4 millisecond. As many as ten of these filters may appear in tandem in the longest talking circuits. The total delay distortion in such cases would then not exceed 4 milliseconds. This is not objectionable since the average listener cannot observe the effect of delay distortion unless it exceeds about 10 milliseconds. A representative filter schematic is shown in figure 2. The condenser shown by the dotted line at the left is used only in the two lowest frequency filters to obtain an impedance transformation internal to the filters and thereby permit the use of crystals of practical thicknesses for these filters. However, the equivalent circuit for each of the 12 filters is the same. In the system the filters work in parallel at one end and between terminating impedances of 600 ohms. The two condensers appearing in the series arms at the left end of the filter schematic are used in obtaining satisfactory operation of the filters in parallel and otherwise might be omitted provided the inductance at this end was made smaller at the same time.

Figure 2 indicates the separate physical elements and the manner in which these are connected in the filters. In considering the performance of the filters the crystal elements are replaced by their equivalents, an inductance and capacitance in series, shunted by a second capacitance as shown in figure 3. Also, the condensers in shunt across the filter are shown inside the lattice combined with the direct capacitance of the crystals and the inductances are relocated in series in each lattice arm. In making this conversion, however, the effective resistance of the in-

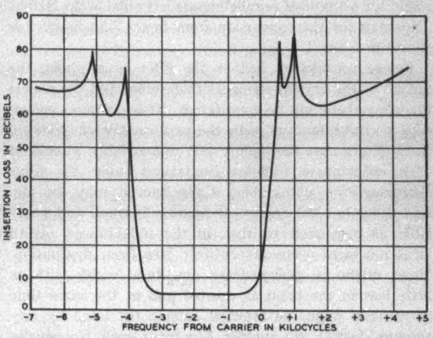

Figure 1. When plotted in cycles removed from the carrier frequency, the insertion loss frequency characteristics of each of the 12 crystal channel filters are for all practical purposes identical

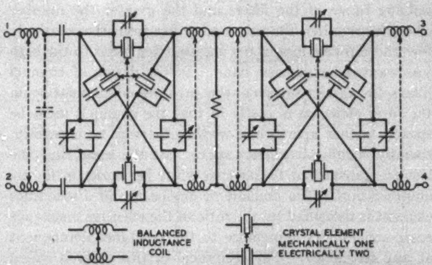

Figure 2. The schematic circuits of each of the 12 filters are the same except for the addition of the condenser shown by the dotted lines which appear only in the two lowest-frequency filters

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Paper number 37-180, recommended by the AIEE committee on communication and presented at the AIEE winter convention, New York, N. Y., January 24–28, 1938. Manuscript submitted October 29, 1937; made available for preprinting December 18, 1937.

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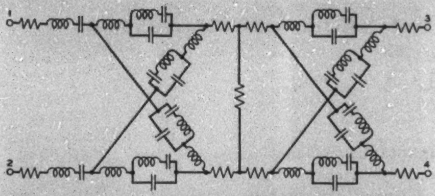

Figure 3. The schematic circuits of the filters are each equivalent to two lattice sections with a resistance pad between them, resistances at each end, and at the paralleling end a coil and condenser which resonate at the midband frequency

ductance coils are, for reasons which will appear later, shown remaining outside the lattice. Also the capacitances and the portion of the inductance which are used solely for purpose of paralleling are left outside the lattice. The basis for the conversion from figure 2 to figure 3 is shown in figure 4.

Before considering further the filter as a whole, the nature of the crystal elements themselves and the reason for using them will be considered. It is common knowledge to those familiar with the performance of electrical wave filters that the energy loss unavoidably associated with inductances imposes limitations upon the filter characteristics obtainable. Capacitances may be designed so that the energy dissipation is small and negligible as compared to that in the inductances. With ideal reactance elements entirely free from dissipation, filters might be designed for any band width with as little loss in the band as wanted and at the same time frequencies might be rejected outside the band by any amount desired, no matter how near such frequencies were to the edges of the transmitting band. Of course the sharper the filter cutoffs, other requirements being the same, the more complex the filter structure would be even neglecting dissipation. The greater the dissipation in the filter elements, the greater the loss in the transmitting range of the filters and the greater the number of cycles required for this loss to rise from the relatively low and uniform loss in the transmitting band to the high loss wanted outside the band. In the design of channel filters for carrier systems, the presence of dissipation in the filter elements is costly in that the channels must be spaced farther apart than would otherwise be necessary, thereby wasting frequency space. At the same time the loss to transmitted frequencies must be made up for by amplification. The amount of dissipation in a reactance element is measured by the ratio of the effective resistance component of its impedance to the reactance component at any frequency. The reciprocal of this ratio is called the Q of the reactance and hence is a measure of efficiency or freedom from dissipation. In the design of inductances in the form of wire-wound coils, it is generally not practical to obtain Q's much in excess of 200 or 300 at any frequency. The quartz crystal element used in the filters as previously stated is equivalent electrically to a two-terminal reactance consisting of an inductance and capacitance in series shunted by a second capacitance. For the Q of the inductance in the equivalent circuit of the crystal element a value of 15,000 or more can readily be obtained. It is for the purpose of utilizing this high Q inductance and obtaining the benefits therefrom that crystal elements are used in these filters.

The filter schematic in figure 2 shows crystal elements in each filter section; the two in the lattice arms and the two in the series arms in each case are identical. Electrically there are four crystals in each section but for reasons of economy and for convenience in handling and adjusting the crystals those in corresponding arms are physically one. This is possible since the filter is a balanced structure and the two like crystals vibrate in unison. Figure 5 is a photograph which shows a representative double crystal element taken from each of the 12 filters. The four crystals in the lowest-frequency filter range from 40.2 millimeters to 41.8 millimeters in length and those in the highest-frequency filter from 23.8 to 24.3 millimeters. The thickness of the crystals in all four of the lowest-frequency filters are 0.63 millimeters, in the next four filters 0.82 millimeters, and in the highest-frequency filters 1.1 millimeters. Uniformity in thickness is maintained as far as practicable since it contributes to economy in manufacture of the crystal. Within the range using the same crystal thickness the impedance and frequency differences, called for by the design of the different filters, can be provided by variations in width and length of the crystals. The ratio of width of the crystals to their length ranges from about $^1/_2$ to $^4/_5$.

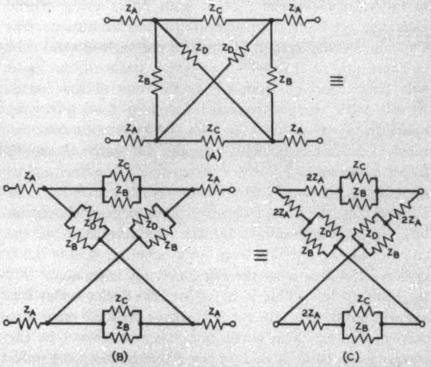

Figure 4. It does not alter the transmission properties of a network such as shown in figure 4A to remove the impedances in shunt and outside the lattice and replace them by impedances of equal magnitude in shunt across each lattice arm nor by removing the impedances in series with the lattice and replacing them by series impedances inside the lattice of twice the magnitude of those removed

The major surfaces of the crystals are plated with a thin layer of aluminum deposited by an evaporation process. This plating is divided along the center line lengthwise of the crystals to form the two electrically independent crystals. Since the crystals vibrate longitudinally with a node across the middle, they are clamped at this node in mounting. Figure 6 shows the orientation of the crystal plates with respect to the natural axes of the quartz from which they are cut. The plates are cut as accurately as practical to the dimensions computed making a small allowance in length and then the crystal is finally adjusted in an electrical circuit by grinding the end of the crystal until the resonant frequency falls within five or ten cycles of that desired.

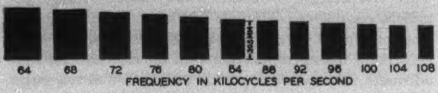

Figure 5. The length of the crystal elements used in the different filters varies about inversely as the frequency of the filter band location

Considering again the filter schematic as a whole, figure 3, and neglecting the dissipation in the crystals and condensers, the filter may be regarded as made of two lattice filter sections having ideal reactance elements, that is, elements free from dissipation. The location of the effective resistance of the coils outside the lattice, for purpose of performance analysis, shows, how, at the end of the filter these resistances may be regarded as part of the terminating impedance between which the filter works and how between the filter sections the resistances may be combined with a shunt resistance to form a resistance pad which matches the image impedance of the two filter sections. The effect, then, of the coil resistances is primarily to provide a flat loss over the entire frequency range and does not affect appreciably the shape of the loss characteristic furnished by the reactance inside the lattice sections.

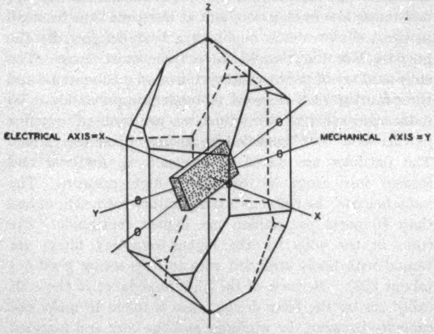

Figure 6. The crystal elements are cut with their major surfaces perpendicular to the electrical axis of the natural quartz, with their side surfaces making a small angle with the mechanical axis, and with their end surfaces making a small angle with the optical axis

In considering the performance of lattice-type filter sections, it is common practice to sketch together the frequency reactance curve of the two lattice arms Z_x and Z_y. This is done for one of the filter sections and is shown in figure 7. In the frequency range where the two curves are of opposite sign the filter transmits and where they are of the same sign there is attenuation. At the point where the two curves intersect there are attenuation peaks of very high loss. The reactance curves of figure 7 are for the filter section accountable for the pair of attenuation peaks shown in the filter characteristic which are the closer to the edges of the transmitted band. For the other section the cross-over points of the two reactance curves are farther away from the band, since this section is responsible for the outer pair of attenuation peaks. The design of the filters consisted in determining values for the inductance coils, condensers, and crystals, such that the reactance curves of the lattice arms of the filter passed through infinity and intersect with each other at the desired frequencies and, at the same time, of determining the impedance level for all of the elements such that the filters would have the right image impedances.

The curves of figure 7 would seem to indicate a somewhat greater band width for these filters than shown by the insertion loss characteristic of figure 1. The reason

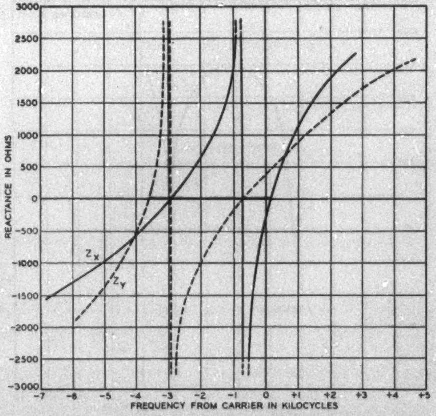

Figure 7. In a lattice filter section transmission occurs for frequencies where the impedances for the two arms of the lattice are of opposite sign and attenuation peaks of very high loss occur where the impedances cross

for this can best be explained by referring to the image impedance of one of the filter sections as shown in figure 8. Within the band the image impedance is, of course, a pure resistance which varies with frequency. It is about 800 ohms at midband frequency and falls rapidly to zero near the edges of the band. Assuming the effective resistance of the coils which is about 100 ohms as belonging to the terminating impedances, the filter sections actually work between impedances of about 700 ohms. This means that large reflection losses occur at each end of each filter section near the edges of the transmission band where the image impedance of the filter is very small. It is these reflection losses that are responsible for the actual transmission band being much narrower than it would be with the filter sections terminated in their actual image impedances. The filter sections are designed with 800 ohms image impedance at midband frequency instead of 700 ohms to make the band flatter and somewhat wider than it would be otherwise.

When a number of band filters are operated in parallel it is generally necessary to connect across the paralleled end a two terminal network to correct for the distortion that would otherwise be present in the highest- and lowest-frequency filters in the group. A circuit of the network used for this purpose with the channel filters is shown in figure 9.

The filters employ crystal elements in order to obtain abrupt discrimination between wanted and unwanted frequencies and at the same time to secure low and uniform loss in their transmitting bands. This characteristic must not only be obtained at the time the filters are assembled and adjusted but must be maintained throughout the service life of the filters and not appreciably affected by temperature variations. This imposes severe stability requirements upon the elements used in the filters. The crystal elements themselves are very stable when properly designed and once adjusted will retain at a given temperature their frequencies of resonance within one or two cycles seemingly indefinitely. Their temperature coefficient is only about 25 parts per million per degree centigrade which is not objectionable. The obtaining of suitable inductance coils and condensers for use in conjunction with the crystals that were adequate in stability required considerable development effort. The inductance coils were required to have not only a high degree of stability with respect to temperature and time but also a high ratio of reactance to effective resistance, low modulation, and at the same time be small in size. Air-core coils might have been designed for the purpose but they would have been quite large. The coils used are of the toroidal type wound on about one and three-fourths inch rings of molybdenum-permalloy. To reduce eddy-current losses the cores are made of very fine powder and then annealed to reduce hysteresis losses. The particles are mixed with insulating material and formed into rings by extremely high pressure. The inductance of the coils has a temperature coefficient of less than 40 parts per million per degree centigrade. The cores of the coils for the higher-frequency filters are wound with finely stranded wire to help secure good Q's (about 225). Because of the high impedance of the coils called for by the filter design, care is taken to make the capacity between the windings and the core and between the windings and the case as low as practical and also to make stable all such small capacities as must be present.

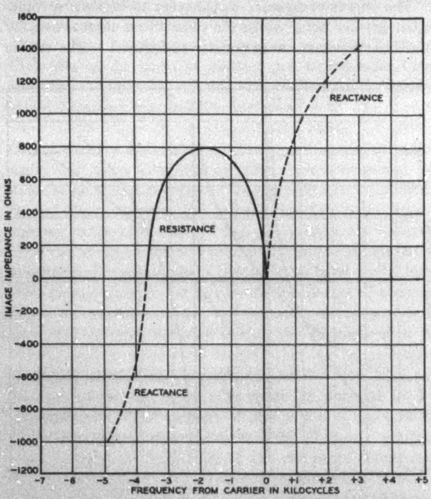

Figure 8. The large reflection losses occurring within the transmission band of the filters and near the edges of this band has the effect of narrowing the width of the transmission band

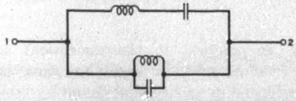

Figure 9. A two-terminal reactance network is connected in shunt across the filters at their paralleling end to improve the characteristic of the highest- and lowest-frequency filters

The two extra condensers used at one end of each filter for paralleling purposes are of a high-grade mica type. The other condensers are all quite special. The fixed ones, ranging in magnitude from about seven micromicrofarads to 100 micromicrofarads are made by plating short lengths of high-grade glass tubing inside and outside with silver. Because of the intimate association of electrodes with the surfaces of the tubes and the low expansion coefficient of the glass used a condenser is obtained that has a temperature coefficient comparable with that of the coils and crystals. No aging effect has been observed. It will be noticed that four small adjustable air condensers appear in each filter section. These are used to secure precise initial adjustment of the filter capacitances.

To protect the filter elements against moisture, the filters are hermetically sealed in a container made from a rectangular section of seamless brass tubing with closely fitting plates soldered in each end. One end plate carries four metal-glass seal terminals and a nozzle through which dry air is blown after the filter is assembled. The other end plate is provided with a small hole for the escape of the drying air. After the drying operation the hole in the nozzle and the hole in the opposite end of the filter are closed by soldering. The elements that make up the filter are assembled on a chassis which is completely wired and then slid into the container in assembly. Figure 10 is a photograph showing this chassis and the arrangement of the elements. The elements are located in such a way as to use very short wiring connections which reduce the magnitude of any stray admittances. The wired filter chassis is carefully adjusted by setting the values of the air condenser such that for each filter section the resonance frequencies looking into each end of each section occur where they theoretically should. This compensates for the effect of small capacitances between the filter parts. In the design of the filter parts care is taken to use no material which absorbs moisture readily since such moisture would later be released and raise the relative humidity of the air inside the filter.

If voltages much in excess of about 20 volts are applied across crystal elements at frequencies near resonance, the crystals will break from the mechanical strain of their vibration. The maximum safe voltage across the channel filters at the resonant frequencies of the crystals in them is considerably greater since at resonance the full voltage does not appear across the crystals. In their normal use in the system the voltages across the filters will be very much less than 20 volts.

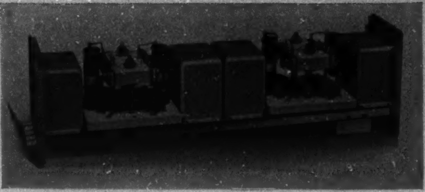

Figure 10. The filter parts are assembled and wired on a chassis which is slid as a unit inside the filter container

Other filters forming part of the terminal apparatus are the group modulator and group demodulator low-pass filters, the channel and group carrier supply filters, and the pilot supply filters. The group modulator and demodulator filters are of the low-pass type employing only coils and condensers as elements. The group carrier supply filter is the same in schematic and mechanical design as the crystal channel filters described. The pilot supply filters and the channel carrier supply filters are equivalent in schematic to one section of the channel filters. Mechanically, of course, they are only about half the size and are hermetically sealed in the same manner.

References

For a further discussion of crystal filters the reader is referred to THE EVOLUTION OF THE CRYSTAL WAVE FILTER, O. E. Buckley, *Journal of Applied Physics*, October 1936, and ELECTRICAL WAVE FILTERS EMPLOYING QUARTZ CRYSTALS AS ELEMENTS, W. P. Mason, *Bell System Technical Journal*, July 1934.

Discussion

For discussion of this paper see page 259.

Crosstalk and Noise Features of Cable Carrier-Telephone System

By M. A. WEAVER NONMEMBER AIEE R. S. TUCKER ASSOCIATE AIEE P. S. DARNELL NONMEMBER AIEE

CROSSTALK and noise are important factors in cable carrier transmission as outlined in the paper "A Carrier Telephone System for Toll Cables" by Messrs. C. W. Green and E. I. Green. Crosstalk and noise limit the number of carrier channels which can be utilized in any one cable, not only by limiting the number of channels which can be placed on a single pair, but by limiting the number of pairs which can be used. Noise also controls the transmission loss which can be permitted between repeaters. Without the crosstalk and noise reduction measures described in this paper, the number of carrier channels per cable would be so few and the spacing between repeaters so short, that the type-*K* carrier system would be impracticable.

Crosstalk

To utilize existing toll cables in the Bell System for frequencies up to 60 kilocycles required the solution of many new crosstalk problems because: (1) crosstalk increases rapidly with the frequency; (2) nonloaded carrier pairs due to their high speed of propagation are especially suitable for very long distances and hence the crosstalk requirements per unit length are relatively severe; (3) the large gains of the carrier repeaters amplify certain crosstalk currents much more than in the case of voice frequency circuits.

Two general effects need to be considered: intelligible crosstalk must be prevented; and a large number of circuits crosstalking into a particular circuit must not contribute an undue amount of noise. The second effect is called babble, since it consists of a multiplicity of unrelated voice sounds which, in the aggregate, are unintelligible.

An important feature is the use of different cables for opposite directions of transmission. This makes the major crosstalk problem the reduction of crosstalk between pairs in the same cable used for transmission in the same direction. The crosstalk currents due to transmission at one end of a disturbing circuit through the distributed couplings with a disturbed circuit tend to arrive at the distant end at the same time since the currents via any of the couplings travel substantially the same distance. This makes it possible to greatly reduce the total effect of these distributed couplings by the use of small adjustable mutual inductance coils connected between pairs at one point in each repeater section.

If nothing more were done, there would still be objectionable crosstalk since currents from the outputs of carrier repeaters could crosstalk into voice-frequency circuits and these circuits could then again crosstalk into other carrier-frequency circuits at points near their repeater inputs. This effect is minimized by transposing the carrier pairs from one cable to the other at carrier repeater points.

At common voice-frequency and carrier-frequency repeater points there would be an unsatisfactory crosstalk path from a carrier repeater output into all the wires not used for carrier frequencies and from them through coupling between office wiring into similar wires in the other cable and finally into carrier repeater inputs in the second cable. This crosstalk is minimized by the use of carrier-frequency suppression coils in the voice-frequency circuits. These coils also serve the purpose of preventing carrier frequency noise originating in voice frequency circuits from being transmitted into the cables and inducing noise at points near carrier repeater inputs.

NEAR-END CROSSTALK

Near-end crosstalk is the result of coupling between circuits transmitting in opposite directions, while far-end crosstalk is the result of coupling between circuits transmitting in the same direction. Near-end crosstalk coupling between different carrier circuits of the same frequency must be kept very small, particularly near a repeater point, since crosstalk from the output of a repeater into an opposite directional pair near the input of its repeater will be greatly amplified by this repeater.

Crosstalk between carrier circuits within the offices is kept low by careful shielding, segregation, suppression of spurious paths through battery supply, common grounding arrangements, etc.

Since the type-*K* system operates on a "four wire" basis, different electrical paths are used for opposite directions of transmission. Satisfactory near-end coupling in the outside plant is obtained, therefore by placing east bound pairs in one cable and west bound pairs in another. When two cables have relatively heavy sheaths as in the larger Bell System cables, their coupling is sufficiently small even with the two cables in close proximity.

INTERACTION CROSSTALK

The crosstalk currents from a carrier repeater output into voice-frequency circuits in the same cable must be limited, since they crosstalk again into carrier circuits near repeater inputs and, consequently, are amplified by the high-gain repeaters. Intermediate circuits most responsible for crosstalk of this type are made up of combinations of pairs and phantoms and the sheath, i.e., longitudinal paths.

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1. For all numbered references, see list at end of paper.

One case of crosstalk of this kind would occur if the same cable were used for carrier pairs transmitting in the same direction on both sides of a repeater. This is prevented by transposing carrier pairs from one cable to the other at each repeater point, as shown on figure 1.

A second interaction crosstalk problem is encountered at the common voice and carrier repeater points and involves coupling between cables as well as in the same cable. Here the coupling path is from carrier repeater outputs to intermediate circuits in the same outside cable, back into the common office over these intermediate circuits and then via office coupling to intermediate circuits in a second outside cable and from there to carrier repeater inputs connected to pairs in the second cable. Referring to figure 1, a set of noise (and crosstalk) suppression coils is encountered in this path. The high longitudinal circuit impedance of these coils minimizes this interaction crosstalk.

FAR-END CROSSTALK

Far-end crosstalk currents are subjected to line attenuation and amplification similarly to the main transmission currents, and do not have extra amplification as in the case of near-end crosstalk. Furthermore, far-end crosstalk currents due to couplings at different points along the line tend to arrive at the distant end of the disturbed circuit at the same time. Hence a considerable portion of the far-end crosstalk over the type-*K* frequency range, which occurs between circuits transmitting in the same direction in the same cable, can be neutralized by introducing compensating unbalances at only a comparatively few points, such as one per repeater section. The far-end crosstalk reduction problem is greatly simplified because phantom circuits are not used for carrier operation.

Theoretically, for the same precision of match between the impedances in the two directions at the balancing point, the crosstalk reduction would be about the same whether the balancing is done at an intermediate point or at either end of a repeater section. Balancing will be done at repeater inputs rather than at an intermediate point, such as the middle, because it is practicable to obtain repeater impedances matching the average line impedance sufficiently well so that the effectiveness of balancing is reduced only slightly.

NATURE OF FAR-END CROSSTALK COUPLING

The coupling between two cable pairs in a short length may be represented by a mutual admittance and a mutual impedance. The former is due almost entirely to capacitance unbalance, which varies but little with frequency, so that its effect could be practically balanced out by means of a simple condenser. The latter, however, involves a complex mutual inductance of the form $M_a + j M_b$, because of the proximity effect of the wires of a pair and of other cable conductors.[1] As shown on figure 2, both components vary considerably with frequency; M_a on the average decreasing as the frequency increases while M_b in the general case is of negative sign and reaches a maximum value at 56 kilocycles.

TYPE OF BALANCING

To obtain maximum reduction in crosstalk it would be necessary to use a condenser for balancing the mutual admittance and an inductance coil for balancing the

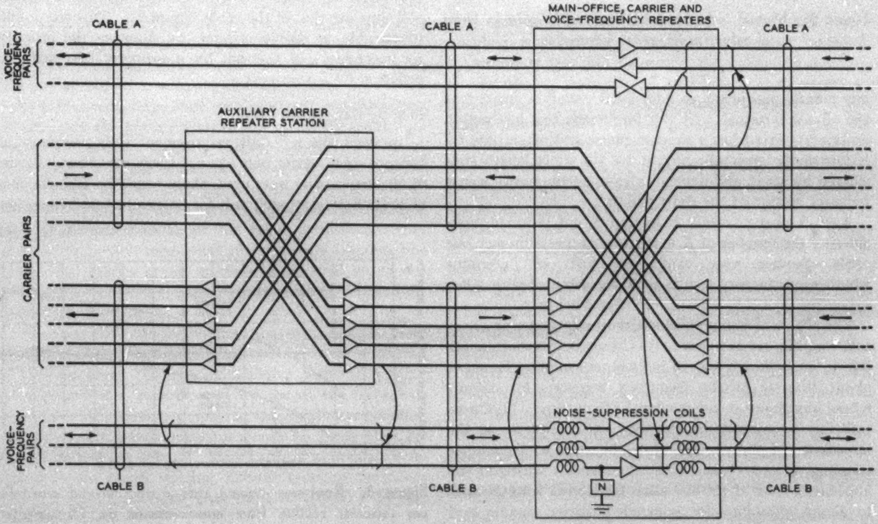

Figure 1. Schematic showing carrier pairs transposed between cables, coupling paths and noise suppression coils

mutual impedance or to use some equivalent complex network. Experimental balancing in a particular cable using the coil-condenser method reduced the mean crosstalk over the type-*K* range about 20 decibels, which is close to the maximum reduction possible with a universal type of balancing unit. The reduction is limited by the fact that two pairs having identical crosstalk couplings in each of two short lengths at different points in the cable will not produce two identical elements of crosstalk current at a circuit terminal because: (1) Cable circuits are not perfectly smooth. Reflections, as at junctions of reel lengths or at terminals, alter the two crosstalk currents differently. (2) The propagation constants of each circuit vary slightly from reel to reel in a random fashion and therefore the two crosstalk currents are of slightly different phase and magnitude. (3) In any short length the disturbing circuit produces crosstalk currents in intermediate circuits, which are propagated along these circuits and crosstalk again into the disturbed circuit at various points, producing an additional crosstalk current at the circuit terminal. At any frequency, this interaction crosstalk current has a random phase and magnitude relation to the crosstalk current for the short length considered by itself, and depends also upon the position in a repeater section of the short length.

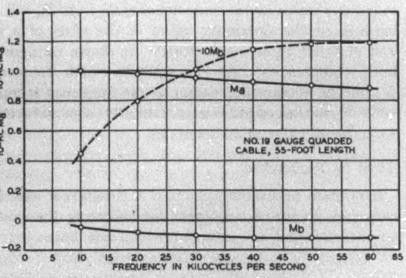

Figure 2. Mutual inductance between cable pairs in terms of value for M_a at ten kilocycles

A 20-decibel crosstalk reduction is not required, considering the number of *K* systems anticipated in any one cable. Studies were made, therefore, to determine whether satisfactory results could be obtained with a less expensive type of balancing, as outlined below.

The effects of frequency and circuit impedance on crosstalk coupling are as follows: (1) Crosstalk in a short length due to capacitance and to inductance coupling increases about directly as the frequency increases for circuits whose impedance is independent of frequency. (2) Crosstalk due to capacitance coupling varies directly as the impedance of the circuits while that due to inductance coupling varies inversely as the impedance. Changing the impedance from about 800 ohms for loaded voice circuits to about 135 ohms for nonloaded carrier circuits and changing the frequency from about four kilocycles to 60 kilocycles increases the crosstalk due to capacitance coupling by a factor of about 2.5 and that due to inductance coupling by a factor of about 90.

Capacitive coupling in existing cables was reduced by design to as great a degree as practicable, particularly for the most closely associated circuits, because it is of most importance in the loaded voice frequency case. These same design measures also reduce inductive coupling but not to the same extent. Capacitive coupling decreases rapidly with separation due to the shielding effect of copper in intervening circuits while inductive coupling is not much affected by intervening copper wires. To minimize magnetic coupling it is necessary to use different lengths of twist for the pairs. Existing cables have relatively few lengths of pair twists.

As the net result, capacitance coupling is no longer all important, inductance coupling at 60 kilocycles actually predominating by a factor of about three to one in existing cables. Capacitance balancing should, therefore, be less effective than balancing designed to reduce the inductance coupling. Tests have shown that capacitance balancing alone gives a crosstalk reduction of about 11 decibels while inductance balancing alone gives a reduction of about 16 decibels. Since the latter reduction is sufficient, except possibly for small cables or special cases, the type-*K* balancing has been designed on this basis. Far-end crosstalk currents due to the two kinds of coupling have phase relations not differing from zero or 180 degrees by more than about 15 to 40 degrees, depending on whether the upper or lower type-*K* frequencies are considered. There is, therefore, a tendency for either type of balancing unit to annul both kinds of coupling.

To obtain as much as 16 decibels reduction it is necessary that the frequency characteristic of the balancing coil simulate that of the cable (figure 2). This was found practicable, as discussed later, by shunting the primary (or secondary) of the coil by a properly designed impedance.

SIZE OF BALANCING COIL

To meet the crosstalk requirement it is necessary to balance each carrier pair against every other carrier pair. If 50 carrier pairs were used, there would be 49 balancing coils connected to each pair for balancing to all the other

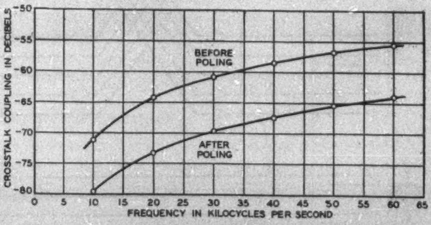

Figure 3. Root-mean-square side-to-side far-end crosstalk per repeater section from measurements on 14 repeater sections

pairs, a total of 1,225 coils. For convenience, adjustable coils having the same mutual-inductance range and the same self-inductance are used. Hence, the insertion loss per coil, resulting from the self-inductance and resistance of the coils, must be kept small. In addition, the self-inductance of the coil presents a problem from the impedance standpoint. To keep the impedance at any point in the balancing panel as nearly like the average cable impedance as practicable, the self-inductance of a series of coils must be neutralized by capacitances shunted at suitable intervals. It is very desirable, therefore, to use coils whose self- and mutual-inductances are no larger than actually essential. Consequently, an attempt has been made to keep the maximum crosstalk before balancing low.

Due to special measures, described below, it appeared that the maximum inductance unbalance per repeater section could be kept below about 1.3 to 1.5 microhenries, with the possible exception of side-to-side unbalances, and trial balancing coils were designed accordingly.

Crosstalk Reduction Before Balancing

Changes in the original splicing are made at approximately 6,000-foot intervals, i.e., at points where voice-frequency loading coils must be removed from the carrier pairs. In most existing voice-frequency toll cables the 19-gauge quads were spliced as three groups, one, a two-wire circuit group, one, an eastbound four-wire circuit group, and the third, a westbound four-wire circuit group. Ordinarily, the carrier pairs will be selected from the four-wire groups because these groups are usually larger than the two-wire group and since the quads within a group are spliced at random there is less chance of a large value of coupling between pairs of different quads, i.e., two pairs are less apt to be recurrently in a relation of high coupling. The carrier pairs are divided evenly between the two four-wire groups, in order that the least number of four-wire voice circuits will be lost.

In cables with large four-wire groups it is satisfactory to maintain the grouping arrangement on the pairs converted to carrier. In such cables, however, one four-wire group is in the center or core of the cable and the other group in the outer periphery. In order that all circuits will have about the same velocity and attenuation and be subjected to about the same temperature conditions for both transmission and crosstalk reasons, one (four-wire) carrier group in these cables will be spliced to the other (four-wire) carrier group and vice versa at the 6,000-foot intervals.

In cables with relatively small four-wire groups, there is more chance of two pairs being recurrently in a relation of high coupling. To reduce this chance, a special splicing arrangement has been devised for use at the 6,000-foot intervals. With existing splicing the maximum coupling in cables with small groups is about 2.5 times that for cables with large groups. This ratio is appreciably reduced by the special splicing, likewise reducing the maximum mutual inductance that must be supplied by the balancing unit.

The foregoing was with particular reference to crosstalk between pairs in different quads. Crosstalk between pairs in the same quad (side-to-side crosstalk) is an additional problem. A quad consists of two twisted pairs of wires which are twisted together to permit the use of voice-frequency phantom circuits. Since the two sides of a quad are so closely associated, side-to-side crosstalk is generally much greater than that between pairs of different quads. The electrical size of the balancing unit, therefore, is determined by the side-to-side crosstalk, which is reduced by "poling."

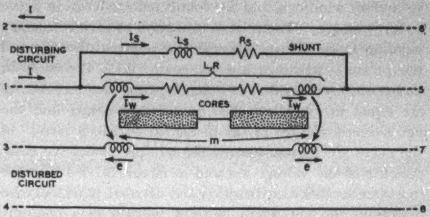

Figure 4. Schematic of a simple balancing coil designed to produce a complex mutual impedance

To apply poling, the quads are carried through as quads for an entire carrier repeater section. From measurements of side-to-side crosstalk in phase and magnitude, quads in one half repeater section are chosen and spliced to quads in the other half in such manner as to partially neutralize the side-to-side crosstalk. In effect, quads in one half-section serve as balancing units for the other half.

In most existing toll cables the side-to-side capacitance coupling was reduced when the cables were installed, by means of test-splicing within the 6,000-foot sections. Obviously, for poling to be effective it is necessary to operate mainly on the inductance component. The poling measurements, therefore, are made at about one kilocycle where an approximate measure of the inductance component can be obtained directly since the capacitance and inductance components of the crosstalk are at an angle of almost 90 degrees at this frequency. Figure 3 shows the crosstalk results obtained by means of one-kilocycle poling on 14 repeater sections. It has been shown that this 9-decibel reduction is within two to three decibels of the maximum reduction possible with much more complicated poling involving measurement and consideration of both components at carrier frequencies.

After side-to-side poling, coil balancing cannot be expected to give as much as 16 decibels reduction in crosstalk. This is unimportant, however, as long as the required reduction can be obtained more economically by the combined methods rather than by balancing alone.

Crosstalk Balancing Coil

Since the voltage which causes the crosstalk current in the disturbed circuit may be in either a clockwise or

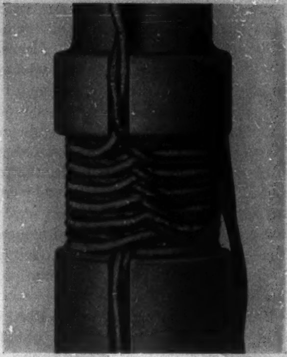

Figure 7. Arrangement of inner winding of trial balancing coil

counterclockwise direction, the balancing device, for flexibility reasons, should be capable of establishing voltages in either direction. A balancing coil was developed, therefore, which in operation may be likened to that of two separate transformers with simultaneously movable cores. The primary windings are in series, as are the secondary windings, and are connected as shown in figure 4, for example. The relative direction of each secondary winding is the same, whereas the relative directions of the primary windings are reversed. With the cores in mid-position, the voltages induced in the two secondaries are equal in magnitude but opposite in phase, and the net induced voltage in the disturbed circuit is zero. As the cores are moved toward the left the respective components of the voltage induced in circuit 3-7-8-4 increase in a counterclockwise direction and decrease in a clockwise direction, the net result being a voltage in a counterclockwise direction. Such a setting of the balancing coil would be used to counteract a clockwise crosstalk voltage, the amount of departure of the cores from mid-position being dependent on the magnitude of the crosstalk voltage being counteracted. Movement of the cores toward the right produces the opposite effect.

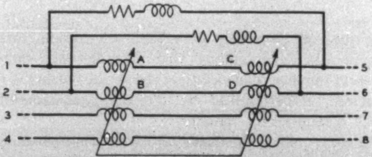

Figure 5. Schematic of winding arrangement of trial balancing coil

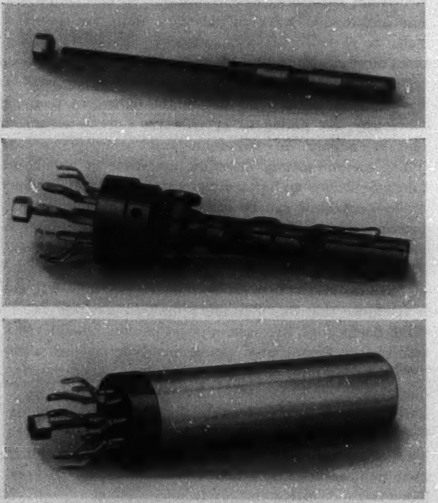

Figure 6. Trial balancing coil construction. Container is four and one-half inches in length and one and three-eighths inches in diameter

This device, disregarding any proximity effects therein and the effects of the shunt, acts to set up a net voltage e which is in phase quadrature with the disturbing current I. Hence,

$$e = -j\omega m I \tag{1}$$

in which m is the net mutual inductance of the device. To obtain the required mutual impedance characteristic, the primary (or secondary) windings of the coil are shunted by an inductive resistance. Let the effective self-inductance and resistance of the line windings (primaries) be denoted by L and R, respectively, and the current through these windings by I_w. Let the effective self-inductance and resistance of the shunt be denoted by L_s and R_s, respectively. At balance, that is, when no crosstalk current flows in 3-7-8-4 due to I (the disturbing current), the current I_w is

$$I_w = \left[\frac{R_s(R+R_s)+\omega^2L_s(L+L_s)}{(R+R_s)^2+\omega^2(L+L_s)^2} - j\frac{\omega(R_sL-RL_s)}{(R+R_s)^2+\omega^2(L+L_s)^2}\right]I \tag{2}$$

$$= [a - jb]\, I \tag{3}$$

where a and b are, respectively, the coefficients of the real and imaginary parts of the expression. Hence, with a shunted coil the voltage induced in the disturbed circuit is:

$$e = -j\omega m I_w = -j\omega(ma - jmb)\, I \tag{4}$$

The mutual impedance, Z_m, equals $j\omega(ma - jmb)$, or the effective mutual inductance M of the balancing coil may be written

$$M = M_a + jM_b \tag{5}$$

wherein $M_a = ma$ and $M_b = -mb$. Assuming R, L,

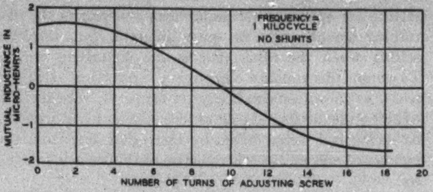

Figure 8. Mutual inductance of trial balancing coil

L_u, and R_s to be constant with respect to frequency of current and position of the cores, it is seen from (2) and (5) that for any core setting, M_a and M_b are functions of frequency only and their ratio at a given frequency is theoretically constant throughout the operating range.

To keep inductance L constant irrespective of the mutual inductance settings, the length of the coil windings, the length of the magnetic cores, and their spacings with respect to the winding spacing are so related that the change in inductance of one primary (or secondary) winding caused by motion of its associated core is equal and opposite to the change caused by the movement of the core associated with the other primary (or secondary) winding. To keep R low over the type-K frequency range, cores of finely powdered molybdenum permalloy pressed into a cylindrical form are used.

Because of other requirements which a balancing coil must satisfy, the winding arrangement actually employed is shown in figure 5. The simple device of figure 4 is not balanced from the standpoint of longitudinal crosstalk for any coil setting except that of zero mutual inductance. The figure 5 arrangement is such that theoretically there is no magnetic coupling between the two circuits for longitudinal currents in either one, regardless of the coil setting. Unless the capacitance between primary and secondary windings can be kept very small, the resultant admittance unbalance produces crosstalk which is not completely balanced out when the coil is adjusted. The turns of conductor in the figure 5 coil are so located that this side-to-side capacitance unbalance is less than five micromicrofarads. The capacitances between wires of either the primary or secondary winding do not affect the unbalance but contribute a part of the capacitance loading which compensates for the line inductance of the coils.

In the actual balancing coil, shown in figure 6, the windings are located in channels cut in a fiber tube which is secured to a head carrying the winding terminals and a bushing through which passes the threaded brass rod supporting the two cores. Below the head are small spool forms on which the shunts are wound. Insulating material such as bakelite is used to obtain proper spacing of the two cores.

The rather unusual manner in which the turns are applied is illustrated in figure 7, which is a close-up view of the two wires forming the inner winding. These two wires alternately cross over each other, progressing along the axis in opposite directions of rotation. The outer winding is similarly applied. This type of winding eliminates all splices within the coil, removing hazards incident to interior splices.

The complete coil assembly is enclosed by an aluminum container which serves the dual purpose of a shield and a convenient means of holding the coil for mounting purposes as this container fits snugly into an aluminum cup riveted to the assembly panel. The windings are dried and impregnated and the space between the coil assembly and container is filled with insulating compound.

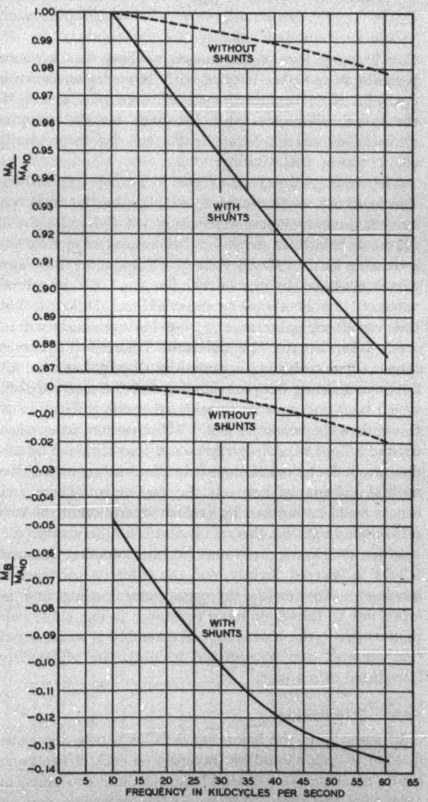

Figure 9. Variation of M_a and M_b components of trial balancing coil with frequency

The mutual inductance of a typical coil varies as shown in figure 8 as the cores are moved. The range, with the shunts disconnected, is approximately +1.6 to −1.6 microhenries, which is covered in about 16 turns of the screw (a total core travel of 0.5 inch). With the shunts connected, the effective mutual inductance at a given set-

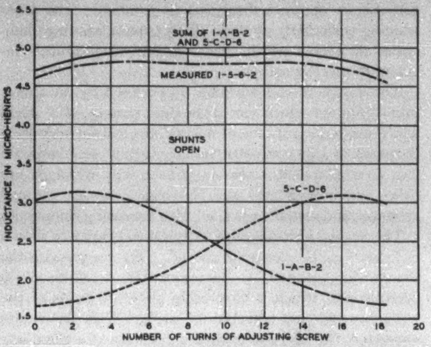

Figure 10. Series inductance of nonshunted trial balancing coil

ting becomes less as the frequency rises, the two components, M_a and M_b, varying with frequency as shown in figure 9. To determine the proper values of L_s and R_s for the shunt, allowance must be made for the complex mutual inductance inherent in the coil due to proximity effect within the windings.

The series inductance of the balancing coil without shunts varies as shown in figure 10. As the cores are moved, the inductances of windings 1-*A*-*B*-2 and 5-*C*-*D*-6 (figure 5) behave as shown by their respective curves, one increasing as the other decreases. The sum of these two curves is shown by the dotted line, and the measured value of 1-5-6-2 is shown by the solid line. It is seen that the over-all self-inductance of 1-5-6-2 is constant to within ±0.1 microhenry. The difference between the curves (about 0.1 microhenry) is caused by the slight mutual inductance existing between winding 1-*A*-*B*-2 and 5-*C*-*D*-6, which is negative owing to reversed winding direction in this side of the balancing coil. The measured inductance around 3-7-8-4 would slightly exceed that obtained by adding the inductances of the two sections owing to positive mutual inductance between the two ends. These end effects could be reduced by greater separation of the two sets of windings, but this refinement is not necessary.

When the shunts are connected, the inductance around 1-5-6-2 is lowered slightly, and the effective resistance is increased. To simplify the capacitance loading and in order not to introduce more resistance in one cable pair than another, the balancing coil assembly is so arranged that shunted and nonshunted windings are alternately introduced into a pair.

BALANCING PANELS

In assembling the balancing coils on panels, the same number of coils should be traversed on each of two pairs before reaching the coil that balances these two pairs, in order that the phase shift up to this balancing coil on one pair will be essentially the same as that on the other pair. If these phase shifts differed materially, the coil setting for minimum crosstalk when one pair is the disturbing circuit might be quite different from the best setting when the other pair is the disturbing circuit. To obtain this equality objective a "criss cross" arrangement, as shown schematically on figure 11, was devised, whereby the number of coils on one pair up to a particular balancing coil never differs by more than one from the number of coils on the other pair up to this same balancing coil.

For economic reasons it is undesirable to install a complete panel for the ultimate number of pairs, possibly 100 in some cases, but rather to install sections conforming more closely to the circuit growth. The placing at different times and properly connecting of sections obtained from the 100-pair criss-cross panel and at the same time maintaining service on operating circuits appeared rather formidable. This problem was solved by the use of two types of criss-cross panels; an intragroup panel for balancing within one group of carrier pairs and an intergroup panel for balancing pairs in one group against pairs in a second group of equal size. In the present design, an intragroup panel takes care of 20 pairs (190 combinations) and an intergroup panel of the 400 combinations between two 20-pair groups. To maintain phase equality through a number of panels, it is necessary to install them following a definite pattern. Figure 12 shows a suitable pattern for the 15 panels required for 100 pairs.

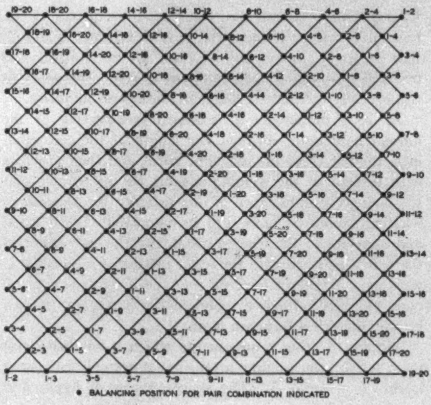

Figure 11. Schematic of criss-cross wiring for 20-pair balancing panel, designed to maintain phase equality of coils

In the criss-cross scheme (figure 11) the side-to-side combinations, which are those marked 1/2, 3/4, 5/6, etc., appear twice, i.e., along the left and right edges of the panel. Advantage of this is taken by installing balancing coils at both locations. This is done because one side-to-side coil of about 1.3 microhenries may not be large enough in all cases in spite of the fact that the mean side-to-side crosstalk has been reduced nine decibels by poling.

Balancing Procedure

As stated above, the far-end crosstalk in a repeater section cannot be balanced out completely over the frequency range with a single balancing unit. To determine the balanceable as distinct from the nonbalanceable crosstalk, involves crosstalk measurements in phase and magnitude at a number of frequencies, using each pair of a two-pair combination as a disturbing circuit in turn. The balanceable crosstalk may then be separated from the nonbalanceable crosstalk by computation. Balancing by this method would be impracticable because of the time required. As a practical scheme, it has been shown that balancing at a frequency of about 40 kilocycles will produce satisfactory results over the type-*K* range even though part of the nonbalanceable crosstalk may be neutralized at this frequency. This is theoretically undesirable since the crosstalk reduction at other frequencies is impaired.

To prevent undue interference into operating carrier circuits when balancing, a frequency falling between the transmitted bands must be used. For this reason, the balancing coils are adjusted at a test frequency of 39.85 kilocycles and a measurement to check the suitability of the adjustment is made at 28.15 kilocycles. Figure 13 shows the crosstalk versus frequency before and after coil balancing by this method on three repeater sections.

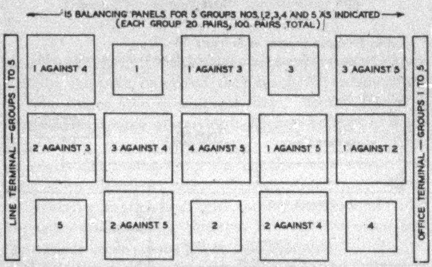
15 BALANCING PANELS FOR 5 GROUPS NOS.1,2,3,4 AND 5 AS INDICATED
(EACH GROUP 20 PAIRS, 100 PAIRS TOTAL)
LINE TERMINAL — GROUPS 1 TO 5
1 AGAINST 4
1
1 AGAINST 3
3
3 AGAINST 5
2 AGAINST 3
3 AGAINST 4
4 AGAINST 5
1 AGAINST 5
1 AGAINST 2
5
2 AGAINST 5
2
2 AGAINST 4
4
OFFICE TERMINAL — GROUPS 1 TO 5

Figure 12. Allocation of balancing panels designed to maintain phase equalization of coils at all stages. Panels with suitable cross-connections between them are installed in following order

For first group—Install 1
Add second group—Add 2, and 1 against 2
Add third group—Add 3, 1 against 3, and 2 against 3
Etc.

Additional Crosstalk Remedial Measures

Although poling as well as balancing is done to reduce side-to-side crosstalk, this crosstalk is still considerably greater than the pair-to-pair crosstalk. For this reason, side-to-side crosstalk is diluted among the pair-to-pair combinations by a system of quad splitting at repeater points.

The crosstalk after balancing (figure 13) is considerably higher at the upper end of the frequency band than at the lower end. Consequently, if circuits were set up to use the same channel throughout, the crosstalk in the upper-frequency channels would be materially greater than that in the lower-frequency channels. In order that all circuits may be equally satisfactory from the crosstalk standpoint, a system of special channel assignments in successive intervals, say 500 to 1,000 miles, can be used. This will tend to equalize both the crosstalk and the noise on all circuits, thus permitting a somewhat cheaper design than if each channel had to meet the crosstalk and noise limits by itself.

Noise

Besides babble (noted above) many other sources of noise need to be considered in cable carrier design. Figure 14, which shows the approximate magnitude of several of these if no means are taken to suppress them, indicates the noise at the end of a single 17-mile repeater section when amplified by a repeater whose gain equals the hot-weather line loss. Curve *A* shows the unavoidable lower limit of noise, that produced by thermal agitation of the electrons in the cable conductors and the repeater.[2] This amounts to about 2×10^{-17} watts per telephone channel per repeater section, at the repeater input. If there were no other noise sources, the repeater section length would necessarily be limited by this effect. Curve *B* shows the sum of thermal noise and noise due to the vacuum tubes in the repeaters, which is little in excess of thermal noise alone. The other three curves show noises of considerably higher magnitude which require suppression in order to arrive at an economical carrier system. Curve *E* shows the order of magnitude of noise on carrier circuits due to connecting open-wire pairs directly to noncarrier pairs in the outside cable near the carrier repeater input. The source of the noise is heavy atmospheric static of a magnitude experienced several times during the summer. The other curves show typical magnitudes of noise originating in the existing telegraph and voice frequency telephone plant; this is generated in existing repeater stations and transmitted by the noncarrier pairs to the outside cable where it is induced into the carrier pairs. Curve *D* rep-

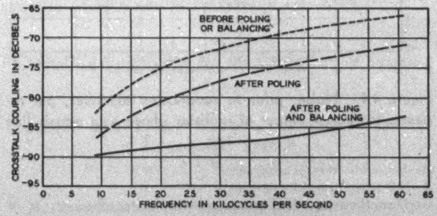

Figure 13. Root-mean-square far-end crosstalk per repeater section from measurements on three repeater sections

resents the situation at a combined telephone and telegraph repeater station, and curve *C*, the situation at a station where there are no telegraph repeaters.

Figure 15 indicates the results after suppression measures have been applied. As shown, at the top frequency, which controls the carrier repeater section length, these sources of noise have been reduced to be well below thermal plus tube noise. It is also shown that the noise due to heavy atmospheric static induced directly into a carrier pair in the outside cable is below thermal plus tube noise at the top frequency.

There are additional types of noise, not shown, whose sources lie within the carrier system: e.g., modulation in amplifiers, intersystem cross-induction, battery noise. While control of such noise is an integral part of the fundamental carrier system design, it is not the purpose of this paper to cover this class of noise.

CONDUCTORS TAPPING THE CARRIER CABLE

Carrier noise may come from open-wire pairs which connect to conductors in the cable. Its sources may be static; corona on power lines; power-line carrier or other carrier-frequency voltages on power lines paralleling the open wire; induction from radiotelegraph stations; or carrier frequency voltages arising in the office to which the open wire is connected, such as voltages generated by d-c telegraph or telephone signaling systems. The limited experience to date indicates that, in a long cable carrier system, the effect of heavy static will be larger than that of the other sources if telephone and power supply plants are co-ordinated so as to be satisfactory from voice frequency and low frequency standpoints. Branch cables connected to the carrier cable have a similar but generally smaller effect than that of open-wire taps.

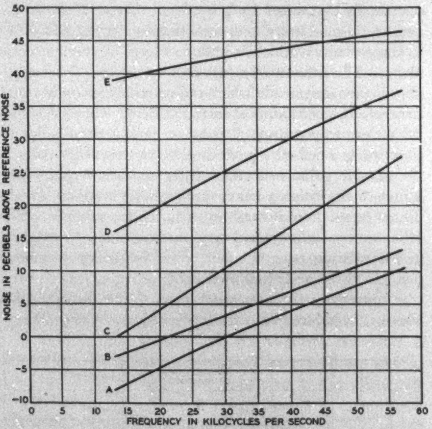

Figure 14. Noise, prior to suppression measures, per repeater section at output of repeater whose gain equals line loss

A—Noise from thermal agitation
B—Thermal agitation plus tube noise
C—Noise from voice frequency telephone repeater office
D—Noise from telephone and telegraph repeater office
E—Noise from heavy static on open-wire tap close to carrier repeater input

Figure 16 illustrates the path followed by this induction. A voltage to ground impressed on the open-wire pairs passes by secondary induction over to the carrier pairs in the cable, and, on account of the unbalance to ground of these pairs, produces a metallic voltage on these pairs at the repeater input. The effect may be greatly reduced by interposing a filter at the junction of the open wire and the cable.

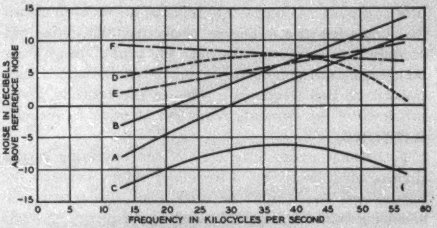

Figure 15. Noise, subsequent to suppression measures, per repeater section at output of repeater whose gain equals line loss

A to *E*—Same sources as in figure 14
F—Noise from heavy static induced directly into outside cable

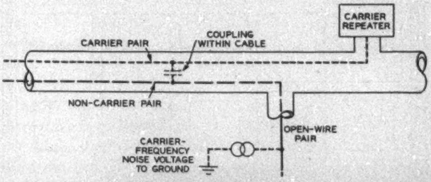

Figure 16. Schematic of path followed by induction from open-wire taps

It is necessary to filter only the longitudinal circuit at an open-wire tap, because; (1) the voltage to ground on the open wire is much larger than the metallic circuit voltage, and (2) the coupling between the longitudinal circuit and the disturbed carrier pair is greater than the coupling between metallic circuits.

Figure 17 is a schematic diagram of the longitudinal filter developed for a phantom group. It consists of two longitudinal retardation coils and a set of condensers connected between the line wires and the cable sheath. This filter has relatively high carrier frequency longitudinal impedance to minimize effects of impedance in the ground connection. The major portion of the carrier frequency impedance of the coils is obtained by designing them to have high core loss at these frequencies. The filter has little effect on voice frequency transmission, precaution having been taken to hold the transmission loss, crosstalk and unbalance to ground to low values.

NOISE ARISING IN EXISTING REPEATER OFFICES

The noise caused by carrier frequency voltages generated in existing repeater offices is due to d-c telegraph, telephone speech and signaling voltages, power supply, etc. Figure 1 shows the path by which they reach the carrier plant and the means used to suppress them. In this figure, *N* represents a source of carrier-frequency voltage in a repeater office, connected to a voice frequency pair which transmits this voltage into the outside cable where it is induced on the carrier pairs. These voltages are reduced by inserting suppression coils in the longitudinal voice frequency paths at the junction between the office and the outside cable connected to carrier inputs.

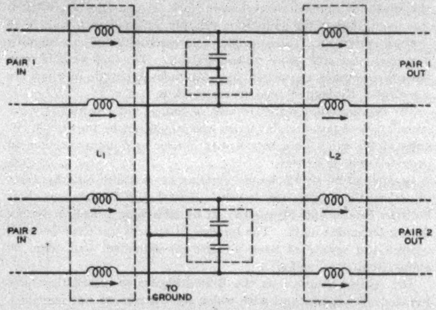

Figure 17. Schematic of longitudinal filter

The design of coils giving the requisite carrier-frequency suppression without appreciably affecting voice-frequency transmission on the circuits in which they are connected was difficult. One coil is used for each phantom group. Each coil has sixteen windings, four for each line wire. These windings are so paired and disposed about the core as to make possible very small side-to-side and phantom-to-side crosstalk between line windings. They also permit obtaining very small leakage flux in both the sides and the phantoms; hence the coils introduce very small transmission loss in their voice-frequency circuits. The leakage impedance of the coils plus the impedance of the cable stub used to connect them into the circuit is held down so that the effect on repeater singing and echoes in the voice circuits is very small. The coils are so wound that their longitudinal inductance is in antiresonance with their distributed longitudinal capacitance at approximately the top cable carrier frequency, resulting in a large increase in their suppression in this critical frequency range. The longitudinal impedance of one of these coils, and the approximate suppression which a set of them provides, are shown in figure 18.

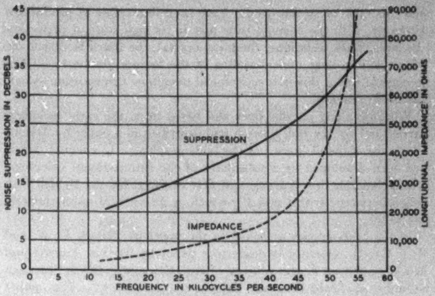

Figure 18. Longitudinal impedance and suppression of noise suppression coils

In addition, the carrier circuits are carefully separated, electrically and physically, from existing voice-frequency circuits in common repeater stations. To this end the carrier pairs in the outside cable are brought out on the line side of the noise suppression coils into a separate cable connected directly to a sealed terminal. From this terminal they are carried in shielded wire to the units in the carrier office and then to a similar sealed terminal leading to the outside cable in the opposite direction. Filters for filament and plate battery supply are included in the carrier amplifiers and additional filament battery supply filters are provided at the carrier fuse panels.

References

1. CABLE CROSSTALK—EFFECT OF NON-UNIFORM CURRENT DISTRIBUTION IN THE WIRES, R. N. Hunter and R. P. Booth. *Bell System Technical Journal*, April 1935.

2. J. B. Johnson, *Phys. Rev.*, 32, 97 (1928); H. Nyquist, *Phys. Rev.*, 32, 110 (1928).

Discussion

A Carrier Telephone System for Toll Cables

C. W. Green and E. I. Green

Cable Carrier-Telephone Terminals

R. W. Chesnut, L. M. Ilgenfritz, and A. Kenner

Crystal Channel Filters for the Cable Carrier System

C. E. Lane

Crosstalk and Noise Features of Cable Carrier-Telephone System

M. A. Weaver, R. S. Tucker, and P. S. Darnell

W. H. Capen (International Telephone and Telegraph Corporation, New York, N. Y.): The authors of this group of papers on carrier-cable developments should be highly complimented on the excellent presentation of this most interesting subject. Not many years ago this type of system would have been impossible of production. However, with the vacuum tube now in its high state of perfection, the latest filter technique, the piezo crystal, the Black feedback amplifier, and a clear understanding of the factors involved and how to control them, this new economical telephone transmission system has been produced.

Working along parallel lines and based upon the early technique carried out by the Bell System and described in a paper by Messrs. Clark and Kendall (item 1 of the bibliography), a 12-channel system has been developed by a subsidiary of the International Telephone and Telegraph Corporation, and first installed for the British Post Office between Bristol and Plymouth, a distance of approximately 123 miles.

In the bibliography attached to the paper by Messrs. C. W. and E. I. Green, reference is made to a paper by Messrs. Angwin and Mack, published in the *Journal* of the IEE, and entitled, "Modern Systems of Multi-Channel Telephony on Cables." This paper includes a description of the English 12-channel system. As there are some differences between the British system and that described in the papers under discussion, a brief review of the Bristol-Plymouth system may be of interest. The system makes use of a frequency band extending from approximately 12 to 60 kilocycles with a carrier spacing of four kilocycles. Only the lower sideband is transmitted.

Separate cables are used for the "go" and "return" circuits and these were designed and installed especially for the application of the 12-channel system. Each cable is a paired cable; i.e., pairs were not twisted together in groups of two to form quads as is usual in voice-frequency cables. The chief advantages of this feature are that the higher couplings between pairs within a quad are avoided and that the effective resistance at 60 kilocycles for a given capacitance is less than in a quadded cable. However, future cables will probably be mainly of the star quad type because of the better space factor. Each cable comprises 19 40-pound pairs, which was the maximum number that would permit of the two cables (with special protection) being drawn into the normal three-inch duct lines. The outside diameter of each cable is 1.1 inches. Each pair was made with a different length of twist to minimize mutual impedance between pairs. To minimize increased resistance due to eddy currents induced in the sheath by pairs near the sheath, the lead sheath was spaced from the outer layer by a thick layer of insulation.

At 60 kilocycles the contribution of skin and proximity effects to the effective resistance of pairs in telephone cables tends to become the dominating factor when the wires are increased in diameter, and it was found that little advantage would be gained by increasing the gauge beyond 40 pounds per mile, which was decided upon. The maximum spacing of repeaters was fixed at 22 miles, with an average of 18 miles, corresponding to an attenuation of somewhat less than 60 decibels per repeater section.

In addition to the special design of the cable, it was, of course, necessary to balance the various pairs against each other in order to reduce crosstalk. It was found, however, that sufficient reduction could be obtained by the use of condensers alone. These were installed at the midpoint of each repeater section. By means of these condensers the average far end crosstalk was reduced from about 72.5 decibels to 85.5 decibels, while the maximum crosstalk was reduced from 63 decibels to 75 decibels.

The filters are of conventional design electrically, consisting of inductance coils and condensers. In view of the higher frequencies involved, the filter sections must be made more accurately than formerly, and so each arm of a filter comprising a coil and condenser in series (or in shunt) has been made into a unit and resonated to a predetermined frequency by means of a small variable condenser, before being assembled on the filter mounting plate. In this way the accuracy of the filters has been improved so that the characteristics of the highest channels are similar to those formerly obtained at frequencies not higher than 30 kilocycles.

The amplifiers follow closely the principles described by H. S. Black of the Bell Telephone Laboratories. A three-stage feedback amplifier is used, the feedback circuit being connected between the output of the last stage and the input of the first stage. A maximum over-all gain of 65 decibels can be obtained.

The over-all line characteristics taken at carrier frequencies on the whole repeatered circuit from Bristol to Plymouth showed a total variation, down to a frequency of 20 kilocycles of less than one decibel. The total variation of output level at any repeater station was found not to exceed two decibels over the range required for the 12 channels, and the over-all equalization was such that the effect on the frequency characteristic of any one channel was almost negligible.

In order to make some estimate of the effect of connecting systems in tandem, arrangements were made on one of the Bristol-Plymouth systems to loop back the channels on the two-wire sides. Using three channels in tandem, the quality as judged by ear was no different from that obtained on the normal circuit between Bristol and Plymouth. With six channels in series, the quality was still good but some slight deterioration could be detected. As a final test the whole 12 channels were looped back on one another, when it was found that, if the gains were suitably adjusted so as to give an over-all equivalent of three decibels, the speech, although considerably degraded, was still easily understandable. This last case is equivalent to a circuit approximately 1,500 miles in length, in which the signals pass through 96 repeaters in tandem.

The degradation referred to was, of course, due chiefly to the accumulation of terminal distortion and would not be present on circuits of the same total length with carrier modulating equipment provided only at its ends.

Equipment for six 12-channel systems was installed initially, three of the systems operating between Bristol and Exeter and three between Bristol and Plymouth; 31 circuits were placed in service during December 1936. The number of circuits has since been increased and several of these circuits are extended, four wire, on audio circuits to London.

The results obtained on the Bristol-Plymouth installation have justified the expectations with which the experiment was launched, and in the planning of the trunk network of Great Britain 12-channel carrier has already taken its place as the premier means of providing additional long circuits.

Plans are now actively under way for installing this type of carrier system on nearly all of the main routes from Plymouth in the south, to Aberdeen and Inverness in North Scotland. A total of nearly 17,000 two-way channel miles of this type of system are now in operation. Several thousand additional two-way channel miles are being installed, and further systems ordered.

Glen Ireland (American Telephone and Telegraph Company, New York, N. Y.): We are greatly indebted to the authors of this group of papers for this story about the development of the cable-carrier system.

The first commercial installation of the cable-carrier system in this country will be placed in permanent service between Detroit, Mich., and South Bend, Ind., a distance of about 200 miles, by the spring of this year. Other projects will follow and by the end of this year, cable carrier probably will have been applied along 830 miles of toll-cable route. Along about 560 miles of the route, the cable carrier will utilize conductors in two existing cables of the type previously placed for voice-frequency use. For the remaining part of the distance where only one such existing cable is available, a small paralleling cable of a special design is being provided. In connection with the engineering of these routes, it has been necessary to select the pairs most suitable for unloading and carrier usage, to survey the cables throughout to make sure that they were suitable in all respects and that necessary noise filters were provided for the open-wire and cable taps. Sites for 34 auxiliary repeater stations have been selected and suitable buildings of the type described in the Green and Green paper either have been or soon will be constructed. To the extent practicable, standardized equipment layouts are being provided in both the main and auxiliary repeater stations.

Cable carrier systems in the future are expected to find their principal field of use along the present long toll cable routes, where conductors in existing cables may be utilized and where there will be a demand for large numbers of additional circuits over 80–100 miles in length. A survey of the present toll cable network indicates that there are some 2,900 miles of route having at least two cables and 6,500 miles of single-cable route that might be considered as potential candidates for the application of some form of cable carrier. Among the more important routes having at least two existing cables are New York-Chicago, Boston-Washington, New York-Buffalo, and Pittsburg-Terre Haute, Ind. The application of cable carrier to the existing toll cable network will be greatly aided by the fact that through previous standardization toll cable of known characteristics and quality and of a given gauge is available throughout the network. This is particularly important in connection with the crosstalk balancing work and the equalization of the high-frequency line.

The general introduction of cable carrier should result in marked improvement in the telephone toll service. Its availability, together with that of other broad-band carrier systems, has been one of the principal factors which makes it practicable for the Bell System to consider the introduction of improved transmission standards for the longer toll circuits. The greater velocity of transmission provided by the nonloaded cable conductors and the resulting reduction in delay effects are important factors in making this possible. The wider band of frequencies which are effectively transmitted by the cable carrier is also an important factor in improving transmission, this latter factor contributing materially to the naturalness and ease of conversation. The cable carrier systems also are expected to be very quiet. This is due in part to careful design and in part to the relative freedom of the high-frequency cable carrier circuits from the interfering effects of neighboring power systems.

Some of the maintenance arrangements mentioned in the paper are of considerable interest to the operating people. In particular the arrangement where amplifiers or a spare circuit link may be quickly substituted with little or no interruption of service is expected to be of considerable value to the field maintenance forces.

E. I. Green (for all authors): The authors have noted with interest the comments on the papers, particularly the discussion by Mr. Capen of the application of cable carrier between Bristol and Plymouth and elsewhere in Great Britain. As Mr. Capen has indicated, the British system follows to a considerable extent along the lines of the experimental system installed at Morristown, N. J., as described to the Institute in 1933, and differs in a number of respects from the present type-*K* system. The British system is applied to new cable installed underground. This cable is of special design and is nonquadded with 16-gauge conductors. The type-*K* system, on the other hand, is applied to existing cables installed either aerially or underground, and employs ordinary 19-gauge quadded conductors. The use of existing toll cables for the type-*K* system greatly augmented the problems of crosstalk and transmission regulation which had to be solved.

Harmonic-Current-Restrained Relays for Differential Protection

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DIFFERENTIAL relaying is the commonly accepted means of protecting large power transformers, a-c generators, and station bus systems against internal faults.[1] The methods and circuits employed in various instances, while differing in detail, are all fundamentally the same in basic principle. Briefly speaking, this principle consists in continuously comparing in each phase the current entering the protected equipment with that leaving. As illustrated in the generalized single-line diagram figure 1, the comparison is made by means of current transformers of suitable ratios placed in all the power circuits connecting with the protected equipment. If current transformation takes place within the equipment as in the case of a power transformer, the ratios of the current transformers placed in the various circuits are chosen to have relative values corresponding to the transformation ratios so that the current transformer secondary currents may be compared on a 1:1 basis. The secondary windings of all the current transformers in each phase are connected in parallel with each other to a special kind of current relay usually called a "differential" relay. The current transformers are all connected in the same polarity with respect to the direction of the protected zone so that currents entering and leaving the protected zone will be represented in the secondary circuit by currents of opposite polarities. Normally, with sound equipment these positive and negative components will be equal, except for negligible exciting currents, and their algebraic sum which by the connection is applied to the relay coil will be essentially zero. When a fault occurs within the protected zone, however, the balance is upset and a difference current proportional to the fault current flows in the coil of the relay, causing it to operate and trip circuit breakers in all the connecting circuits, removing the faulted equipment from service.

It is important that the faulted equipment be isolated as quickly as possible after the fault occurs, not only to limit damage to the equipment but also to minimize the length of time that the voltage is depressed. A prolonged period of low voltage is likely to result in loss of synchronism between rotating machines. When this occurs the excessive current drawn by the machines out of step often causes other protective relays to operate, tripping out circuit breakers until the power system is split apart and a major interruption of service results. A prolonged period of low voltage also results in loss of important loads due to operation of undervoltage devices. It is just as important that loaded equipment should not be needlessly cut out of service due to false operation of the relays when no internal fault is present. Unbalanced currents may sometimes flow in differential relay circuits when no internal fault is present due to causes which will be discussed below. These will cause false operation of the differential relay unless means are used to prevent it. The means hitherto employed and their shortcomings will be discussed, and a new method, free of these shortcomings, and suitable for general application will be described.

Causes of False Differential Currents

A. Departure of Current Transformer Ratios From Their Normal Values

When heavy current, due perhaps to an external fault, is drawn through the equipment protected by differential relaying, the current transformers become saturated and their ratios depart from their normal values by an amount depending on their designs, the amounts and power factors of their secondary burdens and the magnitude of the fault current. With full rated burden connected to the current transformer secondary and with currents of 20 times normal load value passing through the primary the ratio change may be so serious that only half the current calculated from the normal ratio flows in the secondary circuit. With lighter secondary burdens or lower values of primary current the departure is less severe but still considerable. In the special case where only two current transformers are involved, if the characteristics are matched, the secondary current sum will still be zero and no difference current will flow in the relay. With more than two current transformers, however, differences in the current-transformer characteristics or unequal distribution of the primary currents between the several current transformers will cause a difference current to appear which tends to operate the relay.

The usual method of preventing relay operation on differential currents due to current transformer saturation has been to use what is known as a "percentage differential relay." This type of relay compares the differential current with the through current and requires that the differential current exceed a certain percentage of the through current before the relay operates. This is accomplished by connecting a restraining coil in the relay in series with each of the current transformer secondaries. This type relay has been widely and successfully used on a-c generators and on two- and three-winding transformers.[2] Where only two circuits are involved, as in the case of the generator and the two winding transformer, the relative directions of the currents in the two current

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L. F. Kennedy is employed in the relay engineering section of the General Electric Company, Philadelphia, Pa., and C. D. Hayward is development engineer for the same company.

[1] For all numbered references, see list at end of paper.

transformer secondaries is always the same when the equipment is sound, so both restraining coils may be placed on a single restraint magnet acting on the armature. In the general three-winding transformer where the relative direction of current flow in the three windings may be different under different conditions, it is necessary that the three restraint coils be placed on three separate magnets acting on the armature separately. Where more than three windings are involved or similarly in the case of bus systems where there are many circuits, it has not been found practical to use this type of relay because of the complications involved in building a relay with so many restraint magnets.

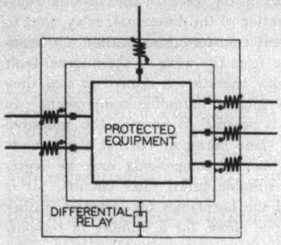


Figure 1. Fundamental circuit for differential relay protection

B. Magnetizing Inrush Current in Power Transformers

It is a well-known fact that when a switch is closed, suddenly applying potential to a winding of a previously de-energized transformer, the transient magnetizing current which flows may for a time reach very high values depending upon the amount of residual magnetization left in the transformer core from former operation and upon the point of the voltage wave at which the switch is closed.[3] In comparison with normal steady-state magnetizing current which for large power transformers is in the neighborhood of one per cent of full-load current, this transient magnetizing inrush current often reaches values of eight to ten times full load current (800 to 1,000 times normal magnetizing current). This magnetizing current flows in the transformer primary winding only and hence causes a corresponding difference current to flow in the differential relay.

The tendency of differential relays to operate on magnetizing inrush currents in power transformers has been resisted in the past either by using a slow acting relay, or by desensitizing the relay (or rendering it altogether inoperative) during a definite time following the application of potential, thus enabling it to ride over the magnetizing inrush period. This latter means has been accomplished by the use of a time-delay voltage operated auxiliary relay energized from a potential transformer connected across the power transformer winding and having contacts connected to remove a shunt around the operating coil (or to close a trip circuit contact in series with the differential relay contacts). Both these methods are unsatisfactory when judged by the standards of modern protective relay practice, the first because of its slowness on internal faults and the second because of the length of time that reduced protection is given the equipment. Cases are often encountered, particularly in large 25-cycle transformer installations where the inrush transient period may last for several seconds. Considerable damage can be done both to the transformer and to the system stability if a fault already present in the transformer when potential is applied is allowed to remain on for this length of time. Another objection lies in the failure of this method to prevent operation of the differential relay in cases occasionally encountered where magnetizing inrush currents appear when the transformer voltage, after being momentarily depressed by a close external fault, suddenly rises again as the fault is cleared.

C. Magnetizing Inrush Current in Current Transformers

Magnetizing inrush currents of considerable magnitude may also appear in current transformers. Heavy currents suddenly drawn through current transformers previously carrying little or no current cause magnetizing inrush currents to flow in much the same way as the sudden application of potential does in the case of power transformers. The magnitude of the inrush current depends upon the design of the current transformer, the magnitude of the fault current and the amount of secondary burden as well as the point of the wave at which the heavy current begins to flow and the amount of flux present in the core at that time. This inrush current flows in the primary of the current transformer only. The secondary current appears as a transformation of the primary current with the inrush current subtracted. In the differential relay connection the several current transformers will in general draw varying amounts of inrush current on through faults so that a difference current due to this cause will often flow in the relay.

Figure 2. Differential current due to an internal fault

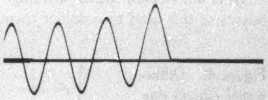
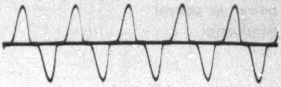


Figure 3. Differential current due to saturation of current transformers on heavy through fault current

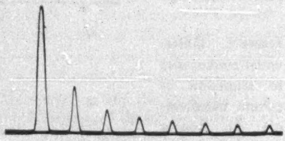


Where the percentage differential relay may be used, as in the case of a-c generators and two- or three-winding transformers, the chance of having false relay operation due to current transformer magnetizing inrush currents is reduced by the use of a liberal percentage of through current restraint. This is not altogether satisfactory, however, since it reduces the sensitivity of the relay to internal faults. In applications where the use of a percentage differential relay is impractical, as in the case of many winding transformers or busses involving many circuits, the only recourse has been to the use of relays set for high pick-up current or to operate slowly enough to ride over the inrush period. This system of course leaves much to be desired.

Harmonic Current Restraint

A new method of preventing the false operation of differential relays on unbalanced currents due to any or all the causes described above has recently been applied. We have called it the principle of harmonic current restraint. It takes advantage of the difference in wave form between the differential current caused by a true internal fault and that caused by current transformer saturation or magnetizing inrush currents to restrain the relay from operating except when a fault exists within the protected zone. Oscillograms illustrating typical wave forms of these various currents are shown in figures 2, 3, and 4.

Figure 4. Differential current due to magnetizing inrush to either power or current transformer



It is convenient when working with non-sine current waves of this kind to consider them as being composed of an infinite series of sine wave components of ascending order of frequencies according to the Fourier expansion:

$$I(t) = I_0 + I_1 \sin(\theta_1 + wt) + I_2 \sin(\theta_2 + 2wt) + I_3 \sin(\theta_3 + 3wt) + \ldots.$$

where

$I(t)$ = the given non-sine wave current, varying as a function of time.
I_0 = the constant or direct-current component.
$I_1, I_2, I_3, \ldots$ = the maximum values of the fundamental, second harmonic, third harmonic, etc., sine-wave current components.
$\theta_1, \theta_2, \theta_3, \ldots$ = the phase angle position of the various component waves at the time $t = 0$.
w = $2\pi f$.
f = frequency of fundamental in cycles per second. For the waves we are considering this will be the power line frequency.

A number of methods for evaluating the magnitude coefficients $I_0, I_1, I_2, I_3, \ldots$, etc., as well as the angle coefficients $\theta_1, \theta_2, \theta_3, \ldots$, are available. Some of these employ graphical means,[4] others mechanical.[5]

Values of the magnitude coefficients computed for the first cycle of each of the waves shown in figures 2, 3, and 4 are given in table I. These are expressed in per cent of the magnitude of the fundamental frequency component. For comparison the crest value of each of the waves in per cent of the crest value of its fundamental component is also given.

While it is understood, of course, that these percentages will vary when the conditions under which the currents appear vary, still for the purpose of our present discussion they may be regarded as typical. The wave due to an internal fault is shown having a rather large displacement, due to the point of the wave at which the fault was initiated. With a symmetrical wave the analysis would show 100 per cent fundamental with zero direct current and negligible harmonic current components.

It will be observed that the waves of the currents which cause undesired operation of the differential relay, that is, currents due to current transformer saturation and magnetizing inrush, differ from the wave of the internal fault current for which relay operation is desired in that they contain considerably greater harmonic components. In a practical relay it is possible to prevent the undesired operations by using these harmonic current components for restraint. The various components may be segregated by means of suitable tuned filter circuits. It is preferable to use all the harmonics for restraint, rather than to select any one alone, for a number of reasons. First of all, it will be observed from Table I that in the two different types of waves for which restraint is desired, different harmonics predominate. Hence, a relay designed to restrain on the second harmonic component only, for example, while suitable for magnetizing inrush currents, would not be properly restrained on currents obtained from saturated current transformers. On the other hand, if the relay were made to restrain on third harmonic only, proper restraint would be obtained on saturated current transformer currents but not on magnetizing inrush. The use of the d-c component alone would not be satisfactory because this component is absent in the saturated current transformer current and also because this component may be lost even on magnetizing inrush waves due to cancellation between phases when the current transformers in a wye-delta power transformer are connected delta-wye, as is the usual practice. Fur-

Table I. Harmonic Wave Analyses of Typical Currents Appearing in Differential Relay Circuits Due to Various Causes

Wave Component		Differential Current Due to: Internal Fault Figure 2	Saturated C. T.'s Figure 3	Magnetizing Inrush Figure 4
I_1	Fundamental	100 per cent	100 per cent	100 per cent
I_0	Direct current	38 per cent	0 per cent	58 per cent
I_2	Second harmonic	9 per cent*	4 per cent	63 per cent
I_3	Third harmonic	4 per cent*	32 per cent	22 per cent
I_4	Fourth harmonic	7 per cent*	9 per cent	5 per cent
I_5	Fifth harmonic	4 per cent*	2 per cent	34 per cent
I_6	Sixth harmonic	6 per cent*	1 per cent	4 per cent
I_7	Seventh harmonic	2 per cent*	3 per cent	3 per cent
	Wave Crest	145 per cent	126 per cent	244 per cent

* NOTE: It may seem surprising to those used to thinking of an offset fault current wave like figure 2 as being composed only of fundamental frequency plus an exponentially decaying direct current to see small quantities of the harmonics listed. This is because the d-c component is assumed by the analysis to have a constant value. The harmonics represent the exponential variation of the d-c component from the beginning to the end of the cycle. Expressing the analysis in this manner is proper because it is the way that the wave appears to the tuned filter circuits of the relay.

thermore, the use of the d-c component alone would result in a relay slow to operate on internal faults where the current wave is appreciably offset.

Practical Application—A Bus Differential Relay

A practical relay with harmonic current restraint, intended primarily for bus differential service is shown in figure 5. The operating characteristics of this relay are as follows:

Pick-up = 5 amperes on sine-wave 60-cycle current.
Operating time = one cycle average (60-cycle base) at twice pick-up or higher.
Burden = 1 volt-ampere at 5 amperes.

This relay was designed with the relatively high 5-ampere pick-up in order to avoid operation of the relay on normal load current in case one of the current transformers in the differential connection should inadvertently have its secondary open circuited. Bus faults invariably draw high enough currents to make extreme sensitivity unnecessary. The internal circuit is shown in figure 6. In this circuit the differential current supplied to the relay is first stepped down by means of a small internal current transformer to a value readily accommodated by standard ratings of the capacitors and rectifier used in the relay.

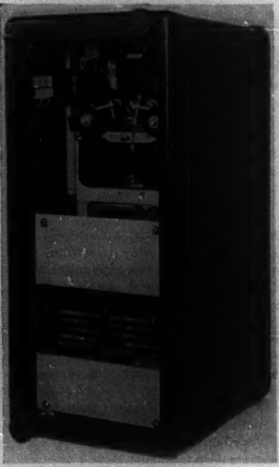


Figure 5. Bus differential relay with harmonic current restraint

Across the secondary of this transformer two parallel circuits are connected, one including the operating coil of the relay and the other the restraining coil. The operating coil circuit branch contains a wave trap made up of a capacitor C_1 and a reactor L_1 in parallel. This trap is tuned to block the predominant harmonic frequency, while the over-all circuit including this trap, the operating coil of the relay and the reactor L_3, is tuned by the capacitor C_3 to have minimum impedance to fundamental frequency. The restraining coil circuit branch has a similar wave trap made up of capacitor C_2 and reactor L_2 in parallel which is tuned to block fundamental frequency. The over-all circuit is untuned. The operating coil circuit branch therefore readily passes current of the fundamental frequency but blocks currents of the predominant and to a large extent the other harmonic frequencies. The restraining-coil circuit branch passes the harmonic frequencies with moderate impedance but blocks the fundamental. Sustained direct current is prevented from flowing in the relay coil circuits by the relay current transformer. Transient current surges may flow in the secondary circuits, however, due to the sudden application of the d-c component when the differential current starts. These current surges have comparatively steep wave fronts and hence pass through capacitors with little impedance. These surges divide between the operating and restraining coil circuit branches essentially in inverse proportion to the relative inductance of the two branches with all capacitors considered as being short circuited. Tests demonstrated that the addition of the reactor L_3 in the operating coil circuit prevents momentary operation of the relay due to these surges on magnetizing inrush currents. The restraint current, having an alternating wave of different frequency from that of the operating current, would, if unrectified, pass through zero at different times than the operating current. At these times the restraining pull would be released and the relay armature would tend to move by momentary flicks in the operating direction. Tests showed these flicks were often of sufficient magnitude to cause contact operation. The use of the rectifier aided by the inductance of the restraint coils was found to be effective in smoothing the restraint current and hence the restraint pull sufficiently to correct this trouble.

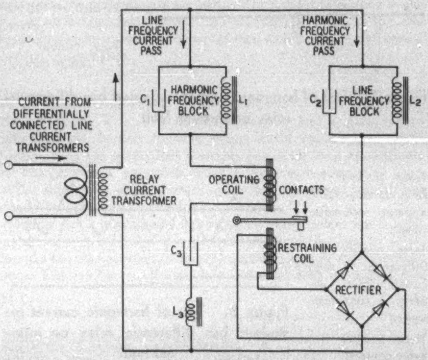

LINE FREQUENCY CURRENT PASS
HARMONIC FREQUENCY CURRENT PASS
C1
HARMONIC FREQUENCY BLOCK
L1
C2
LINE FREQUENCY BLOCK
L2
CURRENT FROM DIFFERENTIALLY CONNECTED LINE CURRENT TRANSFORMERS
RELAY CURRENT TRANSFORMER
OPERATING COIL
CONTACTS
C3
RESTRAINING COIL
RECTIFIER
L3

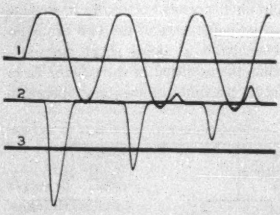
Figure 6. Circuit of differential relay with harmonic current restraint

A differential current due to an internal fault, like figure 2, flows principally in the operating coil and hence causes high speed operation of the relay. A differential current produced by current transformer saturation, like figure 3, or by magnetizing inrush, like figure 4, on the other hand will flow partly in the operating coil and partly in the restraining coil. The restraint magnet air gap is shorter than that of the operating magnet so that the restraint pull predominates and the relay does not operate.

Tests

The relay shown in figure 5 has been thoroughly tested with successful results, under conditions closely simulating those found in actual bus differential relay installations. Two typical oscillograms are shown in figures 7 and 8.

The oscillogram in figure 7 was obtained on a setup representing a through fault. A current of 10,000 amperes root-mean-square was carried into a bus through two lines in parallel and out through a single line. Bushing-type current transformers of 600/5-ampere ratio, one in each of the three lines, were differentially connected to the harmonic current restrained relay. A resistance of three ohms was placed in series with the secondary of the current transformer in the single outgoing line. No burden other than that imposed by the relay and short connecting leads was placed on the other two current transformers. Trace number 1 on the oscillogram is the fault current derived from a shunt, trace number 2 the differential current flowing through the relay, and trace number 3 is of the current in the relay contact circuit. Even though the crest value of the first current loop shown on trace number 2 is 116 amperes, no relay operation occurred.

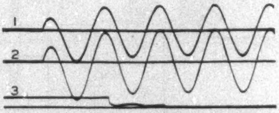


Trace number 1—Fault current; 10,000 amperes root-mean-square

Trace number 2—Differential relay current. Crest value of first peak = 116 amperes.

Trace number 3—Relay contacts

Figure 7. Test of harmonic current restrained bus differential relay on through fault



Trace number 1—Fault current; 7,200 amperes root-mean-square

Trace number 2—Differential relay current; 59.5 amperes root-mean-square

Trace number 3—Relay contacts

Figure 8. Test of harmonic current restrained bus differential relay on internal fault

The oscillogram in figure 8 was obtained on a set up representing an internal fault on the bus. This setup was similar to the one described above except that the outgoing current flowed through a line connected directly to the bus instead of passing through the third current transformer. The fault current as shown by trace number 1 was 7,200 amperes root-mean-square, the differential current as shown by trace number 2 was 59.5 amperes root-mean-square, while trace number 3 indicates that the relay operated and closed its contacts in approximately one cycle.

Conclusions

1. Differential relaying systems as used for the protection of large power transformers, a-c generators, and station bus systems are subject to false operation on differential currents due to causes other than internal faults in the protected equipment.

2. The means hitherto available to prevent false operation, while satisfactory in some cases, are either impractical or give reduced protection in others.

3. The use of the harmonic components of these currents for restraint is an effective and generally applicable means of preventing false operation while allowing complete, continuous and high-speed protection against internal faults.

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Discussion

J. H. Neher (Philadelphia Electric Company, Philadelphia, Pa.): I have had the privilege of watching this relay grow from just a good idea into its present form, and I wish to congradulate Messrs. Kennedy and Hayward on their development of this device and the excellent paper describing it.

The Philadelphia Electric Company was recently faced with the problem of obtaining a differential relay for the protection of the 220-kv bus sections at Plymouth Meeting having an operating time comparable with that of the high-speed relays employed on the lines, which was required by stability considerations, and which at the same time would be immune to operation by transient currents which might appear in the differential circuit on through faults. While high-speed overcurrent relays might have been employed with a sufficiently high setting to override these transients, there is no practical method of determining the maximum value which these transients may assume. It would seem, therefore, that the proper solution of the problem is not to try to guess the value of the transient, but to so arrange the system that the transient, if it appears, will act in such a way that the relay margin is increased rather than diminished thereby. The importance of correct operation of bus differential relays is usually such that the additional complication is well justified.

As a result of this reasoning, we have, or will have shortly, all of our 220- and 66-kv bus sections protected by means of the relay described. These installations are so recent, however, that no operating experience has been obtained to date.

Since both our 220- and 66-kv systems are solidly grounded, the pickup value of the relay could be sufficiently high to make additional restraint against inequality of current transformer ratios unnecessary. If the relay is required to respond to single-phase-to-ground faults on systems where such fault currents may be limited, then a more sensitive relay will be required and its response to the differential currents produced by inequality of current transformer ratios on heavy through faults must be considered. While this type of relay can be constructed with percentage restraint in the usual sense, and the equivalent effect of as many restraining coils as are necessary can be readily provided, nevertheless this complication may not be necessary. The point is that when the secondary currents delivered by the current transformers reach such proportions that the differential current produced by their inequality may equal the normal pickup value of the relay, then these secondary currents may be so rich in harmonics that the effective pickup of the relay may be raised considerably above its normal value, and in effect, percentage restraint is automatically provided. I hope that the authors can give us further information on this point.

There is a unique feature of the relay which the authors have not mentioned and which may be of interest. In a number of differential relay installations, provision has been made for checking the state of balance of the differential system by an arrangement whereby the operator can insert a low range ammeter in series with the differential relays. In the case of the harmonic-restrained relay described, this same result can be achieved by an arrangement whereby the operator can connect a high resistance rectifier type voltmeter across the secondary of the current transformers contained within the relay. This arrangement is more desirable because the a-c connections to the relay are not disturbed.

R. M. Smith (Westinghouse Electric and Manufacturing Company, Newark, N. J.): This paper should be of unusual interest to relay engineers as it proposes one novel solution for the problem of high-speed protection of electrical equipment.

It is interesting to note that with the exception of the magnetizing inrush of power transformers, the problem of high-speed differential protection is made difficult only because of unfaithful response during faults of the current transformers connected to the protective relay. With proper attention paid to the factors affecting current transformer performance, many simple high-speed relays are available for differential protection, such as the percentage differential relay or even the ever-faithful overcurrent relay. Unfortunately most of the applications for existing equipment finds current transformers ill equipped to handle their burden during maximum through short-circuit conditions without one or more departing severely from true ratio.

The relay described in the paper is apparently adapted for bus differential protection only. The only problem here is saturation of one or more of the current transformers, thus producing current in the differential circuit on a through fault. As shown in figure 3 of the paper this current may be distorted and the relay apparently distinguishes between an internal and external fault by the ratio of the harmonic current to the fundamental. We fail to see why an internal fault will produce a differential current free from harmonics if one or more of the current transformers is saturated. It would appear that there must be some definite ratio in the design of the relay between the amount of fundamental frequency component tending to operate the relay and the harmonic currents tending to restrain the relay, which will produce a balance. In table I the analysis shows that for the differential current due to saturated current transformers there is 32 per cent third harmonic for 100 per cent fundamental for the particular case under consideration. We presume that much less than 32 per cent of harmonics will adequately restrain the relay from operating under this condition so that some safety factor will be provided. The question arises in case of an arcing fault on a bus, when the relay should trip, what per cent of the relay current will be harmonics due to the arc. It would appear that if harmonics were introduced by such an arcing fault in any appreciable magnitude, the relay would fail to trip. Since the paper states that the relay has been thoroughly investigated we believe that the paper would have been made more interesting if pertinent data representing cases of arcing faults had been presented.

The figures showing the test conditions show rather simple wave forms. It is to be noted that an almost infinite variety of wave shapes are encountered in such a differential circuit, depending upon the quality of the current transformers, the amount of short-circuit current and the amount and time constant of the d-c transient. Of this tremendous variety of wave shapes to be encountered quite a considerable variation from the values given in table I will be found. We presume that the entire field of possible ratios between the fundamental and the various harmonic components have been explored. Such an investigation should have produced enlightening data to determine in the design just how effective the restraining electromagnet must be in order to prevent the relay from ever operating on through faults. Data showing the percentage of harmonics in the differential current, which will accomplish this purpose, would then determine the percentage of harmonics which could be allowed in the case of an internal fault, without causing the relay to fail to trip, the harmonics being intruduced either through arcing faults, or saturated current transformers.

R. L. Webb (Consolidated Edison Company of New York, Inc., New York, N. Y.): The authors have described a relay system which should develop to have many useful applications. It is judged, from the paper, that the relay has so far been applied only for bus differential protection, using a nonsensitive operating point of five amperes to avoid incorrect operations if a current transformer should become open circuited.

While such a relay should be applicable to bus protection in some instances, such as, where it is desired to use existing current transformers having imperfect or poorly matched accuracy characteristics under high-current conditions, it would seem that its most fertile

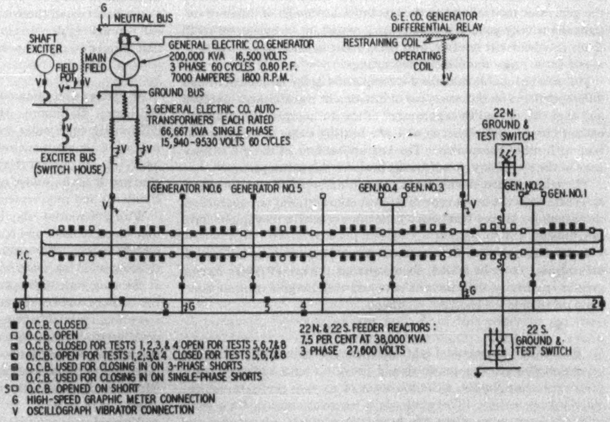
6
NEUTRAL BUS
SHAFT EXCITER
FIELD POT.
MAIN FIELD
V
GENERAL ELECTRIC CO. GENERATOR
200,000 KVA 16,500 VOLTS
3 PHASE 60 CYCLES 0.80 P.F.
7000 AMPERES 1800 R.P.M.
GROUND BUS
3 GENERAL ELECTRIC CO. AUTO-TRANSFORMERS EACH RATED
66,667 KVA SINGLE PHASE
15,940-9530 VOLTS 60 CYCLES
EXCITER BUS (SWITCH HOUSE)
G. E. CO. GENERATOR DIFFERENTIAL RELAY
RESTRAINING COIL
OPERATING COIL
22 N. GROUND & TEST SWITCH
GENERATOR NO. 6
GENERATOR NO. 5
GEN. NO. 4
GEN. NO. 3
GEN. NO. 2
GEN. NO. 1
S
F.C.
G
8
7
6
5
4
2
22 N. & 22 S. FEEDER REACTORS :
7.5 PER CENT AT 38,000 KVA
3 PHASE 27,600 VOLTS
22 S. GROUND & TEST SWITCH
O.C.B. CLOSED
O.C.B. OPEN
O.C.B. CLOSED FOR TESTS 1,2,3,& 4 OPEN FOR TESTS 5,6,7,& 8
O.C.B. OPEN FOR TESTS 1,2,3,& 4 CLOSED FOR TESTS 5,6,7,& 8
O.C.B. USED FOR CLOSING IN ON 3-PHASE SHORTS
O.C.B. USED FOR CLOSING IN ON SINGLE-PHASE SHORTS
S O.C.B. OPENED ON SHORT
G HIGH-SPEED GRAPHIC METER CONNECTION
V OSCILLOGRAPH VIBRATOR CONNECTION

Figure 1. Short-circuit tests—number 7 unit, Hudson Avenue

field would be in the protection of rotating machines and transformers since high sensitivity is usually much more important in protecting such apparatus than in protecting busses.

The authors have pointed out some of the difficulties that must be overcome in applying differential protection to power transformers. We have come upon a type of phenomena found to exist in the secondary of current transformers used for generator differential protection which must also be taken into account when applying relays for this purpose. These phenomena make it appear that a harmonic restrained relay might easily be worked into a protective scheme for such a machine as this generator, but the restraint obtained from the harmonics would probably have to be quite high. It might be worth while considering the use of a small amount of fundamental restraint as well, when a relay of such sensitivity is built.

A few years ago, some short-circuit tests were made on one of the later generators installed at Hudson Avenue Station. Oscillograph elements were connected into the operating-coil circuit of the percentage differential relays as a check on the current transformer characteristics for "through fault" conditions. Figure 1 shows the connections for the test. Both three-phase and single-phase faults were applied, beyond feeder reactors, with the machine operating alone at no-load and also when paralleled with the system and carrying a small percentage of rated load.

Figure 2 shows the results of the three-phase unloaded fault. Both *A* and *B* phase currents had a large d-c component. The effects on the *A* and *B* phase relay-operating-coil currents are plainly shown.

The results were similar to these for the single-phase-to-ground fault and for the tests made under light-load conditions.

In all of the cases, where the fault current had a large d-c component, it was noted that the relay-operating-coil current was rather high for a few cycles and had considerable 60-cycle as well as other frequency components in it. The current transformers, located in the generator neutral and at the bus breaker, though of different designs, have very good accuracy characteristics, at 60 cycles, up to 15 times rated current or more, and the current in these faults was only about three times the transformer ratings.

It is most probable that the d-c component in the fault current had different effects on the two types of instrument transformers involved and that this caused the appearance of the d-c component in the secondary current difference, as well as a healthy value of 60-cycle current with other harmonics. The late appearance of the d-c component in the secondary circuits is the basis for this assumption.

The relays in use, having a ten per cent differential operating characteristic with the usual type of current restraint, did not operate on these faults. If they had been a little more sensitive they might have functioned.

In any case, the use of harmonic restraint, even if some fundamental restraint must be added, should permit the use of relays having greater sensitivity than present ones and thereby give us more complete protection for machine windings.

G. B. Dodds (Duquesne Light Company, Pittsburgh, Pa.): The relay described in this paper should prove a useful addition to the protective-relay family, although some of us may prefer to use the standard percentage differential relays where applicable, for a while until we learn more about the harmonic restraint advantages and limitations.

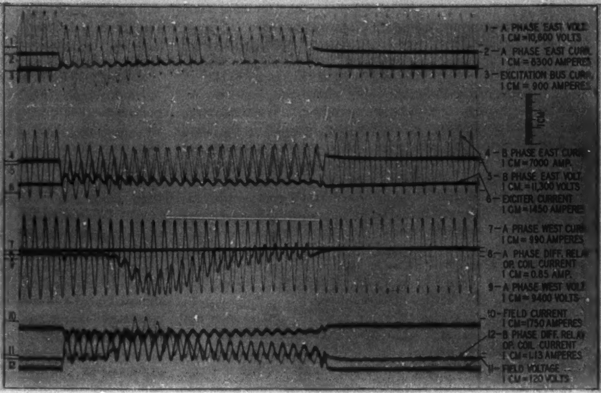

Figure 2. Number 7 unit short-circuit tests

As has been mentioned before, some of us have been wondering whether one-cycle relays are always desirable, and whether there is not frequently a desire for a two- or three-cycle delay. Of course, this can always be obtained by an interposing auxiliary relay.

In figure 7 is shown an oscillogram of fault current and differential relay current on a through fault. The fault current in this oscillogram is far from completely offset, and we are wondering whether tests were made with completely offset waves under this same current transformer and relay setup. From tests we have made, completely offset fault current adds enormously to the current obtained in differential circuits.

While the five-ampere relay will prove satisfactory in many cases, there are also frequent applications where a relay with lower operating current would be desirable; for example, where it is desired to have a fast ground relay in the neutral of the current transformers which must carry the magnetizing inrush to a large power transformer. In cases where the current-transformer ratio is large due to the size of the power-transformer bank, a one-half-ampere pick-up would be desirable. In other cases, where fairly sensitive ground protection is desired and there is danger of obtaining current by asymmetrical through fault currents, a harmonic-restrained relay with one-half ampere to one ampere sensitivity would be very useful.

T. W. Trice (Consolidated Gas Electric Light and Power Company of Baltimore, Baltimore, Md.): The introduction of a high-speed differential type relay unaffected by transients would indeed be welcomed by relay engineers. Methods heretofore used to prevent incorrect differential-relay operation generally result in delayed clearing of faults under correct operation when compared with the standards and requirements of modern relay practice.

With differential-relay installations the need for high-speed operation becomes more and more desirable as the magnitude of fault current increases. For example, a bus assumes greater importance as the connected load, and hence the possible fault currents increase, but at the same time high-speed fault clearing is essential to prevent serious damage and reduce system disturbance. This is an unfortunate condition, for while a reduction in relay time becomes more desirable with increased fault currents, it is necessary to approach high-speed operation with caution because it is more subject to incorrect operation than the slower relay settings. This leads to certain comments with regard to the relay introduced by Messrs. Kennedy and Hayward.

With reference to part *C* pertaining to magnetizing inrush current in current transformers, the authors' statements seem to discount the fact that the current in the primary of a current transformer is determined solely by the load or fault current in the power circuit. The primary current in a series transformer is not determined by the excitation conditions as it would be in a power transformer connected across the line. In other words, a current or series transformer is subject to the magnetizing inrush current of the main circuit only and does not have a primary magnetizing inrush current of its own. Another statement in this paragraph—"the secondary current appears as a transformation of the primary current with the inrush current subtracted"—seems an obscure way of describing the fact that the secondary current of a current transformer is proportional to the primary current except when the primary saturates the core, in which case the inrush current above saturation cannot be reflected in the secondary. In view of this, we are somewhat in doubt as to just what the authors have in mind and would suggest that the phenomena associated with the so-called magnetizing inrush be clarified.

In the operating-coil circuit, tuned to pass fundamental frequency, within what limits of frequency will the circuit permit fundamental current to flow? While system frequencies are normally relatively stable, it is possible to have on the equipment or bus differentially protected, a frequency above or below normal, during or immediately following severe system disturbances, particularly if this part of the system has been separated. If the circuit is too sharply tuned, it is possible that the relays might remain inoperative on a fault occurring during this off-normal frequency period.

It is assumed that the relay is self-contained, not requiring field adjustments, all inherent parts being co-ordinated and assembled at the factory. However, the importance of reliability on differential protective installations dictates that relays used for this purpose should have periodic checking. In the case of a harmonic-restrained high-speed relay, it would seem of utmost importance to periodically reaffirm the effectiveness of the restraining circuit to prevent operation during transient conditions, particularly since the time of operation has been reduced to a minimum. While no difficulty should be encountered for checking correct operation on normal frequency, it appears somewhat difficult to check the restraining circuit to see that it continues to function properly on harmonic frequencies.

In this connection, some information as to the type of condensers utilized in the relay and the effect of temperature change and age on their performance should be of value. Furthermore, what effect will a condenser failure have on the operation of the relay? Having the relay completely inoperative under the condition of condenser failure, depending upon back-up relays for protection would seem to be more desirable than incorrect operation. If a faulty condenser could cause incorrect operation, there is even more reason why periodic checking of relays should be carried out. Likewise, a description of the rectifier used with information as regards its resistance to aging, is desirable.

It is believed that a relay of this type has a definite place in high-speed differential protection practice, particularly on new installations. However, it might be pointed out that there are many differential installations in use on systems throughout the country. From an economic standpoint, it would be highly desirable to modernize many of these installations by eliminating the delay features and reducing the over-all relay time. For this reason it is believed that attention should also be given to the development of a harmonic-frequency filter that could be externally connected to present differential relays.

E. L. Harder (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The satisfactory solution of a relaying problem involves the determination of a discriminating function or quantity which will adequately discriminate the desired trip conditions from the desired nontrip conditions, and then development of a relay element or structure which will respond to this particular quantity. These discriminating functions or quantities are of two general types. First are those which theoretically and practically separate the trip and nontrip conditions by a wide margin, preferably having zero value for the nontrip condition. Relays may then be built having corresponding operating characteristics and applied without any great amount of application effort. Ability to determine the necessary setting analytically is of considerable importance. The second type of quantity or function is that requiring statistical analysis to prove its ability to segregate the trip from the nontrip conditions. This empirical type of solution involves infinitely more application effort and proof of adequacy than the former. For it is necessary not only to show that in general a dividing line exists, but also to show for any individual application where that dividing line is. In other words, it is necessary to determine empirically at what value of the discriminating quantity the relay must be set.

In this particular instance the relaying quantity is of the statistical variety and consequently proof of its adequacy over the range of operating conditions is necessary. It will be of interest to point out the factors of variation affecting the proportion of harmonics present under various conditions. It is also important, as pointed out by the authors, that the relay does not actually respond to harmonics, as calculated from the wave forms by conventional methods, but rather has a response affected by harmonics but altered by the shock application thereof. The factors which would appear to affect the wave shape and hence operation of this type of element are as follows. Consider first factors tending to produce harmonics during internal faults (faults on the protected bus section).

1. The primary current wave shape. For internal faults harmonics may be accentuated by restriking arcs, system resonances, or circulation of harmonic currents between generators of dissimilar wave form. Such circulating currents may have high harmonic content.

2. Saturation of current transformers. During internal faults the largest harmonics would be expected with a critical degree of saturation, greater or less saturation resulting in lower harmonic content. The degree of saturation present is affected by the relay burden, length of leads, internal burden (or secondary leakage impedance) of the transformers, type of iron, iron section, length of iron path and turns, and the amount of a-c current and amount and duration of the d-c transient. These factors vary widely from one application to another.

The other condition of interest is that having maximum tendency to operate the relay when it should not be operated. This would require a differential current above the five-ampere setting with low harmonic content. Such a condition would be most likely to occur when heavy through faults cause a differential current in the relay but where saturation, and hence harmonics, are low. It may also occur at very severe saturation where the current transformers act nearly as air core devices over a fairly large part of each cycle. At intermediate saturations the restraint may be more favorable.

The root-mean-square of harmonics in columns 1 and 2 of table I are 14 per cent and 33.5 per cent. In view of this fairly close margin, for typical conditions, the margins under limiting conditions as mentioned would be of particular interest.

E. H. Bancker (General Electric Company, Schenectady, N. Y.): The conversion of an antagonist into an ally is an accomplishment worthy of remark. In this paper the authors tell how the aid of one of the chief obstacles to successful differential protection has been enlisted to bring about better performance. The inability of current transformers to maintain a constant ratio at high overcurrents has been a major hindrance to a wider adoption of bus differential relaying because it was feared that there was too great a hazard of losing busses for external faults.

It has been known for a long time that whenever current transformers saturate because of a d-c component in the primary current or because the a-c current is too large for the particular transformer with the particular burden, the magnetizing current is large and full of harmonics. D-c saturation produces a preponderance of even harmonics and a-c saturation chiefly the odd harmonics. It was also known that when the secondaries of several current transformers saturated to different degrees were differentially connected, the magnetizing current differences passed through the differential relay. It remained, however, for the authors of this paper to put these known facts to work for the good of mankind. The ingenious separation of differential currents into their fundamental and harmonic components and the use of the harmonic content for restraining operation make possible high-speed differential relaying without the attendant danger of unnecessary losing a bus or piece of equipment for some fault external to it. This marks another advance along the path that relaying usually follows, i.e., to recognize the peculiarities and limitations of associated equipment and despite them all, to provide the required protection and service continuity.

Time alone will tell what other uses will appear for the harmonic restraint principle enunciated here. One is immediately apparent, transformer differential protection, where the troublesome harmonics arise from saturation in the power rather than in the current transformer. The application of harmonic restraint to transformer differential relaying will avoid the difficulties now existing in the use of high-speed relays for this purpose. Magnetizing inrush may be rendered innocuous without impairing the speed or sensitivity of the protection. Similarly, sensitive generator protection may be retained without the danger of false tripping or external faults having a large and slowly decaying d-c component.

H. P. Sleeper (Public Service Electric and Gas Company, Newark, N. J.): There can be no quarrel with the contention of the authors that the use of standard differential relays will tend to give improper operation on either bus or transformer differential protection because of the saturation of the current transformers or the sustained high charging current when a transformer is energized. However, the writer believes that it may not always be necessary to install relays of the types described by the authors of this paper to prevent such improper operation.

When straight differential relays, as compared with percentage differential relays, are used for such protection, the common way to avoid such a difficulty is merely to raise the trip-current settings. The apparent difficulty became increased when percentage differential relays were introduced, since the pick-up of these relays with one side de-energized is usually considerably lower than the normal setting of a straight differential relay. The obvious remedy is to merely increase the trip setting under these conditions. This applies to both transformer and bus percentage differential relays and ordinarily can be done without danger of failure since faults requiring the operation of such relays usually involve high values of power, and the operation of the relay elements under such fault conditions are accumulative, thus assuring positive operation.

It is recognized that the above suggestions are expedients only to take care of undesirable operating conditions. One of the simplest methods of preventing improper operation of percentage differential relays used for bus protection is to make certain that there will always be power in at least one restraining coil. This means that sufficient restraining coils must be present in the relays or certain current transformers must be paralleled to effect this result.

There is certainly a field for a differential relay which will not trip out a transformer bank incorrectly due to the inrush of magnetizing current when energized and yet will operate correctly and instantaneously when the protected unit becomes faulted. No such relay is on the market, to the writer's knowledge, which will or can be made to completely fulfill these specifications. If this new relay meets these specifications, the writer will be the first to cheer.

Since no characteristic curves of these new relays have been presented in this paper, it is impossible to analyze their operation; but it is the writer's opinion that the proper use of restraining coils, as in the manner described above, may still be required with the harmonic current restrained relays described in the Kennedy-Hayward paper.

L. F. Kennedy and **C. D. Hayward:** We feel very gratified indeed at the amount of interest aroused by this paper as evidenced by the unusually large number of discussions presented. The subject is apparently one of live and widespread interest. Since some of the points discussed were mentioned by more than one individual we think it better to review the discussion by topics rather than by the individuals presenting them.

Several questions were asked regarding possible fields of application. There seemed to be a feeling on the part of some that the harmonic restraint relay is intended only for bus protection. While the particular relay application described in the latter part of the paper was one of bus protection, still it should be understood that the harmonic restraint principle may just as readily be applied to relays intended for any other form of differential protection. As a matter of fact, the idea was first conceived for transformer protection. The recent interest in and demand for better bus protection lead to its first practical application being made in that field.

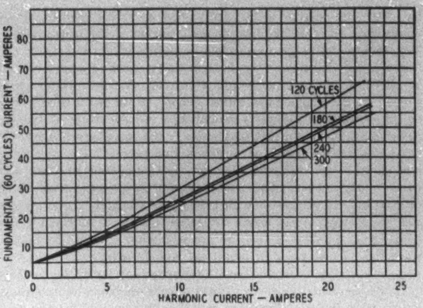

Figure 3. Harmonic-current-restraint characteristics of bus differential relay

Showing amount of fundamental-frequency current required to cause pick-up over the restraining effect of a given amount of harmonic-frequency current when the two are mixed in the input to the relay

Application to transformer and generator differential protection will require a relay having lower pick-up current than the five-ampere value required for the bus relay.

Some applications may require a certain amount of through current restraint in addition to the harmonic differential current restraint. This feature may be added by any of several means, for example, an additional restraining magnet energized by the through current might be used.

In regard to the question as to whether the high-speed operation is always necessary, we would like to give a very broad answer: If the reliability of a given type of protection is not impaired by high-speed operation, then high-speed operation is always desirable even though it may not be necessary. The necessity for high-speed operation usually arises from stability considerations but it is always desirable since it means reduced damage. We feel that with the harmonic-restraint principle the utmost reliability is obtained and that therefore the relay should be made to operate at as high speed as possible.

Several discussors asked for additional information on the operating characteristics of the relay, particularly as regards the percentage of harmonic restraint used and the effect of ordinary variations in the

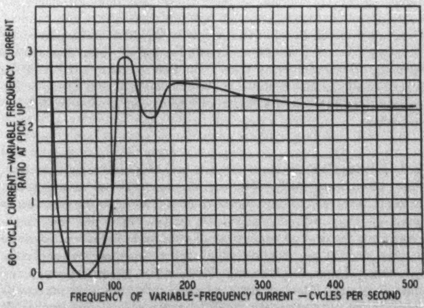

Figure 4. Restraint ratio versus frequency of current mixed with fundamental-frequency current

Slope of curves of the type shown in figure 3 plotted against frequency system frequency. This can best be shown by reference to two additional figures. The curves in figure 3 were taken by mixing a current of a given harmonic frequency with current of fundamental frequency in the input to the relay. For each value of harmonic-frequency current the amount of fundamental frequency current to cause pick-up was determined. Curves for the second, third, fourth, and fifth harmonics are shown. The similarity of these curves to the slope characteristic curves of the familiar percentage differential relay should be noted. A small difference in the slope of the curves for the various harmonics is apparent. This is due to the characteristics of the filter circuits employed in the relay as is better illustrated by figure 4. Here the slope of the curve, that is the ratio of fundamental current to harmonic current required for pick-up, is plotted against the frequency of the harmonic. The shape and height of this curve may be modified by design if occasion demands it. The curve is fairly flat at 60 cycles, indicating that small variations in system frequency will have negligible effect on the pick-up.

Periodic testing of the relay after installation to check the effectiveness of the harmonic restraint can be accomplished by passing current of a known harmonic content through the relay. Such a current is readily produced by means of a suitable small portable saturating reactor. A measure of the degree of restraint can be obtained by simultaneously passing a variable amount of sine wave current controlled by a rheostat through the relay coil, mixing it with the current produced by the saturating reactor and observing the proportion of the two at pick-up.

The capacitors used are of the "pyranol" type while the rectifier is of the copper-oxide-disk type. Both are of the best quality and are energized only momentarily when a fault occurs. Under these conditions exceptionally long life is anticipated. Any possible deterioration with age would, of course, be detected by the periodic tests.

The possibility of incorrect relay operation due to harmonics produced in the primary line current by various causes such as line oscillations, circulating currents due to differences in generator waveforms and arcing faults has been suggested. First of all, it should be appreciated that any harmonic present in the line current in the case of an external fault, even assuming it was not entirely cancelled out in the differential connections of the current transformers, can only produce a small additional restraining effect on the relay, which is not undesirable. It seems unlikely that any large amount of harmonic current will be produced in the differential circuit in the case of an internal fault by any of these causes. At any rate, the examination of many oscillographic records taken under a wide variety of conditions fails to show any noticeable amount present due to these causes after the first cycle. From this it would seem that the worst effect these harmonics could have would be to delay tripping for at most a cycle.

The successful application of the harmonic restraint relay requires knowledge of the relay characteristics and of the current transformer performance. At the present time, detailed information regarding the harmonics produced by various types of current transformers under all conditions is incomplete. A program of investigation is under way to obtain this information as quickly as possible. Until this information is complete applications will have to be made with unusual care and will therefore require individual consideration.

Some Schemes of Current-Limiting-Reactor Applications

By F. H. KIERSTEAD
MEMBER AIEE

CURRENT-LIMITING reactors are so detrimental to the normal operation of an electric system, and yet such a valuable contribution to its safety during short circuits, that it is very much worth while to try again and again to devise schemes which will use the ability of reactance to limit fault current and yet minimize the inevitable increase in regulation. With this in view, this paper reviews some well-known schemes of reactor application.

It is hoped that in exciting discussion, not only of the few schemes described here, but also of allied schemes, a trend toward better current-limiting protection may develop. This paper is written from the viewpoint of a designer, and the author realizes that application and operation engineers may find valid objections to the suggested uses of the schemes discussed. The author feels, however, that the chief value of the paper to the art will not be in its contents, but will be in the discussion it may excite.

The following schemes will be discussed:

A. Reactors shunted by circuit breakers.
B. The saturable-core reactor.
C. The split-circuit reactor.

A. Reactors Shunted by Circuit Breakers

In order to alleviate the difficulty of opening a faulted feeder, it has been proposed to use two circuit breakers in series, the one with a shorter time delay being shunted with a reactor, so that when it opens its reactance is automatically inserted in the circuit—thus lessening the duty on the second "main" circuit breaker. Such a scheme can be operated successfully and, therefore, the economic aspects of it will be discussed below.

The interrupting duty on a breaker shunted by a reactor depends, among other things, upon the amount of reactance in shunt. The duty is reduced by the shunt reactor to the extent that the voltage across the breaker is reduced below that which it would have been if the reactor had not been present—in other words, to the extent that the recovery voltage is reduced by the presence of the reactor.

In order to evaluate the reduction in duty on a breaker affected by a shunt reactor, let us compare two 60-cycle, 6,800-kva, 13.2-kv cable feeders—one equipped in the conventional manner with a circuit breaker and a three per cent reactor, and the other equipped, in addition to the above breaker and reactor, with a breaker shunted by a reactor. These two circuits are shown in figure 1 and the pertinent circuit constants are given below.

Generator reactance (X_G) = 10 per cent and 20 per cent reactance on 50,000-kva base.
Series reactor (X_R) = 3 per cent reactance on 6,800-kva base.
Shunt reactor (X_S) = 15 per cent reactance on 6,800-kva base.

The final recovery voltage across the breaker of the conventional feeder and that across the shunted breaker of the proposed feeder are given below:

Generator Reactance	10 per cent	20 per cent
	Final Recovery Voltages	
Breaker in conventional feeder	100%	100%
Shunted breaker in proposed feeder	77%	71%

This table shows that the breaker shunted by the reactor must, in the cases assumed, be approximately three-quarters of the capacity of the breaker in the conventional feeder. The shunt reactor was chosen large so that the series breaker which finally opens the circuit could have a low interrupting capacity. This breaker, however, must have sufficient capacity to carry the initial current until the shunt breaker opens. Of course, if the reactance of the shunt reactor were made less, the breaker which it shunts could have smaller capacity, but the series breaker would then have to have greater capacity.

The series reactor could, in some cases, be made smaller where a shunt reactor is used, but this would result in higher current to be interrupted, and consequently in shunt and series breakers of higher interrupting capacity.

The table below lists the equipment required for both methods. The interrupting duty of each breaker is given in per cent of that required for the breaker of the conventional feeder. The short-time current capacity of the series breaker is neglected in this comparison.

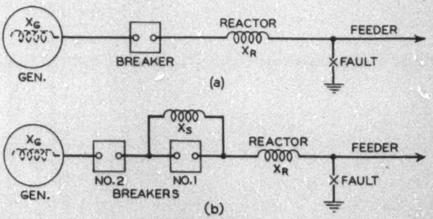

Figure 1

(a)—Diagram of the conventional feeder with a reactor and circuit breaker

(b)—Diagram of feeder same as (a) except an additional reactor shunted by a circuit breaker

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Actually this consideration may call for a breaker approaching 100 per cent in size.

	Reactors		Breakers	
	Shunt	Series	Series	Shunt
Conventional feeder	No	Yes	Yes—100%	No
Proposed feeder	Yes	Yes	Yes—25%	Yes—75%

Since the proposed feeder requires an additional reactor and breaker, it appears to be more expensive than the conventional feeder.

Considering the use of the "proposed feeder" from the standpoint of regulation, the series reactor might be reduced and thus improve the regulation. This would, of course, increase the current to be interrupted and, therefore, increase the required capacity of both the shunt and series breakers. The same reduction in the series reactor, and hence the same improvement in regulation, could be obtained in the conventional feeder by the use of a larger circuit breaker. It would appear that the same improvement in regulation would entail about the same increase in cost in each scheme, and that the conventional scheme would have the lowest cost in any case.

There seems to be no economic justification for the "proposed feeder" from either the viewpoint of cost or improvement of regulation.

There does, however, appear to be a very limited application for a circuit breaker shunted by a reactor in the case of a substation where the short-circuit kilovolt-amperes has grown beyond the capacity of the feeder circuit breakers. The feeder bus can be sectionalized from the source of power by a large breaker shunted by a reactor, and all the feeder breakers interlocked with the large breaker so that they cannot open until the large breaker opens, and then the breaker on the faulted feeder interrupts the fault current, which has been reduced to the capacity of the breaker by the insertion of the reactor.

Before such an application is made, thorough consideration should be given to the following factors:

1. **Have the existing feeder breakers sufficient short-time current capacity?**

2. **Is the increased time required to open two breakers in succession (instead of one) with the consequent liability of greater destruction at the fault justified?**

3. **Is there greater liability of motors on the other feeders dropping out of step due to the longer time before the short is cleared?**

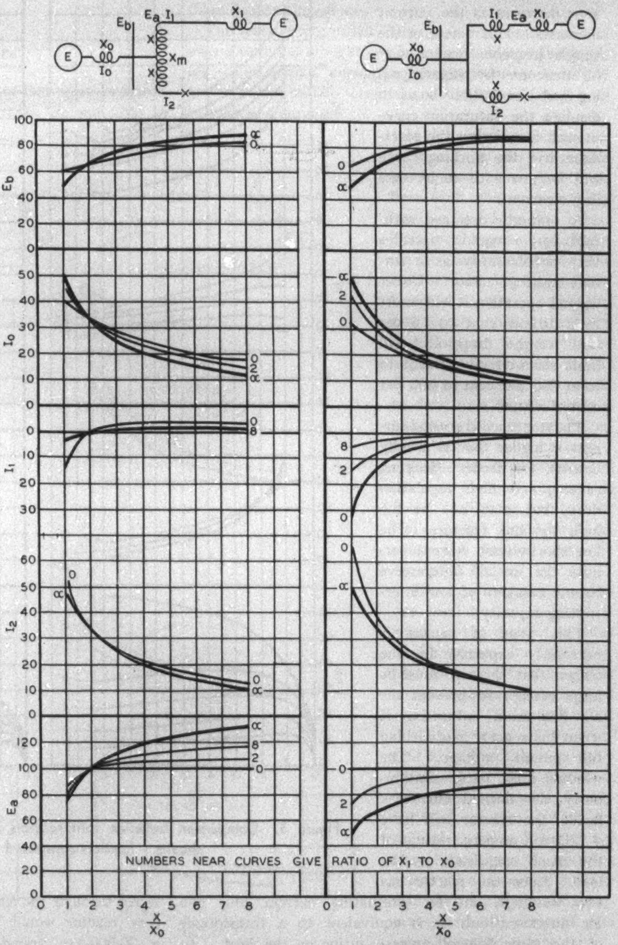


Figure 2. Comparison between split reactors and conventional reactors for feeder circuits. Feeders interconnected to other sources of power

In general, higher interrupting capacity breakers are a better solution.

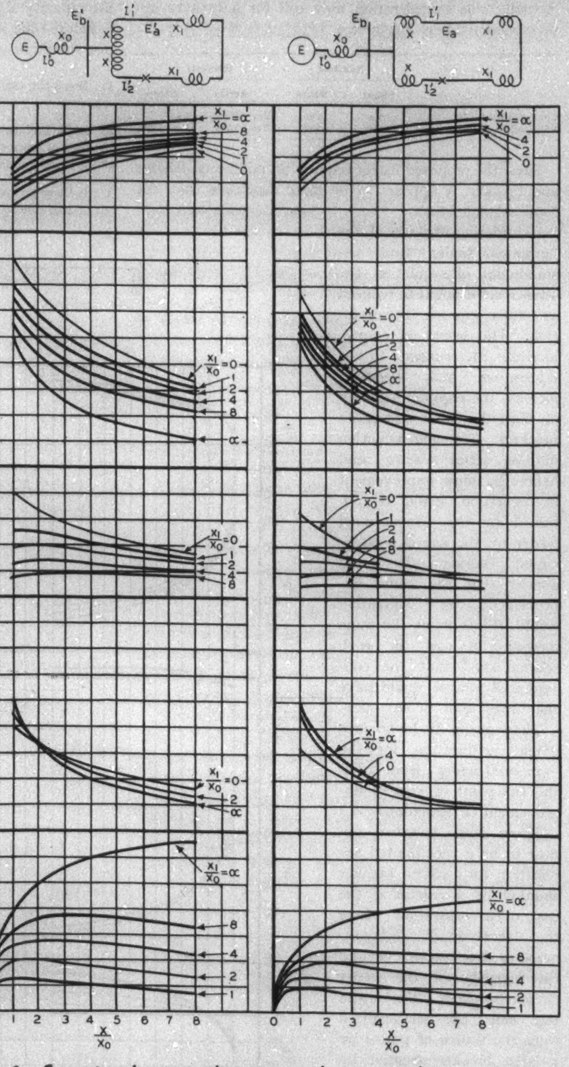


Figure 3. Comparison between split reactors and conventional reactors for feeder circuits. Feeders connected to same substation

B. The Saturable-Core Reactor

The inherent characteristic of an iron-core reactor is just the reverse of the desirable characteristic for short-circuit protection, in that its reactance *decreases* as the current increases. This defect of the simple iron-core reactor may be overcome by superimposing a d-c excitation[1] so as to displace the saturation curve in such a way that the reactance over the working range will *increase* with an increase in a-c current.

A properly designed saturable-core reactor operates very satisfactorily as a current-limiting reactor. Under normal operation it will insert from 10 per cent to 15 per cent reactive drop, and will limit short-circuit current to from 250 per cent to 350 per cent of normal.

The reactance at normal current is higher than is usually desired for feeder reactors, causing too high regulation drop, but may not be too high for bus reactors. The low short-circuit current permits the use of inexpensive circuit breakers of low interrupting capacity.

This type of reactor is inherently expensive for the reason that the core must be large enough to prevent the a-c flux from saturating it when the reactor is absorbing full circuit voltage. The winding must carry continuously the normal current—hence, the reactor will have a kilovolt-ampere rating of the same magnitude as the load. Since the reactor has two windings, one for alternating current and one for direct current, it is equivalent to a transformer of the same kilovolt-ampere rating as the load. In order to compare this type of reactor with the conventional air-core reactor, assume that the load transferred through the reactor is 30,000 kva. The saturable-core reactor would be equivalent in size and cost to a 30,000-kva transformer. If air-core reactors were used they would have 10 per cent to 15 per cent reactance, and since they have no iron core their physical size, and therefore their equivalent rating is based on the normal operation. Their rating would, therefore, be 3,000 kva. Since reactor kilovolt-amperes cost much less than transformer kilovolt-amperes, it is clear that a saturable-core reactor costs many times more than an air-core reactor. It is, of course, true that the circuit breaker used with a saturable-core reactor would cost much less than that used with an air-core reactor, due to the reduced short-circuit current. The reduction in circuit-breaker cost has not in the past been commensurate with the increased cost of saturable-core reactors, and for this reason they have not found a field of application as current-limiting reactors.

C. The Split-Circuit Reactors

The split-circuit reactor consists of two reactors placed so closely together that the reactance of one (due to mutual reactance) is affected by the current in the other. When used as current-limiting reactors they are connected so that with normal current in each reactor, their reactance is less than with current in only one. It is expected that under short circuit the current in the reactor connected to the fault will be very much greater than that in the other and, as a result, the reactance will be substantially greater at short circuit than at normal operation.

In some cases the current in the reactor not connected to the fault may be reversed in direction due to the fault, resulting in making the reactance of each reactor greater than if operating alone.

The reactors are usually wound as one continuous coil with a terminal brought out from the middle of the winding. The normal current is brought from the source of power to the middle terminal and divides, and part goes to each end terminal.

In the case of feeder reactors, it would be very desirable to separate the two coils from each other so as to fully insulate one from the other. This would permit placing a circuit breaker between the reactor and the bus. This, however, very materially reduces the mutual reactance and does not appear to be economically feasible in the dry type reactors, but could be done in a liquid-insulated reactor where the insulating distances are so much less.

In the dry-type reactors 50 per cent mutual reactance is about as high as is practical, while in the liquid-filled reactor the mutual may be somewhat higher.

The reactive drop across one section (one reactor) is equal to the current in that section times its reactance, plus the current in the other section, times the mutual reactance. The mutual adds, if the direction of the current in one section is positive compared to the current in the other section, and subtracts if negative. With the mutual reactance 50 per cent of the self-reactance, the resulting reactance of a section is 50 per cent with equal opposing currents, and 150 per cent with equal boosting currents.

With both sections carrying opposing currents, the

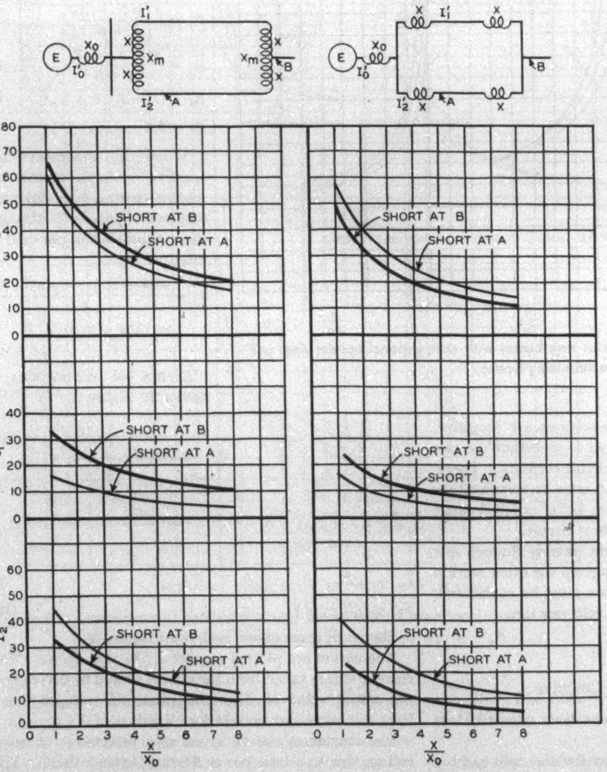


Figure 4. Comparison between split reactors and conventional reactors for feeder circuits. Feeders connected to same substation and reactors identical at each end

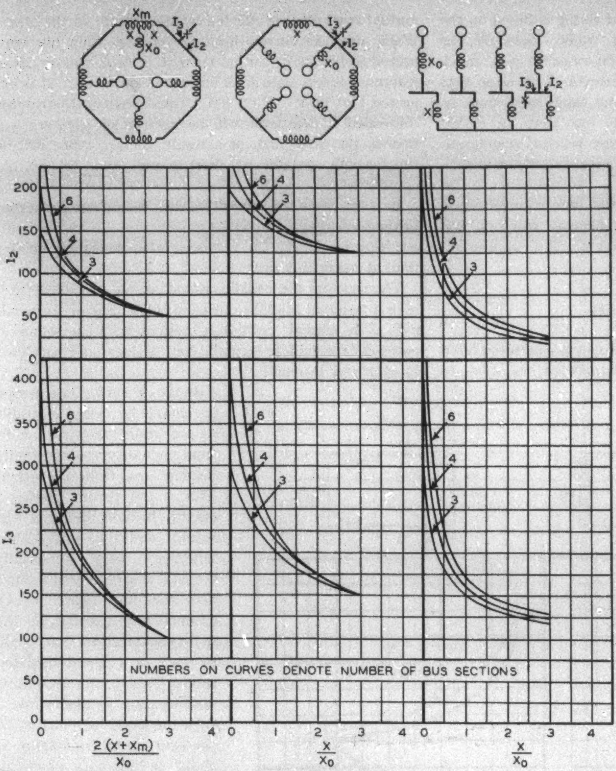


Figure 5. Comparison of split-reactor ring busses with conventional reactor ring and synchronizing busses

magnetic forces between the two sections are repulsive, and may be very great if the circuit is such that the two sections can carry opposing short-circuit currents. Therefore, in general, split circuit reactors must be very strongly tied together to prevent them from being expelled apart by the magnetic force incident to short circuit.

Due to mutual reactance, current passing through one section causes reactive rise in voltage on the other section which may be serious, and therefore must be considered.

There are two general cases for split reactors:

1. For feeder circuits.
2. For bus sectionalizing.

1. Split Reactors for Feeder Circuits

Three general feeder circuits have been considered as is shown in figures 2, 3, and 4.

The values given by the curves for the split-reactor feeder have been calculated from the formulas given below. These formulas are simple algebraic expressions, and the insertion of their proofs here is not considered warranted. The calculation of the curves for the conventional feeder reactors made use of standard formulas and are not given.

The value of mutual reactance used in all these calculations was 50 per cent.

The formulas 1 to 5, inclusive were used in calculations for split reactor feeders in figure 2.

Calculations for the split-reactor feeders shown in figures 3 and 4 made use of formulas 6 to 10, inclusive.

$$I_1 = \frac{X_m - X_0}{A + B} \qquad (1)$$

$$I_2 = \frac{X + X_1 + X_0}{A + B} \qquad (2)$$

$$I_0 = \frac{X + X_m + X_1}{A + B} \qquad (3)$$

$$E_a = (I_2 - I_1)(X + X_m) \qquad (4)$$

$$E_b = I_2X - I_1X_m \qquad (5)$$

Where all reactances are reactive voltages across the reactors with a base kilovolt-amperes passing through them expressed in per cent of rated voltage.

$$A = \frac{X(X + X_1) + X_0(2X + X_1)}{100}$$

$$B = \frac{X_m(2X_0 - X_m)}{100}$$

Sketch of connections is shown in figure 2.

$$I_1' = \frac{X + X_m}{A + B} \qquad (6)$$

$$I_2' = \frac{X + X_m + X_1}{A + B} \qquad (7)$$

$$I_0' = \frac{2(X + X_m) + X_1}{A + B} \qquad (8)$$

$$E_a' = I_1'X_1 \qquad (9)$$

$$E_b' = I_2'X - I_1'X_m \qquad (10)$$

Sketch of connections is shown in figure 3.

The curves are plotted against X/X_0 and X_1/X_0. The current values taken from the curves should be divided by the actual value of X_0. The values of voltages taken from the curves are correct for all values of X_0.

The reactances are all given as a percentage of rated voltage due to a base power flowing through them. The currents are given as the number of times the current of the base power can be divided into them.

An examination of these curves shows that the amount of feeder reactance (X) must in general be greater for the split reactor than for the conventional reactor; however, with equal currents in both sections, the effective reactance is equal to the self-reactance minus the mutual reactance, and since the mutual is 50 per cent, the effective reactance is only half of the self-reactance. This is considerably less than that required by the conventional reactors to limit the fault currents to the same values.

The mutual reactance helps to balance the load in parallel feeders due to raising the voltage on the underloaded feeder and lowering it on the overloaded one.

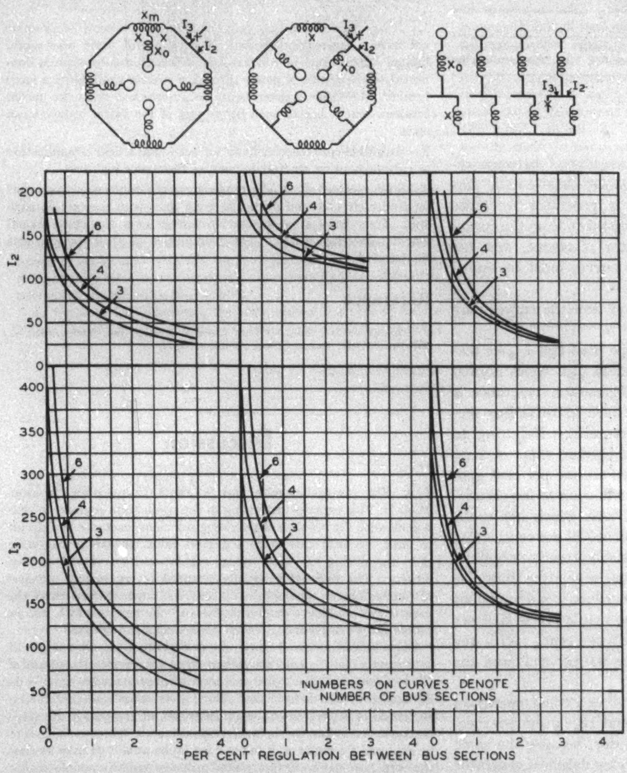


Figure 6. Comparison of split-reactor ring busses with conventional reactor ring and synchronizing busses based upon $X_0 = 1$ and $X_m = {}^1/_2X$

Whether or not there is sufficient merit in the split reactors to warrant their use in particular feeder circuits depends on the individual characteristics of the system and on the value that can be ascribed to reduced regulation in the particular system.

With these factors known, the curves make a rapid comparison possible. The circuits shown in figure 4 are particularly favorable to split reactors.

The case of split reactors for feeders may be stated as follows:

The advantage of lower regulation and better division of load between the parallel feeders must adequately balance the following disadvantages:

1. Higher cost due to a larger reactor and to the additional expense of supporting it to resist high repulsive forces.
2. Rise in voltage (E_a) which may be appreciable, as is shown in figure 2, if feeder reactance (X_1) is high.
3. The necessity of using liquid-filled reactors if circuit breakers must be placed in the individual feeders between the reactors and the bus.

2. Split Reactors for Bus Sectionalizing

In order to ascertain if the split reactor has any value for bus sectionalizing, its use in ring busses has been compared with conventional reactors in ring and synchronizing busses. The split reactor is not suitable for use in synchronizing busses.

In figure 5 the ratio of bus reactance to generator reactance is plotted against short-circuit currents for the split-reactor-ring bus, the conventional ring bus, and the conventional synchronizing bus. Three, four, and six bus sections are included. The curves are based upon one per cent generator reactance. In practical cases the generator reactance is of course always greater than one per cent, and the correct short-circuit current is obtained by dividing the current values taken from the curves by the generator reactance. The split-reactor curves are based on $X_m = {}^1/_2X$.

In figure 6 short-circuit currents are plotted against the regulation due to load transfer. The calculation of the regulation is based on the following conditions:

1. That the kilovolt-ampere supply connected to each bus section is equal.
2. That the kilovolt-amperes transferred to one bus section from

all the other bus sections is equal to the kilovolt-ampere supply connected to one bus section. This means that the load on one bus section is equal to the kilovolt-amperes connected to two bus sections.

3. That these bus sections each contribute an equal amount of the transferred kilovolt-amperes.

4. The regulation is the reactive drop between extreme bus sections due to the load transferred.

5. That the reactance is the per cent of normal voltage across the reactance with the kilovolt-amperes of one bus section passing through it.

The formulas for the calculation of the regulation are given below:

Number of Bus Sections	Regulation Drop		
	Split-Reactor Bus	Conventional Ring Bus	Conventional Synchronizing Bus
3	$X + X_m$	$0.5X$	$1.5X$
4	$1.33\ (X + X_m)$	$0.667X$	$1.33X$
6	$1.80\ (X + X_m)$	$0.9X$	$1.2X$

In the curves in figure 6 the regulation between extreme bus sections is plotted against short-circuit current. The curves are based on a generator reactance of one per cent. The current values taken from the curves must be divided by the generator reactance, and the regulation values taken from the curves must be multiplied by the generator reactance.

The following examples will show how the curves may be used.

Suppose a six-section bus is to be used and I_2 (see diagrams in figure 6) is to be limited to nine times normal current of one generator and the generator reactance is 15 per cent. The regulation between extreme bus sections and the bus reactance is required. Referring to figure 6 and entering curves with current value of $9 \times 15 = 135$ (because curves are based on one per cent generator reactance), the split-reactor bus curve for six sections gives a regulation value of 0.49, which has to be multiplied by 15, giving a regulation of 7.35 per cent. The through reactance, $2(X + X_m)$, of the bus reactor obtained from the regulation by use of the regulation formulas is 8.18 per cent. By the same procedure it is found that with the conventional ring bus the regulation is 30.0 per cent and the bus reactance is 33.3 per cent. With the synchronizing bus the regulation is 9.0 per cent and the bus reactance is 7.5 per cent.

It should be noted that figure 6 gives a true comparison of relative short-circuit currents and regulation irrespective of the reactance of the generator, because the same reductions have to be made for varying values of generator reactance for all three bus schemes.

Since the reactance of generators will always be larger than the mutual reactance of the split reactor, the bus voltage will not rise above normal.

Figure 6 shows that the split-reactor bus has a marked advantage over the conventional ring bus. A comparison with the synchronizing bus is much more difficult because the two schemes are quite different. In addition to the currents shown on the curves, consideration must be taken of the short-circuit currents on the synchronizing bus itself, as these currents are very large. In many cases, bus reactance sufficiently high to limit fault currents on the synchronizing bus to safe values may make the regulation so high that the synchronizing bus scheme is difficult to operate.

The author realizes that the choice between these schemes depends a great deal on the individual conditions of particular stations, and has given these curves to assist in making the choice. The curves indicate that the split reactor bus should be given thoughtful consideration.

Summarizing

1. It has been shown that reactors shunted by circuit breakers do not have a general application. A very careful study may reveal limited applications in which a feeder bus of a substation is connected to the source of power through a reactor shunted by a large breaker, the large breaker being so interlocked with the feeder breakers that it has to open before any of the feeder breakers can open.

2. Saturable-core reactors have not yet found a field of application as current-limiting reactors because of their very high cost.

3. Split reactors show advantages over the conventional reactors for feeder circuits, and particularly for lines with reactors on each end. They have a substantial advantage over the conventional reactors for ring busses. The possibilities of their use warrants careful study.

Reference

1. THEORY OF D-C EXCITED IRON CORE REACTORS AND REGULATORS, A. Boyajian. AIEE TRANSACTIONS, volume 43, 1924.

Discussion

J. A. Elzi (Commonwealth and Southern Corporation, Jackson, Mich.): This paper is interesting to the application engineer in that it deals with the problem of providing sufficient reactance in a circuit to limit the fault current to a desired value without at the same time introducing excessive voltage regulation for normal load conditions. The engineer is usually tempted to try various schemes of connections to accomplish this, and the paper demonstrates the necessity of making a careful analysis of the results which can be obtained by using various reactor connections and designs.

The conclusions given in the paper regarding the advantages of split reactors for feeder circuits where there are reactors at each end of the feeders seem quite logical, but the curves presented in figure 4 do not appear to demonstrate this. For a given set of reactance values, the currents I_0', I_1', and I_2' apparently are all greater for the split-reactor scheme than for the conventional-reactor scheme. This is to be expected for faults at *B* but not for faults at *A*. It may be that the curves are not being interpreted properly and an example similar to that presented for the six-section bus of figure 6 would be helpful in clearing up this point.

S. I. Oesterreicher (Metropolitan Device Corporation, Brooklyn, N. Y.): Due to first cost, complications in circuit arrangement and control in scheme *B*, as well as greater space requirements for scheme *A*, there are, no doubt, good reasons from the operating standpoint for objecting to these two schemes as described in Mr. Kierstead's paper.

However, the tee-circuit reactor—also known by the names of split- or twin-circuit, double or bifurcated reactor, has certain operat-

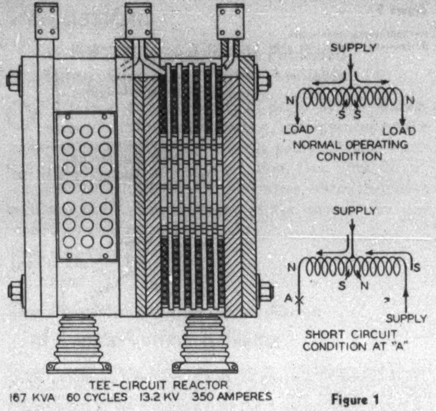

SUPPLY
N
S S
N
LOAD
LOAD
NORMAL OPERATING CONDITION
SUPPLY
N
S N
S
A
SUPPLY
SHORT CIRCUIT CONDITION AT "A"
TEE-CIRCUIT REACTOR
167 KVA 60 CYCLES 13.2 KV 350 AMPERES

Figure 1

ing characteristics for which it should in the future receive greater popularity than during the past.

Much of the distrust toward the tee-circuit reactor is due, no doubt, to some shortcomings of early designs. After correction, it functioned in accordance with the intent of its inherent characteristics. Some of these later tee-circuit reactors have over 12 years of satisfactory operating record to their credit.

As stated by Mr. Kierstead, the advantage of this type of reactor is that during normal operation the voltage drop across the reactor is reduced by the amount of mutual inductance between the two windings. During short circuits the subtractive mutual inductance becomes additive, and thus the reactance through each coil half increases and there is automatically a greater amount of short-circuit protection inserted into the respective circuit.

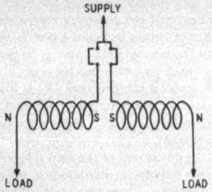


Figure 2. Tee reactor for two independent circuits

Unless there is a simultaneous short circuit in both load legs of the reactor, the reactance value through the reactor will always be greater during short circuits than during normal operation. This is true even when one of the two circuits is open beyond one of the reactor load side terminals.

While simultaneous short circuits in both reactor legs are exceptions from normal rule, nevertheless, the reactor has to be designed for just such emergencies. The repelling forces between the two halves during such short circuits, especially with such high mutual inductance values as Mr. Kierstead assumed in his calculations, may reach well into hundreds of thousands of pounds. I presume this value was assumed to illustrate the advantages of the *T* reactor to its fullest extent. The mutual inductance in our coils was limited close to 30 per cent.

I believe the *T* reactor would have its greatest field of application on secondary distribution feeders up to the five-kv range. In these circuits as little as possible reactance would be most welcome from the standpoint of regulation. With increase in generating capacity and load, the usually inexpensive breakers on these circuits would require greater protection during shorts, which could be readily provided by the *T* reactor. The reactor being compact, takes less space, is lighter, and does not cost more than two single-phase coils. To eliminate the disadvantage of having two circuits out of service at the same time in case of disturbance on only one side of the *T* reactor, the two feeder circuit breakers may be placed upon the supply side of the circuit beyond the common terminal of the reactor. Beside the regular equipment of the two feeders this arrangement would require only one additional terminal upon the reactor.

P. N. Sandstrom and **E. L. Michelson** (Commonwealth Edison Company, Chicago, Ill.): This paper presents interesting data regarding the application of the split reactor on feeder circuits and for bus sectionalizing. Some time ago, it was proposed by the Commonwealth Edison Company to use the split reactor in a somewhat different manner, shown in figure 3*a* of this discussion. In this diagram, busses 1 and 2 represent the main busses in a large station; these busses are normally separated, but the split reactor can be used to connect a source of power to feed both busses simultaneously, and at the same time maintain a certain degree of separation between the two busses. In this way, it possible to obtain a low value of reactance between the source and the load, and yet have a high value of reactance between the two main busses in the station. It is recognized that this application is a special form of the feeder circuit described in the paper.

Figures 3*c* and 3*d* of the discussion show values of fault current corresponding to I_2 of figure 2 in the paper, for values of x/x_0 from 0

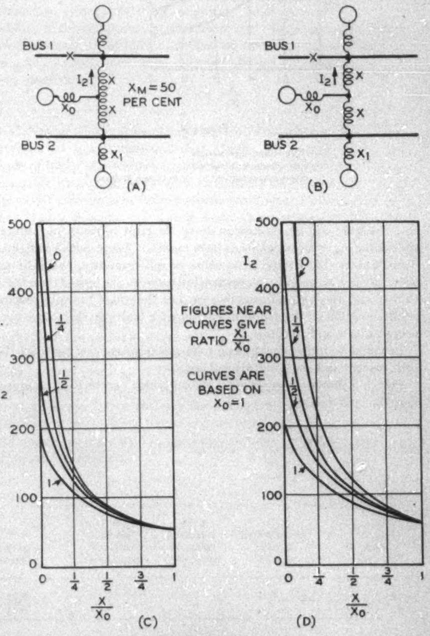


Figure 3

A—Split-reactor connection
B—Conventional design
C—Short-circuit currents with split reactor
D—Short-circuit currents with conventional reactor

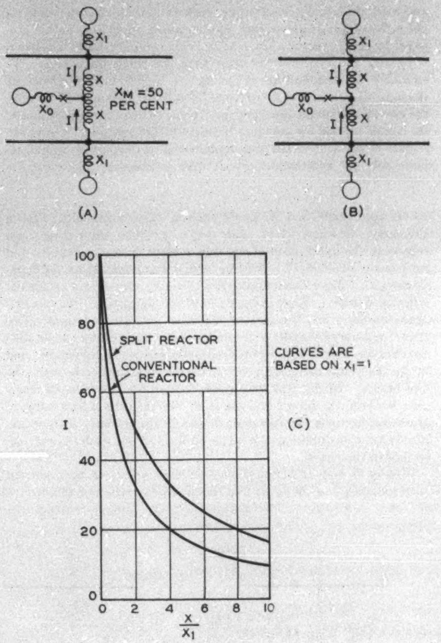


Figure 4

A—Split-reactor connection
B—Conventional connection
C—Short-circuit currents for faults shown in *A* and *B*

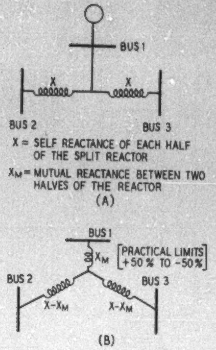
to 1. Figure 4 of the discussion shows the fault currents for faults in the source of power which feeds both busses. These curves in figures 3 and 4 show that for the same value of self-reactance, the split reactor gives a more effective separation between the busses than is obtained with the conventional reactor, but the use of the split reactor results in higher values of fault currents for faults on the source connected to the split reactor.

It would be interesting to know the comparative cost between the split reactor and the conventional reactor.

Figure 5 shows the equivalent circuit which can be used to represent the split reactor.

Figure 5

A—Split-reactor connections
B—Equivalent circuit for split reactor


BUS 1
X
X
BUS 2
BUS 3
X = SELF REACTANCE OF EACH HALF OF THE SPLIT REACTOR
X_M = MUTUAL REACTANCE BETWEEN TWO HALVES OF THE REACTOR
(A)
BUS 1
X_M
[PRACTICAL LIMITS +50 % TO -50 %]
BUS 2
BUS 3
$X-X_M$
$X-X_M$
(B)

In this circuit, X_M should be taken as positive if the mutual reactance has a positive effect; that is, if fault current in one circuit results in a drop in voltage on the second circuit. X_M should be taken as negative if the mutual reactance has a negative effect; that is, if fault current in one circuit results in a rise in voltage on the second circuit. This is in accordance with the accepted method for dealing with such circuits. It will be noted that this is contrary to the method used in this paper and in figures 3 and 4 of the discussion, since a negative mutual effect has been expressed as a positive quantity.

F. H. Kierstead: In accordance with Mr. J. A. Elzi's request, I wish to give an example covering a comparison of the split reactor with the standard reactor. In the split reactor assuming the reactance of one section (X) is four per cent and the generator reactance X_0 is one per cent, and a fault at *A*, the curves in figure 4 give I_2 = 21.5, I_1 = 7.5, and I_0 = 28. Since the mutual inductance is 50 per cent, the reactance to rated balanced load current is two per cent.

Referring now to the standard reactor, if its reactance (X) is 3.5 per cent and the generator reactance is one per cent and a fault at *A*, curves in figure 4 give I = 21.5, I_1 = 6.5, I_0 = 27. Reactance to rated balanced load current is 3.5 per cent.

Figure 3 of this discussion shows the advantage of the split type of reactor for a fault on bus 1, while figure 4 shows its disadvantage with a fault on the midpoint of the split reactor. These two curves show that the split reactor must be properly applied to obtain its benefits.

Mr. Oesterreicher shows practical applications of the split-type reactor. In the split-type reactor for independent circuits care must be taken that the two sections are insulated from each other for the full voltage, and if one of the circuits is taken out of service to be worked on, it must be grounded at both ends so that the mutual inductance does not induce serious voltages in the idle reactor.

Discussions

of AIEE Technical Papers Published Before Discussions Were Available

ON THIS and the following 13 pages appear discussions submitted for publication, and approved by the technical committees, on previously published papers presented at the AIEE winter convention, New York, N. Y., January 24–28, 1938. Authors' closures, where they have been submitted, will be found at the end of the discussion on their respective papers.

The Application and Performance of Carrier-Current Relaying

Discussion and authors' closure of a paper by Philip Sporn and Charles A. Muller published on pages 118–24 of this volume (March section), and presented for oral discussion at the relays and reactors session of the winter convention, New York, N. Y., January 25, 1938.

T. G. LeClair: For discussion see page 291.

R. M. Smith (Westinghouse Electric and Manufacturing Company, Newark, N. J.): While the paper by Messrs. Sporn and Muller deals primarily with the problems to be solved by the application of carrier-current relaying, we believe that some comments on the particular scheme of carrier relaying are worth while.

In reviewing the steps in the development of carrier relaying toward faster operation it is apparent that one outstanding contribution assisted in the realization of speeds in the order of one cycle. This improvement consisted in the use of a normally blocked trip circuit by the means of a receiver relay whose tripping contacts are normally open. This method of handling the carrier-operated blocking function was first used by the authors of the paper and I believe that due credit should be given them for this contribution to the development of carrier-current relaying.

One thing to be noted in the one-cycle scheme described is that the speed of clearing faults has been gained at the expense of reliability in blocking on external faults. This comes about, of course, from the use of circuit-opening contacts to remove carrier rather than circuit-closing contacts. This means that on quick reversals of power which occur on parallel lines and loop systems, carrier is stopped by a circuit-opening contact and established at the opposite end of the line by a circuit-closing contact. The opening contact will ordinarily beat the closing contact and there will be a short interval in which no blocking signal is present. The authors have circumvented this difficulty by the use of an additional temporary blocking relay. It is questioned whether this is good practice since an undesirable time delay will be introduced in clearing an internal fault which occurs immediately after an external fault. Would it not be preferable to slow up clearing times by, say, 0.25 to 0.5 cycle and use this much time interval as a definite margin of safety in blocking?

It is observed that the contacts which start carrier and the tripping contacts are on the same overcurrent element. This means that for external fault currents, which are near the minimum pick-up value of the relays, the relay at one end of the line may operate, while, due to different current-transformer characteristics, the relay at the other end may not operate. If the relay at the line end where power flows into the bus is the one which does not pick up, then a blocking signal is not sent to prevent unnecessary tripping at the other end. The obvious thing to do is to set both relays the same and considerably below the minimum fault current expected. On phase-to-phase faults it is generally not difficult to do this, but on ground faults where fault resistance or distance may limit the current to very low values, it may be very difficult to set the relay pick-up value below the lowest possible fault current. One solution of this trouble is to place the carrier start and tripping contacts on separate elements. The starter element can then be set considerably below the tripping element, thus assuring that whenever the currents for external faults approach the minimum pick-up value of the tripping element, they will be safely above the pick-up value of the carrier start element.

Since a three-phase directional element requires that the ground fault current be twice the magnitude of the load current, to indicate fault power correctly, there is grave possibility that the relay scheme, as described, will fail to clear internal ground faults involving low currents. Of course, this characteristic of the three-phase directional element has been encountered before on conventional relaying, but its seriousness in a carrier scheme is much aggravated because the failure of only one directional element to function will block both ends from tripping. We feel that single-phase directional elements for phase, and a separate ground directional element, are much more desirable in a carrier scheme. It may be argued that the single-phase element is liable to go wrong on a system with a multiplicity of ground points, or due to load currents, but this is taken care of by supervising the directional element through an impedance element and the use of a ground preference scheme as described in the paper by Harder, Lenehan, and Goldsborough, whereby on ground faults the ground relays can take command of the situation from the phase relays.

Messrs. Sporn and Muller have cited several of the advantages in the application of carrier relaying and have described actual examples of the applications which have served to bring out the specific features to be gained by the use of carrier relaying. The record of their experiences with this type of protection should be of great interest to system-protection engineers, as it demonstrates the reliability of the equipment in actual service. They have stimulated a great interest in this form of protection and their paper shows that from an operating standpoint their faith in this type of protection has been justified.

O. C. Traver (General Electric Company, Philadelphia, Pa.): I appreciate the frank exposition that Messrs. Sporn and Muller have made of both their sorrows and their joys in connection with a commendable pioneering work with carrier-current protection. To me the major interest is in table I where much more information exists than is at first apparent. I should like to analyze these figures in a search for the reason back of the authors' boundless faith indicated by their remarks at the end of section II concerning eventual "perfect performance."

In this review I will offer no alibi and only cold figures will be used, directly from table I without modification. I understand from the text that the heading "Operations on Internal Faults" might be more properly expressed as "Number of Internal Faults." As these were invariably cleared correctly, we can say that there were at least two correct breaker operations for each fault. I am therefore going to double all quantities given under that heading so as to approximate the number of correct operations for direct comparison with those that are incorrect. On the other hand, I will ignore the recorded 2,000 possible chances for operation as being of debatable concern.

Year Installed	Total Correct Breaker Operations	Total Incorrect Breaker Operations	Approximate Years in Service	Ratio Correct
1936	18	1	2	95 per cent
1937	12	0	1/2	100 per cent

Observe that the over-all ratio definitely and consistently *improves* with years of service, indicating that troubles with a given equipment are rapidly eliminated.

The installations of the last two years have not been in action long enough to show a trend.

But there is real comfort here again because it indicates that later days relays have been designed for this work, and we can now confidently expect to start from a higher plane at the very outset, and with the vision of such a prospect perhaps the authors are justified in looking forward to results which in comparison with their precarrier experience might not be improperly considered as "perfect performance."

The authors have told of having corrected certain troubles. Let us see what actual effect followed. We will admit that one may expect difficulties the first year or so, and then check results after these have had a chance to be ironed out. For example, the operating record for the years 1935, 1936, and 1937 for all relays installed in 1933 shows at least 74 correct operations and one incorrect. This gives 98.6 per cent correct operations, and for the speed involved that looks excellent. Even better is the entire record for 1937 with 96 correct and one faulty operation giving 99 per cent correct. But we won't confine ourselves to scattered illustrations. Maybe I just looked for good ones, so how about the rest of the figures?

If troubles met in the field are truly eliminated in the earlier years, there should be progressive improvement in equipments that have been installed for longer periods. Yearly variations are apt to be irregular and confusing. Accordingly let us lump together, for example, all the operations for the years from 1933 to 1937 of just those relays installed in 1933 and then similarly for each succeeding year and so examine all the figures broken down in groups based on the year of installation.

Year Installed	Total Correct Breaker Operations	Total Incorrect Breaker Operations	Approximate Years in Service	Ratio Correct
1933	174	5	5	97 per cent
1934	170	17	4	91 per cent
1935	32	4	3	89 per cent

E. E. George (Tennessee Electric Power Company, Chattanooga): All relay engineers realize that there have been great improvements in carrier current protection in the last few years. The present paper is therefore very welcome at this time—particularly because it includes a schematic and circuit description of the latest practice. The detailed operating record given is also of interest to both manufacturers engineers and operating engineers.

Experience with a recent installation of carrier-current relay protection indicates that the total cost per terminal is now between $4,000 and $4,500 installed, depending upon the type of backup protection, if any. Taking the lower figure of $4,000 and applying the percentage of overheads commonly used by utilities for major equipment, one gets a total carrying and operating charge of something over $40 per terminal per month, which means that on the above basis the cost of carrier protection would pay for 20 to 30 miles of pilot wire between two terminals. This figure should be reduced slightly on account of the cost of the simpler pilot wire relays. However, it is obviously improper to apply the same percentage of depreciation, obsolescence, and maintenance to carrier-current equipment and relay equipment that is applied to power houses and transmission lines, so we may say that the figure derived above is too small. As another approach to the subject let us take the carrying and operating charges generally applied by communication companies to relay, switchboard and carrier equipment which works out to be about 30 per cent per annum—and this is admittedly insufficient on special equipment requiring particularly skilled maintenance. On such as basis this would give an operating and carrying charge of $100 per month per terminal, which would lease 25 miles of toll circuit per terminal or 50 miles of circuit between two terminals. Experiments are now being made to determine the utility of ground return or composite telegraph circuits (which takes a considerably lower rental for pilot wire purposes).

It is thus apparent that the length of the transmission circuit to be protected is a primary factor in determining the economic feasibility of carrier-current protection as compared with other methods. Among other factors are the availability of existing coupling equipment for carrier-current dispatching communication, the existence of trained personnel familiar with electronic maintenance, and the availability of high-grade communication channels between the power terminals, etc.

The industry owes much to the development of equipment and methods carried on by Mr. Sporn and his associates. While any endeavor to advance the art and science of electrical transmission may frequently subordinate economics during the development period, sooner or later economic considerations will determine how rapidly new developments are accepted into general practice. Any criticism of pilot wire relaying in a carrier relay paper invites such comparisons. Although the author of this discussion is just now making his first installation of carrier-current relay protection, his experience with pilot wire protection leads him to suggest that to the seven conclusions of Messrs. Sporn and Muller should be added the following:

8. Carrier-current relaying is now becoming the accepted standard on long transmission lines. On urban tie lines and short transmission lines (regardless of voltage) pilot-wire protection is more economical and is adequately reliable. On transmission circuits of intermediate length the decisions between the two types will depend upon many factors, such as existing dispatching communication practice, existing maintenance personnel, existing commercial communication channels, availability of new capital, and other considerations which are more largely economic than technical.

O. A. Browne (Western Massachusetts Companies, Turners Falls, Mass.): The paper presented by Mr. Sporn and Mr. Muller expresses a great deal of enthusiasm for the carrier-controlled relay which one can hardly appreciate unless one has had experience with it. Our system has had these relays in service at 26 line terminals since spring of 1933, and although some troubles have been encountered similar to those mentioned in the paper, they have all been fairly easily corrected and we are just as enthusiastic as the authors.

Our relays are all of the older four-cycle type used on line sections where either one or two tapped stations were involved. This, with other complications, made any type of overcurrent or distance type of relaying difficult to work out without cascading time and current settings to the point where lines were often burned off.

Since the installation of carrier equipment in four years we have had only one case of lines burned off during a lightning storm. Emergency line patrol and repair work on these lines during lightning storms has only averaged one case a year.

We found it desirable, as the authors did, to incorporate a time delay so that on the termination of a fault the receiver relay would not close the contacts blocking tripping until the ground and phase relays had a chance to open their contacts. We accomplished this by the installation of a capacitor in series with a resistor connected across the operating coil of the receiver relay. On the receipt of a carrier signal the capacitor is charged up. On termination of the signal the capacitor is discharged through the operating coil of the receiver relay, thus keeping it from resetting for four to five cycles. This seems to be a more positive method than the introduction of another relay with instantaneous opening and time delay closing, although it may not be applicable to the newer relays.

Reliability of tubes is borne out by our experience. In four years of operation, only one failure to block tripping could be attributed to a tube failure.

E. H. Bancker (General Electric Company, Schenectady, N. Y.): There is an intermediate stage in most developments after the principles have been outlined and the first installations made when people hesitate about adopting something new and look for the story of results secured by those who first dared to try it. The significance of this paper lies in the fact that it gives the results of several years' experience of an early user of many terminals of carrier relaying, a development that has been the subject of several previous papers, but all of which dealt with design features. Now we have the "proof of the pudding."

As a distinguished New Yorker was wont to say, "let's look at the record." First the adoption of carrier relaying was the only practical solution to certain operating problems. In other cases it was found to be the most economical solution. In still others it was the best, and even though something of lower cost might have served, the better performance expected from it was apparently thought worth its slightly higher cost. Thus, as was said of carrier relaying in its development stages, it has furnished the system designer and operator with a new tool of great utility. Secondly, it has come up to expectations in performance. As part III of the paper shows, the over-all accuracy on the comparison of correct and incorrect operations is not as good as might be desired although it compares favorably with other forms of high-speed relaying. The interesting and encouraging things are: (1) that the carrier system was called upon to function many times and the percentage failures were so small that the over-all performance was about as good as other relaying used for similar purposes; and (2) that every cause of a false operation was found and remedied so that the 1937 performance was better than could be expected of other forms of relaying. This indicates that while there may be growing pains, the adult is a healthy citizen.

Finally the carrier part of the equipment accounted for only half as many failures as were attributable to the relays. Certainly the relays cannot be said to be any more difficult to adjust and maintain than any others, and assuming what is probably true, that they were well maintained, it will be seen that the carrier part has an even better record of performance than relays. This means that the carrier equipment is even more dependable than relays, which should allay some of the doubts as to the reliability of a carrier pilot channel.

Naturally the results of experiences such as those cited in this paper are incorporated in the design as quickly as they are made known. Coupling-capacitor gaps are now enclosed in a housing at the base of the capacitor where they are protected against the weather. Line-trap capacitors are mica with no appreciable temperature error, and they are protected against damage from overvoltage by a distribution arrester having a characteristic that does protect even up to the highest traveling wave currents that may be expected.

In the modern schemes the contacts of the receiver relay are normally open and are mainatined so during an external fault so there is no race between them and the tripping relay after the fault has been cleared. Directional-relay contacts have been greatly improved by the addition of antibounce features. Moreover they normally close to stop carrier rather than open as in the authors' scheme where the additional tripping speed of a fraction of a cycle gained thereby was considered necessary. Directional relays themselves have much increased torque and speed with lower coil burdens as a result of the induction cylinder development.

The necessity for so many correct blocking operations has been greatly reduced by diminishing the area in which the occurrence of a fault causes an attempt to trip, through the use of distance relay elements.

With these many improvements, whose origin lay within actual experience such as is reported in this paper, results at least as good as those obtained in 1937 should be realized right from the start in any new installations.

A. F. Rose (American Telephone and Telegraph Company, New York, N. Y.): The first paragraph, in referring to the use of pilot wires for relaying, states: "Neither of the above schemes, however, has ever met with any great favor in the United States on account of the prohibitive cost of pilot-wire circuits, particularly on long lines, and because in the long run the scheme itself is no more reliable than the pilot wire. Experience has shown that the reliability of the latter is not quite at the 100 per cent level desired and mandatory for proper service." The authors apparently feel, therefore, that the carrier method is superior to the use of pilot wires. With respect to pilot wires leased from the telephone companies the following facts may be of interest. These facts relate both to relative costs and reliability.

With respect to costs, figures have been presented which indicate that the use of power-line carrier channels involves a first cost installed of about $4,000 more per terminal than the pilot-wire plan. The added annual expense for two terminals, including interest, insurance, administration, taxes, depreciation, and maintenance is probably not less than $2,500 (This figure is supposed to cover the allocated cost of all work, some of which might conceivably not add to "out of pocket" cost in an individual case but which would form part of an engineering comparison) which would seem to indicate that, in general, the use of leased channels would be economical up to 75 to 150 miles. The capital outlay and risk of obsolescence with the carrier scheme would thereby be avoided as would also the necessity of specially trained personnel for maintaining the equipment.

With regard to reliability, figures are presented in the latter part of the paper pertaining to the performance of a number of carrier systems over a period of several years. These figures show that out of about 2,000 occasions when called upon to transmit a signal, the carrier channels failed to function in 27 instances, only nine of which are, however, attributable to failure of the actual carrier channel. Converting the authors' figures to a percentage basis, which eliminates any need for consideration of the number of systems or years of service, we find that the carrier channels were out of service one-half per cent of the time, or putting it in another way, each circuit was on the average out of service about 1.5 days per year. Extensive summaries of failures on commercial telegraph channels, which should be representative of the type of channels which would be leased for pilot-wire relaying, indicate "out of service periods" of about an hour and a half per year. This is less than one-twentieth of the "out of service" time indicated for the power line carrier arrangement.

Philip Sporn and **C. A. Muller:** Referring to the statement of Mr. R. M. Smith that in the one-cycle system described by the authors the speed of clearing faults has been gained at the expense of reliability in blocking on external faults: Our opinion is that this is not true but, on the contrary, the scheme described is more reliable than the scheme employing circuit-closing contacts to stop transmission of carrier. It is evident that in the scheme described by the authors the employment of the auxiliary lockout relay prevents any false tripping from occurring on external faults due to quick reversals of power which occur on parallel lines and loop systems. On the other hand, in the scheme employing curcuit-closing contacts to stop transmission of carrier, the prevention of false tripping on external faults due to sudden power reversals is based on the premise that the directional relay contacts at one end of the line will open before the corresponding contacts at the other end of the line will close. It is the authors' opinion that under certain conditions the circuit-closing contacts may close before the circuit-opening contacts open, in which case false tripping would occur on external faults. The reason for this is that the relay whose function is to close its contacts on sudden reversal of power, has a much higher voltage applied to its potential coils than the directional relay whose function is to open its contacts. This is very apt to happen if the contact spacing of the directional relays is so adjusted as to obatin a maximum relay time of one cycle for the clearing of all types of internal faults.

In regard to Mr. Smith's comment of the auxiliary lockout relay introducing an undesirable time delay in clearing an internal fault which occurs immediately after an external fault, we wish to point out that it has been the authors' experience that this occurs very infrequently. However, when it does happen this extra time delay will amount to approximately three to four cycles, which is not considered very serious due to it occurring so seldom. Moreover, the employment of this auxiliary lockout relay gives out of step protection to the scheme without the addition of a complicated relay arrangement usually recommended for out-of-step protection on carrier current relay systems when such protection is desired.

The question of employing one set or two sets of fault detector relays, referred to by Mr. Smith, to prevent the possibility of incorrect tripping on external faults when the fault currents are near the minimum pickup value of the relays was thoroughly studied by the authors, and it was their conclusion that only one set of fault detector relays was necessary; the adoption of this obviously results in a simplified relay scheme. For phase protection it will be noted that the impedance-type fault detectors are employed. In case of an external fault, the fault detector at the line end where power flows into the bus bars has a lower voltage applied to its potential coil than the fault detector at the opposite end of the line has applied to its potential coil. Hence, this fault detector will have a lower current pickup than the fault detector at the other end of the line. However, this same fault detector starts transmission of carrier, blocking tripping action at both terminals of the line. Therefore, for phase-to-phase external faults no incorrect tripping should occur. For external ground faults, there is a possibility of incorrect tripping occurring when the ground currents are near pickup value of the ground fault detectors due to inaccuracies of current transformer ratios. However, the authors' experience up to the present time has not indicated an incorrect operation from this cause, thereby proving the infrequent occurrence of this condition. If future experience should show this to be serious, we expect to employ two ground fault detectors.

The authors believe the employment of a three-phase power directional relay is more satisfactory than the use of three single-phase power directional relays described by Mr. Smith. Our experience has been that a three-phase power directional relay is more reliable in its operation for phase-to-phase faults near the station bus bars. When applied to carrier-current relaying a much simpler relay arrangement is obtained than by the use of three single-phase power directional relays in conjunction with a ground directional relay. The type-*CBP* relay illustrated in the paper having only three phase elements is only applied when the minimum ground currents are far in excess of maximum load currents. In other words, it is only applied on the line section where ground-directional relays are not necessary. On line sections where ground-directional relays are necessary, the type-*CCP* three-phase directional relay is applied having three phase elements and a ground element operating on the same shaft. In this relay the ground element is made very sensitive and no difficulty is experienced with overcoming the torques produced by load currents. Furthermore, the employment of the type-*CCP* eliminates the use of a ground preference scheme, thereby simplifying the relay arrangement.

Finally, after carefully comparing the relay arrangement described by the authors with that described in the paper by Harder, Lenehan, and Goldsborough, referred to by Mr. Smith, the authors are of the opinion that the scheme they describe is a more simplified arrangement requiring less maintenance and having less likelihood of false operation. To an operating man the scheme described in the paper by Harder, Lenehan, and Goldsborough appears to be a Christmas tree setup with relays in profusion acting as the decorating medium.

Messrs. Rose and George both raise the question of the relative costs of carrier-current channels versus leased pilot wires from the telephone companies. The following figures may be of interest: The first cost for the system employing carrier-current channel is approximately $4,000 per terminal, which includes the cost of protective-relay equipment. Of this amount $2,400 represents the installation cost of carrier-current channel per terminal, and $1,600 represents the installation cost of the protective-relay equipment. The annual carrying charges, including interest, taxes, depreciation, and maintenance, for the carrier-current channel would be approximately $1,000 for two terminals. This represents a fixed and maintenance charge of over 20 per cent. Assuming a rental for leased pilot wires of $50 per mile per year for two conductors, then it is evident that the use of leased channels will be economical up to twenty miles instead of 75 to 150 miles as stated by Mr. Rose.

With regard to reliability of operation of carrier-current channels versus leased pilot wires, the following analysis may be of interest: The operating record of carrier-current relaying given in the paper covers 16 line sections which have been in service for an average time of 3.3 years. In other words, the operating experience given in the paper covers 53 line-section years. During this period only nine incorrect operations occurred due to the failure of the actual carrier channel. Extensive summaries of failures on commercial telegraph channels, which should be representative of the type of channel which would be leased for pilot-wire relaying, indicate "out of service periods" of about an hour and a half per line-section year. Then for the period covered by the operating experience in the paper, namely, 53 line-section years, the total outage period would amount to 79 hours. We believe that at least half of this outage period would occur during severe lightning and sleet storms just at the time that faults occur on transmission lines. The authors believe, and operating engineers in general will agree, that with the leased pilot-wire channel out of service 40 hours during severe lightning and sleet storms, that more than nine cases of trouble could be expected on the transmission system during that distress interval. There would of necessity, have to result more than nine incorrect relay operations chargeable to the leased pilot wire channel. It needs to be kept in mind, too, that all of the incorrect operations due to failure of the carrier channel occurred in the first two years of carrier-current experience, and that during the remaining two and a half years of carrier-current operation experience no incorrect operation occurred due to the failure of the carrier channel. Furthermore, these nine failures were all attributable to faulty design which has since been corrected, and no further failures from these sources are to be expected in the future. This has been abundantly demonstrated by the last two and a half years' operation, which was 100 per cent correct in so far as the carrier channel is concerned.

The authors are very gratified to find that the experience of Mr. Browne, both as regards to benefit obtained from carrier relaying and as to reliability of component parts of the equipment, has been the same as theirs.

Mr. Bancker's analyses of the cause of the original mild difficulties with carrier are interesting, and, the authors believe, to the point. They very definitely cannot quarrel with the conclusion reached by him, since they believe their experience definitely bears this out, and that results at least as good as those obtained on their system in 1937 can be realized in any new and modern carrier installation. The same thing holds true with regard to the discussion of Mr. Traver. The authors believe with him that the results that are being obtained with carrier today are something that neither they nor any other practicing relay engineer of a decade ago even dared to envision, and the results that are obtainable today with carrier are as close to perfect performance as there is a call for in a system aiming to give the very highest type of service. Carrier relaying very definitely has reached the point where its performance, given proper application engineering, can be predicted with the utmost certainty. That performance is in general so much of an improvement over anything that can be obtained by any other single relay scheme, that there does not appear to be any doubt that as its performance and functioning becomes better known, its use is bound to spread more and more widely.

A System Out of Step and Its Relay Requirements

Discussion of a paper by Leslie N. Crichton published on pages 1261–7 of volume 56, 1937, AIEE TRANSACTIONS (October 1937 issue of ELECTRICAL ENGINEERING) and presented for oral discussion at the relays and reactors session of the winter convention, New York, N. Y., January 25, 1938.

E. H. Bancker (General Electric Company, Schenectady, N. Y.): Man's easiest way of learning seems to be through much repetition. Not many of us have frequent occasion to study loss of synchronism and yet it has an important effect upon the action of many types of relays. Therefore, although the information presented in this paper is not new, it is given in a new and easily understood manner, and as it is knowledge we relay engineers need, it bears repetition. Perhaps if we read or hear it enough times we may be able better to interpret records of peculiar relay action or to determine in advance what may be expected to happen to some of our relays when systems swing or lose synchronism.

A little work has already been done in determining how loss of synchronism affects relays, especially the distance type. This indicated that there was some danger of sectionalizing at several points between the machines out of step, thereby leaving areas without a source of supply. Several solutions have been proposed, most of them including means for preventing distance relay action under the circumstances and requiring an out-of-step relay to separate at the most convenient point.

As the author says, most out-of-step relays must wait for three or four slip cycles to differentiate completely between loss of synchronism and every other condition of operation. This should usually be satisfactory, but it may be of interest to know that it is possible, if necessary, to construct an out-of-step relay that will operate during the first slip cycle and for no other condition except loss of synchronism.

This paper will have served its purpose if it merely helps a few of us to get a better mental picture of the quantities existing when machines go out of step. The simple, direct analysis is a big help in acquiring an understanding of this complicated phenomenon.

H. R. Paxson (Philadelphia Electric Company, Philadelphia, Pa.): In Mr. Crichton's paper the graphical method of treatment of the conditions at different points on a system running out-of-step enables the rather complex current and voltage relationships to be readily visualized. This method of representation would have been extremely useful to us early in 1935 when the problem was forcibly presented to us by a large frequency converter pulling out of step and continuing to run out of synchronism until considerable damage to the machine resulted.

Previous to this occurrence we believed that induction-type overcurrent relays would furnish protection against a condition of this kind. This belief was based on the fact that our smaller frequency converters equipped with induction type overcurrent relays had been clearing successfully during out of step conditions. Since the induction relays in the case of the large frequency converter did not perform as expected on the occasion in question, an analysis of the problem was made for the purpose of developing suitable out-of-step protection. As the action of induction relays was first investigated a few words on them is appropriate at this time.

The reset time of an induction relay on zero current is constant for any given lever setting while its operating time is a function of the current so that on a varying current such as results from a synchronous machine running out of step, the relay contacts will swing back and forth but will not actually close until the current swings become great enough for the relay disk to travel farther forward than backward on each swing. The time taken by the relay is therefore the time required for the disk to oscillate back and forth until the summation of the differential amounts of travel on each swing equals the total travel.

For a given setting the inverse time-current characteristics of induction relays makes the number of poles which may be slipped before tripping a function of several variables which include the maximum value of the current surges as a percentage of the relay tap setting and the speed with which the disk resets on currents below the tap setting. On the basis of using relay settings determined necessarily by the maximum load and the need for selectivity with other relays, it was found that if the generating capacity was high compared to the rating of the frequency converter, induction disk relays will eventually trip on out-of-step conditions but if the generating capacity should be low they may not trip at all. As three pairs of poles had been established as the maximum number which the protective relays should allow the frequency converters in question to slip, induction relays were obviously inadequate for this application.

Fortunately an oscillograph record was obtained. This showed that two pairs of poles were slipped during the first second after clearing the fault which initiated the disturbance and that the number of pairs of poles slipped, increased to seven per second at the end of the ninth second where the record stopped. With this background a study of out-of-step relay protection was undertaken. A method of analysis somewhat different from that presented by Mr. Crichton was followed but substantially the same conclusions were reached with respect to the requirements which the relays should meet in order to distinguish an out-of-step condition from hunting, overload swings, and faults elsewhere on the system.

Several relay designs were considered but at the conclusion of the study the out-of-step protective scheme described toward the end of Mr. Crichton's paper was chosen and installed on both the 25-cycle single-phase generators and 60-cycle three-phase motors of two 30,000-kw and three 15,000-kw frequency converters. The out-of-step relays consist of a duodirectional watt element whose contacts are in series with an overcurrent element. The two elements working together operate a notching train of auxiliary relays so that three complete power reversals accompanied by overcurrent are required to pick up all the relays in the notching train and close the tripping circuit; a timing device automatically resets the notching relays.

The overcurrent elements were set to pick up on currents ten per cent above the continuous ratings of the machines and the timing device was adjusted to reset the notching train in five seconds on the basis that under out-of-step conditions more than three surges would always occur in this interval. These relays were installed in November 1936 and although a number of system disturbances have occurred their performance to date has been 100 per cent correct which tends to confirm the correctness of the principles that were developed.

F. C. Poage (Ebasco Services, Incorporated, New York, N. Y.): Rewriting the equation from the author's figure 1 (*F*)

$$I = \frac{E_G - E_M}{Z}$$

We have

$$I = \frac{E_G}{Z} - \frac{E_M}{Z}$$

Let us analyze the expression E_G/Z. E_G/Z represents a fictitious short-circuit current supplied by the generator which would flow in the system *if* the internal voltage of the *motor* were removed and replaced by a short circuit as in the author's figure 4.

Likewise, analyze the expression $-E_M/Z$. $-E_M/Z$ represents a fictitious short-circuit current supplied by the motor which would flow in the system *if* the internal voltage of the *generator* were removed and replaced by a short circuit.

Thus in this case the total current I, through a given portion of the circuit, is the sum of the two fictitious short-circuit currents whose directions are, one positive and the other negative with respect to an arbitrary positive reference direction.

The voltage between conductors (or between phase wire and neutral as in conventional polyphase calculations) at any given point in the system is the *sum* of the fictitious voltages existing at this point in the two cases, first: if the internal voltage of motor is eliminated and replaced by a short circuit, as in the author's figure 4, and second; if the internal voltage of the generator is eliminated and replaced by a short circuit.

So far this analysis has followed the standard, but not commonly used method of network analysis by the method of superposition.

The author's power system may thus be considered the summation of two systems, each having but a single generated voltage (internal voltage of motor or generator) with the other generated voltage removed and replaced by a short circuit. It is apparent that each of these two systems may operate at its own frequency, and in operating at its own frequency may be out of synchronism with the other system, producing pulsating or alternating variations in voltage, current, power, etc., at beat frequency (or slip) equal to the difference between the two machine frequencies.

The illustrations and discussions offered by the author clearly illustrate the results obtained when two such systems are combined in a step-by-step synthesis from the component systems. This method may be extended to calculate the currents and voltages existing in a complicated system having (*n*) generators or sources of internal voltage by building up from (n) systems, each of which has but a single generator or source of internal voltage with all other internal voltages removed and replaced by short circuits.

This analysis of the out of synchronism operation requires that voltages, currents, impedances, and power quantities be determined for each of the component systems operating at its own frequency. Each of such component systems may be analyzed by the standard methods of short-circuit analysis now well established.

It should be noted that the principal difference between the equations cited in this discussion and those used by the author is the exchange of polarity signs in the equations for voltage and current, which change should not be cause for confusion to the reader.

Temperature Limits for Short-Time Overloads for Oil-Insulated Neutral Grounding Reactors and Transformers

Discussion and author's closure of a paper by V. M. Montsinger published on pages 39–44 of this volume (January section), and presented for oral discussion at the relays and reactors session of the winter convention, New York, N. Y., January 25, 1938.

F. Von Voigtlander (Commonwealth and Southern Corporation, Jackson, Mich.) This paper suggests standards of time-temperature limits based on allowable insulation aging. The statement is made that devices with closed-type coils (that is, without oil ducts) could operate during a fault at somewhat higher temperatures than could devices with open-type coils (that is, with oil ducts), and the temperature limits for the closed-type coils are shown to be considerably higher than for the open-type coils, tables II and III. Under leif expectancy, however, it is shown that for a given set of conditions, ten operations per day are permissible with the open-type coils but only 1.3 operations per day are permissible for the closed type coils for the same conditions. The premises for the comparison of the performances of these two types of coils and their allowable temperatures do not seem clear.

From time to time it becomes necessary or desirable to use standard power of distribution transformers for grounding banks. The question then arises as to what is the permissible rating of the transformer whose rating under normal loads is known, when it is used on a one-minute of perhaps an even shorter basis as a ground bank. Could not a similar attack, as that developed in this paper, be used to determine the ratio say between the continuous rating and a short time rating of such transformers?

D. R. MacLeod (General Electric Company, Erie, Pa.): This paper by V. M. Montsinger is of value in determining the ratings of transformers for electrified railway systems. It is easy to show that two different load cycles having the same root-mean-square current will result in different temperatures. This paper and its predecessors show that the aging of transformer insulation is a function of the temperature and the time that the temperature exists.

The thermal characteristics of motors, rectifiers, converters, etc., differ widely from those of transformers and the temperature limitations of all these equipments differ. In selecting transformers for use in railway substations and on locomotives, multiple-unit cars, etc., the most economical design will only be secured by studying the load cycles on which the equipment operates. The ratings should be determined by calculating the temperature-time curves and interpreting these results in terms of the life of the equipment. This is of particular importance on rolling stock where space and weight is at a premium. Higher temperatures can be tolerated than on central station systems with the same aging because the load cycle is such that the transformers are idle or running cool a great part of the time.

This method of calculating aging units enables the engineer of a railroad to calculate the relation between the life he will obtain on an emergency load cycle with the life which he would obtain with a load cycle under normal conditions. He can evaluate the cost of reserve equipment in terms of reduced life on necessary equipment.

In short it enables the engineer to apply a transformer intelligently to fit the enconomic design of his particular system. The old method of determining ratings by the root-mean-square current failed entirely in this respect.

V. M. Montsinger: In connection with the point raised by Mr. Von Voigtlander, the higher temperatures for the closed type of coil were predicted on the fact that closed-type coils cannot be used to carry a load continuously as can open-type coils. It is, of course, true that for carrying current only during faults, open-type coils could attain higher temperature limits. For example, if 160 degrees is the safe limit for one minute for open-type coils, the one-minute temperature limit for closed-type coils to give the same aging should be approximately 135 degrees centigrade. This is covered in the paper. On the other hand, if 160 degrees centigrade is safe for closed-type coils, the temperature limit could be raised to approximately 190 degrees centigrade for open type coils. 160 degrees centigrade may be too conservative for open type coils but, as pointed out in the paper, we have no experience to justify increasing the temperature limit above 160 degrees centigrade for one minute.

I agree that it is possible to determine the ratio between the continuous rating and a short-time rating of a standard power or distribution transformer used as a grounding transformer. To determine the temperature rise of a power transformer when used as a grounding transformer, it will be necessary to obtain from the manufacturer the winding losses in terms of watts per pound or amperes per square inch. Any standard formula can be used to calculate the temperature rise.

I am glad to learn from Mr. MacLeod's discussion that he intends to make use of the method followed in the paper to obtain the ratings of electric locomotive transformers which carry heavy overloads for only a short time and with light loads or no load the remainder of the time.

While the method used in the paper was applied to a specific kind of service—faults on neutral grounding devices—there is no reason why it cannot be applied to any kind of power service.

It should be understood, however, that unless a transformer has been built to carry short-time excessive overloads, similar to locomotive transformers, there may be limitations other than the aging of the insulation. That is, the lead joints, bushings, load ratio control parts, etc., may cause trouble.

Characteristics of the New Station-Type Autovalve Lightning Arrester

Discussion and author's closure of a paper by W. G. Roman published on pages 819–22 of volume 56, 1937, AIEE TRANSACTIONS (July 1937 issue of ELECTRICAL ENGINEERING) and presented for oral discussion at the lightning protection session of the winter convention, New York, N. Y., January 25, 1938.

K. B. McEachron (General Electric Company, Pittsfield, Mass): Mr. Roman's description of the use of the mica spacers and carborundum insert for the purpose of decreasing time lag is of interest to me since I experimented with both of these arrangements some years ago. I am sure it would be of interest if Mr. Roman could give an explanation of the apparent reduction in time lag when using the carborundum. Our tests seem to indicate that the effect was rather erratic and could not be depended upon to always give the expected reduction.

There has been considerable discussion concerning high currents through lightning arresters in recent years, and I notice that Mr. Roman makes some mention of it in connection with the discharge capabilities of the new autovalve arrester. It seems to me that we have to be rather careful about our thinking and consequent practice with regard to high currents through arresters. Distribution arresters do not as a rule have the benefit of the shielding effects of overhead ground wires, and in many cases the insulation level is as high as the usual wood line designed transmission voltages, and in many cases will have as high or higher level than the steel-tower line. This indicates that the data already obtained for frequency of occurrence of currents of different magnitudes through distribution arresters may also be used for station arresters, at least until better data are available.

In general, it is safe to state that the difference between station arresters and line arresters is not altogether based upon the proposition that arresters in stations must carry more lightning discharge current than the so-called line type or distribution arresters. The choice must include factors such as protection level afforded, stability of performance, endurance for repetitive discharges and other factors of over-all reliability, but current-carrying capacity alone is not to be regarded as the criterion. As a matter of fact, when overhead ground wires are properly applied, close to the station at least, the current-carrying capacity of the arrester is not important, beyond certain limits which I will discuss.

The paper by McEachron and McMorris, presented also at this Convention, dealing with field measurements of discharge currents through distribution arresters, indicates in figure 1 that 95 per cent of the discharges recorded during 4,036 installation-years were less than 10,000 amperes and 99 per cent were less than 21,000 amperes.

Considering the fact that the wave shapes possible with lightning are not known, although it now appears from unpublished work that many shapes will be found, it seems wise from the point of view of getting the most reliable station protection to make all of the discharges conform to a certain maximum. Although we cannot control lightning itself, we can control the voltage which can reach the station by shielding the station, so that direct hits of unknown characteristics do not reach the apparatus. If waves of the 2,000 kv per microsecond, mentioned by Mr. Roman, are to be allowed in the station, then rates of rise of four kv per foot must be allowed for the distance between the arrester and the transformer, if the transformer itself were hit. Thus, with only 50 feet separation, 200 kv would have to be added to the arrester voltage. This simple calculation indicates how necessary it is to shield the station from at least the bad strokes, and this will of course shield the mild strokes also, if advantage is to be taken of the good characteristics of a lightning arrester.

It would seem that a proper policy is to pass the lightning discharge through a sieve, as it were, which is the effect of placing an effective ground-wire system over the station and the incoming circuit for at least 2,000 feet. Sufficient information is now available so that such a shielded area may be built so that one is certain that strokes to conductors within the area will be practically nil, even over a great many years. Such a construction places on the arrester a definite duty, instead of an indefinite and unknown duty, with the result that the protection afforded is predictable, and co-ordinated insulation may be designed to a definite protection level which may be demonstrated. In case the entire transmission line is not protected by overhead ground wires, a protector-tube installation can be made for most voltage and current ratings, so that flashover may be prevented when a wave of high magnitude meets the lower insulation level of the protected section.

With such an arrangement, the arrester and the overhead ground wire, together with the protector tubes if used, become a co-ordinated protection designed to function with certainty for the 220,000-ampere stroke as well as the 50,000- or 5,000-ampere discharge.

Furthermore, it seems to me, with as many multiple discharges as are now known to contact transmission lines, it does not appear to be wise to stress the arrester up to its limit when the protective shield over the station and adjacent lines will limit the duty on the arrester, maintaining greater factors of safety and increased reliability.

What I have said is not to be construed as an argument that arresters should not have a reasonable current-carrying capacity. They should, but, for station protection the arrester will do its job with greatest accuracy and precision if it is not called upon to discharge severe direct strokes of lightning. Furthermore, an arrester, which held the voltage down at its terminals to even normal line potential could not protect the apparatus if unlimited steepness of wave were permitted, unless the arrester were directly at the transformer terminals.

The best solution at present, therefore, for important installations at least, is to control both the magnitude and the maximum steepness of the incoming wave, by forcing it to travel through at least 2,000 feet of line conductor before reaching the arresters.

V. M. Montsinger (General Electric Company, Pittsfield, Mass.): I wish to call attention to what appears to be an error in line *D* of figure 9*b*. This gives the 115,000-volt class impulse strength of the transformer major insulation as 700 kv. The maximum impulse applied to a 115-kv transformer is 630 kv or 5 per cent over the bushing flashover under standard atmospheric conditions. If line *D* represents the demonstrated transformer strength it should show 630 kv instead of 700 kv. Furthermore, according to my experience in obtaining volt-time curves, it should turn up for times less than approximately three microseconds. The one-microsecond value should be about 720 kv. If line *D* does not represent the test level it does not mean anything.

D. D. MacCarthy (General Electric Company, Pittsfield, Mass.): Mr. Roman's paper "Characteristics of the New Station-Type Autovalve Lightning Arrester" contains a very interesting description of the construction and performance of this arrester.

The multigap structure which normally insulates the arrester from the power line is called a "switch gap." The individual gaps in this structure are porcelain spaced, are without shunting resistors, and have carborundum inserts in the face of alternate electrodes for the purpose of improving the impulse ratio. It would be of interest to learn from Mr. Roman how these inserts act to reduce the impulse breakdown, if they are equally effective for all rates of rise of applied voltage, and if the effectiveness is reduced by the condensation of metal which may be boiled from the electrodes by severe surges such as he shows in figure 8.

It is believed that tests made by the writer to determine the effect of carborundum inserts on gap spark potential will be of interest. The spark potential was measured on five single gaps in series with an impulse voltage rising at 50 kv per microsecond.

The following results were obtained:

Test Number	Carborundum Inserts	Impulse Ratio			Number of Measurements
		Maximum	Minimum	Average	
1	In place	2.36	1.49	1.87	10
2	Removed	2.04	1.45	1.85	10

Previous to test 1, these gaps had been sparked over ten times with 60-cycle voltage, 20 times with 1,500-ampere impulses, and 43 times with impulse and power voltage simultaneously applied. It will be observed that on these tests the carborundum inserts had no beneficial effect and that the average impulse ratio is about 40 per cent higher than the average shown by Mr. Roman in figure 4.

The multigap structure used in the new autovalve arresters to interrupt the follow current is referred to as a "quench gap." A high 60-cycle breakdown has been obtained by shunting the individual gaps with resistances, and a low impulse breakdown is obtained by spacing the gap electrodes with mica washers. Since a high 60-cycle and low impulse breakdown are equally desirable in the switch gap, it would be interesting to learn why the same type of construction has not been used in both gaps. It is of interest to note that resistance shunts have been used in the quench gap in the new arrester. The benefit of resistance shunts has been realized for some time and this principle was applied to the Thyrite station arrester brought out in 1930. Unlike the new autovalve arrester, the Thyrite arresters have always used resistance in parallel with each individual gap in the arrester, which resulted in several benefits. The resistors control the distribution of the normally applied 60-cycle voltage so none of the individual gaps is overstressed. This permits each gap to carry its predetermined share of the applied voltage. It makes the voltage on each gap relatively independent of the effects of water or sleet on the outside of the arrester housing and also independent of the position of the arrester with respect to grounded structures. The resistance shunts also permit the design of an arrester which has a lower impulse breakdown than would otherwise be possible. It would seem that resistance shunts would be even more necessary in the switch gaps of the autovalve arrester than in the quench gap.

The valve element blocks in the *SV* arresters are cemented with a chlorinated wax. It would be of interest to know if Mr. Roman's tests agree with tests made by the writer and his associates showing that such waxes may generate explosive pressures in the event of an arrester failure.

Mr. Roman presents data in figures 6 and 7 showing that the voltage across the valve element during the discharge of impulse current depends upon the rate of rise of current as well as upon the current magnitude. Data are not given for the fastest rates of rise for low currents. Surges of this type will occur in service when the front of a steep surge is chopped by spark-over of line insulation resulting in low current amplitude but steep rate of current rise applied to the arrester. If data are available for low currents with very short fronts, it would be of value for Mr. Roman to extend the range of data of figure 7 to include currents as low as 1,500 amperes at rates of current rise of 10,000 and 20,000 amperes per microsecond. Based upon figure 7 it would seem the arrester voltage for low-current steep surges may exceed the voltage for much higher currents which have a slow rate of rise.

Figure 8 shows current and voltage oscillograms from tests on valve element blocks which were apparently made without the gap unit and without power voltage. The total time duration of the 53,000 and 79,000 ampere surges is about 12 microseconds to current zero. It is possible that surges of longer duration will impose a more severe thermal duty on the valve element even though the current magnitudes were relatively low. Mr. Roman states that "the current in the majority of the strokes is below 50,000 amperes." He also states that "we find that the new *SV* arrester will discharge the majority of direct strokes without damage to the arrester, and the arrester while discharging these high currents will offer a fair degree of protection to the connected equipment." To justify this last statement, Mr. Roman must have other data not presented in his paper. The necessary supporting facts would include data on the time to crest and duration of lightning strokes as well as on the current magnitude, the length of the circuit between the arrester and protected equipment so that the voltage drop on the circuit inductance could be evaluated, and performance data on the actual protection afforded by complete high voltage arresters taken when actually discharging typical lightning strokes with rated power voltage applied. Also subsequent test data should be obtained on the arrester to determine the effect of actual direct stroke currents upon arrester performance.

I. W. Gross (American Gas and Electric Service Corporation, New York, N. Y.): The quench-gap station-type lightning arrester described by Mr. Roman is interesting for several reasons. First, it would appear from figure 7 that the arrester has actually passed lightning currents in the order of 30,000 to 80,000 amperes, and I assume passed them successfully. Thus it would appear that we are not far remote from an arrester that can handle successfully direct stroke currents of the higher order of magnitude.

Second, rates of current rise as high as 20,000 amperes per microsecond are mentioned, as compared with our present AIEE standards for lightning arresters of 150 amperes per microsecond. While we have no knowledge that such high rates of current do exist in the field, it is gratifying to observe that they can be handled without undervoltage rise at the arrester when they do appear.

I would like to ask Mr. Roman if the characteristic performance of the arrester given in figures 4, 5, 6, and 7 is to be taken as the maximum, minimum, or average values, and what are the maximum deviations to be expected from the curve data.

In figure 9*b* I am unable to follow Mr. Roman's story on the protection of 115-kv service with the 121-kv arrester. The transformer is shown with an impulse strength of 700 kv and the lightning arrester with an *IR* drop of 530 kv, leaving 170 kv margin between the arrester and the transformer, which certainly seems reasonable. It should be noted, however, that the lightning-arrester terminal voltage is shown as a wave approximating a $1^1/_2 \times 40$ full wave. Now the full-wave impulse test on a 115-kv transformer is 540 kv, not 700, which leaves a margin between the arrester and transformer of ten kv, i.e., less than two per cent, which is less than the tolerance of measuring either the test voltage on the transformer or on the arrester.

Although the author represents the major insulation of the transformer as being 700 kv, it is not conceivable that he proposes to protect the insulation to ground which may have a strength of 700 kv and not the turn to turn and coil to coil insulation which may be only 540 kv.

Again in figure 9*b*, curve *B* depicts the time lag curve of a $31^1/_2$-inch gap, called by the author a "co-ordinating" gap. The minimum impulse flashover of this gap is shown as 525 kv, and as mentioned above the full-wave transformer test is 540 kv, leaving a margin of less than three per cent. Further, this gap flashover of 525 kv is for a positive wave and if a negative impulse should inadvertently appear in service the voltage at the gap could rise to about 585 kv, i.e., about eight per cent above the full-wave test on the transformer.

I hope Mr. Roman can prove the suitability of the new quench-gap station-type lightning arrester without working to margins of two to three per cent between the lightning arrester characteristic and the tested strength of apparatus, and also without the use of an auxiliary air gap to back up the arrester.

W. G. Roman: Mr. Montsinger and Mr. Gross have questioned the data of figure 9*b*. Since this paper was prepared the transformer subcommittee of the AIEE committee on electrical machinery has authorized the use of kilovolts rather than gap spacing as a measure of transformer strength. Since co-ordinating gaps are no longer considered desirable, curve *B* of figure 9*b* can be eliminated and the demonstrated transformer substituted. This does not change the protection picture. In the paper it was pointed out that the arrester would not protect the $31^1/_2$-inch gap. The voltage across the arrester is approximately the same as the demonstrated transformer strength of 540 kv for long waves. In other words, a traveling wave of 5,000 kv results in an arrester drop approximately the same as the demonstrated transformer strength. As pointed out by Mr. Gross, we cannot expect to co-ordinate our protection with small differences between the arrester drop and the transformer strength. If practical, the station should be designed to prevent waves of this magnitude entering the station.

In answer to Mr. Gross's question, the curves of figures 5, 6, and 7 are average values and variations of plus or minus five to ten per cent can be expected. The data in figure 4 show the actual variation during a life test on four gap assemblies. These are representative gaps and indicate the variation to be expected.

Mr. MacCarthy's tests on gaps having silicon carbide inserts show higher impulse ratios than we have found. The tests were probably made under conditions which resulted in an unequal distribution of voltage across the gaps. Under these conditions, we have found the 60-cycle breakdown of the gap assembly will be reduced more than the impulse breakdown and the impulse ratio will, of course, be increased. In the autovalve arrester the gap tube and outer casing tend to distribute the voltage uniformly across the gap assembly.

In the higher voltage arresters, grading rings are also used for this purpose. Since the switch gap is concentrated at the top of the arrester where the highest gradients normally appear, this is not as difficult as it would be if the switch gap were distributed through the length of the arrester. We have found the silicon carbide inserts effective in obtaining a reduction in impulse ratio and their effectiveness does not seem to be reduced by either high surge current or power follow current. The chart of figure 4 shows the results of a life test using a surge of 4,000 amperes and power follow. There is no apparent change.

The resistance-shunted quench gap was specifically designed to have a high power follow interrupting ability and at the same time a low impulse ratio. But, since the low impulse ratio is apparently due to corona or a similar discharge at the point of contact between the electrode and mica gap spacer, we felt it desirable to insulate the quench gap from normal voltage. Also, by insulating the gap from line voltage the shunting resistance can be made low enough to divide the voltage equally across the quench gaps without the aid of external shields. Since there is no continuous loss due to leakage current to be dissipated, it is not necessary to ventilate the arrester.

A flashover of the autovalve blocks resulting in an arc through the chlorinated wax cementing the blocks in place can, of course, result in the generation of gas heat and can rupture the porcelain casing. However, it has been our experience that a power arc over the outside of the blocks, even without the wax, will usually result in bursting the porcelain casing.

Mr. MacCarthy asks if we have data for very high rates of rise and low crest currents. The points shown on the curves of figure 6 show the maximum rates of rise we were able to obtain at the particular crest currents shown. By using a more compact surge generator, or by placing a gap across the arrester to chop the wave on the rising front, these curves could be extended. By extrapolating the curves of figure 6 Mr. MacCarthy's point can be demonstrated. For example, the crest voltage across the blocks will be about the same for a 1,500-ampere surge rising at 7,500 amperes per microsecond as for a 10,000-ampere surge rising at 1,000 amperes per microsecond.

I believe the data presented in the paper, together with the available data on stroke currents, justify the statement that the arrester will discharge the majority of direct strokes without damage to itself and at the same time offer protection to connected equipment. Of course some of the more severe strokes may result in failure of the arrester, connected equipment, or both. The 5,000-kv surge entering a 115-kv station discussed above represents a border line case. In a paper describing arrester characteristics I do not see why we must consider inductive drops, ground resistance, etc. These are problems for the station designer and must be considered in applying the arrester, not in designing it.

I agree with Mr. McEachron that an arrester will do its job best if the station is so designed that the arrester will not be called on to discharge the more severe strokes. But I also believe it is well to know what the voltage across the arrester will be and its chance of surviving if it does discharge a severe stroke. The use of ground wires over the station and for some distance out over the lines is good practice. It is also important to place the arrester as close as possible to the equipment to be protected.

Some Engineering Features of Petersen Coils and Their Application

Discussion and author's closure of a paper by E. M. Hunter published on pages 11–18 of this volume (January section), and presented for oral discussion at the lightning protection section of the winter convention, New York, N. Y., January 25, 1938.

R. E. Hellmund: See discussion, page 304 of this volume (June section).

J. R. Eaton: See discussion, page 304 of this volume (June section).

J. R. North and **F. Von Voigtlander** (The Commonwealth and Southern Corporation, Jackson, Mich.): Mr. Hunter's paper gives a general résumé of some of the engineering features of Petersen coils and their application and it should be decidedly helpful to those concerned with the use of these devices.

There are many factors which must be carefully considered in determining the probable effectiveness of Petersen coils in a given application and, judging from our experience, these include:

1. Relative number of single-line-to-ground faults versus faults involving two line conductors.
2. Relative number of permanent ground faults versus transitory ground faults.
3. Magnitude of the in-phase component of fault current due to line resistance, insulator leakage, corona, etc.
4. System arrangement—radial, loop, multiple lines, relative location of lines.
5. Dynamic and transient over-voltages as may occur with faults at different locations, and the ability of the system insulation to withstand them.
6. Protective relay scheme and necessity for automatically clearing permanent ground faults.

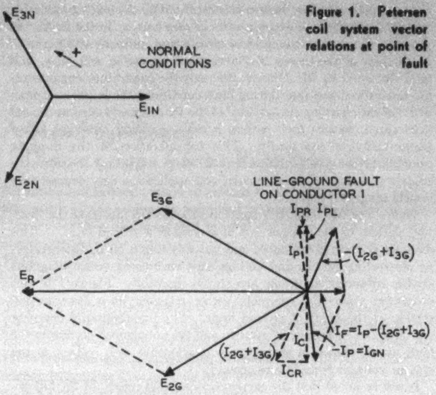


Figure 1. Petersen coil system vector relations at point of fault

E_{NG} = neutral-to-ground voltage
E_{1G}, E_{2G}, E_{3G} = line-to-ground voltages at fault ($E_{1G} = 0$)
E_R = residual voltage to ground = $E_{1G} + E_{2G} + E_{3G}$
I_C = quadrature component of charging current to ground from sound phases 2 and 3
I_{CR} = equivalent resistive component of current to ground due to conductor resistance, earth resistance, corona, etc.
$I_{2G} + I_{3G}$ = resultant line-to-ground current from phases 2 and 3
I_{PL} = inductive component of Petersen-coil current
I_{PR} = resistive component of Petersen-coil current
I_P = Petersen-coil current, ground to neutra
I_F = fault current

Petersen coils have been in operation for some years on our operating systems and we are of the opinion that they have a definite field of application in improving the performance reliability of transmission lines. However, this experience, combined with analytical studies and field tests, does not bear out some of the broad generalized statements made in Mr. Hunter's paper regarding the effectiveness of Petersen coils and it would appear that some of these statements should be qualified.

For example, the statement is made on page 11 of the TRANSACTIONS that the Petersen coil "*eliminates* transient overvoltages of the isolated-neutral system and the short circuits of the grounded-neutral system." We believe that the author here intends to convey the thought that the Petersen coil may eliminate the *effects* of the transient overvoltages. The coil automatically extinguishes the ground fault arcs initiated by the transient overvoltage (or other causes) and maintained by the dynamic overvoltage and so prevents these arcs from spreading to other phases to cause service interruptions. The thought should not be entertained that Petersen coils *eliminate* transient or dynamic overvoltages as this can be refuted by calculation, by test, and by operating experience. The use of Petersen coils may tend to limit the extremes of both the transient and dynamic overvoltages and a properly designed application may result in a considerable reduction of these overvoltages. Nevertheless, for certain conditions of system layout, fault locations, coil locations, etc., line-to-ground voltages well in excess of the normal line-to-line voltages may still be obtained during ground faults, even though the coil is in tune and successful arc extinction is obtained. Furthermore, the coils mitigate only the effects of transitory single-line-to-ground faults and have no effect on two-line-to-ground faults or line-to-line faults.

The statement is made on page 12 that "theoretically the leading charging current of the system is neutralized by the lagging inductive current from the coil, which results in zero current in the fault. In practice, this is approximated as closely as conditions will permit." Evaluation of the theory of Petersen coil operation will show, as is later discussed by Mr. Hunter, that only the quadrature components are neutralized and that during fault conditions the in-phase components of the charging current and of the Petersen-coil current are not neutralized, so *zero fault current* is never actually obtained, either theoretically or practically. This consideration of the in-phase components of the fault current is very important because the success or failure of the particular coil application may depend very largely upon this quantity.

Figure 1 of Mr. Hunter's paper is intended to illustrate the "vector relations of the components of current in a ground fault." This diagram is quite abbreviated and possibly might be misinterpreted. In illustrating current and voltage relationships by vector diagrams double subscript notations are always desirable. Figure 1 of this discussion shows the complete vector relations on a Petersen-coil system at the point of ground fault. The line-to-ground charging currents, the Petersen-coil current, and the unbalanced current in the fault are all shown in their proper relationship with respect to the system voltages from line to ground.

It will be noted that the current components consist of the following vector quantities:

$I_C = (I_{2g} + I_{3g}) - I_{corona}$
I_P = inductance + resistive components of coil current
I_F = vector resultant of I_C and I_P

It is mentioned on page 13 of Mr. Hunter's paper that test results may give charging currents as high as 50 per cent in excess of calculated values. This should be true only if certain important factors such as system layout, earth resistivity, overhead ground wires, and other factors affecting the zero-sequence impedance are not taken into account. When all these factors are considered, our studies on 44-kv and 138-kv systems have indicated that the charging current may generally be calculated with an accuracy of about ten per cent plus or minus and this has been substantiated by tests. The in-phase current components may also be predicted, but with somewhat less accuracy.

In the discussion of general applications on page 14, the author observes that theoretically one Petersen coil is sufficient to protect all parts of an entire system which are metallically interconnected. In an extensive system, the use of only one coil may permit such large in-phase components of fault current to exist that the effectiveness of the one coil would be considerably mitigated, whereas by the use of a number of smaller coils the charging current may be more fully compensated in the network branches.

Table I under "Remarks" should be corrected to read, "140 miles of line without ground wires," instead of "but 40 miles of line is without ground wire."

Referring to the "Summary" in Mr. Hunter's paper, item 3 may not be exactly clear as to the level of insulation referred to. With the overvoltage conditions which have been found by test to exist on Petersen-coil systems, it would seem that caution should be exercised in applying these coils to a system insulated only for what might be considered solidly grounded neutral operation with "full neutral displacement."

Item 4 should also be qualified, since the effectiveness of the Petersen coil depends upon a number of other factors such as: corona current, in-phase component due to unbalanced conditions, etc., as well as the size of the system and its circuit arrangement.

Mr. Hunter has called attention to the importance of the rate of rise of system recovery voltage at the fault location. This factor must be carefully considered in analyzing coil operation. Apparently, from the information available in oscillographic records of Petersen-coil systems, there is a relatively slow rate of rise of recovery voltage after the arc breaks and thus but little tendency for arc restriking. This rate of rise of recovery voltage is a governing factor in determining the maximum amount of fault current which may be successfully interrupted.

This discussion is intended to emphasize and to elaborate upon some of the statements made in this paper. It is not intended to in any way disparage the effectiveness or the desirability of Petersen coils when all the pertinent factors have been considered in their application to a power system. It is our purpose merely to point out the necessity for considering all the factors involved, particularly if the power system be of any extent.

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): One of the discussers has apparently become so enthusiastic about the use of Petersen coils that he seems to think they might be used to advantage on underground cable systems operating at between 3 kv and 33 kv, especially since Petersen coils have been used on some underground systems in Europe. Almost all of the multiple-conductor cable purchased in this country has been obtained on the basis of neutral grounding, and the insulation thickness between the conductor and ground has been made substantially less than the insulation thickness between conductors. If the Petersen coil is used, then there may be a fault on one phase with the other phases operating with the full delta voltage between conductor and sheath for a considerable time. Such operation may be permissible for some cables, but it is certainly outside of the contractual arrangements and probably is not feasible for many installations from the standpoint of the strength of the insulation. In Europe there are comparatively more systems where the insulation thicknesses are the same between conductor and sheath as they are between conductors.

It is to be noted that the use of the Petersen coil on an underground cable system is not so beneficial as on an overhead system, because the fault does not clear itself. A faulted three-conductor cable usually could not be kept in operation very long, even with the help of the Petersen coil, since the residual unbalance current through the fault would very quickly burn the insulation of the other two phases and result in a two-phase or three-phase short circuit. The larger the system, the quicker the other phase would become involved so there is little likelihood of being able to keep a faulted three-conductor feeder in service until it is convenient to repair it, at least for our cable system.

The use of the coils on a system having both overhead and underground distribution such as our four-kv system does not seem to have any attraction from the standpoint of diminishing the effects of lightning. The overhead construction is such that flashovers limit the surges on the overhead system to 300 or 400 kv, which means that the resultant lightning impulses traveling into the cable are only a small fraction of the impulse strength of the cable insulation. When all kinds of troubles were taken into account, we found that only about ten per cent of our four-kv circuit interruptions might have been eliminated by the use of Petersen coils.

E. M. Hunter: Messrs. North and Von Voigtlander have summarized very nicely some of the factors which should be considered in estimating the probable improvement in service to be obtained by installing Petersen coils to an existing power system. They have very aptly pointed out that the success of a Petersen coil as a service protective device depends upon the faults being single phase to ground. On many systems at least 70 per cent of all the faults are single phase to ground On some systems it is possible to raise the degree of protection obtained by such supplementary aids as down wires on wood poles and low footing resistances which tend to keep the flashovers single phase.

On the other hand, service protection as such is not always the major issue. The reduction in maintenance on connected apparatus has more than justified the addition of Petersen coils to some systems which were formerly operated with the neutral isolated.

These discussers have stressed repeatedly the importance of the in-phase component of current in the fault. The Consumers Power Company 140-kv system which has Petersen coils has a relatively large in-phase component of current resulting from corona. It is probable that the magnitude of this component of current on this system is in excess of that to be expected on many systems. As far as I have been able to learn, of the more than 1,700 applications of Petersen coils in the world today, all have successfully functioned to quench transitory arcs to ground without the aid of any means to reduce the in-phase component of current. Fault tests have been made on a 33-kv Petersen-coil-equipped system with more than 1,300 miles of interconnected lines without excessive in-phase components of current being recorded. However, there undoubtedly is some maximum limit to the current that will successfully extinguish itself even with the relatively unstable arc that is obtained when the Petersen coil is properly tuned and where corona is present, the in-phase component of current warrants consideration.

The statement in my paper in connection with the accuracy of calculating system charging current is to the effect that measurements on 33-kv systems give currents that are as high as 50 per cent in excess of those calculated by the aid of Maxwell's coefficients. For those who wish to check this, a measured value of charging current in a ground fault on a 33-kv line with three-foot delta spacing and 4/0 conductor mounted on 30-foot poles was 0.22 amperes per mile. On high-voltage lines, there has been in general a closer agreement between the currents calculated by these coefficients and those obtained by test.

In connection with vector diagrams, the one in the paper has been drawn by viewing the system from the neutral where the Petersen coil is located to the point at fault while the discussers' is apparently drawn from a position at the fault looking into the system to the neutral where the Petersen coil is located. An examination of these two vector diagrams shows that in general they both correctly represent the neutralizing of the current in a ground fault. For example, on the discussers' diagram, although not shown, the system neutral is located one-third the way up the vector E_R from the point of fault. Drawing in a voltage vector from the neutral to phase one E_{1n}, on the discussers' diagram, will place the vector relations of system charging current and Petersen-coil current in the same relative positions that they occupy in the diagram in the paper.

The two diagrams differ in the presentation of the system residual voltage. The discussers' diagram shows the residual voltage E_R as three times the magnitude of the system line-to-neutral voltage E_{1n}. It is well known that such a voltage exists on a system only on the secondaries of those potential transformers which are connected to measure residual voltage. To show the residual voltage as three times the system line-to-neutral voltage and three times that actually existing on a system may lead to the erroneous conclusion that the application of a Petersen coil to a system will cause to be impressed upon connected apparatus three times the line-to-neutral voltage during each ground fault. Since this is decidedly not the case, this vector diagram with its residual voltage E_R should be shown with an explanation of what it actually represents.

Mr. J. R. Eaton has given a very interesting discussion of the recovery-voltage problem as it is associated with Petersen coils. It is to be hoped that he will extend this investigation and present a more detailed story of the correlation of his theory with actual tests.

Mr. Herman Halperin discusses the application of Petersen coils to cable systems. At the present time there are no Petersen coils on cable systems in the United States so that we have no operating experience from which the efficacy of this type of application can be judged. However, several installations of Petersen coils on cable systems have been made in England in the last year or so and since in the past English grounding practice has closely paralleled our own, it is of interest to learn of their experience with Petersen coils on cable systems. This has been given in a recent paper presented in London in December 1937, "Line Protection by Petersen Coils, With Special Reference to Conditions Prevailing in Great Britain," by H. Willott Taylor and P. F. Stritzl, which is to be published soon in the Institution of Electrical Engineers *Proceedings*. This paper reports in particular three separate single-phase faults on three-phase 11-kv cables, which remained single-phase for extended periods. In each case, the Petersen coil permitted the cables to be kept in service until repairs could be made. From these service reports, it appears logical to conclude that the application of the Petersen coil to a cable system warrants consideration.

Mr. R. E. Hellmund is of the opinion that the Petersen coil is not the best solution of a protective problem under *all* conditions, and I am fully in agreement with him on this. Duplication of circuits, overhead ground wires and counterpoises, expulsion protective tubes, and immediate reclosure of oil circuit breakers should all be considered with the Petersen coil and the one best suited for the application selected. Comments on these various other methods of protection will be found in the article "The Application of the Petersen Coil" by E. M. Hunter, in the *General Electric Review* for December 1936.

A New High-Speed Distance-Type Carrier-Pilot Relay System

Discussion and authors' closure of a paper by E. L. Harder, B. E. Lenehan, and S. L. Goldsborough published on pages 5–10 of this volume (January section), and presented for oral discussion at the relays and reactors session of the winter convention, New York, N. Y., January 25, 1938.

T. G. LeClair (Commonwealth Edison Company, Chicago, Ill.): Like all other improvements in the art, carrier relaying brings up additional and related problems which must be solved. Some of these problems which have arisen in connection with application of carrier relaying in Chicago may be of interest because they are likely to occur on other transmission systems.

INDUCED CURRENTS

For the protection of two long overhead lines which parallel closely, a ground fault on one line introduces serious relaying problems due to induction. On the particular application in Chicago the current induced in the parallel line under maximum conditions will be greater than the fault current due to an actual fault on this line at minimum conditions.

In the case of two parallel lines supplied by a single bus and transformer, the induction is no problem, because the relative direction of the induced current and the transformer neutral current is used to obtain the directional effect. However, in the case of two lines frequently operated on separate bus sections supplied by separate transformers, the induced current flows through both the line and transformer neutral circuits and may trip the relay in error. This problem has been solved quite simply in one case by polarizing the directional relay on each line with the vector sum of the neutral currents in the transformers on both lines. In this case current in the faulted line and the net polarizing current (fault current minus induced current) are in the same direction on the faulted line and in opposite directions on the unfaulted line.

The problem is more complicated where the two lines have a parallel throughout most of their length, but have different destinations. For this case a solution which will work in some instances is the use of phase-to-ground voltage relays to prevent tripping on overcurrent unless the voltage on one phase has been reduced below a predetermined value. Also, negative-sequence current or voltage relays may be used. Another suggestion which has been made, and on which we would be interested in comments by the authors, is that of balancing out the zero-sequence current in phase-directional power relays. This can be accomplished by the installation of three three-to-one secondary current transformers in the neutral circuit of the secondaries of the main-circuit current transformers and con-

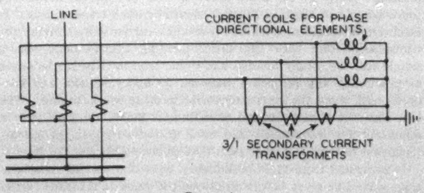

LINE
CURRENT COILS FOR PHASE DIRECTIONAL ELEMENTS
3/1 SECONDARY CURRENT TRANSFORMERS

Figure 1

necting each of these three-to-one current transformers to the current circuit of the phase-directional relays, as indicated in figure 1 of this discussion.

This application should be satisfactory where the overhead line has the normal number of transpositions so that the induced current will be distributed equally in the three phases. By the addition of these secondary current transformers, which add a third of the total ground current to the current from each main current transformer, the induced current will be balanced out and fail to appear in the relay. However, by actual fault on the line where the ground current appears in one phase only, this will not be the case and the relay will operate satisfactorily.

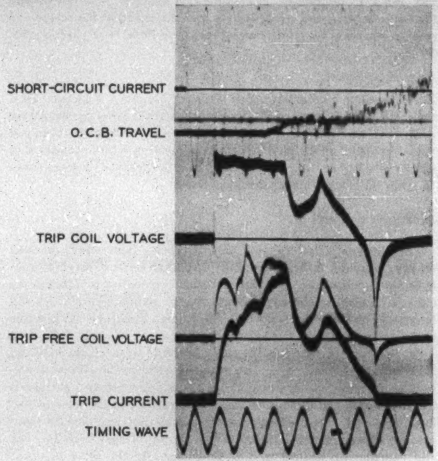


Figure 2. Performance of normal trip circuit, sustained fault

APPLICATION TO UNDERGROUND CABLE

The application of carrier relaying with which we are concerned at the moment is that of a line having underground cable over six miles of its length and an overhead line for approximately 30 miles. It has been found that in some cases it is quite difficult to send the carrier frequency current through the underground cable. For the Chicago application we had a specially favorable opportunity to compare the transmission characteristics of an underground line with and without sheath-bonding transformers to reduce cable-sheath losses. We have two lines in parallel in the same underground duct section, each consisting of three single-conductor lead-covered cable. One of these lines is solidly bonded. The other has insulated joints in the cable sheaths, with the sheaths across the joints connected together through sheath-bonding transformers. It has been found quite difficult to transmit a carrier wave through the transmission line with the sheath-bonding transformers. On a simple test the solidly bonded line would transmit the carrier wave at practically any frequency between 50 kilocycles and 140 kilocycles with about the same attenuation as on an overhead line. The line with the sheath bonded transformers would transmit only a relatively few frequencies, and even at these frequencies the attenuation was much greater than that on the solidly bonded line.

It is evident from these preliminary tests that careful considerations should be given before an attempt is made to transmit carrier frequencies over underground transmission lines with an artificial impedance inserted in the cable sheath circuit. It is apparent that the circuit for the carrier wave is normally the conductor of the transmission line and that the return path is the sheath of the same cable instead of the general ground plane.

H. P. Sleeper (Public Service Electric and Gas Company, Newark, N. J.): This paper describes a system of carrier-pilot relay protection which compares the indications of instantaneous directional relays at the two ends of a transmission line and trips or blocks in accordance with this information which is communicated between terminals by means of a carrier-pilot channel which is normally not operative. It is now considered the up-to-date relay protection of this general type and relay systems operating on this principle have been in service for several years and are quite successful.

This scheme, which has been boiled down from about 15 years' experimentation with carrier-pilot relaying, has apparently arrived at a stable and fairly dependable stage. Having been until very recently, only an interested observer of this type of relaying, but having followed it on paper for many years, there is to my mind only a single fundamental fallacy in the present perfected scheme. I refer to the fact that when a fault occurs on a power system with this type of protection on all lines, it is necessary for the relay protection on all good lines, as well as all faulted lines, to operate correctly to secure selective relaying of the defective line. This is, of course, only an extension of any ordinary system of directional relays where direction alone locks out the operation of nearby breakers on good lines, but by virtue of the transmission of information of such elements in the carrier scheme to the far line ends, it requires the correct operation of considerably more elements than with standard relaying. Unfortunately, I have no solution for the problem, but there seems to be opportunity for further development work.

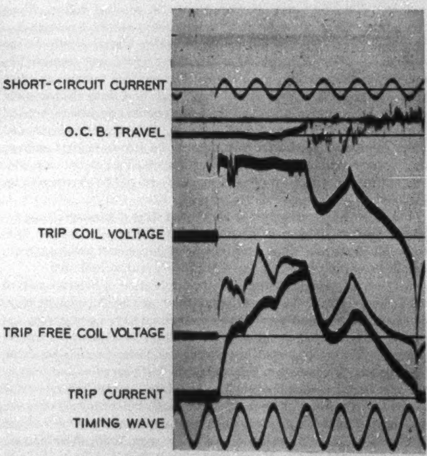


Figure 3. Performance of normal trip circuit, one-cycle fault

The authors of this paper recommend the use of several steps of distance relays to effect protection of a line unit; one to operate independently of the carrier system for a considerable number of faults on the line and back-up protection of the carrier; a second step to operate for end zone line faults and bus faults at the end of the protected section; and third step for general back-up protection for all faults beyond the line unit in question.

To the writer it would seem to constitute more efficient, dependable, and economic relaying to omit all back-up features and depend upon the carrier system alone for protection for all faults on the line in question. Similarly, the carrier protection on the next section of line would be likewise depended upon for its proper operation and no back-up relays provided in the carrier equipment. The busses would have their own differential relay protection which would be depended upon to clear its own faults, preferably, of course, with sectionalizing breakers to maintain service on the good part of the bus. Furthermore, the omission of all accurate distance-measuring elements on each line enables high-tension potential transformers to be omitted and only low-tension potentials used on the fault detector element, thereby effecting the saving of a considerable amount of money, at the same time obtaining more dependable operation of the directional elements by virtue of the impedance drop through the step-down power-transformer banks.

One problem which arises in connection with high-speed carrier relaying, or any type of high-speed relaying, and which has been recently solved by this writer in a manner believed to be novel, is that of preventing improper operation of such relays during transient short circuits, either phase-to-phase or -to-ground. There are many causes for such transient faults, which were very completely described in a paper entitled "Power System Faults to Ground" by Gilkeson, Jeanne, and Davenport, published in the April 1937 issue of Electrical Engineering. Such faults, which ordinarily last only a portion of a cycle, but may continue for as much as one cycle, will cause improper operation of circuit breakers on at least one end of a line with any system of modern high-speed relaying. Since expulsion tubes will nearly always cause such operation, it is absolutely necessary that the relays be capable of distinguishing between this and a legitimate fault which will be sustained, if not relayed out.

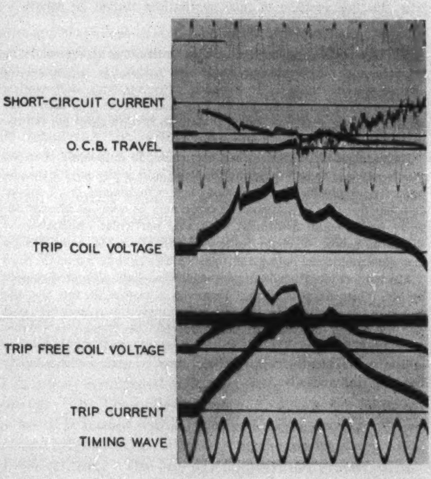


Figure 4. Performance of modified trip circuit, sustained fault

The solution worked out by the writer consists of the use of a reactor and condenser in the d-c trip circuit of the high-speed relays. Without such auxiliaries the reactance of this circuit is sufficiently low to allow the direct current to build up almost instantly when the relay closes contacts; and even though the contacts open a cycle later when the fault disappears, the arc will be maintained sufficiently long across the contacts to permit the high-speed breaker to open. The introduction of a properly proportioned additional reactance in series with the breaker trip coil and the application of a proper capacitance across the relay contacts prevents this in the following manner. When the d-c circuit is closed at the relay contacts, the trip coil current starts to build up, but very slowly because the L/R time constant of the d-c circuit has been considerably increased. Hence, the current to be broken on the relay contacts is much smaller than formerly after one cycle of time. However, this alone is not depended upon but the current is actually reduced to zero at the relay contacts by the creation of an oscillatory circuit caused by the exchange of energy between the condenser and the inductance of the trip coil circuit. Actually, the oscillogram shows that several oscillations occur, and of course, the direct current arc is broken at current zero.

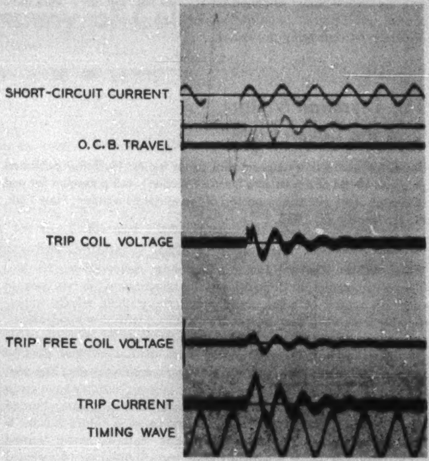


Figure 5. Performance of modified trip circuit, one-cycle fault

In actual operation the use of a two-microfarad condenser and a reactance of one henry, when used in the trip coil of a standard 37-kv circuit breaker, gave dependable operation so that a one-cycle fault was always removed without causing the circuit breaker to trip. Likewise, of course, a one- to two-cycle delay is introduced into the normal tripping operation of the breaker when a sustained fault occurs. The operation of the scheme is clearly shown in figures 2, 3, 4, and 5.

B. E. Lenehan and **S. L. Goldsborough:** Mr. LeClair's suggestion for removing the zero-sequence components from star-connected phase relays may have certain applications. The delta connection used in the paper removes the zero-sequence components. When system conditions are such that ground relays are not required, it is a suitable method of preventing induced current from tripping.

Sometimes, local conditions require certain limited power oscillations to be permitted without tripping. Star-connected phase relays will occasionally allow these oscillations to occur where otherwise out-of-step blocking would be required.

Where relays are required to operate on ground currents smaller than full-load current, separate ground relays are required. When maximum induced current is greater than minimum fault current, some other factor such as negative-sequence current line-to-neutral voltage or station-neutral current has to be introduced to properly select the operation desired. Each location has to be considered by itself to find the best additional factor to use.

We have carried on experiments in improving the transmission over an underground cable of the Public Service Corporation of New Jersey, which is equipped with sheath-bonding transformer. The joints were by-passed with four microfarad capacitors to simulate a continuous sheath at carrier frequency.

The transmission of the cable was greatly improved, approximately 30 decibels, in the ten-mile length. There are other factors to be considered, voltage induced in the sheath by switching surges and faults, being high enough to require protective gaps on the capacitors to prevent damage to capacitors or cable sheath insulators. A careful study will be required for each individual case to determine whether the power cable should be used for the carrier or a separate channel provided for it. As Mr. LeClair's tests have shown, underground construction does not necessarily bar the use of carrier-current relay protection.

The Thyratron Motor at the Logan Plant

Discussion and author's closure of a paper by A. H. Beiler published on pages 19–24 of this volume (January section), and presented for oral discussion at the electronics session of the winter convention, New York, N. Y., January 25, 1938.

W. C. White (General Electric Company, Schenectady, N. Y.): The references to tube difficulties in this paper are of interest because they are typical of what occasionally occurs along wholly unpredictable lines when new devices are combined with new applications.

Structurally and except for slight differences in characteristics, the tubes used in this thyratron motor are identical to those used by the hundreds in thyratron control of seam welding during the past few years. In this welding-control application, the tubes have given satisfactory life and the nature of failure "cathode heater burned open," which is listed as the most common cause of failure in table I, has been absent. Careful analysis of this "cathode heater burned open" indicates that it is basically a mechanical breakage of a welded connection.

On the basis of all known factors, this thyratron-motor application represents less severe service on the tubes than that encountered in welding-control practice. Certainly the failure of welds, which commonly results from frequent intermittent operation and marked mechanical vibration, is absent in welding control. However, the fact remains that such failures did occur to an abnormal degree and, therefore, there must have been a special contributing factor in this thyratron-motor cubicle, which was probably a mechanical vibration of some unsuspected frequency. This might have originated either from the machinery in the vicinity or connected with the actual operation of the cubicle controls. A very simple and minor change in this small welded connection appears to have entirely eliminated this difficulty.

A paper on such unpredictable occurrences in other electrical fields, in addition to those that have been encountered from time to time in connection with tube applications, would undoubtedly prove interesting reading. Often first-class detective work is essential to get at the bottom of such unforseen combinations of factors.

Philip Sporn (American Gas and Electric Service Corporation, New York, N. Y.): The thyratron motor, like many other tube applications that have been developed in the past decade, again demonstrates the ease with which the solution of problems involving variation in flow of current for the obtaining of proper equipment characteristics, whether they be voltage, torque, speed, or related quantities, can be obtained by the use of electronic devices. The operating experience cited by Mr. Beiler clearly shows that everything expected in the way of performance at the time the application was first developed has been attained in practice. However, it also strikingly brings out the one serious weakness of electron tubes—their short life—due mainly to the lack of sufficient experience in the building of tubes, particularly those carrying fairly heavy current. I have on other occasions pointed out that until we get to the point where engineers begin using these tubes in quantity, we cannot expect major improvements either in cost reduction or in the qualities of reliability and life comparable to that attained with other more standard classes of equipment. The experience with the tubes on the motor again demonstrates this; further it shows that the causes of the difficulties can in most cases be determined and having been determined, remedies worked out. However, what there is lacking is the integration of sufficiently extensive experience under sufficiently varying conditions. Given that experience there is no reason why this one problem in tube applications cannot be solved.

A. H. Beiler: Mr. Sporn's point about additional experience being required with electronic control is well taken. It is natural for central station engineers to hesitate before applying such control to large power station auxiliaries, but once the ice is broken there should soon be available sufficient data to furnish clues for the detective work which Mr. White mentions.

Regarding Mr. White's other comments, it is possible that the shocks incident to removing and replacing cells in the cubicle may be partially responsible for the cathode-heater breakages, but I cannot help but feel that a contributing cause may be some at present obscure phenomenon associated with this particular application of thyratron tubes. Perhaps studies of the effects of commutation may shed additional light on this problem.

Mechanical vibration from an unsuspected frequency as suggested by Mr. White is a possibility. However, in view of the fact that the motor and its draft fan are the only rotating apparatus anywhere near the cubicle and that they run very smoothly this appears as a rather unlikely explanation. Within the next year there should be evidence indicating whether the new method of connecting the cathode heater has been really effective.

Test and Operation of Petersen Coil on 100-Kv System of Public Service Company of Colorado

By W. D. HARDAWAY
MEMBER AIEE

W. W. LEWIS
MEMBER AIEE

THE HISTORIC Shoshone-Denver transmission line of the Public Service Company of Colorado connects the Shoshone Hydro Plant with Denver and serves important loads in the Leadville and Idaho Springs districts in Colorado (figure 1). The line traverses mountainous country for most of its length, crosses the Continental Divide three times (figure 2), and is badly exposed to lightning. The design and construction work on the line was begun in 1906 and 1907, making this line the pioneer in its voltage class. Conductor spacing and tower clearances are very close in comparison with modern standards, and although progressive improvements have been made, the operating record of the line shows an average of 60 tripouts per year, of which 42 are chargeable to lightning.

Tapping this line at Leadville is a wood pole H-frame line supplying the Climax Molybdenum Company at Fremont Pass and the Empire Zinc Company at Gilman. Since this line is an integral part of the Shoshone-Denver system, operating records of both lines have been added to give the performance record shown in table VI, and the combined five-year average is 69.8 tripouts per year, of which 50.8 are chargeable to lightning.

The Shoshone-Denver line is a single-circuit line approximately 154 miles long with conductors horizontally arranged on steel towers. The conductors vary in size from 1/0 to 4/0 and are spaced 10 feet $4^1/_4$ inches to 10 feet 8 inches apart. A short section at the Denver end (2.22 miles) is on wood H-frames with 12-foot conductor spacing. The Leadville-Gilman tap, 30.5 miles long, is on wood H-frames with 13-foot conductor spacing. The average height of the conductors at the towers is approximately 40 feet. The line has no overhead ground wires with the exception of the 2.22 miles of H-frame line at the Denver end, which has one overhead ground wire.

Transpositions in both the Shoshone and Gilman lines are irregular, and were installed to accommodate occasional telephone parallels, rather than to balance charging current. For example, one conductor of the Shoshone line occupies the center position for 68 of the total 154 miles of line.

During recent years rapid growth of the mining load in the Leadville and Idaho Springs areas, served by these lines, has thoroughly justified steps toward improvement in service. The two obvious methods, overhead ground wires with effective tower grounding, or expulsion protective gaps, were investigated, but did not appear to apply, due to limitations imposed by the design of the existing towers. The additional structural loading of overhead ground wires appeared excessive, and the close tower clearances made successful application of expulsion gaps very difficult. The installation of a Petersen coil appeared to be a possible solution, and was particularly attractive as its cost estimated at considerably less than that of either ground wires or gaps.

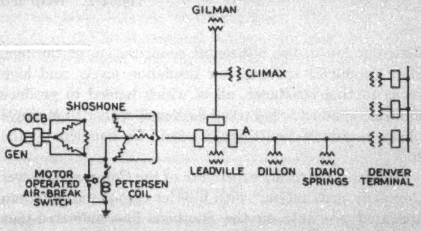


Figure 1. One-line diagram of system as normally connected

One apparent difficulty was that the line operates normally with appreciable corona loss, as the conductor is relatively small and the average elevation above sea level is 9,000 feet, with portions exceeding 13,000 feet (figure 2). Corona loss current, of course, is not compensated for by the coil. Also it was believed that the coil should be located at the center of the line to provide more accurate average tuning for faults at any point. The only neutral transformer point available was at Shoshone at the end of the line, so the cost of a grounding bank for intermediate location at Leadville or Dillon would have materially increased the cost of the project. Another

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Paper number 37-100, recommended by the AIEE committee on protective devices, and presented for oral discussion at the AIEE winter convention, New York, N. Y., January 24–28, 1938. Manuscript submitted October 6, 1937; made available for preprinting December 3, 1937.

W. D. HARDAWAY is superintendent, hydroelectrical production and transmission, Public Service Company of Colorado, Denver; W. W. LEWIS is electrical engineer, central station department, General Electric Company, Schenectady, N. Y.

4. For all numbered references, see list at end of paper.

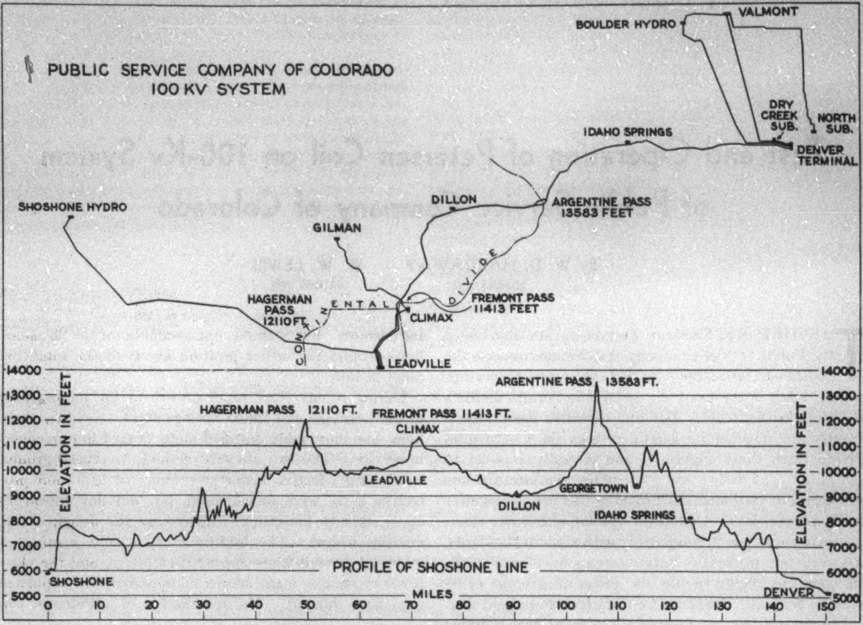

PUBLIC SERVICE COMPANY OF COLORADO
100 KV SYSTEM
BOULDER HYDRO
VALMONT
DRY CREEK SUB.
NORTH SUB.
DENVER TERMINAL
IDAHO SPRINGS
SHOSHONE HYDRO
DILLON
ARGENTINE PASS 13583 FEET
GILMAN
CONTINENTAL DIVIDE
HAGERMAN PASS 12110 FT.
FREMONT PASS 11413 FEET
CLIMAX
LEADVILLE
ELEVATION IN FEET
14000
13000
12000
11000
10000
9000
8000
7000
6000
5000
ARGENTINE PASS 13583 FT.
HAGERMAN PASS 12110 FT.
FREMONT PASS 11413 FT.
CLIMAX
LEADVILLE
GEORGETOWN
DILLON
IDAHO SPRINGS
PROFILE OF SHOSHONE LINE
SHOSHONE
DENVER
MILES
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

Figure 2. Map and profile of system

difficulty lay in the horizontal configuration of the line, close conductor spacing, low insulation levels, and high tower-footing resistance, all of which tended to produce multiple conductor lightning flashover, rather than single phase-to-ground faults which could be handled by the coil.

However, operating experience of the Consumers Power Company installation,[4] with heavier corona current than appeared probable on the Shoshone line, indicated that the corona problem was not particularly serious. Also, the Maine tests and Hunter's theoretical work,[8] indicated that a coil tuned for a fault at any point on a line would be in tune for faults at all other points on the line. This knowledge then permitted the selection of Shoshone for the coil location.

The problem of multiple-conductor flashovers still remained. Operating records were not very definite, but indicated that somewhat more than 50 per cent of the faults relayed out as phase-to-phase troubles. It appeared probable that some of these originated single phase-to-ground and developed into phase-to-phase faults before tripping. It appeared, therefore, that the coil would have a chance to extinguish some 50 per cent of the faults, and the installation appeared to be worth while on this basis. Furthermore, work was already under way on reconductoring and reinsulating portions of the line for mechanical reliability, which permitted increasing the normal suspension-string flashover distance from 30 inches to 38 inches, also a relatively inexpensive scheme of tower grounding or counterpoise was devised which would greatly reduce tower-footing resistances, and thereby the number of multiple-phase flashovers.

Under these conditions it appeared that a Petersen coil might initially quench up to 50 per cent of line flashovers, with the possibility of handling 75 per cent of the faults after the tower grounding and reinsulation work were completed. This was definitely economic, as no other scheme available approached this percentage of service improvement per dollar of investment.

Description of Petersen Coil

In the fall of 1936 some tests were made with the neutral isolated at Shoshone. A fault was placed on one conductor at Denver, each phase being grounded in turn, and the average of the three readings gave 123 amperes to ground at 95,000 volts line-to-line. With one conductor grounded at Leadville the charging current to ground was about 138 amperes at 95,000 volts line-to-line. Finally, a test was made on the Shoshone-Leadville section isolated from the remainder of the system. Extrapolating the results for the whole system gave 101 amperes charging current to ground at 95,000 volts line-to-line.

The Petersen coil as finally designed was rated 193 amperes for 10 minutes. The arrangement of the windings is such that there is practically a uniform voltage distribution throughout during transient conditions. The coil is insulated for the standard high potential and impulse tests for the 69-kv class (leg voltage of 115 kv). A shunting Thyrite resistor is installed inside the tank and designed to limit transient overvoltages to a safe value for the coil insulation.

Figure 3 shows schematically the arrangement of the coil and shunting Thyrite resistor. Ten taps are provided on a ratio adjuster for tuning and to take care of contemplated future extensions of the line, also disconnection of portions of the present line for maintenance.

Table I gives the approximate reactance values for the various taps, also the current rating for the three terminal tap connections. Based on an assumed value of 120 amperes for the present line from the preliminary tests previously mentioned, the coil will take care of a range of line from approximately 100 miles on the 855-ohm tap to 300 miles on the 285-ohm tap.

Figure 4 is an exterior view of the coil, showing the handwheel for adjusting the taps. Figure 5 is an interior view, showing the assembled core and coils and the shunting Thyrite resistor.

The coil was received in April 1937, and installed as shown in figure 1. It will be noted that the coil is installed with a stick-operated disconnect for isolation, and with a motor-operated air-break switch for by-passing the coil and solidly grounding the neutral. Although not so equipped initially, this motor-operated switch is now equipped with a timing relay which automatically closes the switch at the end of the pre-set time interval (approximately three minutes). The timer is controlled by a current relay in the Petersen coil lead, so that phase-to-ground faults not cleared by Petersen coil action are, at the end of three minutes, converted into single phase short circuits and the faulty section is relayed out.

During the period of April 21 to May 1 tuning and other tests were made on the system with the Petersen coil in circuit.

Tuning Tests

Figure 1 is a one-line diagram of the system as normally connected. All of the tests unless otherwise stated were made with this connection.

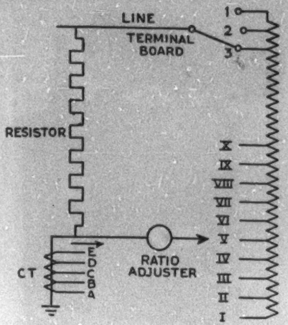


Figure 3. Schematic arrangement of coil, showing taps and shunting Thyrite resistor

Table I. Petersen-Coil Taps

Tap Number	Terminal Tap I		Terminal Tap II		Terminal Tap III	
	Ohms	Amperes	Ohms	Amperes	Ohms	Amperes
I	855	64.3	794	69.3	738	74.5
II	740	74.3	705	78.1	663	83.0
III	675	81.5	640	86.0	602	91.3
IV	620	88.8	587	93.7	547	100.5
V	570	96.5	530	103.8	485	113.3
VI	515	106.8	475	115.7	431	127.5
VII	468	117.5	425	129.4	382	144.0
VIII	440	125.0	395	139.2	347	158.5
IX	410	134.1	367	150.0	312	176.4
X	390	141.0	338	162.6	285	193.0

The coil was adjusted or tuned with the system in two different ways: First, with no grounds on the conductors. In this case the ohms were varied and the amperes through the coil read on an ammeter. When the coil and system are in tune, the amperes are maximum. The amperes flow by virtue of a residual voltage caused by the unbalance in the capacitance of the three conductors to ground. The circuit consisting of the residual voltage, the coil reactance, and the capacity reactance of the system constitutes a series resonant circuit when in tune (figure 6*a*).

Second, with a ground on one conductor, the ohms of the coil are varied and the amperes read at the line fault. When the coil and system are in tune, the amperes are a minimum, and in fact would approach zero if there were no resistance, leakage, or corona loss. The circuit consisting of the line-to-neutral voltage, the fault, the Petersen-coil reactance, and the capacity reactance of the system constitute a shunt or a multiple resonant circuit when in tune (figure 6*b*).

Table II. Tuning Petersen Coil With No Grounds on Conductors

Tap	Ohms	Petersen Coil Amperes	Petersen Coil Volts	Product Ohms × Amperes
I	855	0.6	675	513
II	740	1.0	760	740
III	675	1.3	936	878
IV	620	1.92	1,250	1,196
V	570	3.2	1,870	1,823
VI	515	4.9	2,490	2,523
VII	468	3.15	1,380	1,475
VIII	440	2.25	980	990
IX	410	1.7	680	697
X	390	1.4	528	546

Petersen coil terminal on tap I.

Tuning Tests With No Grounds on Conductors

In this test the total system was in circuit with Petersen-coil tap I connected to the line. The ohms were varied through taps I to X, and the current in the coil was read on an ammeter.

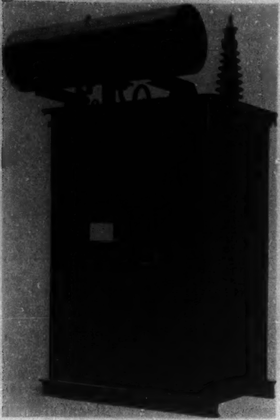


Figure 4. Exterior view of Petersen coil, showing hand wheel for adjusting taps

Table II and figure 7 give the results of the tuning tests with no grounds on conductors. In this test the maximum current was obtained on tap VI.

Tuning Tests With Ground on One Conductor

In these tests a ground was made on one conductor and the current in the fault read while the ohms of the Petersen coil were varied. Two sets of tests were made, one with a ground at Denver and one with a ground at Leadville.

Ground at Denver. In this test a ground was placed on the west phase at Denver by means of the transfer bus and an oil circuit breaker. The Petersen-coil ohms were varied and the current at the fault measured. The data are given in table III and plotted on figure 8.

Minimum current of about 22 amperes root mean square is obtained on tap VII, but the tuning is rather broad, taps VI and VIII giving only a slightly larger residual current.

Table III. Tuning Petersen Coil With One Conductor (West Phase) Grounded at Denver

Tap	Ohms	Petersen Coil Amperes	Fault Current at Denver (Root-Mean-Square Amperes)	Initial Fault Current From Oscillograph (Crest Amperes)	Product Ohms × Petersen Coil Amperes
IV	620	76	34.3	132	47,100
V	570	83	27.2	123	47,300
VI	515	90	23.1	126	46,350
VII	468	96	21.6	126	44,900
VIII	440	101	23.1	126	44,500
IX	410	107	29.0	83	43,900

Petersen-coil terminal on tap 1.

Table IV. Tuning Petersen Coil With East Conductor on Gilman Tap Grounded at Leadville

Tap	Ohms	Petersen Coil Amperes	Fault Current (Amperes)	Ungrounded Phase Kilovolts at Denver*	Product Ohms × Petersen Coil Amperes
III	675	71	39	55.9–103.4	47,900
IV	620	75	33	56.0–102.5	46,500
V	570	83	27	56.0–102.1	47,300
VI	515	90	23		46,350
VII	468	98	22.1		46,300
VIII	440	105	24.1		46,200
IX	410	113	29.5	56.7– 99.4	46,350
X	390	120	36	56.5– 99.0	46,800

* First reading before conductor grounded and second after conductor grounded.
Petersen coil terminal on tap 1.

Table V. Insulator Flashover at Denver Terminal

Test Number	Insulator String	Petersen Coil Tap	Fault Duration (Cycles)	Amperes Fault Crest by Oscillograph	
16–1	5 Hewlett	VII	54	154	2d loop
				31	30th cycle
16–2	7 Hewlett	VII	267*	260	1st loop
				31	30th cycle
16–3	7 Hewlett	VII	48	142	2d loop
				31	30th cycle
16–4	8 Hewlett	VII	57	148	2d loop
				31	30th cycle
16–5	7 Hewlett	VI	56	222	1st loop
				31	30th cycle
16–6	7 Hewlett	V	83	179	2d loop
				43	30th cycle
16–7	7 Hewlett	VII	Fuse blew in oscillograph control circuit		
16–8	7 Hewlett	VII	840	74	2d loop
				37	30th cycle
16–9	7 Hewlett	VI	57	265	1st loop
				31	30th cycle
16–10	7 Hewlett	IX	322		Off scale
				37	30th cycle
16–11	7 Hewlett	IV	272	222	2d loop
				56	30th cycle

* Opened by breaker.
Insulator string in test 16–4 dead end, in other tests suspension.
Petersen-coil terminal on tap I.

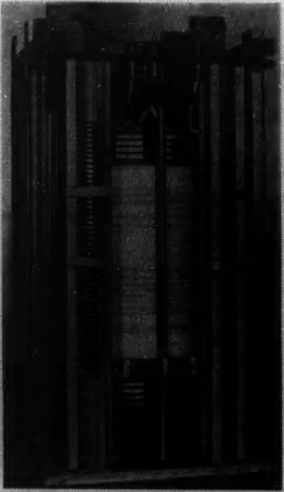
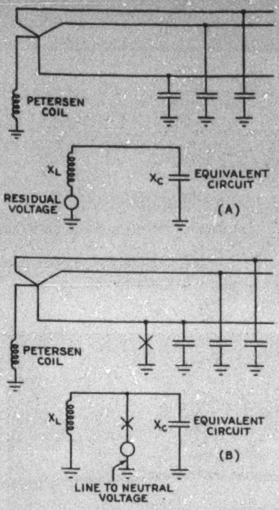


Figure 5. Interior view of Petersen coil, showing assembled core and coils and shunting Thyrite resistor



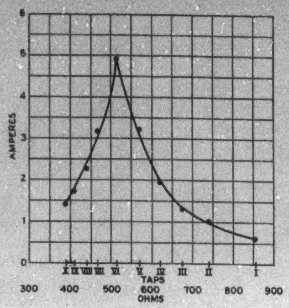


Figure 7. Tuning curve with no grounds on conductors

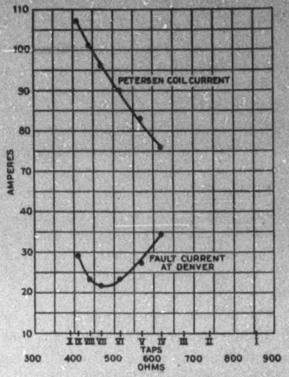


Figure 8. Tuning curve with ground on one conductor at Denver

Figure 6 (left). Equivalent circuits for tuning of Petersen coil

(a) No grounds on conductors
(b) Ground on one conductor

Conductor Grounded at Leadville. For this test the line connections were as shown in figure 9. The conductor was grounded at point *G*. The data are given in table IV and are plotted in curve form on figure 10. Here again the minimum current is obtained on tap VII and is about 22 amperes.

Arcing Tests

Several sets of arcing tests were made at Shoshone, Leadville, and Denver with the Petersen coil in service. Of these the most interesting were the tests with expulsion protective gaps at Leadville and the insulator flashover tests at Denver.

Conductor Grounded Through Expulsion Protective Gaps

A series of arcing tests was made at Leadville using three 115-kv expulsion protective gaps of different bore.

The line connections were as shown in figure 9. The Leadville-Gilman line was connected to the main Leadville-Denver line at Robison switch rack. The Leadville-Gilman line was opened toward Leadville at Climax. This allowed the Climax and Gilman loads to be fed from the main line and gave a stub line from Leadville to Climax controlled through a breaker at Leadville for testing.

The tests were made at the first structure out of the Leadville substation on the stub line. This is a three-pole, wood dead-end structure.

The external gaps were set at approximately 13 inches and a fuse wire was placed across the external gap and through the tube of the expulsion protective gap. The Gilman line oil circuit breaker was then closed, thus throwing a fault on the east conductor at the point *G* through the expulsion protective gap.

Five tests were made on large-, medium-, and small-bore tubes with various settings of the Petersen-coil taps. In four tests the arc was extinguished in 0.5 cycle or less and in the remaining test in two cycles. In all cases the current interrupted was considerably under the minimum rating of the tube. In general the arc and the report were rather mild. Figure 11 is one of the oscillograms taken during these tests.

Table VI. Tripout and Flashover Record of the Shoshone-Denver Line for Five Years 1932–1936 Inclusive, and Portion of 1937

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
Average number of tripouts, Shoshone-Denver line and Gilman tap, years 1932–1936, inclusive	1.8	0.6	2.6	2.0	6.4	7.6	26.8	15.4	3.0	1.4	1.2	1.0	69.8
Portion of above caused by lightning	0	0	0	0	4	7.0	24.4	14.0	1.2	0	0.2	0	50.8
Tripouts, 1937	1	2	2	1	10	8	8	2	0	1			
Petersen-coil operations, 1937					0*	7*	14	10	3	1			
Total flashovers, 1937	1	2	2	1	10*	15*	22	12	3	2			
Per cent of flashovers extinguished by Petersen-coil operation					0*	47*	64	83	100	50			

* Period of preliminary operation.

INSULATOR FLASHOVERS AT DENVER

At Denver terminal one five-unit suspension string, one seven-unit suspension string, and one eight-unit strain string were set up and arranged so that they could be thrown onto the transfer bus by means of an oil circuit breaker. Fuse wire was placed across the string to ground and this ground fault thrown on the bus by the breaker. All strings were composed of Hewlett disks.

Eleven tests were made, one on the five-unit suspension string, one on the eight-unit strain string, and the remainder on the seven-unit suspension string. This latter string has been adopted as standard for the sections of the Shoshone-Denver line which are being reconductored.

The tests on the five- and eight-unit strings were made with the Petersen coil on tap VII. The tests on the seven-unit string were made with various taps on the Petersen coil ranging from IV to IX. Table V gives the important data in regard to these tests.

Figure 12 shows the arc on the five-unit suspension string, with the Petersen coil on tap VII. The arc lasted 54 cycles. The exposure was for the full duration of the arc.

Figure 13 shows a one-fiftieth-second exposure of a typical arc over the seven-unit suspension string. In this particular test the Petersen coil was on tap V and the arc lasted 83 cycles.

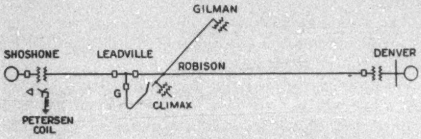


Figure 9. Line connections for arcing and tuning tests at Leadville

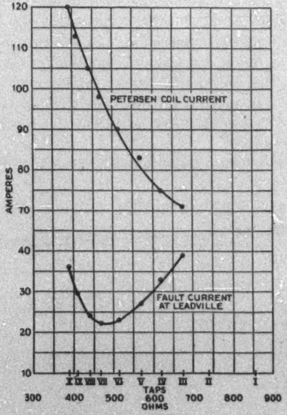


Figure 10. Tuning curve with ground on one conductor at Leadville

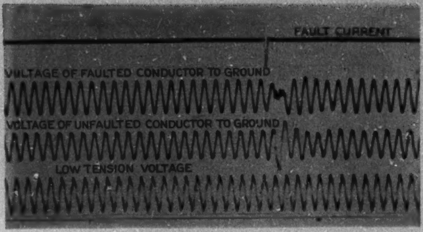


Figure 11. Oscillogram taken at Leadville during operation of expulsion protective gap with Petersen coil on tap I

Figure 12. Flashover of five-unit suspension string at Denver. Petersen coil on tap VII (test 16–1). Exposure for full duration of arc

During these tests there was practically no wind blowing. The small current in the arc would cause very little heating and consequent blowing and elongation of the arc. It is probable that these two circumstances together allowed the arc to persist from three-fourths of a second to 14 seconds in one case. Under lightning conditions there will usually be a wind, and the arc will probably snap out on an average in less time than in some of the tests.

An examination of the insulator units showed no damage to the porcelain or hardware as a result of these tests. There was no noticeable effect on the station lights nor any disturbance on the system with the exception of the ground-return telegraph circuit, which parallels the transmission line on the same right of way. This was disturbed in all of the arcing and tuning tests by induced voltage and circulating current. Figure 14 is a typical oscillogram taken during these tests.

Operating Experience

The coil was placed in service on May 1, 1937. From this date until July 1 may be considered as a period of preliminary operation. During this time unsuccessful attempts were made to work out relaying ideas at Leadville. It was hoped that an arrangement could be made which would select the faulty section of line and start a timer, which would disconnect the faulty section if the ground fault proved to be permanent, and was not cleared by Petersen-coil action. This idea proved not feasible with the equipment available, and was abandoned after some tests. Also, later operation has indicated that such a set-up, even if successful, would be of doubtful value, as the number of cases of trouble not cleared by coil action or normal phase relaying will probably be less than one per year.

An attempt was then made to arrange the ground relaying at Leadville so that it would be inoperative while the coil was in service but would be operative with the coil by-passed and the Shoshone neutral solidly grounded. This was readily successful on the Denver breaker at Leadville, but the relays on the Shoshone breakers there gave so many faulty operations that they are now kept blocked in normal operation and are manually made operative only when the Petersen coil is by-passed.

During the period of preliminary operation weak points on the system insulation were being discovered and eliminated. Two such cases involved flashovers on the Leadville transformers, which resulted in erratic relaying and interruptions.

By the first of July these difficulties were fairly well ironed out and the operation since then appears to be normal. It should be noted, however, that reconductoring, reinsulation, and tower-grounding work on the line has been progressing, and no doubt affects the operating results.

Figure 13. Flashover of seven-unit suspension string at Denver. Petersen coil on tap V (test 16–6). Exposure one-fiftieth second

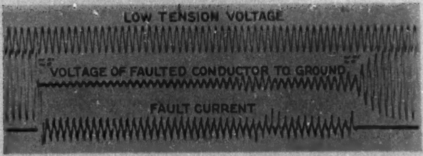


Figure 14. Oscillogram taken during flashover of seven-unit suspension string at Denver (test 16–3). Petersen coil on tap VII

Coil operation has been detected by a recording ammeter in the coil lead at Shoshone, by an automatic oscillograph at the Denver terminal, and by the effect of the communication circuit which closely parallels the transmission line.

For reasons which are not entirely clear, coil operation in service has been much faster than its performance in the staged tests, and it seems probable that some successful operations during the preliminary period passed unnoticed.

It has not been possible to keep the oscillograph on this duty continuously and some records have been missed. Oscillograms have been obtained of 21 operations. The duration of the arc ranged from 2 cycles to 216 cycles, and averaged 40 cycles.

Since July 1 all breaker operations, with one exception, have indicated as phase-to-phase or multiple phase-to-ground faults. The exception was a five-disk insulator-string failure within an indoor substation. This arc maintained for three minutes, then the Petersen coil was by-passed and the faulted section relayed out. All other single phase-to-ground arcs have been quenched by Petersen-coil action, without system disturbance.

The shock to the system at the time of coil operation is only that caused by the sudden increase in corona loss, two to three thousand kilowatts, and is negligible. However, the private telephone and telegraph circuit which is on the same right of way with the transmission line, becomes noisy and inoperative during the period of arcing.

None of the oscillograms obtained have recorded line-to-ground voltages in excess of normal line-to-line voltage.

Table VI gives a summary of the tripout data on the Shoshone-Denver and Leadville-Gilman lines for the years 1932 to 1936 inclusive; also the tripout and flashover record for 1937, during a portion of which the Petersen coil has been in operation. Between July 1 and November 1 the coil extinguished 28 out of 39 flashovers, or about 72 per cent, without circuit-breaker operation. The tripout in October was caused by a two-phase short circuit, no ground current being indicated. The fault was attributed to wet snow unloading and crossing the conductors at a transposition tower.

Conclusions

1. The Petersen coil and system were tuned by two different methods: (*a*) no grounds on conductors, varying coil ohms, and reading current through coil, and (*b*) ground on one conductor, varying coil ohms, and reading current at fault. Theoretically these two methods should give tuning on the same tap. Practically the second method sometimes requires lower ohms, because more coil current is required to balance the ground current, which is in excess of normal under this condition. The excess ground current is caused by increased capacitance due to corona, the corona current itself, and the harmonics in the charging current.

2. With one conductor grounded, tuning was on the same coil tap for a line ground at Denver and a line ground at Leadville. This confirms the conclusion reached after the Maine tuning tests, namely, that tuning would be the same for line ground at any point on the system.

3. The actual charging current at the fault was not measured during any of the tests. Assuming that it is approximately equal to the Petersen-coil current measured with a fault on one conductor, this will be about 97 amperes. This is about 1.65 times the calculated normal charging current of the system.

4. The Petersen-coil current with a ground at Denver and one at Leadville measured about the same (96 and 98 amperes respectively). Theoretically the current with ground at Denver should be considerably smaller on account of the increased line reactance. Voltage conditions at various parts of the system may have been sufficiently different during the two tests to equalize the Petersen-coil current.

5. The combination of Petersen coil and expulsion protective gap proved to be very fast and positive. In four out of five tests the duration of the fault did not exceed one-half cycle, although the current interrupted was only a fraction of the minimum rating of the tube. This combination may be valuable in locations where numerous line-to-line faults are experienced.

6. In the arcing tests over insulators, all arcs were successfully extinguished by the Petersen coil (one fault was removed by breaker at the end of four seconds). Time to extinguish the arc varied from 47 to 840 cycles. Air conditions were very quiet during some of the tests. The operating record indicates that during lightning conditions, there is usually sufficient wind to snap the arc out much quicker than in some of the tests. The observed average arcing time of 21 operations was 40 cycles.

7. The coil has been in service since May 1, 1937. In the first two months operation, trouble was experienced with faulty relay operation and breakdown of weak points in the system. In the following four months, the coil operated successfully to extinguish all single-phase-to-ground flashovers with one exception. The percentage of all flashovers extinguished by Petersen-coil operation in these four months averaged approximately 72 per cent.

References to American Literature on Petersen Coils

1. The Petersen Earth Coil, **R. N. Conwell and R. D. Evans.** AIEE, 1922.

2. The Neutral Grounding Reactor, **W. W. Lewis.** AIEE Transactions, volume 42, April 1923, pages 417–34.

3. Operating Performance of a Petersen Earth Coil, **J. M. Oliver and W. W. Eberhardt.** AIEE Transactions, volume 42, April 1923, pages 435–45, and volume 45, February 1926, pages 165–8.

4. Petersen Coil Tests on 140 Kv System, **J. R. North and J. R. Eaton.** AIEE Transactions, volume 53, 1934, pages 63–74.

5. The Petersen Coil, **W. W. Lewis.** *G. E. Review*, April 1935.

6. The Application of the Petersen Coil, **E. M. Hunter.** *G. E. Review*, December 1936.

7. Petersen Coil Installed on 100-Kv Colorado Line, **W. D. Hardaway.** *Electrical World*, September 11, 1937.

8. Some Engineering Features of Petersen Coils and Their Applications, **E. M. Hunter.** AIEE Transactions, volume 57, 1938, pages 11–18 (January section).

Discussion

H. M. Trueblood (Bell Telephone Laboratories, Inc., New York, N. Y.): Mr. Hardaway and Mr. Lewis state that none of the oscillograms have shown line-to-ground voltages in excess of normal line-to-line voltage. These are presumably the 21 oscillograms from the automatic oscillograph at Denver, which I assume is a magnetic oscillograph. The period of time covered by these observations is of course rather short. It is therefore interesting to recall[1] that in a series of 166 similar records extending over two years on one of the systems listed in table I of another paper,[2] the sound phase voltage to ground recorded by a magnetic oscillograph was less than normal line-to-line voltage in about 90 per cent of the cases.

There has been occasional skepticism regarding the reliability of the magnetic oscillograph for work of this sort, i.e., the recording of dynamic overvoltages at times of ground fault. Recent work of this nature in which the Joint Subcommittee on Development and Research has been participating indicates that such skepticism is hardly warranted. We have been finding that the magnetic oscillograph, with string vibrators tuned to about 3,000 cycles per second, appears to be quite a dependable piece of apparatus. It has been found to give good checks—within a few per cent on the average—with an automatic cathode-ray oscillograph, the tendency being apparently for the cathode-ray oscillograph to read slightly lower than the magnetic oscillograph.

It seems surprising that the observed coil current, as reported by Messrs. Hardaway and Lewis, should have been as much as 65 per cent in excess of the calculated normal charging current of the system, although this observed figure checks quite well with the empirical rule given[2] for medium voltage systems. Using 97 amperes as the charging current for the 187-mile system at 95 kv line voltage, the direct capacitance between one phase-wire and ground figures out to be about 0.0084 microfarad per mile. While data for an exact calculation of this capacitance from Maxwell's coefficients are not at hand, we estimate, from data for a somewhat similar system, that this figure of 0.0084, which presumably takes account of corona capacitance, is not more than 20 per cent above the theoretical value, without allowance for corona. I would be glad to learn the authors' views regarding this apparent discrepancy, which of course may have some simple explanation.

We infer, from a European publication, that an allowance of 10 per cent or so is considered about right in Europe (for a 100-kv system) for the various uncertain factors in capacitance calculations mentioned[2] exclusive of corona.

References

1. **Gilkeson and Jeanne,** AIEE Transactions, volume 53, 1934, pages 1301–09.

2. Some Engineering Features of Petersen Coils and Their Applications, **E. M. Hunter.** AIEE Transactions, volume 57, 1938, page 11 (January issue).

J. R. North and F. Von Voigtlander (The Commonwealth and Southern Corporation, Jackson, Mich.): This paper presents in a very concise manner the results of the tests made in connection with the application of a Petersen coil to a 100-kv system. It is of particular interest to us in view of the similar tests and studies made in 1931 when considering the application of Petersen coils to a section of the Consumers Power Company's 140-kv isolated-neutral system. The Colorado and the Michigan Petersen-coil systems have somewhat similar characteristics.

The Colorado system which constitutes a total line mileage of about 185 miles is subject to very severe lightning exposures. It is located in a very high altitude, mountainous region where both the earth resistivity and the contact resistances are undoubtedly very high. The combination of these factors, coupled with the exposure to lightning storms of considerable intensity, probably contributes to the large number of trip-outs, averaging 38 trip-outs per year per 100 line miles. With the modernization of the line by adding more insulation, reconductoring with larger conductors and installing the counterpoise in addition to the Petersen coil, the number of trip-outs per year on the line should be reduced greatly.

The method of tuning the coil with and without faults on the line conductors is identical with the method followed during the Michigan tests, and corresponding conclusions are reached. Referring to our paper, "Petersen Coil Tests on 140 Kv System," presented at the winter convention in January 1934, it will be noted that the results of our tests definitely show that tuning of either one or both of the coils was not affected by the location of the fault nor by the particular conductor faulted. This fact was thus demonstrated long before the Maine tests (mentioned in conclusion number 2), although the complete mathematical proof of this fact may not have been worked out at that time.

Under "Operating Experience," the authors advised that by-passing the Petersen coil for the condition of a permanent fault, thereby operating the system essentially with a solidly grounded neutral during this period, was successful on the Denver breaker at Leadville, but not on the Shoshone breaker there; therefore, the relays on the Shoshone breaker at Leadville are kept blocked in normal operation and are manually made operative only when the Petersen coil is by-passed. Relaying by short-circuiting the Petersen coil during times of sustained faults has proved quite successful on the 140-kv system of the Consumers Power Company so that some question arises as to what the limitations of this method of operation were found to be in the Colorado installation. Was it due to the fact that with a ground fault on the Shoshone line section the relays at Leadville would only have charging current from the other sections to operate them?

It would be interesting to have explained the control for the coil by-passing switch and the relaying for sustained ground faults on the Colorado system, particularly for such faults between Leadville and Shoshone.

The statement is also made under the heading "Operating Experience" that none of the oscillograms obtained have recorded line voltages in excess of normal line-to-line voltage. In order to obtain maximum voltages on the system as described, with the Petersen coil located at Shoshone, a ground fault should be applied to one phase of the 100-kv system at that point, the maximum voltages appearing at the distant end of the line, at Denver. Test results under this condition of maximum voltage are not reported. Calculations which we have made and later substantiated by field tests show that line-to-ground voltages of as much as 130 per cent of normal line-to-line voltages cannot only be obtained, but can reasonably be expected in extensive systems, even though the system be equipped with a properly tuned saturating-core Petersen coil.

H. P. Sleeper (Public Service Electric and Gas Company, Newark, N. J.): Why is a Petersen coil like a French telephone? Because it required 20 years' use on the Continent to introduce it to the country. In retrospect this seems rather odd, particularly in view of the excellent operating results reported abroad. On the other hand it should not be forgotten that the Petersen coil was tested experimentally in this country as long ago as 1921 and found wanting as judged by United States standards of operation (see paper by Conwell and Evans, AIEE TRANSACTIONS, February 1922, volume 41). However, the present perfected device is a far cry from the experimental model tested by Conwell and Evans and does not have the drawback of sharp tuning characteristics and the predilection to series resonance of the early design. The iron core with its broad-top tuning curve permitting detuning of the system up to 25 per cent seems to have overcome the main theoretical difficulty. The saturation of the iron core and the presence of a surge suppressor across the windings prevent the propagation of high voltages throughout the system; this latter phenomenon being the most serious practical difficulty reported by the early investigators. But the device is not yet perfect and in spite of the glowing results reported by Hardaway and Lewis the time is not yet here when the operating engineer can prescribe the Petersen coil as an antidote for all ills that can befall the operation of a modern high-voltage power system. In this writer's opinion the reported result of 72 per cent correct Petersen coil operation seems optimistically high and it would seem that the sample of operation in this case is yet too small to give accurate data which can be used generally as a criterion of expected operating results over the wide range of operating conditions presented by the various voltages, types of pole and tower line construction, terrain grounding conditions, and many other possible variables that exist here as compared with similar practice abroad. The problem presented by the all-wood-pole line is an admittedly difficult hurdle for the Petersen coil and one which will unquestionably reduce the per cent of effective operation of this device. That is not to say that the Petersen coil should not be economically applicable to that type of system, but it will require the tempering of one's expectations of close to 100 per cent operation for this device.

However, while the Petersen coil may not be the ultimate solution, it does seem to the writer that it is at least the arrow that points the direction to a new phase of high-voltage transmission system operating psychology. The practice of relaying out a faulted member of a power system is a practice universally used in this country today and has been since such operation began. This is the doctor's method of cure by amputation. But surely it is more advanced psychology to cure the patient by palliative doses rather than by operation. The same psychology applies to the maintenance of service on power systems. Why relay out a defective unit if the fault can be removed without disconnecting it from service? This might be called the method of fault removal versus fault disconnection. The superiority of the former system over the latter is unquestionable. That it cannot at this time be attained 100 per cent in no wise reflects on the soundness of the new psychology. Neither is the method of fault disconnection 100 per cent effective, and frequently its efficiency is zero per cent from a maintenance of service standpoint. Furthermore, the ultimate economic effect of the new method will always exceed that of the old since theoretically no duplicate service is now required since no disconnections will occur. Of course this is a practical fallacy, but it is undeniable that a higher degree of service should be rendered by equal facilities, and an equal degree with fewer facilities. Obviously there is some point of economic balance to be worked out in the practical use of this theory.

I don't think that the Petersen coil is by any means the ultimate answer in the application of this new psychology of power system operation. Rather it is merely a sample, or introduction. It is inherently limited to the problem of one-phase-to-ground faults. This means that its theoretical efficiency is at once limited to some value between 25 to 90 per cent, depending upon many factors. Hence, the remaining 10 to 75 per cent of faults must still resort to the old method of treatment, namely, fault disconnection. But is it beyond the bounds of conception that phase-to-phase short circuits cannot be similarly treated? It is not, of course, because it is being done today with expulsion tubes. Hence, although it may not be economical, it is possible. It does not seem unreasonable to assume that it will be done in the not too distant future by this or some other means which is well within the limits of economical feasibility for more or less universal application.

Looking forward a bit, it seems probably that the Petersen coil as we now know it will be further perfected, or maybe it will be superseded by some brain child yet unborn. Likewise the phase-to-phase problem will probably be similarly solved economically by some perfection of existing devices or others not yet developed. But, whatever form they may take, it is the writer's belief that they will merely further justify the psychology of fault removal which has been demonstrated and practically effected by the Petersen coil. And regardless of what the form may be, or where perfected, it is hoped that it will not take operating engineers in this country another 20 years to acknowledge recognition.

J. G. Hemstreet (Consumers Power Company, Jackson, Mich.): The paper by Messrs. Hardaway and Lewis, dealing with the application and operating experience of a Petersen coil on the 100-kv system of the Public Service Company of Colorado, is of particular interest to us, since the conditions are quite similar to those on the Petersen coil section of the Consumers Power Company's 140-kv system. The 100-kv system in Colorado is not quite as long as the Consumers coil section, nor does it operate at as high a voltage. Also, there are two Petersen coils in operation on the 275-mile section in Michigan.

With the thought that the operating experience over a seven-year period with the two coils on the 140-kv system would be of interest and of value in this discussion, we are summarizing it in table I of this discussion.

It will be noted that during the six-year period when automatic oscillograph records were available, 94 per cent of the line-to-ground faults were cleared without breaker operation, whereas only four per cent of the short-circuit faults were cleared without breaker operation. Of the total faults, both during the six-year period and also throughout the total period of seven years, 72 per cent of all types of faults have been cleared without breaker operation and

Table I. Petersen Coil Operation on Consumers Power Company's System

	Six-Year Period, 1931 to 1936, Inclusive, With Automatic Oscillograph		Sept. 24, 1936, to Dec. 31, 1937	Seven-Year Period Through 1937	
	Number	Per Cent	(Number)	Number	Per Cent
Number of faults					
Due to: Lightning	219	88	44	263	85
Sleet	3	1	1	4	1
Other causes	28	11	14	42	14
	250		59	309	
Line-to-ground faults	129				
Cleared without breaker operation	121	94			
Cleared with breaker operation	8	6			
Short circuits	45				
Cleared without breaker operation	2	4			
Cleared with breaker operation	43	96			
Not definitely determined	76				
Cleared without breaker operation	58	76			
Cleared with breaker operation	18	24			
Totals all types	250		59	309	
Cleared without breaker operation	181	72	42	223	72
Cleared with breaker operation	69	28	17	86	28

only 28 per cent required automatic sectionalizing of the faulted section.

It is interesting to see that this 72 per cent is the same as the experience in Colorado, as indicated in the paragraph immediately preceding the heading "Conclusions" of the paper by Messrs. Hardaway and Lewis.

We should also like to point out that during our tests, some of the arcs persisted for a considerable period. The actual operating experience with the Petersen coils has been somewhat better than that obtained during the field tests.

W. C. DuVall (University of Colorado, Boulder): In company with Professors Palmer and Eastom of the electrical engineering staff of the University of Colorado, I had the pleasure of witnessing the retuning of the Petersen coil of the Public Service Company of Colorado on the night of January 9, 1938.

If one witnessed this with the idea of seeing a spectacle, keen disappointment was in store; however, if it was viewed as an engineering achievement it was a keen delight. An arc that lasts for only two cycles is not much to see. I feel that enough has not been said about the slow recovery voltage of the faulted conductor. Oscillograms show that it takes about ten cycles for the voltage to reach normal. This gives ample time for the ionized air to dissipate and the insulation to recover to its normal value before normal voltage is reached. This slow recovery is inherent with the Petersen coil and seems to have its maximum time of recovery for the point of sharp tuning.

In the case of the Public Service Company tuning was found to be on tap number 6. Oscillograms taken between the faulted conductor and ground on taps above and below number 6 showed an oscillation, one beat was present. On taps number 5 and number 8 the voltage was found to pass through normal at about one cycle increasing and reaching a maximum overshoot at about three cycles, then decay and reach normal voltage in about ten cycles. This overshooting, or oscillation, is not present for sharp tuning. By using an automatic oscillograph and finding the tap that produces no beat is a third method of tuning and is probably the most accurate method. The reasons for this slow recovery voltage and the beat are rather complex. In returning to normal conditions the faulted conductor has its potential raised from a low value (zero in most cases) to phase voltage, conductor to ground. The clear conductors have their potential lowered from line voltage to ground, to phase voltage to ground. The charge on the faulted conductor is raised from zero to normal, the charge on the sound conductors lowered from that produced by line-to-line voltage to that produced by phase voltage.

If this automatic slow voltage recovery could be incorporated into into circuit breakers they could be redesigned more effectively. One more point: the *A. E. G. Progress*, April 1929, page 113, states that in Germany secondary cable networks of potentials as low as 3,000 volts have been successfully protected by the Petersen coil. Not to protect against surges produced by lightning necessarily, but faults produced by accidental grounding.

The relative cost of Petersen coils versus relays and circuit breakers warrants some real investigation on the application to cable networks especially at voltages from 13,000 to 30,000. This is particularly apropos in the face of some recent serious cable failures.

R. E. Hellmund (Westinghouse Electric and Manufacturing Company, Pittsburgh, Pa.) [**Editor's Note:** This discussion covers also "Some Engineering Features of Petersen Coils and Their Application," E. M. Hunter, *EE, Jan. '38, Trans. p. 11–18*]: Various grounding means, particularly the Petersen coil, were the subject of extended discussion at the International Conference on Large Electrical High-Tension Systems in Paris last June. Unquestionably a rather strong case was made for the Petersen coil, and as a result even greater use of this coil may be expected in Europe in the future. Although I discussed some of the solutions common in this country, such as grounded neutral with high-speed relays and breakers, the use of deion gaps on the line, and the use of quickly reclosing breakers, it was obvious that the Petersen coil was favored by a large majority of those present at the meeting. However, regardless of this, subsequent inspection of apparatus and power systems did not convince me that the Petersen coil is the best solution under all conditions. For example, it is difficult for me to believe that the addition of Petersen coils to a well-sectionalized double line with high-speed relay systems and high-speed breakers would be justified economically. This would make it necessary to use transformers with a neutral fully insulated against ground, which, particularly in the case of tap-changing transformers, would lead to complicated and expensive designs. The European arrangements of this kind, while highly ingenious, did not impress me favorably as compared with the American transformer designs for solidly grounded neutral.

There was also a good deal of evidence that in some applications of Petersen coils the problem of lightning protection by arresters leads to considerable difficulty. It seems that during an arc to ground, which according to the papers presented today may last a great many cycles, overvoltages occur which are hard to handle by available designs of arresters. This has led to the practice of applying arresters with a greater margin wherever the insulating level of the system makes this possible. Frequently, however, the insulating level of European lines seems to be so low as to force close application of lightning arresters and consequent frequent damage to arresters. Nevertheless, there are a great many places, especially for medium voltages, where the Petersen coil can be applied to good advantage and its use should be given serious consideration in this country. As experience with the Petersen coil is accumulated for conditions prevailing here, a gradual increase in its use may be expected.

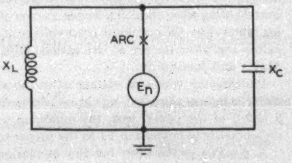
J. R. Eaton (graduate student, University of Wisconsin, Madison) [**Editor's Note:** This discussion covers also "Some Engineering Features of Petersen Coils and Their Application," E. M. Hunter, *EE, Jan. '38, Trans. p. 11–18*]: Mr. Hunter, in his paper, makes several statements regarding arc extinction on a Petersen-coil system, which on first examination appear contradictory. However, by a more careful analysis of the factors determining fundamental frequency recovery voltage across the arc, the statements are found to be in general quite correct. Refer to figure 1 which is figure 6 of the Lewis and Hardaway paper. Those authors have demonstrated that this circuit represents at least approximately the equivalent Petersen-coil system. It may be noted that this is a parallel resonant circuit energized by a generator connected to it through an arc. When properly tuned, the frequency of the resonant circuit is equal to that of the generator. Suppose the arc were to go out at current zero, as it always does at least until reignition occurs. The recovery voltage will be zero, because the voltage between the arc electrodes will be zero and will theoretically remain zero indefinitely. That is, the resonant circuit will continue to oscillate and the voltage of the condenser-inductance junction would be forever equal to, and in phase with, that of the line-to-neutral generator. Suppose next, that as in figure 2, resistance R_1 is inserted in series with X_L, and R_2 in parallel with X_C. When the arc goes out the resonant circuit will no longer oscillate indefinitely but will die down exponentially with time. The voltage across the electrodes will again be the vector sum of neutral and condenser voltages or

$$e = E_n \sin wt - [E_n \sin wt]\epsilon^{-at}$$

where a is dependent on the losses in the resonant circuit, and the rate of rise of recovery voltage will be

$$\frac{de}{dt} = E_n[w \cos wt - w\,\epsilon^{-at} \cos wt + a\epsilon^{-at} \sin wt]$$

Figure 1



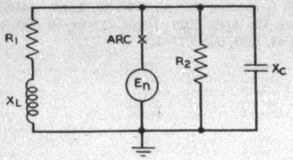


Figure 2

At $t = 0$ this becomes

$$\frac{de}{dt} = a\,E_n \sin wt$$

Suppose again that the tuned circuit has a frequency slightly different $(w - b)$ from the generator frequency—that is, the Petersen-coil system is slightly out of tune. At the instant the arc goes out at current zero, the voltage appearing across the electrodes will be approximately

$$e = E_n \sin wt - [E_n \sin (w - b)\,t]\epsilon^{-at}$$

The recovery voltage is now dependent on both a and b.

Although apparently ignored in the past, a rather complete record of recovery voltage appears at the end of every oscillogram showing a ground fault on a Petersen-coil system. It may be observed that following the clearing of the arc, system conditions do not return to normal immediately, but change gradually as the transient in the oscillating current dies out. As the losses in the circuit determine the rate of decay of the transient, it is possible to calculate from the rate of decay alone, the ratio of the in-phase uncompensated current to the quadrature compensated current.

A study of recovery voltage on the basis of the above approach, shows quite strikingly that arc interruption is much more probable on a Petersen-coil system, whether in tune or not, than would be found on a simple *LR* or *CR* circuit having arc current of the same magnitude and phase position. Moreover, it shows that for best operation Petersen coils cannot be located at any point on the system but should be located near the major part of the system capacitance, rather than being tied through a long transmission line of high ohmic resistance. Any factor which tends to decrease the time constant of the oscillating system will result in higher rates of rise of recovery voltage and hence less probability of arc extinction.

W. D. Hardaway and W. W. Lewis: Doctor Trueblood expresses surprise that the observed Petersen-coil current was as much as 65 per cent in excess of the calculated normal charging current of the system. Measurements on five systems, varying in voltage from 33 to 140 kv, have shown the ratio of Petersen-coil current to calculated normal charging current to range from 1.75 to 2.15, with a fair average of about 1.9. The normal charging current is based on the capacitance to neutral, which is susceptible of fairly accurate calculation. The capacitance to ground is more difficult to calculate accurately, as it depends on the location of the ground plane, the presence of ground wires, the proximity of trees, the increase in apparent conductor diameter due to corona, etc. Tests on a 110-kv system consisting of one horizontal circuit with two overhead ground wires showed the average measured zero-sequence capacitance (capacitance of one conductor to ground when all three conductors are in multiple) to be 1.5 times the average calculated zero-sequence capacitance. The average measured zero-sequence capacitance of one vertical 110-kv circuit with one overhead ground wire was 1.2 times the average calculated zero-sequence capacitance. The calculated capacitance in this case was based on the actual geometrical relations at the towers, assuming the ground plane to be at the earth's surface, but with no allowance for trees, connected apparatus, or influence of corona on conductor diameter.

Messrs. North and Von Voigtlander asked for information as to the control for the Petersen coil by-passing switch and method of relaying for sustained ground faults, particularly between Leadville and Shoshone.

The Leadville relaying, before the Petersen-coil installation, included normal directional relaying for phase-to-phase faults, which is of course still operative. Phase-to-ground relaying was made selective between the Shoshone and Denver breakers by the magnitude of the fault current. That is, the residual or zero-phase-sequence current at Leadville for faults on the Denver side was much higher than for faults on the Shoshone side, and a satisfactory selectivity was obtained by blocking the Shoshone breaker trip when residual currents exceeded a definite value. When the current was below this value, a residual voltage relay was permitted to make a contact and trip the Shoshone breaker.

With the Petersen coil in service, it was not possible to find current values for the blocking relays which would be inoperative during the successful Petersen-coil operation, but which would still function when the coil was by-passed. The ground relaying on the Denver side has continued to function satisfactorily. The phase-to-phase relays will trip the Shoshone breaker under the condition of a single-phase-to-ground fault on the Shoshone side with the coil by-passed, although they are slower than is desirable.

The control of the coil by-passing switch is very simple. A current transformer in the Petersen-coil ground lead actuates an overcurrent relay, which starts a timer, set to make contact at the end of three minutes. The timer contact controls the closing circuit of a motor-operated air-break switch. Opening this switch is a manual operation. The performance of this air-break switch appears to be satisfactory, and the installation was much less expensive than an oil switch would have been.

As to the question in the last paragraph of the discussion concerning line voltages: As there are no high-tension breakers at Shoshone, we were not able to make tuning or grounding tests at that point. However, we made tuning tests at Leadville by throwing a ground on one conductor and during these tests we read the voltage at Denver on one of the ungrounded phases. These readings showed that the voltage jumped from approximately 56–57 kv to approximately 99–103 kv. In actual operation during the summer of 1937, the automatic oscillograph at Denver was in service with one vibrator on a potential transformer, which was connected between one high-tension conductor and ground. Oscillograms of 21 operations did not show any voltages in excess of normal line-to-line voltage, as mentioned in the paper.

Mr. Hemstreet gives some very interesting information in regard to the operation of the Petersen coils on the Consumers Power Company's system. In regard to his statement that the actual operating experience has been somewhat better as to duration of the arcs than obtained during staged tests, this checks with the Colorado experience. In the staged tests of May 1937 insulator strings were flashed over in 11 cases at Denver, the arcs continuing from 48 to 840 cycles. In a similar set of tests made at Denver in January of this year, consisting of ten insulator string flashovers, the arcs lasted from one to 688 cycles. In actual operation during the 1937 season in 21 cases the duration of the arc varied from two to 216 cycles with an average of 40 cycles. The increased persistence of the arc during staged tests is probably partly due to metallic particles and vapor from the fuse wire used to start the arc. On the other hand, during actual operation in lightning storms there is usually considerable wind, while in both sets of staged tests the air was quiet.

Tests of May 1, 1937

Test	Tap	Ohms	Maximum Crest Voltage of Grounded Conductor: Before Arc	After Arc
16–11	IV	620	100	123
16–6	V	570	100	104
16–9	VI	515	100	98
16–5	VI	515	100	100
16–8	VII	468	100	104
16–2	VII	468	100	104
16–1	VII	468	100	104
16–4	VII	468	100	104
16–3	VII	468	100	104
16–10	IX	410	100	119

Tests of January 9, 1938

Test	Tap	Ohms	Maximum Crest Voltage of Grounded Conductor: Before Arc	After Arc
1	IV	620	100	120
2	V	570	100	109
2*A*	V	570	100	113
3	VI	515	100	100
3*A*	VI	515	100	100
7	VI	515	100	100
4	VII	468	100	105
5	VIII	440	100	112
5*A*	VIII	440	100	113
6	IX	410	100	127

We are glad that Professor DuVall noticed and commented on the slow rate of recovery of the voltage at the faulted conductor and the overshooting when the coil is under- or over-tuned. The accompanying tables show the maximum crest voltage of the grounded conductor before and after the arc at the various taps, for the staged tests of May 1 and January 9.

Plotting the values of voltage after the arc as ordinates against ohms or taps as abscissas, we obtain curves similar to those of figures 8 and 9 of the paper, with the minimum points at tap VI. This undoubtedly indicates tap VI as the proper tap for tuning.

A possible explanation for the oscillation and overshooting of the voltage of the faulted conductor after the arc breaks is contained in a paper by one of the authors in the section headed "Arcing Tests" ("The Neutral Grounding Reactor," W. W. Lewis, AIEE TRANSACTIONS, volume 42, April 1923, pages 417–34, or AIEE *Lightning Reference Book*, 1937, pages 77–94).

Discharge Currents in Distribution Arresters—II

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TWO YEARS ago, the authors of this paper described to the AIEE a co-operative investigation by the Boston Edison Company, the Commonwealth Edison Company of Chicago, the Detroit Edison Company, the Georgia Power Company, and the General Electric Company, having as its object the determination of data relative to lightning discharge currents through distribution lightning arresters. At that time the results were based upon records obtained over a 13-month period from August 1934 to August 1935, inclusive. The object of this paper is to present the results from August 1934 to September 1937.

About 1,350 installations have been made for measuring lightning discharge currents, each installation consisting of two magnetic links supported by a wooden bracket, with the links spaced approximately three-quarters of an inch and two and one-eighth inches from the arrester ground lead in most cases. The use of two links not only permits a wider range of currents to be measured, but also indicates any reversal in the polarity of the discharge current.[1]

Table I. Distribution of Surge-Crest-Ammeter Installations

Operating Company	Line Voltage	Transformer Connection	Rural or Urban	Surge-Crest-Ammeter Installations: Present Number	Installation, Years to Date	Number of Records Obtained
Georgia Power Company	120–240		Rural	19	15	19
	2,300	Delta	Urban	37	70	72
	6,900	Delta	Rural	36	165	151
	11,500	Delta	Rural	81	300	192
	11,950	Gr. Y	Rural	97	305	155
Detroit Edison Company	4,800	Delta	Urban	78	119	11
	4,800	Delta	Rural	350	1,004	419
	24,000	Gr. Y	Urban	12	39	39
Commonwealth Edison Company	4,000	Gr. Y	Urban	324	1,002	169
	12,000	Gr. Y	Rural	20	39	16
	20,000		Rural	0	2	0
Boston Edison Company	4,000	Gr. Y	Urban	298	976	365
				1,352	4,036	1,608

As described in the authors' first paper on this subject,[2] the method of measurement used is accurate within ten per cent through a range from 500 amperes to 17,000 amperes for single unidirectional waves or for damped oscillations of the type used in making the laboratory calibration. Values down to 300 amperes and up to 25,000 amperes can be estimated.

When comparing data taken with magnetic links on different systems, or even on the same system, it is recognized that since it is impossible to be sure that links are changed after each stroke, there is bound to be a certain amount of superposition of results, the magnetic link recording the highest value if there has been no change in polarity. With different systems, the method of servicing the links was somewhat different, so that in some cases the effects of several storms may be superimposed. The effort has been to change the links after each storm in so far as this could be done economically.

Data concerning the four systems being studied are given in table I. The classification of circuits as urban or rural is entirely arbitrary. Those which have more than 12 customers per mile are classed as urban, all others as rural.

Results

To secure an over-all picture of the results, it is perhaps best first to examine all of the data from all of the companies, later breaking it down for a somewhat more detailed study.

MAGNITUDE OF CURRENT

Figure 1 shows the distribution of the discharges as to current magnitude, without respect to polarity or to the circuit involved. These results are based upon 1,608 records from 4,036 installation-years. These figures include 95 records and 145 installation-years in which current to ground was measured, but not through individual lightning arresters, as for instance on secondary neutral ground connections or in the common ground lead of three-phase arresters. Twelve records exceeding 25,000 amperes were obtained. These are included in the curve, but since their magnitude is not known, the curve is not plotted beyond 25,000 amperes.

This curve shows that 50 per cent of the discharge currents through arresters may be expected to be at least as great as 1,300 amperes, while only 1.2 per cent may be as large as 20,000 amperes. Although it is rather dangerous to extrapolate data of this sort, it would appear that an occasional record of 30,000 to 40,000 might be expected. A more accurate measure of the duty on an arrester is obtained if the results from the rural and urban locations are considered separately.

POLARITY OF CURRENT

In view of the work already done with respect to lightning currents through transmission line towers,[3]

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[1] For all numbered references, see list at end of paper.

considerable interest attaches itself to the polarity of the currents measured. Of the total of 1,608 records obtained, 1,011 were negative and 597 were positive. Based on the expectancy curves of figure 2, negative currents of 5,000 and 15,000 amperes may be expected 3.7 and 3.5 times as frequently as positive currents of the same magnitude. From the point of view of the performance of the arrester, either in protecting transformers or in keeping itself out of trouble, the polarity of the impulse is of no consequence. However, it appears to be quite firmly established that direct strokes of lightning reaching transmission line towers[3] are about 95 per cent negative, although the polarity of strokes to open country is believed to be more evenly divided between positive and negative. There is laboratory evidence that the so-called cone of protection around a grounded point is greater when the point is positive than when negative. If this can be applied to natural lightning, negative cloud discharges to positive transmission lines would concentrate on the lines a much higher percentage of the total discharges than if the polarities were reversed. As the line becomes lower in height, the positive and negative discharges would approach each other in frequency of occurrence, assuming equal numbers of positive and negative discharges over the terrain involved. This is of course somewhat of a speculation, but does seem to have some foundation.

COMPARISON OF URBAN AND RURAL RESULTS

In figure 3 are plotted expectancy curves for all urban and all rural arrester discharge currents. These results indicate that the rural arrester will be called upon to discharge a current of 500 amperes about twice as often as an urban arrester; at 15,000 amperes the rural arrester must discharge about nine times as often as the urban arrester.

From the standpoint of duty on the arrester, these results indicate that one rural arrester out of 50 installed will discharge a current of 15,000 amperes or more each year, while but one out of 450 urban arresters will be called upon to discharge a current of 15,000 amperes or more in any one year.

The number of records per installation-year for all arrester installations is 0.39, while the corresponding values for rural and urban installations are 0.51 and 0.28. On the basis of equal numbers of installation-years in rural and urban locations, 81 per cent more records of all current magnitudes are obtained on rural circuits. These values are based upon the average of the results for a large number of measuring locations. The severity at certain installations with greater than average exposure may be considerably greater.

DISCHARGE CURRENTS THROUGH URBAN NEUTRAL AND PHASE ARRESTERS

Since the magnetic-link installations in Boston are equally divided between six-kv phase arresters and three-kv or six-kv neutral arresters, a direct comparison between neutral and phase arresters may be obtained from the Boston data. These circuits were originally installed as 2,400/4,160-volt three-phase four-wire ungrounded neutral circuits, but the neutrals have since been grounded without making any changes in the arresters used. One hundred sixty-six phase-arrester current records and 189 neutral-arrester records were obtained. The number of installation-years in each case is 488, giving 0.34 records per installation-year for the phase arresters and 0.39 for the neutral arresters. For all practical purposes, it can be stated that the number of records is the same;

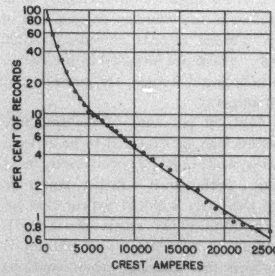

Figure 1. Summary of all records

Half of the 1,608 records were of currents of less than 1,300 amperes. Curve shows per cent of discharges whose magnitude was at least as great as indicated

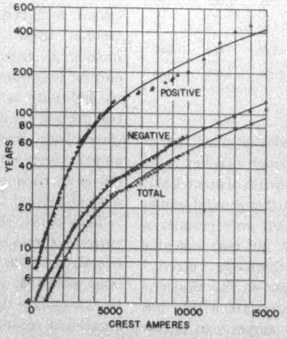

Figure 2. Comparison of 597 positive and 1,101 negative records from 4,036 installation-years

Curves show the number of years that may be expected to elapse for each discharge of at least the magnitude indicated, through any one arrester

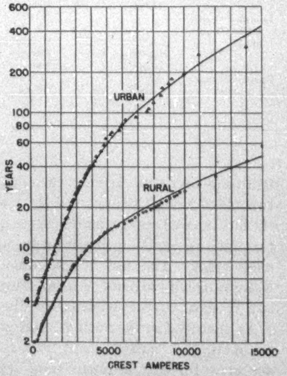

Figure 3. Comparison of 588 urban and 925 rural records from arrester ground leads

Curves show the number of years that may be expected to elapse for each discharge of at least the magnitude indicated, through any one arrester

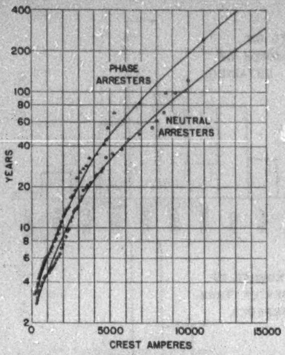

Figure 4. Comparison of 166 records from six-kv phase arresters and 189 records from three-kv neutral arresters on circuits of the Boston Edison Company

Curves show the number of years that may be expected to elapse for each discharge of at least the magnitude indicated, through any one arrester

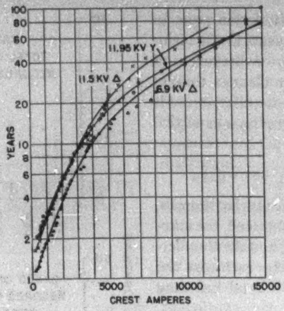

Figure 5. Comparison of results from Georgia Power Company circuits of different voltage ratings

151 records from 6.9-kv circuits
192 records from 11.5-kv circuits
155 records from 11.95-kv circuits

Curves show the number of years that may be expected to elapse for each discharge of at least the magnitude indicated, through any one arrester

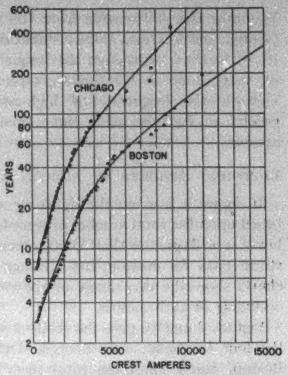

Figure 6. Comparison of 123 urban records from the Commonwealth Edison Company of Chicago and 365 urban records from the Boston Edison Company

Curves show the number of years that may be expected to elapse for each discharge of at least the magnitude indicated, through any one arrester

however, from figure 4 there seems to be a definite tendency for the current to be 20 to 30 per cent greater through the neutral arrester. Some difference in current would be expected in those cases where the two arresters are effectively in parallel, and the discharge current may divide between them according to their resistance. Because of their lower average rating, the neutral arresters would have lower resistance and would pass higher current in such cases.

EFFECT OF CIRCUIT RATING

There seems to be no definite effect of circuit rating upon arrester discharge currents, although as shown in figure 5 there is a slight tendency for the lower rated arresters to carry a current of a given value more frequently. All of the data in this figure come from rural installations of the Georgia Power Company, all within the same state and presumably with about the same degree of exposure. The differences between the curves are too small to be more than an indication of what to expect, since a great many unknowns influence the results.

EFFECT OF LOCATION UPON NUMBER OF RECORDS

For a given locality, the number of records per month seems to vary almost in proportion to the number of storms in that locality. However, as between localities, the correlation is not good.

A comparison between the results of 976 installation-years in Boston and 874 installation-years in Chicago is interesting. In Boston the currents are through three-kv and six-kv arresters used as neutral and phase arresters, while in Chicago the results are based on current measurements through three-kv phase arresters. Three hundred and sixty-five records were obtained in Boston compared to 123 obtained in Chicago. The expectancy curves of figure 6 indicate that a current of at least 10,000 amperes will occur in Boston about three times as often as in Chicago, in spite of the fact that an average of 15 thunderstorms per year were reported by weather bureau stations near Boston, and 45 per year at Chicago during the period of the investigation. These results seem to indicate that as far as records of arrester discharge currents are concerned, some other factor than number of storms per year exerts a major influence. A similar difference was found for the first 13 months of the investigation, and it was suggested by the authors that the difference might be due largely to difference in exposure. In Chicago the shielding is undoubtedly much more effective than is the case in Boston where the installation might more properly be classed as suburban. The difference is possibly due in part to less frequent checking of the magnetic links in Chicago. Most installations there were checked only once a year. However, since most of the links remained unmagnetized even at the end of that period, it does not appear likely that there was much superposition of records.

From the above it follows that the prediction of arrester currents upon the basis of thunderstorm data, in a locality where current measurements are not available, is not likely to lead to a very satisfactory result. The 20-year isokeraunic map[4] of the Weather Bureau indicates about 40 storm days per year in Chicago compared to 20 in Boston.

A comparison may also be made between the records obtained on rural circuits in Georgia and Detroit. Figure

7 summarizes 490 records from 765 installation-years in Georgia and 419 from 1,004 installation-years in the vicinity of Detroit. Although the frequency of discharges of all magnitudes is 1.5 times greater in Georgia than in Detroit, the frequency of high-current discharges is greater in Detroit, being three times as high as 15,000 amperes.

Lightning Severity by Months and by Years

In figure 8 is given the lightning severity by months, based upon the total number of records obtained, the number of records greater than 5,000 amperes, and the thunderstorm data for Boston and Chicago. In each case the charts show the percentage of the year's total lightning occurring in each month.

Figures 9 and 10 give the relative severity by years in the four locations; i.e., Georgia, Detroit, Chicago, and Boston, based upon the total records and those above 5,000 amperes. Values are given in per cent of the overall average for the four locations for the period of the investigation. Here again it should be remembered that links were checked less frequently in Chicago and this may have resulted in some decrease in the apparent severity.

Relative Severity in Different Locations

The greatest number of records per installation (figure 9) was obtained in Georgia each year but 1937, the average to date being about double that of the next highest, the Detroit Edison Company. For records above 5,000 amperes (figure 10) the Detroit Edison Company has the highest average. Although the Georgia records are twice as great in number as the Detroit records, above 5,000 amperes the Detroit records are about 30 per cent more numerous. A similar situation exists between Detroit and Boston, the average number of records being approximately equal, yet the number of records above 5,000 amperes being about four times as great in Detroit as in Boston. This seems to indicate that lightning strokes are more severe in Detroit or else the line construction is such that a higher level of line insulation is maintained. The Commonwealth Edison data cannot be used to good advantage in this comparison, because the circuits are so predominantly urban.

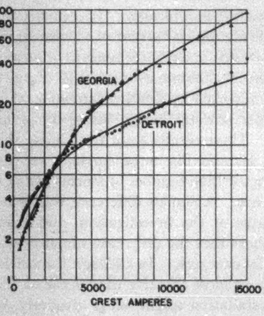

Figure 7. Comparison of 490 rural records from Georgia Power Company and 419 rural records from the Detroit Edison Company

Curves show the number of years that may be expected to elapse for each discharge of at least the magnitude indicated, through any one arrester

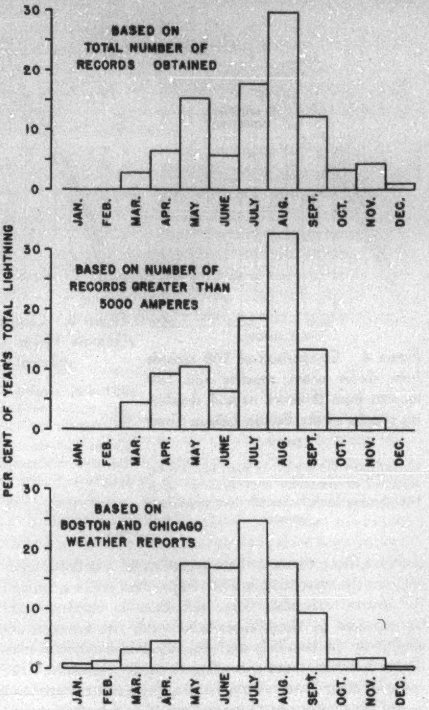

Figure 8. Relative lightning severity by months

There seems to be a persistent feeling among operating men that although the storms are less frequent in the Middle West than in the South, the strokes are more violent and do more damage. The Detroit data tend to support this contention, as far as the severity of strokes is concerned. Damage to apparatus was, however, much more frequent in Georgia.

Arrester and Transformer Failures—Fuse Blowings

There has been much speculation as to the performance of lightning arresters when subjected to natural lightning currents of different values, with regard to the protection offered and the effect on the arrester itself. Figure 11 shows the operating record of arresters for all four systems. With 3,787 installation-years involved, the arrester failure rate is 0.66 per cent per year. These data of course include arresters of all makes and ages. Exclusive of Georgia, the failure rate is 0.27 per cent per year, which is a very satisfactory record.

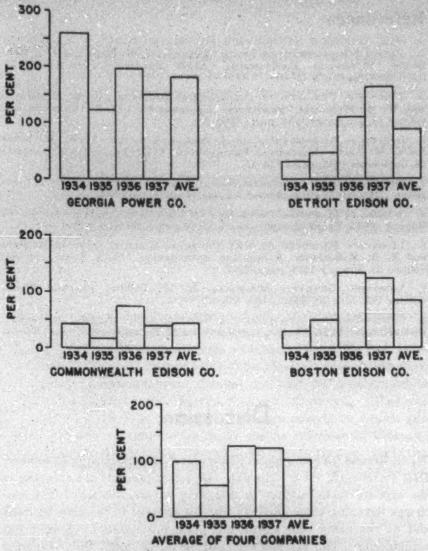

Figure 9. Relative lightning severity by years based on total number of records obtained

Values shown for 1934 and 1937 cover only a part of the year

There is some tendency for the failure rate of arresters per record to increase as the current increases, although this is not apparent to any extent if the Georgia data are excluded. Sixty-eight per cent of the arrester failures occurred in Georgia, despite the fact that Georgia had but 34.5 per cent of the total records.

Of the 12 records above 25,000 amperes, two are associated with one transformer failure. There were no arrester failures associated with these high currents. As to the distribution of these high currents with respect to location, one came from Chicago, two from Georgia, three from Boston, and six were obtained from Detroit.

Eighty-four per cent of the arresters under study were interconnected in 1937, but no segregation of data is attempted since the number not interconnected is too small to yield definite conclusions. Possibly the transformer failure record would have been even better than it is, if a larger number of arresters had been interconnected.

The data show 36 fuse blowings, of which 28, or 78 per cent, were reported from Georgia. There is some tendency for the percentage of fuse blowings per record to increase with increasing arrester current, but as with the arrester data, this effect is practically nonexistent if the Georgia data are excluded. This may be explained by the fact that at some of the Georgia locations, the fuses were ahead of the arresters. They would therefore be called upon to pass all impulse currents which went to ground through the arresters, and the high-current discharges may in themselves be sufficient to blow the fuses.

Of the 17 transformer failures, 13 were in Georgia giving an over-all failure rate of 0.72 per cent of the installed transformers per year, and excluding Georgia, a rate of 0.21 per cent. Why the Georgia trouble rate is higher than the others is not clear, although it is partly explained on the basis of increased frequency of lightning discharges.

Conclusions

1. Within the limits of this investigation, circuit rating seems to have but little effect upon the frequency of occurrence of arrester discharges of a given current magnitude. If any conclusion is to be drawn, it is indicated that the lower rated arresters tend to carry more current, or to carry a given current more frequently.

2. Fifty per cent of the currents measured were found to exceed 1,300 amperes, while 0.75 per cent exceeded 25,000 amperes. In the current range from 5,000 amperes to 15,000 amperes, negative currents were found about 3.5 times as frequently as positive currents. The smaller ratio of negative to positive discharges compared to measurements on transmission line towers may be due to the lower height of the distribution circuits.

3. The rural arrester must discharge currents of the

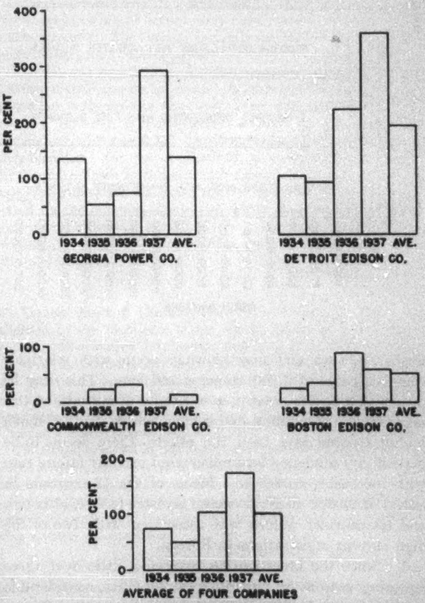

Figure 10. Relative lightning severity by years based on number of records of currents greater than 5,000 amperes

Values shown for 1934 and 1937 cover only a part of the year order of 500 amperes twice as often as the urban arrester, while at 15,000 amperes the ratio is increased to nine to one. The rural arrester must discharge a current of 15,000 amperes or more once in 50 years and the urban arrester once in 450 years.

4. There does not seem to be any satisfactory method for translating Weather Bureau thunderstorm data into the number of arrester discharges to be expected. In a given locality, however, the variation in the number of records agrees fairly well with the variation in the number of storm days.

5. In general the largest number of arrester and transformer failures and fuse blowings occur with relatively small currents of 3,000 amperes and less. This may be due in part to the great preponderance of currents of this magnitude. Excluding one system which had a much higher trouble rate than the others, there seems to be little if any tendency for an increased arrester failure rate with increasing currents. None of the 12 currents in excess of 25,000 amperes caused arrester failure, although one transformer failure was associated with two of the high current registrations in Boston.

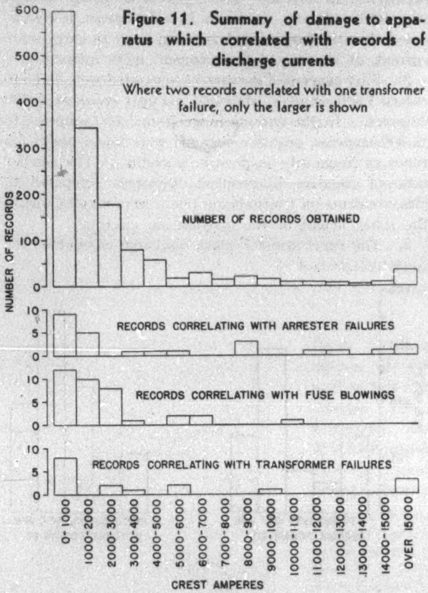

Figure 11. Summary of damage to apparatus which correlated with records of discharge currents

Where two records correlated with one transformer failure, only the larger is shown

6. Since the investigation covered a little over three lightning seasons in four different localities, considerable confidence can be placed on these results for design purposes. Proper consideration must of course be given to the circuits and localities involved, and to the limitations of the methods used in making this study.

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Discussion

W. G. Roman (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The data this paper presents are valuable to the arrester manufacturer in designing an arrester which can discharge lightning surge currents without damage to the arrester itself and at the same time afford adequate protection to distribution equipments. The arrester's record so far is good, but continued efforts should be made to improve it both in design and in the method of application.

It is interesting to note that of the 12 records above 25,000 amperes none were associated with arrester failures. This indicates that the distribution arrester has a discharge capacity sufficient to handle even the more severe lightning surges.

Can the higher trouble rate experienced in Georgia be co-ordinated with the type of protective scheme used or the ground resistance of the average arrester installation? As the authors point out, the lightning discharge frequency is higher in Georgia than on the other systems, but their trouble rate is higher than can be accounted for by this increased frequency.

The magnetic-link surge-crest ammeter has given much valuable data on lightning surge currents. A need is felt for data of this type on the duration of the surge current as well as the crest value. Has the General Electric Company any data on the type of record which would be obtained with a magnetic link inclosed in a conducting or semiconduction sheath? Currents induced in this sheath would tend to prevent the link being magnetized. On short surges this effect would be greater than on long surges of the same crest current and it might be possible to use this link co-ordinated with an unshielded link indicating crest current to give an approximate indication of the length of the surge.

I. W. Gross (American Gas and Electric Service Corporation, New York, N. Y.): The magnitude and frequency of lightning currents in distribution arresters actually measured in the field, as reported by Messrs. McEachron and McMorris, show how far short our present AIEE Standards fall in setting up standards for impulse tests of lightning arresters. The authors' paper (figure 1) shows over 40 per cent of the recorded currents above 1,500 amperes (the maximum value mentioned in the Standards); ten per cent are above 5,000 amperes and five per cent above 10,000 amperes, with maximum recorded currents of 25,000 amperes.

Certainly the users of lightning arresters want to know what degree of protection the arrester is capable of supplying at these higher currents so that they may co-ordinate, on a sound and economical basis, the arrester protective level with the apparatus it is intended to protect. In the none too distant past it must be admitted the application of arresters for protection was more on a catalog basis of system voltage rating rather than on the basis of establishing a protective level known to be below the safe strength of the apparatus.

As more data of the type presented in this paper are obtained, including rates of voltage and current rises of natural lightning on electric systems, it is hoped our knowledge of the protective characteristics of the arrester will be extended to cover at least those conditions which are known to exist in the field.

The authors mention 36 cases of fuse blowings. I would like to ask if they can tell us the current rating of these fuses, or the approximate rating. In other words does field experience, from these tests, give any information on fuse sizes which do or do not blow under known lightning currents?

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): An essentially statistical investigation of this type gains progressively in value as the number of results accumulates. It has now reached the stage where reliable conclusions may be drawn. We feel, however, that the interpretation of some of the results is open to question.

With 976 installation-years, Boston obtained 365 records, while with 874 installation-years Chicago obtained 123 records. This was in spite of the fact that the Weather Bureau reports 15 storms per year at Boston as compared with 45 storms per year at Chicago. Most installations in Chicago were checked but once a year, while the checkings were more frequent in Boston. The fact that most of the links remain unmagnetized at the end of a year is no assurance that there was a negligible amount of superposition of records.

An excess of superposition over the amount computed, assuming the records to obey a statistical distribution, is to be expected, because, first, in a given year certain locations in a large city will be located in a larger proportion of storm centers than others and, second, due to less shielding by neighboring buildings some locations may have a considerably greater a priori probability of being struck than others.

At 22 locations in Chicago the links were checked four times during 1936; 12 records were obtained, assuming no superposition to have occurred in the short intervals between inspections. If the installations had been checked but once in 1936, it appears that only eight records would have been found. In other words, in order to obtain the correct number of records, at least 50 per cent would have to be added to the observed number as obtained on the yearly basis.

This superposition is much greater than would be predicted under the assumption that all locations were equally susceptible to lightning during 1936. The straight application of the Newton formula to the data would indicate that 1.8 records had been lost while the number actually lost was four. The data obtained in 1937 at these same 22 locations gave similar results. In this year the number of records actually lost by superposition was four times the computed number.

It is not our contention that the apparently abnormally low lightning severity in Chicago is completely to be explained by superposition of records. The effect of the better shielding conditions in Chicago, as suggested by McEachron and McMorris, may explain the balance of the discrepancy.

The highest current shown in figure 6 for Chicago, i.e., 9,000 amperes, was for an installation having standard arrester connection and $10\frac{1}{2}$ ohms ground resistance. Apparently at the time of the discharge, an old three-kva transformer failed, but the old-type arrester at this location did not fail. This is another experience showing that the arrester standards should specify a much higher current than 1,500 amperes for acceptance tests on modern-type arresters.

L. G. Smith (Consolidated Gas, Electric Light and Power Company of Baltimore, Md.): The data being collected by the authors are of considerable value in designing the lightning protection of distribution systems and determining the necessary discharge capacity of lightning arresters applied to such systems. However, in order to be of conclusive value it requires the collection of a considerable amount of data. The 1,352 installations constitute a very small sample of the total number of installations of distribution lightning arresters. It is highly desirable for more companies to co-operate so as to increase the number of installations and spread and coverage of the territory represented by these installations. However, before definite conclusions can be drawn, many more years of collection of data will be required. The industry is greatly indebted to the authors for undertaking such a valuable study.

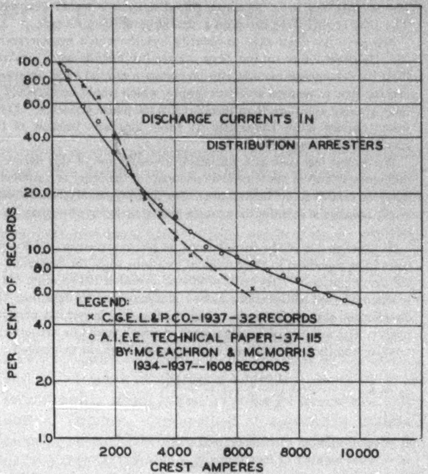

Figure 1

Last summer our company installed surge-crest-ammeter links at 150 distribution-arrester locations. As these installations were completed late in the summer, data were obtained for only two lightning storms. However, 32 records were obtained. The maximum surge amperes recorded were 6,500. Surge currents as low as 500 amperes were recorded. As a matter of interest our meagre data was plotted on the curve shown by the authors in figure 1, in order to determine how closely our data conformed with the more voluminous data collected by the authors. These two curves show remarkably close agreement for the small number of records obtained by our company. We plan to continue the collection of these data over a period of several years.

C. Francis Harding (Purdue University, Lafayette, Ind.): In addition to my discussion of the similar paper by L. G. Smith entitled "Distribution Transformer Lightning Protection Practice—II" it should be noted that the results of this statistical field analysis also confirm the predictions based upon the experimental tests of the four-span standard distribution system subjected to surge generator exposures at Purdue University under the auspices of the Public Utilities Research Commission of Chicago.

The record reported in this paper of relatively high surge currents in the arresters located on rural distribution circuits indicates that the attenuation of the wave along such circuits is not as great as anticipated and that the magnitude thereof should be more seriously considered in establishing higher current testing standards by the lightning-arrester subcommittee.

K. B. McEachron and **W. A. McMorris:** Mr. Halperin presents some interesting results based on data obtained in 1936 at 22 measuring locations, where links were checked four times during the lightning season. It is indicated that with annual inspection of links, the number of records lost by superposition would have been at least four, but the number predicted on the basis of equal exposure of all locations would have been approximately two. If only two less records had been obtained, this discrepancy would disappear. The 1937 results were based on a smaller number of records.

We have not been able to identify exactly which measurements Mr. Halperin refers to, but they apparently include some readings from the few special installations on common ground leads, interconnection ties, or neutral arresters (gaps), which would be expected to have greater exposure than installations on phase arresters. Links were checked more frequently at these locations because of the special installations there.

We do not feel that in a statistical investigation of this nature, a difference of two or three records one way or the other is a sufficient basis for very definite conclusions. The following analysis based on a larger number of installations over a longer period of time may be of interest.

In Chicago 236 of the installations on phase arresters on 4-kv circuits have remained in their original locations since the beginning of the investigation. They have produced a total of 104 records. If it is assumed that all locations have equal exposure to lightning, the distribution of records may be calculated from the law of independent trials. The number of locations each of which would be expected to produce exactly k records may be determined from the expression

$$n_k = n\frac{m!}{k!\,(m-k)!}\left(\frac{1}{n}\right)^k\left(1-\frac{1}{n}\right)^{m-k}$$

where

m = total number of records, and

n = total number of locations.

The actual and calculated distributions for Chicago are as follows:

k Records per Location	Number of Locations With Exactly k Records Each	
	Calculated	Actual
0	151.8	151
1	67.2	67
2	14.7	17
3	2.1	1
4	0.2	0

The agreement between actual and calculated distribution is surprisingly good. The same is true for the Detroit and Georgia records, and to a lesser degree for the Boston records. This agreement indicates that the distribution of records has not been greatly influenced by factors which have not been taken into account and therefore that no great error would result from using the theory of probability to predict the number of records lost by superposition.

When $k = 0$, the above equation reduces to

$$n_0 = n\left(1-\frac{1}{n}\right)^m$$

solving for m,

$$m = \frac{\log n - \log n_0}{\log n - \log (n-1)}$$

This equation will permit us to estimate the number of records lost by superposition. If we let n_0 represent the total number of installations at which links remained unmagnetized after a given period of exposure, then m represents the expected total number of discharges during that period. The number of records obtained would be equal to $(n - n_0)$, so $m - (n - n_0)$ would be the number of discharges which were not recorded because of superposition. The curve in figure 2 herewith is based on this relation. It does not account for the later discharges of a multiple stroke, which are not free to discharge through any path at random, but have a very strong tendency to follow the first discharge of the series.

Several who have discussed this paper have brought out the point that the present standard AIEE impulse tests are not representative of the most severe discharges measured in service. Field data on the arresters involved in the investigation do show, however, that the arresters which have the best protective characteristics on the AIEE test have given the best account of themselves under service conditions.

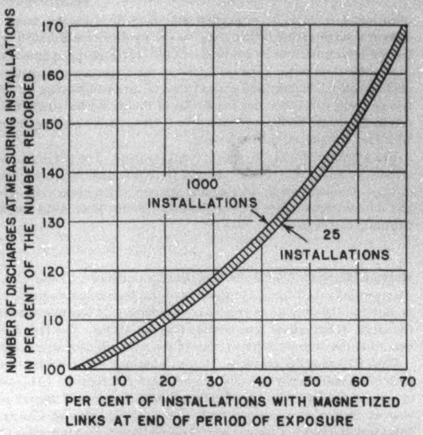

Figure 2. Effect of superposition of records on the number of discharges actually recorded, assuming that it is equally probable for any discharge to appear at any measuring installation

Mr. Gross has inquired about the ratings of fuses which were reported to be blown. These extended from two amperes up to 30 amperes. The rate of fuse blowing was higher for those locations where the fuse was ahead of the arrester, so that arrester discharge currents were carried by the fuses. Some of the fuse blowings are believed to have resulted from overloads due to secondary faults, rather than from lightning currents alone.

There is great need for more data on the duration of impulse current waves, as pointed out by Mr. Roman. We have considered the method of measurement he proposes, as well as other similar methods, for adapting magnetic links to give an approximate indication of wave fronts and duration. Laboratory tests have been made over a period of several months to determine the best constants for the sheath, and to obtain calibration data.

This method is attractive because it is so simple and inexpensive to apply. The interpretation of records is however quite complicated, and only approximate results are obtained, because of the large number of variables involved. It may prove desirable to make installations of this type, but it is hoped that other equally inexpensive methods may be developed for obtaining information more easily interpreted into accurate values for wave shape and duration.

We do not have suficiently complete data available to justify definite conclusions as to the causes of higher arrester and transformer failure rates in Georgia. Some of the ground resistances were, however, relatively high. The use of longer leads from secondary neutrals to arrester grounds may have contributed to the higher trouble rate.

We are pleased that others are undertaking investigations of this nature and that the results already obtained by Mr. Smith are in general agreement with what we have reported. The accumulation of more data will more firmly establish what duty is imposed on a distribution arrester in different types of service.

Since the scope of the investigation has been limited, it may prove more advantageous to alter it to obtain more information on the few discharge currents above the present measuring range, to include other localities or types of circuits, or to obtain information on the wave shape or duration. A given amount of work may bring greater returns to the industry if directed along these lines, than by merely continuing at the present locations with no changes.

Results of Operation of PCC Cars in Pittsburgh

By T. FITZGERALD
ASSOCIATE AIEE

THE Pittsburgh Railways Company has purchased 201 cars of the Presidents' Conference Committee type. By the middle of November 1937, there had been delivered and placed in service 140 of these cars. The new vehicles have exceeded our expectations with respect to their mechanical performance, their reception by employees, and the approval given them by the public.

The company placed its first order, consisting of 100 cars, in July of 1936. Of these, 75 units are equipped with Westinghouse motors and control apparatus, and 25 are General Electric equipped. The same distribution was made on the second order.

Prior to the delivery of any units of the first order, we obtained a sample car which we used to demonstrate to the public the superior qualities possessed by this outstanding advance in vehicle design. We operated this sample car in demonstration service from August until December, in which period we carried 25,000 passengers free, and it was inspected by uncounted thousands on the streets. This period of operation also gave our mechanical department an opportunity to study its design and to devise methods by which to cope with such difficulties as developed. The preliminary study and experience thus obtained by the mechanical group were of great value, in that they were well prepared to test the cars as they arrived, and to adjust them to give the performance of which we knew the car to be capable.

The design of the PCC car incorporates features which we in Pittsburgh had felt for a long time were essential in a trolley car if it were to compete effectively with the private automobile and the bus. The principal deficiencies of the cars we had were excessive vibration and noise; inadequate deceleration; inadequate, jerky acceleration; and unnecessary weight. We had a little speed, which we obtained by rewinding the motors on about half of our cars, which increased their power by 50 per cent and their free running speed from a maximum of 30 miles per hour to a maximum of 40 miles per hour, an increase of $33^1/_3$ per cent. On a six-per-cent grade, the speed improvement of the higher-speed cars was 50 per cent. When the cars were converted to higher speed, we increased the braking efficiency and performed other work on them designed to make the car in general conform with its higher speed characteristics.

It will be noted that the operating characteristics of the rewound-motor cars approach those of the PCC cars, so that we are able to place both types of cars in a schedule without losing the efficiency of the new car, and with reasonable assurance of maintaining the schedule.

Delivery of our first order of cars commenced in late January of 1937. At the outset, we had to decide whether to fully equip a route with the new vehicles, or to equip the base service of the route selected, and use cars with rewound motors to supplement the service for the peak haul. We decided on the latter plan, because we could thereby get a greater coverage of the system, and also keep the new cars in service to a greater extent. If the new cars had been used to fully equip a route, the peak cars would have been idle the greater part of the time. By applying the cars to base and evening schedules only, we were able to equip ten routes with the first group, and we will equip 13 routes with the second 100 cars.

At the middle of November, we had a total of about 3,440,000 miles of operation on the first group of cars, and 60,000 miles of operation on about one-third of the second order, which began to come in during the latter part of October. Our method of application results in securing about 1,000 miles of service per week on each car, or 50,000 to 52,000 miles annually.

We have had less mechanical trouble with these vehicles than with any new car we have had on the property. We have had troubles, of course; none, however, which were fundamentally serious, or for which the engineers have not been able to devise remedies. In the light of the severe operating conditions which the Pittsburgh district offers, and the far-reaching changes in car design incorporated in the vehicle, it is to be wondered why we have had comparatively so little difficulty with them.

Evidence of how the cars are standing up in service is furnished by the weekly mileage of 1,000 miles which they are averaging. At ten miles per hour, that means they are averaging better than 14 hours of service per day, exclusive of time standing as spares, and of time required for routine inspection and cleaning.

Earlier, I indicated that noise reduction was an important advantage which we expected from the new design. The new car has advanced a long way in this regard, but it has made us acute to new noises which will have to be corrected. A short time after the cars were delivered, it was noted that a distinct and irritating rattle developed when crossing special track work, and when going over track irregularities. The source of the noise was found to be the bearings of the brake-shoe hangers on the brakebeam. Rubber was introduced into the design to eliminate the difficulty. Altogether, the noise level of the car is a tremendous improvement but it is not as good as we should seek to make it.

When we purchased the new cars we experienced some anxiety over the hypoid gears with which the cars are driven. Our experience has been such as to convince us that our fears were unfounded. So far, we have had only

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five gear failures and each was traceable to omitted lubrication which maintenance routine will overcome.

We have had no failures of resilient wheels except those which were damaged because hand brakes had not been fully released, or because the required pressure was not properly maintained on the rubber sandwiches incorporated in the wheel.

We have had little or no trouble with motor generator sets. The wear on rail brake shoes has been very slight. We feel we are getting good wearing properties out of our rail brake shoes, largely because our technicians have made the dynamic brake properly do its job. We are obtaining from four to five times the mileage on wheel brake shoes and they cost a little more than twice the cost of regular brake shoes. We originally estimated we would obtain about 60,000 miles of service from the resilient wheels of the PCC cars. Indicated wear at the present time is about 100,000 miles for one-half inch of wear, which is the limit of wear obtainable. This compares with 140,000 miles on regular wheels for two and one-half inches of wear. Due to the limited wear on shoes and wheels, there is considerably less labor required in resetting brakes.

We have had two derailments in 3,500,000 miles of operation, due to mud on the tracks, carried there by severe storms, and one derailment due to a broken propeller shaft. We have had one split switch caused by an object in the switch, and one split switch probably caused by excessive speed. I personally feel that these cars are going to hold the rails longer, not only in the sense of avoiding derailments and split switches, due to the resilient wheel, but also in the sense of demonstrating the value of rails in transit service and re-establishing the prestige of that type of service.

We have not made any changes in schedule speeds on the routes where the cars were installed, largely because the schedules were designed for and operated by the higher-speed cars which I have mentioned. The higher-speed schedules are, however, about 15 per cent better than is obtainable with the standard slow-speed cars.

Power consumption on the PCC car is, by test, about the same as that on the higher-speed car, and it is 25 per cent greater than on a standard slow-speed car. We will save considerable on heat, however, with the PCC car, over the conventional car. We use electrical space heaters on the old cars, and the new cars of course use the heat dissipated by the control resistors during acceleration and dynamic braking to heat the cars. We have ample heat in the new cars; in fact, we are receiving complaints of excessive heat, whereas the old cars are complained of as being cold, on days when extremely low temperatures prevail. We are not in position to estimate the effect of this on power consumption until we go through a winter with the new cars.

The PCC cars we have are all of the foot-control type. Although we had experimented with foot control on seven cars, it is the first time we have had a considerable number of cars of that type in operation. In qualifying trainmen on the new cars, we found that on the average about four hours of practice is all that is needed. The reaction of trainmen to the car has been universally good. They like them because of their smooth performance characteristics, the adequacy of the brakes, and the resultant ease of schedule maintenance, and because lost time can be recovered so much more easily with the new car. The only difficulty in operation so far encountered is glare in the windshield at night, particularly on wet nights. This is due to the bright interior, and we have mitigated this difficulty by various methods, such as dimming the cab lights, cutting off floor reflection, blacking the windshield frames, and on the second order of cars provision is made for a curtain which can be drawn about the operator's position.

Of course, the true measure of the new car rests in the effect upon revenues and the effect upon costs. We have seen enough of the car to lead us to feel that the costs will not be higher than with the conventional car and, if anything, the new car will produce an economy in track maintenance, power costs, and in costs peculiar to the car itself.

So far, the public response to the vehicle has been splendid. We have received many communications from patrons, who have been pleased with the improved service; and from civic and municipal groups, congratulating the company on the new vehicle. We are constantly receiving inquiries as to when the new cars will be installed on other routes in the system, which is another evidence of their popularity. The noise-reduction qualities of the new car have accentuated complaints of noise from older cars. I have for a number of years held the opinion that much of the lack of prestige of street cars was traceable to the noise attending their operation. The items of superiority of the new cars over the older cars most frequently mentioned are its quiet operation and its smooth performance.

I will now turn briefly to the patronage results on the routes where we have installed the new cars. I am sorry that we cannot speak more definitely about the increased patronage they have attracted. Further, I must confine my remarks to results on the routes on which the first 100 cars were installed, because we have had insufficient experience, up to this time, on the second group to record any patronage results.

The picture on three of the routes on which new cars have been installed has been confused by the fact that we made fare concessions at about the time the new car service was inaugurated. On certain of the routes, territory is served which is also served by other routes which do not have the new equipment, so that it is possible that certain patrons may use either route. In the studies which have been made, we have endeavored to adjust as nearly as we can for the factor of competition of the new cars with the older cars. On two routes on which the new cars have been installed and where a fare concession also was made, there has been no reduction of revenue—on the contrary, the route earnings have shown an increase. The fare reduction which was made was of substantial proportion.

It should be remembered that thus far where the new cars were installed, we previously operated higher-speed cars, which in themselves had a tonic effect upon revenues.

The new cars, of course, have provided a much improved performance and appearance, and to that we must ascribe the added traffic obtained. A property which has not speeded up its service will, in my opinion, secure the advantage of traffic stimulation from the greater speed of the PCC car. We began our program of conversion of cars to higher speed at the start of the depression, and we continued the program during the depression. We found that the routes which were equipped with the higher-speed car held their patronage better than others—an advantage, of course, which the recent installation of PCC cars does not give us in the making of comparisons.

The installation of new cars commenced on February 4, 1937. Two days later, on February 6, the rate schedule of the Pittsburgh Motor Coach Company was reduced from 25 cents cash, nine tickets for $2.00, to 15 cents cash, eight tickets for $1.00, and 20 tickets for $2.00. The routes on which the rate change took place had been originally designed to furnish a service supplemental to the rail service, and in most instances they occupied the same streets or closely paralleled rail service. It can be readily appreciated that with such a sharp reduction in fare, traffic on the busses was stimulated. Many of the new car routes have been up against this bus competition, and they have shown revenue improvements despite this unfavorable circumstance.

Data we have obtained to determine increased patronage shows that the new cars have been responsible for improvements ranging from 5 per cent to 11 per cent.

When consideration is given to the elements which affect the comparisons, such as bus competition, previous higher-speed schedule service, and installation in the spring of the year when traffic normally is on the decline, I feel that we have had a very favorable patronage response to the new car.

Our experience with the mechanical and electrical performance of the PCC cars has been better than we expected and is satisfactory. The cars have the performance to attract and hold patronage, and in my opinion they are an adequate answer to the charge of obsolescence of street railways. Continued improvement in the vehicle, which is inevitable, will, I believe, result in complete rehabilitation of the prestige of the street car as the most efficient and economical carrier where traffic demands heavy enough to justify that vehicle are encountered.

Discussion

R. E. Hellmund (Westinghouse Electric and Manufacturing Company, Pittsburgh, Pa.) [**Editor's Note:** This discussion covers also "The PCC Street Car," C. F. Hirshfeld, *EE, Feb. '38, Trans. p. 61–6*, and "Electrical Equipment for Modern Urban Surface Transit Vehicles," S. B. Cooper, *EE, Jan. '38, Trans. p. 50–6*]: The fact that the Presidents' Conference Committee car has attracted considerable attention in Europe may be of interest. Some of the visitors to the World Power Conference in 1936 had an opportunity to inspect the first Pittsburgh car and they were very much impressed not only by the technical features of the car but also by the arrangement which made its development possible. They felt that similar undertakings in Europe might be instrumental in bringing about renewed activity in electric street railways. When I visited Europe in 1937, I was surprised at the many inquiries made by the engineers regarding the PCC car, with reference both to generalities and to details, such, for instance, as the number of steps in the control, voltage per step which could be handled by the two types of control, etc.

As usual, it will for various reasons be difficult to directly apply American designs in Europe. For example, the narrow streets and sharp curves in many European cities are not suitable for a long two-truck car such as the PCC. There also seems to be some hesitancy in Europe to design motors for temperature rises and the very high speeds used in this country because experience along these lines is still lacking in Europe. Although for this reason developments may take a somewhat different course, I nevertheless believe that we can expect to see innovations in Europe which have been stimulated and influenced, at least to some extent, by the PCC car.

Operating Experiences With Gas-Electric-Drive Motor Busses

By R. H. STIER

THE Philadelphia Rapid Transit Company operates a co-ordinated urban transportation system serving the City of Philadelphia and adjacent suburban communities. The system is comprised of surface car, subway-elevated, motor bus, and trackless trolley lines. Motor-bus operation was started in September 1923, with 21 double-deck mechanical-drive busses of the "Fifth Avenue" type. At the present time a total of 323 busses are operated 12 million miles annually, and produce approximately nine per cent of the system passenger revenue. The busses operated include 138 33-passenger single-deck and 161 71-passenger double-deck gas-electric-drive busses purchased during 1925 and 1926. The remaining 24 vehicles are small-capacity 21-to-25-passenger busses, that have been purchased during the past four years.

Operating characteristics of the original 21 mechanical-drive double-deck busses, purchased in 1923, were most unsatisfactory from a passenger or transportation viewpoint. These busses were noisy and rough during acceleration, due to inherent characteristics of the clutch and transmission equipment. In order to determine means of improving the operation of these vehicles, the Philadelphia Rapid Transit Company investigated alternate drive mechanisms that could be applied to the motor bus.

It was learned that the General Electric Company had built ten double-deck gas-electric-drive busses for the Fifth Avenue Coach Company in 1904. In so far as could be ascertained operation of these vehicles was entirely successful. The experiment was abandoned, apparently due to reluctance of any of the automotive manufacturers to undertake construction of the gas-electric-drive bus, probably due to increased cost for the electric drive and limited market for motor busses.

During the interval between 1904 and 1923 the electric drive was not commercially developed, although isolated applications of the electric drive to automotive equipment were made, principally by the United States Army. Information available indicated that the electric drive offered the greatest promise of eliminating the objections of noisy and rough operation experienced with the mechanical-drive vehicle.

One of the "Fifth Avenue" type double-deck busses operated by the Philadelphia Rapid Transit Company was equipped with the electric drive. The apparatus was essentially identical to that still operated in 299 PRT busses, consisting of a separately excited four-pole generator similar to the GE 1098 machine, two series-wound variable-voltage four-pole motors similar to the GE 1079, and a simple three-position drumtype controller. Performance of the bus was improved to so marked a degree that the electric drive was specified for the 352 busses purchased by the Philadelphia Rapid Transit Company during 1925 and 1926.

During the early years of operation the usual trials and tribulations that go hand in hand with the pioneering of a new development were experienced. Operating problems were multiplied by the rapid growth that occurred in the Philadelphia Rapid Transit Company's bus operation, which was expanded from 527,000 miles in 1924 to 8,000,000 miles in 1926.

The problems were accentuated by changes in design of the vehicle, made during the early years of operation, which tended to increase the effective load on the electrical equipment. These changes included the replacement of the original 34-inch solid tires by 40-inch pneumatic tires on the double-deck busses and by 38-inch pneumatic tires on the single-deck bus; and the enclosure of the upper deck with resultant increased vehicle weight. The electrical apparatus was rebuilt to increase its capacity and improve its performance. Increased capacity improved performance of the equipment, but the basic cause of continued failure of the electrical equipment was attributed to use of noncommutating-field motors and generators. Commutator and brush wear was excessive and frequent flashover, with resultant damage to equipment, was experienced in service. After considerable study of the problem, in co-operation with engineers of the General Electric Company, methods of installing commutating field poles in the standard GE 1098 generator and GE 1079 motor were developed. Analysis of service data indicated that cost for this change would be justified in the GE 1098 generator, but it was doubtful whether the cost could be justified in the GE 1079 motor. During 1931 and 1932 the majority of the GE 1098 generators were rebuilt and commutating field poles installed. Mileage operated between reconditioning periods was increased from 32,000 for the noncommutating-pole GE 1098 generator to 107,000 miles for the commutating-pole GE 1098 generator.

Performance of the electrical equipment can be judged by analysis of maintenance expense, shown in table I. Maintenance expense for electric transmission equipment includes all costs for generators, motors, controllers, kilowatt-hour meters, and wiring. It will be noted that installation of commutating poles, during 1931 and 1932, decreased percentage of total maintenance chargeable to electric transmission equipment from an average of 17 per cent to an average of less than 13 per cent of total maintenance expense. Record of maintenance expense for these vehicles and for the electric transmission equip-

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ment, during recent years, is an outstanding achievement that can be largely attributed to the inherent advantages of the electric drive.

The Philadelphia Rapid Transit Company is now confronted with the necessity of replacing the greater portion of its bus fleet, 93 per cent of which is of the gas-electric type and has been operated between 11 and 12 years. In view of the imminent purchase of new bus equipment, the controversial question as to whether or not the electric drive should be utilized must be answered. Since 1924 extensive changes in design of clutch and transmission equipment have been made, that eliminate many of the objections to the equipment that were in part responsible for the original selection of the electric drive in 1924. Although majority of busses that have been operated are of the gas-electric type, operating experience with these vehicles is of little value in determining the type equipment best suited to the service requirements, since the vehicles are of obsolete design and construction, and since comparative service data is not available for the mechanical-drive bus. It is considered that a description of the analysis that has been made of the relative advantages and disadvantages of the electric-drive and mechanical-drive motor bus would be of greater interest than an elaboration of the operating experiences with vehicles of obsolete design.

Discussion of the relative merit of the electric drive will be predicated upon its use in frequent-stop heavy-traffic urban service for which it is basically adapted. The primary problem encountered, in weighing the relative advantages and disadvantages of the electric-drive and mechanical-drive bus is to secure authentic, comparative basic data. In order to secure the fundamental information required, tests were conducted with a number of modern busses of various types. For simplification we have limited the discussion to analysis of representative results obtained from performance tests of two 35-passenger busses of identical design and construction, differing only in that one vehicle was equipped with a conventional clutch and three-forward-speed, constant-mesh transmission; the other vehicle equipped with a GE 1501 self-excited generator and one GE 1205 series-wound motor. Both busses were tested with the equivalent of a seated passenger load, the mechanical-drive vehicle weighing 21,425 pounds and the electric-drive weighing 23,050 pounds.

Table I. Maintenance Expense for Gas-Electric-Drive Motor Busses

Year	Total Maintenance (Cents per Mile)	Maintenance—Electric Transmission Equipment	
		Cents per Mile	Per Cent of Total Maintenance Expense
1926	4.60		
1927	5.59		
1928	5.04		
1929	7.29	1.17	16.1
1930	5.86	0.90	15.4
1931	5.01	0.86	17.3
1932	4.03	0.85	21.1
1933	2.71	0.28	10.3
1934	2.76	0.34	12.3
1935	2.83	0.38	13.4
1936	3.23	0.42	15.0
1937*	2.75	0.30	10.9

* Ten months.

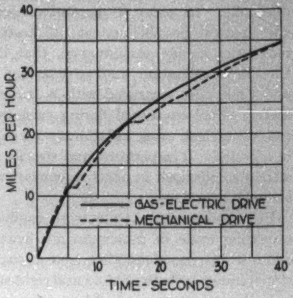


Figure 1. Acceleration characteristics

In order to eliminate, in so far as possible, the human element, a skilled and specially trained driver operated the vehicles during all tests. The test busses were equipped with special apparatus to measure performance. Acceleration and deceleration were measured by a Hasler-Tel recording accelerometer, which is essentially an integrating speedometer that records distance traveled in feet, time in seconds, and mean or integrated speed during each second. The meter was driven by means of a bicycle wheel, mounted on the side of the bus. Combustion efficiency was measured before each series of acceleration runs, with an Orsat exhaust gas testing apparatus. An electric tachometer measured engine speed.

Acceleration runs were operated over a 2,100-foot strip of concrete highway that had a slight but constant grade of 0.7 per cent. Runs were operated in each direction, and the average acceleration characteristics for the gas-electric and mechanical-drive busses are shown graphically, figure 1. It is evident from comparison of these speed-time graphs that instantaneous acceleration values for the mechanical-drive bus are slightly higher than corresponding values for the electric-drive bus. This advantage is more than offset by time required for gear changes in the mechanical-drive bus, with the result that road speed of the electric-drive bus is slightly higher over the operating range. For comparative purposes, the speed-time graphs for the two vehicles can be considered essentially identical. It would be concluded from a comparison of acceleration characteristics that there would be a negligible difference in relative scheduled speed ability between the electric-drive and mechanical-drive busses. This conclusion is contrary to normal operating experience, and further analysis is necessary to determine whether other factors affect the scheduled speed ability of the two types of vehicles.

Fuel consumption of the electric-drive and mechanical-drive busses was measured at varying stops per mile, under rigidly controlled operating conditions. Tests were made over a closed course having a measured distance of one mile. In order to minimize operating variables to the greatest possible extent, all tests were made with maximum engine performance, that is, with wide-open-throttle acceleration and no coasting. Operating speed and time were measured with a Hasler-Tel performance meter. Fuel consumed during each test run was accurately weighed, using a platform scale calibrated to 1/100th of a pound. Temperature and specific gravity of the fuel were also obtained in order to convert weight of fuel consumed into gallons.

Fuel consumption tests were conducted, with a fixed operating cycle of maximum acceleration between stops and a seven-second standing time at each stop, for various stop frequencies between four and eight stops per mile. The mile closed test course had been surveyed and stops were made at fixed points during the test runs. The comparative fuel consumption for electric-drive and mechanical-drive busses is shown in figure 2. Fuel consumption as determined by these tests indicated that use of the electric drive increased fuel consumption 22.9 per cent at four stops per mile and 23.1 per cent at eight stops per mile.

During tests to measure fuel consumption of each type bus, the elapsed time for each run was measured in order that scheduled speed at varying stops per mile could be calculated. The comparative scheduled speed in miles per hour for the electric-drive and mechanical-drive busses is shown in figure 3. It is apparent, despite the fact that the acceleration tests indicated negligible difference between the scheduled speed ability of the two vehicles, that the electric-drive bus constantly maintained a higher scheduled speed than the mechanical-drive bus. Scheduled speed operated by the gas-electric-drive bus was 6.4 per cent faster at four stops per mile, and 8.6 per cent faster at eight stops per mile.

Test data shown graphically, figures 1, 2, and 3, were measured under conditions controlled so as to insure to our satisfaction that measurement of performance characteristics were comparable between the two vehicles and that results could be duplicated between successive runs. We would not defend the accuracy of these measurements from an absolute or scientific viewpoint, but we believe that they may be used as a basis for predicting the relative differences in performance between the gas-electric-drive and mechanical-drive bus.

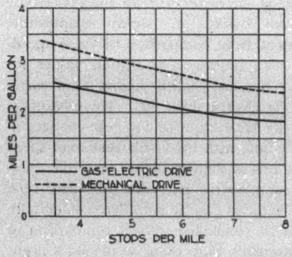


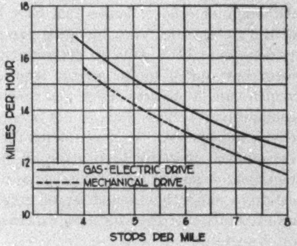
Figure 2. Fuel consumption

Selection of bus equipment is fundamentally a question of economics and in the final analysis the relative advantages of a particular type vehicle can and should be reduced to terms of dollars and cents. Comparison between the electric-drive and mechanical-drive vehicle can be made in terms of dollars and cents, predicated upon the results of the tests conducted, adjusted to take cognizance of operating experience. The problem may be reduced to a simple equation:

Net income per vehicle = total revenue − total expense

Since the only variable under consideration is the drive mechanism, analysis of those factors, inherent in the electric and mechanical drives, and the evaluation of their effect on revenue and expense, will indicate which of the two types of drive mechanism is best suited to the operating requirements. Many of the factors are indeterminate, but a qualitative estimate of their value may be approximated. Evaluation of these factors represents the personal viewpoint of the author, and it is acknowledged that there may be wide divergence of opinion as to the interpretation of the basic information herein presented.

Figure 3. Scheduled speed



Total revenue may be a function of the traffic promotional value or sales appeal of the vehicle. It would seem self-evident that the electrice-drive bus has a greater sales appeal to the public than the mechanical-drive vehicle. The quiet, smooth, and uninterrupted acceleration characteristics resulting from use of the electric drive offer a more attractive ride to the passenger. The operator being relieved from the mechanics of gear-changing operations, is less fatigued at the end of the day's run. Public relations can be improved by use of the electric-drive bus, since driver fatigue is one of the usual causes of frayed nerves, snappy and discourteous retorts, and slovenly operating practices.

One advantage of the electric drive that is seldom considered is the decrease in "gassing" effected by eliminating overdrive of the engine during deceleration. One of the greatest single sources of complaint in bus operation, both from passengers and residents along the bus route, is the obnoxious exhaust gases discharged by the engine. This problem has attained proportions that justify consideration of the decrease in volume and change in characteristics of the exhaust gases discharged by the engine of the gas-electric bus due to its freewheeling action during declerating periods. Evaluation of these advantages accruing from use of the electric drive is obviously impractical, but its is believed that some weight should be given to the inherent "sales appeal" of the electric drive in any analysis of the advantages of that type vehicle.

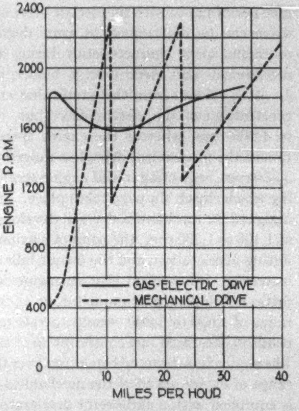
Operating expense is an all-inclusive factor that can be subdivided to minute detail. The major components of operating expense that are appreciably affected by the type drive utilized, include depreciation charge per vehicle, platform wages, gasoline or fuel cost, and maintenance expense. These factors can be estimated to some degree on the basis of known performance for each type bus.

Depreciation charges are a function of the difference in initial cost of the two types of vehicles and their probable economic life. Additional cost for the electric drive will vary, but can be assumed to be approximately ten per cent of the cost for a 35- or 40-passenger bus. Since depreciation charges will normally range from 2.5 to 3.5 cents per mile, use of the electric drive will increase the direct operating expense from 0.25 to 0.35 cents per mile, if we assume equal mileage life for the gas-electric and mechanical-drive busses. However, experience of the Philadelphia Rapid Transit Company would indicate that use of the electric drive prolongs the useful life of the vehicle by decreasing rate of obsolescence and decreasing physical deterioration of the vehicle. Evaluation of the potential increase in useful life of the electric-drive bus is a matter of individual opinion and judgment. It may be considered that the actual increase in depreciation charges incurred by use of the electric drive will vary from nothing to a maximum of 0.35 cents per mile, depending upon specific operating conditions and individual judgment.

The claims and counterclaims regarding effect of electric drive on scheduled speed ability of the motor bus, have tended to obscure the facts behind the smoke screen of controversy. The comparative acceleration characteristics of two identical vehicles, one electric drive, the other mechanical drive, were shown graphically, figure 1. These tests would indicate that there is a negligible difference between the probable scheduled speed abilities of the two vehicles. However, actual tests operated to measure scheduled speed ability, shown in figure 3, indicated that there is a definite and appreciable difference in scheduled speed operated by the two busses. The electric-drive bus operated a faster scheduled speed, varying from 6.4 per cent at four stops per mile to 8.6 per cent at eight stops per mile. These conflicting results can be readily correlated by examination of the conditions under which the tests were operated. The electric-drive bus accelerates automatically, the driver merely depressing the accelerator pedal, and no skill or special technique is required to consistently attain maximum performance. The mechanical-drive bus, however, is accelerated successively through three or more gear ranges. Speed at which gear change is made, and rapidity of gear change affect the acceleration rate of the vehicle. During any acceleration test, the driver is under tension, gear change is made at optimum engine speed, with the minimum of time loss and performance attained is the maximum. During scheduled speed tests there is a natural letdown on the part of the driver, despite the fact that there still exists the necessity for changing gears under controlled conditions. It is apparent that in actual service, the mechanical-drive bus can only approach but never attain maximum performance. The gear-changing operation slows up, and engine speeds at which gear changes are made vary widely from the optimum values. The increased scheduled speeds, shown in figure 3, maintained by the electric-drive bus represent the minimum that could be expected in service. It is concluded that in frequent-stop service, the electric-drive bus could maintain a scheduled speed ten per cent faster than could be maintained by a mechanical-drive vehicle of identical design and construction.

Scheduled speed affects those items of operating expense that are proportionate to bus hours and number of busses operated, provided service frequency permits adjustment of schedules to maintain constant the number of bus miles operated. In short-headway heavy-traffic service, such adjustment is usually possible for any appreciable change in scheduled speed. Increased operating speed will, therefore, decrease in direct proportion, drivers' wages, bus depreciation charges, and certain fixed charges. These expenses will normally average not less than 11 cents per mile for heavy-traffic urban service. An increase in scheduled speed of ten per cent would therefore effect a decrease in direct operating expense of 1.1 cents per mile.

Figure 4. Engine speed during acceleration



Comparative fuel consumption for the electric-drive and mechanical-drive busses is shown graphically, figure 2. These fuel-consumption values were measured at maximum performance, to minimize operating variables, and are not indicative of the values that would be obtained in actual service, although comparison between the two types of busses should be valid. The miles per gallon of gasoline for the mechanical-drive bus were consistently higher than for the electric-drive vehicle, the increase for the mechanical-drive vehicle varying from 22.9 per cent at four stops per mile to 23.1 per cent at eight stops per mile It will be noted that the percentage increase is practically constant over the range of service conditions from four to eight stops per mile. It may be assumed that determination of comparative fuel consumption under known conditions of actual operation will be a measure of difference in fuel consumption for the two types of vehicle. These busses were operated for a period of two months in actual passenger service. Two groups of runs were selected having similar traffic characteristics, and the drivers assigned to these runs were instructed in operation of the test busses. To eliminate, in so far as possible, variables of operation, the busses were interchanged each week between the two groups of runs. At the end of the two months' period, the mechanical-drive bus averaged 4.27 miles per gallon of gasoline and the gas-electric bus averaged 3.50 miles per gallon of gasoline, an increase of 18.1 per cent in miles per gallon of gasoline for the mechanical-drive vehicle.

Difference in fuel consumption between the mechanical-drive and electric-drive busses was measured with standard design engine and standard engine adjustments. The engine design and adjustments are predicated upon requirements of the mechanical drive, which differ radically in many respects from the requirements of the electric drive. Comparison of the requirements for optimum engine performance with the electric drive and the mechanical drive can be determined to some degree by comparison of engine speed characteristics during acceleration of the mechanical- and electric-drive busses, as shown in figure 4. It is evident from this graph that engine speed during acceleration of the electric-drive bus, ranged from 1,600 to 1,800 rpm, whereas the engine speed during acceleration of the mechanical-drive bus ranged from 1,100 rpm to 2,350 rpm, neglecting initial engine accelerating period during which clutch slippage takes place. Since the engine is designed for mechanical drive, it develops maximum torque at 1,100 to 1,200 rpm, the point of maximum power demand during acceleration, and the torque falls off as engine speed increases. It is evident that an engine designed for electric drive, developing maximum torque at the operating speed range of 1,600 to 1,800, would operate more efficiently and result in decreased fuel consumption of the gas-electric bus. The engine is carbureted to perform over the entire operating range of engine speed of the mechanical-drive vehicle, and is equipped with a carburetor accelerating pump to insure pickup at low engine speed and heavy load. The electric-drive vehicle does not require this type of carburetion, and fuel economy can be attained by adjusting carburetion in the electric-drive bus to actual operating requirements. Test data have also indicated that the permissible maximum engine compression ratio that can be operated without detonation, using a given fuel, is substantially higher with the electric drive than with the mechanical drive, due to relatively constant speed operation and the elimination of low-speed operation at heavy load. It is believed that increased use of the electric drive will tend to focus attention on the inherent possibilities of improved economy to be derived from changes in engine design to meet the actual conditions under which the engine operates with the electric drive. Although the test data would indicate that use of the electric drive results in an increased fuel consumption of from 18 to 20 per cent, equivalent to from 0.5 to 0.8 cents per mile, it is concluded that these estimates represent the maximum probable increases in fuel operating expense. Development of an engine design properly adapted to requirements of the electric drive should decrease the differential in fuel expense to a fraction of the indicated value.

Differences in maintenance expense for the electric-drive and mechanical-drive busses can be approximated qualitatively by analysis of the characteristics of the two types of drive mechanisms. It has been shown, table I, that maintenance expense for electric-drive equipment operated by PRT during recent years has varied from 10 to 13 per cent of total maintenance costs. The electric-drive equipment operated is of obsolete design and it is considered that maintenance cost for modern commutating-field equipment would be substantially less than the costs experienced. It is believed that the expense for the electric equipment should not appreciably exceed expense for clutch and transmission equipment of modern busses with remote engine location. There are a number of characteristics of the electric drive that contribute to lowered maintenance costs. Elimination of the shock of clutch engagement tends to minimize wear on axle gearing and lessen the general wear and tear on the entire vehicle. Engine maintenance is definitely decreased due to relatively constant-speed operation and elimination of high engine speeds, as shown graphically, figure 4. In addition, the ability to measure readily engine performance, by rheostatic loading of the power generator at periodic inspection intervals, contributes materially to improved engine maintenance. Free-wheeling action with electric drive further lessens engine wear and tear by eliminating overdrive of the engine. It may be concluded that there is a definite probability of lessened maintenance expense with the electric drive.

There is one other important factor that should be considered in weighing the relative advantages and disadvantages of the electric drive. Although bus operation is a relatively new industry it will be confronted in the future with the problem of increasing age of its employees and the resultant impairment of the physical abilities of individual drivers.

The trend toward increased acceptance on the part of all industry of a social duty toward its employees necessitates adoption of every device that may prolong the useful working life of the employee. The characteristics of the electric drive, in minimizing required driver skill and in decreasing the physical effort necessary to efficiently control the vehicle, permit satisfactory operation of that type bus by individuals that could not operate comparable mechanical-drive busses under the same service conditions. This factor is of paramount importance to those companies that are faced with the problem of extensive substitution of the motor bus for the street car. Many of the displaced car operators can be trained to operate an electric-drive bus who are not capable of satisfactorily operating the mechanical-drive vehicle. Even this advantage of the electric drive has a dollars and cents value, since the alternative to continuing the employment of many of these employees is to retire them on a pension that must be paid from operating revenues.

The effect of operating electric-drive or mechanical-drive busses on operating expense may be summarized as in table II.

It has been stated that selection of the electric drive or the mechanical drive bus is fundamentally a question of economics. The problem was reduced to the solution of the following equation:

Net income per vehicle = total revenue − total expense

Attempt has been made to evaluate the differences between the two type vehicles in order that solutions to this equation might be developed. It is concluded that in frequent stop, heavy traffic urban service, the electric-drive bus will produce a higher net income than the mechanical-drive vehicle, since it has been credited with increased sales appeal, resulting in a higher total revenue, and decreased operating expense.

Table II

	Economies in Operating Expenses (Cents per Mile)	
	Electric Drive	**Mechanical Drive**
Depreciation charges		0.00 to 0.35
Increased scheduled speed	1.10	
Savings in fuel costs		0.50 to 0.80
Savings in maintenance expense	0.25 to 0.50	
Total	1.35 to 1.60	0.50 to 1.15

Modern City Transportation

E. J. McILRAITH

IN MOST cities there are enough automobiles to carry all of the citizens at one time. Yet, there is still a great need for public carriers as a part of city life, and this need increases relatively with increase in size of cities. People pay with great enthusiasm for modern automobiles because they satisfy a desire for new style, more power, greater comfort, and higher speed. Public transportation in its service must recognize these desires.

Life today is streamlined, air-conditioned, finger-tip controlled, and wrapped in striking play of colors and lights. A competing company offers radio harmonizing and another must put in automatic tuning. Then comes "no squint," "no stoop," "no squat." It is an era of dramatic selling.

Not even steel making remains the same. It is alloys we need now. Yet, sales increase because people spend their money for whims, and the whims of yesterday become, through advertising, the needs of today. Trade grows and the keen organization gets its share, while the old shop gets more and more dust and cobwebs.

Transportation has changed, but not fast enough through the last generation. Political control fastened itself upon the street railways and steam railways. Now the other industries have their share of government control, and how they like it. We have grown used to it.

The successive blows of high costs due to the war, automobile competition, the depression, have kept many transportation companies without reasonable earnings for 20 years. Credit has been low, and there has been no sound modernizing plan available to bring back business or build up new lines of trade.

But the long-delayed modernization is now definitely in evidence. The correct solution is not fully understood or established, but experience is molding opinion. A great difficulty is the fact that too many of those who influence transportation are outside of the active management, and seek to find a standard solution applicable everywhere. Even among the operators of transportation systems one finds many expressing fixed opinions as if they were definite and unquestioned axioms.

We hear: Street cars are doomed—two-man car operation is silly—busses are the only solution—small busses are the perfect answer—trolley busses give best service at lowest cost—elevated railways are out.

Some individuals in prominent places believe firmly in the absolute accuracy of one or more of these statements. Each of these statements has definite adherents who are enthusiastically willing to argue for their beliefs, and to spend money based on those beliefs. Actually no one of them is correct in all circumstances, but each of them is true for certain operations, or for certain cities, or for certain circumstances. The great trouble is lack of analysis to determine the different solutions that are needed to give correct answers for the different situations that exist, and to provide the best service with the best operation, and the maximum net return at a reasonable rate of fare. There is no single unit that should be universally used, even in a single city except, perhaps, in cities of very small size.

The transportation industry like any other needs careful thinking and planning, careful analysis and application of sound principles. One doesn't build the same standard factory with an unvarying type of machine tools for a fixed number of workmen to manufacture shoes in one place and containers in another; nor would the same factory be built to manufacture shoes in Chicago as for a factory in Augusta, Georgia. Building a factory or building a transportation system calls for painstaking analysis of the market, the source of supply, the costs of materials, the wage rate, the type of machine tools, the layout of the plant, the operating and working conditions, the trend of public opinion toward the product, the possibilities of molding that opinion, and the probability of producing an adequate net return. Each of these problems has its influence on the success of the whole project.

The final answer that determines the success of an undertaking is the size of the net return after making allowance for the ability to hold the customers by maintaining the quality and price of output. One should not plan for a character of transportation service or kind of unit to be used in giving that service unless it best fits into the production of greater net earnings with satisfaction to the customers. A system should be built around these two objectives, using the best available tools and equipment for each of the many varieties of service within the community served.

Over the years since electric cars have been used, there has been continued progress; better trucks, motors, controls, brakes, doors, bodies, modernized lines but cars were made to last so long that many of the cars you rode on as boys are still with us. Only a pitifully small number of new cars were built each year. That is not real progress.

Ten years ago these facts were recognized and the desire for change finally resulted in the formation of the conference of the presidents of the larger transportation companies and representatives of equipment manufacturers. This conference gave the pressure that comes from large purchasing power to the designing and creation of a truly adequate modern car of two types, for one-man operation and for two-man operation.

Those cars are at last realities operating in several cities and demonstrating definitely that a street car can also be

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thoroughly modernized. The 1937 model car is speedy, sleek, quiet, attractive, and even as modern in appearance as the best bus and much more comfortable to ride. The next models will be better yet.

The new car has excited public interest as an innovation, has attracted many new riders and has gained for the railways new confidence in sound thinking to build a balanced, competent system, using adequate equipment for each transporation need.

What are the transportation needs?

Twenty-five years ago the only public carrier was still thought to be a street car on rails. Then came busses, first as unwieldly, awkard looking, as the early automobiles were, but improving steadily and developing through the years, even as automobiles have developed. Today the modernized bus has reasonable speed, power, comfort and has cleared most of its principal handicaps, except those of heating, lighting, ventilation, fumes of exhaust, and smoothness on uneven pavements.

As bus manufacture became more competent, trolley busses of an efficient type and of adequate performance were developed. Now we have an equally quiet, equally modernized street car for either one-man or two-man operation and surface units run the entire range from private automobiles to small busses to the largest bus, or trolley bus conveniently usable on city streets, into the rail vehicles for lines of heavy density of travel, all running speedily, quietly, smoothly, comfortably.

A full discussion of the relative merits and proper use of these units in a short paper is quite out of the question, but a brief reference to their relative positions is necessary. The principal reason for variety of unit is due to the difference in costs of maintenance, operation, and the various fixed charges to be met.

One cannot depend safely upon general figures on the relative cost per mile of operating street cars, busses or trolley busses, although that is what many managements seek to do, attempting to use approximate estimate as a measuring stick in determining the relative place of each of these types of vehicle. There is no uniform measure that can be correctly used as a generality even for a given city with a single organization and a single scale of wages, working conditions, and costs of material. One must study the cost of each single line and compare the analysis of several types of lines as costs would be for each of these several types of vehicles available to make clear the reason for this statement. The relative importance of operator's wages, milage, costs of maintenance and operation, and fixed charges such as interest on investment, depreciation, insurance, taxes, licenses, and city imposts, vary widely in their effect, dependent upon the characteristics of the operation of the street or line in question.

Unless one has made intimate analysis of proper scheduling of an operation and has found that waste in scheduling is so easily glossed over and ignored, he will not be aware of the significance of intimate study. Very few people understand the possibilities of careful schedule-making, and few schedule departments experience from management intensive understanding, supervision and pressure toward correct analysis and technique.

A few specific illustrations of relative total costs may indicate the need for applying careful, complete analysis to single lines before reaching conclusions as to size of unit or type of operation.

There are three lines in Chicago of nearly the same length but somewhat different riding characteristics. Number 1 carries 53,000 people per day, number 2 60,000, number 3 44,000. Careful detailed analysis of all the factors affecting operation, costs, and investments for each of the various types of operation—two-man cars, one-man cars, trolley busses and gasoline busses—shows that number 1 line would be best served by one-man cars, saving $30,000 per year over the next best type of operation. Number 2 would have the best net return by the use of two-man cars, saving about $25,000 more than by any other method. Number 3 line having somewhat lighter travel and different distribution is best served by trolley busses and these would be best by about $50,000.

By general measures of classification these three lines are alike and directly comparable. It would be easy to create the conviction that the three routes should be treated alike, yet, the net return to the company can be greatly improved by individual decisions and careful application of correct equipment.

The differences are created by the use of the lines by the passengers; that is, the relative intensity of the rush hour as compared to the nonrush period, the relative number in rush hour and nonrush using each of the two directions of travel, the length of ride of the passengers on the line, the zones in which they ride and the number of times the load changes on each trip.

Further, to demonstrate relative costs on one of these lines, index figures for the various types of units are:

Two-man car	118
One-man car	111
40-Passenger gas bus	108
30-Passenger gas bus	114
40-Passenger trolley bus	104

An even more striking illustration can be given by comparing two routes of exactly the same length. One, now a trolley bus line, carries about 19,000,000 passengers per year; the other is a one-man car line carrying 17,000,000 passengers. The average load per car or bus in rush hours in the rush direction is almost exactly the same for the two lines but because of the variations in such factors previously mentioned the total operators' hours paid for on the trolley bus line is 56 per cent higher than on the one-man car line.

Further analysis of the costs of giving gasoline bus service with different sizes of gasoline busses indicates again that general conclusions are likely to result in errors. One line using seven 30-seat busses could be operated with ten 20-seat busses of the same quality and general appearance, except for size, at a saving of around $7,000 per year, and with a reduced investment in busses from $63,000 for the 30-seat bus to $45,000 for the 20-seat bus. A saving of $7,000 with a reduction of nearly 30 per cent in investment is a very substantial accomplishment on so small a line, but the ordinary operating man would be likely to suggest a 30-seat bus if he looked at the line's operation somewhat superficially or without the careful analysis of the complete details of costs.

On another line using at present ten 30-passenger busses the total cost would be $4,000 more per year if smaller busses were substituted, but the investment would be lower for the 20-passenger bus. In each of these cases one is balancing operators' wages, which are naturally higher with the smaller bus, against costs of operation, maintenance and fixed charges, which are likely to be lower on many of the light lines for the smaller bus.

In most any system of transportation prevailing today in cities there are certain lines built many years ago when only street cars appeared to be serviceable for city service, and which have continued to operate with street cars where a substitution could readily be made at a saving by using trolley busses or gasoline busses as the circumstances would indicate. These two newer types of vehicles are better adapted to certain kinds of operation with lighter densities of travel, and should be a part of a modernized system for those lines on which they are appropriate. No one kind of equipment is adequate for best results in all types of service, and the production of net revenue. Rule-of-thumb methods of deciding upon the type of equipment can bring wrong answers.

Of course, another source of error comes from the use of old figures of costs of operation. New methods of maintenance and newer, better equipment are constantly being introduced in gasoline driven busses of all sizes, in trolley busses, and in street cars. The relative comparisons are an ever changing relation and, just as in all business ventures, transportation management needs to be alert in analysis, study, and in modernization.

The new street car generally known as the PCC, or Presidents' Conference Committee Design, has now been in use for more than a year in a number of cities. In most cities it is used as a one-man car; in Chicago only as a two-man car. But the Chicago car is larger in size with different arrangements of doors to make it adaptable for the carrying of larger loads. This car has distinct accomplishments that make it a completely different street car than has been available. The rate of acceleration is not greater than that of some of the best cars previously used, but is very much smoother and more uniform. Jerks are possible only when the motorman must throw off his control while the car is still accelerating at a rather high rate and must immediately put on his brake to avoid an accident. It is, of course, impossible to make such emergency stops without disturbing standing passengers. This is true in any type of vehicle equipped with efficient brakes.

The brake on the new car is a great improvement because braking is not dependent upon wheel adhesion. The combination of dynamic braking with air braking and a battery-operated track brake permits a quicker stop than is possible with an automobile, not withstanding the condition of the rail. Of course, the rate of braking ought to be selected as one that the passengers can tolerate when standing and holding to a support. But since the motorman knows that he is no longer dependent solely on rail adhesion, he may run closer to traffic, and at much higher speeds so that he may hold his place in the street.

The only way in which greater speed can be developed for a vehicle of this sort is by giving the operator complete assurance that he has a thoroughly dependable braking system that will operate at least as rapidly as that of any other vehicles on the street. The success of gaining speed in operation of these cars, then, has been made possible not primarily because of change in accelerating rates, but also because of improved braking.

The new car also has a much better heating and ventilating system, and uses the heat generated in accelerating and in decelerating by dynamic braking for heating the car. This practically eliminates the use of any other electrical heating, and so cuts the cost materially.

The modernized design of equipment in unit assembly has greatly reduced problems of maintenance and has cut costs in both maintenance and manufacture. The weight of the whole car has been materially reduced by better design, and modernized methods of manufacture, made possible by many companies pruchasing from similar specifications, has reduced greatly the manufacturing cost and, therefore, the purchasing price.

However, the outstanding accomplishment is the reduction of noise to a level in keeping with that for busses or automobiles. Without noise reduction the new street cars would be quite incomplete, in spite of the important advantages they have for many lines in all large cities. With the present demonstration of a very quiet street car which can, no doubt, be even further improved in later models, the greatest handicap or source of annoyance has been definitely eliminated.

In the Chicago cars, hand control of the motor circuits and of the brakes is used. The motorman operates the controller with his left hand by a lever that moves vertically through an angle of 22 degrees, the position determining the rate of automatic acceleration chosen. With the right hand he operates the brakes with another lever, also in a vertical plan through an angle of approximately 60 degrees. Here the position determines the automatic rate of deceleration.

This is not quite all the story of the braking controller, because the three forms of braking are not ordinarily all effective at the same time, and the track brakes do not begin to function unless the control handle is moved beyond the middle of its stroke.

The point I wish to make is that we have kept hand control of the movement of the vehicle, while all other properties have gone wholly to foot control in both starting and stopping the car. The principal reason for retaining hand control is that in the operation of a street car no steering is required, so that while the car is running the operator on a two-man car has nothing whatever that he needs to do with his hands, if acceleration and braking are put on foot pedals. Now it isn't good psychology to leave a man at an important job that calls for a high degree of concentration on the smoothness and safety of his operation with his hands idle. One's thinking is too much tied up with what the hands are doing. Hands will not stay idle, and if the operator when operating a car begins to do other things with his hands that take away his attention from safety and smoothness of the car, then the job is being poorly done.

Hand operation with proper kinds of movements can be much more accurate than can foot operation, and due to the flexibility of one's hands, wrists, arms, and body one may make levers move accurately up and down in a vertical direction even though the body is not in a fixed position in a seat.

Other problems remain yet to be settled. All of the new cars in use are front-entrance, center-exit cars. In Chicago a rear exit was also added. But the passengers still tend to concentrate in the front half of the car. It is quite likely that this is inescapable and that in two-man operation it is necessary to have rear-entrance, center-exit, and front-exit, in order that the passengers may be permitted to move in the direction that is psychologically sound, and in which they are most easily trained, instead of trying to force the passengers to go back to alight at points farthest from the crosswalk. The street intersection seems to be their main objective. While passengers are waiting for a car they will move back toward a boarding point where the rear end will be because they are not wasting time in so doing. When they get off they wish to be at the nearest possible point to the street intersection. It seems that this is too strong a tendency to overcome, and that reversal of the present movement for two-man cars will not only accomplish better distribution of the loads, but will also tend to reduce accidents within the car, because the passenger will be facing in a better direction for bracing himself against acceleration or deceleration, and in one to which he has been accustomed over a long period of years of training.

However, the car has been a notable success, and undoubtedly large numbers will be built and used in many cities in modernization now underway. It seems quite important that electrical operation be retained in cities. Certainly the air in the streets is cleaner and better without the annoying fumes of gasoline driven units. Fortunately, on heavier lines electrical vehicles, either street cars or trolley busses, are more economical and perhaps it is much better to have transportation systems largely dependent on central station energy than to be dependent on gasoline or oil, which may become in a few years relatively more difficult to obtain and more costly in use. The source of gasoline is not unlimited. Of course, if Diesel engines can ultimately be developed to be utilized to advantage, the increased use of Diesel oil will also cause a rise in price, but Diesel engines in city service need to be equipped as Diesel-electric type, which produces added complications.

But electrically controlled vehicles served by central stations would have marked advantages from the standpoint of fumes, better heating, lighting and ventilation, lessened noise, greater dependability in power source, and they may also be made larger. Street cars and trolley busses are permitted to be wider than gasoline driven busses because they cannot get away from their city streets into state highways, and when wider may have wider aisles, wider seats, wider doors, and generally better transportation convenience for lines of frequent interchange of passengers.

The conclusion that should be reached from this discussion is that a process of change is going on. Modernization and improvement is vitally necessary, but accurate thinking indicates the need for care in making decisions. Sound transportation planning will utilize all the best ideas and will develop plans that will permit continuing modernization.

Thyratron Reactor Lighting Control

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Synopsis: The introduction of electron tubes and power saturable reactors for the control of theater and mobile lighting has made possible the design and construction of lighting control equipment of greater efficiency and flexibility than was possible with the older methods of control. This paper describes some of the new equipment, systems, and practices.

Several control systems for the pilot controller, the individual, and master units are described and illustrated. The basis for the intensity scale for the control units in the pilot controller is given.

Introduction

IN A PREVIOUS paper,[1] an electron-tube saturable-reactor circuit was described which has proved to be a basic element in the development of a system of theater-lighting control adaptable to a wide range of applications. In applying this circuit it has been possible not only to enlarge former systems of control, but also to develop new and more flexible systems which have proved to be of great practical value. It is the object of this paper to explain some of these systems and the characteristics of some of the component parts.

The theater-lighting control equipment here described is made up of two basic parts, reactor groups and a pilot controller. The reactor groups consist of thyratron-tube panels, distribution panels, saturable reactors, and booster transformers, which regulate the amount of power supplied to the lamp circuits. These reactor groups can be located in any remote place, but are usually situated at a load center. The pilot controller is constructed in the form of a console or vertical panel from which the operator has control of all lighting circuits and is located in front of or at the side of the stage in order that the operator may see the lighting effects which are being produced.

Control Units

The pilot controller consists of individual intensity-control units, master intensity-control units, etc. The number of each used and their arrangement depends upon the control system employed and the number of lighting circuits. In each of these units there is a small lever or knob moving on a calibrated scale for controlling the light circuit intensity.

An outstanding advantage of the thyratron-reactor control over resistance dimmers is the fact that the calibration of this intensity scale serves two purposes:

1. It acts as an index or guide for resetting the lighting circuit and
2. It is a true indication of the percentage of light intensity.

This latter condition is based on an experimental determination[2] of a series of reflecting surfaces which represent a series of equal differences in brightness sensation for the average observer. The intensity scales are arbitrarily marked from zero to ten and correspond to equal divisions of brightness sensation, i.e., zero on the intensity scale indicates no light, five indicates 50 per cent brightness, and ten indicates 100 per cent brightness or maximum light output. Plotting light output in per cent lumens against the intensity scale gives the ideal curve (figure 1, curve *A*). Converting per cent lumens to volts for Mazda *C* lamps[3] provides a convenient basis of comparison for determining how closely control units give results approaching the ideal curve (figure 1, curve *B*).

By referring to illumination levels on a percentage basis, a new terminology is made available to the lighting director since the calibration of the intensity scales is in units which are proportional to the brightness sensation of the human eye.

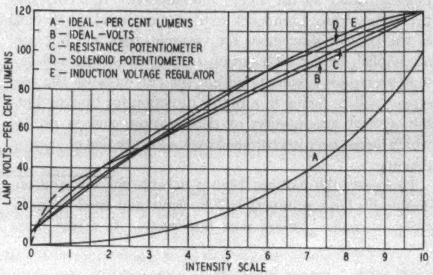

Figure 1. Ideal and actual characteristic curves of individual control units

A—Ideal, per cent lumens
B—Ideal, volts
C—Resistance potentiometer
D—Solenoid potentiometer
E—Induction voltage regulator

INDIVIDUAL INTENSITY-CONTROL UNITS

1. Resistance Potentiometer. A resistance potentiometer was used as the intensity control unit in the description of the theater lighting control circuit previously referred to. The current drawn by this circuit from the intensity control unit is about three milliamperes and hence a resistance potentiometer of small physical size and having a resistance of several thousand ohms can be

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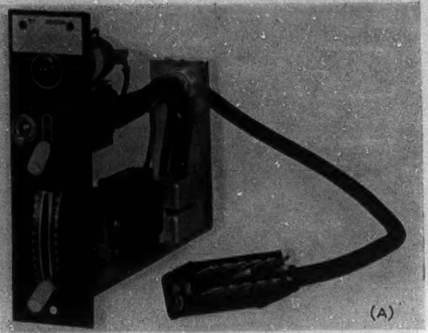
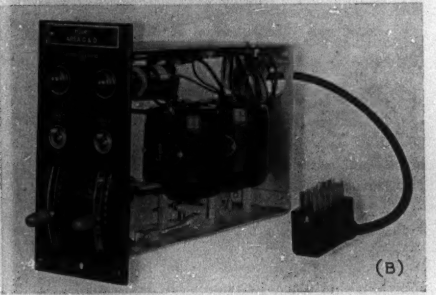
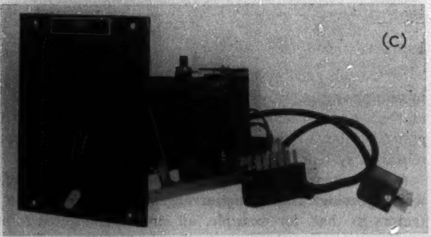
[1] For all numbered references, see list at end of paper.



A—Solenoid potentiometer type



B—Induction voltage regulator type



C—Master (above) *D*—Scene fader (below)

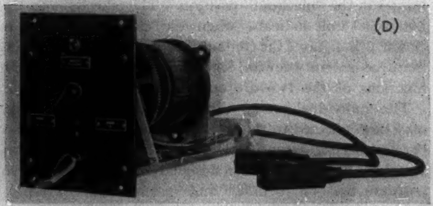
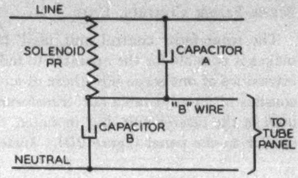


Figure 2. Control units

used. The potentiometer is usually mounted with the shaft perpendicular to the face of the unit so that the control knob is turned to control the lamp circuit intensity. Uniform wire-wound potentiometers are usually used, and the intensity scale calibrated to correspond as nearly as possible to the ideal characteristic (figure 1, curve *C*).

2. *Solenoid Potentiometer.*[4] Figure 2*A* illustrates a typical control unit using a solenoid potentiometer. Control of the lamp intensity is effected as follows:

Figure 3. Connections of solenoid potentiometer



Referring to figure 3, change of voltage between the *B* wire and neutral is obtained through the resonant rise and fall of voltage across capacitor *B* as the inductance of solenoid *PR* is varied from a low to a high value. That is, solenoid *PR* goes from series resonance with capacitor *B* into parallel resonance with capacitor *A* as the inductance of *PR* is increased. From the above description, it is evident that this intensity control consists of a regulating scheme similar to an ordinary resistance potentiometer but employing solenoid *PR*, capacitor *A*, and series capacitor *B* to control the voltage magnitude existing between the *B* wire and the neutral through the variation of the inductance of *PR* caused by the change in position of an iron core rather than by the change in position of a sliding contact. The intensity scale is uniform because the proper selection of solenoid coil and capacitors gives an intensity calibration (figure 1, curve *D*) which closely approaches the ideal characteristic.

3. *Induction Voltage Regulator.* A control unit using induction voltage regulators is shown in figure 2*B*.

The induction voltage regulator consists of a stator winding and a rotor winding. The stator winding, when connected to an a-c source, induces a voltage in the rotor winding depending upon the relative positions of the stator and rotor. In order to use an induction voltage regulator as an individual intensity control unit, it must be of small size. However, it has been impractical to build a regulator whose maximum rotor voltage is as high as that required by the tube unit. Therefore, to increase this voltage a transformer is connected between the rotor and the tube unit. The induction voltage regulator generally is not used singly, but in conjunction with similar regulators in preset control systems. As the curve of output voltage versus angular position of the rotor is sinusoidal, a uniform intensity scale can be used because the proper selection of mechanical linkage gives an intensity calibration (figure 1, curve *E*) which closely approximates the ideal characteristic.

MASTER INTENSITY-CONTROL UNIT

The master intensity-control unit is used to control a group of individual units or other master units. It usually consists of a toroidally wound adjustable autotransformer so arranged that power may be tapped from any turn of the winding. Any voltage from zero to full value may be obtained from this device with but small increments of voltage between steps. One form of a master in which the adjustable auto transformer is operated by a rack and pinion is shown in figure 2*C*.

SCENE-FADER CONTROL UNIT

The scene-fader control unit used in preset systems makes it possible for the operator to fade the light circuit intensities of one scene into those of another. It usually consists of an adjustable autotransformer similar to that used in the master unit, but mounted with shaft perpendicular to the panel (figure 2*D*). Instead of the lever a small crank is used through reducing gears to slowly adjust the transformer.

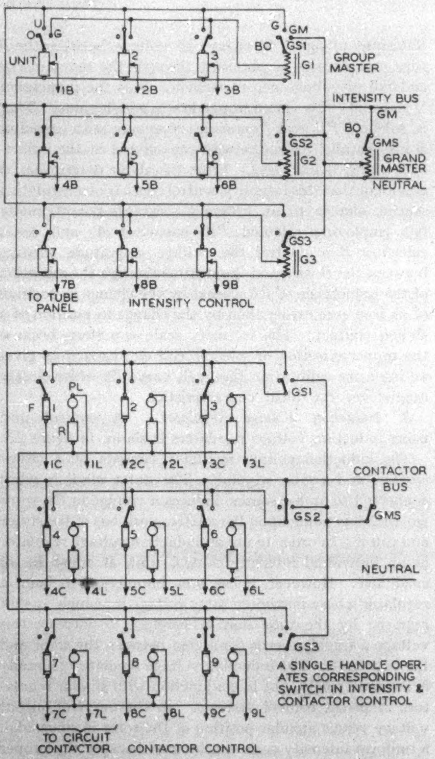

U
O
G
UNIT
1
2
3
G
GM
BO
GS1
G1
GROUP MASTER
1B
2B
3B
INTENSITY BUS
GM
BO
GMS
GS2
G2
GRAND MASTER
4
5
6
NEUTRAL
4B
5B
6B
7
8
9
GS3
G3
7B
8B
9B
TO TUBE PANEL
INTENSITY CONTROL
PL
F
R
1
2
3
GS1
1C
1L
2C
2L
3C
3L
CONTACTOR BUS
4
5
6
GS2
GMS
NEUTRAL
4C
4L
5C
5L
6C
6L
7
8
9
GS3
A SINGLE HANDLE OPERATES CORRESPONDING SWITCH IN INTENSITY & CONTACTOR CONTROL
7C
7L
8C
8L
9C
9L
TO CIRCUIT CONTACTOR
CONTACTOR CONTROL

Figure 4. Schematic of pilot controller—typical rehearsal system

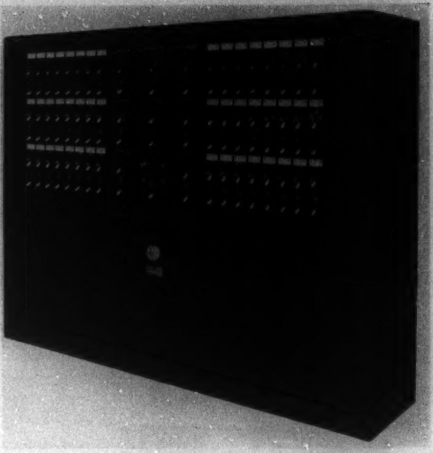


Figure 5. Pilot controller using rehearsal system

Systems

As pointed out above, the pilot controllers consist of individual units, master units, switches, etc. In general, the systems of control for the pilot controllers are classified as rehearsal, preset, rehearsal and preset, or modifications of these.

REHEARSAL SYSTEM

In the rehearsal system, individual units are arranged in groups and may be controlled from a group (or color) master, and these masters may in turn be controlled from a grand master. Figure 4 shows, schematically, a typical rehearsal system.

A contactor control may be included in the system, which provides a switching arrangement for circuit contactors so that, for example, all circuits connected to a group master may be turned off quickly and simultaneously by putting the three-position switch in the group master in the "blackout" position. The pilot light on the individual unit indicates when the circuit contactor is closed. In figure 4 the contactor control is shown separate from the intensity control for clarity, whereas actually the functions of the two are combined.

The individual units, illustrated as potentiometers, may be connected to the "off," "intensity line bus," or a "group master intensity bus" by their three position switches. The customary designations for these three positions are "off," "unit," and "group master," respectively. In figure 4, circuit 6 is in the "off" position, circuit 4 is in the "unit" position, and circuit 5 is in the

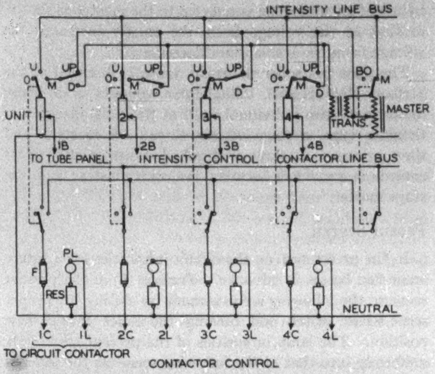

INTENSITY LINE BUS
U
UP
O
M
D
BO
M
UNIT
1
2
3
4
TRANS.
MASTER
1B
TO TUBE PANEL
2B
INTENSITY CONTROL
3B
4B
CONTACTOR LINE BUS
PL
F
RES
NEUTRAL
1C
1L
2C
2L
3C
3L
4C
4L
TO CIRCUIT CONTACTOR
CONTACTOR CONTROL

Figure 6. Schematic of pilot controller—rehearsal system with cross control between individual units and master unit

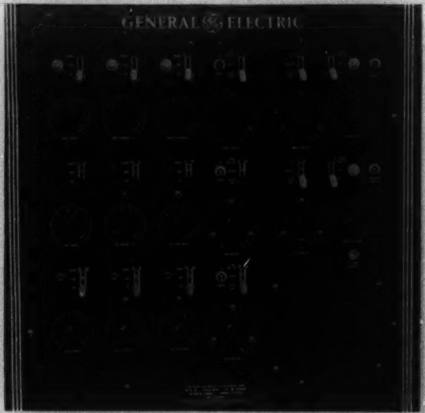

GENERAL ELECTRIC

Figure 7. Pilot controller—Rehearsal system with cross control between color masters and stage master

"group master" position. As the lamp intensity of each circuit depends upon the voltage between the B wire and neutral N for the respective circuits, the lamp intensity for circuit 6 will be zero, for circuit 4 will depend upon the setting for that individual unit, and for circuit 5 will be proportional to the setting of that individual unit and group master 2. For example, if the individual unit and the group master are each set at one-half intensity position, the lamp intensity will be one-fourth. If units 4, 5, and 6 are connected to the group master and one is set at full intensity position, another at one-half, and the third at one-fourth, then as the master unit is increased from zero to full, the intensity of each circuit will increase in proportion, reaching full, one-half, and one-fourth, respectively, when the master reaches the full intensity position.

The group masters are connected to the grand master when their three-position switches are in the "master" position, to the line when the switches are in the "group" position, or off when the switches are in the "blackout" position. The lamp intensity of a lighting circuit, whose control unit is connected to the group master and the

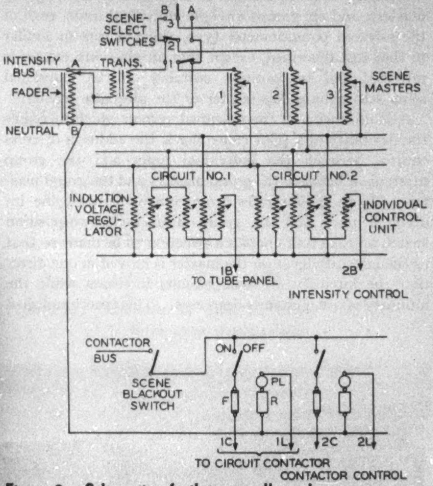

SCENE-SELECT SWITCHES
B
A
3
2
1
INTENSITY BUS
A
TRANS.
FADER
NEUTRAL
B
1
2
3
SCENE MASTERS
CIRCUIT NO.1
CIRCUIT NO.2
INDUCTION VOLTAGE REGULATOR
INDIVIDUAL CONTROL UNIT
1B
TO TUBE PANEL
2B
INTENSITY CONTROL
CONTACTOR BUS
ON
OFF
SCENE BLACKOUT SWITCH
F
PL
R
1C
1L
2C
2L
TO CIRCUIT CONTACTOR
CONTACTOR CONTROL

Figure 8. Schematic of pilot controller—three-scene preset system

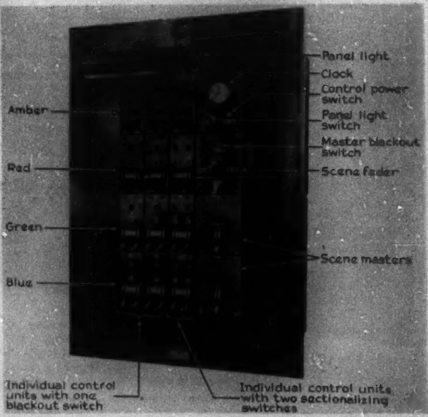

Panel light
Clock
Control power switch
Panel light switch
Master blackout switch
Scene fader
Amber
Red
Green
Scene masters
Blue
Individual control units with one blackout switch
Individual control units with two sectionalizing switches

Figure 9. Pilot controller using two-scene preset system

group master connected to the grand master, will be proportional to the setting of the individual unit, the group master, and the grand master. For example, if the unit, group master, and the grand master each are set at half intensity, the lamp intensity of the lighting circuit will be $1/2 \times 1/2 \times 1/2 = 1/8$. Units which are connected through the group masters to the grand master can be turned off by putting the two-position switch on the grand master in the "blackout" position.

The pilot controller shown in figure 5, installed at the University of Iowa, consists of a grand master, six group masters, and six groups of eight individual units, each of the solenoid potentiometer type, using a system similar to that just described, except that an alternate master is provided and two-position switches on the individual units select the group master or the alternate master.

A modification of the rehearsal system which increases the flexibility of a pilot controller is the addition of cross control between the individual units and the group masters, or between the group masters and the grand master. Figure 6 illustrates cross control between the individual units and the group master. A two-position switch on each unit enables a selection to be made so that as the intensity lever on the master is moved in one direction the intensity of some circuits increases while the intensity of other circuits decreases. This is accomplished by use of a transformer connected to the master as shown, so that as the voltage from the master increases the voltage from the transformer decreases.

The pilot controller shown in figure 7, installed at the Stetson High School, Philadelphia, consists of a house master with two individual units of the resistance potentiometer type, a stage master, three color masters, and three groups of three individual units each. Cross control is provided between the color masters and the stage master.

PRESET SYSTEM

In the preset system, the circuit intensities for a future scene can be set in advance. Transfer from the present scene to the following scene is made by closing the proper scene-select switch and turning the fader to the new position. The lamp intensities of the present scene fade uniformly into that of the following scene as the fader is turned. In this transfer the primaries of the voltage regulators[5] of the present scene are de-energized while the voltage regulators of the future scene are energized. The voltage from the secondary of the regulators depends upon the primary voltage and the angular position of the rotor with respect to the stator. For example, in figure 8 suppose circuit 1 is set at one-half intensity in the present scene (scene 1) and is to be full intensity in the next

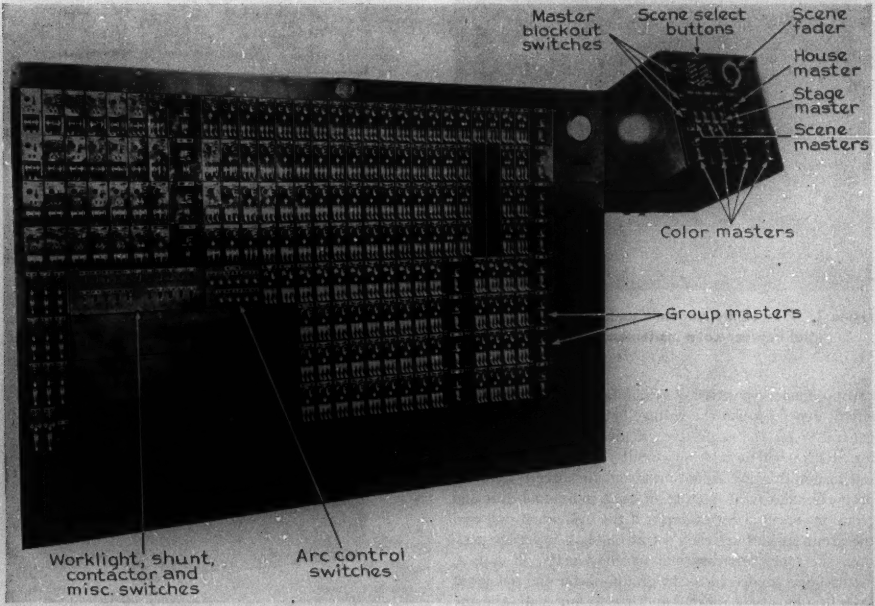


Figure 10. Pilot controller using rehearsal and three-scene preset system

scene (scene 2), and circuit 2 is at full intensity and is to be zero intensity in the next scene. The scene-select switches and fader are as shown, and it is seen that the primaries of the scene-1 regulators are energized and the other regulators are not energized. The scene-2 regulator for circuit 1 is placed in the full intensity position and the scene-2 regulator for circuit 2 is at the zero position. Then by turning the fader from *A* to *B* the voltage decreases on the primaries of the scene-1 regulators from maximum to zero, and at the same time increases the voltages on the primaries of the scene-2 regulators from zero to maximum. The lamp circuit intensity for circuit 1 has thus increased uniformly from one-half to full, and for circuit 2 has decreased from full to zero. All units of a given scene can be controlled by the scene master. For example, the scene-2 master can now be used to dim all units on scene 2.

The scene-select switches in figure 8 are not required if a preset system for only two scenes is used. In this case each individual unit has but two voltage regulators for intensity control. The scene-1 regulator stators are energized and the scene-2 regulators de-energized when the fader is turned to the scene-1 position. On turning the fader to the scene-2 position, scene-1 regulators are de-energized and scene-2 regulators are energized. Figure 9 shows the pilot controller installed at Maple Leaf Gardens, Toronto, Ont., Canada, which is of this two-scene preset type.

PRESET AND REHEARSAL SYSTEM

An extremely flexible control system is made by combining the rehearsal system and the preset system. Such a combination provides a single pilot controller having the advantages of both systems. A three-scene preset and rehearsal control system is used in the pilot controller (figure 10) for the stage lighting circuits at the Metropolitan Opera House. The scene fader, scene-select switches, scene masters, stage masters, and group masters are the same as in the previously described systems. Refer to figure 11. The individual control unit consists of three induction voltage regulators and a three-position switch. Each of the three positions of the switch represents three different connections of the individual control unit: (1) "unit," wherein the circuit intensity is controlled only from voltage regulator number 1; (2) "group," wherein the circuit intensity is controlled from voltage regulator number 1, the group master, or the stage master, as in a regular rehearsal system; and (3) "preset," wherein voltage regulators number 1, number 2, and number 3 are the intensity controls for scenes 1, 2, and 3, respectively.

Summary

Three types of individual intensity control units, a master intensity control unit, and a scene fader for use in theater-lighting pilot controllers have been described. A scientific basis for a rational marking of the intensity scales on the individual units has been given. Three basic systems of connections for pilot controllers and several modifications of these systems have been described. From the foregoing description of the various systems it may be seen that pilot controllers may be adapted to varying requirements for theater lighting control and furnish a wide variety of possibilities in the selection and production of lighting effects.

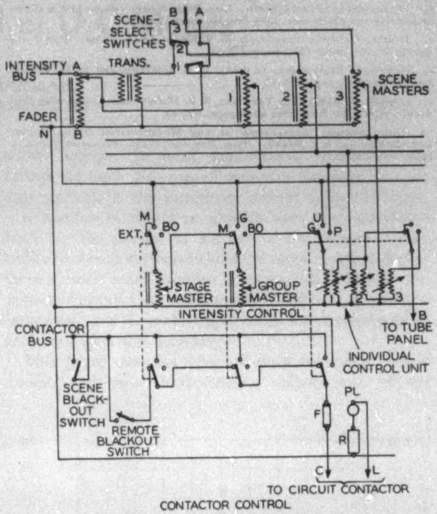


Figure 11. Schematic of pilot controller—rehearsal and three-scene preset system

Nomenclature—Schematic Diagrams

EXT—Extended control from remote blackout switch
F —Fuse in contactor coil circuit
GMS—Grand master switch
GS1 —Switch on group master number 1
PL —Pilot light
R —Resistor

CONTROL-UNIT SWITCH POSITIONS

BO —Black out
D —Down
G —Group master
GM —Grand master
M —Master
O —Off
P —Preset
U —Unit

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Thyratron Control of D-C Motors

By G. W. GARMAN
ASSOCIATE AIEE

Synopsis: This paper describes the characteristics of new thyratron circuits which have been developed for the control of direct current motors. By means of these circuits it is possible to hold constant motor speed, constant armature voltage, or constant relative speed between two systems. It is also possible to hold constant either an alternating or direct current voltage which is dependent on the speed or position of the armature of a direct current motor.

Introduction

THE USE OF electron tubes in the industrial field has grown steadily for the last ten years. They have found their greatest application where other methods of control could not readily be used or were decidedly inferior. Speed control of d-c motors by means of thyratrons is in direct competition with those using rheostats, vibrating contacts, and other similar forms of field control. The thyratron has the advantage of high efficiency, full control, and quickness of response. The small control energy required, and the quickness of response greatly simplifies the design of precision control circuits and reduces the problem of hunting. A further advantage, in comparison to the rheostat control, is that it is not necessary to make step-by-step calculations of resistor values to obtain uniform control throughout the entire speed range. In general, the power component of thyratron control circuits is limited only by the maximum current to be controlled, and the range of control required. The first determines the size of tubes and power transformers and the second determines the magnitude of the anode voltage. In most applications, since 230 volt motors are used, the range of control is the same, and the only variable is the maximum current to be controlled.

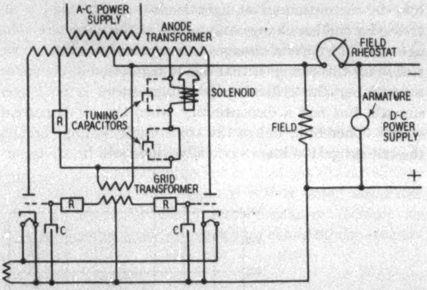

Figure 1. Thyratron field-control circuit with solenoid control

A number of thyratron circuits have been developed both for the control of a-c and of d-c motors. Some utilize feed-back systems to hold the speed constant, but in general, their main function is to provide a smooth adjustable control of the motor speed. There are, however, a large number of d-c motor applications where some type of feed-back system is necessary.

This paper describes some of these feed-back control circuits and their characteristics. Where new circuits were developed every effort has been made toward simplicity and to the use of the same fundamental elements. These circuits can be divided into mechanical feed-back and electrical feed-back systems. In mechanical feed-back systems such as wire reeling applications where the tension of the wire is held constant, the position of a solenoid is varied mechanically, such that the speed of the controlled motor will vary as the reel diameter changes. In electrical-feed-back systems a voltage or current is used as a reference to maintain constant tension or speed or to hold the speed of the controlled motor at some definite relationship with reference to the speed of another motor or system.

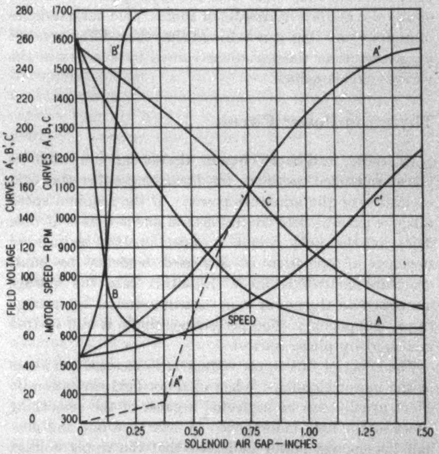

Figure 2. Motor-speed and field-voltage curves for figure 1

Data for this paper were taken on a shunt motor rated

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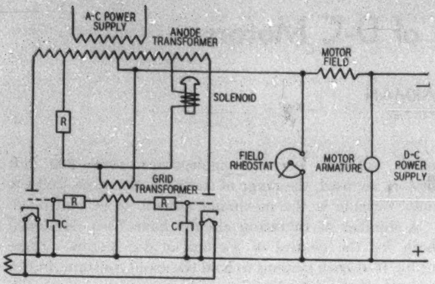


Figure 3. Thyratron field reducing control circuit with solenoid control

one and one-half horsepower, 230 volts, 500/1,500 rpm. In discussing the characteristics of these circuits it has been assumed that these data are typical for the control of any d-c motor. Actually, of course, the characteristics of motors vary, but it is believed that their differences are no greater than the variations caused by changes in the various circuit values.

Thyratron Power Circuit

In these feed-back circuits thyratrons are used as phase-controlled rectifiers for furnishing either the field excitation or the armature power. If the required speed range is narrow, field control is used and if wide or if time delay of the field circuit is objectionable, because of slowness of operation or increased tendency to hunt, armature control is used. In either case the voltage impressed on the motor is not continuous but has a large ripple component. The percentage ripple is still further increased by phase control.

The effect of this ripple voltage is to increase the losses in the motor circuits. When used for field excitation its effect usually can be neglected because of the smoothing effect of the field inductance. A test on a one and one half horsepower motor indicated that the ripple voltage increased the field heating by less than two degrees centigrade.

When used for armature control this smoothing effect is not present and in addition current cannot flow from the thyratrons into the armature circuit until the impressed sinusoidal voltage becomes greater than the counter electromotive force. For this reason unless a smoothing reactor is used the armature current may consist of a series of impulses which may increase sparking of the commutator and cause an abnormal temperature rise in the commutator and particularly in the interpole windings. In some cases for small motors of one horsepower and less, when a biphase rectifier was used, it has been found more economical to use a larger motor rather than to add a smoothing reactor. Tests indicate that for the same load the motor should be approximately 25 to 35 per cent larger than that required if the ripple voltage was not present. The increased heating in the motor depends upon the percentage of ripple current, which in turn depends upon the reactance in the armature circuit, the number of phases in the rectifier circuit and the ripple voltage produced by phase control. The increased rise in temperature is illustrated by table I.

It will be noted that the greatest rise in temperature occurred in the interpole windings and was caused by increased core and copper losses. The field poles normally are not designed for the alternating flux produced by this ripple current.

Table I. Temperature Rise in Degrees Centigrade; 1$^1/_2$-Horsepower 230-Volt 500/1,500-Rpm Motor

	With Direct Current Applied. 100 Per Cent Rated Armature Current	With Smoothing Reactor. 100 Per Cent Rated Armature Current (Average Amperes)	Without Smoothing Reactor. 75 Per Cent Rated Armature Current (Average Amperes)
Field windings	50	52.5	48
Interpole windings	40	42	38
Commutator	31	31.5	18

All readings were taken by thermometer immediately after the motor was stopped at end of test. Biphase type rectifier used for thyratron power circuit.

Even with a smoothing reactor the armature current may be discontinuous at light loads. As the load is increased a sudden change may occur in the armature voltage when the current changes to a continuous value. One reason for this change is that due to regulation in the anode transformer the critical grid voltage curve is no longer smooth, but has a discontinuity when the anode current stops. Therefore, unless the control-grid voltage crosses the critical grid voltage vertically, there will be an appar-

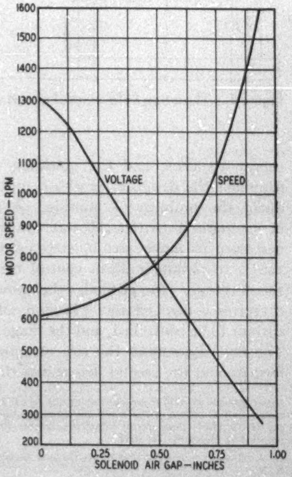


Figure 4. Motor-speed and field-voltage curves for figure 3

ent phase shift in the grid voltage which tends to suddenly increase the output voltage. By using a large inductance this discontinuity can be made to occur at such a low value of current that the control is not affected. Where it is impractical to use a large reactor there are two other methods which can be used to reduce the effect of this discontinuity to a negligible value. A resistor can be connected across the reactor or a phanotron tube can be connected across the output terminals of the rectifier circuit. In the first case the transition point is raised above the maximum load current value, without seriously increasing the heating in the motor, and in the second case the phanotron, because of its low effective resistance, increases the time constant to such a value that the discontinuity occurs at a very small value of current and has the effect of using a reactor of greater size.

Control Circuit

Numerous methods have been developed for varying the apparent phase of the thyratron grid voltage. One method is to superimpose a variable d-c voltage on a fixed a-c voltage; another is to vary the phase of a peaked voltage, but the simplest method is to use a sinusoidal voltage of adjustable on variable phase. The superimposed control has the disadvantage, unless an amplifying tube is used, that the control voltage is so low that the variation in thyratron characteristics, caused either by changes in temperature or by usage, tends to limit their application or to have them discarded before their ultimate life was reached. With variable phase control the grid voltage can be made large enough to compensate for these variations.

A common method of obtaining phase control is to use a variable reactor in a resistance-reactance bridge circuit. The variable-reactor method of control has the advantage that contacts are not required. If a small saturable reactor is used for the variable-reactance element its reactance can be smoothly varied by changing the saturating current. For mechanical feed-back systems the ordinary solenoid is satisfactory for the variable-reactance element.

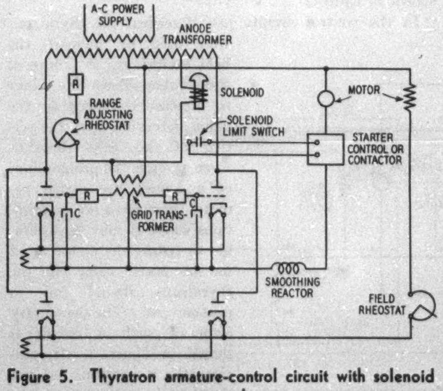


Figure 5. Thyratron armature-control circuit with solenoid control

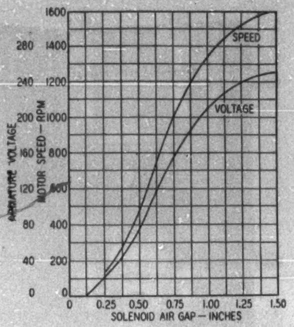


Figure 6. Motor-speed and armature-voltage curves for figure 5

If the reactance is varied from zero to infinity, the voltage from the midpoint of the bridge power supply to the junction of the resistor and the reactor will vary from the in-phase position to 180 degrees lagging with reference to the applied voltage. If the thyratrons supply a resistance load this range of control is required. In d-c motor applications there is always some reactance present which materially reduces this required phase angle shift. In addition, it is possible to still further reduce the required reactance change by partially tuning the reactor with capacitors.

While the data for this paper were taken on a one and one-half horsepower motor, these feed-back control circuits are not limited to motors of this size but can be applied to motors of any size, provided thyratrons of sufficient size are available. At the present time large thyratrons of 100 amperes rating require an appreciable amount of grid power. For this reason the elements in the control circuit must have an appreciably higher rating. The elements as described can be used to control the field excitation of motors up to approximately 100 horsepower and the armature power of motors up to three horsepower. Still larger motors can be controlled either by using small auxiliary thyratrons, or by controlling the field excitation of the generator in a Ward Leonard system.

Antihunting Circuit

In any feed-back system hunting may be caused by time delay in either the control or controlled circuit. It is more apt to occur as the difference in speed from no load to full load is decreased. A common method of eliminating hunting is to slow the response of the control circuit sufficiently to allow the circuit being controlled to respond to the new condition. Slow-speed feed-back systems are inherently stable, but in high-speed systems it may be necessary to either decrease the rate of response or to provide an antihunting circuit.

In electrical feed-back systems one method of eliminating hunting is to feed back into the control circuit a

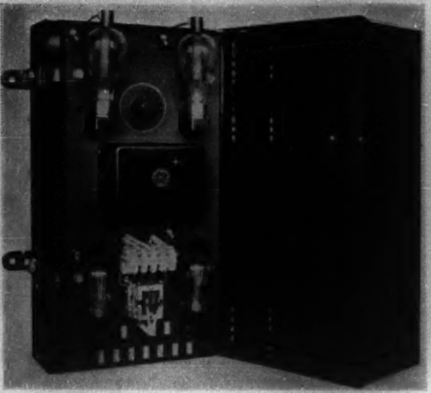


Figure 7. Thyratron control panel for one horsepower d-c motor showing two thyratrons for armature control and two phanotrons for field excitation

voltage which is proportional to the rate of change of one of the circuit voltages. This can be accomplished by impressing one of the circuit voltages on a resistor-capacitor circuit in which the voltage drop in the resistor varies as a function of the rate of change of the applied voltage. This particular method has been found satisfactory in electronic voltage regulators and to be equally as effective in the electrical feed-back systems described below.

Mechanical Feed-Back—Solenoid Control

The circuit of the simple mechanical feed-back system is shown in figure 1. In this circuit the excitation as furnished by the thyratrons is in the same direction as that obtained from the d-c power supply. Should a thyratron fail, the maximum speed of the motor is limited to that value as determined by the setting of the field rheostat and in normal application is approximately ten per cent above the normal maximum speed. The use of this separate source of excitation has still another advantage in that the discontinuity in the field current as mentioned previously is eliminated.

Referring to figure 2, curves A, B, and C show the variation in speed for different settings of the solenoid, and curves A^1, B^1, and C^1 show the variation in the thyratron output or field voltage. By partially tuning the solenoid coil with capacitors, as shown by the dotted lines in figure 1, it is possible to obtain the same degree of control with a smaller variation in the movement of the solenoid core. The untuned characteristics are shown in curves A and A^1 and the limiting tuned characteristics are shown in curves B and B^1. The dotted curve A^{11} shows the effect of the separate field excitation in removing the discontinuity in the field voltage curve. Curves C and C^1 show how these characteristics may be modified still further by changing the value of the fixed resistor in the phase shifting circuit. While these characteristic curves are not linear, their departure usually will not cause any difficulties. In the few cases where a linear relationship is required a correcting cam can be used.

As mentioned previously, the magnitude of the thyratron grid voltage must be large enough to compensate for variations in the critical grid voltage caused by usage and changes in temperature. It is possible to reduce the effect of these variations by replacing one of the thyratrons, figure 1, with a phanotron and to connect one side of the field circuit to the anode of the phanotron instead of to the midpoint of the anode transformer. In this way it is only necessary to control one tube. The secondary voltage of the anode transformer is the same in both cases. This circuit has the disadvantage that the secondary current flows only for one half the time and in only one direction. In addition the circuit is slower to respond for a decrease in excitation because of the short circuiting effect of the phanotron. The anode transformer can be eliminated if the power supply voltage has the correct value. The circuit characteristics are similar to those shown in figure 2.

In the control circuits just described the thyratrons were used to increase the field excitation. A failure of these tubes allows the motor to increase its speed to the value determined by the setting of the field rheostat. Even though the probabilities of a failure are rather remote, there are a few applications where it may be desirable to reduce the motor speed to its basic value if the thyratrons should fail to pass current. The elementary form of such a circuit is shown in figure 3. In this case the thyratrons control the field excitation indirectly

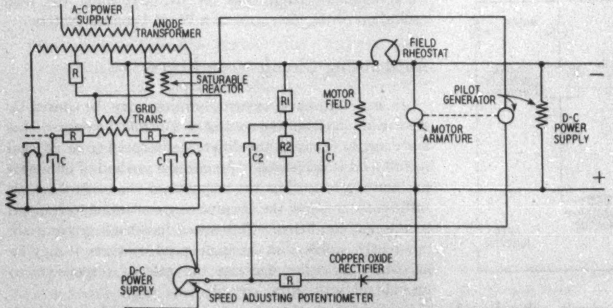


Figure 8. Thyratron field-control circuit, for holding constant speed

by varying the voltage drop in the field rheostat. With a failure of these tubes the motor speed will be reduced to its minimum value as determined by the setting of the field rheostat. The anode transformer must be designed correctly to limit the voltage applied to the field rheostat to such a value that the field current will not be reduced below its safe value, otherwise the motor will run away. The utility factor of this circuit is reduced because the change in motor field excitation is obtained indirectly. The maximum thyratron current will be approximately two times the rating of the motor field. The electrical characteristics of this circuit are shown by figure 4.

The elementary diagram of an armature control circuit in which the solenoid can also be used to vary the output voltage is shown in figure 5. The field excitation can be obtained either from a separate d-c source or from another pair of phanotrons which can use the same anode transformer for their source of voltage.

When armature control is used, some means must be provided to prevent impressing full voltage suddenly on the armature. In some applications this protection is taken care of by using a pair of contacts on the solenoid that are connected in the coil circuit of an anode contactor and are closed only when the solenoid air gap is reduced to zero, corresponding to zero armature voltage. By means of an electrical interlock on the anode contactor, the contactor is held in the closed position after the motor is started.

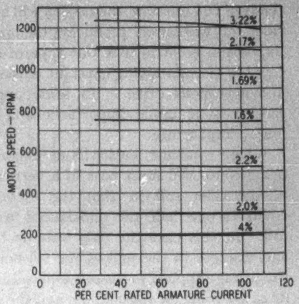


Figure 11. Characteristic curves of armature-control circuit, figure 10, showing speed regulation for different speed settings

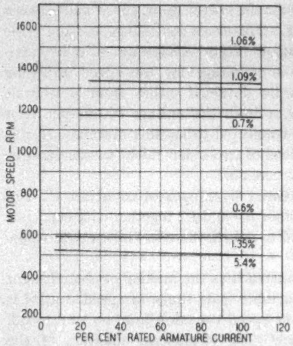


Figure 9. Characteristic curves of field-control circuit, figure 8, showing speed regulation for different speed settings

If this method of protection cannot be used a standard d-c motor starter can be substituted. In either case protection against overload and loss of field excitation can be applied in the normal manner.

The characteristics of this type of control are shown in figure 6. A typical panel with two thyratrons for armature control and with two phanotrons for field excitation are shown in figure 7. An adjustable resistor is used in the phase-shifting bridge circuit to vary the range of control that can be obtained with the phase-shifting solenoid. The adjusting knob for this rheostat, the cathode time-delay relay, and the anode contactor are also shown on the front of this panel. The anode transformer and smoothing reactor are usually wall mounted.

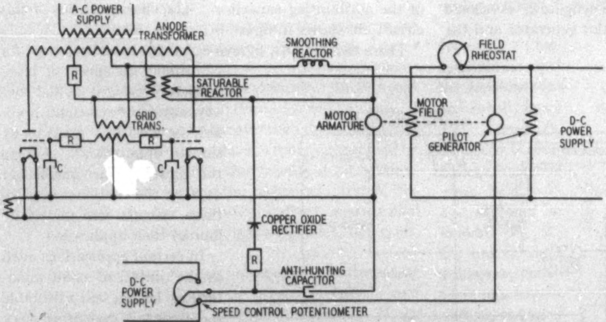


Figure 10. Thyratron armature-control circuit, for holding constant speed

These mechanical feed-back circuits have proved to be very useful in wire drawing, the making of celluloid, rubber and rubber fabrics, and other similar processes. In these applications the problem is to hold the tension constant as the material passes from one part to another part of the same machine, or from one machine to another, even though the driving speed of one part of the machine may vary. For example, if the material is being wound on a reel, it is necessary to change the speed of the reel-drive motor as the diameter increases. In this case the phase-shifting solenoid is mechanically connected to a rider roll which rests on the material. If the tension is above normal, the rider will rise, thereby changing the solenoid air gap, which in turn either increases or decreases the thyratron output voltage, restoring the motor speed to the correct value. If the reel-drive motor tends to run too slow, the tension will be decreased and the rider roll will lower, producing the opposite effect. The important point is that this type of control not only is capable of maintaining constant tension but also provides a synchronous drive. In some applications constant tension is of minor importance and synchronism is of major importance.

Where constant tension is of importance the material being controlled must pass in a vertical plane either side of the rider roll so that the tension will always be equal to one half the unbalanced weight of the rider roll. In this case the only error is that due to friction and changes in the pull of the solenoid core. Normally the voltage applied to the solenoid coil is appreciably below its rated value for solenoid use and the change in pull from zero air gap to the maximum gap is less than two ounces and can usually be neglected.

Electrical Feedback—Saturable-Reactor Control

The previous discussion has covered the use of thyratrons for controlling either the field excitation or the armature power of d-c motors in which a mechanical feed-back control system has been used. There are many other applications where it is desirable to maintain constant speed, constant armature voltage, constant field voltage, to control a motor as some function of either a d-c or an a-c voltage, or to maintain approximate synchronism between two different systems by means of pilot generators. In the following circuits a small saturable reactor is used in place of the solenoid-type reactor. A suitable reactor is one that a change from zero to three milliamperes in the d-c winding will vary the reactance of the a-c winding from 1,000 to 10,000 ohms. If the a-c coils are connected in series this change in reactance can be obtained within a few cycles referred to a 60-cycle power supply. Since this reactor will have the same a-c impedance for a given saturating current of either polarity, it is necessary to limit current flow to one direction by connecting a unidirectional device such as a copper oxide rectifier or a rectifying tube in series with the d-c winding.

The elementary diagram of a field-control circuit for holding constant motor speed is given in figure 8. As shown the speed determining means is a pilot generator and the standard of reference is the voltage drop in a potentiometer. In normal operation the voltage of the pilot generator will be slightly greater than the voltage obtained from the potentiometer. If the load on the motor is increased the accompanying decrease in speed will be reflected by a decrease in the voltage of the pilot generator, which in turn will decrease the current through the d-c winding of the saturable reactor. This reduction in the saturating current will reduce the thyratron output or field voltage, thereby restoring the speed to approximately its previous value. If the load on the motor is decreased, the accompanying rise in speed will have the opposite effect. The speed never will be restored to exactly its previous value, when a change in load occurs, because there always must be a change in the relative voltages of the potentiometer and the pilot generator in order to produce the necessary change in the saturating current. With the correct saturable-reactor design a small voltage, when impressed on the d-c winding, is sufficient to vary the thyratron output voltage from zero to a maximum. By adjusting the arm of the potentiometer the speed of the motor can be held constant within a few per cent from no load to full load over its entire speed range as shown in figure 9. It will be noted that the change in speed from no load to full load for the lowest speed curve was between five and six per cent. This abnormal change was due to the fact that the thyratron output voltage was near its maximum value. The d-c voltage for the speed adjusting potentiometer can be obtained either from a suitable d-c voltage source or from the voltage of another pilot generator which is connected to another system. In this way the relative speed of two independent systems can be held approximately constant.

Hunting is prevented by means of capacitor $C2$ and resistor $R3$. Resistors $R1$ and $R2$ together with capacitor $C1$ have been added to reduce the effect of the field ripple voltage.

In certain processes such as the making of incandescent lamp bulbs an adjustable wide range of speed control is required in which the speed variation from no load to full load is a minimum. The elementary diagram of the armature control, figure 10, fulfills these requirements. Again the control elements are the same, including the use of the antihunting capacitor. The characteristics of this circuit are shown in figure 11.

These two circuits, figures 8 and 10, have been used for controlling the speed of high-frequency generators and conveyor systems and could probably be used in a great many similar applications. Up to the present time insufficient data on the characteristics of these circuits has materially limited their application.

In certain applications such as the control of small wood-turning lathes, an adjustable speed-control system is very desirable. The circuit shown

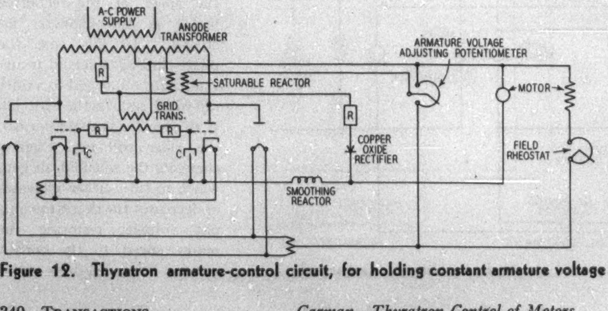


Figure 12. Thyratron armature-control circuit, for holding constant armature voltage

in figure 12 provides an adjustable source of power for the armature, and excitation for the field. The control circuit is the same as that shown in figure 10 except that the voltage applied to the speed-control potentiometer is obtained from the field circuit. Due to regulation in the anode transformer the change in speed from no load to full load is greater than that shown in figure 11. If better regulation is required, power for the potentiometer can be obtained from a separate direct current source or from a small rectifier. The effect of this regulation is shown in figure 13.

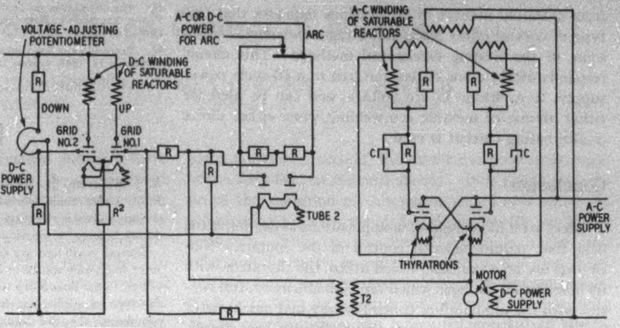


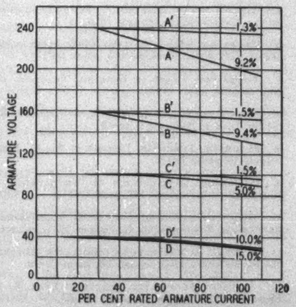
Figure 14. Thyratron control for reversing d-c motor—atomic or metallic arc-welding control

In all of the motor-control circuits which have been discussed the problem was primarily that of either maintaining a definite speed relationship between two systems or of maintaining constant motor speed. The third general classification is the control of a d-c motor as a function of either an a-c voltage or a d-c voltage which is dependent upon the speed or position of the motor. The control of either an arc furnace or an automatic arc-welding head fall in this class. The problem in either case is to maintain constant either the arc voltage or the arc current. The present mechanical method of arc-furnace control is to use either a contact making ammeter or contact making voltmeter together with reversing contactors. These contactors are required to operate a large number of times, consequently the wear is great. Somewhat the same situation applies for automatic arc welding heads but in addition either alternating current or direct current may be used. The circuit shown in figure 14 has been developed for either atomic or metallic arc welding. In this application a small d-c motor is used to feed the electrode wire to the arc. Since this motor is small only two thyratrons are required to run the motor in either direction while two pairs of thyratrons are required to control each motor of an arc furnace.

Referring to figure 14, two small saturable reactors are used to control the two thyratrons. With equal saturation in the two reactors each thyratron will pass the same amount of current, but of opposite polarity. In this case the average value of the torque in the motor will be zero. If the saturation of one reactor is greater than that of the other the motor will run in one direction at a speed determined by the difference in saturation. Conversely, if the saturation is in the reverse order, the motor will run in the opposite direction. The saturation current is controlled by means of a duotriode which has two anodes, two grids, and a common cathode.

In normal operation the arc voltage, either alternating or direct current, is rectified by means of tube 2 and impressed on grid number 1 of the duotriode. If the arc voltage is above normal, as determined by the setting of the voltage adjusting potentiometer, the saturating current through the "up" reactor will decrease, thereby reducing the current through the "up" thyratron with the result the motor armature will run in the down direction. The accompanying reduction in current through resistor *R*2 increases the bias on the grid number 2 in a positive direction tending to still further saturate the "down" reactor. If the arc voltage is low the "up" reactor will become saturated and the motor armature will run in the reverse direction. Due to the unbalancing effect of resistor *R*2 a variation of several per cent in the arc voltage is sufficient to cause the motor armature to run at full speed in either direction. In atomic welding the electrodes burn away so slowly that the motor armature normally stands still. In metallic arc welding the welding electrode burns away much faster and in normal operation the armature gradually turns in the downward direction, but can if necessary run from zero to full speed in either direction. While hunting is normally not present over-control is materially reduced by means of transformer *T*2, which introduces a retarding voltage into the grid circuit when a change occurs in the motor speed. The circuit will respond to a change in arc voltage within a few cycles referred to a 60-cycle power supply. Experience

Figure 13. Characteristic curves of the armature-control circuit, figure 12. Curves A, B, C, and D show voltage regulation when field voltage was used for control. A′, B′, C′, and D′ show voltage regulation when separate d-c voltage was used for control



from a limited number of applications indicates that this type of control offers many advantages in comparison to some of the existing mechanical methods. This circuit responds within a few cycles, referred to a 60-cycle power supply, to a change in arc voltage, and can be used for either atomic or metallic arc welding when either direct or alternating current is used.

Conclusion

There are a large number of applications in the industrial field that require smooth control of d-c motors. New circuits are now available which utilize the thyratron with its inherent advantages which are high efficiency, full control with the minimum of control energy and quickness of response. The elements of these circuits are simple, easy to service, and of conventional design and consequently the circuits are easy to apply. It is possible by means of these circuits to maintain constant speed and constant relative speed between two systems. It is also possible to hold the voltage or current constant in an arc furnace or in an automatic arc welding head.

The use of these circuits in no way limits or alters the use of conventional d-c motor starting on protective systems. These circuits should not only be competitive, but should accomplish results that are difficult if not impossible to obtain with existing mechanical methods.

In the past, cost of the control and lack of confidence have been limiting factors in the application of thyratrons to the control of d-c motors. With increased use and refinements in design the cost will be reduced. Experience has already indicated that confidence in this type of control is increasing as thyratron control is being used with complete success at the present time in certain processes in which a failure of either the control or the thyratrons would cause a shut down of the entire system.

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Discussion

A. J. Williams, Jr. (Leeds and Northrup Company, Philadelphia, Pa.): I was much interested in Mr. Garman's paper, particularly in certain passages referring to antihunting devices which, however, I do not believe he mentioned in his oral presentation.

Several years ago my company had the problem of using a constant frequency source of low power level to control a d-c to a-c converter. One possibility was the use of a thyratron inverter, but for the type of application involved a motor generator seemed better suited since the tube capacity was less, since the output voltage could be made independent of d-c line voltage, and since a tube failure would cause only a loss of constant frequency and not a complete loss of voltage.

We used a shunt motor with thyratron control on the field. The thyratron was supplied with alternating current on the plate from the self-excited alternator and alternating current on the grid from the constant-frequency source. In operation, if the machine slowed down, the plate voltage increased its lag with respect to the grid voltage, more current was passed which weakened the motor field so restoring the phase in part and the speed completely.

We found that full-load variation could be taken care of readily but that the permissible variation in the d-c line voltage was not very great due to the inherent characteristics of a shunt motor with more or less saturation in the field. Anything which increased the range of line voltage stiffened the phase control and increased the tendency to hunt. There was a definite limit which could not be exceeded.

Our next step was to build in an antihunting system. I will not describe this. However, what I do want to mention is the very gratifying results which were obtained from the use of the system. The permissible variation in d-c line voltage was increased by a factor of four.

In closing, therefore, I want to repeat that I was much interested with the short passages in Mr. Garman's paper which referred to antihunting devices and from my own experience, I highly recommend their use for difficult control problems.

G. W. Garman: The prevention of hunting or its elimination as discussed by Mr. Williams, in any closely regulated system, can be a major problem in itself. It can be dealt with only in generalities, except in special cases where all of the constants of the system are known. In a large number of industrial applications, it is sufficient to hold the speed of the controlled motor within one of two per cent, as the load of the motor is varied from its no-load to its full-load value. In these applications hunting can be prevented by the methods illustrated in the paper. In those applications where the speed must be held constant within less than one-half of one per cent, the problem of hunting or of speed variation may become more serious, as other factors, which normally have little effect, now become of importance.

For example: Normally we think of the voltage of a d-c pilot generator as being continuous, except for the small ripple voltage produced by the commutator segments, the effect of which can be eliminated by means of a filter circuit as the equivalent frequency is high. However, the ripple voltage caused by an eccentric armature, even though it is very small, may be sufficient to produce an objectionable variation in the speed of the controlled motor. In addition, it may be difficult to determine whether motor speed variations of this type are caused by an unstable control circuit or by some other factor such as variations in the pilot generator voltage.

It is largely through experience that we learn to appreciate the effect of some of these factors, which normally are negligible, and therefore are able to solve satisfactorily these more difficult problems.

Electronic Speed Control of Motors

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IN THE EARLY years of the electrical industry there was no acceptable a-c motor. After the development of the induction motor and the synchronous motor two machines were available which are unsurpassed in simplicity, reliability, and economy. These motors are, however, both inherently constant speed machines. In later years, a variety of a-c motors have been developed and a number of these have had adjustable or variable speed characteristics. There is, however, as yet, no adjustable-speed a-c motor so flexible as the Ward-Leonard combination, and there is still room for much improvement in adjustable speed a-c motors.

The development of power tubes provides a new approach to the problem of variable-speed motors. This appears to be one of the most promising fields for large tubes and rapid progress may be expected in this direction. It is the purpose of this paper to outline briefly some of the more promising methods of using tubes to control the speed of a motor. A great variety of combinations may be produced, and there will be no attempt to give a complete discussion of this field. The arrangements described, are some of the simplest combinations and provide attractive characteristics.

Thyratron Motor

Those familiar with the art will recognize that a start has already been made in using power tubes to control the speed of an a-c motor. The first tube motor to be developed in America was constructed as a synchronous motor and employed a group of 18 thyratrons as a commutator.[1-3] The Appalachian Electric Power Company has been operating a 400 horsepower motor of this type for over one year. The characteristics and operation of this motor are fully discussed in other articles.[2-4] It is therefore not necessary to dwell further on this method of using thyratrons to produce an adjustable-speed motor.

Rectifier Speed Control for Wound-Rotor Induction Motor

The wound-rotor induction motor with secondary resistance has been used as an adjustable-speed drive in many applications. There are, however, two well-known disadvantages to this use of the induction motor: first, the efficiency of the motor is reduced in proportion to the speed reduction; second, the motor has poor speed regulation with load at reduced speeds. It is possible to improve the operation of this motor at reduced speeds by substituting a rectifier and d-c motor for the secondary resistance. This combination of induction motor, rectifier and d-c motor is quite similar to a Kraemer drive. The d-c motor may be used to help drive the mechanical load, or to drive an auxiliary a-c generator which returns, to the supply system, the power recovered from the induction-motor secondary.

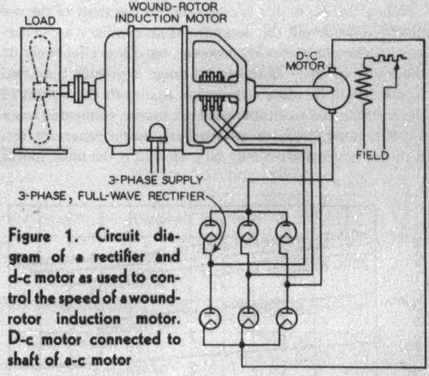

Figure 1. Circuit diagram of a rectifier and d-c motor as used to control the speed of a wound-rotor induction motor. D-c motor connected to shaft of a-c motor

Speed adjustment of the induction motor is obtained through the field of the d-c machine. The action is as follows: assume that the d-c motor in figure 1 is operating at constant speed and with a given field excitation. It has then a definite counter electromotive force and the rectified d-c voltage must match this before current can flow. The d-c voltage of the rectifier bears a definite ratio to the alternating voltage applied to the rectifier which, in turn, is determined by the slip of the induction motor. If, therefore, the a-c motor be operating at synchronous speed it must slow down until the secondary induced voltage is sufficient, when rectified, to match the voltage of the d-c machine. Any further reduction in the speed of the induction motor will cause a current to flow in its secondary and produce a corresponding induction-motor torque. In this manner the induction motor operates at an approximately constant speed or slip determined by the counter electromotive force of the d-c motor.

It is evident from the unidirectional character of a rectifier that a small increase in the speed of the a-c

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1. For all numbered references, see list at end of paper.

motor does not result in a reversed torque. Dynamic braking will, however, occur if the speed of the induction motor exceeds the synchronous speed by a slip equal to the slip required for motor action.

Apparatus Economy

The principal advantage of the rectifier control arises from the fact that the operation and rating of the rectifier are quite independent of the frequency. The rating of the rectifier must approximately equal that of the induction motor. The size of the d-c machine however will be determined by the speed variation required. In case the d-c motor is connected to the same shaft as the a-c motor the approximate rating of the d-c motor may be obtained from the relation,

$$M = \alpha - 1 \qquad (1)$$

where α represents the ratio of the maximum to the minimum speed required and M, the ratio of the rating of the d-c and a-c motors. For a two-to-one speed ratio the d-c motor must equal the a-c motor in capacity.

When the d-c motor is connected to the shaft of the a-c machine, we avoid the necessity of an auxiliary a-c generator. The d-c motor is, however, used less effectively at the lower speeds. If the speed range is greater than two to one it will be more economical to absorb the output of the rectifier in a constant speed d-c motor, connected to an a-c generator, to form an auxiliary motor-generator set. A motor-generator set may be preferred if the main motor operates at a low speed or when its location is such that it is inconvenient to connect the auxiliary d-c motor to its shaft.

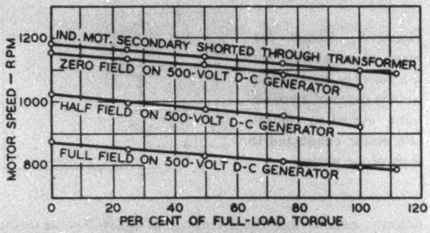


Figure 2. Speed-torque characteristics of a wound-rotor induction motor with rectifier speed control

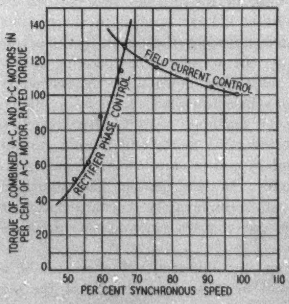


Figure 3. Torque available at rated current when induction - motor speed is reduced by rectifier and d-c motor connected to shaft of a - c motor. Speed control partly by d-c motor field and partly by rectifier phase control

When an auxiliary motor-generator set is used M, the ratio of the rating of the d-c motor to the rating of the main motor is

$$M = \frac{\alpha - 1}{\alpha} \qquad (2)$$

where α as above is the ratio of the maximum to minimum speed. For a two to one speed control the d-c motor must be half the rating of the main a-c motor. The a-c generator in the auxiliary motor-generator set must be large enough to return the power recovered to the supply system. In case of a constant-torque load the a-c generator must have approximately the same rating as its d-c motor. If, however, the load torque decreases with speed as in a fan drive the a-c generator may be considerably smaller than the d-c motor. At high speeds the d-c machine delivers very little power because it has a weak field. At low speeds the d-c motor furnishes little power because it has a small current.

Wye-Delta Secondary Connection

When the load and therefore the current decreases as the speed is reduced it is possible to extend the range of speed control by changing the induction-motor secondary from a wye to a delta connection at some reduced speed. In this way a greater speed range may be obtained from a given d-c motor.

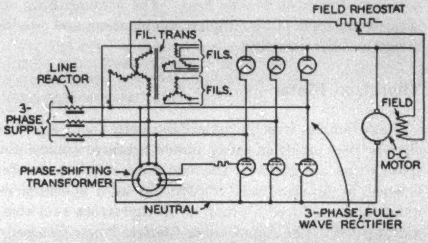


Figure 4. Circuit diagram of a d-c motor operating from a phase-controlled rectifier. Three-phase full-wave rectifier with half-wave phase control shown

Circuit Diagrams and Characteristics

There is given in figure 1 a circuit diagram for a three-phase full-wave rectifier as used to control the speed of a wound-rotor induction motor. The d-c motor is here shown connected to the shaft of the a-c motor. The same rectifier circuit may be used with an auxiliary motor-generator set. The curves on figure 2 show the speed-torque characteristics of the motor shown in figure 1. The speed-torque curves with an auxiliary motor-generator set are similar.

In taking data for the curves shown, a 25-horsepower

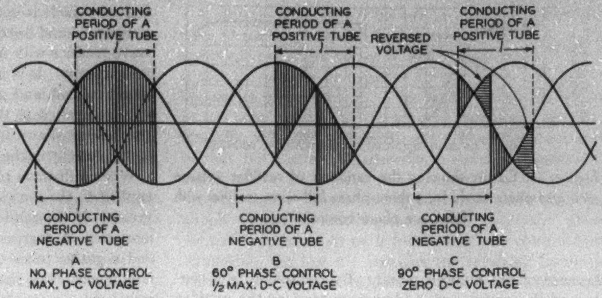

CONDUCTING PERIOD OF A POSITIVE TUBE
CONDUCTING PERIOD OF A POSITIVE TUBE
REVERSED VOLTAGE
CONDUCTING PERIOD OF A POSITIVE TUBE
CONDUCTING PERIOD OF A NEGATIVE TUBE
CONDUCTING PERIOD OF A NEGATIVE TUBE
CONDUCTING PERIOD OF A NEGATIVE TUBE
A
NO PHASE CONTROL
MAX. D-C VOLTAGE
B
60° PHASE CONTROL
½ MAX. D-C VOLTAGE
C
90° PHASE CONTROL
ZERO D-C VOLTAGE

Figure 5. Curves showing the operation of a three-phase full-wave rectifier with full-wave phase control

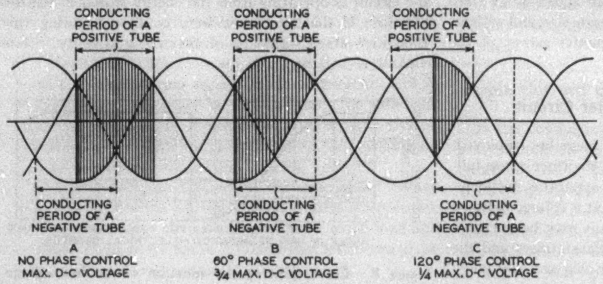

CONDUCTING PERIOD OF A POSITIVE TUBE
CONDUCTING PERIOD OF A POSITIVE TUBE
CONDUCTING PERIOD OF A POSITIVE TUBE
CONDUCTING PERIOD OF A NEGATIVE TUBE
CONDUCTING PERIOD OF A NEGATIVE TUBE
CONDUCTING PERIOD OF A NEGATIVE TUBE
A
NO PHASE CONTROL
MAX. D-C VOLTAGE
B
60° PHASE CONTROL
¾ MAX. D-C VOLTAGE
C
120° PHASE CONTROL
¼ MAX. D-C VOLTAGE

Figure 6. Curves showing the operation of a three-phase full-wave rectifier with half-wave phase control

motor with a 110-volt three-phase secondary winding was used. The secondary voltage of this motor was too low to rectify efficiently. This difficulty may be encountered in some motor designs, but it seems possible to obtain motors of 50 horsepower or greater with a secondary voltage high enough to rectify economically.

In the case of the 25-horsepower motor the secondary voltage was stepped up to 1,100 volts by means of autotransformers. This device would be expensive and cannot be considered typical. The use of the transformers in the tests increased the full load slip somewhat but otherwise did not materially affect the operation.

Phase Control of Rectifier

It is evident that, by phase controlling the rectifier, it is possible to extend the range of speed control. The curves in figure 3 show the variation of torque with speed for an induction-motor control with constant effective current in the primary of the induction motor. In obtaining data for these curves the d-c motor was connected to the shaft of the induction motor. This accounts for the increasing torque available as long as the speed is reduced by motor field control. It is evident from the curve shown in figure 3 that the motor torque, available at rated current, decreases rapidly if the speed is reduced by phase control of the rectifier. When an auxiliary motor-generator set is used, phase control of the rectifier causes the main motor to operate even less effectively than as shown in figure 3.

Starting

When starting an induction motor which operates with a secondary rectifier for speed control, the starting current may be limited by series resistance in the a-c lines. A somewhat simpler arrangement would employ a resistance in the d-c lines. If there is an auxiliary motor-generator set which can be started before the main motor, no other starting device is necessary. The counter electromotive force of the d-c machine, acting with the inherent impedance of the induction motor will limit the starting current to a moderate value.

Phase-Controlled Rectifier and D-C Motor

The d-c motor has long been the standard of excellence as an adjustable-speed drive, but its use is limited because of the prevalence of a-c power. When a motor-generator set is used for converting from a-c to d-c power the Ward Leonard combination gives a flexibility and range of speed control difficult to equal. The cost or low apparatus economy of the Ward Leonard combination is its greatest limitation.

There is little or no novelty in supplying a d-c motor from a rectifier and varying the speed by field control. When, however, the motor and rectifier are considered as a unit, a new degree of freedom may be introduced in varying the armature voltage by phase control of the rectifier. This combination permits a greater range of speed adjustment than can be economically obtained by field control alone. In the upper part of the speed range where the power required is a considerable portion of the motor rating, field control is desirable because this will give the highest efficiency and power-factor. For the lower speed ranges, phase control of the rectifier, to give a reduced armature voltage is desirable. The low efficiency and power-factor produced by phase controlling the rectifier are of little consequence at the low speeds

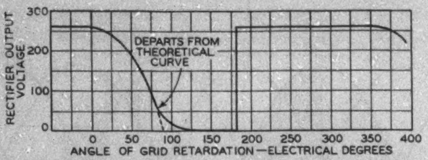


Figure 7. Curves showing the variation of rectifier voltage with grid phase angle for a three-phase full-wave rectifier with full-wave phase control

because of the reduced power. By combining field control of the motor and grid-phase-control of the rectifier it is possible to obtain a 100 per cent speed range (zero to rated speed) with a motor of moderate size, and without unduly increasing the losses and reactive power of the rectifier.

Three-Phase Full-Wave Rectifier Circuit

Any convenient rectifier circuit may be employed. Where three-phase power is available the three-phase full-wave rectifier circuit gives the best apparatus economy. If the three-phase power is available at a voltage between 220 and 600 volts, this rectifier circuit may be operated from the a-c lines without rectifier transformers and the d-c voltage produced will be between 250 and 800 volts. The full-wave rectifier is penalized at low voltage by having two tubes in series and therefore a high arc drop loss. This objection is, however, more than compensated for, by the elimination of rectifier transformers, even though the operating voltage is as low as 220 volts.

The rectifier should be connected to the a-c power supply through line reactors as shown in figure 4. These reactors are very useful in suppressing current and voltage harmonics, and also serve to limit the fault current in case of an arc back or other failure. The three-phase full-wave circuit is attractive in view of the fact that line reactance is twice as effective in limiting the short-circuit current of the a-c system, as in producing regulation of the d-c voltage. A ten per cent reactance in the a-c lines produces only five per cent regulation of the d-c voltage, but limits the current to ten times normal in the case of a three-phase short-circuit.

HALF-WAVE PHASE CONTROL

In the circuit diagram shown in figure 4, phase control is applied to only half of the rectifier tubes. This arrangement is quite desirable. For lack of a better description this operation may be called half-wave phase control of a full-wave rectifier. This method of phase control is superior to the full-wave phase control where all tubes are controlled, in that it produces lower reactive kilovolt-amperes and smaller harmonic currents in the a-c supply system. The advantages of half-wave phase control are greatest at low d-c voltage.

The operation of a rectifier with half-wave phase control is best explained by comparison with full-wave phase control which is somewhat simpler. The curves given in figures 5 and 6 show the operation of a three-phase full-wave rectifier with full-wave and half-wave phase control, respectively. It is important to note that with full-wave phase control, and more than 60 degrees phase shift the voltage applied to the d-c circuit reverses during part of the time as shown in figure 5*C*. This causes low power-factor current in the a-c circuit.

When half-wave phase control is employed the voltage applied to the d-c circuit may go to zero but it never reverses. If the conducting period of the tubes is delayed more than 60 degrees there comes a time when the positive and negative tubes of the same phase are conducting simultaneously as shown in figure 6*C*. During this interval the a-c circuit is effectively disconnected, and the d-c circuit is operating from the energy stored in the d-c reactor. If the a-c circuit were connected during this time its voltage would be of the wrong polarity. It is therefore better that the a-c circuit be effectively disconnected.

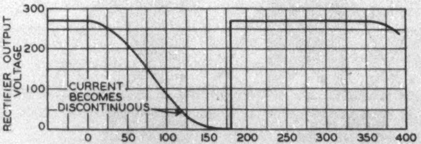


Figure 8. Curve showing the variation of rectifier voltage with grid phase angle for a three-phase full-wave rectifier with half-wave phase control

When a three-phase full-wave rectifier with full-wave phase control is employed E_{dc} the d-c voltage is given approximately by the relation

$$E_{dc} = 1.35\ E_L \left(\cos\theta - \frac{XI}{2I_0}\right) - 2e_t \tag{3}$$

where E_L is the effective a-c line voltage, θ is the angle by which conduction of the tubes is delayed by grid control, e_t is the arc drop of one tube, I_0 is the rated current, I is the actual current in the d-c circuit, and X is the per-unit reactance in the a-c lines. It is evident from this relation and from the curves of figure 5 that the d-c voltage is zero when the phase shift θ is 90 degrees. The relation (3) assumes that the current in the d-c circuit is continuous but it is evident that as the d-c voltage is decreased the ripple voltage increases. The current in the d-c circuit will, therefore, become discontinuous at some reduced voltage, depending on the size of the d-c reactor and the load current. The average voltage in the d-c circuit will then depart from the curve determined by relation (3), because this relation assumes a continuous current and does not hold when the current is intermittent.

Figure 7 shows a typical phase-control curve for a three-phase full-wave rectifier operating with full-wave phase control. The data for the curve was taken with a resistance load in the d-c circuit. If there had been a counter electromotive force load the current would have become intermittent at a higher voltage.

When the voltage applied to the grid circuit of the tubes is sinusoidal a phase shift of more than 180 degrees causes the tubes to be started at the normal time by the lagging tip of the grid-voltage wave. A phase shift of more than 180 degrees therefore, restores full d-c voltage, as shown in figure 7.

When a three-phase full-wave rectifier is operated with half-wave phase control, E_{dc} the d-c voltage is given approximately by the relation.

$$E_{dc} = 1.35\, E_L \frac{\left(\cos\theta + 1) - X \dfrac{I}{I_0}\right)}{2} - 2e \qquad (4)$$

where, as above E_L is the effective a-c line voltage, θ is the angle by which the conduction of the tubes is delayed by grid control, e_t, is the arc drop of one tube, I_0 is the rated current, I is the actual current in the d-c circuit, and X is the per unit reactance in the a-c lines. In case half-wave phase control is employed the grids must be shifted 180 degrees to reduce the d-c voltage to zero, as may be seen from relation (4) and the curves of figure 6.

The curve of figure 8 shows the variation of d-c voltage with grid angle when half-wave phase control is employed. The data for figure 8 was taken with the same load and rectifier circuit as for figure 7, except that half-wave phase control was used when taking data for figure 8.

It is evident from the curve in figure 8 that with a sinusoidal grid voltage full d-c voltage is restored by a grid shift just a little greater than that necessary for zero d-c voltage. This becomes a hazard when operating at a very low d-c voltage, unless some type of self-biasing grid circuit be used so that the positive voltage wave on the grid is shorter than the negative voltage wave. Even with a self-biasing grid circuit too great a phase shift will cause the d-c voltage to increase abruptly to its maximum value.

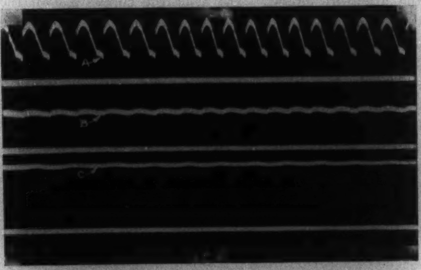


Figure 9. Oscillogram showing voltage impressed on motor field, trace A, field current, trace B, and voltage induced in armature at rated speed, trace C, when the field of a d-c motor is supplied from a three-phase half-wave rectifier

A possible objection to the use of a rectifier with half-wave phase control arises from the fact that the positive and negative current waves are not symmetrically spaced. Either the positive or negative current wave is displaced because of phase control. This leads to even harmonics in the supply current. These even harmonics do not, however, increase the effective value of the harmonic current but the even harmonics replace odd harmonics which would be present if the rectifier were operated symmetrically. The resulting harmonic current in the supply circuit is never greater with half-wave phase control than it would be if full-wave phase control were employed. At low d-c voltages the resulting harmonic current is lower with half-wave phase control than with full-wave phase control. With this picture of the operation of half-wave phase control we may return to a consideration of a d-c motor operating from a phase-controlled rectifier.

Ripple Current in Armature Circuit

If the conducting period of a rectifier is delayed by grid control, the average voltage in the d-c circuit is reduced but the deviation of the instantaneous voltage from the average or d-c voltage is greatly increased. This deviation of the instantaneous from the average voltage is usually referred to as the ripple voltage. The increase of ripple voltage with phase control is obvious in figures 5 and 6. The ripple voltage produced by the rectifier causes a current ripple in the d-c load circuit. An inductive reactance is frequently used to reduce the magnitude of this ripple current.

When the phase-controlled rectifier is supplying a d-c motor, commutation will limit the magnitude of the ripple current permissible. The inductance required for the d-c circuit may be determined from the relation

$$L = \frac{E_r}{\dfrac{dI_{am}}{dt}} \qquad (5)$$

where

E_r = the maximum ripple voltage produced by the rectifier; $\frac{dI_{am}}{dt}$ is the maximum rate of change of armature current consistent with good commutation.

The maximum permissible rate of change of armature current can be determined from the motor design.

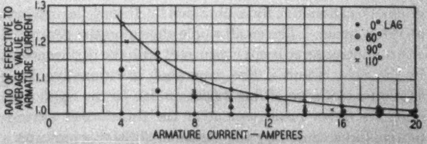


Figure 10. Curve showing ratio of effective to average value of armature current when a d-c motor is supplied from a three-phase full-wave rectifier with half-wave phase control

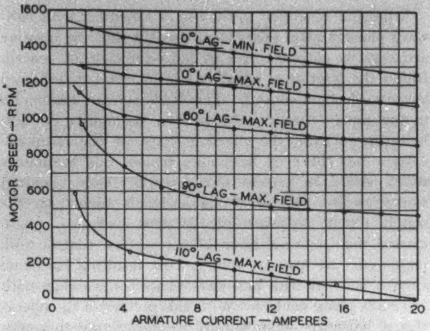


Figure 11. Speed–armature-current characteristics for a d-c motor operating from a three-phase full-wave rectifier with half-wave phase control

Field Excitation for D-C Motor

When operating a d-c shunt motor from a phase-controlled rectifier it is important that the field of the motor be supplied from a reliable source at a constant voltage. It is possible to supply the field from an auxiliary rectifier with a suitable interlock for removing the armature voltage if the field power fails. A more convenient source of field excitation is, however, available when half-wave phase control is used. It is then possible to connect the field between the a-c neutral and the d-c line common to the three tubes not phase controlled. This connection is shown in figure 4.

If the a-c system has no neutral, an artificial neutral can be created by the filament transformers, phase-shifting transformer, or other auxiliary equipment. It is desirable to have the transformers forming the artificial neutral, connected zigzag to avoid saturation.

When the shunt field is supplied by connecting it from the a-c neutral to one d-c line, the voltage impressed on the field will have a triple-frequency ripple of considerable magnitude. This ripple voltage will, however, cause only a small variation in the field current because of the high inductance of the field circuit. Eddy currents set up in the poles and frame of the d-c machine will further neutralize the small ripple present in the field current and maintain the flux through the field sensibly constant. The oscillogram in figure 9 shows the voltage impressed on the field circuit, the field current, and, the generated electromotive force obtained by driving the machine at rated speed. The small ripple shown on the generated electromotive force is not concurrent with the ripple current in the field circuit. The generated electromotive force shows no evidence of flux variation due to the ripple current. A direct measurement of the transformer electromotive force caused by the variation of the flux indicates a voltage of the order of 20 to 50 millivolts in the coil under commutation. An effect of this magnitude will have a negligible influence on commutation.

In case a series field winding is employed, the ripple current in this winding will also tend to produce variations of the flux. Measurements made with a ten per cent series field indicate a transformer electromotive force in the coil under commutation no greater than that caused by the ripple in the shunt-field current; i.e., 20 to 50 millivolts. With a larger percentage of series excitation the ripple current in the series field will be correspondingly reduced by the inductance of the series field. It appears, therefore, that no appreciable commutating trouble may be expected from field-flux variation with either shunt or series field excitation.

Operating Characteristics

The operating characteristics of a d-c motor are quite familiar. When, however, the d-c motor is supplied from a phase-controlled rectifier there are certain modifications introduced by the rectifier. It, therefore, seems desirable to give some operating data. The curves shown in figures 10 to 13, inclusive were taken on a 10-horsepower 550-volt 1,300-rpm d-c shunt motor operating from a three-phase full-wave rectifier with half-wave phase control. The circuit diagram is as shown in figure 4. The rectifier was supplied from a 440-volt, 3-phase, 60-cycle source of power. A picture of the rectifier and its associated equipment is shown in figure 14.

The curves shown in figure 10 give the ratio of the effective to average value of the armature current. The ripple current in the armature contributes nothing to the power of the d-c motor but increases the copper loss of the armature circuit. The armature copper loss of the d-c motor will be increased approximately as the square of the ratio of effective to average current shown in figure 10.

The ripple voltage increases rapidly as the d-c voltage is reduced by grid control, but does not vary appreciably with the load. It is evident, therefore, that with a given angle of phase control the ripple current is fairly constant and independent of the load current. At light loads the ripple current will exceed the load current, with the result

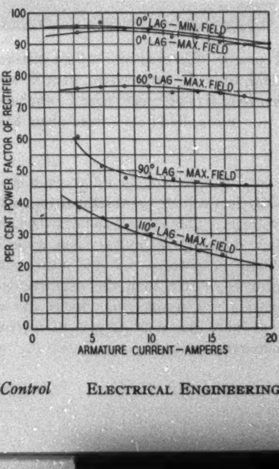


Figure 12. Power-factor curves for a three-phase full-wave rectifier with half-wave phase control. Rectifier used to supply a d-c motor

that the armature current will become intermittent. It is this condition which causes the high ratio of effective to average current at light loads. Fortunately this pulsating armature current does not cause undue motor noise or vibration when the rate of change of armature current is kept within the limit required for satisfactory commutation.

The curves in figure 11 show the variation of speed with load, or armature current. These curves show that the motor has considerable speed regulation largely because of the reactance in the a-c supply lines. When phase control is employed to reduce the motor speed, the speed regulation at light loads is quite large. This is due to the intermittent armature current at light loads. Where less speed regulation is required this may be obtained by using less line reactance or by a number of methods of compounding the rectifier to give a grid-phase shift in response to load. It is undesirable to operate with a very low a-c line resistance because this will increase the harmonic currents present in the a-c supply and lead to more severe fault currents in case of an arc-back.

Power-factor curves are shown in figure 12 for several grid phase angles, selected to reduce the d-c voltage by steps of about 25 per cent. The data for these power-factor curves was taken with 20 per cent reactance in the a-c lines. It is unlikely that a reactance greater than this would be used.

When the rectifier is operating to give full d-c voltage and with about 20 per cent a-c line reactance the power-factor is about 90 per cent with zero grid angle; and this power-factor is maintained even at light loads as may be seen from figure 12.

If the motor speed is reduced by phase control the power-factor of the rectifier is affected quite adversely. At low speed the decrease in power-factor caused by full-wave phase control is even more than that shown in figure 12.

The reduction of power-factor caused by phase control is one of the important reasons for using motor-field control for part of the speed range. It seems desirable to use motor-field control to reduce the speed until the power required by the motor is not greater than half the full-load power at maximum speed. This corresponds to a 50 per cent speed reduction for a constant-torque load and about 20 per cent speed reduction by field control for a variable-torque load such as a fan. If the motor speed is reduced by field control until the motor power is reduced to half its maximum value the reactive and harmonic power required by the motor will never exceed that required at full load.

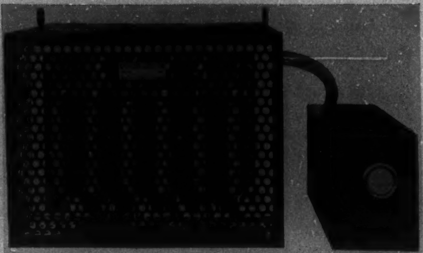


Figure 14. Picture of three-phase full-wave rectifier and associated equipment as used for controlling the speed of a 10-horsepower 1,300-rpm 550-volt d-c motor. One hundred per cent speed range provided, partly by field control and partly by phase control of rectifier

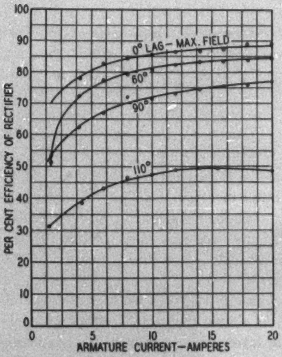


Figure 13. Efficiency curves for a three-phase full-wave rectifier with half-wave phase control. Rectifier used to supply a d-c motor

Efficiency curves are shown in figure 13. These curves are taken for the same conditions of operation as the power-factor curves of figure 12. The efficiency as shown in figure 13 applies to the rectifier only and does not include the motor losses. Since motors may vary considerably in efficiency it seems best to give rectifier efficiency separated from the motor efficiency. The very low efficiencies found at low speeds, result from the fact that the major rectifier loss is caused by the arc drop of the tubes which is approximately a constant voltage drop. As the d-c voltage is reduced by phase control the arc drop represents an increasing proportion of the total power. It should be recognized that the rectifier efficiency will be lower, if a lower a-c supply voltage is employed.

Starting, Reversing, and Braking

Any means of motor speed control which permits the motor to be operated at very low speeds provides the requirements essential for starting. Phase control of the rectifier gives the control necessary for starting the motor and accelerating it smoothly, without producing abnormal currents. A suitable interlock between the phase-shifting transformer in the grid supply and the contactor in the motor circuit will prevent an improper starting sequence.

If a simple rectifier circuit is employed, the unidirectional conductivity of the rectifier necessitates the reversal of the motor connections, either armature or field, when the motor is to be operated in a reverse direction. Regenerative braking is also impossible when a simple rectifier circuit is used because there can be no reverse current to decelerate the motor. When regenerative braking or rapid reversal of the motor is required a double tube-circuit should be employed as shown below.

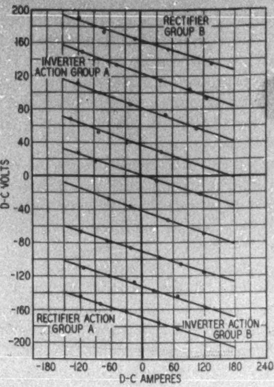
D-C Motor Operated From a Reversible Electronic Converter

There are many motor applications which require either rapid reversal or regenerative braking. Elevators and hoists are two examples, typical of the service where deceleration is an essential requirement. For these and many similar applications, a d-c motor operating from a simple phase controlled rectifier, is inadequate as was pointed out above. It is possible, however, to add the regeneration braking feature, by the addition of a duplicate set of tubes connected for reverse current and with their grids excited for inverter operation.

Regenerative Braking

The circuit diagram in figure 15 shows a double set of tubes with a single-phase a-c supply. One set of tubes is connected for reverse current, which is essential for regenerative braking. The operation may be explained as follows: assume that the *A* group of tubes is rectifying and furnishes current to drive the motor in a clockwise direction. Should the d-c motor be overspeeded a little, the motor counter electromotive force will exceed the rectified voltage and no current can then flow from the rectifier. The *A* tubes are then inoperative.

Figure 16. Curves showing the variation of d-c voltage with load current for a reversible electronic converter



If the inverter tubes, the *B* group, as shown in figure 15, are properly adjusted a small increase in the counter electromotive force of the motor will be sufficient to cause them to begin inverting. They will then draw power from the motor, causing it to operate as a generator with the corresponding retarding torque. The power delivered by the d-c machine is transferred to the a-c system through the *B* group of tubes acting as inverters.

There is of course some speed regulation with load for both the rectifier and inverter tubes. With rectifier or driving operation, the speed regulation is positive and the motor speed decreases with increasing armature current. For inverter or braking operation, the speed regulation is negative, and the motor speed increases with increasing armature current. The curves shown in figure 16 are typical speed armature-current characteristics as the motor operation is changed from braking to driving.

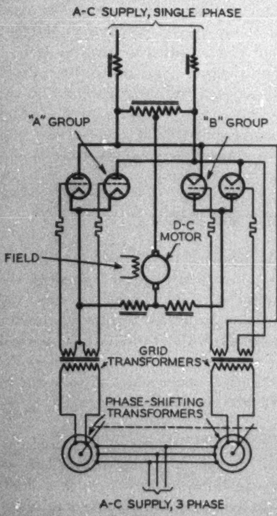


Figure 15. Circuit diagram for a reversible electronic converter for supplying a d-c motor

Reverse Rotation by Phase Control

The action as explained so far would permit the motor to operate at an approximately constant speed over a load range, from full driving to full braking. A constant speed application which requires regenerative braking is, however, seldom if ever encountered. Speed variation and reversal are essential in most applications which require braking. With the circuit arrangement as shown in figure 15 it is possible to vary the motor speed by phase controlling the rectifier and inverter simultaneously. Full-wave phase control is most satisfactory for this service.

When full-wave phase control is employed the rectified voltage is reduced to zero by a grid phase-shift of 90 degrees as is shown in relation (3) and figures 5 and 7. If the grids be retarded still further, the rectifier will be found to have a counter electromotive force and may operate as an inverter. Similarly, advancing the grid phase-angle of an inverter first reduces the counter electromotive force to zero and then produces a forward or rectifier electromotive force.

The grid-control characteristic of a rectifier having full-wave phase control, as shown in figure 7, is shown again in figure 17. In the latter case, however, the operating

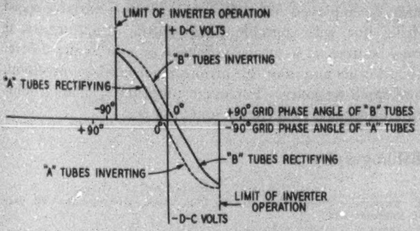


Figure 17. Curves showing the variation of d-c voltage with grid phase angle for a reversible electronic converter

curve of both the *A* and *B* tube groups are shown, and the rectifier phase-control curve is extended to show the inverter range. It is evident from an inspection of figures 15 and 17 that the distinction between inverter and rectifier operation is largely a question of the relative phase of current flow in the a-c circuit. The principal difference arises from the change of the voltage stress across the tube from negative to positive as the operation is changed from rectifying to inverting. The positive tube voltage in the inverter region necessitates a negative grid bias during the entire non-conducting interval. Assuming a suitable grid excitation, however, a group of tubes will function as either a rectifier or inverter. Such a tube group is conveniently and accurately described as a converter.

When the tube converter employs natural or phase commutation, its operating range covers only about 180 electrical degrees. Phase control as a rectifier is then possible only by retarding the phase of the grids. There is a commutating voltage in inverter operation only if the current transfer takes place before the voltage of the incoming phase equals the voltage of the outgoing phase. Fortunately the rectifier and inverter quadrants are adjacent and we may pass from rectifier to inverter action or conversely by a continuous shift of the grid phase-angle.

If the d-c voltage of a rectifier is to be reduced, the grids of the rectifier tubes should be retarded. The opposite is true for an inverter, and the grids of the inverter tubes should be advanced to reduce the operating voltage.

For this reason separate phase-shifting transformers are required for the *A* and *B* tube converters. These two phase-shifting transformers may however be rigidly coupled and moved simultaneously if they are so arranged that the electrical rotation of one is opposite to the other. This is indicated on figure 17 by showing reversed scales for the grid phase-angle of the two converters.

If the phase control characteristic of the *A* and *B* converters are displaced as shown in figure 17, the operating voltage of the inverter will exceed the rectifier voltage for both directions of rotation and there will be a "discrepancy" or change in motor speed when changing from driving to braking. This phenomena can be eliminated by proper alignment of the two phase-shifting transformers. Should the inverter be set for an operating voltage below the rectifier voltage, there will be a circulating current between the rectifier and inverter and corresponding losses. A small circulating current is desirable when a motor speed "discrepancy" between driving and braking is objectionable.

When the phase-shifting transformers are displaced beyond the safe operating limits the tubes acting as inverters lose their commutating voltage. This results in a short circuit between the two converter groups, and should therefore be avoided.

With a circuit arrangement as shown in figure 15 and suitable grid excitation for both the *A* and *B* converters, as discussed above, the motor speed can be controlled through a speed range from full speed forward to full speed reverse. The control required is only a movement of the phase-shifting transformer. At all speeds, forward and reverse, the motor will give regenerative breaking if over-speeded. With the phase-shifting transformers in any operating position there is a speed armature-current characteristic, similar to those shown in figure 16.

The combination of d-c motor and reversible converter just described, is quite flexible and has a very sensitive control in the phase-shifting transformers. This arrangement appears suitable for many applications which have formerly required a Ward Leonard set.

Torque Amplifier

The sensitive and accurate control permitted in the grid circuit of the converters of the motor just described adapt

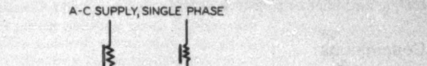


Figure 18. Functional diagram for a torque amplifier employing a d-c motor and a reversible electronic converter

it to many applications not adequately served by a Ward Leonard combination. The arrangement shown in figure 18 gives a scheme for producing a torque amplifier or positioning device. The only change required from the circuit of figure 15 is that the two phase-shifting transformers, which supply the grid power are not manually operated as in figure 15, but are connected to both the positioning control and the main motor, in effect, as through a differential. Referring to figure 18, a displacement of the positioning control acts through the synchro-motor to cause a movement of the grid phase-shifting transformers. This grid shift sets up a current causing the main motor to move. The main motor is connected to the stator of the synchro-motor through a gear. The rotation is such that a movement of the main motor produces a phase shift which cuts off the current causing the motor to move. Therefore, when the main motor has moved through an angular displacement corresponding to the displacement of the position control, the main motor will have corrected the grid phase-displacement which was caused by the movement of the positioning control. In this manner the main motor is kept in close correspondence with the positioning control. If the controlling device is kept in continuous motion, the motor is compelled to follow in a corresponding angular position, but the load or torque burden on the positioning control is only a slight portion of the torque exerted by the main motor in driving its load. The grid control is so accurate and rapid that very close correspondence is maintained between the motor shaft and the positioning device. Automatic tools offer an attractive field of application for a reversible electronic converter arranged to operate as a torque amplifier or position control.

Conclusions

Three different arrangements have been described, each employing electronic devices for controlling the speed of a motor operating from an a-c power supply: viz., the synchronous motor using thyratrons as a commutator (see references 1, 2, 3, 4), the wound-rotor induction motor using a rectifier and d-c motor on the secondary, and a phase-controlled rectifier supplying a d-c motor from an a-c power source. These three schemes all have attractive features and each type of motor will probably find a field of application.

The simple phase-controlled rectifier supplying a d-c motor may use a number of rectifier circuits other than the three-phase full-wave circuit described. For small motors a single-phase rectifier is adequate. The combination of a d-c motor and a simple phase-controlled rectifier provides a very flexible motor operating from an a-c source of power. It is simple and has a high apparatus economy. This motor should find a wide field of application in small and moderate sizes.

If the service demands regenerative braking this feature may be secured by adding a duplicate set of tubes, and the grid excitation necessary to produce a reversible electronic converter.

The combination of d-c motor and reversible converter, may be employed not only as a reversible variable-speed drive but with suitable phase-shifting transformers, it may be used as a torque amplifier or positioning device. As a torque amplifier this arrangement has high precision, and rapid response. For many problems it offers a solution unequaled in accuracy and elegance.

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Discussion

E. F. W. Alexanderson (General Electric Company, Schenectady, N. Y.): Our paper has the object of indicating some of the many possible applications of electron tubes for power transformation. Specifically, it deals with the controlled rectifier, inverter and frequency changer as a power supply for motors. It has not been possible to include in this discussion all the combinations that we have in mind. There is, for instance, one combination which we should like to include and which recent tests indicate to be one of the most promising. This is to absorb the secondary power of the induction motor in a thyratron frequency changer and return the energy to power lines. This combination requires eight tubes if a quarter-phase motor secondary is used.

At this stage is may be interesting to forecast how high motor powers may be conveniently operated by such methods. We may assume that tubes of 200 amperes average capacity will be available. It may also be assumed that the induction motor secondary is wound for 2,300 volts. Such a combination will control a motor of 4,500 horsepower. In this electronic technique we are by the nature of the problem compelled to analyze our expectations for the near future even though the ways and means to meet them are not available for the moment. There is a good reason for this. The basis for this work is the electron tube itself around which all the rest of the apparatus must be designed. But a development of high-power electron tubes is a long and expensive process. To furnish the incentive for this, it is necessary that the electrical engineers who deal with power problems both in design and operation should analyze their needs so that they will be able to point out the directions in which such development will show economic advantages. We hope that a discussion of the subject by the Institute will serve this purpose.

C. C. Shutt (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The authors present an interesting and timely paper on the subject of electronic speed control. It is of particular interest to note that the authors have given considerable space to the wound-rotor type of machine with secondary control. For several years the writer has considered this arrangement preferable to the authors' earlier schemes, which used primary frequency control and a synchronous-type motor, for the following reasons:

1. The number of tubes is reduced and their connections simplified. Only one electronic power conversion required when d-c machine is used as described.

2. A practically standard motor is used with normal densities in copper and iron.

3. At reduced speeds the power factor can be practically as good as with resistance control.

4. Unless the speed reduction is large, the kilowatt rating of the tubes is reduced by the use of secondary control.

With these features in mind, a 350-horsepower unit has been built, which was designed primarily for a fan drive. It uses grid control on the tubes for the lower portion of the speed range. The advantage of grid control for such a drive is that the reduction of the d-c voltage reduces the flux required in the d-c motor at the lower speeds, and thus reduces the size of this machine. It will be recognized that this reduction of d-c voltage is accompanied by an increase in direct current and the current in the secondary of the induction motor. But, with a fan type drive, the current is still well below the rating of the windings at the low speeds. The use of grid control causes a reduction of primary power factor at low speed. But this is not objectionable at light loads which accompany low speeds on a fan drive.

A 1,250-horsepower, 1,200-rpm unit is now under construction.

In common with a Kramer drive, and other forms of secondary control, the drive will develop a maximum of practically constant horsepower when the secondary power is recovered in a d-c motor on the main shaft. It will develop a maximum of practically constant torque when the secondary power is recovered by some other means. These statements assume that the induction motor is worked at normal flux and current densities at all speeds. The equations 1 and 2 of the authors' paper appear to be based on this assumption.

Attention is called to the second sentence of the section under "Apparatus Economy." The authors state "The rating of the rectifier must approximately equal that of the induction motor." This statement is also based on the same assumptions as equations 1 and 2, the correctness of which depends upon whether the tubes are rated in current or kilowatts. Under the authors' assumptions, the tube current is constant regardless of the speed. Neglecting grid control—and it would probably not be used under these assumptions—the d-c voltage increases as the speed decreases. Hence, the kilowatt rating of the tubes is a function of the speed reduction, and certain types of tubes have higher current rating at reduced voltage. Hence, smaller tubes can be used with limited speed reduction.

In closing, the writer would point out that the relatively simple circuit consisting of an induction motor with a rectifier and d-c machine in the secondary only scratches the surface of the possibilities of this type of variable speed drive. Other types of electronic secondary control have been investigated for a range of applications and will be presented in the near future.

L. A. Kilgore (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): This paper gives a good description of several circuits which may be used for a-c variable-speed drives employing electronic tubes, but the description of the motor requirements is confusing at certain points. Equations are given for the relation between d-c and a-c motor ratings. These equations are misleading because they are only applicable to one special type of load, but which special case they are intended to cover is not stated.

Under the heading of "Apparatus Economy" the ratio of d-c motor rating to a-c motor rating is given in equation 1 as: the ratio of maximum to minimum speed minus one, for the case of a d-c motor connected to the same shaft as the a-c motor. This relation would only be true for the special case of constant power; also the ratings would have to be based on the minimum speed.

Also equation 2, which applies to the case where the secondary power is fed through a rectifier to a d-c motor of a motor generator set, is only applicable to the case of constant power; furthermore, it would seem that a maximum speed equal to synchronous speed had been assumed.

The general idea of a rectifier applied to the secondary of a wound-rotor motor to furnish power either to a d-c motor on the same shaft or to be fed back into the line is old, but very little use has been made of it in the past. However, with the recently developed thyratron and ignitron tubes, this type of variable-speed drive holds real promise. One place where it is particularly applicable is in the drive of large fans. For this case, smooth speed control from zero to 90 per cent of synchronous speed is possible with a d-c motor rated only about 40 per cent of the total power required at full speed, and for ignitrons the rectifier rating would be approximately 60 per cent of the maximum power output if the capacity is based on the increased voltage rating at low speeds.

Grid-controlled rectifiers have been used for some time for the purpose of speed control of d-c motors with or without the use of field control of the motor. The authors put particular emphasis on the full-wave three-phase circuit, which avoids the use of a transformer when a suitable a-c voltage source is available. Besides the advantages mentioned by the authors, this circuit has the additional advantage where the blocking of grids or ignitor circuits may be used to interrupt the a-c supply to an arcback, for unless a tube in both groups arcs back at the same time, there will be no back feed from the d-c motor.

The authors show some improvement in power factor at low d-c voltages for the particular arrangement which they call half-wave phase control of the full wave rectifier. However, this is accomplished at the expense of an increase of the magnitude of, and the introduction of, lower frequency components in the d-c voltage ripple. It may be found that where improvement in power factor is desired, it can be obtained more economically some other way.

J. H. Cox (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): This paper constitutes a *review of several methods* of controlling the speed of motors by electronic means, and as such is a valuable contribution. However, since the paper *mentions only thyratrons* as the electronic device to be used, and since thyratrons are only practical in relatively small sizes, the inference is left that this method of speed control is only applicable in small capacities. *Actually, the group with which I am associated* has carried out tests with a 350-horsepower drive using a similar method of control and is now building a 1,250-horsepower drive.

Throughout the paper the authors' choose the *three-phase full-wave circuit* and this circuit is much more convenient for this application than the more conventional power rectifier circuits, though the latter are somewhat more efficient. However, the three-phase full-wave circuit demands that the *cathodes associated with at least three* of the anodes be separated from the others and *there are serious objections* to building the conventional metal tank rectifier in single-anode tanks. However, *ignitrons are normally built in single-anode units,* and where no vacuum pumps are required they are amenable to any electrical circuit desired. Therefore, the methods of speed control described by the authors are now conveniently available up to sizes requiring electronic power that can now be provided by ignitrons with sealed-off construction. As mentioned in Mr. Kilgore's discussion, for a blower type of load, the electronic capacity required is only 60 per cent of the maximum power output at maximum speed.

W. R. King (nonmember; General Electric Company, Schenectady, N. Y.): I am particularly interested in the system described by the authors for controlling the speed of a d-c motor by means of a phase-controlled rectifier. I have had some experience in applying electronic equipment to the control of d-c motors and in the past have been forced to conclude that, primarily for economic reasons, the electronic systems were limited to the field of applications where some very special motor-control functions were required. Numerous successful applications have been made but practically all have been to problems extremely difficult if not impossible to solve by more conventional means.

However the new system described by the authors holds promise for obtaining with reasonable economy the advantages of adjustable speed motor operation where only alternating current is available. Thus it opens to the electronic control the field of applications in which the problem is merely to obtain adjustment of motor speed, rather than to obtain some special and more or less automatic control.

The system provides not only a source of direct current for operation and control of the motor by the conventional field rheostat method, but also an adjustable armature voltage. The wide speed range thus afforded makes it applicable to drives requiring a wider range than the conventional three-to-one or four-to-one adjustable-speed d-c motor. I believe this will make the system quite useful in certain machine tool applications, where greatly reduced speeds are sometimes needed.

For machine-tool application, the system provides one distinct advantage over the use of a motor generator set and exciter. This is that the equipment can be built up into a small cabinet which can be mounted directly on or in the machine in the same manner that magnetic controllers are now mounted. This is quite important in view of the decided trend toward making machine tools completely self-contained so that they may be readily moved for new production setups. Against this advantage, of course, is the fact that user must consider the cost of occasional tube replacements.

The armature current-motor-speed curves in the authors' figure 11 show a regulation at reduced speed which may be excessive for some applications. I do not believe that the slope of the main part of the curves will be excessive for most applications but the large increase in speed at very light loads may be objectionable. Individual applications would have to be studied to determine the suitability of the system in this respect. As the authors point out, the regulation may be improved by various compounding methods but of course this will complicate the system to some extent and one of the big economic advantages of the system is its inherent simplicity.

One other disadvantage, inherent to the simple rectifier systems, is that regenerative braking cannot be obtained without a very substantial increase in the cost and complexity of the equipment. For some applications only rapid braking is required and in such cases dynamic braking can be obtained by addition of suitable braking resistor and magnetic control. However, other applications require rapid deceleration to a reduced operating speed and the method of obtaining this operation automatically is more complex.

Although it is true that the system as described may be used to start the motor from rest without additional magnetic control, I believe that there may be some objection to this arrangement, particularly on machine tools in the high production industries. It would be difficult to educate the operators to accelerate the motor at a proper rate by turning a control knob when they have already been educated to push a button and obtain acceleration at the proper rate automatically.

In general, though, this new system is a distinct forward step and I believe numerous applications for it will be found.

C. H. Willis: The authors appreciate the interest shown in their paper by those giving discussions. It is evident from this wide interest that electronic devices may find a wide field of application in motor control. It is to be hoped that this field will develop rapidly.

In some of the discussions there appears to be a divergence of opinion as to the rating of the control equipment, where a rectifier and d-c motor are used to control the speed of a wound-rotor induction motor. Since this equipment is required to carry a maximum current under one condition of operation and also must produce a maximum voltage under some other condition, the authors believe that the proper rating is the product of the maximum current and voltage. While the control equipment may not operate at this rated load it must be capable of so operating and this rating gives the most accurate and conservative estimate of the apparatus.

Discussions

of AIEE Technical Papers Published Before Discussions Were Available

ON THIS and the following three pages appear discussions submitted for publication, and approved by the technical committees, on previously published papers presented at the AIEE winter convention, New York, N. Y., January 24–28, 1938. Authors' closures, where they have been submitted, will be found at the end of the discussion on their respective papers.

The PCC Street Car

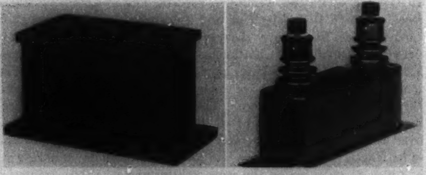
Discussion of a paper by C. F. Hirshfeld published on pages 61–6, of this volume (February section) and presented for oral discussion at the modern electric vehicles session of the winter convention, New York, N. Y., January 24, 1938.

E. J. Allen (General Electric Company, Pittsfield, Mass.): The author is to be commended for his description of the salient features of the modern Presidents' Conference Committee street cars, over 500 of which have been in operation during the past three years. Low operating cost and low maintenance expense to the transit company is obtained through applied research and careful design, yet securing modern appearance, speed consistent with good riding quality, good illumination, and ventilation.

The PCC street car, like its earlier predecessors, derives its motive power from overhead trolley lines, which are very frequently exposed to lightning, even in urban areas. Consistent with the objectives of low first cost, and low maintenance expense in general design of the PCC car, a d-c capacitor-type lightning arrester is installed in one of the compartments. To meet the demand for a d-c lightning arrester, which does not require maintenance and one that is unaffected by winter and summer temperatures, the d-c capacitor type arrester has been made possible only through successful research in dielectrics during the past few years.

Figure 1*a* shows a type of 750-volt four-microfarad d-c capacitor arrester which has been used on PCC cars. Another d-c capacitor-type arrester having the same electrical characteristics, but enclosed in a drawn steel housing, is shown in figure 1*b*.

This arrester is directly connected between positive trolley and car frame, and depends entirely upon the effect of a large amount of capacitance for its impulse protective characteristics. While the protective voltage maintained by the capacitor-type arrester depends on the wave shape and amplitude of the impulses applied on the trolley circuit, this protective level is only 3,000–4,000 volts crest for typical applied waves on trolleys supported by cross spans on steel or guyed wood poles. Moreover, the sloping of applied impulse wave front accomplished by the capacitor-type arrester is essential in order to reduce the turn-to-turn impulse stresses in the motors and car equipment.



(*a*) (*b*)

Figure 1. Typical d-c capacitor-type lightning arresters for 600-volt d-c service

The protective characteristics afforded by d-c capacitor arresters as shown by calculation and tests are rapidly being substantiated in service by the virtual lack of lightning failures reported on PCC car equipments.

W. I. Rodgers, Jr. (nonmember, Brooklyn and Queens Transit Corporation, Brooklyn, N. Y.): In Brooklyn, the Presidents' Conference Committee car was first introduced to the riding public. The 100 new cars replaced old equipment on four lines starting October 1, 1936. New schedules were written, headways reduced, and an improved service resulted. Rider endorsement followed quickly and the mails brought hundreds of appreciative comments.

To date, the cars have operated over five million miles. Comparing this with average mileages for older equipment we find that the new cars have traversed one-third more mileage over equal periods. As these cars are able to do more work per day, it follows that operation costs are lowered.

Much of the electrical and the mechanical equipment of the cars had been tested in Brooklyn on the widely known "million dollar" experimental car. Many more improvements were selected for the production cars. Fundamentals had been well established by the PCC research group. It remained for actual service operations on a wide scale to prove the equipment. The early experiences in Brooklyn with the first production cars from the factory quickly brought out the necessity for changes of slight importance in themselves but highly desirable for maintenance reasons. Both construction and engineering detail reflected the careful planning and long research to provide a modern vehicle that could be priced attractively to operating companies.

In the desire to build such a vehicle to these standards, no sacrifice of sturdy, dependable equipment and accessories was permitted. Thus, our operating difficulties have been minor and have not in any way interfered with smooth, successful performance during the year.

Our maintenance costs have not yet reached their level because of the changes and improvements applied after the delivery of the cars. It is confidently expected that costs will shortly match the system average and ultimately be less than the costs of maintaining the conventional street car of former days.

Reduced track costs are anticipated by the use of the resilient wheels and much elimination of the hammer-blow pounding of steel wheels on the rail seems assured. All lateral movement and swing of the car body is well controlled by the rubber coned springs at the four corners of the trucks and the body swing links. These design factors should go far toward reducing destructive forces which cause excessive track maintenance.

Coupling Between Parallel Earth-Return Circuits Under D-C Transient Conditions

Discussion and author's closure of a paper by K. E. Gould published on pages 1159–64 of volume 56, 1937, AIEE TRANSACTIONS (September 1937 issue of ELECTRICAL ENGINEERING) and presented for oral discussion at the communications session of the winter convention, New York, N. Y., January 24–28, 1938.

G. Wascheck (Bell Telephone Laboratories, Inc., New York, N. Y.): Experimental measurements made in connection with the d-c transient induction tests presented in Mr. Gould's paper afforded opportunities for studying certain aspects of the problem of determining local earth resistivities and structures—which problem is an important one in predicting the a-c coupling between earth-return circuits. If the earth were uniform, such resistivity would be independent of separation or frequency. If the earth were not uniform but varied as some function of the depth, the resistivity would vary with frequency because of the lesser penetration of the current to the lower depths and the increased concentration closer to the surface as the frequency is increased—a phenomenon closely allied to that of skin effect with which we are all familiar as applied to metallic conductors. This is also reflected in a change of the in-phase and quadrature components of the coupling from that of a uniform earth structure. Thus, a series of tests over a large frequency range would show variations in the resistivity indicated at the surface which would be a function of the depth, and from which interpretations of the structure may be made—provided, of course, that such variations are roughly in a horizontal plane and not too deeply submerged for the frequencies employed. The resistivity indicated at the lower frequencies and hence influenced by the resistivity of the lower depths is not necessarily the actual resistivity at the lower levels but rather a composite of the entire cross section of the area through which the current is distributed. Therefore, whereas the trend will qualitatively indicate resistivity variations, comparisons with theoretical calculations for simple structures are required in order to determine the nearest equivalent structure.

For example, from the Jennings, La., test results, a curve of resistivity based on the magnitude of the mutual impedance (differing slightly from the curves in figure 13 of the paper, which are based upon the reactive component of the mutual impedance) was plotted against frequency, and indicated variations as follows: a resistivity of about 15 meter-ohms at 20 cycles, increasing to over 30 meter-ohms at 100 cycles, and then decreasing eventually to about ten meter-ohms at 3,200 cycles. Calculations show that this trend may be explained on the basis of a horizontally stratified earth, and geological studies substantiate the existence of such a structure in this region, which is in the so-called delta country around the mouth of the Mississippi River. This area has been built up by the deposition of silt over a great number of years and is probably quite uniform in a horizontal plane. In addition, experimental results were available, of resistivity measurements made in a 2,000-foot oil well a few miles distant, which indicated a surface layer over a hundred feet thick of rather low resistivity, a second layer below this several hundred feet thick with a resistivity about three to four times as high, and below this a gradual change to a very low resistivity, which low value is no doubt due to the salt water which underlies this territory.

A simpler method for determining earth resistivity and structure, requiring less time and apparatus than the a-c tests and easily analyzed when the structure is not too complex, is that employing direct current. Such tests were also made at this site and comparisons with the above results indicated a close correlation with the general resistivity trend and structure.

The d-c procedure, which has been used for some time in detecting variations in the earth's structure, particularly in geophysical prospecting work, consists in applying direct current (periodically reversed through a commutator to reduce polarization and stray voltage effects) to a pair of ground electrodes, called the current electrodes, and measuring the potential between two intermediate electrodes, called potential electrodes. This determines the mutual resistance between the two circuits from which, with the known geometrical arrangement of electrodes, the apparent uniform earth resistivity may be determined directly by a simple formula. To determine possible variations of resistivity with depth, use is made of the fact that the direct current penetration into the earth becomes progressively greater as the spacing between the electrodes is increased. When these variations with spacing assume certain definite trends, calculation and tests on small models have shown that a fairly reliable interpretation of the structure may be made. The tests at Jennings, which were made with current electrode spacings from a few feet up to nearly 10,000 feet indicated a resistivity of little over ten meter-ohms at the closer spacings which increased to 35 meter-ohms at the intermediate spacings and then dropped off rapidly at the last measured point, indicating about six meter-ohms with the trend still downward. These results agree essentially with the oil well data and with the a-c results, where a continuation of the measurements below 20 cycles quite likely would have shown a progressive drop in resistivity to the d-c value for a very wide spacing of electrodes.

Thus, provided the coupling characteristics for the resultant structure are known, or an equivalent earth resistivity can be substituted for it at a given frequency, results from d-c exploration tests enable a close estimate to be made of the a-c coupling. Tests in many places have indicated that where the structure is not too complex, an equivalent uniform or a two-layer stratified structure may be postulated from d-c exploration data, and for these cases the variations of coupling with separation can readily be calculated. More complex structures may often be reduced to a simpler structure, though invariably in heterogeneous territory, irregularities in a horizontal direction are also present. These introduce difficulties in assigning an average resistivity, especially for a long circuit, when only exploratory or sampling measurements are made. This is true when measurements are made either by the a-c or d-c method and a larger portion of the territory must be explored. However, where sampling methods are deemed satisfactory, the d-c procedure provides a useful method for predicting a-c coupling between earth-return circuits.

A New Single-Channel Carrier Telephone System

Discussion and authors' closure of a paper by H. J. Fisher, M. L. Almquist, and R. H. Mills published on pages 25–33 of this volume (January section) and presented for oral discussion at the communications session the winter convention, New York, N. Y., January 24–28, 1938.

Glen Ireland (American Telephone and Telegraph Company, New York, N. Y.): The members of the Institute may be interested in some brief comments regarding the application of the type-*H* system in the Bell System plant.

About 35 type-*H* systems have been placed in commercial service and engineering has been completed on a great many more. Some of the systems are being applied for distances as short as 25 miles, others are being used for distances up to 160 miles. In the engineering of these systems, the small size of the *H* terminals is being found to be of considerable advantage particularly in offices where floor space is at a premium. In the case of one office it was possible to defer the enlargement or rearrangement of the toll terminal room and at the same time take care of the toll circuit growth by removing three low bays containing three type-*D* terminals and in their place installing two low bays containing 4 type-H terminals. The type-D terminals are to be reinstalled elsewhere where floor space is not a controlling factor.

Experience with the *H* systems already in service although limited, has been very satisfactory. The circuits are providing good quality, are very quiet and relatively stable. The systems are simple and easy to maintain, relatively few routine maintenance tests being required. In particular the use of varistors as the modulating and demodulating elements is helpful since it results in greater freedom from carrier leak and avoids the tests and adjustments of carrier leak required rather frequently in the case of previous types of systems employing vacuum tubes.

E. E. George (Tennessee Electric Power Company, Chattanooga): In October 1937 The Tennessee Electric Power Company leased from Southern Bell Telephone and Telegraph Company a type-*H* carrier system for use over the power company owned telephone line between Nashville and Chattanooga. This carrier system has provided one of the finest talking channels that we have ever used. The quality and volume are above the levels on the best toll circuits and the noise level is almost zero. This latter point is particularly remarkable in view of the fact that the power company telephone line over which this channel operates is "a hot telephone line" for most of its length. This telephone line consists of 165 miles of open wire of which 85 miles are on the same right-of-way as 110-kv transmission line. The line has several way stations with the usual drainage coils and insulating transformers. A portion of this circuit constitutes the side leg of a phantom with mid-tapped insulating transformers used for phantom coils. At one end there is a few hundred feet of entrance cable.

During the installation of the type-H carrier channel it was assumed that the two insulating transformers in the main line would have to be by-passed with high voltage condensers but it was found possible to operate without by-passes provided the ringing gain was increased sufficiently. By-passing insulating phantom transformers was found to make the phantom circuit useless due to the effect of the condensers in transmitting longitudinal noise frequencies. The circuit has been operating satisfactorily several months with one insulating transformer by-passed and one without by-pass.

The line has several way stations and the only changes made at these stations were to install small low pass filters ahead of the subsets on the station side of the connecting key or cord circuit. These filters are thus normally kept off of the line.

At present the power supply for the carrier terminals is derived from 115 volts a c although 130 volts d-c is available near by. For communication under emergency conditions such as dispatching, it would be highly desirable if the equipment could be modified to work off 130 volts d-c with no intermediate taps and no separate 24-volt battery. When the installation was first made some of the way station filters were omitted temporarily. Although the use of the voice channel created noise on the carrier channel and vice versa, the modulation was apparently inverted and the crosstalk was unintelligible. For this reason filters have been omitted at the patrol booth stations which are seldom used.

To power engineers familiar with the usual weaknesses of copper oxide rectifiers in the matter of heating, aging, overloading, etc., it seems remarkable that rectifiers have been found to be satisfactory as modulators and detectors in a high grade communication circuit. Elsewhere it has been stated that in some respects the copper oxide rectifier is superior to the vacuum tube for such purposes and it would be interesting to hear the authors of this paper discuss this factor in a little more detail. As noted the space requirements of the new "H" terminal are small, and its low cost makes it very attractive to power companies and others even on relatively short telephone lines. A type "H" carrier terminal has recently been installed by a neighboring company over a 20 mile telephone circuit on power line right-of-way where rectifier noise made the voice frequency circuit unusable. The channel has been so satisfactory that a second one is being installed.

J. E. Smith (nonmember; RCA Communications, Inc., New York, N. Y.): During his presentation Mr. Almquist stated that ten copper-oxide disks were series connected in each branch of the modulator bridge arm. Was the multiple-disk varistor used to permit high-signal-level inputs, and, if so, what was the maximum signal level?

M. L. Almquist: Mr. George suggested that some further discussion of the advantages of copper-oxide varistors over vacuum tubes as modulators and demodulators would be of interest. The principal advantages are better balance, greater stability, lower cost, smaller size, and reduced power consumption. The modulator circuit in the type-H system consists of a copper-oxide varistor and a single tube amplifier. To accomplish the same result with vacuum tubes would require the use of three tubes, two tubes operating as a single-balanced modulator and one tube operating as an amplifier. In order to obtain a good balance in the vacuum-tube modulator, which is necessary in order to reduce carrier leak and unwanted modulation products, it is frequently necessary to select pairs of tubes whose characteristics closely match each other. The balance will change as the tubes age and will also vary with changes in the power supply voltages. The copper-oxide varistor as manufactured has a balance which is better than can be obtained with vacuum tubes and this balance is maintained over long periods of time, thus avoiding the periodic rebalancing which is required when vacuum tubes are employed. Furthermore, with a double-balanced modulator such as is used in the type-H system, the input frequency does not appear in the output and this simplifies the filter requirements.

In reply to Mr. Smith's question, the primary purpose of using a multiple disk varistor is that it affords an economical way of obtaining units which are uniform in their performance, well balanced, and stable with time. The load-carrying capacity is determined largely by the amount of carrier which is applied; in order to obtain good performance, the carrier voltage across each disk must be larger than the speech voltage peaks at the same point. In this system, the modulator operates at a level which is four decibels below and the demodulator at a level 13 decibels below that at the toll switchboard. The power which can be carried by the modulator without noticeable distortion is of the order of five to ten milliwatts.

A System of Electric Remote-Control Accounting

Discussion and author's closure of a paper by L. F. Woodruff published on pages 78–87 of this volume (February section), and presented for oral discussion at the communications session of the winter convention, New York, N. Y., January 24–28, 1938.

E. R. C. Coe (The National Cash Register Company, Dayton, Ohio): I should like to ask Mr. Woodruff how a customer would identify himself in the event of a lost "charge token." It would seem that permission to set up the account number by hand at the sales counter somewhat destroys the safety of the system.

A. B. Smith (Associated Electric Laboratories, Inc., Chicago, Ill.): This paper deals with the general structure and operation of a system of accounting which is a combination of mechanical and electrical devices. In harmony with the scope of the paper, the author does not treat of these devices.

The business man of today has been fully acquainted with complicated mechanical devices, and he has learned to depend upon them to a very large extent. Beginning with the typewriter, he has been led on to the use of simple adding machines, and from them to the more capable machines like the Comptometer and the several calculating machines. As to their essential functions they are purely mechanical, and though he does not know what is in them, the business man has learned to rely on them. His confidence is justified.

In the accounting system described in this paper, there is the added element of many electrical circuits and devices. The business man may rightly ask, "To what extent are these electrical devices reliable, and what will their failure do to me?"

As most electrical engineers know, for many years the railroads have been using block systems for the safety and expedition of trains. They have made increasing use of interlocking plants to control switches, derails, and signals intersections. In all of these electrical appliances, the circuits are so designed that any failure will result in such a signal aspect as to give a more restrictive indication. Each failure is on the side of safety.

In the case of this electrical accounting, it is of interest to the prospective user to know if similar safeguards exist. He will also like to know the frequency with which electrical faults actually occur, or may be expected to occur. Experience with automatic switching of telephone lines has revealed the high degree of accuracy and dependability which relays and magnets can furnish.

L. F. Woodruff: Doctor Smith's remarks touch on a most pertinent aspect of the system, and one which has been of foremost importance in the development of the individual machines and circuits. This is primarily the elimination of possible error in transmission, and secondarily, the assurance of continuity of operation.

Twin contacts in parallel are used on all circuits. Contacts through the holes in the tags at the transmitter are made at two points. The coding of the numbers to be transmitted has been selected so that just two holes comprise the code for each digit. A contact failure, or an extra hole, will result not in an erroneous transmission, but the control circuits have been designed so that this event would cause a stoppage of transmission. The record would be incomplete, and the price tag would not be stamped. The transmitter would remain locked, and would have to be released by a special key. The record would have to be transmitted over again. If the transmitter were actually defective it would of course have to be replaced.

Mr. Coe has asked how a customer would identify himself in the event of a lost charge token. I agree with his point that setting up the number by hand at the sales counter would somewhat reduce the safety of the system, and we do not recommend that method. We advocate providing at a service station on each floor a place where the customer, upon proper identification, can be provided with a temporary token, good for that day only. A duplicate of this token would be filed for use in making correction in event an error was made in punching.

There are other possibilities for handling this problem, and the final choice must rest with the store management.

Design and Test of High-Speed High-Interrupting-Capacity Railway Circuit Breaker

By WILFRED F. SKEATS
MEMBER AIEE

THIS PAPER describes an oil circuit breaker built to the following specifications:

Rated voltage	15,000 volts
Rated current	1,500 amperes
Frequency	25 cycles
Interrupting capacity	65,000 amperes
Over-all duration of short circuit	1 cycle

Being intended for railway trolley service, which is rather severe from the standpoint of frequency of short-circuit operation, it is expected to handle 50 short-circuit operations without internal inspection or change of oil.

Like breakers of somewhat lower interrupting rating previously supplied for this type of duty,[1] this breaker operates upon the impulse principle. A spring-driven piston forces a blast of oil across the arc, bringing about its extinction with a very small contact separation, and thus resulting in an arc of very short duration. The arrangement differs from that previously used in two important respects:

1. The earlier breakers used the radial blast, in which the oil moves in radially upon the arc and exhausts through a hole in one of the contacts; the present breaker employs the cross blast, in which a passage causes the oil to sweep directly across the arc path.
2. In order to supplement the action of the piston at high currents, a short pressure-generating break has been added. A check valve is employed to prevent the additional pressure from this break from being expended in reversing the piston.

The manner in which these operations are performed is shown in figure 1, which is a sectional view of the lower part of the breaker. While the breaker is in the closed position, the moving contact *1* presses against the intermediate contact *2*, holding it, in turn, against the stationary contact *3*. The piston *4* is at the upper end of its stroke. Upon tripping, the moving contact moves rapidly upward and the intermediate contact follows to the limit of its stroke. At the same time, force is applied to the piston and at moderately low current values it starts downward, forcing oil ahead of it down the piston cylinder up through the check values *5*, through the contact chamber, and through the horizontal passage *6* in the baffle *7*. This passage leads the oil directly across the arc path at *8* and exhausts both oil and gases from the arc into the air space at *9*. The velocity of the oil through the cross-blast passage is augmented slightly by the action of the pressure-generating break.

At high currents, the performance differs in that the pressure generated by the pressure-generating break is greater than that caused by the piston. The pressure-generating break then supplies all of the oil for the blast and the check valves remain closed and so prevent oil from the contact chamber from flowing down through them instead of through the cross-blast passage.

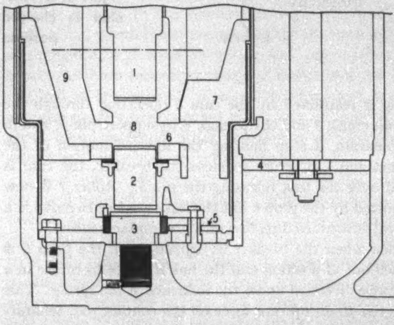


Figure 1. Sectional view of piston and interrupting element

Control of the length of the pressure-generating arc is accomplished, as shown in figure 2, by means of the metal ring *2* placed around the stationary contact *1* and so arranged that any current brought to it by the arc is led radially outward as indicated by the arrows. When the arc strikes the ring, this current path sets up a magnetic field which tends to move the arc back to the stationary contact. Thus the length of the arc can never greatly exceed the separation of the contacts.

Paper number 37-163, presented at the AIEE Middle Eastern District meeting, Akron, Ohio, October 13-15, 1937, and recommended for publication by the AIEE committee on protective devices. Manuscript submitted October 13, 1937; released for publication April 2, 1938.

WILFRED F. SKEATS is technical assistant to engineer, General Electric Company, Philadelphia, Pa.

[1] For numbered reference, see end of paper.

The mechanism is similar in principle to that used in the earlier breaker. It is shown schematically in figure 3. Energy for both the closing and opening operations, as well as contact pressure in the closed position, is supplied by the spring *1*. When the breaker is charged and open, this spring is restrained by the cam *2* operating through the pitman crank *3* and the pitman *4* to support pin *5*, and in combination, a stop limiting the upward motion of the contact rod. In order to close the breaker, the cam is rolled over the top, releasing the pin *5*. Roller *7* is now supported by the prop *6* but the links are free to move in a general downward direction until the contacts meet.

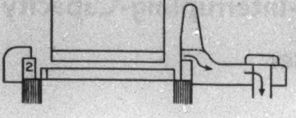


Figure 2. Arc-control arrangement at pressure-generating break

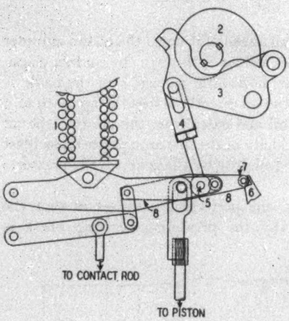


Figure 3. Mechanism in charged position

Then when the breaker is tripped open, the prop *6* is pushed out of position and the link *8* is free to rotate in a clockwise direction under the influence of the spring. This gives rise to an upward force on the contact rod, separating the contacts, and a downward force on the piston, creating the oil blast.

Immediately following the opening operation, a motor operating through a gear reduction mechanism turns the cam through about 300 degrees, which recharges the spring, placing the breaker in position to be reclosed in about seven seconds from the time of tripping.

The breaker and its control panel are assembled in the factory in a steel housing, with a door in front opening into the breaker compartment and a door in the rear opening upon the control panel. Figures 4 and 5 show the front and rear views with the doors open.

The testing of this breaker presented an interesting problem. In a one-cycle breaker, the severity of the duty is associated, to a large extent, with what may be called the waiting period, that is, the period after the breaker has set up conditions for clearing and while it is waiting for a current zero. It is obvious that the duration of this period is largely dependent on the frequency. Hence it is essential that the test frequency be the same as that at which the breaker is to be applied. As this was only 25 cycles while the normal testing-plant frequency is 60 cycles, the testing-plant generator had to be slowed down to about 40 per cent of its normal speed with a corresponding reduction in the output. Under these conditions the power available from the plant by the conventional testing circuit was only about 30 per cent of the breaker rating.

To overcome this situation, a special testing circuit similar to that used on the 287-kv breakers for the Boulder Dam–Los Angeles transmission line was employed. In addition to the breaker under test and the generator, this circuit comprises an auxiliary breaker with the same characteristics as the breaker under test, a transformer, a sphere gap, and, for the present tests, a synchronous closing switch, all arranged as shown in figure 6. The operation of this circuit is as follows. Initially the auxiliary breaker and the test breaker are in the closed position, and the synchronous closing switch is open. In order to initiate the short circuit, the synchronous closing switch is closed, and current flows through the auxiliary breaker, the test breaker, and the synchronous closing switch in series, and the full short-circuit current of the 3,000-volt generator connection is available. The test breaker and the auxiliary breaker both trip promptly and, in about three-quarters of a cycle, both set up conditions to clear the circuit on the next current zero.

With the breakers operating according to specification, the circuit is cleared in both at this current zero and in a few microseconds the voltage at the generator terminals

Figure 4. Breaker assembled in housing

has risen to approximately its normal peak value. This voltage is also applied at the low voltage terminals of the transformer, stepped up by the transformer in accordance with its ratio, and applied to the gap combination. This is sufficient to break down the gap and the transformer voltage is then applied to a point between the two breakers. Thus it appears directly across the test breaker. This entire process takes place, as shown by the cathode-ray oscillograph, within about 25 microseconds after the current reaches its zero value, or in about half the time required for the 10,000-cycle recovery transient to reach its first peak.

Thus, except for a period of about 25 microseconds during which no current normally flows and the voltage across the breaker is comparatively low, current interrupted by the breaker and recovery voltage are the same as in an operation at full voltage and full current.

For proper operation of the circuit, it is necessary that the gap combination should not break down appreciably before the time of current zero, and as the arc voltage when stepped up by the transformer ratio, had occasional voltage surges comparable with the recovery voltage in magnitude, a special gap arrangement was introduced to insure proper operation.

The design of this arrangement was facilitated by the fact that the breaker always interrupted after either one or two loops of current, according to the point on the voltage wave at which the short circuit struck. Since both the point on the voltage wave and the polarity of the first loop of short-circuit current were subject to control by the synchronous closing switch, it will be obvious that the polarity of the last loop was likewise controllable. The gap arrangement consisted of two sphere gaps in series shunted by equal resistances of about 120,000 ohms, but one of the two gaps was also shunted by a resistance of 1,000 ohms in series with a rectifier, the polarity of the rectifier being made such as to prevent current flow in response to arc voltage, but to permit flow in response to recovery voltage, which is of the opposite polarity.

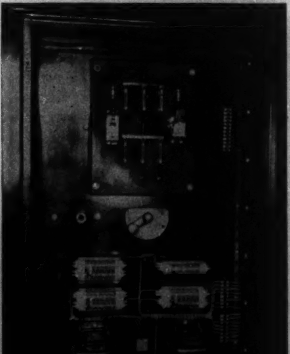


Figure 5. Rear view of housing showing control panel

Thus during the arcing period, the voltage is substantially equally divided between the two gaps by the higher resistances and the combination has a maximum breakdown strength. For the recovery voltage, however, one gap is shunted by a resistance of only 1,000 ohms, so that practically all of the applied voltage appears across the other gap. When this gap breaks down, the transformer supplies voltage to the breaker through a resistance of 1,000 ohms, which is low enough so that the drop across it is negligible, and thus, in effect, the gap combination breaks down on recovery voltage at about half the value required on arc voltage. As a matter of interest, the second gap usually breaks down either immediately or after about 50 microseconds when the voltage is falling in the second half of the first recovery oscillation.

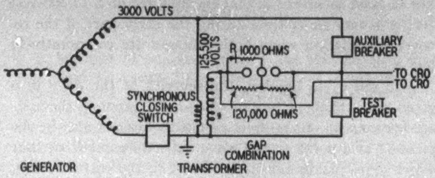


Figure 6. Special testing circuit

The magnetic oscillograms of figure 7 and the cathode-ray oscillogram, figure 8, indicate the performance of both breaker and circuit. Curves 3 and 6 show the short-circuit current, which has a very small minor loop followed by a large major loop. Within 0.05 cycle after the start of the major loop, the trip current responds, and at 0.6 cycle, motion of the contact rod starts (curve 2). Pressure (curve 7) is quickly generated by the primary break, and at the end of the first cycle the circuit is cleared. Curve 5 shows the voltage across the breaker, and it will be noted that the appearance of the curve immediately after interruption is no different from that corresponding to a conventional test at 12,500 volts.

The application of voltage to the breaker immediately after interruption is shown in more detail by the cathode-ray oscillogram. Two curves are shown on this record. The lower curve shows the voltage at the transformer side of the gap. Irregular oscillations occur in this curve during the arcing period; then, at the cessation of current, the voltage changes its polarity and swings into the recovery oscillation. A slight discontinuity is noticeable in this curve as it rises to the first peak of this recovery oscillation. This indicates the time of breakdown of one gap. Another discontinuity slightly after the peak indicates the breakdown of the second gap.

The upper curve shows the voltage across the breaker. During the arcing period, the deflection is barely perceptible. As the recovery voltage rises, however, the first gap breaks down and from that time on, except for a short time after the first peak when the breaker voltage is somewhat higher than the transformer voltage, the breaker voltage follows the same course as that of the voltage on the transformer side of the gaps. Thus the only period after current zero during which the voltage across the breaker is not at least as severe as that encountered on a 12,500-volt test is a very short time during the early part of the recovery oscillation when both voltages are comparatively low.

In a one-cycle breaker, the severity of the duty in a given test is likely to be a function of the point on the voltage wave at which the short circuit strikes and may be the greatest at a point on the wave quite different from that which gives maximum displacement to the current wave, or maximum root-mean-square value of the total current. This was the case with the present breaker, and may be explained roughly as follows.

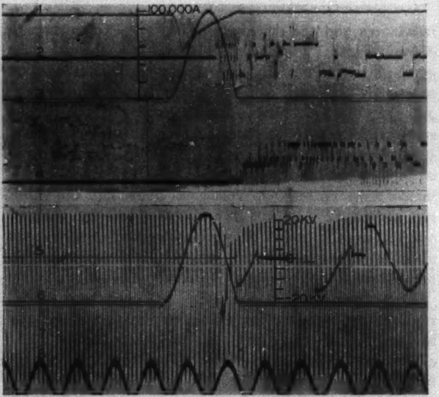


Figure 7. Magnetic oscillograms of an interruption at 12,500 volts, 60,000 amperes by special testing circuit

Curve 1—Trip-coil current
Curve 2—Contact travel: one-fifteenth inch per step
Curve 3—Current through breaker
Curve 4—Piston travel: one-fifteenth inch per step
Curve 5—Voltage across breaker
Curve 6—Current through breaker
Curve 7—Pressure in contact chamber
Curve 8—Timing wave—60 cycles

It may be expected that the severity of the duty will be a more or less direct function of the gas generation. This will depend approximately upon the ampere-seconds from the time of contact separation to the time of interruption, which will usually be the next current zero. Now when there is decay of the d-c component, as is always the case, and the contacts separate about 0.6 cycle after the inception of short circuit, the situation is as shown in figure 9. Here curve *a* shows a current wave initially fully displaced, and curve *b* a current wave about 50 per cent displaced, with the minor loop first. While curve *a* gives a higher root-mean-square value of current measured over the full duration, it comes to zero, on account of the decrement, somewhat before a full cycle has elapsed, and runs below curve *b* from 0.65 cycle on. Thus it gives a lower value of ampere-seconds in the arc than curve *b*. The greatest internal pressures were found to be generated in the breaker in cases similar to curve *b*, and, as will be seen in the tabulation of tests given below, care was taken that this particular condition was well represented in the test series.

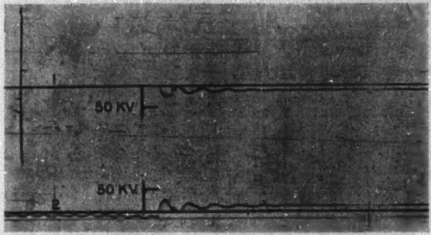


Figure 8. Cathode-ray oscillogram taken with figure 7

Curve 1—Voltage across test circuit breaker

Curve 2—Voltage at transformer side of gap

As railway service is likely to involve more frequent short-circuit operations than either central-station or industrial service, these breakers were built to withstand a large number of short-circuit interruptions and the breaker was subjected to the complete series of tests shown in table I without changing contacts or oil.

In all but one of these tests, the fault was cleared within one cycle after its inception. This exception was one of the light-current tests which started with a minor loop too small to trip the breaker. The breaker tripped on the second loop, however, and cleared the circuit on the third.

Figure 9. Comparison of fully displaced current wave with 50 per cent displaced wave having minor loop first

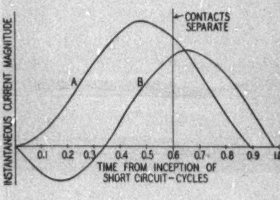


Figure 10 shows the condition of the contacts after test, and it will be noted that the burning is quite moderate for

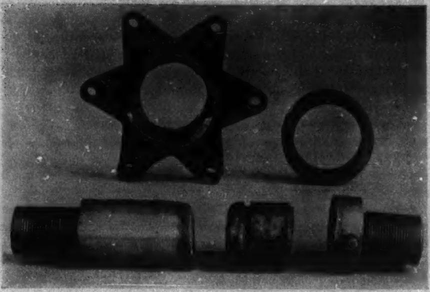


Figure 10. Contacts from impulse oil circuit breaker, 15,000 volts, 1,500 amperes, after test

the large number of severe short circuits interrupted.

Following the short-circuit tests, and without changing the oil or any breaker parts, a high-potential test was applied to the breaker resulting in a breakdown at 49 kv. The path at this breakdown, however, was through a hole which was drilled in the insulating lining to permit pressure measurement during the interrupting-capacity tests and when a sound lining was used together with the oil from the interrupting-capacity tests, the breaker withstood 54 kv for 55 seconds.

Table I. Tests

Number of Tests	Opening or Closing-Opening	Voltage	Current	Testing Circuit	Point on Voltage Wave
10	CO	12,500	3,400– 5,500	Conventional	Random
20	CO	3,000	38,000–70,000	Conventional	Random
10	O	3,000/12,500	36,000–68,000	Special	Equally spaced over 180 degrees
10	O	3,000/12,500	Approx. 40,000	Special	Selected for highest tank pressure
4	O	3,000/12,500	Approx. 67,000	Special	Selected for maximum displacement

After the first high-potential test, a heat run was made on the breaker, using the burned contacts and the dirty oil, with a maximum temperature rise of 28 degrees centigrade.

Conclusions

On the basis of the foregoing, the following conclusions may be drawn:

1. In an exceptionally thorough test program the breaker has shown itself adequate for the severe service for which it is intended, interrupting currents from 3,000 amperes beyond its rating of 65,000 amperes within one cycle, over 50 times without inspection of contacts or change of oil.

2. The testing scheme used, by a four-to-one increase of effective plant capacity, has more than overcome the handicap which the low frequency placed upon the testing plant.

Reference

1. The Impulse High Speed Circuit Breaker, W. K. Rankin. *G. E. Review*, volume 34, October 1931, pages 553–8.

Discussion

J. B. MacNeill (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The method of testing oil circuit breakers with a high current at low voltage followed immediately after the interruption by the superposition of high voltage across the open contacts of the switch is questionable, because the conditions in the arc as the current approaches zero are different when this scheme is employed from the conditions that would apply under a circuit giving the same amount of current at the high voltage initially. We are all hoping that ways will be found for improving circuit breakers for large interrupting capacities by the use of relatively small testing equipments, as the handling of switchgear business with a normal cost probably will demand something of this kind for the future. The greatest care must be used, however, to assure reliability of such "synthetic" methods.

A review of past test experience will throw some light on this subject. It is definitely known that test circuits vary in severity and that circuit interrupters which are wholly adequate on some circuits are entirely inadequate on others. The speed of restored voltage and the maximum value to which restored voltage goes are major factors. However, the condition in the arc as the current approaches zero is influenced by the voltage available across the circuit during the interruption and must be given consideration.

Let us look at data taken on the same circuit breaker on two different test laboratories with practically the same applied voltage and approximately the same currents interrupted. Table I shows such a comparison, with the arcing time on the more severe laboratory averaging 3.2 cycles and on the less severe laboratory averaging 2.1 cycles. Again, table II shows a similar comparison on a circuit

Table I. Low-Current Tests

100,000 Kva, 15-Kv Breaker

	Laboratory 1		Laboratory 2			
Test number	35702	35703	2-5807	2-5809	2-5809	2-5810
Voltage	12,000	12,000	13,200	13,200	13,200	13,200
Current (amperes)	830	830	900	900	1,200	1,200
	880	1,060	Three	Three	Three	Three
	870	1,300	Phase	Phase	Phase	Phase
Duration of arcing (cycles)	2.0	2.5	3.0	3.0	3.5	3.5
	2.0	2.0	2.5	3.5	3.5	3.0
	2.0	2.0	3.0	3.5	3.0	3.5
Average current		960		1,050		
Average arcing time		2.1		3.2		

Table II. Medium-Current Tests

	Laboratory 1		Laboratory 2			
Test number	35706	35707	2-5811	2-5812	2-5813	2-5814
Voltage	12,000	12,000	13,200	13,200	13,200	13,200
Current (amperes)	2,870	3,120	2,500	2,500	3,500	3,500
	2,910	2,410	Three	Three	Three	Three
	3,200	3,060	Phase	Phase	Phase	Phase
Duration of arcing (cycles)	2.0	2.0	3.3	3.5	3.0	3.0
	1.5	2.0	3.5	3.5	3.0	3.5
	1.5	2.0	3.5	3.5	3.0	3.5
Average current		2,960			3,000	
Average arcing time		1.8			3.3	

breaker tested in the same two laboratories with the same applied voltage and approximately the same currents interrupted. Here the average arcing time on the more severe laboratory is 3.3 cycles on a 60-cycle circuit and on the less severe laboratory is 1.8 cycles.

These differences in arcing times result in wide differences of arc energy as the last half cycles carry more arc energy and more destructive effects on the contacts and oil. If two laboratories designed for the same specific purpose of circuit breaker testing give such widely different results when operated at the same voltage and applying the same short-circuit current, then we are bound to have reservations on any wide extrapolation such as is caused by testing at a low voltage and superposing across the open contacts a voltage several times as great. If the device under test is good for the given current at the higher voltage, then of course the test method used is of secondary importance. If, on the other hand, there is doubt of the ability of the device to handle the current at the high voltage, very little added assurance is given of the ability of the device by the superposed voltage method.

A preferred method of demonstrating the ability of higher power interrupters is that of dividing the interrupting device into several duplicate parts, each of which can be separately demonstrated to the limit of capacity to which it will be subjected on the complete apparatus. This requires means for assuring proper division of the duty between the several units, but such means have been adequately developed. For the present, however, service requirements do not call for extended subdivision of interrupting devices so that this method also like the one discussed by Mr. Skeats is of limited application.

Any assumption made from tests on 3,000 volts alternating current looking toward verification of higher voltage devices must be carefully reviewed. Three thousand volts alternating current is easy to switch, especially at high current. Under oil a plain break one inch long is adequate. Even in open air certain modifications of standard carbon breakers will do the work. This however, gives no feeling of security regarding 12,000-volt operation, which is a more difficult problem.

W. F. Skeats: Mr. MacNeill presents a table showing considerable difference in the arcing times of the same breaker in tests in different laboratories, claiming this difference as evidence of unreliability of the testing method described in the paper. However, an examination of the differences in testing conditions which might be responsible for this difference in behavior reveals three:

1. A probable difference in the rate of rise of recovery voltage;
2. A difference in the magnitude of recovery voltage as evidenced by the higher test voltage in the second laboratory;
3. A somewhat higher system voltage during the period of current flow.

Of these, items 1 and 2 are admitted by Mr. MacNeill to be of major importance, and they may be well simulated in the testing method described. The third item must derive what effect it has from any distortion introduced into the current wave by the increase in the relative importance of the arc voltage when the system voltage is decreased. In the usual case, this distortion is not great. On the basis of the evidence presented, therefore, it appears that the important differences in Mr. MacNeill's test conditions are those which can be readily taken care of in the test method described in the paper.

Again, Mr. MacNeill points out that 3,000 volts alternating current is easier to switch than 12,000 volts and that any assumption based on tests at the lower voltage must be carefully reviewed before being accepted as valid with reference to performance of the tested device at the higher voltage. This is of course true, but here Mr. MacNeill does not even make the statement that the difference does not lie in the recovery characteristic, which is certainly where it is to be expected and which is simulated by the test method described.

As a matter of fact, a few tests made on an early arrangement of the breaker by the conventional method and also at the same voltage and current, by the scheme described indicate this scheme to be somewhat more severe than the conventional method. This is attributed to the fact that this scheme overemphasizes the increase caused by the arc voltage amplitude of the recovery oscillations.

Printing Telegraph Operation of Way Wires

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Synopsis: Although most commercial telegraph trunk circuits have been operated by printing telegraph methods for some years, Morse operation has been generally adhered to on way wires. Arrangements for operating way wires by printing telegraph have been developed, and are described. These include a polar simplex system, a neutral way-wire system, a code calling bell arrangement, and a multistation customers' printer circuit that provides secrecy by preventing the connection of more than one customer's printer to the line at a time. The paper also describes a new single-line repeater arranged to repeat polar as well as neutral signals, and a "dotter" for transmitting uniform test signals.

COMMERCIAL telegraph messages are transmitted between major cities over a network of trunk lines, usually operated by the multiplex printing telegraph method. These trunks terminate at main "relay offices" of the telegraph company, from which lines radiate to smaller cities and towns. Some of these radiating lines are direct trunks to cities having a considerable amount of traffic, and are equipped with start-stop printers, operated either single or duplex. In other cases, the traffic to a single place is not enough to justify exclusive assignment of a line conductor, and intermediate offices or "way stations" are connected into the circuit to increase the traffic load. Such circuits are called way wires.

The manual or Morse method of operation has been used on these way wires, because of its simplicity and low line-signal frequency and because of the cheapness and ease of adjustment of the apparatus. The way-station operator will usually be the manager of the office, and to provide continuous service a second operator should be available to cover lunch periods and to permit the manager to take care of other business.

The advent of telephone train dispatching and the increasing use of printers on commercial telegraph circuits and on private leased wires has reduced the supply of available Morse operators, so that it is becoming difficult to obtain Morse operators. It is usually better to hire a suitable manager, who can be trained easily and quickly to operate a printer. A messenger can learn in a short time to operate a printer well enough to act as relief operator. It is evident, therefore, that it is desirable in many cases to substitute printer operation of way wires for Morse.

Another use for printing telegraph operation of multistation circuits is for connecting private customers' offices to a telegraph office, where the customers are located at a considerable distance from the telegraph office, but are so placed that they can be connected in series on a single circuit. The total traffic received from all customers may justify the cost of such a line, where it would be uneconomical to provide a separate line to each customer. In such cases, the system must be so arranged that messages to or from any customer cannot be copied by other customers' printers on the same circuit.

Multistation Leased Lines

By "leased line" is meant the type of circuit used for so-called "private wire service," where a subscriber contracts for the exclusive use of a telegraph circuit, over which he transmits and receives his own messages. Multistation leased lines have some points of similarity with the commercial telegraph way wires discussed in this paper, but in other respects their requirements are quite different. A previous paper[1] describes this "private wire service" and points out the fact that the start-stop printer or teletypewriter is particularly suitable for this type of service because of its simple sending, receiving, and timing arrangements, and its automatic ability to take care of varying lags in signals transmitted from any sending station to any receiving station. For the same reasons, the start-stop printer is suitable for way wires.

In many cases the messages sent from any station on these private leased-wire circuits are intended to be recorded at all other stations, as, for example in the case of press or police circuits. In other cases, messages are intended for a particular station and it may be desirable to use a selective calling arrangement. The Gill telegraph selector[2] has been used in some cases for automatically starting and stopping the printer motor at the desired station, when the proper code signal combination is transmitted. This type of selector usually operates on all message signals that pass over the circuit, but it does not close its contacts unless a particular combination of signals is received. This continual operation of the selector mechanism causes wear and maintenance expense. This difficulty has been avoided in other selective calling systems[3] where telephone dials are used to operate step-by-step switches and relays. In these systems, the selective equipment is normally disabled while printing signals are being transmitted and is cut in by slow acting relays when the printers are shut down.

These multistation leased line circuits may extend for long distances, through several telegraph central offices where telegraph repeaters are located, with trained personnel for adjusting them. The individual stations are connected to these central offices over comparatively short loops, which present no severe transmission problems. The cost of the long line circuits and the large volume of traffic transmitted, justify the use of elaborate repeating

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1. For all numbered references, see list at end of paper.

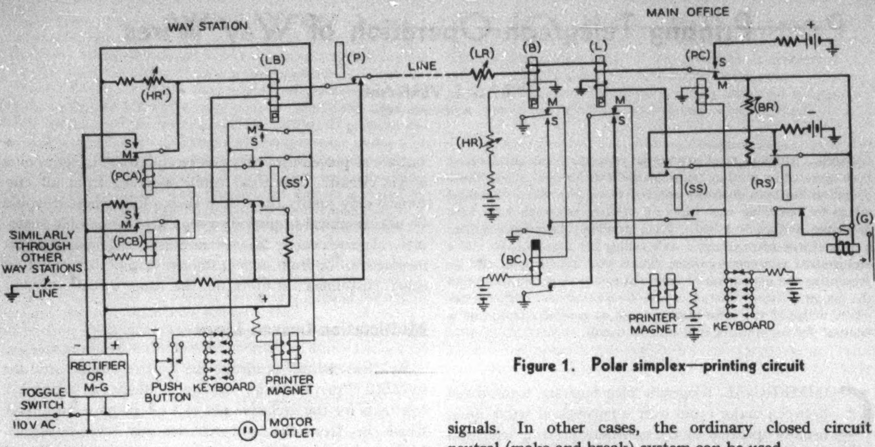


Figure 1. Polar simplex—printing circuit

and selecting equipment, if these are needed to provide good service.

Printer Way-Wire Requirements

The requirements for operating printers over commercial telegraph way wires are quite different from those for operating over private wire multistation lines. These way wires extend from a main telegraph office, through one or more intermediate "way stations," to a terminal office, which may also be considered as a way station. Traffic is exchanged between the main office and each way station, but messages are seldom, if ever, transmitted directly from one way station to another.

The way stations are usually located at a considerable distance from any main telegraph office and skilled attendants are not available for balancing duplex sets or for making any but the most simple adjustments. For these reasons, the way-station apparatus must be simple and reliable. To compare favorably with Morse operation, the cost of the equipment must be kept down, expensive selecting equipment which may require frequent maintenance attention should be avoided, and the power consumption should be reduced to a minimum.

The best wires are not ordinarily assigned to these way-wire circuits. They are usually open-wire ground-return lines and are frequently subject to severe leakage during wet weather and to induced interference from other telegraph circuits or nearby power lines. Start-stop printer signals, at the usual transmitting speed of about 60 words per minute, are approximately twice as fast as the usual Morse signals. For satisfactory printer operation, signal distortion, particularly bias, must be kept below definite limits. These factors impose severe requirements on the system of signal transmission used, and in some cases make it necessary to use polar (+ and −) signals. In other cases, the ordinary closed circuit neutral (make and break) system can be used.

Means should be provided for calling the main office from any way station and for calling any particular way station from the main office, so that the printers can be shut down when not in use. This avoids having messages intended for one way station printed at all other stations, reduces the wear on the printer mechanisms and saves power. A simple code calling arrangement, with single-stroke a-c bells, is being used instead of selectors, so that no d-c supply is required during idle intervals at the way stations, except in a few cases where it is needed at terminal way stations for line current or for operating repeaters.

Polar Simplex System

In this system,[4] polar (+ and −) signals sent from any one station are received by polar relays in series with the line at all other stations, and switching relays are provided for automatically connecting and disconnecting the source of transmitting current to reverse the direction of transmission, so that a station formerly receiving can send back to a station that was formerly transmitting. Any receiving station can "break" or interrupt the sending station, the operation of the printers being exactly the same as on a single closed-circuit line. While sending polar signals from either terminal station, the line is grounded at the other terminal station; and while sending polar signals from an intermediate station, the line is grounded at both terminal stations. Whenever sending stops, the circuit returns to the normal, idle condition, with line current supplied from the main office.

While the circuit is idle, all printer motors can be switched off, as well as the power rectifiers at all way stations. Any way station can call in, operating a signal lamp and buzzer at the main office, and the main office can call any way station by operating a single-stroke bell at each station.

Figure 1 is a simplified diagram of that part of the circuit that is concerned with sending and receiving printer signals. It shows a main office and one way station. The line may extend through several way stations, but as they are identical, only one is shown. Certain parts, such as call bells, signal relays, and the local operating circuits of the switching relays have been omitted from this figure to simplify the drawing.

Negative "marking" battery is normally connected to the line at the main office. When the keyboard is operated, pole changer *PC* transmits (+) spacing and (−) marking signals over the line. Polar line relay *L* responds to these signals and operates the main office printer. Polar relay *LB* at the way station also responds and operates the way station printer. Switching relays *SS* and *RS*, at the main office, operate and remain operated until transmission stops. They have no effect however except to open the biasing winding circuit of relay *L*. Break relay *B* is held by a current through its lower winding equal to about one and one-half times the line current. If the line circuit has considerable capacity, line current surges may operate this relay momentarily at the beginning of each spacing signal, but this will have no effect, as relay *BC* is of the slow-release type and the short circuits momentarily applied by relay *B* to relay *PC* occur at a time when its operating circuit is open at the keyboard.

Signals sent from the way station keyboard operate the printer magnet as well as pole changers *PCA* and *PCB*. Switching relay *SS′* operates during the first spacing signal and remains operated until the way station operator stops sending. It switches the marking contact of *PCA* from ground to the (+) terminal of the rectifier. The rectifier is connected in series with the line and is reversed by the pole changers to transmit polar signals over the line toward the main office and also over the line toward the terminal way station. It should be noted that the direction of current in the line, for marking and spacing signals, respectively, is the same regardless of whether the main office or a way station is sending.

Relay *LB* does not respond to signals from the pole changers at its own station, as marking signals transmitted through both windings in series hold its armature against the marking contact, and spacing signals are sent to line through only one winding while marking current continues through the other winding and is adjusted by rheostat *HR′* to be about one and one-half times the line current. As in the case of relay *B*, relay *LB* may be operated momentarily by spacing current surges due to line capacity. This will open the circuits of the printer magnet and pole changers at a time when they are already open at the keyboard contacts.

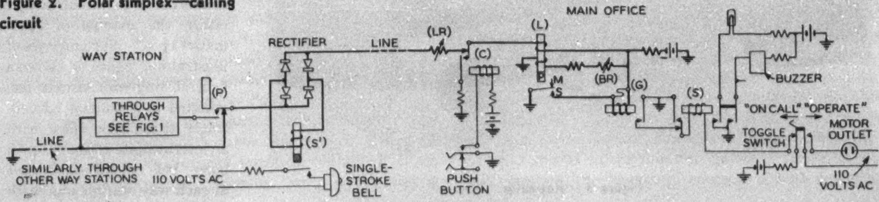
As (−) battery is normally connected to the line at the main office, the first spacing signal from the way station reduces the line current approximately to zero, at first, instead of reversing it. Relay *L* is now operated, however, by biasing current through its lower winding, regulated by rheostat *BR* to be only slightly less than the normal line current. Relay *L* immediately grounds the line through the winding of relay *G*, allowing the line current to reverse. Relay *G* operates switching relay *RS* (over circuits not shown). Relay *RS*, which remains operated as long as relay *L* responds to signals, disconnects the (−) battery from the line and grounds the line through both windings of relay *L*. Relay *L* operates the main office printer magnet causing the printer to record the message being transmitted from the way station.

When the way-station operator stops sending, relay *RS* releases and applies battery to the line at the main office and relay *SS′* releases and disconnects the rectifier from the marking contact of *PCA* at the way station, so that the circuit is automatically restored to its normal idle condition with line current fed only from the main office. Relay *RS* is timed so that it always releases before relay *SS′*, so that marking signal current is always applied to the line at the main office before it is removed at the way station. If the way station operator sends in a slow or hesitant manner, relay *RS* may release occasionally during the message. The momentary double strength marking current will have no bad effect, however, as relay *RS* applies the bias current to relay *L* at the same time.

Relay *P* is operated by direct current from the rectifier. It serves to disconnect the way station printer set from the line when the power is switched off. The arrangement is such that any way station set can be switched on and off without momentarily opening the line or otherwise interfering with other stations that may be using the line.

Any station can interrupt transmission from another station by operating any key on the keyboard several times in succession or by opening the keyboard circuit with a push button "break" key such as that shown at the way station. When a spacing signal from the "breaking" station occurs at the same time as a spacing signal from the transmitting station, the double strength current in the line operates the armatures of the *B* and *LB* relays to their spacing contacts, overcoming the holding currents through rheostats *HR* and *HR′*. Relay *LB* releases pole changers *PCA* and *PCB* as well as the printer magnet. As long as the double-strength current continues, the

Figure 2. Polar simplex—calling circuit



contacts of relay *LB* will remain open and the pole changers and printer magnet will remain released, thus giving a definite indication of the break signal at the way station.

At the main office, relay *L* opens the printer magnet circuit to indicate the break at that station, relay *B* releases relay *BC* and both relays connect a ground through contacts of relay *SS* to short-circuit pole-changer *PC*. The double strength break current continues, therefore, as long as relay *SS* remains operated. When the main office operator stops sending on the keyboard, relay *SS* releases and allows pole changer *PC* to operate and send marking current to the line. This allows relays *B* and *LB* to operate and restore the circuit to normal.

Figure 2 shows the calling-signal arrangement. This apparatus is included in the line circuit of figure 1, but is shown in a separate diagram to simplify the drawings. Slow-release relay *S′* at each way station is connected to the line through a small copper-oxide rectifier so that the line current always flows in the same direction through the relay. This relay remains operated on all printing signals but releases when the line current is stopped by depressing the push button at the main office. The main station can operate the call bells at all way stations, regardless whether the power is turned on the way station sets or not.

To call the main office, the operator at any way station switches on the local power and depresses the push button or any key on the keyboard. Relays *L*, *G*, and *S* respond to operate the buzzer and light the signal lamp at the main office. These signals are released when the main office operator turns on the printer motor to answer the call.

The only operating adjustments required with this system, are the settings of the three rheostats at the main office and the one rheostat at each way station. If properly adjusted, when the circuit is first installed, they usually require no further adjustment, except that, in cases where considerable interference or very heavy leakage is experienced, it may be necessary to readjust bias rheostat *BR*, at the main office, occasionally. The adjustments of break holding rheostats *HR* and *HR′* are not critical, and even if incorrectly adjusted they are not likely to affect the operation of the printers but they may interfere with the proper action of the "break" feature.

The main-office circuit can be arranged so that the circuit can be extended through a concentrator to one of a group of main-office printers or over a duplexed line or carrier circuit to a distant city. The concentrator permits grouping the polar simplex way wire with other lightly loaded circuits and connecting these circuits to the main office printers, only when needed for sending or receiving messages. This reduces the number of main-office printers required and improves the operating efficiency at that office. The extension to a distant city can be used for special purposes such as the transmission of news directly from the way station without rehandling at the main office.

Neutral Way-Wire Printer System

Where line conditions are not too severe, the simpler "neutral way wire" system can be used. This system uses neutral printing signals and reversed current or polar signals for operating the call bells. If the line resistance is not too great, the line may be connected directly to ground at the terminal way station, the line current being supplied from the main office. Positive battery is normally connected to the line at the main office, and for longer lines, a reduced negative voltage can be connected to the line at the terminal way station. As this is of the same polarity as the bell signal potential at the main office, the bell signal current will be reduced to a comparatively low value, normally about 15 milliamperes, but this is sufficient to operate the sensitive polar relay that rings the call bell.

For still longer lines, a single-line repeater can be installed at an intermediate station. A special repeater for this purpose, is shown in figure 3. Ordinary single-line repeaters can repeat neutral or "make and break" signals only. If equipped with fast relays, they can be used to repeat start-stop printer signals. The repeater shown in figure 3, however, is arranged to repeat the polar bell signals as well as the neutral printing signals.

At the main office, the line may be connected permanently to a printer set, if there is enough total traffic to justify it, or it may be connected to a concentrator of either the manual or automatic[5] type. For purposes of illustration, figure 4 shows a neutral way-wire circuit terminated in a manual concentrator at the main office, with an intermediate repeater, with polar bell signal equipment at each way station and with

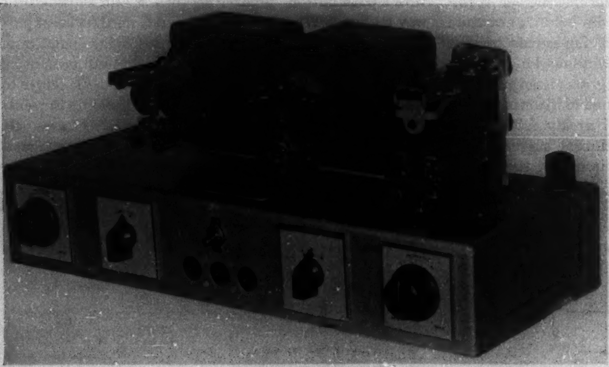


Figure 3. Repeater

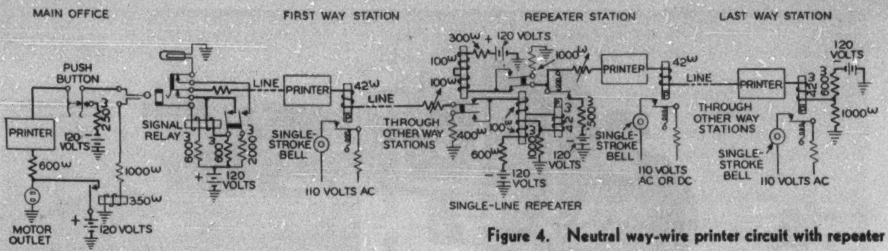


Figure 4. Neutral way-wire printer circuit with repeater

a potentiometer for supplying a reduced line potential at the terminal way station.

High-Speed Neutral Relay

The repeater uses a new fast neutral or nonpolar relay, two of which are shown mounted on the repeater in figure 3. This relay has a laminated core, an aluminum armature lever pivoted on a stainless-steel knife edge, operating two pairs of contact springs. The contacts are of molybdenum, mounted on beryllium copper springs, and are so arranged that they can be adjusted for either break-before-make or make-before-break operation. The latter adjustment is applied, when the relays are used in this repeater. The relay contact springs, and the adjustable spring, are so designed that the increase in spring tension, as the relay operates, closely balances the increase in magnetic pull, as the armature approaches the poles of the magnet core; with the result that the operate and release adjustments of the relay can be brought close together, without adjusting the airgap, as is necessary in the usual type of neutral telegraph relay. Only one simple spring adjustment is required, therefore, to adapt the relay to meet a wide range of operating conditions, including operation over lines subject to considerable leakage. This design and method of adjustment also maintains heavy contact pressures at all times and increases the reliability of the relay.

Single-Line Repeater

Referring to figure 4, with positive battery connected to the line at the main office, the contacts of the polar relays at the repeater and at the way stations are held open by the line current and when the line is opened and closed in transmitting printer signals they are held in the same position by their armature biasing spring. Each of the neutral relays of the repeater responds to printing signals received over one line section and repeats them into the other line section. Each relay also controls a locking circuit for the other relay to prevent it from releasing during the open or spacing intervals in the outgoing signals. As previously explained, the relay contacts are flexible, and are so adjusted that, when the relay releases spacing contact *S* closes before marking contact *M* opens, and when the relay operates, contact *M* closes before contact *S* opens. Therefore, when either relay is repeating signals, the other relay will remain continuously operated, as its circuit always remains closed, either through the *M* contact and line circuit or through the *S* contact and locking resistance. As each relay closes the local locking circuit of the other relay only during spacing signal intervals, the repeater is in condition, during marking signal intervals to transmit a "break" signal from the receiving to the sending station, thus permitting any station to interrupt transmission, if necessary.

During idle periods, any way-station operator can call by depressing his "break" key, momentarily opening the line, releasing the signal relay, and lighting the signal lamp at the main office. The main office operator responds by plugging a printer set cord into the jack and answering on the printer keyboard.

The main-office operator can call any way station by plugging in and depressing the push button the proper number of times. This transmits a reversed current of reduced strength (about 15 milliamperes) over the line operating the call bells at any way stations between the main office and the repeater. Opposing currents now flow through the two windings of the right hand repeating relay, releasing it, and the reversed current operates the polar relay, thus removing the positive potential and connecting a negative potential to the outgoing line and ringing the bells at way stations beyond the repeater.

Referring to figure 3, the single-line repeater consists of a pair of high-speed neutral relays mounted on a base or chassis, together with certain auxiliary equipment. The chassis is an aluminum-finished steel box with a detachable base plate. It contains the resistors, spark killers, rheostats, switches, and jacks and mounts a receptacle for a power-supply plug, a terminal strip for external wiring connections and sub-bases into which the relays are plugged. The polar bell signal relay is mounted on a bracket at the right side of the repeater and there is room for mounting a milliammeter, if required, at the left side. When installed at a way station, this repeater can be mounted on any convenient table or shelf, or if installed at a main office it can be mounted on a standard repeater rack.

A jack is provided in each line for inserting a milliammeter to measure the line current, which can be regulated by a rheostat. The center jack is in a local test circuit used for checking the adjustment of the relays. When the left-hand switch on the repeater chassis is turned to the "cut" position, the "marking" contacts of both repeating

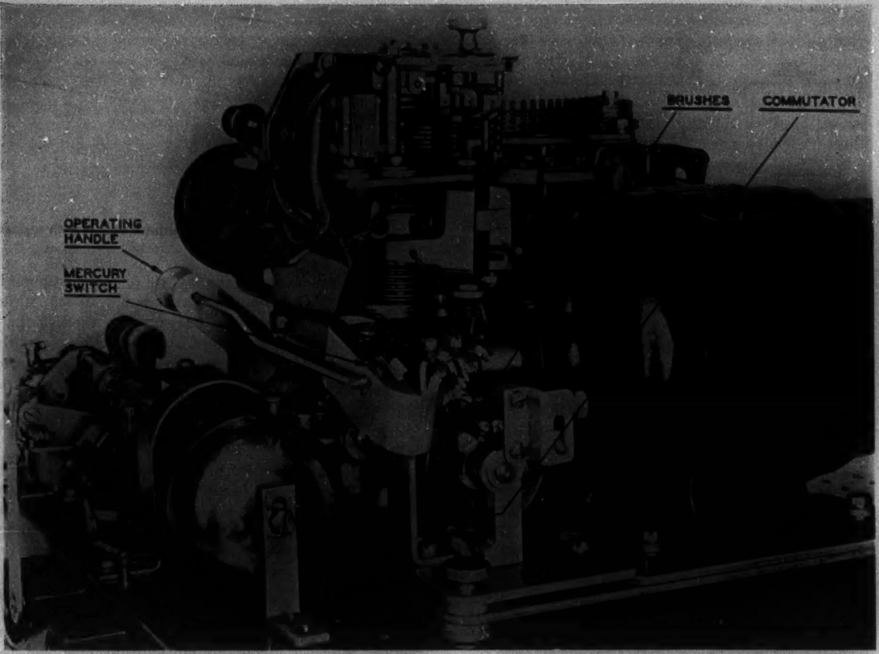


Figure 5. Rear view of printer with dotter attachment

relays are connected in series with this local test circuit. To line up the repeater, the current is measured in this local test circuit and then uniform dot signals are transmitted from an interrupter or "dotter" at the main office. The armature spring of the repeating relay is then adjusted until the milliammeter indicates half the closed-circuit current in the local test circuit. Dotter signals are then sent from the terminal way station to adjust the other repeating relay in the same way. The adjustment should be checked for both wet and dry weather conditions. It has been found that, in most cases, a mean adjustment can be obtained that will operate satisfactorily for long periods without change.

Figure 5 shows a "dotter" that will transmit uniform signals at the same frequency as the printer signals. To reduce its cost, it has been designed as an attachment to a standard tape printer. It consists of a small commutator driven by the printer motor. A link extending to the front of the printer lifts the brushes off the commutator and tilts a mercury switch to short-circuit the dotter when it is not in use.

Figure 6 is an oscillogram illustrating the action of the repeater in repeating printing telegraph signals. The upper curve shows signals as received by the repeater over an artificial open wire line equivalent to 260 miles of number 9 (American Wire Gauge) copper with leakage of one megohm per mile. The signals were transmitted from a "dotter" at a speed of 50 dots per second (100 bauds), which is approximately twice the usual printer signal speed. The lower curve shows the outgoing signals as repeated into a noninductive resistance circuit. It should be noted that, although the windings of one relay are included in the outgoing line, the inductance of the windings does not prevent the outgoing signals from having an approximately square wave shape. This is because the winding always has current through it, as previously explained, and there is no large change in the current through the relay while the other relay is acting to repeat signals. In this particular case, the relay-locking current was slightly greater than the normal line current, and the inductance of the relay winding has an effect opposite to that usually expected. It causes the line current to rise rapidly at the start of each closed circuit signal to a value approximately equal to the locking current. Even if the inductance of another relay or printer magnet is included in the outgoing line, the outgoing signals will more closely approach a square wave shape than if they were transmitted directly into the outgoing line without going through the repeater.

These oscillograms are included solely for the purpose of illustrating the action of the repeater and do not represent the extreme conditions as regards signal speed, etc., under which the repeater can operate.

Two-Wire System With Secrecy Feature

In the previously described systems, the printer can be turned on at any way station to copy messages being transmitted by other stations. This prevents their use for circuits to private customers where privacy is required. To meet the need for a multistation line for connecting customers' printers to a main telegraph office, the circuit shown in figure 7 was developed, and has been used for the last year and a half. As previously mentioned, this arrangement is suitable for cases where the customers' stations are so located that they can be connected in series along a line. It can also be used, to advantage, where customers' stations are located in a group close together but at a considerable distance from the telegraph office.

The number of stations that can be connected on one circuit is limited only by the amount of traffic to be handled. Too many stations should not be connected to one circuit, so as not to delay appreciably any customer's messages. The circuit is normally arranged for a maximum of ten way stations, but could be readily modified to accommodate more.

This circuit requires two wires, which extend from the main office, in series through the way stations to ground. One of these wires, the "control" wire, is used for transmitting calls from the way stations to the main office and for controlling, from the main office, the starting and cutting in of the printer at any particular way station. The second wire, the "printing" wire, is used for transmitting printer signals and for stopping the printer motors and cutting out the way-station printers.

A special form of selector is used at the way stations, consisting of two contacts operated by cams on the shaft of a small self-starting synchronous motor. The motor is somewhat like an ordinary electric clock motor and has reducing gears such that the cam shaft runs at a speed of four rpm. One of the cams is permanently fixed to the shaft, and its contacts serve the purpose of insuring that the motor cam shaft will make a complete revolution after it is started. The second cam is adjustable, and is set at a different position at each way station. Its contact serves to operate the motor-control relay, if the proper signal is received from the main office. A somewhat similar device is used as a sending selector at the main office. This has, in place of the adjustable cam, a distributor or commutator, with one segment for each station to be selected. A manually operated ten-point switch is provided to connect the transmitting circuit to the particular segment corresponding to the desired station.

To call a way station, the main-office operator first sets the ten-point switch to the terminal corresponding to the desired station and then operates the key to the "call" position. This applies positive potential through the main office printer to the printing wire, releasing the "stop" relays and lighting the busy signal lamps at all stations. It also closes a circuit to start the sending selector motor. Immediately after it starts, a cam contact closes, to insure that the shaft will make a complete revolution. When the brush crosses segment *S*, the sending relay is operated momentarily, thus sending a positive impulse over the control wire and momentarily operating the "start" relay and the "selecting" relay at each way station. The start relays start the receiving selectors, and the cam contacts insure that their shafts make a complete revolution. After the start impulse, the sending relay sends negative current to the control wire until the sending selector brush crosses the segment selected by the ten-point switch, when a brief positive impulse is transmitted, momentarily operating the start and selecting relays again.

At the desired station, the contact spring drops into the notch of the adjustable cam while this positive impulse is being received. At this station the motor-control relay operates and locks itself, starting the printer motor and removing a short circuit from the printer. At all other way stations, the adjustable cam is set at a different point, and, therefore, the motor control relays do not operate. At the main office, the control relay operates in series with the sending relay, at this time. It locks itself through the key contacts, starts the printer motor, and opens the starting circuit of the selector motor. After transmitting the positive selecting impulse, the sending relay continues to send negative current over the control wire, as the selector brush continues its revolution to the stopping point.

The milliammeter indicates a slight reduction in current at the instant when the way station printer is cut in. This occurs at the same time as the starting of the main office printer motor, and indicates to the operator that the way station selector has responded properly to the call. Messages can now be transmitted.

It should be noted that the starting of the customer's printer motor is controlled entirely from the main office and that no customer can start his own printer motor or remove the short circuit from his printer magnet. Therefore, only one customer's

Figure 6. Oscillogram of repeater signals at 50 cycles per second

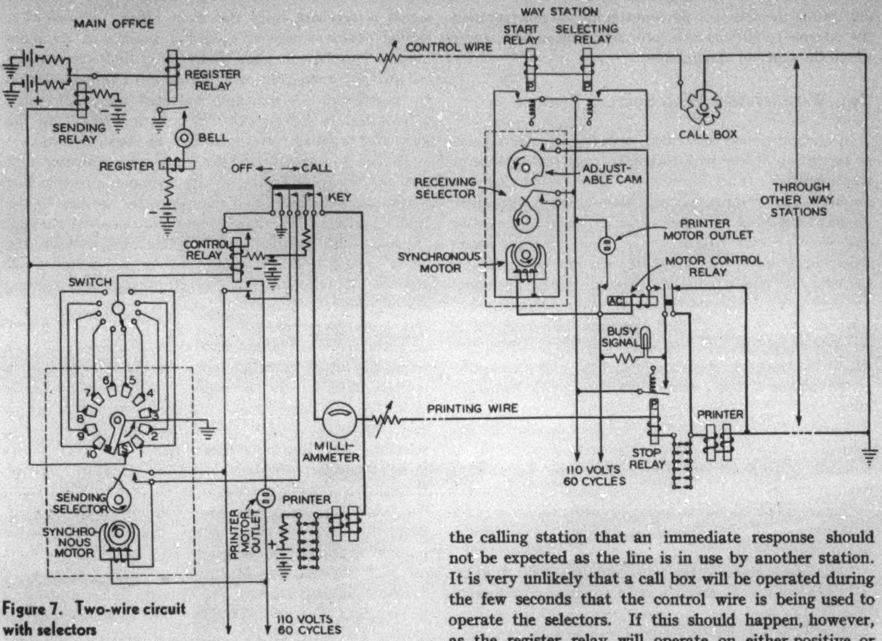

MAIN OFFICE
REGISTER RELAY
SENDING RELAY
BELL
REGISTER
OFF
CALL
KEY
CONTROL RELAY
SWITCH
SENDING SELECTOR
SYNCHRONOUS MOTOR
PRINTER MOTOR OUTLET
PRINTER
MILLIAMMETER
110 VOLTS 60 CYCLES
CONTROL WIRE
PRINTING WIRE
WAY STATION
START RELAY
SELECTING RELAY
CALL BOX
ADJUSTABLE CAM
RECEIVING SELECTOR
SYNCHRONOUS MOTOR
PRINTER MOTOR OUTLET
MOTOR CONTROL RELAY
THROUGH OTHER WAY STATIONS
AC
BUSY SIGNAL
PRINTER
STOP RELAY
110 VOLTS 60 CYCLES
1
2
3
4
5
6
7
8
9
10
S

Figure 7. Two-wire circuit with selectors

printer can be connected, in operating condition, to the line at a time and it is impossible for one customer to copy another customer's messages. The control equipment is installed in a locked box to prevent tampering with it.

After the messages have been sent and acknowledged, the operator restores the key, thus releasing the control relay, stopping the main office printer motor, and applying negative potential to the printing wire. This operates the "stop" relays at all way stations, extinguishing the busy lamps and releasing the motor control relay to cut out the previously selected station. The key must be restored to the "off" position, and reoperated, in order to start the sending selector, if another way station is to be called immediately. This prevents the operator from calling a second way station by mistake, without first cutting out the station previously called.

Any way station can call the main office by operating the call box. This is like an ordinary messenger call box and transmits a series of open circuit impulses, different for each station. These impulses operate a single stroke bell and a tape register at the main office, and the signal indicated by the bell and recorded on the tape identifies the particular way station that called. In response to such calls, the main office operator selects and calls this station in the same manner as for sending outgoing messages.

It should be noted that calls can be sent in and registered while the line is busy. The busy signal indicates to the calling station that an immediate response should not be expected as the line is in use by another station. It is very unlikely that a call box will be operated during the few seconds that the control wire is being used to operate the selectors. If this should happen, however, as the register relay will operate on either positive or negative current, and as the call-box signals are much faster than those sent out by the sending selector, such calls will be correctly received and usually will not interfere with the proper operation of the selectors. Even if they should prevent the proper starting of the motor at the called station, the milliammeter will indicate this to the main office operator, who can repeat the call by restoring and reoperating the calling key. In any case, incoming calls cannot cause the selection of a wrong station.

Conclusions

The conversion of way-wire circuits from Morse to printer operation has been retarded by lack of suitable, simple, and reliable equipment for operating such circuits, for calling individual way stations and for preventing one way station from recording messages intended for another. It is expected that the apparatus and systems described in this paper will meet this demand and permit more rapid expansion of printer way wires.

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A New Type Vacuum Seal

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Synopsis: A new type of lead-in structure for vacuum-, gas-, or oil-tight chambers is described in which porcelain bushings are sealed to metal by means of a glass which serves as a bonding agency. The method of assembly is extremely simple, requiring no skilled workmen or elaborate equipment. Some of the features and some of the places where the seal is being applied are discussed.

Introduction

THE PROBLEM of bringing insulated electrical conductors into metal containers which must be hermetically sealed, oil-tight, or vacuum-tight, has for many years been a formidable one. This is attested by the large number of methods now in use, the voluminous literature on the subject and the many patents which have been issued. The most obvious type of lead-in construction is of the gasket type, where a porcelain or glass insulator is used with gaskets between the metal and the porcelain. The whole assembly is held together by some compression arrangement, and the seal depends for its effectiveness upon a tight fit between the metal and gasket, and the ceramic and the gasket. Seals of this type have always been extremely difficult to make perfectly tight. Usually the gasket material, or the structure, is such that it is impossible to heat them to even a few hundred degrees without ruining them. Consequently, it has long been recognized that the most desirable type of lead-in structure is one in which the porcelain, glass, or other ceramic which is used for the insulator, is actually sealed to the metal, either directly or by means of an intermediate sealing material. Seals of this type may be conveniently divided into four classes as follows:

I. Glass or porcelain sealed to metal by means of resins or gums.
II. Glass or porcelain metalized and attached to main metal member by means of low-melting-point metal alloys.
III. Glass sealed directly to metal.
IV. Porcelain sealed to metal by means of a glass.

Seals in class I may be made using ceramics and metals of quite different expansion characteristics due to the plasticity of the resins or gums. They are limited to low temperature applications.

Seals in the second class are usually made between a metal and porcelain which do not match in expansion. The bond is made at the melting point of the alloy and upon cooling to room temperature the structure is quite highly stressed due to the difference in expansion of the components. It is essential that the seals are so designed that all high stresses in the porcelain are compression, as the ceramic materials are very strong for that type of stress.

Seals of this type are mechanically strong and are used extensively, but they are limited to moderate temperatures due to the low melting point of the sealing alloy.

There are many different seals which fall into class III. One of the early seals of this type was made by Housekeeper.[1] At the time this type of seal was developed, available metals, which could be easily worked and glasses were of widely different coefficients of expansion. Under most conditions, if they were fused together at the high temperature necessary to work the glass, the stresses which resulted upon cooling due to the differences in expansion caused the glass to crack. Housekeeper circumvented this difficulty by using a metal which had a low yield point, (copper) and thinning it to a very small section where it sealed into the glass. In this way he caused the metal to flow and prevented the development of further stress in the glass before the stress in the glass reached a high enough value to cause it to crack. One of the main disadvantages of this type of seal, however, is the weakness caused by the thinning of the metal member.

Many advances have been made in the science of sealing metals to glasses, due primarily to the fact that various metals and alloys have been developed which have coefficients of expansion matching those of certain glasses.

One of the greatest and most recent advances in the field is due to H. Scott.[2] From his study of the low expansion anomaly of the iron-nickel-cobalt alloys,[3,4] he found that one particular alloy, which he called Kovar (Patent No. 1,942,260. The alloy Kovar is also known as Fernico. Hull and Burger in a paper[5] on glass-to-metal seals discuss seals to this alloy and many other metals), could be made to match quite closely one of the hard glasses (Corning *G*-705-*AJ*; Corning code designation for this glass is number 705) from room temperature to above the annealing range of the glass, and can, therefore, be sealed into glass in large sizes. This alloy can be rolled, drawn, and subjected to all the usual forming operations of steel. It is not attacked by mercury and consequently is one of the most important factors in the development of the "all metal" type mercury-vapor thyratrons.

For many types of structures, rather large and intricate glass pieces must be sealed to metal, for which considerable skill is required by the glass worker. It would often be far easier if a porcelain could be substituted for the main body of the glass, and then only a small amount of glass used for bonding the metal to the porcelain. The porcelain will maintain its shape to much higher temperatures

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1. For all numbered references, see list at end of paper.

than the glass and consequently only a minimum amount of skill would be required to make the joint. Seals of this type fall into the fourth class.

The desirability of such a seal has long been recognized and many attempts to obtain a satisfactory one have been made. The successful ones have all been of the "Housekeeper" type.[6] Low-expansion porcelains and glasses having approximately the same coefficient of expansion and which consequently make a stress-free seal to each other have been used. The seal has always been made to a metal of higher coefficient of expansion. In order to prevent cracking of the seal it has been necessary to thin the metal member to a very small section so that the stresses developed would be below the breaking strength of the glass and porcelain. Seals of this type are, therefore, weak where joined.

The new type of seal to be described also falls into the fourth class. The materials have been so selected and the structure so designed that it is no longer necessary to make the "Housekeeper" type seal.

General Principles

The prerequisites for making an ideal seal between metal, glass, and porcelain are:

1. Both the metal and porcelain must be wet by the glass.
2. The metal, glass, and porcelain must all have the same coefficients of expansion up to the temperature where the glass is sufficiently viscous to relieve stresses quickly.
3. All component parts must be vacuum-tight.

A satisfactory seal may be made, even though condition 2 is not exactly fulfilled. It may, therefore, be rewritten as follows: The metal, glass, and porcelain should have coefficients of expansion sufficiently alike that upon cooling after the seal is made the stresses developed due to the differences in thermal contraction are not great enough to cause a fracture of the glass or porcelain, or leave the completed seal mechanically weak.

Wetting

The bonding of glass to metal or what is known as the wetting of the metal by a glass, takes place at elevated temperatures and, in general, only when the metal is oxidized. At the elevated temperature where the glass is viscous and plastic, it dissolves some of the oxide of the metal, and thus adheres tightly to it. In order that the joint between the metal and the glass should be vacuum-tight, the oxide which forms on the metal should be a tightly adherent one. A scaly oxide would naturally give a very weak seal. Care should also be taken as to the thickness of the oxide. If the oxide is too thin, it may all be dissolved by the glass, changing the composition of the glass near the junction, and the resulting glass may not, in fact probably will not, have the same or as great a strength of bonding between itself and the metal as the original oxide of the metal exhibited. A thick oxide on the metal is likely to cause trouble also. In general, a metal and its oxide do not have the same coefficient of expansion, consequently the oxide may develop small fissures or cracks very similar to the cracks in an enameled metal surface when the enamel and metal do not match. In addition, the oxide may be of a spongy nature permitting a slow leakage of air through it.

The wetting of the porcelain by the glass takes place in much the same manner as the wetting of the metal oxide. The porcelain is already an oxide material, and consequently needs no further oxidation. When the glass and porcelain are heated to a sufficient temperature they tend to diffuse into each other and form a most intimate, tenacious bond which cannot be broken free.

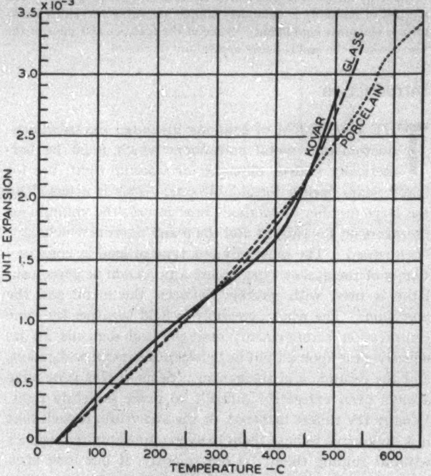


Figure 1. Expansion characteristics of component parts of metal-glass-porcelain seal

Thermal Expansion

The apparatus used for determining the expansion characteristics of the materials was similar to that used by Scott in his study of iron, nickel, cobalt alloys.[3,4] In order to insure a uniform temperature distribution, a special heating furnace was built which had a large copper core. Thermocouples attached along the specimen showed the maximum variation in temperature to be less than one degree.

Expansion curves were obtained for many different metals, glasses, and porcelains, but for the purpose of this paper, only one particular combination will be discussed. The materials are, the iron-cobalt-nickel-alloy, known as Kovar, the glass *G-705-BA* (Corning code designation for this glass is number 704), and standard electrical porcelain. As will be shown, these materials all have approximately the same expansion characteristic over a considerable range, and can be satisfactorily sealed together. Although there are other combinations of metals, glasses, and porcelains which also meet the requirements of a satisfactory seal, these particular ones have been chosen because of their availability. All components are standard materials and are readily obtainable. Figure 1 gives the total thermal expansion per unit length of the materials as a function of the temperature. They are shown together for comparison, and as may be seen they have very nearly the same average coefficient of expansion up to about 500 degrees centigrade.

Relief of Stresses

In order to show that a stress-free seal can be made between Kovar and electrical porcelain, it is necessary to show that *G*-705-*BA* glass has a low enough viscosity at a temperature around 500 degrees centigrade to permit the Kovar and porcelain members to move with respect to each other at a rate sufficient to compensate in a reasonable time for the difference in expansion between the Kovar and porcelain at temperatures above 500 degrees centigrade. If, then, all stress is relieved at around this temperature, no appreciable further stress will be developed upon cooling to room temperature, as all components contract practically the same amount.

There are two particular temperatures defined in glass technology which are of importance in connection with relief of stresses in the glass. These are known as the *anneal point*, and the *strain point*. The anneal point is that temperature at which 90 per cent of the stress in the glass will be relieved in 15 minutes. The strain point is that temperature at which the same percentage will be relieved in four hours. The range between these temperatures is usually spoken of as the annealing range. The Corning Glass Works, in published data on their commercial glasses, gives these temperatures. For *G*-705-*BA* glass they are 484 degrees centigrade and 450 degrees centigrade for the anneal point and strain point, respectively.

It would seem that, if a metal-glass-porcelain seal were made, and, before cooling, held at around 484 degrees for 15 minutes or longer, most of the stress would be relieved. That this is not true will be shown.

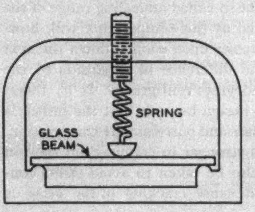


Figure 2. Glass beam loaded by means of a device having considerable elastic deformation

There are a number of methods which have been used for determining the annealing constants of glass, but only one method[7] which clearly shows the implications of these data will be described here. A small rectangular plate of the glass to be studied is supported at opposite ends, and stressed by tightening down a screw in the center between the two supports. The loading mechanism is made very rigid so that no appreciable elastic deformation will take place, and so that the glass will be permanently constrained to a fixed position. This is all set up inside a uniform temperature furnace and so arranged that light can be directed through the specimen, and relief of stresses determined directly for any particular temperature by a photoelastic method. Thus the two temperatures are found which satisfy the definition of the anneal point and strain point.

Further study has shown that these temperatures correspond to points where the glass has definite values of viscosity,[8] and that the rate of relief of stresses may be found if the viscosity is known. It was then experimentally determined that the common logarithms of the viscosity measured in poises at the anneal and strain points are 13.4 and 14.6, respectively. Since the determination of viscosity from a study of the mobility of glass fibers is a much simpler method of finding the anneal and strain points, it has almost completely supplanted the older, more direct methods. However, we must still remember that, no matter how these temperatures for rates of annealing are determined, they apply directly only to the case where the glass is constrained to some definite shape, and the stress is relieved while the glass remains in that position.

Consider a glass beam which instead of being loaded by a rigid screw is stressed through a stiff elastic spring similar to that shown in figure 2. If this is held at an elevated temperature, say the anneal point, the stress in the glass will tend to be relieved. However, as the stress in the glass is relieved, it will be still farther strained due to the elasticity of the spring. Consequently the stress in the glass cannot be completely relieved until it has deformed so far that it has permitted the elastic deformation of the spring to be completely relieved. It is at once apparent, therefore, that the annealing of a glass structure constrained by an elastically deformable means, will be greatly dependent upon the rigidity of this mechanism.

In the construction of metal-glass seals, the elastic deformation of the metal may be a very important consideration in determining the rate of relief of stress in the glass. For a simple bead seal, where a layer of glass is melted around a quite rigid metal rod, the deformation of the rod is negligible under the stresses produced. It is, therefore, substantially correct to assume that, at the annealing temperature, 90 per cent of the stresses due to any differences in expansion between the metal and the glass will be relieved in 15 minutes. It is incorrect, however, to assume that an annealing schedule which is correct for a bead on a heavy rod will be satisfactory for all other shapes of seals. Thus, for example, a bead of the same size made on a thin wall tube having the same outside diameter as the solid rod, will anneal quite differently. The initial stresses will be lower due to the fact that the metal deforms elastically, but the rate of relief of stresses will also be slower. It can be shown that after a time t the stress in the glass on the metal tube, may actually be higher than in the glass on the solid rod, even though the initial stresses on the rod seal were higher. This is due to the fact that the rate of relief of stress is lower for the seal on the metal tube. A graphic illustration of stress versus time is shown in figure 3.

In order to have the same rate of relief of stress for the tube seal as for the rod seal, it is necessary that the viscosity of the glass should be lower. In consequence the temperature of annealing would have to be higher. It may be seen, therefore, that it is desirable to have the metal and glass match up to temperatures above the so-called annealing point of the glass.

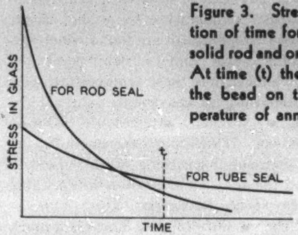


Figure 3. Stress in glass as a function of time for a bead seal on a solid rod and on a thin-walled tube. At time (t) the stress is higher in the bead on the tube. The temperature of anneal is the same for both seals

Since Kovar and *G*-705-*BA* glass match quite closely even at temperatures well above the anneal point, little difficulty is encountered in obtaining a satisfactory anneal in a short time for most any type of structure.

For a seal between Kovar, glass, and porcelain, conditions are somewhat different. Let us consider the case of a moderately thick-walled tube of Kovar sealed to a porcelain tube with a thickness *a* of glass between them, as shown in figure 4. The seal is made at a high temperature (usually between 900 degrees centigrade and 1,100 degrees centigrade, depending upon the design), and consequently the Kovar and glass have expanded considerably more than the porcelain. After the seal is made and cooling is started, the metal tends to contract faster than the porcelain, and consequently stresses are set up in the seal. In order that these stresses may be relieved at the lower temperature, the glass must permit the metal to move with respect to the porcelain by an amount equal to the difference in contraction between the metal and porcelain in cooling from the temperature at which the seal is made to the lower temperature. As soon as this relative motion of the parts is required during the annealing process, we see that the anneal point of the glass no longer defines the temperature at which 90 per cent of the stress will be relieved in 15 minutes. Either a higher temperature is necessary, or a longer time will be required.

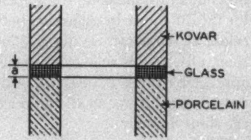
Now it is known that the rate of flow with respect to each other of two surfaces in shear and separated by a viscous medium is proportional to the distance of separation (*a* for our case) and inversely proportional to the viscosity. Consequently the greater the thickness of the glass, the faster the relief of stresses for a given temperature.

If a seal is made using a thin layer of glass, and is cooled quite slowly, the stresses developed are so high that they usually crack the porcelain.

Figure 5 shows another way in which Kovar and porcelain might be sealed together by means of glass.

After the seal is completed and cooling started, the metal tends to shrink down on the porcelain. Now in order that the resulting stresses may be relieved, the glass must be squeezed out somewhat so that the metal can move toward the porcelain. This is analogous to a long thin strip of viscous material in compression between two plates. Doctor Nadai has shown (this is from an unpublished Westinghouse research report entitled "The Distribution of the Pressure in Thin Layers of Viscous Material Under Compression," by A. Nadai) that for this case, the velocity with which the plates move together is approximately proportional to $\frac{1}{\eta}\left(\frac{a}{l}\right)^3$ where η is the coefficient of viscosity. It is, therefore, important that the ratio l/a should not be large. For the case of the long thin strip in compression, the pressure distribution is not constant along the dimension. It has been shown that it takes on a somewhat parabolic shape, dropping very low at the edges. This is advantageous, as it tends to cut down the danger of stress concentration at the edge of the seal which might break the porcelain.

Figure 4. Kovar and porcelain sealed together by means of a thickness of glass (a)



Calculations, of the rate of relief of stress for several simple structures, have been made, but exact solutions even in the simple cases, were found quite impossible due to the complexity of the mathematics involved. The approximate solutions indicated, however, that the range of temperature, in which the greater part of the stresses developed upon cooling from a higher temperature can be relieved in a moderate length of time, is around 100 degrees centigrade higher than the so-called annealing range of the glass. Any seal annealed at this temperature will, however, develop further stresses upon cooling down the next 100 degrees, due to the difference in expansion of the components above 500 degrees centigrade. It is, therefore, necessary that the design be such that the principle residual stresses in the glass and porcelain are compression, as both are many times stronger in compression than in tension. Care should also be taken to avoid stress concentrations which might cause cracking of the glass or porcelain.

Another factor which undoubtedly enters into the relief of the stresses in seals annealed at high temperature is the relaxation in the Kovar. Scott has shown[3] that at temperatures as low as 400 degrees centigrade, Kovar loses 50 per cent of the maximum stress applied by bending, in 15 hours. At the higher temperatures, this takes place at a much greater rate, although no specific data are available.

Any types of seal which might be made between metal, glass, and porcelain are modifications or combinations of the structures in figures 4 and 5. Consequently, from the foregoing analysis of annealing of seals, it may be seen that *it is quite impossible to obtain complete relief of stress in a metal-glass-porcelain seal in any reasonable length of time. It has also been shown that the rate of relief of stresses in metal-glass seals, and in metal-glass-porcelain seals is a function of the design constants of the structure, as well as the viscosity of the glass and relaxation characteristics of the metal.* Rough analysis and experimental verification show that by so adjusting the seal design that stress relief will continue to as low a temperature as practically feasible, a satisfactory strong seal can be made.

Vacuum Tightness of Components

Finally, in order to have a good seal it is necessary that all components of the structure, and the joints between them be vacuum-tight. Nothing further need be said about Kovar and glass, for their extensive use in the metal radio tubes, metal thyratrons, etc., speaks for them. Tests made on many types of porcelain show that all those that are well vitrified are vacuum-tight, and that a vacuum-tight seal between them and a glass of the proper expansion may easily be made. The tests were made by closing off one end of a tubular porcelain structure and sealing an ionization gauge onto the other end, after which the whole structure was baked and treated under vacuum and then tipped off. Gauge readings were then taken from time to time to see if there was any leakage. No pressure change has been detected in seals made over a year and a half ago.

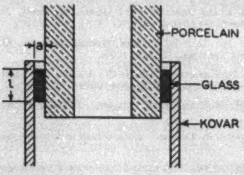


Figure 5. Kovar and porcelain sealed together by means of a thickness of glass (a) and length (l)

Tests made in a somewhat similar manner on completed metal-glass-porcelain seals show them to be vacuum-tight.

Assembly of Seals

Metal-glass-porcelain seals may be assembled in a number of ways. One of the most obvious ways is the utilization of the usual glass blowing technique. The porcelain member is brought up to temperature slowly and when it is sufficiently hot a glass bead is worked upon it at the point where the seal is to be made. The Kovar portion is then heated to a dull red heat for a few seconds to give it a slight oxidation and sealed to the glass.

For commercial production, such a method is too slow and expensive. A method is desirable by which larger quantities can be made at one time and without the assistance of skilled workmen. Such a method will now be described, as applied to the assembly of terminals for oil-filled capacitors.

Figure 6 is a photograph of the components of one of the terminals. The Kovar cap and flange, parts *A* and *E*, respectively, are drawn from sheet material, and the glass rings *B* and *D* are cut from a standard-gauge tubing. The parts are stacked together on a vertical spindle of heat-resisting steel and are ready for the furnace.

The sealing cycle depends upon the size of the porcelain pieces, and whether a batch or a conveyor type furnace is to be used. The final seal is made in the temperature range 950–1,100 degrees centigrade. If the porcelains are quite small it is sometimes possible to put them directly into the furnace at this temperature, although in general this will not be the case. Porcelain is a very poor conductor of heat, and the resulting high thermal gradient and consequent high stress is usually sufficient to crack the porcelain. Rapid heating is especially bad for the porcelain in the temperature range from about 500 to 575 degrees, where the expansion shows a marked increase. Consequently it is usually found advisable in batch assembly of small terminals to preheat, holding the furnace temperature slightly above this dangerous zone (about 600 degrees centigrade). As the porcelains come up to temperature their rate of heating becomes less and less and they are carried through the dangerous zone without cracking.

Above this temperature the expansion coefficient is lower, and the porcelain will stand higher thermal gradients. The assembly may, therefore, be transferred directly from the preheat furnace to the sealing furnace, which is at a temperature of around 1,100 degrees centigrade. Except for very small terminals which can be heated rapidly it is usually desirable to have the sealing operation take place in an oxygen-poor atmosphere in order to prevent overoxidation of the Kovar. An atmosphere of tank nitrogen is satisfactory.

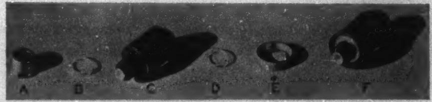


Figure 6. Component parts of a metal-glass-porcelain terminal seal and a completed terminal

A—Kovar cap B and D—Glass rings C—Porcelain
E—Kovar flange F—Completed terminal

After the seal is made the terminal should again be brought to an intermediate temperature furnace at around 600 degrees centigrade. It is during the interval of cooling to this temperature, and while at this temperature, that the major portion of the annealing takes place.

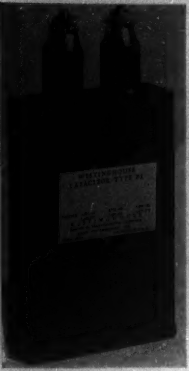


Figure 7. Inerteen-filled capacitor using two metal-glass-porcelain terminals

It also may be desirable to maintain a hydrogen atmosphere in this furnace in order to reduce the surface oxides on the exposed Kovar surfaces.

The terminals are next moved to a cooler zone in the furnace and after they reach a temperature around 300 degrees centigrade where no appreciable oxidation of the Kovar will take place, may be removed from the furnace.

It has been found possible to use the same furnace for both the preheating and for the cooling and annealing. Figure 6*F* is a completed terminal which was assembled using the following cycle: seven minutes, preheating; 15 minutes, sealing; and 20 minutes, cooling and annealing. The correct cycle to be used is largely a matter of size and shape of the porcelain and may best be determined by experiment. If the quantity is sufficient to warrant it, a seal of this type is admirably suited to production in a continuous-type furnace.

Applications

When work was started on the development of metal-glass-porcelain seals, it was intended primarily for application to vacuum-type equipment. It soon became apparent, however, that the demand for oil-tight and gas- and vapor-tight terminals was greater, as the needs of those using vacuum containers were quite well met by metal-glass seals. It is in consequence of the relative demand, that the first designs of the metal-glass-porcelain seal were for use on oil-filled capacitors.

Figure 7 shows an Inerteen-filled capacitor using two metal-glass-porcelain terminals of the type just described. The terminals are soldered into the can and the leads from the capacitor brought out through the opening in the caps. These are soldered to the caps at the same time the threaded studs are soldered in. Terminals for capacitors are standard in three different sizes, and others are being used experimentally. The same terminals are being considered for other applications such as small oil-filled transformers, and oil-filled portable X-ray equipment. They have a distinct advantage over other types, in that they permit bringing the leads out of the container below the oil level.

The possible uses of a seal of this type are so many and varied that it would be impossible for me to enumerate them. In fact, it may be used practically any place where an electrical lead into a hermetically-sealed container is desired. The individual reader will undoubtedly be able to select from his own experience many places where it might be applied.

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A Stabilized Amplifier for Measurement Purposes

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ASSOCIATE AIEE

Synopsis: This paper describes a stabilized amplifier with an over-all amplification ratio that remains essentially constant in magnitude and phase for all changes in tube characteristics, line-voltage variations, and output load variations to be expected during normal operation. This stability is obtained by the introduction of two stabilizing circuits which act independently, yet are complementary when used together. During a period of eight weeks' use two such amplifiers with their associated indicating instruments maintained their original calibration within two-tenths per cent of full scale without readjustment.

Introduction

DUE TO ADVANCES in electron tubes and amplifier technique electronic devices have found a rapidly expanding field of application. They have been used quite extensively in measurements and with the aid of amplifiers[1] it is possible to measure smaller quantities than ever before. Such quantities as voltage, current, time, sound, light, and distance can be measured and, moreover, the measurement can be made without appreciably disturbing the circuit characteristics by the introduction of the measuring device.

However, past experience has indicated that when an ordinary amplifier is used to extend the sensitivity of an indicating instrument, such as a voltmeter or ammeter, the accuracy, reliability, and ease of operation of the amplifier do not begin to approach those of the indicating instrument. Although the electron-tube amplifier has remarkably linear properties, variations in tube characteristics between different tubes, with the age of tubes, and with changes in plate voltage and cathode temperature ordinarily cause the over-all amplifier characteristics to change from time to time. Accuracies in the order of one-half per cent are available in present-day instruments and if the over-all amplifier characteristics are not to extend this limit it is necessary either to control the variables during operation or else provide means for a special calibration to check each reading. Either method is inconvenient from the operator's viewpoint and in many cases the resulting uncertainty in measurement does not permit the use of such an amplifier. Either special instruments are built or special ways of using instruments are devised.

In such applications where accuracy and reliability are important, together with sensitivity, it is desirable to have an amplifier whose characteristics are essentially independent of the aforementioned variables.

Specifications

One of the major problems encountered during the recent design of a 480-cycle network analyzer[2] was the measurement and instrument system. A network analyzer consists essentially of a large number of variable circuit parameters conveniently arranged for interconnection such that various circuit problems which are too complicated or involved for direct mathematical calculation may be set up in miniature and the actual results measured by trial (volts, amperes, watts, phase angle, etc.). Such analyzers are used primarily in making load and stability studies of power systems, but the circuits are flexible enough so they can be used for a great variety of problems.

For such an analyzer it is desirable to have instruments that are: easy to read without undue eye strain up to distances of several feet, rapid in response, able to withstand overloads and abuse, easy to use, and accurate to one-half per cent with direct-reading scales. Now these requirements alone would not preclude the direct use of indicating instruments, but it is also necessary to reduce the disturbance caused by inserting the instruments into the network to a point where it can practically always be neglected even when the network is lightly loaded. In an effort to reduce the size, weight, and cost of this analyzer, the power consumption of the total network is made small. Thus, the allowable instrument power consumption is reduced to such a small value that it is hopeless to attempt to use directly any kind of an indicating instrument even if some of the other desirable characteristics are sacrificed. For instance, it is desirable to have an ammeter voltage drop that is not in excess of 0.010 volt with various full-scale currents of 0.010–0.050–0.250–1.000 ampere. The voltmeter should not require over 0.000150 ampere (150 microamperes) when measuring full-scale voltages of 15–30–75–150 volts. Having given these specifications for sensitivity, the primary considerations are accuracy and reliability.

In the network-analyzer application current is measured by inserting a low-resistance shunt into the circuit and then amplifying the voltage drop across this shunt. The amplifier input impedance in parallel with this resistance is made high enough so it may be neglected. For instance, the current amplifier input impedance is not less than 5,000 ohms (maximum resistance of shunts is one ohm) and with the full-scale voltage drop of 0.010 volt the amplifier input is some 0.000002 ampere (two microamperes). This input corresponds to an amplifier output of 0.100 ampere so the current amplification is about 50,000.

Voltage is measured by using a tapped high-resistance voltage divider, each tap having sufficient resistance to

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[1] For all numbered references, see list at end of paper.

limit the current to 0.000150 ampere at the desired full-scale voltage. The voltage drop across a 3,000-ohm portion of the divider is amplified for an indication. The amplifier input impedance is not less than one megohm, which at full scale requires about 0.00000045 ampere (0.45 microampere). This input corresponds to an amplifier output of 0.050 ampere, the over-all current amplification being about 100,000. However, while the actual amplifier input is in the order of 0.1 microvolt-ampere; the relative disturbance caused in any circuit by the introduction of these amplifiers for making measurements must be measured by the over-all volt-ampere requirements of the complete unit and not the amplifiers alone. In this case, the volt-ampere requirement of the resistance shunt is predominant. A current of 0.010 ampere is measured with 0.010-volt drop and a voltage of about 0.5 volt can be measured with a current of 0.00015 ampere; about 0.0001 volt-ampere (100 microvolt-amperes) being required for full scale in each case.

To get a better conception of the meaning of these figures consider table I and compare the volt-ampere requirements of those high-quality a-c instruments in common use with that required in the network-analyzer instruments. The instrument voltage and current ratings given were chosen as typical values that might be expected of an average quality portable instrument in general use. The ammeter voltage drops were derived from the instrument impedance and current ratings and the voltmeter current requirements were derived from the impedance and voltage ratings. The volt-ampere requirements give an approximate idea of the relative disturbance caused when that type of an instrument is introduced into a circuit. To further facilitate this comparison a column is included in which the volt-ampere requirements are expressed in relative values considering the new ammeter value as unity. For convenience in comparison the wattmeter potential- and current-element volt-amperes are added together.

Table I. Volt-Ampere Requirements at 480 Cycles of Representative 60-Cycle Instruments Compared With New Amplifier Type

Instrument	Characteristics at 480 Cycles: Volts	Amperes	Volt-Amperes	Ratio to Amplifier Type
Ammeter				
Electrodynamic	2	5	10	100,000
Moving Iron	1	5	5	50,000
Thermocouple	0.25	0.05	0.012	120
Rectifier	2	0.01	0.02	250
Amplifier	0.01	0.01	0.0001	1
Voltmeter				
Electrodynamic	150	0.07	10	100,000
Thermocouple	1	0.001	0.001	10
Rectifier	1	0.0005	0.0005	5
Amplifier	0.7	0.00015	0.0001	1
Wattmeter*				
Electrodynamic				
Current	1.5	5	7.5	
Potential	100	0.02	2	
			9.5	95,000
Amplifier				
Current	0.01	0.01	0.0001	
Potential	0.7	0.00015	0.0001	
			0.0002	2

* Volt-amperes of potential and current elements combined for convenience.

Of those instruments listed the electrodynamic and the moving iron are probably in greatest general use because of their high accuracy and reliability, but note that they require the largest volt-ampere input for operation. Also, due to the increase in reactance drop at higher frequencies, the volt-ampere requirements and errors increase quite materially. Particularly does the wattmeter accuracy suffer at higher frequencies because of various phase shifts and mutual effects in the windings. This is especially important in this application because the wattmeter must maintain its accuracy even at very low power factors; a requirement exceedingly difficult to obtain even at 60 cycles. So these volt-ampere requirements tell but a small part of the difficulties encountered in building a wattmeter. A conventional thermocouple instrument is useful even at very high frequencies but is extremely slow in response and quite easily burned out with overloads. Descriptions of thermocouple wattmeters have been published but such wattmeters are not in general use. The accuracy of the rectifier type of instrument is not high enough for this application and the wattmeter is not commercially available.

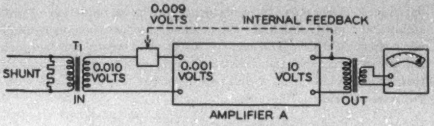

Figure 1. Internal feed-back circuit

So, after giving due consideration to the various instrument characteristics, it is apparent that to satisfy the requirements of speed, accuracy, reliability, low burden, etc., it is advisable to use the electrodynamic or moving-iron type of instrument either of which is suitable on every score except sensitivity, and then supply this sensitivity with an amplifier. Of course, the amplifier must be as reliable in every way as a high-quality indicating instrument. The input-output ratio and phase angle must remain constant within one-half per cent for all changes in tube characteristics, plate voltages, etc., to be expected during operation. Also, as it has turned out, the amplifiers must be able to correct some of the inherent inaccuracies found in a moving-coil-type wattmeter.

Description

The over-all amplifier input-output ratio is made essentially independent of most changes that occur within the amplifier by introducing two stabilizing feedbacks. Each acts independently yet they have a compounding effect when used simultaneously. The specific amplifier referred to in the text is designed only for a frequency of 480 cycles (±2 per cent); however, with suitable modifications the feed-back principles described can be applied to other single-frequency or broad-band amplifiers.

In the first stabilizing scheme, an internal feedback similar to that used by H. S. Black[3] and associates, the amplifier is deliberately built with several times the necessary amplification, and then, by feeding a portion of the output back on the input in such a way as to throw away the excess amplification, the amplifier will acquire new properties, the most important of which is constancy of amplification and freedom from nonlinearity.

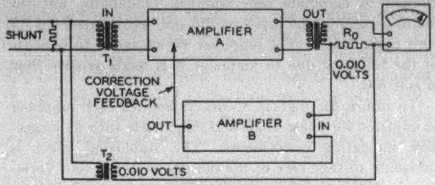

Figure 2. Null-balance feed-back circuit

The principle of such an amplifier can be explained quite readily by considering a specific example. Assume, as shown in figure 1, that 0.010 volt is available as amplifier input to give a ten-volt output. *A* represents an amplifier that delivers ten volts output with 0.001-volt input, but instead of using it as an ordinary amplifier part of the output voltage, namely, 0.009 volt, is introduced back into the input circuit such that the amplifier input of 0.001 volt represents the difference between the feed-back voltage of 0.009 volt and the impressed voltage of 0.010 volt. Amplifiers are subject to normal variations in the order of ten per cent so assume that for some reason or other the amplifier in question should change and now require an input of 0.0011 volt instead of 0.0010 volt, an increase of ten per cent. The output voltage remaining constant, the feed-back voltage will be 0.009 volt. Adding to this the new amplifier input of 0.0011 volt, the total input required is now 0.0101 volt instead of 0.010 volt as before. This represents an increase of one per cent in input volts corresponding to an error of ten per cent in the amplifier ratio, so with a feedback of ten times, the errors are reduced to one-tenth. By increasing the amplification and increasing the feedback, the error can be reduced still further, until a point is reached where the feedback is so large that the over-all amplification characteristics are substantially unaffected by most changes in the amplifier. It is far from a simple proposition to employ feedback this way because of the tendency for oscillation and the very special control which is required of phase shifts in the amplifier and feed-back circuits over a wide range of frequencies above and below the useful frequency band. However, once having achieved a design in which proper phase relations are secured the performance obtained is perfectly reliable.

When adjusted properly this scheme exerts a remarkably stabilizing effect on the amplifier but a second means for obtaining stability is used which serves as a check on the first. This second means consists of an external feedback, which is so-called because it is external to amplifier *A* and which is nothing more or less than a special use of the familiar and extremely accurate null-balance system. As shown in figure 2, a resistance R_0 is introduced in the output circuit of such value that 0.010 volt appears across it with the proper full-scale current flowing through the indicating instrument. This voltage is then compared with the input voltage of 0.010 volt and they should be identical. However, should either one increase or decrease more than the other or change phase angle with respect to the other there will be a voltage difference. This difference, or error as it is in this case, can then be amplified by an amplifier such as *B* and introduced directly back into amplifier *A* in such manner that it will adjust the output of *A* to the correct value corresponding to the input voltage.

There is another advantage in the use of this circuit that might not be evident at first glance. The secondary of transformer T_1 (figure 2) is supplying power to the grid of the first tube in amplifier *A*. This power is small to be sure, but due to the internal grid-to-cathode capacitance of the tube, there is sufficient leading current to cause a slight phase shift in the output of transformer T_1. This introduces an error which is not corrected by use of the first feed-back system alone. The secondary of transformer T_2 feeds into the grid of the first tube of amplifier *B*, but its voltage output is balanced against the voltage across R_0, so the only current that can flow in this transformer and cause phase shift will be due to the difference between these two voltages. Normally this difference will be very small and, therefore, the secondary of T_2 will be essentially open-circuited and its secondary voltage will be truly representative of the voltage drop across the shunt regardless of variations in tube grid characteristics, and can be balanced against that portion of the output voltage appearing across R_0 for a true comparison of the input and output of amplifier *A*.

Obviously the effectiveness of this method depends on how much the output voltage must depart from its cor-

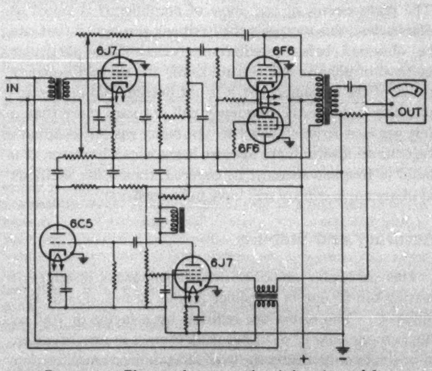

Figure 3. Electrical circuit of stabilized amplifier

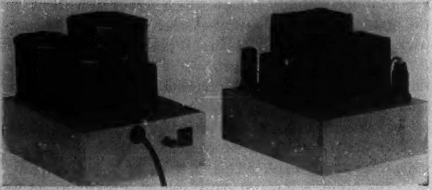

Figure 4. Stabilized amplifier and associated power-supply unit for General Electric 480-cycle a-c network analyzer

rect magnitude and phase to produce the necessary corrective effect. The smaller this voltage the more accurate the balance; the limiting condition being when self-sustained oscillations occur.

The complete amplifier circuit incorporating both systems of feedback is shown in figure 3. The main amplifier consists of a pentode driving a power pentode with a second power pentode connected to the output transformer in such manner as to cancel the d-c flux in the output-transformer core. The internal feedback is made from the plate of the output pentode through a resistance voltage divider to the cathode of the input tube. This connection is necessary to secure proper phasing. The output circuit is tuned with a series capacitor to bring the current in proper phase with the input voltage. The null-balance amplifier consists of a pentode driving a triode with a voltage divider in the triode output to allow control of the correction voltage introduced into the main amplifier. The input pentodes are used because of their low input capacity together with their high amplification and the triode is used in the null-balance amplifier to keep the resistance value of the voltage divider as small as is practicable. Commercially available parts are used throughout. A photograph of the amplifier and associated power supply is shown in figure 4.

The amplifier circuit constants are quite conventional. The plate circuit of one stage of amplifier B is tuned at 480 cycles; this increases the feedback through B that can be obtained before oscillations occur. This particular method of phase-angle control limits the useful frequency range of the amplifier, but it is not detrimental in this application because the frequency will not vary more than a few per cent from 480 cycles. In many cases it is advantageous to discriminate against harmonics; however, if a band of frequencies must be covered, then other methods of phase angle control will have to be used.

Accuracy and Stability

This particular amplifier utilizes feedbacks of approximately ten to one in amplifier A and 70 to 1 due to amplifier B. The errors are reduced by a factor of 700, so the extreme case of a 70 per cent change in amplification of one tube only affects the over-all stabilized amplification by one-tenth per cent. The stabilizing effect is so great that the over-all amplification characteristics are substantially unaffected by changes in the amplifier.

It would be possible to increase the feedback in amplifier A to many times the above value,[2] but in this particular application the extra stability was not necessary and the simplicity of a two-stage amplifier seemed advantageous. Likewise, the effectiveness of the feedback due to amplifier B can be increased providing that the various phase shifts occurring in amplifiers A and B can be controlled to eliminate self-sustained oscillations, but the extra complications involved did not seem warranted for this application. The amplifier shown tends to oscillate if the feedback due to amplifier B is made greater than about 100.

To insure the highest accuracy possible each amplifier is adjusted for phase angle and magnitude with its respective indicating instruments. The stabilizing effect is adequate to take care of the normal ±10 per cent variation in instrument reactance between zero and full scale.

The input-output ratio can be maintained so precisely by using these two stabilizing systems that ordinary instruments are not accurate enough to use in obtaining the initial adjustments. By virtue of the null-balance circuit the input to the null-balance amplifier represents any overall error in magnitude or phase angle between the input and output, so the initial adjustments are made until the output of this amplifier is zero. The ratio of amplifier A is controlled by varying the amount of internal feedback and the phase angle of the output current is adjusted to compensate for the inductance of the indicating instrument by means of a series capacitor.

An indicating device can be connected to the output of the null-balance amplifier and used as an indicator to show if the amplifier is working properly. In the network analyzer application this was not thought to be necessary because a circuit is provided, to which the amplifiers may be connected, and in which the voltage and current are adjusted to give predetermined deflections on the voltmeter, ammeter, and wattmeter. By connecting both amplifiers to this "standard circuit" they can be checked one against the other. This is not used as an absolute calibration for the instrument system but it does serve to increase the reliability for, if the readings do not check, something must be wrong with one or both measuring circuits.

Several of these amplifiers have been made and calibration tests indicate an accuracy of better than one-tenth per cent in magnitude and ten minutes in phase angle. The amplifiers are rated one-half per cent and 20 minutes. Tubes can be interchanged freely, and line-voltage variations on the amplifier power supply of 20 per cent do not change the calibration; however, when using high-speed light-beam instruments, severe transients in the 60-cycle power supply will cause the light beams to jump, after which the reading returns to the correct value. To eliminate this, a regulator can be placed in the 60-cycle supply to the amplifier power units to remove these transients. After being in constant use for eight weeks, two amplifiers and their associated indicating instruments were found to have a maximum deviation from their original calibration of two-tenths per cent of full scale without any readjustment.

Conclusion

With the proper use of stabilizing circuits, electron-tube amplifiers can now be built that are just as accurate and reliable as, and in many respects better than, the indicating instruments with which they are to be used. Such amplifiers combine the characteristic low power-input requirements of vacuum tubes with the accuracy and reliability found in high quality measuring instruments.

The use of stabilized amplifiers may be extended to any application where accurate measurements are required of quantities too small to be measured any other way. Their proved accuracy and reliability should go a long way toward getting around the average engineer's inhibitions against the use of electron tube apparatus whenever accurate and reliable measurements must be made.

References

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3. Stabilized Feedback Amplifiers, H. S. Black. Electrical Engineering, January 1934, volume 53.

Discussion

H. L. Hazen (Massachusetts Institute of Technology, Cambridge): This paper, in my opinion, describes a significant development in the field of electrical measuring instruments. By the technique here described it is practicable to attain a new order of speed of response and accuracy in rugged, low-burden instruments. It appears that the possibilities of this technique are numerous where the accuracy and speed of standard portable instruments are desired with burdens that are a minute fraction of those imposed by such portable instruments themselves.

Apart from the significance and utility of the design as a whole, there are several questions regarding the details on which I should like to comment in this discussion. There are two types of feedback employed in this amplifier which for the most part are mutually helpful. One of these feedbacks—the linear feedback—is actuated by the output voltage. The other—that due to amplifier B—is actuated by output current. It appears that in a device of this sort, in which the indicating instrument inherently responds to current rather than to voltage, the amplifier should be so designed as to maintain the desired mutual conductance of the unit as a whole. That is, for any given input voltage a definite current proportional to this voltage is desired in the indicating instrument. It appears that feedback based entirely on output current rather than on voltage or a combination of voltage and current, is therefore desirable.

A theoretical analysis of this amplifier has been made with Mr. G. S. Brown to determine the effects of these feedbacks and to compare them with other feed-back schemes. One basis of comparison is the ratio of the fractional change in mutual conductance to the fractional change in another variable, such as the gain of amplifier A, the impedance of the output circuit, or the internal impedance of amplifier A as viewed from its output terminals. With the scheme as used, it appears that the fractional change in the mutual conductance is about one seven-hundredths of the fractional change occurring either in the gain of amplifier A or in its internal impedance as viewed from its output terminals. In contrast, the fractional change in the mutual conductance is not one seven-hundredths, but one-seventieth of the fractional change in the load impedance because the linear component of feedback is based on output voltage rather than output current. If a comparable amplifier is made in which the feedback is all based on output current, the change in the mutual conductance caused by changes in amplifier-A voltage ratio, by changes in amplifier-A impedance as viewed from its output terminals, or by changes in the load impedance—all become the same. It appears, therefore, that for a device of this sort, feedback based on output current rather than on output voltage is desirable.

It is stated in the paper that one advantage of using the type of feedback involving amplifier B is that it tends to make the voltage actually applied to the input terminals of amplifier A approach zero, and therefore reduces the current drawn at the input terminals of this amplifier. This statement seems open to discussion on two counts: First, it can be seen that if amplifier A and its linear component of feedback are suitably adjusted, the input to amplifier B is zero, and its output voltage is therefore necessarily zero. Hence it appears that amplifier B does not have any effect on the magnitude of the amplifier-A input voltage unless amplifier A and its linear component of feedback are out of adjustment.

Second, any degenerative feed-back scheme—linear or otherwise—serves the purpose attributed to amplifier B, namely, it puts a voltage in series with the signal input voltage which has such polarity and magnitude as to make the sum of these voltages small compared with either. Thus, assuming the amplifier-B scheme of feedback to be functioning, it does nothing that any other feed-back scheme would not do on this count.

Another way of considering the relative magnitudes of signal voltage and actual amplifier input voltage is this: For a given amplifier output and amplifier gain, the input voltage to the amplifier is fixed. If the amplifier mutual conductance is N times the desired ratio between output current and actual signal voltage, then the net amplifier input voltage is necessarily only $1/N$ of the signal voltage. This, as was pointed out above, is characteristic of any degenerative feed-back amplifier and not merely of the particular type of feedback represented by amplifier B in this circuit.

In raising the foregoing questions I do not wish to detract in any way from the value of this development. On the contrary, the contribution seems so significant in the measurement field that it is worth while to give attention to matters which may be relevant to the design of similar units in the future. The foregoing comments are based primarily on theoretical considerations, and I shall be glad to have the author's comments, both from the theoretical and from the practical point of view.

F. K. McCune (General Electric Company, Lynn, Mass.): Mr. Thompson's paper should be of universal interest to those concerned with electrical measurements. The combination of circuits described appeals to me as particularly ingenious since it compensates not only for magnitude of current but for phase angle as well. I feel that this latter feature is deserving of special comment since the accuracy of correction is such as to be perfectly adequate for the great majority of measurements. The value of ten minutes of phase angle corresponds to one-half per cent at 50 per cent power factor, however, and is therefore too large to neglect for very accurate measurements of power or vars. It will be of interest to see whether the continued use of these circuits shows greater accuracy possible.

A possible field of application for this circuit would seem to be the measurement of extremely small currents such as are dealt with in noise and vibration measurements. Here the effect of "pick up" from extraneous sources, both of noise and vibration, may affect the readings obtained, due to the disturbances they produce in the amplifiers. If the frequency range of the circuits described can be made large enough, it would offer a possible solution to many troublesome problems of this sort.

It would seem from a rough examination that while phase-angle errors in A would be corrected by B, that phase-angle errors in B would result in considerable errors. For example assume currents in the shunt and in R_0 initially ten minutes out of phase. If amplifier *B* should have a ten-minute phase angle and the result should be subtractive, an out-of-phase current with a magnitude of a fraction of one per cent would be impressed on *B* and would be there amplified and impressed on *A* exactly in phase. The result would be no improvement in phase angle and an increased error in magnitude.

Probably this is only a theoretical possibility, but calibration data on the circuits will, no doubt, be taken periodically and will give the real answer to all such considerations.

M. S. Mead (General Electric Company, Schenectady, N. Y.): In the discussions of Mr. Thompson's paper, "Stabilized Amplifiers for Network Analyzer," there appear to be four points which should be answered. Mr. McCune mentioned that in a wattmeter, even a very small phase-angle error will result at low power factor in considerable error. This is a well-recognized difficulty in all wattmeters and in this case the answer simply is that extreme accuracy in power or var measurements at low power factor is not required. So far, in the operation of the board, the accuracy of the watt and var measurements has been great enough, even at low power factors, so as not to handicap the operation of the board in any way.

The second point he raised was in connection with the effect of phase-angle error in amplifier *B* on the phase-angle error of amplifier *A*. Without taking the time to submit a proof, it may be stated that with large amounts of feedback, any small departure from 180-degree feedback will result in an infinitesimal reduction in the benefits secured from the feedback. Therefore, we conclude that a small phase-angle error in amplifier *B* is unimportant.

I know that Mr. Thompson appreciates the nice things Prof. Hazen said about the development. In his discussion, Prof. Hazen mentioned two things. He stated that it would have been better to have designed the amplifier so that 700 times current feedback was used alone without the voltage feedback. Others have offered the same comment. We are in entire agreement with this thought. However, it is extremely difficult to build an amplifier including an output transformer and having a feedback ratio of 700. Confronted by limitations of time and facilities, nevertheless, a feed-back ratio of 700 was secured by using the compound effect of two separate feedbacks, each of which was quite easy to get.

The second point Professor Hazen mentions apparently resulted from some misinterpretation of the text. The paper stated that there was a distinct benefit secured from using external feedback and a reference voltage derived by an additional transformer across the current shunt. This advantage is not concerned with the amount of grid voltage input to amplifier *A* as Professor Hazen understood was claimed, but rather to amplifier *B* whose input voltage approaches zero and consequently the amplifier cannot act in any manner as a load on the transformer supplying the reference voltage.

H. A. Thompson: The discussion seems to center around the question of just which feedback is the simplest and best. Considering that the particular instruments used respond to current rather than voltage, as Professor Hazen has mentioned, it is advantageous to use current feedback and control the transconductance of the amplifier rather than the voltage amplification.

The time at our disposal to develop this amplifier was limited and a conservatively rated two-stage amplifier with feedback appeared to be the simplest and most reliable unit we could assemble and still get the required amplification. The null-balance amplifier *B* was added to serve as a check on the performance of amplifier *A* and also to compensate for the variation in instrument inductance and the mutual voltages introduced in the wattmeter coils. No trouble was experienced due to the effects of phase shifts in *B*. If necessary, it would have been a simple matter to stabilize *B* with negative voltage feedback similar to that used in *A*. As a matter of fact, tests indicated that the tuned circuit used in amplifier *B* could be detuned considerably without any harmful effects.

Doubtless, if the time and money were available, this amplifier could be improved, but for this particular application the accuracy and reliability needed did not warrant any further development. Operating experience has indicated that the amplifiers perform well within the operating requirements of the network analyzer. In fact, the wattmeter has been used at power factors as low as seven per cent with acceptable accuracy, and phase angle measurements are made using the wattmeter at zero power factor.

Phanotron Rectifiers as a D-C Supply for Elevator Motors

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Introduction

THERE ARE a number of public utilities that have instituted programs for curtailing and in some cases completely eliminating the d-c power supply to concentrated load areas. The change from d-c to a-c power supply for elevators in the affected areas requires the substitution of a-c elevator equipment or the use of converters to supply d-c to the existing elevator equipment. Elevators may be operated equally well from either alternating current or direct current, so the choice is dependent upon the cost of the change-over.

Where a converter is to be used, a phanotron rectifier possesses many advantages. Its first cost is competitive with other forms of converters. It has no rotating parts and so requires no special foundations, and it is quiet in operation. These two characteristics permit greater freedom in selecting the location of the converter. It is more efficient than a motor generator set. It is semiautomatic in operation, thus requiring little attention from the building attendant.

The phanotron rectifier used for elevator power supply is of particular interest as it is the first industrial application of the modern type of metal electron tube in power sizes. In addition the automatic features incorporated in these rectifiers have resulted in a very successful service record and consequently merit the careful attention of those interested in applying electronic equipment for power purposes.

Before describing the phanotron rectifier and considering its features, the power requirements of elevators will be reviewed.

Power Requirements of Elevator

Elevators with few exceptions are simple counterweighted hoists in which counter weight generally equals that of the cab and its appurtenances, plus 40 per cent of the cab capacity. The motor load is characterized by peak power consumption during the starting and accelerating period followed by a very much reduced load during the running period, which depends upon the number of passengers in the cab, and may be regenerative in character, followed by regenerative braking to a slow speed or stop. The current required to accelerate full load may vary from 100 to 250 per cent of the current required to hoist full load. The usual maximum regenerative load on gearless installations is approximately 48 per cent of that required to hoist full load, and on geared installations, 23 per cent.

Where there is more than one elevator in an installation, it is impossible to predict the load demand from the speed and capacity of the various elevators. The average elevator does not operate more than 50 per cent of the time that it is in service. Since the loading is usually directional with respect to time of day, maximum load will not be required more than 25 per cent of the operating time. Consequently, the ratio of maximum to average load will be high where only a few elevators are involved in a given plant. As the number of elevators increases, the ratio reduces and the short time power peaks become less pronounced. The type of motor used also has a considerable effect on the shape of the power demand curve. A typical elevator load is shown in figure 1.

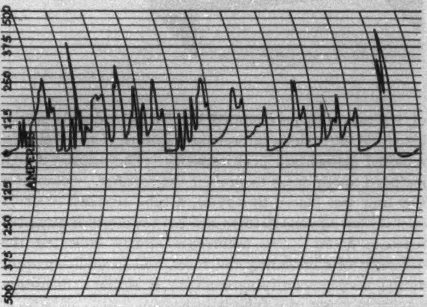

Figure 1. Typical load on a bank of three elevators with 35-horsepower motors

If load curves can be obtained during times when the elevators carry heaviest loads, the maximum momentary peak current, the average current, and the root mean square current may be estimated and used as a basis for determining the size of the phanotron converter.

The First Installation

In the fall of 1935, a trial installation was made in Chicago using a four-tube rectifier operating from a power supply of 208 volts, 3 phase, 60 cycles, and delivering direct current at 230 volts. Operating data obtained before the change-over showed instantaneous peaks in excess of 200 amperes. These peaks and the fact that the only phanotron tube in this current range was rated 30 amperes average with a momentary peak commutating capacity

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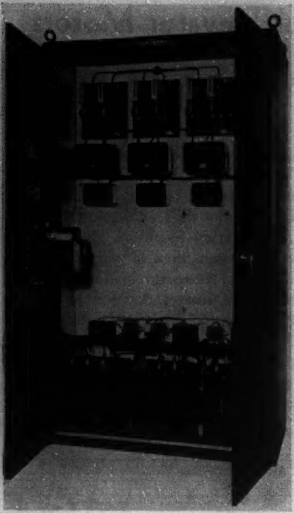


Figure 2A. Front view of rectifier with doors open

of 150 amperes, necessitated the use of a double single-phase circuit with an interphase transformer to obtain the required momentary peak capacity. The original design included anode fuses to protect the rectifier. Soon after this unit was placed in service, a tube arc-back or short circuit blew the anode fuses and left the elevator stranded between floors. This occurrence clearly indicated the necessity of eliminating the fuses, and in addition, providing reserve rectifying capacity in order to maintain service in event of a tube failure.

Phanotron Rectifier

A modified rectifier design using six phanotron tubes, anode contactors, thermal overload relays, and reclosing timers was then built. It operated so successfully that all subsequent rectifiers for elevator service have followed this same fundamental design.

The general appearance of the rectifiers is shown in figure 2. It consists primarily of phanotron tubes mounted in the middle section, with power transformers and interphase transformers mounted in the base. The associated control and protective equipment is mounted upon a panel within the cubicle, and the cubicle is provided with two full length hinged doors to give ready access to all parts of the rectifier. The tubes are enclosed in an individual section at the back of the cubicle, which extends the length of the rectifier, access being provided by two small hinged doors inside the enclosure. This compartment is in effect a chimney providing the proper ventilation to maintain the tubes at the desired operating temperature. The screened opening through which the ventilating air enters is located at the rear of the cubicle.

To date, there have been built seven 230-volt units and three 550-volt units. These range from 25 kw for one hour to 40 kw continuously at 230 volts, and 50 kw for one hour at 550 volts.

These rectifiers are usually described by a one-hour kilowatt and a peak-current rating as the elevator load is directional and highly concentrated with respect to time of day.

Phanotron Tubes

The phanotron tube used in the elevator type of rectifier is of the directly heated hot-cathode type, with a metal envelope, internal insulated anode, and mercury vapor. The tube (This tube bears the trade designation *FG*-166.), figure 3, is designed to operate at condensed mercury temperatures of from 20–80 degrees centigrade. This range corresponds to an ambient temperature of from 20 degrees to 50 degrees centigrade. Before a rectifier of this type can be placed in service, the proper vapor conditions must be established inside the tubes and the filaments must be brought to the emitting temperature. To accomplish this, the filaments should be energized for a short period of time, one to two minutes under ordinary conditions or a longer time if low ambients are encountered. If the rectifier is to be placed in a position where the ambient goes below five centigrade, thermostatically controlled dampers or louvers are used in the ventilating system to assist in maintaining the proper operating temperature of the tubes.

The average expected life of this type of tube is 10,000 hours when operated under conditions described here. Four installed in October 1935 have already given approxi-

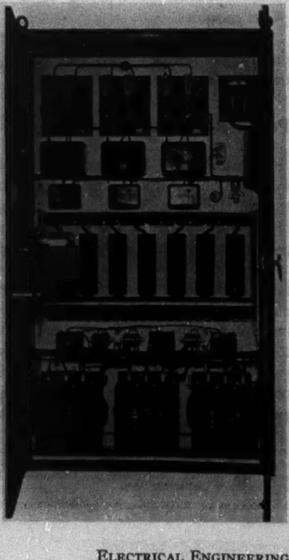


Figure 2B. Front view of rectifier with main door and tube-compartment door open

mately 16,000 hours service and twelve installed in January 1936 have been operating continuously with no failures.

Triple Single-Phase Full-Wave Circuit

In a single-phase full-wave connection, each tube carries current for 180 electrical degrees. The tube used in the rectifiers has an average rating of 30 amperes with a peak rating of 150 amperes. Thus, such a single full-wave rectifier would have an average current rating of 60 amperes and a maximum momentary peak rating of 150 amperes. Likewise, three such rectifiers, one connected to each phase of a three-phase supply, and having their cathodes paralleled through interphase transformers, would have a combined rating of 180 amperes, and a momentary peak rating of 450 amperes. This arrangement of connections will be called a triple single-phase circuit. The triple single-phase full-wave circuit has the additional advantage of balancing the load on the three-phase supply. Also this arrangement increases the ripple frequency and reduces its magnitude because of the 120-degree displacement resulting in a smoother d-c voltage. Considering motor generator sets, however, I would say that the set had a continuous rating of 180 amperes and a maximum commutating peak capacity of 450 amperes which the generator could safely produce. The single-phase circuit makes possible the maximum utilization of the current carrying capacity of the tube resulting in greater reliability of operation and longer tube life than is obtainable in any other type of circuit so far available for intermittent service.

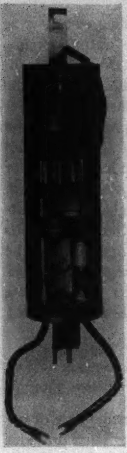


Figure 3 (left). FG-166 hot-cathode tube used in rectifiers

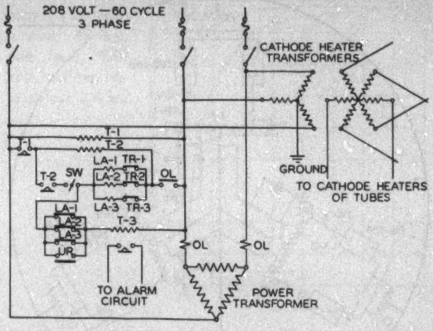


Figure 4 (right). Elementary a-c control diagram

T-1—Cathode-heater timing relay
T-2—Reclosing timing relay
T-3—Alarm relay circuit
*LA*1, 2, 3—Thermal relays
*TR*1, 2, 3—Thermal relays
OL—Overload relay
SW—Control switch
UR—Unbalance relay

The experience gained on the four-tube rectifier indicated the desirability of using a circuit which would have enough reserve capacity to keep the elevators in service in the event of a tube failure. The triple single-phase full-wave circuit accomplishes this purpose as it may be operated as a double single-phase full-wave circuit and produce two-thirds of its rated output simply by opening the circuit having a faulty tube as an inspection of the elementary circuits will show this possibility.

The power and control circuits are shown on figure 4, elementary a-c control diagram; figure 5, elementary diagram of power circuit; figure 6, elementary d-c control diagram.

Functions of Semiautomatic Control

In case a tube arcs back, the a-c overload relay (*OL*) opens the coil circuit to the anode contactors which immediately open. After two seconds the anode contactors are reclosed by reclosing relay (*T*2). If the trouble persists after several reclosures, one of the three double-pole temperature overload relays (*TR*-1-2-3) in the faulty circuit will open the coil circuit of its associated anode contactor and operation will be continued with the remaining phases and four tubes. The tubes are removed in pairs because the failure of one tube in a single-phase full-wave circuit produces saturation in its anode transformer and also reduces the average output voltage resulting in the remaining tube carrying practically no current. The opening of the contactor in one single-phase circuit changes the ripple frequency causing the operation of unbalanced relay (*UR*). The operating coil of this relay forms a part of a series tuned circuit connected across the d-c output (see figure 6). The unbalanced relay (*UR*) operates a relay (*T*3) to close the user's alarm or indicating circuit. If the disturbance has been one of repeated arcbacks, the defective tube can be identified through its element of the temperature relay in that phase.

If the tube fails to conduct because of lack of emission, the unbalanced relay (*UR*) will operate the alarm, no other relays operating. In this case the anode contactors will not be open, but the defective tube can be identified by its lack of the characteristic blue glow as seen through the anode seal at the top of the tube. In order that a spare tube may be placed in operation, provision is made for holding it between the active tubes so that the radiated heat will keep it warm and the mercury properly distributed.

On the occurrence of a severe overload the control functions in the same manner as that for an arcback. If the overload persists for less time than required for the thermal relays to operate the anode contactors will reclose and normal operation will be resumed.

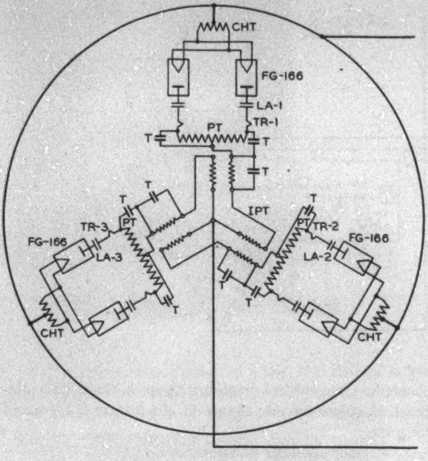


Figure 5. Elementary diagram of power circuit

CHT—Cathode-heater transformer
FG-166—Phanotron tube
IPT—Interphase transformers
PT—Power transformers
*LA*1, 2, 3—Anode contactors
T—Thyrite arresters
*TR*1, 2, 3—Thermal relays

When connecting the rectifier to the source of supply, the cathode-heating transformers (*CHT*) (figures 4 and 5), are energized and supply excitation to the tubes. A timing relay (*T*1) is energized and after a predetermined time applies voltage to a reclosing timer (*T*2). In two seconds, voltage is applied to the closing coils (*LA*-1, 2, 3) of the three-tube anode contactors provided the control switch (*SW*) is closed. The closing of these contactors completes the circuit and makes power available.

The rectifier may be removed from the d-c load by opening switch (*SW*) which de-energizes contactors (*LA*-1, 2, 3) and opens the power circuit of the rectifier. However, the cathode-heater transformers remain connected to the power supply so that upon closure of *SW*, the rectifier is immediately reconnected to the load. Opening of the a-c power supply de-energizes the timing relay and power transformers. For the resumption of operation upon reclosure of the power supply switch, the timing cycle must be repeated after which the rectifier can be returned to service.

Overhauling Load

When the elevator is lowering a sufficiently heavy load, it holds speed by regenerative braking. Since the elevator rectifier cannot return this power to the supply circuit, it is necessary to provide a loading resistor which will absorb it. The loading resistor (*R*3) is shown in elementary d-c control diagram, figure 6.

The operation of the regenerative loading circuit is as follows: if power is supplied to the d-c output circuit, the coil of timing relay (*T*4) is energized through the loading resistor (*R*3), which relay completes the circuit of the instantaneous voltage relay (*VR*). When the rectifier operates with an overhauling load, the voltage on the d-c bus rises sufficiently above no load voltage to actuate voltage relay (*VR*) which in turn closes the loading resistor contactor (*L*) and allows it to absorb the overhauling load. The closing of contactor (*L*) also de-energizes the operating coil of the timing relay (*T*4), which after four seconds drops out, opening the voltage relay coil circuit so that the control is ready to repeat the cycle if the regenerative condition persists. As the regenerative power usually exists for much less than four seconds, generally one operating cycle is sufficient.

Power and Interphase Transformers

The transformers employed in these rectifiers are of the natural-draft air-cooled type as this construction provides the most economical design. As the transformers are located inside the cubicle (see figure 2) individual enclosing cases are not required. Transformers operating in rectifier circuits are subjected to transients during faulty operation. To protect the windings against any surge voltages that may be induced at such times, thyrite resistors (*T*) in figure 5 are connected across power transformer secondaries to neutral, and across one winding of each interphase transformer.

R-1—Loading resistor
UR—Unbalance relay
CAP—Capacitor
VR—Voltage relay
R-2—Variable resistor
T-4—Timing relay
L—Contactor
R-3 — Regenerative loading resistor

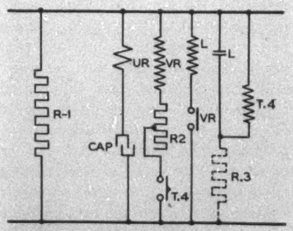


Figure 6. Elementary d-c control diagram

The triple single-phase circuit requires three interphase transformers. Each transformer has two independent sections connected zig-zag in order to cancel the d-c ampere turn under balanced load. The connection is equally effective with either two or three single-phase circuits in operation.

The design of the interphase transformer is of special importance in this particular service as the magnetizing current determines the point of inflection of the voltage regulation curve, figure 7. Slight manufacturing variations will naturally be encountered in similar circuits of identical design; hence, the d-c current supplied by each rectifier circuit will be different. To prevent saturation due to any small unbalance that may be present, the interphase transformers are constructed with an air gap. The magnetizing current being roughly proportional to the air gap, a small air gap is desirable in order that the inflection points occur at as light a d-c load as possible.

To provide the desired interphase transformer characteristics, the air gap is slotted, that is there is a small section of iron in the gap which produces a low-impedance flux path at light load, but which saturates very early under increased load, and so effectively lengthens the air gap. To prevent a high no-load voltage, a small loading resistor (*R*-1) (figure 6) rated 1.6 amperes is permanently connected across the d-c output. Its loss is only a small percentage of the no load loss of the rectifier.

Voltage Regulation

With the loading resistor in the circuit, the voltage at no load is about 257 volts and the voltage regulation of the elevator rectifier is approximately ten per cent as shown by the flat portion of solid line in figure 7. The dotted curve in figure 7 shows the regulation which would be obtained if one branch of the triple single-phase circuit were not operating. This latter regulation has been found to be sufficiently good to maintain substantially normal operation of the elevators.

Efficiency and Power Factor

The efficiency curve of the 40-kw, one-hour, 230-volt rectifier is very flat as shown in figure 8. This is materially better than can be obtained from a motor generator set of like rating, particularly at the light loads which are usually encountered with most elevator plants because of their low load factor. No-load losses are approximately 2.4 kw.

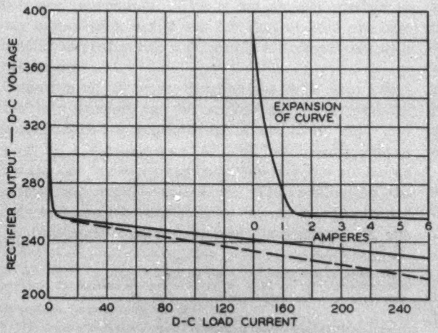
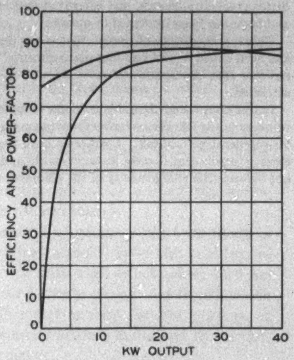


Figure 7. Voltage-regulation curve

Figure 8 also shows the input power factor. It will be noted that this is very good even at no load, where the power factor is approximately 0.77. This is contrasted with a motor generator set having a no load loss of approximately 3.5 kw at a power factor of 0.25.

Figure 8. Power factor and efficiency 25 kw continuous; 40 kw, one hour, 240 volt, rectifier; no-load loss 2.4 kw



Summary

The triple single-phase phanotron rectifiers at the present time are built in capacities up to 40 kw at 230 volts and 75 kw at 550 volts. Their application has not been confined to elevator loads, one having been applied to general machine shop load including cranes, and another handling the entire building load elevators, ventilating fans, pumps, etc.

The units are self-contained with all live parts enclosed and if desired the cubicle may be locked. The installation wiring is limited to bringing in the a-c supply and connecting the d-c load. Installation costs are thus held to a minimum.

The operation of the seven rectifiers in service has to date been entirely successful. Additional rectifiers have been purchased by the original users after one year of use, indicating the suitability of the rectifier for hoisting service.

Discussion

T. G. Glenn (General Electric Company, Chicago, Ill.): There are at present five 40-kw and two 25-kw, 250-volt phanotron rectifiers furnishing power for elevator equipment in the Chicago Loop area. These rectifiers supply power for elevators, both freight and passenger, which in the majority of cases are in service 24 hours a day, seven days a week, so that the phanotron equipment is energized continuously. Because of this continuous service it is essential that the equipment operate with high no load efficiency.

The installations in the Chicago Loop area have been very successful and given remarkably satisfactory service. As the buildings in which they are installed are of a smaller office and manufacturing loft type, the electrical maintenance force is not of the caliber to give equipment the type of maintenance that rotating apparatus would require. In fact the equipments since their installation have seen little or no maintenance, and several of them are covered with dust and have not been inspected in months.

This is a condition, of course, that is not desirable and one which has been called to the building managements' attention, but it does indicate that the phanotron rectifiers will operate with little or no attention.

It has been found desirable to make complete load checks before recommending the size of conversion apparatus, or the size of resistor for regenerative braking. Load tests have been of an advantage as elevator equipment in most instances is old and nameplate ratings do not indicate the true capacity of the equipment.

As these phanotron rectifiers were installed in buildings where most of the available space is in use, rectifiers must be installed where space is available, sometimes this in the the basement and sometimes in an out of the way corner on one of the floors of the building. Our experience has shown that locations that were not suitable for rotating apparatus could be used for the rectifiers.

At first the electrical contractors who in general are installing the phanotron rectifiers, were skeptical of the equipment. This attitude perhaps was due to the equipment being new and unfamiliar to them, and contained electronic devices. However, the extreme simplicity of installation and wiring appeals to them. For instance, the wiring of a phanotron rectifier equipment requires only the installation of a three-wire incoming a-c line and a disconnect switch, and an outgoing two-wire d-c line to connect the load sides of the rectifier with the existing d-c mains in the building.

The long tube life experienced in these installations indicates a low maintenance cost.

The operating experience of the phanotron rectifiers in the Chicago area has shown that these units if properly installed, will furnish service comparable to that supplied from a 250-volt d-c system of the power company.

Transmission Line Transients in Motion Pictures

By L. F. WOODRUFF
MEMBER AIEE

IN SELECTING subjects for educational treatment by motion pictures at Massachusetts Institute of Technology, one of those favored was that of traveling waves on transmission lines. Reasons for the selection included the mathematical complexity of the subject and the difficulty of teaching it without spending an undue amount of time, and the fact that the phenomena which are involved are functions of both time and distance, and hence difficult to illustrate or visualize by ordinary methods. Experience in the use of the films with classes has shown that students' general understanding and appreciation of the subject are indeed greatly improved by viewing the motion pictures. The films which have been prepared have been and are now available in both the 16- and the 35-millimeter widths for free loan to other technical schools and associations.

In undertaking the work, it was decided that a true picture should be presented as far as was reasonably practicable. Waves computed on the basis of neglecting resistance and leakage would be very easy to show, since the resulting waves would be purely rectangular in shape. This method could not even allow showing the settling to the steady state. Computing the waves from the formulas for the distortionless line case ($R/L = G/C$) would be relatively easy, but the results would not be very appropriate since this condition presupposes an amount of leakage which is much greater than that present on any ordinary line. Furthermore, both of these foregoing methods involve neglecting skin effect and any other phenomena which result in the departure of the wave front from the vertical.

A better method of computing, but one which still neglects the factors which lead to a rounded wave front, is the one which results from the analysis based on uniformly distributed line constants of resistance, inductance, and capacitance. This is not an easy method to apply numerically, but it was felt that at least one case should be treated in this way, if only for comparison with other results.

Computation of wave distributions with skin effect taken into account can be worked out by brute-force methods, such as representing a suddenly impressed voltage by a Fourier's series of period sufficiently long to allow the line to settle practically to a steady state before the following impulse comes on. This settling to the steady state may be hastened by having the line loaded by a resistance equal to its surge impedance. A tremendous number of harmonics however must be handled even to approach fairly closely to the actual solution for a single time and position on the line. When it is considered that literally thousands of complete curves of distribution are required for the motion pictures, and further that it is desirable to treat cases in which the reflection at the load would add further great complexities in the calculation, it is seen that this method is not possible to apply in any reasonable time or at reasonable expense. Steinmetz[1] has made a very rough use of this method, but without the device of adding the surge-impedance load to hasten subsidence.

A rather meager amount of progress has been made in more direct mathematical attacks upon the problem of traveling wave behavior with skin effect taken into account, but there is no formula as yet developed which yields a solution sufficiently precise for the purpose at hand, or sufficient economy of time and effort to warrant its use for such a purpose.

Attention was turned therefore to methods of determining the data experimentally. The ideal source for the data would be a long actual power transmission line, but the practical difficulty of getting the use of such a line over a long enough period, and of taking observations at a large number of points along the line separated from one another by many miles, were very large.

Fortunately there have been available for a number of years in the electrical engineering laboratories at Technology, artificial transmission lines with distributed constants, designed by F. S. Dellenbaugh, Jr.[2] Comparisons of oscillograms taken of transients on these artificial lines and on actual overhead lines have shown that the characteristics of the laboratory lines are very nearly the same as those of actual lines. It was decided therefore to use, as the chief source of data, oscillograms taken at various points along one of these laboratory lines. Various types of load could be connected, and the transients and reflections resulting from complicated cases could be handled as easily as those from simple cases. A length of 250 miles was selected as being about equal to that of the longest existing lines. Relatively simple cases were selected for first treatment, since the purpose was an educational one.

On account of the rapidity of the transients and the steepness of the wave fronts, it was essential to use a cathode-ray oscillograph. In order to secure a standing image of the transient on the screen of the cathode-ray tube, and to obtain a time datum and comparative voltage datum with respect to the sending-end voltage, a special commutating device was developed to make it possible to give in effect a plurality of traces, of different currents or voltages, simultaneously on the screen. This device has been described in an earlier article.[3]

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1. For all numbered references, see list at end of paper.

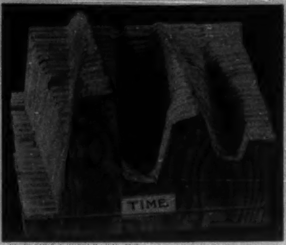


Figure 1. Voltage - time - distance surface representing d-c switching transient on open line

It was decided that the films should be confined to showing variations of voltage, rather than bringing in current variations also, for several reasons. On power lines, when damage is done by traveling waves, it is by high-voltage rather than by high-current effects, and hence the voltage is practically more important. It is also easier to understand the entire phenomenon by watching a voltage wave rather than a current wave, because with the proportionality which exists between voltage and charge on a smooth line, an idea of the current is obtained readily by mentally applying the analogy of a mass of liquid moving along as a wave or surge. The only disadvantage in using voltage rather than current is that in the computation (which was used in one case for comparative purposes) the voltage is much more difficult and tedious to calculate. This disadvantage however can be overcome, as will be seen later.

Each oscillogram is a record of the time variation of voltage at some definite point on the artificial line, whereas the type of curve needed in preparing the motion pictures is a distribution of voltage along the entire length of the line, at a definite single instant of time. It was necessary therefore to replot the oscillographic data, transferring one ordinate from each oscillogram taken under a given set of line conditions, to a distribution curve. This was done by carefully ruling off all the 32 oscillograms which were taken for each case, by equal time intervals, and making use of a pair of proportional dividers to transfer and scale down the voltages recorded, at a particular length of time after energizing the line, on all the 32 oscillograms, to the distribution curve for that particular instant. Corrections for nonlinearity of the sweep time axis and other distortions were made in this process.

Direct Voltage on Open Line

The first case treated was that of the open-end line, with direct voltage suddenly impressed on the uncharged line at the sending end.

As an aid in visualizing the three-dimensional variation involving distance, time, and voltage, tracings of the oscillograms were made and glued to sheets of plywood, and then cut out along the curves by means of a jig saw. When these pieces are assembled in order, they form a three-dimensional plot of the variation of voltage, during the transient period, with both time and distance. A photograph of this solid figure for the open-line case is reproduced in figure 1.

With reference to this solid figure, each voltage distribution curve may be visualized as a thin slice of the figure made in a direction perpendicular to the plane of the plywood pieces and to the base.

The voltage distribution curves for a number of instants of time are shown in figure 2. It is seen that the wave undergoes attenuation and distortion during its travel, and the wave front becomes progressively less and less steep. Complete reflection at the open end takes place without a change of sign, resulting in a voltage nearly twice the impressed voltage for a short time. Reflection

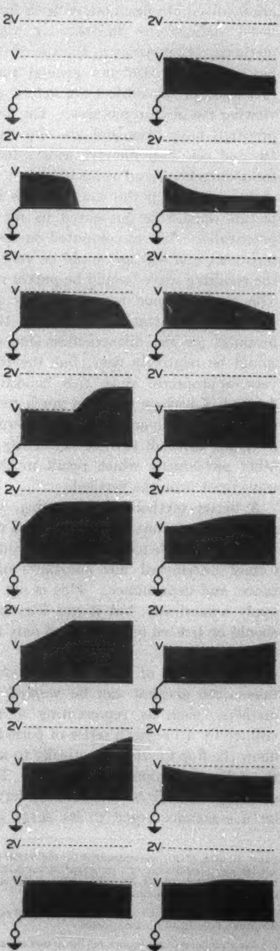


Figure 2. Direct-voltage waves on open line

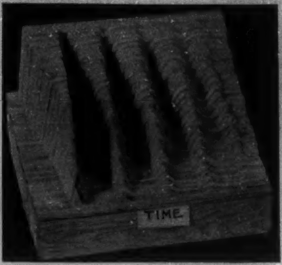
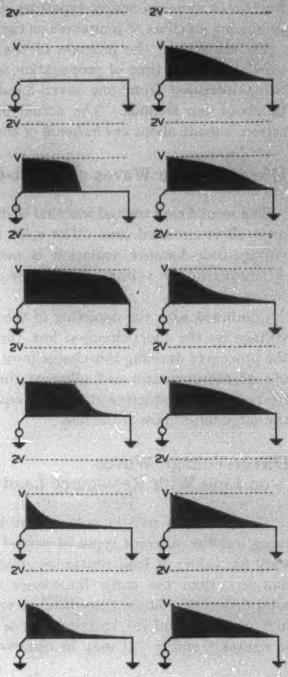


Figure 3. Direct-voltage switching transient on short-circuited line

Figure 4. Direct-voltage waves on short-circuited line



at the sending end occurs with a reversal of sign, and after several trips of the wave over the line the distribution is obviously approaching the steady state of a voltage along the whole line equal to the impressed voltage. This is in the absence of any appreciable leakage on the line.

Technique of Actual Photography

When distribution curves had been plotted in the manner already explained, dark paper cutouts were made with scissors to fit the distributions. These were then glued to white paper background sheets, which had perforations to locate them precisely over pins on a platform under a motion-picture camera. The procedure was quite similar to that used in animated cartoon motion pictures. The background drawings of the transmission line, scale lines, generator, and terminal connection in the animation scenes were made on celluloid. These were then printed photographically on film to avoid trouble due to chipping off the ink, which occurred when the original drawings on transparent celluloid were used repeatedly. Transparencies were used also in obtaining a number of special effects, including rotating commutators, closing switches, sections of wave moved and inverted to illustrate the action of complete reflection.

In order to economize on the labor required, the distribution plots were not made independently for each frame of the motion picture. One was made for every eighth frame, and the distributions in between were determined by graphical interpolation first into halves, then quarters, and finally eighths. One hundred twenty-eight different views were prepared to produce a transit of the wave from one end of the line to the other. As the steady state was approached and the motion became very slow, it was found possible to use one distribution diagram

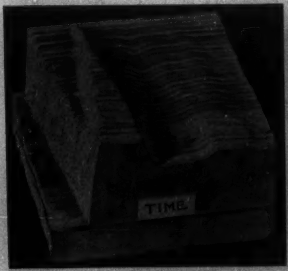


Figure 5. Direct-voltage switching transient on line loaded with resistance greater than surge impedance

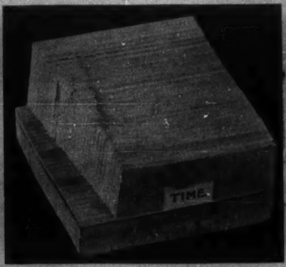


Figure 6. Direct-voltage switching transient on line loaded with resistance equal to surge impedance

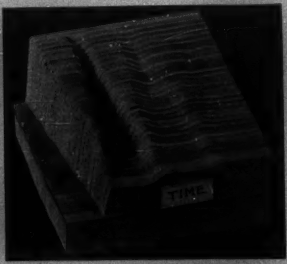


Figure 7. Direct-voltage switching transient on line loaded with resistance less than surge impedance

for two or more successive exposures on the film, without producing an effect of jumpiness on the screen.

On the screen, the progress of the wave is shown in slow motion, its time of propagation on the 250-mile line being increased from one seven-hundred-fiftieth second to about ten seconds. The actual linear speed on the screen is made about *one billionth* of the actual speed.

Direct-Voltage Waves on Short-Circuited Line

The second case treated was that of direct-voltage waves on a short-circuited line. The solid figure showing the voltage-time-distance variation is pictured in figure 3. A number of the voltage distributions along the line are shown in figure 4.

Combined with the depicting of the building up of the voltage on the short-circuited line, there was shown also the process of draining this charge from the line by switching off the generator and allowing the line to discharge through a noninductive resistance equal numerically to the surge impedance of the line.

Direct-Voltage Waves on Line With Resistance Load

Three cases of resistance load were treated in order to bring out the different types of partial reflection. These were for values of load resistance greater than, equal to, and less than the surge impedance of the line. The solid figures of voltage-time-distance variation are shown in figures 5, 6, and 7. Distributions of voltage are shown in figures 8 and 9. It may be observed in figure 8 that partial reflections without a change of sign occur at a resistor termination whose resistance is greater than the surge impedance. No reflection occurs when the termination has a noninductive resistance equal to the surge impedance. Partial reflections with change of sign are shown in figure 9, which is for the line with termination having noninductive resistance less than the surge impedance.

Direct-Voltage Waves on Line With Inductive Load

The case of voltage waves striking an inductive termination was treated. The three-dimensional plot, made by packing together 33 oscillograms pasted on plywood and cut out along the curves, is pictured in figure 10. Plots of voltage distribution are shown in figure 11.

It may be observed that when the voltage wave first strikes the inductive load, there is an approximately complete reflection without a change of sign, just as is the case when the wave strikes an open end. The reason for this is that the flow of current along the line accompanying this voltage wave is temporarily halted by the inductance. In order for the current to flow on without delay into the inductance, an infinite rate of change of current there would be required, if there were no distributed capacitance in the inductor itself. The rate of change of current build-up can occur only at a rate such that the back emf produced by the inductance will equal the voltage impressed on the inductance. This initial stoppage of the current causes the charge to build up just as in the open-line case, but soon the current begins to flow in appreciable

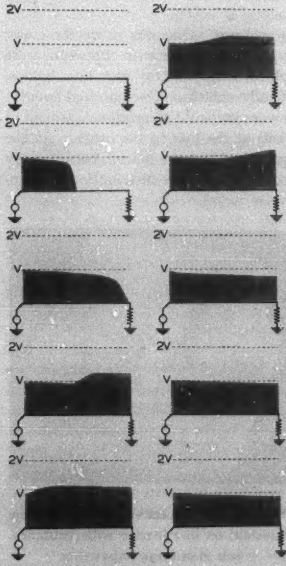


Figure 8 (extreme left). Direct-voltage waves on line loaded with resistance greater than surge impedance

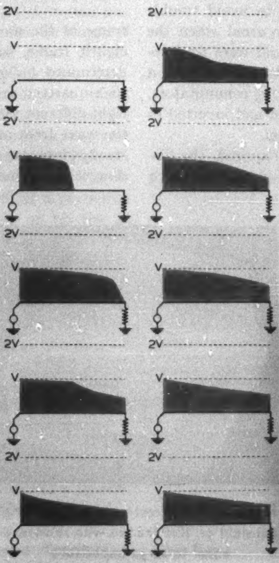
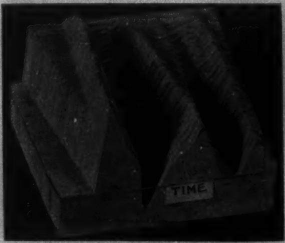


Figure 9 (left). Direct-voltage waves on line loaded with resistance less than surge impedance

Figure 10 (below). Direct-voltage switching transient on line with inductive load



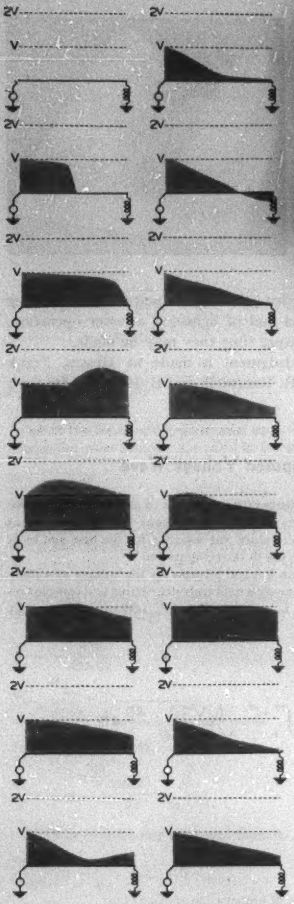


Figure 11 (left). Direct-voltage waves on line with inductive load

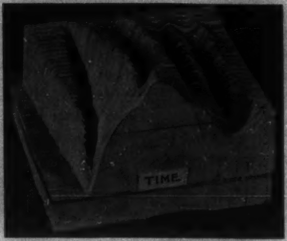


Figure 12 (above). Direct-voltage switching transient on line with capacitive load

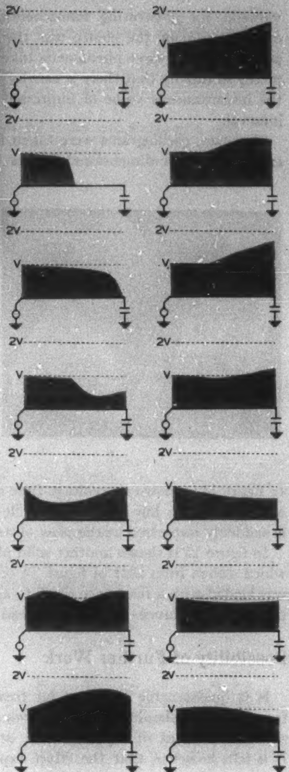


Figure 13 (right). Direct-voltage waves on line with capacitive load

quantity through the inductance, draining some of the charge off the line and reducing the voltage. The size of the inductance used as a load in these tests was approximately the same as that of the entire 250 miles of line—about one henry.

When the original wave has reflected from the load, and the reflection returned to the sending end, its sign becomes reversed, and when it travels down toward the load end again, its sign is negative. This negative wave reaches the load and undergoes a negative reflection, at a time when the voltage at and near the load has been greatly reduced by the continuously increasing current which is being drained away by the inductive load. As a result, the net voltage at the load and near it goes below zero and takes on a very considerable negative value.

Direct-Voltage Waves on Line With Capacitive Load

Another case treated was that of direct-voltage waves on a line with a termination of capacitance. The solid figure of voltage-time-distance variation for this case is shown in figure 12; and plots of distribution at different instants of time in figure 13. It will be observed that the initial reflection is similar to that on a short-circuited line, since the load end is maintained at zero voltage until a finite and sufficient time has elapsed to allow a finite quantity of charge to accumulate in the condenser. The action as this charge accumulates can be traced by the rise of the voltage behind the wave front of the first reflection as it makes its way back toward the sending end.

Alternating-Voltage Waves

No motion pictures have been made as yet of the alternating-voltage waves. It was felt that the comparative simplicity of the d-c phenomena made the d-c case more valuable for illustrating fundamental phenomena. Another reason for the choice was the fact that for most power lines the wave phenomena take place and are completed so rapidly that there is usually very little change in the instantaneous value of impressed voltage during the interval.

However, oscillograms were taken for a number of a-c cases, and three-dimensional figures built up as before.

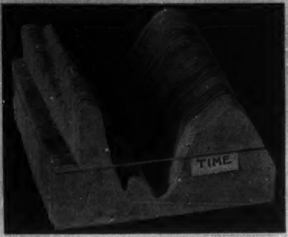


Figure 14. Alternating - voltage switching transient on open line

In figure 14 is shown the voltage-time-distance relation on a 250-mile open line when a 60-cycle-per-second voltage is suddenly impressed at the peak of its wave.

In figure 15 is shown another solid plot for the a-c case, which differs from that of figure 14 only in that the line was loaded with a resistance equal to approximately twice the surge impedance of the line, instead of being open.

Possibility of Further Work

It is questionable just how far treatment of different types of wave should be carried, owing to the very considerable amount of expense, time, and effort involved. It is felt, however, that the effort expended on the films so far produced has been well invested.

Additional oscillographic data have been taken of waves on a large lead-covered telephone cable which was generously lent for the purpose by Bell Telephone Laboratories, Inc. Tests were made without loading and with loading coils at the usual intervals, for direct current and for 1,000-cycles-per-second alternating current impressed. These data are in form sufficiently complete to form the basis for a number of traveling-wave films showing telephone-transmission phenomena, if they should be undertaken in the future.

If these films were to be made for the a-c case it appears that the best method of procedure in the preparation of the master distribution diagrams would be to work from the d-c distributions by applying the superposition formula,[4] using a mechanical method of integration. By doing this, various frequencies may be treated, as may the impressing of the voltage at different parts of the cycle, and even complex waves representing speech. Voltages of two widely separated frequencies, superposed and traveling along the line, would illustrate effects of amplitude and phase distortion.

It would undoubtedly be most interesting to see films of waves produced by lightning on power lines, showing the effect of corona and of lightning arrester operation. Perhaps at some future time they may be made.

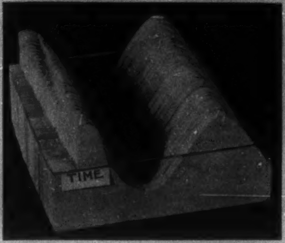


Figure 15. Alternating - voltage switching transient on line loaded with resistance greater than surge impedance

Grateful acknowledgment is made to Messrs. Frank H. Conant, Philip H. Ramsdell, and B. E. Battit for their parts in the work.

Appendix—Computed Voltage Wave

It was desired for comparative purposes to show a voltage wave computed on the basis of assumed constant line parameters, as already stated. The literature was searched for the best and most practicable formulas to use in the computation.

J. R. Carson[4] gives an integral formula for the voltage as a function of x and t, on a line with uniformly distributed and constant resistance R, inductance L, and capacitance C (leakage zero) per unit length, as follows:

$e(x,t) = 0$ for $vt < x$.

For $vt \geq x$,

$$e(x,t) = E\epsilon^{-A} + EA\int_A^T \frac{\epsilon^{-T_1} \cdot I_1(\sqrt{T_1^2 - A^2})}{\sqrt{T_1^2 - A^2}}\, dT_1$$

where

$$A = \frac{R}{2}\sqrt{\frac{C}{L}}\,x$$

$$T = \frac{R}{2L}t$$

$v = \dfrac{1}{\sqrt{LC}}$ = velocity of propagation

E = size of suddenly impressed d-c voltage

V. Bush[5] gives the following solution, following Heaviside[6]:

$$e(x,t) = E\epsilon^{-\rho t}\left[1 + \left(\frac{x\sigma^2}{2v} + \rho\right)\left(t - \frac{x}{v}\right) + \left(\frac{x^2\sigma^4}{2^2 2!^2 v^2} + \frac{x\sigma^2\rho}{2^2 v} + \frac{\rho^2}{2!}\right)\left(t - \frac{x}{v}\right)^2 + \ldots\right]$$

noting that it is useful for computation only when $(t - x/v)$ is small. In the above, the new symbols have the following definitions:

$$\rho = \frac{R}{2L} + \frac{G}{2C}$$

$$\sigma = \frac{R}{2L} - \frac{G}{2C}$$

It may be noted that when the leakage G is equal to zero, $\rho = \sigma = R/2L$.

Now a fairly simple formula is available for the computation of the *current* wave on a nonleaky line, and in view of the simplicity of the relation which exists between current and voltage, it was decided to attempt to determine the voltage by performing the necessary operations on the known current solution.

We have from Heaviside and may later writers the current-wave formula for the nonleaky line, as follows, for $x \lesssim vt$:

$$i = E\sqrt{\frac{C}{L}}\,\epsilon^{-\frac{Rt}{2L}} I_0\left[\frac{R}{2L}\sqrt{t^2 - \frac{x^2}{v^2}}\right] \tag{1}$$

In equation 1 I_0 designates the Bessel function of the first kind and zero order, for pure imaginary argument. Let the abbreviation Z be introduced, equal to

$$Z = \frac{R}{2L}\sqrt{t^2 - \frac{x^2}{v^2}} = \frac{R}{2Lv}\sqrt{(vt - x)(vt + x)} \tag{2}$$

so that (1) simplifies to

$$i = E\sqrt{\frac{C}{L}}\,\epsilon^{-\frac{Rt}{2L}} I_0(Z) \tag{3}$$

One of the fundamental differential equations upon which transmission-line wave solutions are based is, for zero leakage,

$$\frac{\partial e}{\partial t} = -\frac{1}{C}\frac{\partial i}{\partial x} \tag{4}$$

The value of i given in equation 3 will be substituted into equation 4, and then solution for e will be made. Throughout the following, it is assumed that $0 < x < vt$. It is known of course that for $x > vt$ the voltage $e = 0$. It is assumed that we are dealing with an infinite line, although of course this solution is applicable to finite lines by proper handling of the reflections.

$$\frac{\partial Z}{\partial x} = -\frac{R^2x}{4L^2v^2Z} \tag{5}$$

$$\frac{\partial Z}{\partial t} = \frac{R^2t}{4L^2Z} \tag{6}$$

From (3), (4), and (5),

$$\frac{\partial e}{\partial t} = E\,v\epsilon^{-\frac{Rt}{2L}}\frac{\partial I_0(Z)}{\partial Z}\cdot\frac{R^2x}{4L^2v^2Z} \tag{7}$$

We have the following recurrence formula[7] for Bessel functions:

$$\frac{\partial I_n(Z)}{\partial Z} = \frac{nI_n(Z)}{Z} + I_{n+1}(Z)$$
$$= I_{n-1}(Z) - \frac{nI_n(Z)}{Z} \tag{8}$$

From the resemblance which equation 7 bears to $\partial i/\partial t$ it can be seen that by adding and subtracting the quantities in the brackets below, a portion of the expression can be made a perfect differential with respect to t.

$$\frac{\partial e}{\partial t} = E\epsilon^{-\frac{Rt}{2L}}\frac{\partial I_0(Z)}{\partial Z}\frac{R^2x}{4L^2vZ} + E\left[\epsilon^{-\frac{Rt}{2L}}\frac{\partial I_0(Z)}{\partial Z}\frac{R^2(vt-x)}{4L^2vZ} - \epsilon^{-\frac{Rt}{2L}}\frac{\partial I_0(Z)}{\partial (Z)}\frac{R}{2L}\sqrt{\frac{vt-x}{vt+x}} + \frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}I_0(Z) - \frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}I_0(Z)\right]$$
$$= E\left[\epsilon^{-\frac{Rt}{2L}}\frac{\partial I_0(Z)}{\partial Z}\frac{R^2t}{4L^2Z} - \frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}I_0(Z)\right] - E\epsilon^{-\frac{Rt}{2L}}\frac{\partial I_0(Z)}{\partial Z}\frac{R}{2L}\sqrt{\frac{vt-x}{vt+x}} + E\frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}I_0(Z) \tag{9}$$

By the recurrence formula, equation 8, it is seen that $\partial I_0(Z)/\partial Z$ in the second term of equation 9 may be replaced by $I_1(Z)$. We have further the general relation that

$$\int u\,v\,dw = u\,v\,w - \int u\,w\,dv - \int v\,w\,du \tag{10}$$

which it is convenient to apply to this same second term, with

$$u = \frac{\partial I_0(Z)}{\partial Z} \text{ or } I_1(Z)$$

$$v = \sqrt{\frac{vt-x}{vt+x}}$$

$$dw = -\frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}dt$$

The entire expression for $\frac{\partial e}{\partial t}$ is then integrated with respect to t, giving:

$$e = f(x) + E\epsilon^{-\frac{Rt}{2L}}I_0(Z) + E\epsilon^{-\frac{Rt}{2L}}I_1(Z)\sqrt{\frac{vt-x}{vt+x}} + E\int\left\{-\epsilon^{-\frac{Rt}{2L}}\frac{\partial I_1(Z)}{\partial Z}\frac{R^2t}{4L^2Z}\sqrt{\frac{vt-x}{vt+x}} - \epsilon^{-\frac{Rt}{2L}}I_1(Z)\frac{vx}{(vt+x)^2}\times\sqrt{\frac{vt+x}{vt-x}} + \frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}I_0(Z)\right\}dt \tag{11}$$

Using the recurrence formula (8) again and substituting for $\partial I_1(Z)/\partial Z$ and $I_0(Z)$ there results, for the integral term:

$$e = \ldots. + E\int\left\{-\epsilon^{-\frac{Rt}{2L}}I_1(Z)\frac{R^2t}{4L^2Z^2}\sqrt{\frac{vt-x}{vt+x}} - \epsilon^{-\frac{Rt}{2L}}I_2(Z)\frac{R^2t}{4L^2Z}\sqrt{\frac{vt-x}{vt+x}} - \epsilon^{-\frac{Rt}{2L}}I_1(Z)\frac{vx}{(vt+x)^2}\sqrt{\frac{vt+x}{vt-x}} + \frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}\left[I_2(Z) + \frac{2I_1(Z)}{Z}\right]\right\}dt \tag{12}$$

Substituting in (12) the value of Z from (2), and simplifying, there results:

$$e = \ldots. + E\int\left\{-\epsilon^{-\frac{Rt}{2L}}I_1(Z)\left[\frac{v^2t}{(vt+x)^{3/2}(vt-x)^{1/2}} + \frac{vx}{(vt+x)^{3/2}(vt-x)^{1/2}} - \frac{2v}{(vt+x)^{1/2}(vt-x)^{1/2}}\right] + \epsilon^{-\frac{Rt}{2L}}I_2(Z)\left[\frac{R}{2L} - \frac{R\,vt}{2L(vt+x)}\right]\right\}dt \tag{13}$$

The first two terms of the first bracket in (13) have a common denominator, and together they equal half the negative value of the third term. These terms come from the term in equation 11 containing $\partial I_1(Z)/\partial Z$ and the succeeding term. If the signs of these terms in (11) are reversed, the result is a doubling of the two terms referred to in (13), which will cause the entire first bracketed expression to become zero. In order to compensate for this change in sign, we must add

$$E\epsilon^{-\frac{Rt}{2L}}I_1(Z)\sqrt{\frac{vt-x}{vt+x}}$$

to the integral portion of e, as may be seen by inspection of (10) and

(11). We then have:

$$e = f(x) + E\epsilon^{-\frac{Rt}{2L}} I_0(Z) + 2E\epsilon^{-\frac{Rt}{2L}} I_1(Z)\sqrt{\frac{vt - x}{vt + x}} +$$

$$E\int\left\{\frac{R}{2L}\epsilon^{-\frac{Rt}{2L}} I_2(Z)\left[1 - \frac{2vt}{vt + x}\right] + \right.$$

$$\left.\epsilon^{-\frac{Rt}{2L}}\frac{\partial I_0(Z)}{\partial Z}\frac{R}{2L}\sqrt{\frac{vt - x}{vt + x}}\right\}dt$$

$$= \ldots + E\int\left\{-\frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}\left(\frac{vt - x}{vt + x}\right) I_2(Z) + \right.$$

$$\frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}\sqrt{\frac{vt - x}{vt + x}}\, I_2(Z)\ dt \qquad (14)$$

Now we may take the integral term at the end of equation 14 and operate upon it in a manner similar to that used in integrating and reducing the last two terms of equation 9. There results:

$$E\int\left\{-\frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}\left(\frac{vt - x}{vt + x}\right) I_2(Z) + \frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}\sqrt{\frac{vt - x}{vt + x}}\, I_1(Z)\right\}dt =$$

$$2E^{-\frac{Rt}{2L}}\left(\frac{vt - x}{vt + x}\right) I_2(Z) + E\int\left\{-\frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}\left(\frac{vt - x}{vt + x}\right)^{3/2} I_3(Z) + \right.$$

$$\left.\frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}\left(\frac{vt - x}{vt + x}\right) I_2(Z)\right\}dt \qquad (15)$$

It is seen that each integration results in another integral of the same character but of order one higher, and a by-product of an additional term outside the integral. The evaluation of the integral may be generalized for the *n*th term as follows:

$$E\int\left(\frac{vt - x}{vt + x}\right)^{n/2}\frac{R}{2L}\epsilon^{-\frac{Rt}{2L}}\left[-I_{n+1}(Z)\left(\frac{vt - x}{vt + x}\right)^{1/2} + I_n(Z)\right]dt =$$

$$2E\epsilon^{-\frac{Rt}{2L}}\left(\frac{vt - x}{vt + x}\right)^{\frac{n+1}{2}} I_{n+1}(Z) + E\int\left(\frac{vt - x}{vt + x}\right)^{\frac{n+1}{2}} \times$$

$$\frac{R\epsilon^{-\frac{Rt}{2L}}}{2L}\left[-I_{n+2}(Z)\left(\frac{vt - x}{vt + x}\right)^{1/2} + I_{n+1}(Z)\right]dt \qquad (16)$$

The relation expressed in (16) was obtained by induction, but a direct proof of it may be had by differentiating it with respect to *t*.

The voltage wave may be expressed:

$$e(x, t) = f(x) + E\epsilon^{-\frac{Rt}{2L}} I_0(Z) +$$

$$2E\epsilon^{-\frac{Rt}{2L}}\sum_{n=1}^{\infty} I_n(Z)\left(\frac{vt - x}{vt + x}\right)^{n/2} \qquad (17)$$

If the line is initially uncharged, $f(x)$ is equal to zero. There is a discontinuity at $x = vt$, and equation 17 indicates a wave front (which exists where this condition holds) of magnitude

$$E\epsilon^{-\frac{Rt}{2L}}$$

The magnitude of the current wave front, found by substituting $x = vt$ in equation 3, is

$$E\sqrt{C/L}\,\epsilon^{-\frac{Rt}{2L}}$$

The wave front is advancing at a velocity $v = 1/\sqrt{LC}$, and it is seen that this flow of current is just sufficient to charge up the line at this rate and to the magnitude indicated as being required by the expression for the voltage wave front. The voltage solution is therefore compatible with the current solution at the point of discontinuity.

For $x > vt$, the voltage is everywhere zero, since no charge has yet been brought to that part of the line by the oncoming current wave.

Let the new symbol *s* be introduced, defined as:

$$s = \sqrt{\frac{vt - x}{vt + x}} \qquad (18)$$

Then the expression for the original voltage wave, due to a direct voltage *E* suddenly impressed on a transmission line with constants *R*, *L*, and *C*, is:

$$e = E\epsilon^{-\frac{Rt}{2L}}\{I_0(Z) + 2sI_1(Z) + 2s^2I_2(Z) + 2s^3I_3(Z) + \ldots)\} \qquad (19)$$

where

$$I_n(Z) = \frac{Z^n}{2^n n!}\left\{1 + \frac{Z^2}{2^2 1!(n + 1)} + \frac{Z^4}{2^4 2!(n + 1)(n + 2)} + \ldots\right\} \qquad (20)$$

In a similar manner, the more complicated case of the line with all four constants *R*, *L*, *C*, and *G* may be worked out in terms of Bessel functions. The result is:

$$\begin{aligned}e = E\epsilon^{-\rho t}\{&I_0(Z) + 2rsI_1(Z) + (4r^2 - 2)s^2I_2(Z) + \\ &(8r^3 - 6r)s^3I_3(Z) + (16r^4 - 16r^2 + 2)s^4I_4(Z) + \\ &(32r^5 - 40r^3 + 10r)s^5I_5(Z) + (64r^6 - 96r^4 + 36r^2 - 2)s^6I_6(Z) + \\ &(128r^7 - 224r^5 + 112r^3 - 14r)s^7I_7(Z) + \\ &(256r^8 - 512r^6 + 320r^4 - 64r^2 + 2)s^8I_8(Z) + \ldots.\}\end{aligned} \qquad (21)$$

In the above, the symbols have all already been defined except *r*, which is defined as $\frac{\rho}{\sigma}$, and *Z* is made equal to

$$\sigma\sqrt{t^2 - \frac{x^2}{v^2}}$$

Equation 19 is the one which was used in the computation of the voltage distributions for the motion picture.

A table of Bessel functions of orders from 0 to 11, and covering the range of argument required, is available in the Report of the British Association for 1896.

In the artificial line used, which was made up of 32 sections of the single-phase line described in Dellenbaugh's paper,[2] the constants were:

	Per Section	32 Sections
Resistance	7.9 ohms	252.8 ohms
Inductance	0.0326 henry	1.043 henries
Capacitance	0.0565 microfarad	1.808 microfarads
Equivalent length	7.87 miles	251.84 miles

The computation for voltage distribution was carried out for a range of *x* and *vt* between 0 and 12 times the length of the line (0 and 384 sections), at intervals of one quarter of the length or eight sections. A few of the early results are given in table I, which is reproduced only in small part on account of the large size of the complete table.

For the open-line case, the reflection from the end is complete and without change of sign. To illustrate the numerical computation, let $vt = 40$ sections, or one and one-fourth times the length of line. Then the section of the wave indicated in table I as lying between $x = 32$ and $x = 40$ has actually turned around and gone back from 32 to 24. The wave front is at section 24 and moving toward the generator. The distribution is

x(section) =	0	8	16	24	32
e/E	= 1.000	0.962	0.925	0.887 1.699	1.700

The first three figures and the upper figure under section 24 are

Table I. Values of Computed Voltage e(x,t) per Volt Impressed

vt (Sections) \ x (Sections)	0	8	16	24	32	40	48
0	1.000	0	0	0	0	0	0
8	1.000	0.959	0	0	0	0	0
16	1.000	0.960	0.920	0	0	0	0
24	1.000	0.9605	0.921	0.883	0	0	0
32	1.000	0.9613	0.923	0.885	0.847	0	0
40	1.000	0.962	0.925	0.887	0.850	0.812	0
48	1.000	0.963	0.926	0.889	0.852	0.816	0.779

taken directly from table I. The other entry under section 24 is the sum of the table I entries for sections 24 and 40. The entry under section 32 is twice the corresponding entry in table I. The two entries under section 24 represent the values of the wave at the top and bottom of the wave front there.

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Discussion

John Riordan (nonmember; Bell Telephone Laboratories, Inc., New York, N. Y.): I wish to present a verification of the expansion given as equation 21 in the appendix to Mr. Woodruff's paper; this is the general result of the development in the appendix. I should explain at once that the process I use is not a substitute for Mr. Woodruff's original method; given the form of the expansion by Mr. Woodruff, it will determine the coefficients and verify the form, but it requires either this advance information or an act of inspiration to be complete. However, I am able by its means to give a generating function, a literal formula, and a recurrence formula for the coefficients of the general expansion, equation 21, thus completing Mr. Woodruff's work. The coefficients turn out to be simply related to the Tchebycheff polynomials.

Using the notation of the paper with a slight modification, the expansion to be verified is as follows:

$$e = E\sum_{n=0}^{\infty} C_n(r)s^nI_n(\sigma z)\exp(-\rho t)$$

where

$C_n(r)$ = function of r and n which is the coefficient of the nth term in equation 21.

$r = \rho/\sigma$

$z = [t^2 - x^2/v^2]^{1/2}$

Using the Campbell and Foster table of Fourier integrals (Bell Telephone System Technical Publications, Monograph B-584), by pair 870.5:

$$\mathfrak{M}[s^nI_n(\sigma z)\exp(-\rho t)] = \frac{(2\sigma)^n\exp\left[\frac{-x}{v}\sqrt{(p+\rho)^2-\sigma^2}\right]}{\sqrt{(p+\rho)^2-\sigma^2}\,[\sqrt{p+\rho+\sigma}+\sqrt{p+\rho-\sigma}]^{2n}}$$

where the script $\mathfrak{M}$ is to be read "mate of" or "Fourier transform of."

Hence the original expansion may be written

$$\mathfrak{M}(e) = E\frac{\exp\left[\frac{-x}{v}\sqrt{(p+\rho)^2-\sigma^2}\right]}{\sqrt{(p+\rho)^2-\sigma^2}}\sum_{n=0}^{\infty} C_n(r)u^n$$

where

$$u = \frac{2\sigma}{[\sqrt{p+\rho+\sigma}+\sqrt{p+\rho-\sigma}]^2} = \frac{\sigma}{p+\rho+\sqrt{(p+\rho)^2-\sigma^2}}$$

But the mate of e is known otherwise (Campbell and Foster, l.c. table II, case 3) as:

$$\mathfrak{M}(e) = \frac{E}{p}\exp\left[\frac{-x}{v}\sqrt{(p+\rho)^2-\sigma^2}\right]$$

Hence $C_n(r)$ is determined, assuming the form of the expansion is correct, by:

$$\sum_{n=0}^{\infty} C_n(r)u^n = \frac{\sqrt{(p+\rho)^2-\sigma^2}}{p}$$

If the right-hand side may be expressed as a function of u and r the expansion is correct and the expression so obtained is the generating function of $C_n(r)$. Writing $W = \sqrt{(p+\rho)^2-\sigma^2}$, for convenience, this may be done as follows:

$$\begin{aligned}\frac{W}{p} &= \frac{W(p+W)}{p(p+W)}\\ &= \frac{p(p+W)+2\rho p+\rho^2-\sigma^2}{p(p+W)}\\ &= 1+\frac{2\rho}{p+W}+\frac{(\rho^2-\sigma^2)(p+W)}{p(p+W)^2}\\ &= 1+\frac{2\rho}{p+W}+\frac{\rho^2-\sigma^2}{(p+W)^2}+\frac{W}{p}\,\frac{\rho^2-\sigma^2}{(p+W)^2}\end{aligned}$$

Hence, noting that $p + W = -\rho + \sigma/u$:

$$\frac{W}{p}\left[1+\frac{(1-r^2)u^2}{(1-ru)^2}\right] = 1+\frac{2ru}{1-ru}-\frac{(1-r^2)u^2}{(1-ru)^2}$$

or

$$\frac{W}{p} = \frac{1-u^2}{1-2ru+u^2}$$

which is the required generating function expression.

But the Tchebycheff polynomials $T_n(r)$ (Courant and Hilbert, Methoden der Math. Physik, Springer, Berlin, 1931, pages 75 and 76) defined by

$$T_n(r) = \frac{1}{2^{n-1}}\cos[n\cos^{-1}r]$$

have the generating identity:

$$\frac{1-x^2}{1-2rx+x^2} = \sum_{n=0}^{\infty} T_n(r)(2x)^n$$

Hence

$$C_n(r) = 2^nT_n(r) = 2\cos[n\cos^{-1}r]$$

and for $n \geq 2$:

$$T_{n+1}(r) = rT_n(r) - \frac{1}{4}T_{n-1}(x)$$

$$C_{n+1}(r) = 2rC_n(r) - C_{n-1}(r)$$

the last of which is the recurrence relation for the coefficients. For convenience of comparison with Mr. Woodruff's result, the first few coefficients are as follows:

$C_0(r) = 1$
$C_1(r) = 2r$
$C_2(r) = 4r^2 - 2$
$C_3(r) = 8r^3 - 6r$

The first three follow from the literal formula, the last from the recurrence formula; they may also be obtained by simple long division development of the generating function.

In the special case $G = 0$, that is $\rho = \sigma$ and $r = 1$, the nth coefficient is

$$C_0(1) = 1$$

$$C_n(1) = 2n \mp 0$$

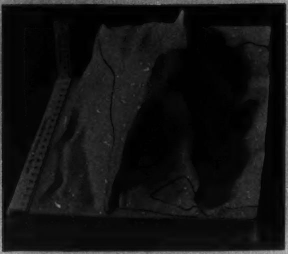
which verifies equation 19. This result follows immediately either from the generating function or the formula for $C_n(r)$.

I am indebted to my colleague S. O. Rice for pointing out the connection of the coefficients with the Tchebycheff polynomials.

L. V. Bewley (General Electric Company, Pittsfield, Mass.): Professor Woodruff has shown two very interesting and instructive methods of teaching transmission line transients by means of motion pictures and voltage-distance-time models. These should prove of great value to schools in which traveling wave phenomena is taught; because it gives the student a perfect visual picture of what is going on during the travel and reflection of a wave.

Along about 1929 we calculated the wave shapes for lightning surges and worked out the reflections for a number of terminal conditions, ("Traveling Waves Due to Lightning," AIEE TRANSACTIONS, volume 48, 1929). At that time F. W. Peek, Jr., was directing the lightning research at Pittsfield and he asked J. T. Lusignan, Jr., to supervise the making of an animated motion picture based on the calculated wave shapes. This film (General Electric *AX-353-B*) was shown by Mr. Peek before many different AIEE Sections throughout the country. It depicts the formation of a charged cloud; the lightning stroke; the formation and development of the traveling waves; the attenuation due to corona; reflections from open and grounded terminals and from a resistor equal to the surge impedance of the line, also from transformer terminals and series inductances with and without shunt resistors; and finally the internal transient oscillations taking place inside a transformer winding and their elimination by means of shields.

Figure 1



Several years ago D. C. Morgan at Pittsfield conceived the idea of illustrating the internal transient of a transformer by a topographical voltage-distance-time model, just as Professor Woodruff has now done for transmission lines. Figure 1 is a photograph of Morgan's plaster-cast model.

Discussions

of AIEE Technical Papers Published Before Discussions Were Available

ON THIS and the following 21 pages appear discussions submitted for publication, and approved by the technical committees, on previously published papers presented at the AIEE winter convention, New York, N. Y., January 24–28, 1938. Authors' closures, where they have been submitted, will be found at the end of the discussion on their respective papers.

Alternator Short-Circuit Currents Under Unsymmetrical Terminal Conditions

Discussion of a paper by A. R. Miller and W. S. Weil, Jr., published on pages 1268–76 of volume 56, 1937, AIEE TRANSACTIONS (October 1937 issue of ELECTRICAL ENGINEERING) and presented for oral discussion at the electrical machinery session of the winter convention, New York, N. Y., January 27, 1938.

F. H. Pumphrey (Rutgers University, New Brunswick, N. J.): I have been very much interested in the effort of Professor Miller and Mr. Weil to digest the work which has been done in machine transients and to present it in somewhat simpler form. The necessity of conforming with the Institute publication standards has prevented them from making a complete statement of assumptions. It thus becomes rather difficult to follow some of the steps and so the paper loses some of its advantage of completeness as well as simplicity.

In this connection, I have been unable to interpret physically the very low frequency element of armature current given on page 1269 I wonder if it might not be obtained equally accurately by a constant term with a different decrement? I should appreciate any help the authors could give on this interpretation.

Another question has arisen with respect to the change of resistance and reactance of the field circuit of the machine used as an illustration, between the first and second papers. Could the authors state the basis of this change?

D-C Machine Stray-Load-Loss Tests

Discussion and author's closure of a paper by Victor Siegfried published on pages 1285–9 of volume 56, 1937, AIEE TRANSACTIONS (October 1937 issue of ELECTRICAL ENGINEERING) and presented for oral discussion at the electrical machinery session of the winter convention, New York, N. Y., January 27, 1938.

R. F. Franklin and **T. M. Linville** (General Electric Company, Schenectady, N. Y.): Stray-load losses of d-c machines are of particular interest to designers because by their reduction in value increased efficiency and decreased heating are obtained. Every effort, therefore, should be made to measure this loss as accurately as possible and to determine its manner of variation.

Designers have recognized the existence of stray-load loss for a number of years and many attempts have been made to evaluate its magnitude both by test and calculation. Unfortunately the nature of stray-load loss is such as to preclude its direct measurement. The usual method of measurement is to measure the total losses under load and from this measurement subtract all of the known losses which can be measured or calculated either directly or indirectly. This difference is called the *stray-load loss*. But some of the known losses cannot always be determined very accurately and vary in value from time to time. The errors thus introduced together with the inherent inaccuracies of testing methods, have lead to considerable difficulty in determining the stray-load loss.

Because of these difficulties, the AIEE Standards have devised a conventional segregated-loss method of evaluating these losses in which arbitrary methods of determining stray-load loss, brush-friction loss, brush-contact loss, and I^2R losses are given. These methods of determining these losses are, of course, to some degree in error, and attention was given to the possible error at the time the methods were devised. It is possible that with present better knowledge of these losses or possibly by some further investigation other methods could be devised that would give a more accurate determination of these losses in the majority of cases. It should be borne in mind, however, that the present method serves as a very practical and convenient method of determining individual losses which makes possible the approximate calculation of total losses without excessive cost to the trade.

On the other hand, it is also very desirable to have a more accurate method of determining these losses, even though the method may be more expensive and require more elaborate testing equipment. Such a method should prove of particular value to designers as the division of losses is of particular interest to them. It is probable that such a method would be too costly and impractical for general use. It would, however, afford a means of improving designs and furnishing data for a modification of the more convenient segregated-loss method which would make that method more accurate without increasing its cost and practicability. The booster-pump-back method described in these papers seems the most practical method yet devised. Its use is greatly handicapped by the requirement of duplicate units as it would be difficult to have machines calibrated for load loss available at all times. Nevertheless, an intensive study of this method in order to simplify the test procedure, find methods of increasing the accuracies of meter readings and reduce their number to a minimum and a determination of its limitations might be profitable. There are also certain inaccuracies in the determination of the various losses when using this method that should receive further study.

One of the most difficult losses to measure accurately is that of the brush losses. It is well established that the brush friction loss is sometimes reduced 50 per cent or more between no load and full load. Yet the brush friction loss is measured at no load and assumed to be the same under load. Because of this assumption the stray-load loss may actually measure negative at light values of load. It should, therefore, be recognized that under the present method of testing for stray-load loss that it contains a component of brush-friction even though this component may have a negative value.

Another loss that is difficult to measure is that of the brush-drop loss. Because of the uneven current distribution in the brush and the uncertainties of voltage measurements made on a moving commutator the actual brush-drop loss may be quite different from the loss measured. The difference between the two should be recognized as a component of stray-load loss.

A third uncertain loss measurement is that of the armature copper I^2R loss. This loss cannot be directly measured and so must be determined indirectly as accurately as possible. Since the magnitude of this loss is greatly influenced by the temperature of the armature winding, one of the problems is to accurately measure this temperature or else maintain it at the time the loss is measured. Either of these is difficult to accomplish and the inaccuracies introduced by the determination of this loss should also be recognized as a component of the stray-load loss.

The above discussion leads to the important conclusion that the stray-load loss cannot be measured without at the same time specifying how the other losses are to be determined. In those cases where the other losses cannot be accurately measured under the exact conditions of load at a given time, provision must be made to either measure them in a definite manner at no load under arbitrary conditions or to specify a definite manner for their calculation. In either case it should be recognized that any inaccuracies in the determination of any of the other losses will have an influence on the magnitude of the stray-load loss.

In analyzing the data given in the papers attention should be called to the distinction between the per cent stray-load-loss values given as compared to the one per cent stray-load loss specified in the AIEE Standards. The one per cent value is established in conjunction with the calculation of brush friction as a function of commutator speed, allowance of a definite value of volts brush drop and calculation of the armature I^2R loss at 75 degrees centigrade. The stray-load loss determined in the papers was not obtained in this manner. It is incorrect, therefore, to use the stray-load loss as measured in the papers as a basis for judging the conventional allowance of one per cent.

Commercially, only total losses are important and there should be little argument against the method of measuring the total losses described in the papers. Both papers recommend a booster for circulating current between the machines being tested. Unfortunately, the booster must carry the full current of the machines at a comparatively low voltage. Such a machine is not likely to be always available. It would be less expensive and more practical to adopt the conventional pump-back test as a sufficiently accurate determination of the total losses. This may be done entirely apart from a consideration of stray-load losses.

In conclusion, it appears that further study should be given to the measurement and calculation of the known as well as the stray-load losses. A better knowledge of these losses will not only be helpful to the designer in obtaining improved machines but will be helpful in increasing the accuracy of determining the total losses by the convenient segregated-loss method. The present papers show there is interest in the problem and have contributed valuable test data and presented improved methods of test. It is hoped further interest can be aroused in this subject so as to stimulate further investigation and progress.

E. W. Schilling (Michigan College of Mining and Technology, Houghton) and **R. J. W. Koopman** (University of Kansas, Lawrence): It is gratifying to know that at least one other party has answered the Institute's request that research work be started and papers presented on stray-load losses in d-c machines. Professor Siegfried has made tests using the so-called Kapp method and also using the Hutchinson test in which the two machines to be tested are direct connected and arranged in a pump-back connection using a booster to circulate the current. This last test is very similar to one of those used by us.

Referring to Professor Siegfried's figure 2, page 1286, Electrical Engineering for October 1937 [AIEE Transactions, volume 56] it is interesting to make a comparison of his method and our method of allocating the losses to the individual machines. Since we plot loss in watts as ordinates against average armature current as abscissae for the two machines, it is necessary to replot his curve *A* as shown in figure 1. Since his data plotted on logarithmic paper produces a straight line as shown in figure 2, the curve can be expressed by an equation of the form $L_s = kI_a^n$. The exponent is found to be 1.255. Now consider a point on his curve at 160 amperes. The distribution of current between the two machines is not given but for our purpose here we will consider $I_g = 75$ amperes and $I_m = 85$ amperes. The loss for one machine, at 75 amperes for example, will be

$$L_g = L_s \frac{I_g^n}{I_m^n + I_g^n} = 500 \frac{\overline{75}^{1.255}}{\overline{85}^{1.255} + \overline{75}^{1.255}} = 242 \text{ watts at 75 amperes}$$

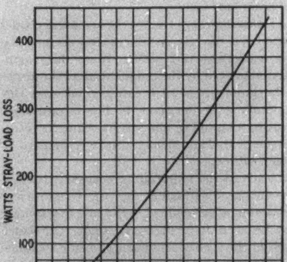


Figure 1. Curve A of Siegfried's figure 2 replotted to show average armature current for the two machines

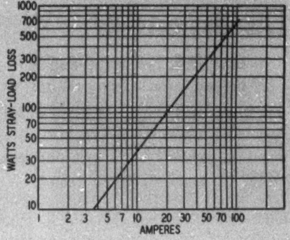


Figure 2. Determination of the exponent n

An interesting portion of Professor Siegfried's paper is the application of paraffin to the commutator to reduce friction loss. We can readily agree that some such procedure would be desirable, provided that the contact drop would be unaffected. However, it is hard to believe that such would be the case. Test data taken on slip rings with and without paraffin should be made to determine whether or not this very valuable aid to the technique of making stray-load loss tests may be safely used under all conditions.

In our tests brush drop was not taken at standstill. Contact drop measured at standstill will not under all conditions be the same as when the machine is running. The difference may be so small that it can safely be neglected. Contact drop curves taken at standstill and at various speeds should be valuable in determining whether this simplification in the test procedure can be used without too great loss of accuracy.

In the last paragraph of "Test of Machines," page 1288 of Professor Siegfried's paper, it appears that the no-load loss was measured when the two machines were running as motors taking the same armature current. The booster was then included in the circuit and the load current increased. A final reading was then taken of the no-load loss. The exact procedure followed here is not perfectly clear. According to the test code, paragraph, 2.112, the core loss shall be taken "when excited to produce a voltage at the terminals corresponding to the calculated internal voltage." We presume that this was the purpose of the second reading. However, if such was the case, what was the purpose of the first reading?

Edward Hughes (Technical College, Brighton, England): The results discussed in this paper are very useful in showing the close agreement between the Kapp and the Hutchinson methods. Their value, however, would have been greatly enhanced if the author had analyzed the reliable data collected from the various tests in a manner that would indicate the fundamental relationships between the stray loss and the factors which affect its magnitude. In a paper published in the *Journal* of the Institution of Electrical Engineers, volume 63, 1924, pages 35–50, I investigated the effect of flux distortion on iron loss by measuring the iron loss in an armature with its winding on open circuit, the flux being distorted by passing current through a compensating winding. Figure 3 shows the results obtained at 2,000 rpm. Similar curves were obtained for speeds varying between 200 and 2,400 rpm. It was found that over this range of speed and for cross ampere-turns varying between zero and those corresponding to full-load and for field excitation ranging from zero to about 50 per cent above the normal excitation, the total iron

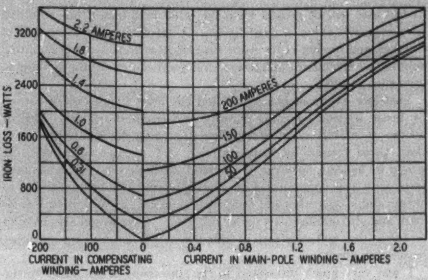


Figure 3

loss could be represented very closely by the following simple expression:

$$kf^{1.5}\{V_t(B_1^2 + B_2^2) + V_aB_a^2\}$$

where

k = a constant derived for undistorted flux
f = frequency in cycles per second
V_t = total volume of teeth
B_1 = mean flux density in tooth under strengthened pole tip
B_2 = ditto under pole tip at which the flux is reversed (if flux is not reversed, this term is omitted)
V_a = volume of armature core carrying flux
$B_a = \dfrac{\Phi + 2\Phi_c}{A}$
Φ = resultant flux per pole
Φ_c = cross flux (if any) re-entering pole shoe
A = sectional area per pole of magnetic circuit through armature core

The iron loss in armature teeth due to the flux of the commutating pole can be calculated from a similar formula.

The above expression has since been confirmed by R. G. Isaacs (*J.I.E.E.*, volume 69, page 1303). It is seen from the above expression that the stray iron loss is an integral part of the total iron loss. Consequently, if the stray loss is expressed as a separate quantity, it is impossible to relate it satisfactorily to the factors upon which it is dependent.

It is hardly necessary to emphasize that it is just as important to be able to predetermine the stray iron loss as it is to measure this loss after the machine has been constructed; and I would express the wish that the author should again analyze his results either to confirm the above expression or to suggest another that may fit in more satisfactorily with his own data.

T. H. Morgan (Worcester Polytechnic Institute, Worcester, Mass.): These papers are important and valuable in that they show that stray-load loss in d-c machines can be measured to a high degree of accuracy. The authors of both papers are to be complimented on their splendid painstaking work. Old and well-established methods are here applied to determine a loss which only recently has become recognized as one which cannot be ignored.

The results described in these papers indicate that we are well on our way in the matter of being able to definitely specify a simple straightforward method of determining stray-load loss in d-c machines. The load-back method is desirable because it gives a measurement of the loss with the machines under actual operating conditions. I am of the opinion that using boosters or other machines to supply certain losses separately and thus keep both machines under test in exactly similar conditions of load and field current is not necessary.

It is most desirable that a code method be as simple as possible without sacrificing the required degree of accuracy. It seems to me that these papers indicate that a test might be used in which one single voltage is applied to the machines operating in parallel on the same shaft with load conditions controlled by adjustment of the field currents. With this condition existing we would have the machines operating with slightly different armature currents and also with different field currents. The apportioning to each machine of the correct amount of its loss due to different armature currents can be conveniently and accurately taken care of by the method devised by Paul M. Narbutovskih for similar use in induction machines. If the law of variation of the stray-load loss as a function of saturation were sufficiently well established so that correct apportioning due to difference in field currents could be accomplished the problem would be solved and the whole matter of testing greatly simplified. The information given on this point in the papers would seem to indicate that for a given armature current the loss variation as a function of saturation might prove to be a very simple law throughout the range required for the test. Possibly the assumption of a straight line relationship would give the required degree of accuracy. Application of this law might be considerably simplified if one machine were over-excited to the same degree that the other was under-excited and the average excitation of the two similar machines used as the field current for this case. Further work along this line would be of great help in determining the accuracy of this proposal.

Coming back to the question of the test code I have never been able to understand a philosophy that says we will set up a code which will specify methods of testing for all the losses in a machine so that we may determine an accurate value of each, and then goes on to arbitrarily allot one loss as a fixed percentage of the input. I do not think that such procedure can be justified as being logical, scientific, or even practical, for in a matter of this kind practicality requires a fair degree of accuracy and not simply arbitrary action. It would seem that the authors of these papers have helped very materially by showing that it is possible to specify a definite method for the accurate measurement of stray-load loss in d-c machines.

L. E. Miller (Reliance Electric and Engineering Company, Cleveland, Ohio): I feel that Mr. Siegfried is to be complimented on the clearness of his paper. In view of the clearness and conciseness with which he has condensed the facts which might very easily cover many times the space of his paper, it seems like any discussion on the subject might be construed as near carping. I would like, however, to make some comments.

I believe that Mr. Siegfried has placed his finger upon the cause of considerable apparent inaccuracies which occur in many tests, when he attributed to the variability in the performance of the commutator. The very wide change in brush friction is very well brought out by Doctor S. W. Glass in his paper on "Measurement of Frictional Characteristics of Brushes," which was published in the July 1937 issue of *Iron and Steel*. Mr. Glass brings out in this article the possibility of a three-to-one change in brush friction for certain changes of temperature which are well within the range of a commutator temperature, and also a possibility of a variation as great as three-to-one occurring from sanding the commutator. He also brings out various rather wide ranges for various other phenomena which he classes as the effect of commutator poisons. In view of Doctor Glass' paper, I am forced to wonder as to whether it is not possible for paraffin, as mentioned by Mr. Siegfried, to cause certain inaccuracies which are not brought out into normal operation of the machine.

Siegfried also brings out the necessity of measuring the complete terminal to terminal drop, rather than assuming a constant value of brush contact drop. Presumably he makes this measurement while the machine is stationary. This brings up the question as to whether the stationary drop approaches any more closely the actual drop occurring while the machine is operating than the assumed value given under AIEE specifications. There is no doubt in my mind that the assumed value is inaccurate, but I question as to whether the stationary drop is not just as inaccurate.

Another point brought up in Mr. Siegfried's paper is the suggestion of leaving out series winding, which may be on the machine during these tests. Certainly the use of such a series winding does effect the field strength, and does vary as the load varies. On the other hand, I think it is quite possible to get inaccurate results when producing the same number of ampere turns in the shunt fields as the series field would produce under the conditions simulated. We have found in actual test that ampere turns from the shunt field do not produce the same resultant flux as ampere turns produced by the series field. The reason for this is no doubt due to the location of the shunt in series field on the pole and the differences in leakage resulting therefrom.

Mr. Siegfried very carefully points out the hysteresis effect that is noticeable and causes a different result when changing from a higher field current to a lower field current from that which is obtained when changing from a lower field current to a higher field current. I would like to add that on test floor we have also noticed what appears to be a hysteresis effect from changing the armature current. This is particularly noticeable under weak field conditions, and is probably due to the distorting action of the armature on the main field. I would suggest that in making a test of this sort where accuracy is very essential, that the tester be warned always to change the armature current in the same direction just previous to making readings.

Apparently from the results obtained by Mr. Siegfried the stray load loss is much greater at the high speed of an adjustable-speed motor where the high speed is obtained by weakening the field. From the curve shown it appears to be six or seven times as great as the full field low speed condition when operating at two and one-half times the low speed, and perhaps is ten to fifteen times as great on a four or five-to-one speed range operating at high speed. It is theoretically correct that we should get a much higher stray load loss at weak field and high speed than we should at full field and low speed. However, I would like to bring up for what it is worth, the fact that in making dynamometer tests, attempting to determine the difference in losses between those shown by the conventional method and the actual losses which are obtained in a machine, we have found that we could not establish any fixed rule for variation in the stray-load losses. The results were quite erratic, sometimes showing a greater stray-load loss at full field and low speed, sometimes showing a greater stray-load loss at the weakened field and highest speed, and sometimes showing a greater stray-load loss at some in-between value. These tests were made a year or more ago, and since the commutators were not treated in any way but were simply used in the condition existing at the time the tests were made, it is possible that the difference in commutator losses more than accounted for this erratic behavior.

I note in Mr. Siegfried's paper that he does not say anything regarding the possibility of shaft current causing part of the stray-load loss. I would like to inquire as to whether, during his test, he found any evidence of shaft current existing.

In closing, I would like to inquire as to whether Mr. Siegfried found any material difference in percentage of stray-load losses on larger machines than were present on smaller machines. In other words, is the present neglect of stray-load losses on smaller machines justified, or do the facts of the case show that this should be taken into account on smaller machines as well as on larger machines?

R. E. Hellmund (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): It is not at all surprising that close agreement on load losses has been obtained with the various methods discussed in the papers on this subject. Although the current and voltage conditions in the two machines on test differ to some extent with the simpler methods, the differences are not large enough to affect the results materially. It is nevertheless very gratifying to have the results of these careful investigations presented here, as any of the methods proposed can now be used with considerable confidence.

I am firmly convinced that most of the previous unsatisfactory results were due to variations caused by brush friction and perhaps, to a less extent, to variations in contact resistance. Since brush friction decreases with load and temperature, it may well be that in some machines the difference between brush friction at no load and at full load will be greater than the other load losses thus resulting in a negative value of load losses. A machine with a number of brushes, and having at the same time small armature distortion, well subdivided armature conductors, and good commutation, is likely to give such results.

Professor Siegfried has eliminated the irregularities often caused by brush friction by using paraffin on the commutators. This of course, is entirely satisfactory when analyzing the load losses other than those caused by brush friction and contact resistance. It would be helpful, however, if he could continue his work and make tests with the commutator in normal condition, being careful to obtain stable operation and temperature conditions as applying to no load, and also similar tests under full load. Care would also have to be taken that other factors of influence, such as humidity, are kept reasonably constant.

I was very much interested in the statement in the paper to the effect that current conduction between the brush and commutator through the paraffin film takes place because of carbon particles embedded in the film. In a previous discussion at last winter's convention, I suggested the possibility of such embedded particles being a factor with the usual oxide film on the commutator even under normal conditions. I am therefore wondering whether Professor Siegfried has any evidence supporting this theory, such as microscopic inspection for example.

Victor Siegfried: The author is indeed gratified by the interest shown in this subject, as evidence by the amount and the character of the discussion. It is apparent that the matter of stray-load loss in d-c machines is by no means a closed issue, and that much remains to be learned concerning its variation with the factors upon which it depends. The author is fully aware that many of the points touched upon in this paper and in its companion by Professors Schilling and Koopman, and by the discussers, show the need for further investigation of particular factors affecting stray-load loss. It must be made clear, however, that the author is here concerned primarily with establishing *methods* for determining the loss, and only secondarily with investigating the nature of the loss, in so far as it affects the procedure of the tests. It is shown by these papers that the loss can be measured accurately by a convenient and simple method, and that it is no longer necessary or proper to adopt an arbitrary percentage value to be assigned to it.

In considering matters relating to stray-load loss, it is very important that it be defined clearly. The definition given in the Test Code for D-C Machines is not sufficiently general, as it specifically mentions losses to be included, but does not allow for all factors which should be considered as a part of stray load loss. Stray-load loss is the difference between total losses as they occur under load conditions and the values of the individually known losses determined separately from measurements made under no-load conditions. It is thus a residual loss which includes all variations of machine losses under load which cannot otherwise be accounted for by conventional methods. It is made up of numerous component losses which change in differing degrees with load, in such a way as to preclude establishing any one general law for it.

The author is in complete agreement with Professor Morgan and Messrs. Franklin and Linville that the load-back test (without boosters) is the most practical and simplest method to be used. Although the two machines do operate thereby at different field currents, a division of the loss can be made, if the excitation is assumed to be represented by the average of the two machines, with reasonably accurate results over the normal range of excitations necessary for this test. For apportioning the loss between machines carrying different armature currents, the graphical separation method, devised by Mr. Narbutovskih and alluded to by Professor Morgan (1. Morgan and Narbutovskih, ELECTRICAL ENGINEERING, volume 53, 1934 pages 286–90) was used in this paper. As Professors Schilling and Koopman show, the results agree with their method, which involves the determination of the exponent of the function representing the total losses. It does not always follow that the exponent is constant over the entire range of currents in the machines, so that this method suffers from the limitation that the exponent must be determined in the range for which the separation is desired. The difference in methods is principally that between graphical and analytical approaches, as both perform the same operations upon the function; the choice is thus largely one of personal preference.

Mr. Hellmund points out the possibility of a decrease in brush friction with increasing values of load, and also that negative values of stray load loss may thus be indicated in certain instances. Such results must be interpreted in the light of the general definition of the loss, and in no way be allowed to reflect on the validity of the test. It was with full appreciation of the extreme variability of brush performance that the expedient of paraffining the commutator was adopted in making these tests, and it must be admitted that the results do not include any consideration of the possible decrease in this so-called constant no-load loss. Such procedure was permissible under the Test Code definition of stray load loss, which did not allow for any such variation. The comparison of methods as presented in the paper, and the general relations of stray load loss to operating factors, are in no way rendered less valid by this procedure, but are in some measure enhanced by the removal of the one factor which might otherwise mask the true picture of the more important components. The author's mental concept of the mechanism of commutator-brush conduction apparently agrees with that of Mr. Hellmund, although no effort has been made to substantiate it by microscopic examination of the paraffin film. The only evidence that can be offered is the blackening of the wax film either by carbon particles from the brushes or by carbonization of the paraffin itself.

Messrs. Franklin and Linville have very capably pointed out the possible inaccuracies involved in making the separation of the total losses under load into known and stray-load losses whatever the method for determining them. Again the matter is one of definition, and final efficiencies should have correct values so long as the procedure is consistent. They also state that it is well established that the brush-friction loss is sometimes reduced by half between no-load and full-load. While this is quite conceivable, it would be helpful to a fuller appreciation of the part played by brush friction in the total losses to have further investigation of this factor presented in published form. The available information on this point is quite limited, and it may be that what is considered as common knowledge among manufacturers is as yet unreported in a form to make it available to independent investigators. The author here makes the plea that the manufacturers, whose designers are the ones to benefit most by the full discussion of this subject, themselves undertake and publish the results of careful and open-minded investigations. The investigation of the known as well as the stray-load losses would certainly aid in bringing the whole consideration of d-c machine efficiencies up-to-date and in line with the growing importance of the d-c machine.

The percentage values presented in the papers are not strictly in accordance with a general definition of stray load loss, in that they do not account for certain variations in "constant" losses which might work for the reduction of the total stray load loss. These "known" losses were subtracted from measured total losses exactly as they occurred at the time of the test. The values are at least consistent with the original Test Code definition, and form a valid basis for comparing machines within a certain size class. The author himself would dislike to have these figures or any other figure so presented used as an argument in favor of the one per cent rule. Any agreement of the measured stray load loss with this one per cent figure must be taken as accidental, as the only satisfactory means of arriving at stray load loss is to measure it.

It has been proposed from time to time that a measurement of stray-load loss might be had by determining the increase in power needed to drive the rotor of a machine when it is short-circuited and sufficient excitation applied to circulate full-load current in the armature. Curve *E* of figure 3 of the paper shows the trend as the excitation is reduced, with a stray load loss 50 per cent greater at 28 per cent excitation than at normal excitation. Subsequent tests of the same machine by this short-circuit method show a loss of 300 watts, or practically double that for normal operating conditions. The conclusion is inescapable that this method has no place in the array of possible methods for determining stray-load loss.

This discussion does bring out the need for a more complete treatment of stray-load loss in the Proposed Test Code, so as to make the definitions general and correct, and also to specify rather definitely what procedures are to be followed in determining stray-load loss by measurement.

The discussion offered by Mr. Miller is very much appreciated, as it amplifies some points given only brief mention in the paper. The wide variability in performance of the commutator is easily demonstrated by almost any operation that is performed upon it, or by any conditions to which it is subjected. The continual fluctuation of armature current under no-load conditions is indicative of this variation, and sanding of the commutator produces notable and long-lasting increases in loss, with accompanying heating effects. Likewise, paraffining produces a marked decrease in loss, and it was observed in some instances that the total rotational loss was decreased by as much as 20 per cent. This represents a considerable decrease in actual power for any machine, and the cooler operation of the commutator at no-load was very apparent. The paper by Doctor Glass, which was not available at the time of writing the paper, is a very fine treatment of the subject and is worthy of considerable study.

As to the terminal-to-terminal drop, it was not stated just how this was obtained, and the conclusion that it was measured while stationary is not surprising. However, the armature was actually rotated slowly by hand so as to procure average sliding contact conditions. It has been the author's experience that this represents the running contact conditions with sufficient accuracy that it is far preferable to measuring the contact drop at full speed. Although this is not in accordance with code provisions, the intent was to measure the drops as they actually occurred at the time of the test, so as to measure the actual stray-load loss.

The series winding was left out for purposes of the comparative tests for loss with different methods where the armature current conditions could not possibly be duplicated, but the commutating winding was included at all times. This established the magnetization as being entirely from an independent source and not dependent on the variable armature currents. That the ampere-turns of shunt and series fields are not of equal effect in producing flux is open to little question. It may be possible that, once the validity of the test method has been established, the series winding should be left in so as to give its full effect as under operating conditions. It is interesting to note that this will require reversing the series coil on the machine which serves as the generator of load-back couplet.

The experience with regard to hysteresis effect when changing armature current is well included at this point. It was mentioned briefly in the paper that this same effect was present in the machines tested, particularly when the low excitation run was made. It is felt that the effect is of minor importance at normal saturations of the machine, but is well worth keeping in mind as it explains some manipulative difficulties that are otherwise surprising. As to shaft currents, no attempt was made to determine the presence of shaft currents, or in general to separate out the relatively smaller components of the stray-load loss. The size of the machines tested seems wholly unrelated to the amount of the loss measured, and it is the author's feeling that even on small machines, where efficiency guarantees are of any importance at all, some tests must be made to measure the actual value of that loss.

A detail of test procedure has been questioned by Professors Schilling and Koopman, and it is sufficient to say that both first and last readings of no-load loss were taken in accordance with standard procedure, the duplication being made to be certain of no major change in rotational loss during the test. Any difference of minor order was either prorated according to the time elapsed, or taken as an average of the two readings.

The author is very glad to have the discussion from Mr. Hughes, of Brighton, England. He raises a question on a point which is often forgotten in the consideration of stray-load loss; namely, the original source of what is included in this rather comprehensive term. Certainly a sizeable portion of the stray-load loss is due to a change in the total iron loss of the machine with the increase in load current. It does not seem possible, however, to relate the stray iron loss to flux conditions when it is practically impossible to separate the loss into the many components which of necessity are included in it. The test suggested by Mr. Hughes is one which would appear to have some promise for getting at the iron loss component, provided the machine in which one is interested has a compensating winding. The analytical expression for total iron loss should be helpful in considering the problem from this rather fresh point of view.

Too often the original source of the approximate methods of dealing with anything become so obscure that the very obscurity tends to strengthen the belief that those methods are exact, and it remains for one from afar to break down the time-honored acceptance. It is this desire, academic though it may be, to bring a fresh consideration of the stray-load loss to the d-c machine which prompted the author to undertake this work originally, and it is his hope that further progress can be made until the accurate treatment of the d-c machine is as well established as that for any other, particularly in view of the present major trend to d-c machines for large mill drives where certain inherent characteristics make them indispensable.

Unsymmetrical Short Circuits on Water-Wheel Generators Under Capacitive Loading

Discussion and author's closure of a paper by C. F. Wagner published on pages 1385–95 of volume 56, 1937, AIEE TRANSACTIONS (November 1937 issue of ELECTRICAL ENGINEERING) and presented for oral discussion at the electrical machinery session of the winter convention, New York, N. Y., January 27, 1938.

C. Concordia (General Electric Company, Schenectady, N. Y.): There are two points which should be brought out in connection with this paper. First, because of the test method, i. e., the application of the capacitor or reactor load at the moment of short circuit, the transients are different from those associated with an actual fault on a system in which the capacitance or other load is continuously in the circuit. Also, due to generator regulation, even the steady state overvoltages observed are not the same as those obtained with similar permanent loads, the apparent overvoltages being high for capacitance load and low for inductance load. Near the point of third-harmonic resonance this difference may be about 30 per cent, but can of course easily be corrected by calculation. In spite of these differences, the test overvoltages are undoubtedly qualitatively the same as those which would have been obtained in tests simulating more exactly the condition of an actual fault.

Second, it should be emphasized that the ratio x_q''/x''_d, upon which the overvoltages due to line-to-line faults largely depend, is not simply dependent upon whether or not the amortisseur end rings are connected between poles, as might be inferred from the paper. In the machine used in Mr. Wagner's tests, an unconnected damper winding gave a relatively large ratio, but as the machine size, in particular the relative axial length, is increased the difference between unconnected and connected damper windings becomes smaller. Thus, for large low-speed water-wheel generators, unconnected damper windings may be much more effective than would be indicated by tests on a small machine. Moreover, the amortisseur design may have considerable effect on the ratio x_q''/x_d'' aside from the connection between poles. Finally, the many other functions [see "Overvoltages Caused by Unbalanced Short Circuits" (Effect of Amortisseur Windings), by Edith Clarke, C. N. Weygandt, C. Concordia] of amortisseur windings should be considered in every application.

Because of these factors, the electrical performance characteristics of an amortisseur can be more accurately described in terms of the ratio x_q''/x_d'' than in terms of whether it is connected or unconnected between poles.

W. V. Lyon (Massachusetts Institute of Technology, Cambridge): This comprehensive analysis of Mr. Wagner's is particularly interesting to me. In 1926 Rene Brosens working under my direction obtained oscillograms of the open-phase potential when a line-to-line short circuit occurs on a three-phase generator delivering a condensive load. As a generator he used a wound-rotor induction motor having a leakage coefficient of 0.046, and consequently the ratio x_q/x_d' was about 20. For this reason Brosens obtained a much higher open-circuit potential than Mr. Wagner. Brosens noted a peak potential of about 18 times normal peak line-to-line potential. By applying the simplified theory proposed by Boucherot in 1911 which neglects resistance and is consequently similar to the "constant flux linkage" theory of Doherty, Brosens checked the peak values of potential within about 10 or 15 per cent. He made no analysis of wave form. A brief report of Brosen's work was sent to a few operating companies calling their attention to the possible danger of overpotentials with these conditions of operation. Apparently at that time the results were considered of only theoretical importance, and so far as I know no attention was paid to the warning.

Fundamentally the excessive steady-state potential on the open phase is due to the difference in the transient reactances in the two axes. Even with a salient-pole machine it is theoretically possible to have a damper winding for which these reactances are equal.

C. F. Wagner: Mr. Concordia is quite right in his point that the transients resulting by suddenly applying a short circuit across two terminals and a capacitor across the sound phase simultaneously will give a transient different than if the machine had been loaded before the application of the short circuit. I had made tests (see figure 11) with the capacitor connected before this short circuit and had come to the conclusion, however, that the difference is rather small. The general question of the effect of load regulation is discussed to some extent in my paper. It likewise will make a difference. As a matter of fact it is possible to enumerate a host of factors that make a greater difference (and a much greater difference) than the factors of whether this capacitor had been connected before the fault or whether the regulation had been included properly. In practice it rarely occurs that the ideal conditions of an unloaded line to which a short circuit is applied are satisfied. Usually the machine under consideration is feeding power over a line to the end of which other synchronous machines are connected. These machines will usually have a damper winding or its equivalent. Upon the inception of an unbalanced fault, abnormally high voltages may be produced but if the breaker at the receiving end of the line opens first, then a condition is satisfied under which much higher voltages may be produced. But in the transition to this, the important condition, note that an intermediate circuit condition was passed through. The transients during the beginning of this final circuit condition would in all probability be entirely different than if a similar unbalanced fault was applied to the unloaded machine, for example, while it is connected to the capacitance of the line.

Mr. Concordia also infers that many of these conditions can be calculated easily. It must be remembered that the resonance conditions are the important points. But for this condition the armature resistance is very important in determining the maximum voltage. A method of analysis which will predict the maximum voltages must be capable of introducing the effect of resistance. Further, in the absence of a knowledge of these resistances it is impossible to predict the maximum voltages. As shown in my paper the resistance varies with the frequency and while these data were obtained for a machine without damper windings a similar relation should apply for machines with damper windings. In this case the current distribution between bars and within the bars will vary and affect the resistance. A quantitative analysis should therefore include this factor quantitatively and must apply for the resonance points. A comparison of figures 13 (*b*) and (*c*) of my paper gives some idea of the effect of armature resistance near a resonance point.

With regard to Mr. Concordia's second point that the inference might be taken from my paper that the effect of damper windings is simply dependent upon whether the damper winding is connected or not connected, I wish to call his attention to my statement near the end of my discussion of this subject (page 1391) which reads: "In any case figures 8*d* and 8*h* show that dangerously high voltages can be attained with this kind of damper unless precautions are taken to insure that the ratio x_q''/x_d'' is made more nearly equal to unity." With nonconnected windings much can be done by properly locating the end bars to reduce this ratio considerably.

But after all most of these points are more of academic than of practical importance. The analysis of this subject either by the method presented by Clarke, Weygandt, and Concordia or by the method presented by myself indicates that the general factors affecting the problem are pretty well understood. The practical solution appears to be to install non-connected damper windings with as low values of x_q''/x_d'' as is practical to obtain.

Professor Lyon's discussion is very interesting. It is unfortunate that Brosens did not extend his analysis at the time to salient-pole machines. It is quite probable that his theories in this case would have received a more favorable reception.

Stray-Load Losses of D-C Machines

Discussion and authors' closure of a paper by E. W. Schilling and R. W. Koopman published on pages 1487–91 of volume 56, 1937, AIEE TRANSACTIONS (December 1937 issue of ELECTRICAL ENGINEERING) and presented for oral discussion at the electrical machinery session of the winter convention, New York, N. Y., January 27, 1938.

R. F. Franklin and **T. M. Linville:** For discussion, see page 401.

T. H. Morgan: For discussion, see page 403.

R. E. Hellmund: For discussion, see page 404.

Victor Siegfried (Worcester Polytechnic Institute, Worcester, Mass.): This paper is a valuable contribution in that it gives further proof of the need for a renewed consideration of the matter of stray load loss in d-c machines. The fact that a test method for determining these losses is available in which confidence can be placed is demonstrated by the results that Schilling and Koopman present. The writer is in whole-hearted agreement with their conclusion that the loss must be measured rather than assumed to have some arbitrary percentage value.

The authors do not state the ratings of the machines used, nor whether series or commutating windings were in use during the tests. This information would be helpful in interpreting their results. Assuming, however, that the machines are rated at 110 volts, and taking rated current as 16.85 amperes, it would appear that rated full-load input is 1,854 watts. Reading from curve *B* of figure 10, a value for stray load loss of 35.5 watts per machine, the loss is thus 1.91 per cent of full load input. Such a figure shows that the loss is one which cannot be neglected even for small machines, and that the one per cent rule of the Standards is wholly inadequate to take account of this loss.

Schilling and Koopman are to be congratulated on the consistency of points on the curves presented in the paper. This shows careful work in the conduct of the tests, and further demonstrates that stray-load loss is actually measurable. Too long has the feeling been expressed that stray-load loss is not measurable. Their paper does present one method for determining it with results checked by an independent method, as shown in their figure 7. The experience of this writer is much the same—that stray-load loss can be measured, that consistent results are obtainable, and that independent methods give practically identical results.

E. W. Schilling and **R. W. Koopman:** In his discussion Professor Siegfried raises the questions of rating, and connection of the series and commutating-pole windings. The machines were rated two horsepower, and the commutating-pole windings were connected in the usual manner. The series fields, however, were not used.

The writers are greatly pleased by the interest in this paper shown by engineers of the larger manufacturing companies. Messrs. Hellmund, Franklin, and Linville point out that brush friction changes between no load and full load. In order to measure bearing and brush friction at conditions as near to actual load conditions as possible the machines were run until constant temperatures were reached before loss measurements were made, and friction loss was measured at no load immediately thereafter. Thus brush friction was measured at very nearly the same temperature, commutator surface conditions, and humidity as prevailed under load. As to the question of reduction in brush friction due to increase in current which is raised by Messrs. Franklin and Linville we have no data; however, it is evident that the change was not great enough to cause the low values of load-loss to swing negative. The writers admit that it is hard to determine accurately the brush-drop loss; however, the values obtained at rated current checked closely the conventional two volts. At any rate, by making allowance for increase in brush drop with increase in current, more accurate data for stray-load-loss were obtained than would have been obtained had the conventional value been used.

If armature I^2R loss had been calculated at the conventional temperature of 75 degrees centigrade and brush drop taken as two volts, some of the lower values for stray-load-loss would have certainly been negative. This would have been logical procedure had one definition of load loss been adopted; however, the writers attempted to adhere to the definition for stray losses given in the table under paragraph 2.105 of American Standards for Rotating Electrical Machinery, *C*50.

The writers do not feel that they have presented adequate data to question the magnitude of the present percentage taken for stray-load loss, but they do believe that they have presented sufficient evidence to show that the use of a constant factor for all conditions of speed and field is quite inadequate.

Switching Surges With Transformer Load-Ratio Control Contactors

Discussion of a paper by L. F. Blume and L. V. Bewley published on pages 1464–75 of volume 56, 1937, AIEE TRANSACTIONS (December 1937 issue of ELECTRICAL ENGINEERING) and presented for oral discussion at the power transmission session of the winter convention, New York, N. Y., January 27, 1938.

R. C. Van Sickle (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The data presented in this paper are of interest to those concerned with switching surges in general, as mentioned by the authors, so a few remarks on their relation to other available circuit-breaker data are pertinent. It is particularly necessary to point out that although the phenomena in high-voltage circuit breakers are similar to that presented here, the magnitudes are much different and in particular the voltage surges are much smaller.

The load-ratio control switches were tested on power sources of 1,000 volts, 20 amperes alternating current and 125 volts, 25 amperes direct current. The speed of opening was relatively fast and currents of 15 amperes were unstable and extinguished without waiting for the normal current zero. The overvoltages produced by them were enormous. By the interruption of direct currents, overvoltages of 67 times normal were encountered and of alternating currents overvoltages of 18 times normal. Such voltages on high-voltage lines would be disastrous so it is fortunate that certain limiting influences exist.

The maximum dielectric strength between contacts of a high-voltage breaker is relatively less than that of contactors. Moreover the increase in the gap between contacts during the period of current zero is also relatively smaller. Consequently, if the arc is interrupted before a normal current zero, by one of the usual forms of high-voltage circuit breakers, the maximum voltage which can be produced is limited by the dielectric strength of the arc path to a magnitude which does not damage other apparatus. A paper "Breaker Performance Studies by Cathode Ray Oscillograms" presented before the Institute in 1935[1] contains an illustration, figure 4, which shows an excellent example of this voltage phenomena which occurred during the interruption of 20-kv, six amperes by a high-voltage breaker. The arc was unstable and restruck about 30 times before a current zero. However, the contacts had not separated sufficiently to enable the breaker to withstand the recovery voltage at this current zero so the arc restruck and continued as an unstable restriking arc until the next current zero. During this time the current flowed through the breaker as a series of impulses with a time interval between them less than 0.0001 second. On the magnetic oscillogram it looks like a normal half-cycle of current during an arcing period. Even with this extremely unstable arc, the maximum overvoltage produced by the circuit breaker was about ten per cent of the normal crest value of voltage.

The effect of having a relatively higher dielectric strength is shown in figure 6 of the same paper. It is a cathode-ray oscillogram of the interruption of a 70-kv 20-ampere circuit by an experimental 287-kv high-speed breaker. In this case, the arc is unstable for approximately two-thirds of a cycle but due to asymmetry of the current two normal current zeros occur in this period and the arc restrikes after the first one. The rapid speed of operation and high dielectric strength of the breaker produce overvoltages approximately 3.5 times the normal peak of the 70 kv applied. However, a pole of a 287-kv breaker must open 87 per cent of 287 kv which is a voltage having a crest value equal to the overvoltages obtained on this test.

The paper by Attwood, Dow and Krausnick on "Reignition of Metallic A-C Arcs in Air" presented before the Institute in 1931[2] contains similar material and was based on arcs in low-voltage circuits. However, the overvoltages obtained were not as high as those given in this paper because the arcs were produced between stationary copper plates eight inches square and three-fourths inch apart. Some of these arcs became unstable and extinguished before the normal current zero. However, the arc terminals were not being separated rapidly and consequently the maximum voltages produced by the current rupture were limited to about the normal crest voltage either by a glow discharge or by a complete dielectric breakdown between electrodes. On the basis of the oscillograms in these papers it is reasonably safe to conclude that unstable arcs in high-voltage circuit breakers do not produce as great overvoltages as are shown in the paper under discussion and that they probably are limited to considerably less than twice the normal voltage crest.

The maximum value of the current at which the arc becomes unstable is given as five amperes for the a-c tests. This is higher than the 0.6 amperes for the arcs between stationary copper plates where conditions were less favorable for arc extinction. However, in the 70-kv 20-ampere circuit previously referred to, where conditions were more favorable for arc extinction, the arc was unstable through the small loop of an asymmetrical wave during which the magnetic oscillogram indicated a smooth loop with a crest value of 19.2 amperes.

The interruption of a 44-kv 330-ampere short circuit with the current becoming unstable at 11 amperes is shown in a paper, "Arc Extinction Phenomena in High Voltage Circuit Breakers" presented before the Institute in 1933, figure 6.[3] These data indicate that the current at which the arc can suddently extinguish is a function of the effectiveness of the deionizing activity and the rate of current decrease.

The maximum value of direct current at which the arc extinguished abruptly is given as 15 amperes on page 1464 and as 20 amperes on page 1470.

The reported erratic recovery of dielectric strength may be due in part to the interpretation of the data. It was assumed, as given in the conclusion, that "Restriking was so abrupt as to be practically instantaneous and was followed by an immediate dielectric recovery." The discharge of a current through the arc space during a restrike requires the ionization of the arc path and, although the dielectric strength of the path does not fall to zero, it must be appreciably reduced. The magnitude of the ionization is indicated by the oscillograms which show that these restrikes pass the equivalent of a steady current of several amperes. Consequently, the assumption that the envelope of the restriking voltage is the curve of dielectric recovery against time and that it gives the dielectric strength at any instant is not justified. The first figure referred to in this discussion illustrates this point also. The recovery voltage builds up to about 25 per cent of the normal voltage crest following the relatively long deionizing period at the first current zero. (At the left of the oscillogram.) After the restrike, the value of voltage at which the arc restrikes is only about eight per cent of the normal voltage crest. The ionization produced by the restrike has definitely reduced the dielectric strength.

The curve of the recovery of dielectric strength as a function of time should be a broken curve with an abrupt drop at each restrike and with the following rise indicating the recovery. However, due to the various factors controlling the deionization, the recovery will still be erratic but not in the manner indicated in this paper.

REFERENCES

1. ELECTRICAL ENGINEERING, volume 54, 1935, pages 178–84.

2. AIEE TRANSACTIONS, volume 50, 1931, number 3, pages 864–8.

3. AIEE TRANSACTIONS, volume 52, 1933, pages 850–60.

D. C. Prince (General Electric Company, Philadelphia, Pa.): The investigations and data obtained by these authors disclose some very interesting phenomena applicable to currents and voltages of the order of magnitude of load ratio control, the field in which they have worked. In endeavoring to extend the reasoning to other fields, particularly the circuit-breaker field, a certain amount of caution must be observed. Arc phenomena of the nature of those which they are investigating in general are very different if the scale is changed. It is therefore not in general safe to assume that overvoltages would be multiplied by the same amount if different current values were used, or that they would be multiplied by the same amount if different voltage values were used.

For both higher currents and voltages, circuit-breaker experience over a considerable period of years shows that overvoltages, where they occur, are of relatively small magnitude. In the range covered by the authors however these phenomena are very interesting and have counterparts in other fields such as mercury-arc-rectifier arc backs, etc.

R. D. Evans and A. C. Monteith (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): It is significant that three of the papers presented at this session deal with switching transients. This is indicative of an increasing interest in the problem not only from the theoretical point of view but particularly from the practical standpoint because of problems actually encountered in the operation of power systems. It should, however, be clearly recognized that the paper by Messrs. Blume and Bewley deals with a very special type of transient problem. These remarks are not to be construed as a criticism of the paper because the authors have indicated this fact in the title and have stated it in the early part of their paper.

Concerning the authors' investigation, we would like to comment on certain specific points. In reviewing the paper we find no mention of the characteristics of the source, a factor which we believe to be of importance. If the abrupt forcing of current zero takes place there will be an oscillation on the source side as well as on the load side of the circuit-interrupting device and the voltage appearing across the device will be the vector sum of the oscillating voltages appearing on both sides.

The authors have divided the recovery voltage problem into three parts: first, the normal-frequency component; second, the high-frequency component dependent on the capacitor voltage; and, third, the high-frequency component depending on the reactor current. Our paper "The Determination of Recovery Voltage by Analytical and A-C Calculating Board Methods," presented at the

1937 summer convention, dealt with this case and the results for interruption at other than current zero were discussed in connection with figure 25 of that paper. In considering these factors it is believed that the third is only of importance in dealing with relatively low current values, such as discussed by the authors. In a normal power circuit with typical values of fault current, it is believed the third factor becomes less significant, especially on the higher-voltage circuits.

The authors have inferred the same "dielectric-recovery curve" whether there is or is not restriking. The shape of the dielectric-recovery curve will be materially modified when restriking takes place and the shape will be dependent upon the number of restrikes. In high-voltage circuit breakers it will depend on the contact separation. Therefore, the phenomenon is not as simple as indicated in figures 6, 7, and 8.

Production of high-frequency transients on a large scale is not to be expected on power systems with conventional types of current interrupters from the factor of the forcing of the current zero as described by Blume and Bewley. Instead, it is our opinion that such voltage, if encountered, will be due to the successive re-establishment of the arc path at such intervals as to produce cumulative oscillations. In this connection it should be pointed out that the process of producing cumulative oscillations requires certain rather definite relations between the system recovery-voltage time curves and the insulation recovery-time curves to make it possible. If the shapes of these curves differ from that which is required, the oscillations do not become cumulative and the voltages will be limited to about the value corresponding to a simple arc suppression or to that of one restrike. It is because of this difficulty in setting up the essential conditions for cumulative oscillations that has made their probability so small that they are rarely encountered.

Summarizing, it is our opinion that if high voltages due to switching transients on main power circuits are to be encountered they will take place principally through the mechanism of restriking just described. Furthermore, the mechanism of producing overvoltages by the sudden forcing of current zero appears to be a factor of small importance on high-voltage apparatus and circuits but of increasing importance as the voltage is reduced until it becomes a major factor in the special type of problems to which the authors have limited their investigation.

J. D. Cobine (Harvard Graduate School of Engineering, Cambridge, Mass.): The authors are to be commended for their instructive presentation of the nature of the voltage appearing across an impedance when its current is interrupted by a switch.

The curves of figures 6 and 7 assume that the duration of current subsequent to a restrike is zero and there is no change in the dielectric recovery curve, while the oscillograms of figures 9, 10, and 11 show that the voltage falls at approximately a uniform rate for a measurable interval of time during which current is apparently flowing. Now if an arc current flows after restrike the dielectric strength of the gap has obviously been reduced to a low value and an entirely new dielectric recovery curve must be plotted starting from the time the new current flow is interrupted. Thus in figures 6 and 7 a new dielectric recovery curve must be drawn for each restrike. These dielectric recovery curves will be different from the first because of two important factors, namely, the current of the new arc will have changed and the length of the gap at the time of its interruption is different. The new arc current will be less than the old and therefore the dielectric recovery will increase at a more rapid rate than before. Lengthening of the gap will also raise the recovery voltage curve. Of course, if the restrike current is very small and of short duration there will be little change in dielectric recovery. It would be of great value if the authors could give some values of current that are typical of the restrikes presented.

The shapes of the experimental curves of dielectric recovery, figure 11, are quite interesting, but somewhat obscure as to details. What is the difference between the several curves for the same shunt capacitance? Was the initially interrupted current the same in each case? If so, what was its value?

It may be that the erratic behavior of the restriking voltage of the switch is due to the presence on the contacts of impurities having low work-functions. These impurities would also be vaporized into the arc column and by their low ionization potentials lower the reignition potential. Some research of the writer (ELECTRICAL ENGINEERING, volume 53, 1934, page 1081) has brought out the marked effect of impurities in lowering the reignition voltage of a short arc. Small amounts of these impurities were found to produce random variations in the reignition voltage. It is obviousely very difficult to have a commercial device free of impurities so that the switch was probably thoroughly contaminated.

Protection Features for the Joint Use of Wood Poles

Discussion and authors' closure of a paper by J. O'R. Coleman and A. H. Schirmer published on pages 131–40 of this volume (March section) and presented for oral discussion at the power transmission session of the winter convention, New York, N. Y., January 27, 1938.

Sidney Withington (New York, New Haven and Hartford Railroad Company, New Haven, Conn.): This paper indicates the great advantage of broad-minded co-operation between power companies and communication interests, permitting the solution on a purely rational and engineering basis of problems which may be of somewhat controversial nature. Both classes of utilities, the electrical engineering profession, and the public generally are to be congratulated on such co-operation.

The public utilities commission of the State of Connecticut, 1922, stimulated among the utilities in the state the compilation of rules and specifications governing joint wood-pole-line construction carrying power circuits of more than 5,000 volts, and communication circuits. These rules provided for construction on a basis mutually agreeable to all interests concerned and saved both power companies and communication companies a very considerable amount of money in duplicate construction, which would have been necessary if each class of public-service companies had been obliged to construct its pole lines independently of the other. That was one of the earliest instances of such joint action among the utilities interested, and was a notable example of successful co-operative effort in the solution on a rational basis, of problems which might otherwise have continued to be troublesome and expensive to all concerned. It is indeed a source of gratification that such co-operation is being continued.

H. A. Dambly (Philadelphia Electric Company, Philadelphia, Pa.): This paper marks an important advance in the co-ordination of electric power and telephone systems. It was early recognized that for two services which so generally supply the same customers, design based on keeping the two systems widely separated was not practical in the long run, and that any solution which did not permit both systems to occupy the same general area and the same streets and highways was unsound from the long point of view. Joint use as compared to separate lines for many situations is a further step in the recognition of this principle. Further, the advantages of joint use which have been conclusively demonstrated over many years in the lower voltage field clearly indicated the necessity for developing means to permit similar operation at the higher voltages. This paper presents a much needed answer to these situations.

The paper is particularly timely in view of the rapidly increasing demands for electric service which in turn have brought many companies face to face with the problem of raising primary distribution voltages in order to meet these demands. Many of these systems are now operating on a joint use basis at 5,000 volts or less, and the necessary pole plant investment to accommodate both systems has been made. Under these conditions the need for suitable design features which will permit continuance of satisfactory joint operation at the higher voltages is evident.

By co-operative arrangement between the operating companies involved and the joint subcommittee on development and research of the Edison Electric Institute and the Bell System, the protective features outlined in this paper have already been worked out for a number of specific situations throughout the country, and it is believed that reasonable solutions can be reached in any specific situation.

In discussing separate lines versus joint lines, the statement is made that in rural and thinly settled areas it has been generally practicable and desirable to construct separate lines. It is recognized that types of construction widely different from normal span lengths may affect economies. However, it seems to me that in these areas one of the principal advantages of joint use, namely, the saving in avoided investment in poles with consequent saving in annual charges may be obtainable, particularly if planning is done on a broad forward-looking basis. Economies are needed to minimize the cost of both services to these rural customers. While the application of the protective measures in these areas involves somewhat different problems than in suburban and urban areas, the methods outlined in the paper should be applicable in these rural areas.

A. H. Schirmer: No one will question Mr. Dambly's statement that every reasonable economy should be practiced to minimize the cost of power and telephone service to rural customers. However, experience to date in studying specific situations, particularly those involving the long-span type of construction which has recently been used by most power companies in rural areas, indicates that, in general, there is no economy in joint use of lines in such areas, even from the structural standpoint alone. In other words, rural telephone lines on separate poles can, in general, be maintained at an annual charge less than the yearly cost of placing such circuits on jointly used poles. Furthermore, the inductive co-ordination problem in rural areas is frequently a difficult one even where separate power and telephone lines are used, and this difficulty may be definitely increased where both the power and telephone circuits are on common poles.

Lightning Strength of Wood in Power Transmission Structures

Discussion and authors' closure of a paper by Philip Sporn and J. T. Lusignan, Jr., presented at the power transmission session of the winter convention, New York, N. Y., January 27, 1938, and published on pages 91–101 of this volume (February section).

F. E. Andrews (Public Service Company of Northern Illinois, Chicago, Ill.): It is desired to emphasize conclusion 8 in the paper to the effect that the data presented add greatly to the knowledge of the impulse insulating property of wood in transmission structures. This paper appears to provide a basis very much more satisfactory than any information heretofore available on the design of wood-pole transmission and line structures to withstand impulse voltages.

This discussion is presented from the point of view of operating experience of 33-kv wood-pole transmission lines, using pin-type insulators without ground wire, with nine feet of wood between phases and the equivalent in insulating value from line to ground of 800 to 1,000 kv minimum on the basis of information in the paper (from figure 12). This operating experience indicates very greatly improved operation, that is, reduction in line tripouts, with this construction in comparison with conventional construction, that is, construction using steel crossarm braces and without impulse insulation in the guys.

Many cases have been noted where lightning has discharged over a structure without power follow or line interruption, the location of such cases being indicated by splinters on the poles or crossarms. In the cases where power follow has caused tripouts this has usually been found to be due to an arc between phases and in a number of instances, as indicated by burns on the conductor, this has taken place through air in the span away from the pole with minimum phase to phase distance of 7.3 feet. This experience indicates that there is no practical spacing with 33-kv single-pole construction at which power follow will not occasionally occur, although the wide spacing appears to materially decrease the susceptibility to power follow by comparison with results obtained with conventional construction.

In order to have a complete answer to the problem of lightning interruptions to lines of this type so that performance can be definitely estimated or predicted, it seems necessary to obtain in addition to lightning flashover data of the character covered by this paper, data on factors influencing power follow. It is believed that data showing the effect of the variables which govern the tendency for power to follow and maintain an arc after lightning flashover are quite necessary if best use is to be made of the lightning strength data in the present paper.

The use of the data on impulse strength of wood and porcelain to provide maximum resistance to lightning flashover within the dimensional limits of practical construction when combined with information on expectancy of power follow when lightning flashover does occur, should greatly increase the practical value of the data given us in this paper. It is to be hoped that the authors will continue their work to cover the problem of power follow.

P. B. Stewart (Cincinnati Gas & Electric Company, Cincinnati, Ohio): We have reviewed this article with a great deal of interest. We note in particular the reference to the reduced insulating properties due to moisture retained in wood poles.

Several years ago we made a study of the electrical resistance of wood poles. This work was covered in an article prepared for the American Wood Preservers' Association and can be found in their proceedings for the year 1936. In this study we found that the conductivity of wood varied with the moisture content. In most cases creosoted pine poles had lower resistance than cedar poles. This, however, was due to lack of proper seasoning.

It may be that there is a definite relation between the electrical resistance of wood and the impulse insulation strength of wood. This relation might hold true either above or below the fiber saturation point of the wood. In our study we found that a variation of moisture content above the fiber saturation point of wood had little effect on the electrical resistance. After the material had been seasoned below the fiber saturation point, a small variation in the moisture content had considerable effect on the electrical resistance.

Wood is a product of nature and has many variations. For this reason it is very difficult to determine accurate values from laboratory studies unless a very large number of tests are made. It is also very difficult to assimilate natural rain conditions due to the slow absorbing qualities of wood.

This article will be of great value in the design of wood-pole transmission lines.

L. G. Smith (Consolidated Gas Electric Light and Power Company of Baltimore, Md.): A paper of this nature is quite valuable to those designing transmission and distribution lines, since wood is of considerable benefit as insulation in raising the impulse flashover values. The formulation of these data so that they may be used as design information is of particular importance. It is frequently desirable to calculate the impulse level of new designs of lines in advance of their installation as otherwise wood insulation may be used wastefully and balanced designs not obtained. Moreover it is undesirable to have to test each design contemplated. The data given by the authors make design calculations of impulse levels possible. The ideal design is one in which a balanced design is obtained, that is, one in which the impulse flashovers from phase to phase and phase to ground on all phases are approximately equal.

Several years ago tests were made by the Locke Insulator Corporation for the Consolidated Gas Electric Light and Power Company to check certain designs of 13.2-kv, 35-, and 115-kv lines. As a result of these tests balanced designs of lines were obtained. These

Table I. Summary of Impulse Tests on 13.2-Kv Wood-Pole Lines

Double-Circuit Upright Construction With Four-Kv Arm Four Feet Below Lower 13.2-Kv Arm Ground Assumed to Be on Four-Kv Neutral and Secondary Neutrals and on Guy Wire

Voltage Impressed	Flashovers in Kilovolts With $1\frac{1}{2} \times 40$ Microsecond Positive Wave																		
			Guys With Two Porcelain Insulators—Steel Crossarm Braces						Guys With Two Porcelain Insulators—Wood Crossarm Braces						Guys With Wood Insulators Wood Braces				
	No Guys			Horizontal Guy						Horizontal Guy					Horizontal Guy			No Guys	
							At Bottom of						At Bottom of			At Bottom of			
	Steel Crossarm Braces	Wood Crossarm Braces	Thirty-Degree Guy at Bottom Lower Brace Within Two In. of Four-Kv Neutral	One Foot Below Lower Brace	At Bottom of Lower Brace	One Foot Below Lower Arm	Lower Arm	Upper Brace	Thirty-Degree Guy at Bottom Lower Brace Within Two In. of Four-Kv Neutral	One Foot Below Lower Brace	At Bottom of Lower Brace	One Foot Below Lower Arm	Lower Arm	Upper Brace	One Foot Below Lower Arm	Lower Arm	Upper Brace	Exp. Pro Gaps Multiplex Conn. Steel Braces	Pole Grd. Wire Steel Braces
Column No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
A phase to ground	765	840	555	705	655	495	395	345	900	685	600	*495	*395	335	*840		*840	108	555 610 (−)
B phase to ground	920	1,230 1,020 (−)	675	895	*675	795	700 750 (−)	530	1,020	1,190	985	875	725	505	*1,230 *1,020 (−)	*1,230 *1,020 (−)	*1,230 *1,020 (−)	83 91.5	410
C phase to ground	530 445 (−)	710 690 (−)	345	465	*345	445	360	345	600	720	590	485 530 (−)	*360	335	*710 *690 (−)	*710 *690 (−)	*710 *690 (−)	94	345
A phase to *C* phase	705	*705		*705					*705	*705	*705				490	420	415	89.5	*705
B phase to *A* phase	700	*700		*700		*700	*700		*700	*700	*700	*700	*700		*700	*700	*585	78.5	*700
B phase to *C* phase	655	*700	*655	*655	*655	*655	*655		*700	*700	*700	*700	*700		*700	*700	585	81	*655
C phase to *C* phase	335	*705								*705					*705			88.5	*335

(−) Negative impulse wave value. * Measurements were not made—values assumed to be unchanged from other conditions. Where no values are shown flashover will go to ground instead of between phases at minimum voltage. Horizontal guys brought out from center line of pole on same side as the phase wires under test.

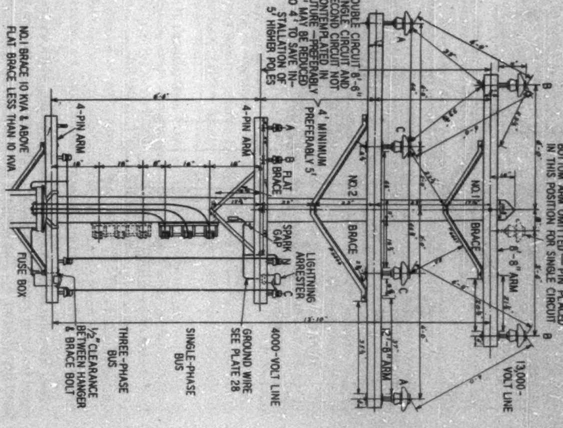

BOTTOM ARM OMITTED — PIN PLACED IN THIS POSITION FOR SINGLE CIRCUIT
B
B
13,000-VOLT LINE
6'-8" ARM
NO. 1
BRACE
A
C
C
A
12'-8" ARM
NO. 2
BRACE
DOUBLE CIRCUIT 8'-6" SINGLE CIRCUIT AND SECOND CIRCUIT NOT CONTEMPLATED IN FUTURE —PREFERABLY 5' MAY BE REDUCED TO 4' TO SAVE INSTALLATION OF 5' HIGHER POLES
4' MINIMUM PREFERABLY 5'
LIGHTNING ARRESTER
A
B
FLAT BRACE
SPARK GAP
N
C
4000-VOLT LINE
4-PIN ARM
GROUND WIRE SEE PLATE 28
SINGLE-PHASE BUS
THREE-PHASE BUS
½" CLEARANCE BETWEEN HANGER & BRACE BOLT
4-PIN ARM
NO. 1 BRACE 10 KVA & ABOVE FLAT BRACE LESS THAN 10 KVA
FUSE BOX

Figure 1

designs are covered in a discussion on page 199, AIEE TRANSACTIONS, 1936. These tests were made on assembled structures. These test results were compared with the data contained in the authors' paper and were found to check very closely.

The results obtained for the types of construction used on 13.2 to 35 kv are shown in table I, for the type of construction illustrated in figure 1. Figure 1 also includes an alternate design for a single-circuit line, using the top crossarm only with a third insulator installed near the pole, which is shown dotted. The results of this type of single circuit construction are shown in table II. An alternative design is shown in figure 2, the results of test on this type of construction being shown in table III.

Table II. Summary of Impulse Tests on 13.2-Kv Single-Circuit Wood-Pole Construction

Ground Assumed to Be on Four-Kv Neutral on Four-Kv Arm, Seven and One-Half Feet Below

Voltage Impressed	Flashovers in Kilovolts With $1\frac{1}{2} \times 40$ Microsecond Positive Wave				
			Angle Guys With Two Porcelain Insulators, Wood Braces		
	No Guys				
	Steel Crossarm Braces	Wood Crossarm Braces	One and One-Half Feet Below Bottom of Brace Within Six In. of Four Kv	One Foot Below Bottom of Brace Within Six In. of Four Kv	At Bottom of Brace Within Six In. of Four Kv
B phase to ground			645	560	400
A phase to *B* phase	630	730			
B phase to *C* phase	570	575			

B phase is on the same side of pole as *C* phase.

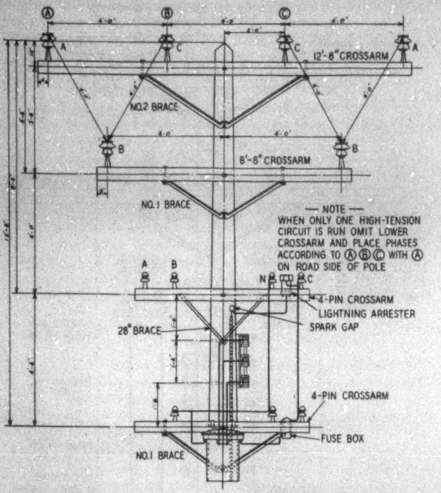


Figure 2 (left)

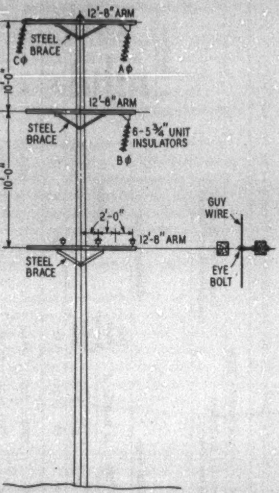


Figure 3 (right)

Tests were also made on a 66-kv type of structure, which was designed for future operation at 110 kv. This type of construction is illustrated in figure 3. The results of the tests on this construction are shown in table IV. Two types of arcing horns were used in these tests, the older type to which reference is made is the conventional type of horn, which had the effect of reducing the flashover of strings. In the new type of horn, the horn was straightened in order to effect the minimum reduction of flashover of the string.

In the tests on wood poles it is interesting to note the difference between the various types of poles. Our tests showed similar results in some cases, the moisture in the pole lowered the resistance sufficiently to permit the impulse generator to discharge through this resistance without obtaining an external flashover. One of the poles tested had an internal moisture content of from 112 per cent to 136 per cent. However, it is believed that if the generator had had sufficient current discharge capacity, external flashovers could have been obtained. Since pine poles immediately after creosote treatment may have a very high moisture content it is quite possible that the trouble noted by the authors may have been similar to that outlined above.

Another point of interest is the relatively small effect of the various sizes of insulators upon the flashover of the combination of pin-type insulators and wood crossarms, shown in figure 10 of the paper. It is believed that this is a very important consideration in the design of low-voltage lines. This feature may be utilized in the practical design of lines by choosing an insulator of sufficient size necessary to give the desired 60-cycle flashover and leakage characteristics. The remainder of the impulse level can then be obtained by the judicious use of wood, which usually will be at a lower cost than the cost for additional porcelain.

I. W. Gross (American Gas and Electric Service Corporation, New York, N. Y.): In the latter part of the paper the authors present impulse flashover data on typical transmission line structures carrying 33-kv and 66-kv circuits (figures 15 and 16). In figures 19*A* and *B* it is shown that the minimum impulse insulation of the line (which was phase to ground) was increased over 100 per cent for the 33-kv structure by the use of wood arm braces, wood bayonets supporting the ground wire, and an offset down lead.

While the gain in impulse strength is clearly apparent from the test results reported, it may be interesting to look at the operating record of this line before and after the above structural changes were made. The line which is 15 miles long is of standard two-circuit vertical configuration carrying a ground wire above the pole top. The ground wire is grounded every third pole, ground resistances ranging from 20 ohms to over 300 ohms. There are 483 structures in the line, 42 of them being guyed structures. The guys were insulated when the wood insulation was added.

During the four years prior to revamping, there was an average of 3.2 single-circuit outages per year due to lightning and 7.2 double-circuit outages per year. For the five-year period following revamping with wood insulation these were reduced to 2.2 and 0.6 on an average yearly basis. That is, the single-circuit outages were reduced 36 per cent and the double-circuit outages 92 per cent.

Where double-circuit lines are used to protect the continuity of service rather than for load requirements, the performance of the lines must be evaluated on the double-circuit outages alone, as a single-circuit interruption does not result in a service interruption. On this basis the improvement in performance of the above revamped 33-kv line is quite remarkable.

Table III. Summary of Impulse Tests on 13.2-Kv Double-Circuit Inverted Wood-Pole Construction

Ground Assumed to Be on Four-Kv Neutral on Four-Kv Arm Four Feet Below Lower Arm

	Flashovers in Kilovolts With $1\frac{1}{2}$x40 Microsecond Positive Wave					
	Angle Guy With Two Porcelain Insulators—Wood Crossarm Braces					
	At Bottom of Lower Brace Within Six In. of Four-Kv *C* Phase,	At Bottom of Lower Arm Within Six In. of Four-Kv *C* Phase		At Bottom of Upper Brace, Within Six In. of Four-Kv *C* Phase	One Foot Below Upper Arm Within Six In. of Four-Kv *C* Phase	
Voltage Impressed	$50\frac{1}{2}$ Degree Angle	50 Degree Angle	40 Degree Angle	40 Degree Angle	25 Degree Angle	14 Degree Angle
B phase to ground	730	605	655	550		575
C phase to ground				550	585	575
C phase to *B* phase			655			

Corresponding records are not at present available for the revamped 66-kv lines, as in some cases the reconstruction work was not carried out on complete lines, and in other cases sections of intervening steel-tower lines obscured the records. An over-all survey of the performance records of these lines, however, seems to indicate a reduction in outages comparable to the impulse-insulation improvement shown for these 66-kv structures in figures 20*A* and *B* of the paper.

R. L. McCoy (Locke Insulator Corporation, Baltimore, Md.): The paper by Messrs. Sporn and Lusignan is of extremely great interest. The authors are to be congratulated for having carried on a well organized and comprehensive investigation on this important subject. Significant light has been shed on many obscure factors which in the past have been given little or no study. This paper gives stimulation to further thought along this line.

In transmission structures we are interested in the over-all impulse insulation afforded by various combinations of wood and porcelain insulation. The data presented bring out in a striking way the fact that to obtain the most benefit from wood as an adjunct to porcelain consideration must be given to the proper co-ordination of these materials. Analyzing the data presented in the paper, that is in figures 10*A*, 12*A*, and 13, it is seen that the positive-polarity impulse strength of wood taken as an adjunct to porcelain insulators varies from approximately 130 kv per foot with pin-type insulators down to approximately 55 kv per foot with ten-unit strings of suspension insulators.

A variation of a somewhat similar nature is noticed in the negative-polarity impulse strength as an adjunct to porcelain in figures 10*B*, 12*B*, and 13. This suggests as the authors point out that particularly in the case of the longer strings of suspension units the stress distribution between the porcelain and wood elements is badly out of proportion to their respective impulse strengths. The authors suggest that this may be due to unfavorable relative electrostatic capacitances of the various elements.

In order to check this proposal a four-foot section of creosote-dipped well-seasoned crossarm having a cross section of four inches by four inches was tested in series with the following dielectric members:

(*a*) One-unit heavy-duty switch and bus insulator having a capacity of 53 micromicrofarads at 60 cycles.

(*b*) A two-unit string of standard five and three-quarter-inch by ten-inch suspension insulators having a capacity of 15 micromicrofarads at 60 cycles.

(*c*) A 13½-inch rod gap which should have relatively very low electrostatic capacity.

These dielectric members were selected because at standard humidity they have approximately equal full-wave impulse flashover values on a 1.5 x 40 positive impulse wave. All tests were made at an absolute humidity of 0.3 inch of mercury-vapor pressure. The four-foot wood section alone gave impulse strengths of 166 kv per foot positive and 154 kv per foot negative. The insulation added by this wood section to the switch and bus insulator, the two unit string of suspension units, and the 13 ½-inch rod gap were respectively positive polarity 153 kv per foot, 158 kv per foot, and 172 kv per foot, negative polarity 155 kv per foot, 148 kv per foot, and 167 kv per foot.

It should be noted that in these measurements practically the full impulse value of the wood itself was added while in the case of the rod gap the wood added more strength than it possessed by itself. This suggests three things, first, that perhaps the relative leakage resistance of the insulation elements is also important, or perhaps the relative surge impedances of the elements which would take into account not only the relative capacities but also the relative resistances. Second, that as the authors' data might be interpreted to suggest the impulse strength of the wood section should be large in comparison with the impulse strength of the insulator element. Third, that the distribution of stress between the porcelain and wood elements may be a function of their relative time lags.

It is a matter of observation that the time lag at the full wave flashover on pin-type insulators and short strings of suspension units,

Table IV. Summary of Impulse Tests on 66-Kv Wood-Pole Lines

Pumphrey to Annapolis Future 110-Kv Line Construction

Six Insulators per String—Pole Ground Wire

	Flashovers in Kilovolts With 1½ x 40 Microsecond Positive Wave															
	Guy Attached Six Feet Below Lower Arm										Guy Attached Four Feet Below Lower Arm at Various Angles to Pole					
	Old Type Arcing Horns				New Type Arcing Horns						New Type Arcing Horns				Six-Unit String Only, Straight Line Fittings	
	Straight Line Fittings	Angle Fittings			Straight Line Fittings			Angle Fittings			Straight Line Fittings, Insulators Vertical					
	Insulators Vertical	Insulators Vertical	30 Deg. Swing	45 Deg. Swing	Insulators Vertical	10 Deg. Swing	20 Deg. Swing	Insulators Vertical	10 Deg. Swing	20 Deg. Swing	17½ Deg. 65 In. Clearance	42½ Deg. 53¾ In. Clearance	51 Deg. 45 In. Clearance	60½ Deg. 36 In. Clearance	Old Type Arcing Horns	New Type Arcing Horns
Voltage Impressed Column No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A phase to ground	*750	725	680	540	*1,025	*940	*835	*995	*895	*770					*520	*580
B phase to ground	*750	755	680	560	1,025	940	835	995	895	770	1,010	810	685	560	*520	580
C phase to ground	750	725	800		*1,025										520	*580

Eight Insulators per String—Pole Ground Wire

	115-Kv Expulsion Protective Gaps, No Arcing Horns	115-Kv Expulsion Protective Gap in Ground Wire, Wood Guy Insulators Above and Below 13-Kv Arm	Guy With Wood Insulators Six Feet Below Lower Arm. Old Type Arcing Horns				New Type Arcing Horns					
			Straight Line Fittings Insulators Vertical	Angle Fittings			Straight Line Fittings			Angle Fittings		
				Insulators Vertical	30 Deg. Swing	45 Deg. Swing	Insulators Vertical	10 Deg. Swing	20 Deg. Swing	Insulators Vertical	10 Deg. Swing	20 Deg. Swing
Column No.	17	18	19	20	21	22	23	24	25	26	27	28
A phase to ground	470 490 (−)	905	*885	840		*490	*1,025	*930	*790	*995	*880	*720
B phase to ground	470 490 (−)	*905	*885	*840	585	490	*1,025	930	790	*995	880	720
C phase to ground	470 490 (−)	945 1,030 (−)	885				*1,025					

* Measurements were not made—values assumed to be unchanged from other conditions. (−) Negative impulse wave values. 13.2 kv on pole grounded in tests with old type arcing horns. No 13.2 kv on pole in tests with new type arcing horns.

say from one to five units, varies from four to eight microseconds, while on long strings of suspension units, ten units or more, it may run from 15 to 25 microseconds.

J. H. Hagenguth (General Electric Company, Pittsfield, Mass.): The authors have given the results of a very thorough investigation of the impulse characteristics of wood alone and wood in series with insulators.

Some tests on wood crossarms made in the high-voltage engineering laboratory in Pittsfield check the results obtained by the authors very closely. For instance, data published in a paper on "Impulse Voltage Strength of Insulators and Materials" by J. C. Dowell and C. M. Foust, figure 30 checks figure 4 of the paper for the flashover of pine at two microseconds almost exactly, while the Pittsfield values for the full-wave flashover are approximately eight per cent higher. Curves of figure 30 of the above reference begin to bend over considerably at about five feet—the flashover voltage of nine feet is only 36 per cent higher than at four and one-half feet—while the authors' figure 7, curve *A* indicates proportionality between flashover voltage and length of wood gap. In my opinion, it seems more reasonable to expect a smaller increase in flashover voltage for great length of wood, because the flashover of wood probably is a creepage phenomenon, which tends to be progressive, once an arc has started and the addition of a few feet of creepage path will not materially increase the flashover voltage.

Some investigation carried out on large-scale pole structures in the laboratory seem to check this contention. During these tests, wood crossarms with a length of as much as 11 feet and insulator strings of 9 to 15 units were in series. The ratio of measured to calculated (similar to table I of paper) flashover voltage was approximately 0.50, if the kilovolt values curve *A* of figure 7 of the paper are used, and 0.80 if the kilovolt values of the Dowell-Foust paper figure 30 for ten microseconds are used. The impulse insulation added per foot of crossarm during these tests varied between 30 and 50 kv per foot. The lower figure usually was associated with the longer wood gap. Lengths of wood poles up to 35 feet were arced over, when in series with an insulator string. During these tests it was also quite well established that the flashover voltage of an air gap 21 feet above ground was approximately the same as the same gap spacing with standard mounting. For instance, a 90.5-inch air gap 21.5 feet high, flashed over at 1,385 kv (1,400 kv standard rod gap) and a gap of 166 inches flashed over at 2,640 kv (2,530 kv standard rod gap). All these tests were made with 1.5x40 waves of positive polarity.

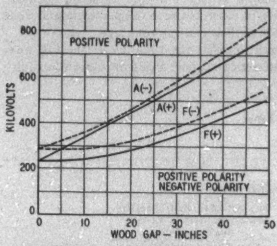


Figure 4. Impulse flashover characteristics of pin insulators plus crossarms—full-wave flashover

A—Creosoted fir crossarm, dry, same as curve *A*, figure 10*A* of the paper

F—New pine crossarm in series with 66-kv pin-type insulator rain test 0.2 inch per minute

Solid line—positive polarity

Dashed line—negative polarity

There appears to be some discrepancy between the results of figures 4, 5, and 7 of the paper. In figure 4, the flashover voltage of a pine crossarm of approximately 20 square inches is given as 530 kv at four feet, while figure 7 shows a flashover voltage of 610 kv for the same length. While this difference may be accounted for by the difference in treatment and general condition of the wood, these results do not seem to check figure 5*A*. The poles used for the test presumably had a larger cross section than the crossarm and should, therefore, be expected to have a relatively lower flashover voltage.

The authors mention very briefly the effect of rain on the insulation strength of wood, stating that a reduction of some 50 per cent might be obtained. Since the lightning disturbances usually occur under rain condition, it naturally would be of real importance to have more data taken under such conditions. It may, therefore, be of interest to compare dry flashover voltages as given in the paper with flashover voltages obtained under rain conditions in Pittsfield with a precipitation of 0.2 inch. The Pittsfield tests were made using a 66-kv pin-type insulator in series with wood crossarms.

Figure 4 (this discussion) shows a comparison between flashover voltages under the two conditions. It is a coincidence that the flashover voltages of the two insulators is the same, although one was tested, when dry, the other under rain. The considerable reduction due to rain in the insulation strength added by the wood is apparent especially for the first 12 inches. This is shown more clearly in figure 5 (this discussion) where the impulse insulation added to the pin insulator by an untreated pine crossarm is plotted, similar to figure 10*C* of the paper. Curve *C* on the figure 5 (this discussion) shows the ratio of wet to dry flashover, which is very low for short lengths of wood increasing to 50 or 60 per cent at about four feet, the maximum length investigated. This ratio is approximately the same for positive and negative polarity. For short lengths of wood, the calculations should, therefore, be based on the

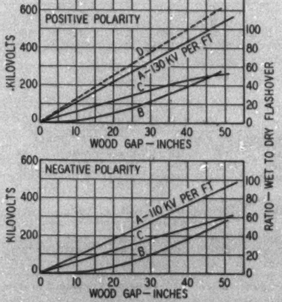


Figure 5 (left). Impulse insulation added to pin insulator by untreated pine crossarm—full-wave flashover

A—Curves for average insulation increase of 130 kv per foot and 110 kv per foot from figure 10*C* paper

B—Values obtained from wet flashover tests 66-kv pin-type insulator in series with pine crossarm, precipitation 0.2 inch per minute

C—Shows ratio of wet to dry flashover voltages

D—Pine crossarm alone

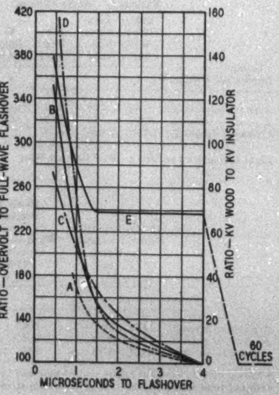


Figure 6 (right). Impulse flashover characteristics of pin insulators plus 36-inch wood crossarm. Positive polarity

A—Fir crossarm, dry, same as figure 11*A* of the paper

B—New pine crossarm, rain test 0.2 inch per minute plus 66-kv insulator

C—66-kv pin-type insulator alone

D—Overvoltage of crossarm alone (when in series with insulator)

E—Increase in strength of wood at short times to flashover. Ratio of kilovolts across wood to kilovolts across insulator wet flashover of the insulator only, for greater wood gaps the strength added by the wood probably will not greatly exceed a value of 50 per cent of those given in the paper. During these tests the overvoltages required to flashover at short times were measured for the insulator alone and for the insulator in series with wood and are shown on figure 6 for a 66-kv pin-type insulator with 36 inches of crossarm. Curve *D* of this figure represents the added strength of the wood in terms of overvoltage obtained by subtracting curve *C* from curve *B* and using the value obtained for the full-wave flashover as 100 per cent. Of considerable interest is curve *E* which shows the ratio between the flashover voltage of the insulator alone to the flashover voltage added by the wood at different times to flashover.

This ratio is almost constant down to 1.5 microseconds and then the insulation strength added by the wood becomes greater than that of the insulator. The other end of this curve indicates zero strength added for 60 cycles. Although the 60-cycle tests were made after the crossarm was subjected to a considerable number of impulses and was badly splintered, it might be expected that the 60-cycle flashover under rain of the added length of wood would be considerably decreased, since the pin of the insulator at this low frequency is practically at ground potential, when the wood is wet and flashover of the combination will occur at the flashover voltage of the insulator.

R. H. Earle (Line Material Company, South Milwaukee, Wis.): This paper is an interesting addition to our knowledge of the electrical properties of wood when used as an insulator on power lines.

This investigation, particularly when compared with former ones, shows rather a wide divergence of numerical test data. Such a range of test results is not at all surprising, however, in view of the other electrical properties of wood.

It is generally recognized that the resistance of wood shows enormous variations and depends not so much upon the species of the wood as upon the amount of moisture that it contains. This is not necessarily free visible moisture, but is rather the moisture which is absorbed by the cells of the wood from the atmosphere and the moisture content of the wood depends upon the humidity of the air to which the wood is exposed. Penetration of this moisture through the wood is rather slow. In ordinary outdoor weather conditions, such as wood structures encounter, the humidity is, of course, constantly changing with the result that the wood is practically always either absorbing or giving off moisture and since the moisture is transferred through the wood slowly, the actual moisture content represents a summary of the weather conditions for perhaps several months past.

It is accordingly seldom that the moisture content of a crossarm or pole will be uniformly distributed and since the electrical resistance of the wood depends so markedly on the amount of moisture that it contains, the pole or crossarm is electrically a bundle of resistors in series and parallel combinations and each resistor has a different value. The over-all resistance of the wood with nonuniform distribution of moisture is apt to be lower than would be the case if the moisture content were uniform throughout the volume of the specimen. Furthermore, in specimens of large cross sections there is likely to be a greater variation in electrical resistance across the section than is the case in pieces of smaller sections.

Applying these facts to some of the conclusions shown by the author's paper, we find a plausible explanation for several of the conclusions. For example, conclusion number 3 states that lower flashovers accompany larger wood sections. This would be expected since in general the resistance of the large sections is less uniform and lower than for the small sections.

In conclusion number 4, it is likely that fir and pine would show the same flashover if they had the same moisture content.

In conclusion number 5, it seems likely that the newly creosoted pine poles had not dried out to the extent that the old ones had, and conclusion number 5 also bears out the belief that impregnation only permits the pole to shed water a little more readily and aside from this does not influence the moisture content.

In conclusion number 6, it appears likely that the effect of the rain was twofold: first, to reduce the resistance of the wood by penetration of the fibers, and second, to form an electrolytic conducting path over the surface of the wood due to the rain dissolving impurities on the surface of the wood and thereby forming an electrolyte.

It appears that the extreme ranges of insulating values of wood crossarms, poles, and strain rods would be obtained by assuming the dry flashover of a corresponding length of air as the upper limit and by running further investigations on water-soaked timbers of some wood that soaks up moisture readily, such as beech, birch, or maple. Due to the comparative ease of impregnating these three woods with water, the results of electrical tests on such specimens would probably be lower than would be obtained in practice where the species would be more likely one of the firs, pines, or cedars.

Philip Sporn and **J. T. Lusignan, Jr.:** Mr. Andrews comments on the ability of wood to help prevent 60-cycle power current from following. This is an aspect of the wood insulation study which is well worth developing, but one which was not included in our program. There were so many factors affecting the impulse breakdown of wood, it was impossible for us to add tests involving 60-cycle follow current.

Mr. Stewart mentions some moisture studies he has made on wood poles and the variation in insulation value observed. We imagine that his work involves the 60-cycle phase of the problem, possibly with respect to pole burning or leakage. The manner in which he found moisture to affect insulation is interesting, but we feel that the behavior under impulse might be different from that encountered under power frequency. Our principal reason for this thought follows from the ability of moisture to serve as a conductor under the two conditions of 60-cycle and impulse surges. In other words, the moisture conduction must be electrolytic and involves movements of relatively slow electrolytic ions which would not respond to impulse voltages, lasting only a few microseconds, but which would serve as fair conductors of current for the longer power-frequency waves. We concur with Mr. Stewart in that a large number of tests are necessary in order to obtain dependable results on wood, chiefly due to the variable nature of that material.

The manner in which Mr. Smith has applied the results of laboratory studies to his line designs is most interesting. It is gratifying also that his results and conclusions concur so well with the authors'.

The tests which Mr. McCoy describes add very interesting information regarding the ability of wood to increase the insulation strength of a transmission structure. All of the reasons why wood does not add its full strength to the porcelain members are not clear to us as yet. We offered the suggestion that capacity distribution was an important factor, since under impulse, the conducting properties of moisture had little time to enter. However, we feel that the effect of the relative time lags of the porcelain and wood which Mr. McCoy mentions to be appreciable also. In fact it was a little easier at the time to explain the shape of some of our oscillograms by considering the possible time-lag curves of the porcelain and wood as well as what we consider to be the relative voltage distributions. The time-lag phenomenon which Mr. McCoy mentions wherein pin-type insulators and short suspension strings have shorter time lags than long strings is something which seems to hold with most long and short air breakdown paths over insulation. In other words, short rod-gap spacings and small switch insulators also have much flatter voltage-time curves than the longer gaps and insulators. We feel that this is probably due to electrode surfaces being relatively larger compared to the shorter air paths, thereby introducing most uniform electrostatic fields and approaching the flat time-lag characteristics of sphere gaps.

The information which Mr. Gross gives us regarding the greatly improved performance of his 33-kv line is very gratifying as it proves the value of laboratory tests in predicting the advantages, or disadvantages, of certain contemplated structure changes.

Mr. Earle goes into an analysis of the moisture factor in wood and its effect upon the electrical breakdown. We concur with his thoughts, although we want to emphasize that the distribution of the moisture has probably as much or more to do with the breakdown voltage as the total amount of moisture itself. In other words, we feel that the location of moisture areas both across the grain and along the axis are an appreciable factor in concentrating stress at certain points and thereby reducing the ultimate strength of the member as a unit. It is probably this factor that would make certain woods behave differently from others, even though they all have the same total moisture content.

Mr. Hagenguth supplements our paper with some very interesting thoughts and laboratory data. He compares the results of the Dowell-Foust paper with our results and in some cases notes agreement. He questions the straight-line curves of our figure 7 which conflict with their figure 30 and suggests that wood flashover is a creepage phenomenon. Although only a few individual pole specimens were tested, an appreciable number of wood lengths were flashed over from one to ten feet so that we believe that our data of figure 7 are fairly reliable, in so far as the specimens tested are concerned. We feel that these specimens were rather dry and typical of those poles to be encountered in service so that the values from the curves could be used rather freely for estimating purposes. We question the existence of much creepage phenomenon, particularly in impulse flashover where, at the short times involved, there is little chance for much creepage. We note a statement at the bottom of the above figure 30 stating: "Whether the wood is dry or wet, soaked in salt water or creosoted, makes little difference." With this our results disagree considerably. The lower ratio Mr. Hagenguth obtains with our values obviously results because of the higher flashover values we measured for wood alone. His results showing no change in air gap above ground checks similar tests made by us after the paper was written.

The disagreement Mr. Hagenguth finds between the crossarm data of figures 4 and 5 and the pole data of figure 7 is something which we noted during the tests. In other words, the cross-section phenomenon of figure 5 applies only to crossarms. Equivalent section areas of poles gave higher breakdown values, particularly with fairly dry poles. We felt that this was due to the unbroken grain structures of the complete poles wherein the discharges were kept along the outside surfaces, whereas the crossarms were cut from inner whole sections and add no continuous outer shells.

The additional wet tests made by Mr. Hagenguth to supplement the few which we made are most worthwhile. The "coincidence" he mentions in regard to the equality of the wet and dry flashover of the insulator alone, we feel to be a reality. On wet impulse flashovers of porcelain insulators we have usually found the voltage values to be about the same as the dry values.

A New Correlation of Sphere-Gap Data

Discussion and author's closure of a paper by D. W. Ver Planck published on pages 45–9 of this volume (January section) and presented for oral discussion at the instruments and measurements session of the winter convention, New York, N. Y., January 26, 1938.

P. H. McAuley (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): A simple and logical correlation of sphere gap sparkover data has been presented. It is particularly interesting to note that it brings out some of the discrepancies which we felt existed in the new AIEE values. The latter are averaged from tests made in different laboratories. Our values for the 50-centimeter sphere gap, for instance, are higher than the AIEE averages for the higher values of S/D. Table I, which includes some unpublished data, shows this condition. Similarly, our experimental curves show a value of 83 kv for positive-wave sparkover of the 6.25 centimeter gap at 3.125 centimeters spacing.

The practice of averaging test data by empirical methods is well justified in the case of the sphere gap. The original data were obtained in several different laboratories. The gaps were made of different metals and no doubt had different surface conditions. Mounting arrangements varied considerably. Clearances were different even for separate tests on the same gap in one laboratory. Considerably more data were available on the 25- and 50-centimeter sphere gaps than on the other sizes. For these reasons, our data were correlated according to average gradient curves for given ratios of S/D. The present author has extended this process with gratifying results.

Table I. Average Sparkover Voltages for 50 Centimeter Spheres; Westinghouse Data Compared to AIEE Values

S/D	Westinghouse Positive	Westinghouse Negative	AIEE Positive	AIEE Negative
0.5	550	520	547	519
0.6	609	575	605	573
0.7	662	623	655	615
0.8	702	662	698	651
0.9	732	694	732	681
1.0	758	721	758	707

S = spacing; D = diameter.

F. J. Vogel (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): The engineer responsible for the design of high-voltage apparatus has long used logarithmic paper for plotting experimental data and design information. This has been true for many years and may be found amongst the writings of engineers associated with several companies. A reference to this fact is given in the paper "Factors Influencing The Insulation Coordination Of Transformers, Part II" by P. L. Bellaschi and F. J. Vogel, ELECTRICAL ENGINEERING, volume 53, June 1934, pages 870–6. This fact does not detract from Mr. Ver Planck's excellent paper since design engineers have not seen fit to call attention to the relationships existing between parts of similar shape.

In figure 1 of Mr. Ver Planck's paper, sphere-gap data are presented using ratios of spacing to sphere diameter as parameters. These data are plotted in figure 1 of this discussion, using given spacings as parameters. For a given spacing, it is seen that, as the sphere diameter is increased, the breakdown voltage becomes constant and presumably approaches that for plain surfaces. Values given in figure 1 of this discussion, for ratios of spacing of sphere diameter of 0.1, by changing the scale as shown, become breakdown values for plain surfaces. For comparison, points taken from Mr. Schumann's book on page 27 are shown. Schumann's values appear to vary as the 0.92 power of the spacing even down to very short distances. The use of the 0.88 power in figure 3 of Mr. Ver Planck's paper does not seem to be justified and perhaps points out an error in the sphere gap data for these distances.

Figure 2 of this discussion shows data established for design pur-

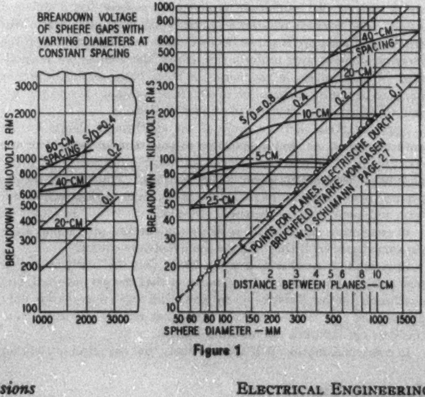


Figure 1

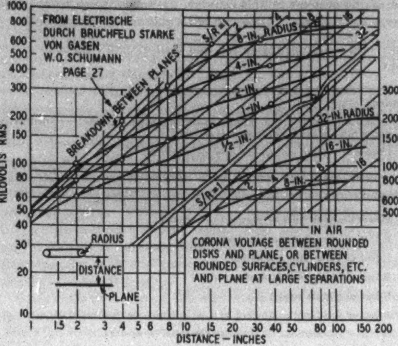


Figure 2

poses some years ago for the dielectric strength of rods and large-diameter parts. These data were obtained using sphere gap calibrations as standards of measurement but using spheres large enough to keep the ratio of spacing to sphere diameter one-half or less. The present calibration is not greatly different from the old one for these ratios, and the data are still sufficiently accurate for design purposes. It is to be noted that these data are plotted in the same manner as that used by Mr. Ver Planck. It is also of interest that these curves result in lower voltages than would be obtained by using the formulae derived by Mr. F. W. Peek, Jr., in his book "Dielectric Phenomena in High Voltage Engineering."

The use of curves, similar to figures 1 and 2, for design purposes, is desirable since they avoid calculation, give the design engineer a picture of the general relationships which exist, and permit the engineer to obtain his results quickly. When these two curves are used to help the engineer in forming judgment as to the clearances required for a given arrangement of parts, it is felt that figure 1 uses a better choice of co-ordinates, since the band showing the data is wider.

P. L. Bellaschi (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): Professor Ver Planck has presented an interesting paper on the sphere gap by plotting the new standard sphere-gap data on logarithmic graph paper and applying the principle of similarity for electrical discharges. It should be stated that the new standard sphere-gap calibrations have been well correlated in the first place, otherwise the good agreement in tables II and III would hardly have been expected.

In practice a number of factors affect the sparkover of the sphere gap and rules in its use are accordingly specified to minimize these effects within a permissible limit. In view of this, the agreement of the calculated values with the standard data is amply close to establish with assurance the various relationships indicated. Among these relationships the factor of relative air density and the probable corresponding corrections are worthy of attention.

Correlation of data naturally will bring to light major discrepancies. For example, in figure 3, the circled point near the 16-kv value, through which the dashed line is drawn, appears out of place. As a matter of fact, this is an extrapolated data point. In the final revision of the new sphere gap standards the few remaining extrapolated data points were subject to close scrutiny. As the result of tests in the various laboratories, the actual data value was established as being 17 kv, thus the dashed curve in figure 3 lines up quite well with the Schumann values.

The correlation of data in high-voltage technique and engineering is not only desirable but even essential. The author makes the statement that the similarity principle for electrical discharges seems not to have been fully appreciated by engineers in this country. This statement appears hardly warranted, for as a matter of fact, the relationship indicated in the paper for the sphere gaps as well as similar relationships for toroidal and cylindrical electrode arrangements have long been known to obey the principle of similarity for electrical design. These relationships have been put to useful application for a number of years by electrical engineers in the design of high-voltage apparatus.

D. W. Ver Planck: As pointed out by Mr. Bellaschi the consistently close agreement shown in tables II and III would have been unlikely if the data had not already been well correlated. In fact this independent check of the regularity of the data is evidence of the excellent work done by the subcommittee who prepared the new sphere gap calibrations.

Two of the three data which were found to be conspicuously out of line have since been revised in a subcommittee report dated December 1937, and they now conform to the new correlation. As a result, the circled point in the lower left-hand corner of figure 3 moves up to the solid line, and the last item in the 50 column of table III becomes 758 instead of 738. The single serious discrepancy remaining is the last item in the 6.25 column of table III, and if Mr. McAuley is right this value would become 83 instead of 81.8, which is within about 1.3 per cent of the calculated value.

Mr. McAuley's suggestion that the AIEE valves for 50-centimeter spheres may be a little too low is supported by the calculations for this size being consistently higher than the data. In fact if the data were revised in the direction indicated by Mr. McAuley, slight changes in the parameters K and α could be made which would result in an even closer agreement between calculations and data for all the other sphere sizes as well.

Mr. Vogel's figure 1 has for its theoretical basis the alternative functional form, equation 2, given in the paper, although generality has been lost by making the horizontal coordinate D instead of $D\delta$. The diagonal straight lines, which are for constant values of S/D, are described by the empirical formula

$$V = G(D\delta)^{\alpha}$$

where

$$G = K\left(\frac{S}{D}\right)^{\alpha}$$

and K and α have the same values given in the paper. These formulas and a table of numerical values for G were included in the paper as first submitted but for brevity were omitted before publication. While this form when plotted gives a greater separation of the lines for small values of S/D, it crowds them more when S/D is large.

Figure 2 indicates a relationship closely paralleling that in the paper and apparently known for several years but unfortunately withheld from publication until now. Here again, as in Mr. Vogel's figure 1 generality has been lost by not taking account of air density through the full use of the similarity principle. This figure would be enhanced in value if it were shown that the parameters of the straight lines for constant values of D/R conformed to an orderly relationship analogous to that given in figure 2 of the paper.

Mr. Vogel's figure 1 shows a considerable difference between Schumann's curve for a uniform field and the diagonal straight line for $S/D = 0.1$. Actually Schumann's data and the most recent revision of the AIEE data for $S/D < 0.2$ determine a single smooth curve. A reduction of Schumann's tabulated data to effective sine wave values gives points which lie from one to four per cent below those in the figure. If the discrepancy is not due entirely to an error in plotting it might be partly explained by a failure to take account of the fact that Schumann's data are for 20 degrees centigrade while those of the AIEE are for 25 degrees centigrade. Correction of Schumann's data to 25 degrees centigrade would shift the curve 1.7 per cent to the right since $\delta = 1.017$ for 20 degrees centigrade.

The use of the exponent 0.88 in equation 4 has been questioned. As stated in the paper the true curve for uniform field and small spacings is not linear when plotted to logarithmic scales, although the curvature is small. Figure 3 shows this curvature and that equation 4 gives a good fit for the range below $(S\delta) = 2.0$. For the range above $(S\delta) = 2.0$ and up to at least 20 the line determined by the parameters in table I for $S/D = 0.1$ is in close agreement with the data as can be seen in figure 3 and the first line of table II. The exponent 0.92 suggested by Mr. Vogel will be found to give good agreement in the intermediate range $0.8 < (S\delta) < 7.0$ if used with the coefficient 31.2.

Although it is much the best for the entire sphere-gap range above $(S\delta) = 2.0$, the power function type of formula, equation 3, is not as good for uniform field and small spacing as the formula

$$V = A\,(S\delta) + B\,(S\delta)^{1/2}$$

a type probably first used by Peek. The author has found that with $A = 24.4$ and $B = 6.8$ it agrees with Schumann's curve over the entire range $0.2 < (S\delta) < 9.0$ to within about 1.0 per cent.

A New A-C Network Analyzer

Discussion and authors' closure of a paper by H. P. Kuehni and R. G. Lorraine published on pages 67–73 of this volume (February section) and presented for oral discussion at the instruments and measurements session of the winter convention, New York, N. Y., January 26, 1938.

W. W. Parker (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): 1. The paper which has just been presented is very timely because of the growing appreciation of the economies and advantages which accrue from the use of such facilities. As distribution systems become more and more complex the solution of their growing problems naturally becomes more difficult. Also the time required for analytical solutions increases beyond all reason unless improved methods are available. So it is that the members of the Institute should find this subject of live interest.

2. Several a-c calculating boards have been built for various companies for their private use and the investment made by these companies has been adequately justified from the excellent results which they have obtained. Along this line the *Electrical World* article of November 23, 1935, by Mr. T. G. LeClair, will probably be of interest. (Several slides were shown showing the a-c network calculator in Chicago, one in New York, one at East Pittsburgh and a new one just installed in Chattanooga, Tenn. This new one is shown in figure 1 of this discussion and employs a 100-volt one-ampere power level which is most convenient to use. This is the very latest in a-c calculating-board design.)

3. It seems that the best contribution that can be made to this discussion and one which will be of interest to the majority will be a brief outline of experience with one of these calculators. In the past eight years approximately 200 studies have been made embracing most of the systems in this country, as well as some from India, Mexico, and Canada. These can be roughly classified as follows:

50 per cent—Voltage-regulation and load-control studies to determine:

(*a*). The effect of new lines or synchronous condensers to improve voltage conditions.

(*b*). Transformer application both as to size and type, i.e., whether quadrature or in phase tap-changing equipment is desired.

(*c*). Choice of system voltage for new installations.

30 per cent—Stability studies both static and transient for determining:

(*a*). Power limits of lines or portions of network.
(*b*). Effect of increased machine inertia.
(*c*). Effect of high-speed excitation.
(*d*). Effect of high-speed fault clearing.
(*e*). Effect of circuit layout.
(*f*). Effect of auxiliary equipment such as reactors, neutral resistors, etc.
(*g*). Effect of connections of "topping turbines" to system.
(*h*). Effect of high-speed reclosing.

In fact, it is very doubtful if the many applications of 30-cycle reclosures on 110-kv systems would have been made if it had not been for the possibility of adequately predetermining just what would happen with an alternating current network calculator.

15 per cent—Short-circuit studies for determining:

(*a*). Breaker application.
(*b*). Relay application.

5 per cent—Miscellaneous studies including secondary-distribution-network studies, theoretical and special design problems.

4. The above classification does not include the voltage-recovery studies which are covered in a paper presented at this convention by Messrs. Evans and Monteith, entitled "Recovery Voltage Characteristics of Typical Transmission Systems and Relation to Protector-Tube Application."

5. These studies have resulted in system economies by obtaining (1) the most effective use of existing facilities, (2) the deferring of capital expenditures, and (3) the determination of the type and location of new equipment.

6. Summarizing, the use of the a-c calculating board or network calculator is being more and more appreciated by planning engineers, system operators, utility managements and even, at least in one instance by a public utility commission.

R. C. R. Schulze (Public Service Electric and Gas Company, Newark, N. J.): There are several points which have not been mentioned yet in the discussion. The principal one I have in mind is the ease of operation of this new G.E. analyzer. There are several features

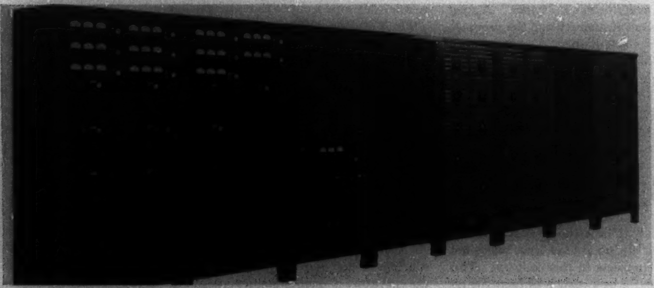


Figure 1

that make operation easier. One is the fact that loads can be set up as resistance and inductive reactance in parallel—so that watts and vars can be adjusted separately. Anyone who has tried to adjust a load with resistance and reactance in series will appreciate the convenience of the parallel connection.

Another factor is the arrangement of jacks and plugs. By making all connections in one (or two) places, the time involved in setting up a network is reduced, it is easier to see just what has been set up, and there is less chance of an error in the connections. We have had ten years of very satisfactory experience on our d-c table with this method of making connections.

The metering scheme is quite flexible, and permits reading almost anything desired. The ability to read watts and vars directly saves a large amount of slide-rule work. This not only saves a large amount of time in the analysis of the data, but it also saves time when adjusting the loads and generator units when making an analyzer setup.

Aside from the technical features of the master meters, the use of light-beam instruments is a distinct help to the analyzer operator. There is a reduction in eye-strain, since it is unnecessary to lean over a meter and squint at the pointer and its reflection with one eye. With the light-beam instrument the reading is essentially the same from any angle, so both eyes can be used. Also, it is possible to read these meters from anywhere in the room, not to the last tenth of a division, of course—but with a fair degree of accuracy. This means that one man can both adjust a load impedance and read the meter from that position, so that it is possible for one man to set up and adjust a network. Of course a second and even a third man would be useful, but the point is that this analyzer can be operated conveniently by one man.

I might add that our experience agrees with that mentioned by several of the previous discussers, in that it is possible to obtain results on a calculating table in much less time and with better assurance of accuracy than can be done by slide rule. This has been demonstrated on several occasions.

Our experience with our d-c calculating table, and with the results obtained on several visits to an a-c calculating table, has been very satisfactory. This experience, plus the complexity of the problems in system layout and operation, has led our executives to purchase the first step of an a-c network analyzer of the type described here.

Mr. Kuehni mentioned the types of problems which are usually solved on the a-c analyzer. A telephone friend of mine inquired about the possibility of studying the inductive coupling between a telephone line and our power lines, and one of our engineers has already suggested the study of transients on our analyzer. It is evident that we are going to have a variety of problems offered for solution on the analyzer.

It is our belief that this analyzer will not only be of considerable help in solving our technical problems, but also the savings resulting from its use will soon pay its cost.

E. W. Kimbark (Polytechnic Institute of Brooklyn, Brooklyn, N. Y.): The network analyzer described in the paper is the third principal design which has appeared. The first one, located at Massachusetts Institute of Technology and built jointly by the Institute and the General Electric Company in 1929, operates on a frequency of 60 cycles per second with a base voltage of 200 and a base current of two amperes. The second design, of which several boards have been built by the Westinghouse Company, uses 440 cycles and a base of about 100 volts, one ampere. The new design uses 480 cycles, 50 volts, 0.05 ampere.

The decrease in base voltage and current of each successive design is closely related to advances in the instrument art. The MIT network analyzer originally used suspended-coil wattmeters and thermocouple ammeters and voltmeters, all of which were sluggish, particularly the thermal instruments. The Westinghouse analyzers introduced dynamometer ammeters and voltmeters, one coil of which was inserted in the network while the other was fed from an instrument phase shifter. By supplying the larger part of the instrument power from the phase shifter, faster instruments were obtained without increasing the burden on the network. In the new analyzer described in the paper a great reduction in instrument burden without sacrifice of speed or accuracy has been effected by the development of suitable amplifiers.

I have had the opportunity of operating both the MIT network analyzer and the new General Electric analyzer and can testify to the improved "usability" (as the authors term it) of the new design. The elimination of calibration tables and plugging diagrams, the centrally located switches eliminating the need for an operator to move the instrument plugs, the fast easily read light-beam instruments, the fact that all three instruments indicate simultaneously, the good voltage regulation of the generator units and their independent continuous phase and magnitude adjustments, all contribute to this usability. Messrs. Kuehni, Lorraine, Thompson, and Slinger are to be congratulated on their excellent design of this new network analyzer.

A frequency in the neighborhood of 500 cycles per second has been chosen by two different groups of network analyzer designers and therefore seems to be well established. This frequency permits the use of smaller reactors and capacitors than those required at 60 cycles with the same voltage and current base. The use of a much higher frequency would increase the difficulty of limiting undesired electromagnetic and electrostatic coupling between network elements to a negligible value.

For experimental work the use of 60 cycles and of the higher voltage and current base has proven very desirable at MIT, as it permits various transformers, rotating machines, and instruments which are not part of the regular equipment of the network analyzer to be pressed into service.

A fair number of studies made on network analyzers have been concerned with transformers for controlling power flow in a loop by means of phase shift, and, although the required results have always been obtained from the analyzers, there has been no really satisfactory and convenient means of representing on the analyzer the phase-shifting transformers of the power system.

A very interesting and ingenious form of network analyzer (described in British patent specification No. 476,164) has been invented by Charles L. Blackburn, and put into service for the firm of Merz and McLellan in London. In the Blackburn network calculator an impedance unit (representing resistance and reactance) consists of two three-winding transformers with ratios controlled by tap switches and serving to connect magnetically four electrically independent circuits, the voltages of which represent respectively in-phase voltage, quadrature voltage, in-phase current, and quadrature current of the represented network. Such an impedance unit is furnished with four six-pole sockets, and the various units are interconnected by six-conductor flexible cords and plugs. This seems very complicated compared with the conventional form of network analyzer. However, the Blackburn calculator affords a ready representation of the following, all of which are obtained with difficulty on the conventional analyzers: (1) negative resistance, (2) transformer of complex ratio one to *a* for representing simultaneous faults, etc., (3) transformer with phase-angle control.

A large portion of the studies made on network analyzers are concerned with transient stability. In these studies generator swing curves are obtained step-by-step. The network analyzers have facilitated what was formerly the most laborious part of the process, the determination of the outputs of the several synchronous machines for given angular positions of their rotors. The other parts of the process, especially the computation of the changes of angular position of the generators in a small time interval, although relatively simple, are now the "bottle neck" of the procedure, keeping as many as four to six men occupied in contrast to one man operating the network analyzer itself. I venture the prediction that in the future generator swing curves will be drawn automatically on an output table, one pencil for each generator. This will be accomplished by an interconnection of a network analyzer and a differential analyzer, two integrators of the latter being used to solve the equation of motion of each generator. The electrical output of each generator unit will control through a watt relay and servo-mechanism the displacement of the first integrator. The output of the second integrator will rotate the phase-angle dial of the generator unit and also actuate a pencil. Such an arrangement would greatly speed up a stability study. One necessary feature of this arrangement, a generator unit having good voltage regulation, has already been developed by the authors of the paper.

H. L. Hazen (Massachusetts Institute of Technology, Cambridge): As a preliminary comment I wish to say that the paper by Messrs. Kuehni and Lorraine describes a fine piece of engineering equipment that has resulted from excellent engineering design. It should aid materially in developing the growing appreciation on the part of power-system engineers of the great power of an a-c network analyzer for the study of power-system operation. When these engineers have once experienced the facility with which they can study the operation of their system in miniature, they usually become enthusiastic about this method of study.

In this discussion I wish to review the development of such a-c network analyzers by stating in broad terms the nature of the essential design limitations in such devices, and to sketch briefly their development as determined by these limitations.

After the development of the technique of power-system representation by static apparatus (reference 6 of the paper) had been made, the design of a device which would be effectively applicable to actual commercial problems was found to be restricted by two primary limitations. The first and most important is that imposed by instrumentation. The second, less fundamental but a rather troublesome detail for the designer, is imposed by the difficulty of proportioning the units—particularly the inductive-reactance units—to properly represent their prototypes.

The effect of the instrument limitation is clearly shown by a consideration of the three principal designs of network analyzers to date; namely, the jointly designed and constructed MIT-General Electric Company analyzer erected at MIT, the Westinghouse design, and the General Electric design reported in the present paper.

In the analyzer at MIT the design with the metering originally provided was developed around the minimum burden for which a-c instruments of good accuracy could be built at the time of its development. These burdens were of the order of 1.5 to three milliamperes for potential elements, and roughly 0.1 volt for current elements. The criterion was adopted that an instrument potential element should not impose a burden, in comparison with a five per cent system load, of more than one or two per cent, and that an instrument current element similarly should not increase the impedance of a series element having a five per cent drop by more than about one per cent. This indicated current and voltage bases for the device of about two amperes and 200 volts, respectively. Assuming the analyzer to be in operation with ten generating stations each capable of 300 per cent of base output, the power requirement for the board is in excess of 12 kva, single phase, which evidently requires a sizable main supply generator in order to maintain reasonable voltage balance on the primary side of the phase-shifting transformers representing the generators. This indicated the use of 60 cycles because of the cost of substituting a higher-frequency generator for the 75-kva, 60-cycle, sine-wave generator already available in the laboratory. The limits of error set for this first design were one per cent for all units and somewhat lower limits for most of the instruments. These tolerances have been found, in the light of subsequent experience, to be unduly severe—a fact which has enabled the subsequent designs to be made somewhat more convenient in operation. The original metering equipment in this design was rather show—a condition that, as mentioned below, has been entirely corrected with recent modernization of the metering equipment of this original analyzer.

While this MIT-General Electric design was in progress the Westinghouse Company started on a somewhat different attack. They reduced the instrument burden by using dynamometer-type instruments and exciting one winding from an independent source that could furnish any desirable amount of power. This resulted in a sufficiently low burden for the other coil of the dynamometer instrument to permit its direct insertion in a network whose nominal current and voltage are somewhat less than those of the first-mentioned design. These instruments have a fast response. Two such instruments were used—one as a voltmeter with a low current requirement, and one as an ammeter with a low potential drop. This design uses a frequency of 440 cycles, which reduces the weight of the reactor units which are necessary to obtain appropriate reactance-to-resistance ratios, and also makes practicable reactor units with decade steps. This design of board has proved very useful. With the separately excited type of instrument the accuracy of readings is directly dependent upon the accuracy of setting of this excitation, both in magnitude and phase, and careful operation is therefore required to get good results. Nevertheless the effectiveness of this design is indicated by the fact that a number of such boards are in successful operation.

In the third design—that described in the paper under discussion—a quite different instrument technique has been used which has resulted in a very marked reduction in the current base. Specifically, the use of precision linear amplifiers in connection with semi-standard instrument elements has permitted the reduction of the voltage and current bases to the 50 volts and 50 milliamperes mentioned in the paper. This use of small base quantities, together with the use of 480 cycles, has resulted in relatively light, small units, decade steps for inductance units, and fast direct-reading instruments. This design also has the additional advantage of a low base current; namely, that the resistance of the long leads required for central plugging can be tolerated. Thus it appears that this third design has succeeded in realizing most of the features sought in an alternating-current network analyzer. This new analyzer is about as simple to set up and operate as is possible in such a device, remembering that the problems to be studied are inherently complicated in the sense of involving huge quantities of detailed numerical data.

It may be mentioned, however, that this new development—significant as it is—has not rendered earlier devices obsolete. The new metering techniques are just as applicable to a board such as the 60-cycle analyzer at MIT. In fact new, fast, direct-reading instruments have been incorporated into this analyzer which are described in a paper in preparation for publication. The new instruments, including a semi-standard portable wattmeter using a linear, degenerative feedback amplifier in the potential circuit, have been demonstrated to be sufficiently fast so that the speed at which readings can be made on this analyzer is determined not by the time which is required for the instruments to furnish indications, but by the rate at which the data can be reliably recorded. Thus the rate at which results are obtained is substantially the rate at which an engineer can record the readings obtained. This older analyzer requires somewhat more man power (estimated at one assistant operator) because the low-impedance base of this device requires interconnecting the units, and plugging for the current measurements, at the units themselves rather than at a central point. Another way in which this older analyzer has required somewhat more time is in the use of calibration charts for the reactance units—a method adopted at the time of the construction of this analyzer when smaller tolerances were prescribed than have proved necessary for the usual system studies. Now a much faster and simpler scheme of determining reactor unit settings is used in which an extreme mis-setting of a unit may amount to about three per cent as against about one per cent for the more accurate procedure. This considerably reduces the total time required for setting up a system on this analyzer. With the new metering equipment which has been in continuous service for some months, and the shorter set-up time involved by using the less precise calibration, the unit at MIT can deliver results very rapidly and, in the hands of good operators, remarkably reliable data are secured in very reasonable lengths of time not greatly in excess of that required on the newer devices.

In the experience obtained over a period of years in the use of this analyzer, both by students of electrical engineering and by the engineers of commercial companies, it has been found that skilled operators add greatly to its effectiveness. This is particularly true in the case of the use of the board by commercial-company engineers who, although thoroughly familiar with the characteristics and peculiarities of their systems, often do not visualize the way that the system conditions in which they are interested can be most easily represented in miniature, or how the information they wish to secure from the analyzer can be most readily obtained. It has been found that the present complement of a skilled operator and an assistant, together with two company engineers—one to direct the study as a whole, and the other to record data—make a very effective crew. The directing engineer states what he wants, and the operator determines the manner of obtaining the desired results. With such a group it is found that work progresses easily and rapidly, and that the mistakes occasionally made are caught before an appreciable loss of time is involved.

Summarizing, I wish to say again that the new development described by Messrs. Kuehni and Lorraine represents the excellent result of first-class engineering design which marks a distinct step forward in the facilities available for studying power-system networks. At the same time, older equipment in which advantage has been taken of newer metering ideas is still very serviceable and competes with the newer equipment on fully equal terms as to accuracy, and on nearly equal terms as to time required.

L. A. Nettleton (Consolidated Edison Company of New York, Inc., New York, N. Y.): Mr. G. C. Crossman, of the Consolidated Edison Company, and I recently had the privilege of using the G.E. network analyzer in connection with some studies of the New York Consolidated Edison system. We found the network analyzer to be simple to set up and operate. The per cent basis for marking the line and load units and instrument scales proved to be convenient since our impedance values which had previously been established on a 100,000-kva base could be transferred directly to the analyzer units with no additional conversion, so that per cent watts and vars read directly as megawatts or megavolt-amperes.

In taking readings, the light beams swing across the instrument scales and become stationary almost instantaneously upon closing the circuit key. After turning the scale-selector switch to give the best scale reading, the voltage, current, and watts are read and the wattmeter switch thrown to read vars. One operator could read all four quantities on successive circuits as fast as they could be recorded. We expected some confusion to result from the large number of scale multipliers but in practice there was no trouble. The voltage multiplier was unity in practically every case and only the current multiplier, which applied also to watts and vars, was recorded with the readings.

The 100- or 150-scale division instruments can be read to about one-fifth of a scale division and the phase-angle dials can be set or read to about one-fifth of a degree. In comparing the readings of six or more circuits tied to a common point it was found that the total watts entering that point invariably checked to within one per cent of the total watts leaving the point.

The generator units proved to have very flat regulation, requiring only slight readjustment when loading conditions were altered. In short-circuit studies it was found convenient to throw the supply voltage switch to one-half or one-quarter normal voltage temporarily in order to check the short circuit current and insure that no circuits would be overloaded on normal voltage.

The instrument system worked perfectly during the two weeks of the study. There is no danger of overloading the ammeter since an off-scale deflection automatically short-circuits the ammeter shunt.

It is sometimes desirable to close two or more circuit keys at the same time to meter the combined circuits. However, it occasionally happens that two keys are inadvertently closed at the same time, joining together two circuits which may be at different potentials. Sometimes this causes a generator fuse to blow, resulting in a few minutes delay to locate and replace the blown fuse. As this condition may not be discovered immediately and as it is not always easy to determine visually whether or not a fuse is blown, it might be desirable to provide some means for indicating the presence of a blown fuse.

In our studies it was sometimes necessary to consider the effect of load-regulating transformers, controlling both voltage and phase angle on parallel feeders. The voltage control feature presents no difficulties since the analyzer is provided with auto transformers which can be connected in the circuit at the appropriate location and adjusted to the desired voltage ratio. However, where there is a difference in the desired phase angle on parallel feeders the problem is not so easy when the system is set up on a "one line" or single-phase basis. There are several methods of obtaining this difference in angle on one of the feeders, but all require considerable adjustment before a balance is obtained. In our case the required displacement was obtained by one generating unit in series with the circuit and the watts or vars supplied to or taken from the circuit by this unit were neutralized by another generating unit together with a series stabilizing reactance connected in shunt with the circuit. For each change in loading conditions these two generators were adjusted in voltage and phase angle until the desired displacement angle was obtained, and until the volts, watts, and vars read the same on each side of that point.

Since these methods require considerable juggling, it would be a great convenience in problems involving such load regulating equipment if devices were available that would introduce the desired phase angle in a circuit and maintain that phase angle difference regardless of voltage or loading.

G. C. Crossman (Consolidated Edison Company of New York, Inc., New York, N. Y.): The increasing size and flexibility of modern power systems has created problems which can no longer be solved quickly and easily by analytical or even d-c calculating-board methods but which require for such solution some sort of a-c calculating device, such as the network analyzer described by Mr. Lorraine and Mr. Kuehni. Continued system growth of many large power companies tends to increase the number of these problems to the point where it becomes desirable for these companies to own and operate such a calculating device themselves rather than rent existing ones. It would seem that the very small currents and high frequency employed in this network analyzer would present rather difficult maintenance problems to such a power company. While most of the testing and maintenance of the line, load, and generator units of the analyzer could probably be handled by the average test department, it would seem likely that the testing and adjustment of the amplifiers and master instruments would require rather special instruments and special training not ordinarily available in such test departments. It was not brought out in the paper whether the trimmer capacitors on the potential shunt are adjusted once and for all at the factory or whether they might require subsequent adjustment; such adjustments would seem to require rather sensitive and special instruments.

It would be interesting to know what instruments and general methods the authors have used or plan to use for the maintenance of their analyzer.

R. N. Slinger (General Electric Company, Schenectady, N. Y.): In their paper entitled "A New A-C Network Analyzer," Messrs. Kuehni and Lorraine have given an excellent description of this new device. As one of those who have had occasion to use the analyzer extensively since it was placed in operation, I have been impressed more and more as time goes on by the many ways in which the device can help engineers solve their problems and it would seem that a few remarks on this subject might be appropriate.

Any power company engineer who has been faced with the necessity of obtaining answers to any of the following problems, and particularly when on an emergency basis, will immediately recognize how helpful it would have been to him to have an analyzer available for his use. A list of some of these typical types of problems for which the network analyzer is especially adapted might include the following:

1. Effect of various amounts of synchronous condenser capacity on bus voltages at important stations under both normal and emergency operating conditions.
2. Determination of best location for new generation.
3. Determination of best arrangement of station or system connections for new generation in order to obtain the maximum degree of system stability and investigation of the effect on breaker interrupting duty.
4. Determination of the actual value of a proposed addition to a transmission or distribution system in terms of improved voltage conditions, increased stability limits, or reduction in the loading of existing circuits.
5. Determination of wattless flow around a system and selection of methods for reducing it to a minimum.

6. Determination of bank ratings in the case of a new transformer bank that is to be connected in a loop circuit or a network or the determination of the ratings of individual windings of a three winding unit for use in a loop circuit or network.

7. Determination of improvement in system voltage conditions to be expected from the addition of a voltage regulating transformer or a phase shifting transformer to a loop circuit or a network.

8. Determination of the time required for a synchronous generator to pull out of step after complete loss of excitation and the effect upon the rest of the system.

9. Investigation of the starting and pull-in characteristics of large synchronous motors connected to a transmission system at some distance from the nearest generating station.

10. Determination of relay currents under three-phase and unbalanced fault conditions, particularly on tie lines during transient disturbances.

All of these problems can be and are studied in part by tedious long-hand calculation methods, but the excessive amount of time and labor involved usually places a serious limitation upon the number of operating conditions that it is feasible to consider. When such problems are solved with the aid of an a-c network analyzer, it is possible to investigate many different operating conditions in a surprisingly short space of time. As a result, studies made in this manner can be and usually are much more comprehensive in nature, covering not only normal system connections and the variations that may be under consideration but also the emergency conditions which unfortunately seem to get overlooked in the average long-hand study until after these contingencies arise.

Perhaps it should also be pointed out that once a system network is set up on an analyzer, such as the one described by Messrs. Kuehni and Lorraine, it is possible to make several different types of studies on the system with practically no change required in the connections except to represent changes in the original conditions. Thus a power-flow study may be made on a system in order to determine circuit loadings, bus voltages, condenser capacity requirements, etc., followed by an investigation of breaker interrupting duties under short-circuit conditions. This may in turn be followed by an investigation of transient stability limits or breaker times required to maintain stability with certain specified power transfers. During the course of the stability study, tie-line currents may be read at troublesome relay locations with perhaps a more comprehensive short-circuit and relay study made immediately afterward. Thus the engineer responsible for system planning and design is able to obtain a much better appreciation than he could otherwise have, of the influence of existing and proposed stations and equipment upon system bus voltages, power flow, stability limits and breaker duty, and he can feel considerably more certain that he has not overlooked something of importance that should have been taken into account.

The analyzer which Messrs. Kuehni and Lorraine describe is particularly helpful in this respect since it enables the power system engineer using it to think in terms of system quantities with which he is already familiar, thereby enabling him to keep his thinking straight. This is made possible by calibrating the entire analyzer including the master instruments in per cent. Thus it is a simple matter to interpret the readings in terms of system quantities; for example, a reading of 65 per cent watts and 40 per cent vars would represent 65,000 kw and 40,000 reactive kva, respectively, on a 100,000-kva base or half those amounts on a 50,000-kva base.

This particular analyzer has been in operation only about six months but in that time it has been used for ten different studies by engineers representing 13 different power companies and industrial concerns. The studies have varied in length anywhere from one day to four consecutive weeks and have covered practically every one of the various types of problems previously listed. It is safe to say that these studies have not only solved specific problems but they have also done much toward giving a clearer picture of system performance to the respective groups than they have ever had before. Furthermore, the results of the studies in some instances were quite different than had been expected and occasionally brought to light new problems that had previously passed unnoticed.

H. P. Kuehni and R. G. Lorraine: The authors are highly pleased by the reception given this paper. The discussion has been of such constructive character as to add materially to its value. The growing importance of the analyzer as an engineering tool, particularly for utility engineers, has been stressed. It is gratifying to know that those who have used the analyzer described in the paper feel that it has the degree of accuracy and the speed, facility, and ease of operation that was expected.

Professor Hazen called attention to the importance of proper instrumentation in the design of an analyzer. It is interesting to learn that advances in measurement ideas were incorporated in the MIT network analyzer to greatly improve the speed of taking readings.

Mr. Parker has presented a comprehensive statistical analysis of the practical types of studies made with the analyzer. Of course, the individual studies are not, in general, quite as clean-cut as one might be led to believe from his per cent tabulation. In practice each study more nearly is a combination of several types, one usually leading to another as the occasion arises. Mr. Slinger described many of the ways in which the analyzer may be of help to the engineer.

Mr. Schulze has given an excellent discussion of the salient design features which contribute to the ease of operation of the new analyzer based upon his wide experience with network analyzers of different types.

There are undoubtedly network studies in which it would be desirable to have available special units to represent phase-shifting transformers as suggested by Messrs. Kimbark and Nettleton. However, these cases are quite rare at present. Because of the cost of such units, it was decided to use generator units for this purpose, even though they are admittedly somewhat more awkward to operate than phase shifting transformers might be. The integrators and servo-mechanisms used with generators for automatically recording swing curves, which Mr. Kimbark predicts will be integral parts of future analyzers, are at present in the same category with the phase-shifting transformers. These devices can be and may be incorporated in the analyzer when there is sufficient economic justification.

Mr. Crossman is of the opinion "that the testing and adjustment of amplifiers and master instruments would require rather special instruments and special training not ordinarily available in testing departments." It is the authors' belief that any one familiar with ordinary instrument calibration routine methods is perfectly qualified to check and adjust the instrument system by following a few instructions of very simple nature. With the exception of perhaps a small portable commercial cathode-ray oscillograph of a type which are so widely used today, no special instruments are required.

All adjustments of the instrument circuits, amplifiers, potential divider, and current shunts are made initially. Many months of experience has shown that with normal operation the adjustments and the calibration of the instrument system remain quite permanent over long periods of time. The parts which may require re-adjustments are designed to be easily accessible and are equipped, for example, with rheostats, trimmer capacitors, or test connection facilities.

In view of the stable performance of the instrument system, occasional simple routine check tests should suffice to indicate that the instrument system is in order. For this purpose a portable voltmeter calibrated at 480 cycles may be used together with the measured resistance part of a circuit unit to indicate that the light-beam voltmeter, ammeter, and wattmeter read correctly.

The potential divider and current shunt phase angles may be checked and if necessary adjusted with the aid of the analyzer master wattmeter, knowing that the power into an analyzer capacitor unit is negligibly small.

The amplifiers have simple rheostat and capacitor adjustments for matching the amplifier input and output in magnitude and phase angle. The cathode-ray oscillograph mentioned above, connected to a suitably located amplifier test jack, is merely used as a sensitive high-impedance null instrument to adjust the output-input voltage differential to zero.

In regard to maintenance of the rest of the analyzer circuits and units, it simply consists of checking and correcting with the analyzer master instruments any abnormal circuit performance. The conservative design of the elements comprising the circuit units and the low resistance contacts and connecting leads assures that the calibration of these elements remains very stable.

Subharmonics in Circuits Containing Iron-Cored Reactors

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Synopsis: The performance of a series circuit consisting of a resistor, a capacitor, an iron-cored inductor, and a sinusoidal impressed voltage is specified by the non linear differential equation

$$\frac{d\psi}{dt} + ri + L\frac{di}{dt} + \frac{1}{C}\int i\, dt = E \sin (t + \theta)$$

in which ψ is an odd function of i. Particular solutions of this equation, showing the effect of varying boundary conditions, are obtained by means of the differential analyzer. An approximate analytical method for determining the types of oscillations possible in this circuit is developed. Results obtained by this method are compared with differential analyzer solutions.

Introduction

THE BEHAVIOR of circuits involving iron-cored inductors, which saturate so that their reactances cannot be considered constant, has been studied for many years. Bethenod[1] in 1907 and Martienssen[2] in 1910 both noted that in such circuits more than one sustained value of current could be obtained for the same value of applied voltage and the same circuit parameters. The current obtained in a given case was found to depend on the way in which the phenomenon was initiated. In 1926, Mauduit[6] obtained oscillograms in which the principle component of current had a frequency which was a submultiple of the frequency of the applied voltage. Such a component of current will be referred to as a *subharmonic*. Mauduit's circuit was studied more completely by Fallou,[7] who suggested that such an arrangement could actually be used as a frequency divider.

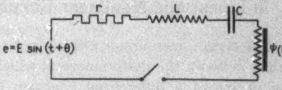
Pederson,[8] who has made a very thorough study of subharmonics in linear circuits in which the parameters vary periodically with time, also mentions the possibility of obtaining subharmonics when the inductance varies with the current.

Recently, the increasing use of series capacitors to overcome the inductive drop in transmission systems has revived interest in subharmonic oscillations. In this case it is desirable to know the range of circuit parameters for which these subharmonic oscillations can exist, and the kinds of shock excitation which may initiate them, so that they can be avoided in the engineering of series capacitor installations. This aspect of the problem was treated in some detail in a recent paper by J. W. Butler and Charles Concordia.[26]

I. Statement of the Idealized Problem

This paper is limited to considerations of the simplest known circuit which exhibits the phenomenon. This is a series circuit consisting of a constant resistance, a constant inductance, a constant capacitance, and an inductor which varies due to magnetic saturation of its core. A sine wave voltage of unvarying maximum value is impressed. This circuit is shown in figure 1.

Figure 1. Idealized circuit



The following idealizing assumptions regarding the inductor will be made:

1. The winding resistance and distributed capacitance are equal to zero.
2. All hysteresis effects in the inductor may be neglected. Hence the saturation curve is a single-valued function.
3. There are no eddy currents in the core.

Any resistance actually present in the inductor may be lumped with the external resistance. Consistent with these assumptions the voltage across the reactor may be expressed as:

$$e = \frac{d\psi}{dt}$$

where ψ is the flux linkages of the inductor. The flux linkages and the current through the inductor are related in the way expressed by the familiar *B-H* curve of a circuit containing iron. Since this relationship is an odd function, the current may be expressed by the series

$$i = a_1\psi + a_3\psi^3 + \ldots. + a_{2m-1}\psi^{2m-1} + \ldots. \qquad (1)$$

The equation of the circuit is

$$ri + L\frac{di}{dt} + \frac{1}{C}\int i\, dt + \frac{d\psi}{dt} = E \sin (t + \theta)^* \qquad (2)$$

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[1] For all numbered references, see list at end of paper.

* The treatment in this paper is entirely mathematical; the symbols r, L, i etc., may be regarded as representing the physical quantities in any consistent unit system including per-unit.

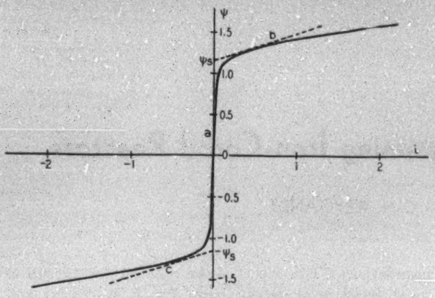

Figure 2. Experimental saturation curve and the straight-line approximation used in this paper

The problem is to determine under what conditions sustained currents of frequency lower than the impressed frequency can exist and to calculate the values of such currents.

II. The Effect of Initial Conditions in Linear and Nonlinear Circuits

The term *linear circuit* will be used to describe an electrical network the performance of which may be specified by means of a differential equation having coefficients which are not functions of the dependent variable, i.e., the coefficients are constants or functions of time only.

The term *nonlinear circuit* will be used to describe any circuit to which these conditions do not apply.

Equation 2 evidently may be transformed into a differential equation of the second order. Since ψ contains i to powers other than the first, the equation is *nonlinear*. It can be proved by application of the Lipschitz[25] condition that there exists one and only one solution of this equation corresponding to a given set of initial conditions.

Before considering the solution of the nonlinear equation let us recall the nature of the solution of a similar *linear* differential equation with constant coefficients. Here it is well known that the solution corresponding to any set of initial conditions is unique. The solution consists of two parts, a complementary function and a particular integral. In the case of a second-order equation, the complementary function contains two constants which depend upon the initial conditions whereas the particular integral is independent of the initial conditions. In equations representing practical electric circuits the complementary function dies out as time goes on, so that after sufficient time has elapsed the solution of the equation, for all practical purposes, is given by the particular integral alone. Therefore, in a circuit having constant parameters, the current which flows after a time long compared with the time constant of the circuit is independent of the initial conditions. This current is ordinarily called the *steady-state* current. That part of the current which becomes sensibly zero after a sufficient interval of time, is called the *transient* current. The division of the total current into *steady-state* and *transient* components is possible mathematically by virtue of the property peculiar to linear differential equations known as superposition.

In nonlinear equations this principle of superposition does not hold; the solution for any value of the independent variable, however large, will in general depend upon the initial conditions. Physically, this means that the current which flows in a nonlinear circuit a week after the switch was closed may depend on the phase angle of the impressed voltage at the instant of closing the switch. An example of this, in a special case where an analytical solution is possible, is given below. In many physical systems the solution after a relatively short interval of time has elapsed is periodic. It is therefore possible, by extrapolating this periodic solution back to the instant at which the phenomenon was initated, to split the solution into two parts, the periodic part which may be called a *steady state*, plus a part which dies out rapidly with time and hence might appropriately be called a *transient*.

Neither of these separate parts can be derived independently from the differential equation and the specified boundary conditions, as is possible with linear circuits. Furthermore, there is no reason to expect the periodic part to be independent of the initial conditions. This revision of the usual concept of "steady state" and "transient" is necessary if the terms are to be applied to currents which flow in nonlinear circuits.

Example Showing the Effect of Initial Conditions

Consider equation 2 in the special case when $E = 1$; $r = 0$; $\psi = Ki^{1/3}$:

$$L\frac{di}{dt} + \frac{1}{C}\int i\,dt + K\frac{d(i^{1/3})}{dt} = \sin(t+\theta) \tag{3}$$

Now let us assume the solution

$$i_a = A^3\cos^3\frac{t+\theta}{3} = \frac{A^3}{4}\left[3\cos\frac{t+\theta}{3} + \cos(t+\theta)\right] \tag{4}$$

where A is an undetermined coefficient.

Substituting the value i_a from (4) in (3) and equating coefficients of sines and cosines of like multiples of $(t+\theta)$ the following relationships can be derived:

$$K = \frac{3}{4}A^2\left[\frac{9}{C} - L\right]$$

$$4 = A^3\left[\frac{1}{C} - L\right] \tag{5}$$

Now let us assume another solution:

$$i_b = B^3\cos^3(t+\theta) = \frac{B^3}{4}\left[3\cos(t+\theta) + \cos 3(t+\theta)\right] \tag{6}$$

In the same way this gives rise to the relationships:

$$1 + KB = \frac{3}{4}B^3\left[\frac{1}{C} - L\right]$$

$$3L = \frac{1}{3C} \tag{7}$$

Any set of values of the parameters L, C, and K, which satisfies both the relationships (5) and (7) will form an equation having at least two solutions of the forms (4) and (6).

Such a set of values is:

$$L = 0.1$$

$$\frac{1}{C} = 0.9$$

$$K = 17.5$$

Equation 3 then becomes:

$$0.1\frac{di}{dt} + 0.9\int i\,dt + 17.5\frac{d(i^{1/3})}{dt} = \sin(t+\theta) \tag{8}$$

The two solutions are:

$$i_a = [-1.39\cos(t+\theta) + 0.462\cos 3(t+\theta)] \times 10^{-4} \tag{9}$$

$$i_b = 3.75\cos\frac{t+\theta}{3} + 1.25\cos(t+\theta) \tag{10}$$

That these are both solutions may be readily verified by substituting them in (8). They are entirely different in character, as one contains a term of higher frequency than the applied voltage, and the other a term of lower frequency than the applied voltage.

If the origin of time is taken as the instant when the current is passing through zero, we have for (9) the boundary conditions:

$$\theta = \frac{3\pi}{2} \qquad i_0 = 0 \qquad q_0 = 0.000124$$

The corresponding boundary conditions for (10) are:

$$\theta = \frac{3\pi}{2} \qquad i_0 = 0 \qquad q_0 = -10.0$$

If the switch were closed at the instant at which the voltage is at a negative peak and with an initial charge of −10 on the capacitor, the solution is given by equation 10. If the switch were closed at the instant at which the voltage is at a negative peak and with an initial charge of −0.000124 on the capacitor, the solution is given by equation 9. In general, if there were any other initial charge on the capacitor, or if the switch were closed at any other instant of time, a still different solution might be expected.

III. Physical Mechanism of Subharmonic Production

The physical nature of the phenomenon of subharmonic production is readily visualized if we consider the behavior of the circuit during the first few cycles after closing the switch. For simplicity let resistance be neglected, and let the boundary conditions be those of equilibrium. Let the phase angle θ be taken equal to zero. If the flux linkages in any constant inductor are included in ψ, the term $L\frac{di}{dt}$ does not appear, hence we have:

$$\frac{d\psi}{dt} + \frac{1}{C}\int i\,dt = E\sin t \tag{11}$$

from which

$$\psi = -E\cos t - \frac{1}{C}\iint i\,dt^2 + K$$

and since for $t = 0$ we have $\psi = 0$ and $i = 0$ it follows that $K = E$, hence

$$\psi = E(1-\cos t) - \frac{1}{C}\iint i\,dt^2 \tag{12}$$

The magnetization curve representing ψ as a function of i will be taken as shown by the solid curve in figure 2. The value ψ_s is the flux linkages at the knee of the curve.

Since the current during operation on the unsaturated portion of the ψ versus i characteristic is negligible, no appreciable current flows until $\psi = \psi_s$. From the closing of the switch until $\psi = \psi_s$ the integral $\frac{1}{C}\iint i\,dt^2$ is therefore negligible and ψ is given nearly enough by $\psi = E(1-\cos t)$. The end of the interval during which this holds is indicated in figure 3B by T_1. The value of T_1 is given by

$$T_1 = \cos^{-1}\left(\frac{E-\psi_s}{E}\right) \tag{13}$$

Above the saturated value ψ_s the saturation curve may, to a reasonable approximation, be represented by a straight line. Let L[*] be the slope of this line and let λ be the time measured from the instant T_1. Then during the succeeding brief interval the differential equation

$$L\frac{di}{d\lambda} + \frac{1}{C}\int i\,d\lambda = E\sin(\lambda + T_1) \tag{14}$$

holds, subject to the boundary conditions:

$$\lambda = 0 \qquad i = 0 \qquad q = 0.$$

The general solution of equation 14 is:

$$i = \frac{CE}{LC-1}\sin\left(\lambda + T_1 \pm \frac{\pi}{2}\right) + A\cos\left(\frac{1}{\sqrt{LC}}\lambda + \phi\right) \tag{15}$$

For the purpose of this illustration it will be sufficient to restrict the solution to the case in which the negative sign is taken in equation 15. This corresponds to the physical case of $L > \frac{1}{C}$. Inserting boundary conditions in (15) we have

$$i = \frac{CE}{1-LC}\cos(\lambda + T_1) - \frac{1}{1-LC}\sqrt{\frac{C}{L}} \times \sqrt{LC(E-\psi_s)^2 + \psi_s(2E-\psi_s)} \times \cos\left[\frac{\lambda}{\sqrt{LC}} + \tan^{-1}\sqrt{\frac{\psi_s(2E-\psi_s)}{LC(E-\psi_s)^2}}\right] \tag{16}$$

This equation holds only until the flux linkages are reduced to the value ψ_s. At this time the current effectively stops and whatever charge has been accumulated on the capacitor is trapped there until the flux linkages again reach the value $\pm\psi_s$. During the interval from T_1 to the instant at which current stops flowing, the value of ψ is given by equation 12, in which the term $\frac{1}{C}\iint i\,dt^2$

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[*] Note that L as used in this section is the sum of the saturated inductance of the reactor plus the external linear inductance.

must be evaluated by substitution of the proper function for i. However, after the charge is trapped on the capacitor, this term becomes $\frac{Q}{C}t$. Hence the flux linkages are given by the expression

$$\psi = E(1 - \cos t) - \frac{Q}{C}t - \psi_a \tag{17}$$

where ψ_a is the value of flux linkages corresponding to $\int\int i\,dt^2$ when current ceased to flow. It is evident that equation 17 represents an oscillatory flux linkage which has a negative component proportional to time. A value $-\psi_s$ will therefore be reached in a time which is short if Q is large and relatively long if Q is small. It is possible that Q may be so small that several cycles of the impressed frequency may have elapsed before another pulse of current occurs. *It is the trapping of charge on the capacitor which gives rise to subharmonic oscillations in circuits of this type.*

IV. Summary of Available Methods of Solution

Step-by-step methods such as that given in the preceding section are very laborious and give the solution only for the parameters and initial conditions for which they are carried out. Such methods, however, have the advantage of shedding some light on the physical nature of the phenomena which the equation describes.

The differential analyzer furnishes a very convenient method of solving equation 2. While analyzer solutions are subject to the same limitations as those obtained by the step-by-step method, in that each solution is good only for the parameters and initial conditions for which it is made, the labor of carrying out a solution is not great. Therefore, it is possible, by carrying out a large number of particular solutions, to generalize to a certain extent. A severe limitation of this method arises from the fact that the analyzer insists on giving the whole solution, including the transient, and refuses to produce a steady state until the transient has died out. In circuits in which the damping is slight, which unfortunately are those of greatest interest, this involves a considerable loss in time as well as a sacrifice in accuracy due to accumulated errors in runs of long duration.

The various other methods which have been applied to nonlinear circuits, Taylor's series expansion, isocline and other graphical methods, the analytical method of Kryloff, etc., do not seem to be of very great value in the solution of the present problem.

Recursion formulas for the boundary conditions which may produce subharmonics are developed in the appendix. This method is an extension of the method used by Boyajian[17] for calculation of this circuit when the initial conditions are such as to produce no subharmonics. The method suffers from the fact that a considerable idealization of the problem is necessary in order to carry out the mathematical details, but has the peculiar advantage, not found in other analyses of the problem, of expressing the possible steady states in terms of the circuit parameters without regard to the method of initiating the phenomenon. Although not applicable to all cases of subharmonic production, this scheme seems to be the most powerful yet developed in those cases where it applies. A combination of differential analyzer and recursion formula solutions, the differential analyzer to give the solution in the initial period and the recursion formula to give the solution after a considerable elapsed time, is perhaps the most revealing analysis possible at present.

In figure 3, a comparison of solutions by the differential analyzer, step-by-step calculations, and by the recursion formula method given in the appendix, is shown.

Figure 3*A* shows the current, applied voltage, flux linkages, and charge on the capacitor as obtained by the differential analyzer. In this case, the relation between ψ and i is that shown by the solid curve in figure 2. The other parameters are

$E = 0.7$ $\qquad L = 0.197$ $\qquad C = 4.66$

It will be noted that at time T_B all the quantities have substantially the same values as at the time T_A. Therefore, from the point T_B on, the curves will be a periodic repetition of the portion from T_A to T_B. The parts of the curves from the start to the point T_A contained *transient* portions of the solution in the sense in which the word is used in this paper. After point T_A, the solutions are *steady state* in the sense in which the term is used here.

The curves in figure 3*B* were obtained by the step-by-step method used in the preceding section to illustrate the physical nature of the phenomenon. For this calculation it was assumed that the resistance was negligible. The value of the flux linkages at the knee of the saturation curve was taken to be $\psi_s = 1.15$. The value of inductance used was the sum of the external inductance and the slope of the saturation curve above the knee; this value was taken as $0.197 + 0.303 = 0.500$. The three terms of equation 17 are shown by the dotted curves; the resultant flux linkages are shown by a solid curve. This method could be carried on indefinitely, or until at the end of some interval all the boundary conditions were found to be the same as at the end of some previous interval, in which case the remainder of the solution would of course be a repetition of the part between points of indentical boundary conditions.

In order to eliminate a prohibitive amount of piece-by-piece calculation to find repetitive boundary conditions (quite conceivably for certain circuits and initial conditions none might exist) the method worked out in the appendix is used for calculating the steady state. This yields the curves shown in figure 3*C*. The solution turns out to be a subharmonic oscillation designated in the appendix as the third kind. A comparison with the upper curves shows that the agreement with the analyzer solution is very good in spite of the idealizing assumptions made.

Appendix—Recursion Formulas for Steady State Subharmonics

In order to circumvent mathematical difficulties, resistance will be neglected in this analysis. Further it will be assumed that the

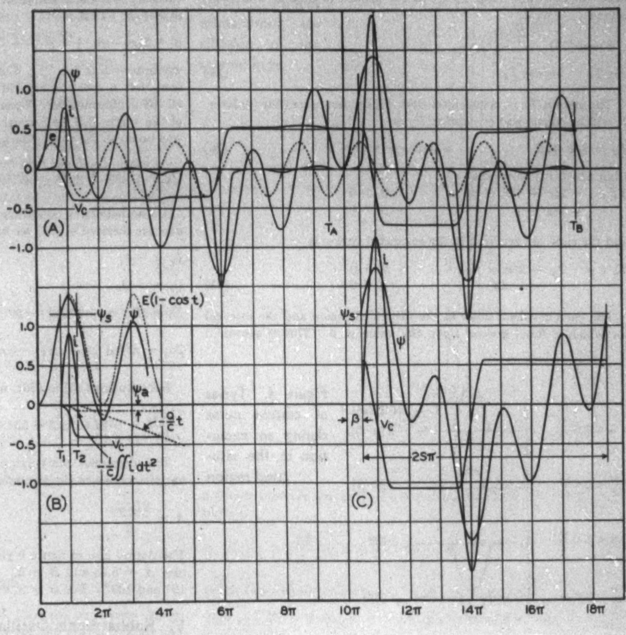


Figure 3

(*A*)—Differential-analyzer solution of the equation

$$\psi(i) = \int\left[0.7\sin t - 0.05i - 0.197\frac{di}{dt} - 0.214\int i\,dt\right]dt$$

(*B*)—Approximate check of analyzer solution by step-by-step calculation

(*C*)—Approximate steady-state solution calculated from recursion relations

Scales:

Flux linkage is shown in per unit

Time is shown in radians of the impressed sinusoid

To obtain current in per unit multiply scale shown by 0.542

To obtain capacitor voltage in per unit multiply scale shown by 0.27

Maximum value of impressed voltage is 0.7 per unit

magnetization curve of the reactor may be represented by the dotted straight lines shown in figure 2. The curve consists of three portions labled in the figure *a*, *b*, and *c*. Curve *a* consists of the ψ axis between the values $-\psi_s$ and ψ_s. For curve *b* we have $\psi = Li + \psi_s$, $i > 0$. For curve *c* we have $\psi = Li - \psi_s$, $i < 0$.

I. Operation on Curve (a)

When operation is on the ψ axis the current is zero, hence the charge is constant. The flux linkages are given by the equation

$$\frac{d\psi}{dt} + \frac{Q_0}{C} = E\sin t \tag{18}$$

Integrating

$$\psi = -E\cos t - \frac{Q_0}{C}t + K \tag{19}$$

in which Q_0 is the charge trapped on the capacitor at the start of operation on (*a*). If at the start $t = T_0$ and $\psi = \psi_0$, we have

$$K = \psi_0 + E\cos T_0 + \frac{Q_0 T_0}{C} \tag{20}$$

The general expression for ψ during operation on (*a*), in terms of the boundary conditions for any given traverse of (*a*), is

$$\psi = -E\cos t - \frac{Q_0}{C}t + \psi_0 + E\cos T_0 + \frac{Q_0 T_0}{C} \tag{21}$$

II. Operation on Curve (b) or (c)

Assume that the capacitance has small effect in comparison with the inductance when operation is on either curve (*b*) or (*c*). Then

$$L\frac{di}{dt} = E\sin t \tag{22}$$

and

$$q = -\frac{E}{L}\sin t + At + B \tag{23}$$

Let the boundary conditions be taken as

$$t = T_k \qquad q = -Q_k \qquad i = 0 \tag{24}$$

then we have

$$A = \frac{E}{L}\cos T_k \tag{25}$$

and

$$B = -Q_k + \frac{E}{L}\sin T_k - \frac{ET_k}{L}\cos T_k \tag{26}$$

The general expressions for the charge and current during operation on (*b*) or (*c*), in terms of the boundary conditions for any given traverse of (*b*) or (*c*), are

$$i = \frac{E}{L}(\cos T_k - \cos t) \tag{27}$$

$$q = \frac{E}{L}(\sin T_k - \sin t - T_k\cos T_k + t\cos T_k) - Q_k \tag{28}$$

III. Time Interval and Change in Charge During One Excursion in the Saturated Region

It will be assumed that when *i* becomes zero, operation on the saturated portion of the characteristic ceases (in exceptional cases this need not be true). Let T_{k+1} be the instant of time at which this occurs.

From (27)

$$\cos T_{k+1} = \cos T_k \tag{29}$$

The instant T_{k+1} occurs later than T_k, but not more than 2π later. It will be convenient to express T_k as

$$T_k = 2r\pi + \beta \qquad \text{where } -\pi < \beta < \pi \tag{30}$$

from which it follows that

$$\begin{aligned} T_{k+1} &= 2(r+1)\pi - \beta \qquad && \beta > 0 \\ &= 2r\pi - \beta && \beta < 0 \end{aligned}$$

and the time of excursion in the saturated region is

$$\begin{aligned} T_{k+1} - T_k &= 2(\pi - \beta) \qquad && \beta > 0 \\ &= -2\beta && \beta < 0 \end{aligned} \tag{31}$$

The current which flows in the saturated region and the interval for which it flows depend upon the value of β. This is shown in figure 4. The change in charge due to the current flowing during this interval may be calculated from (28):

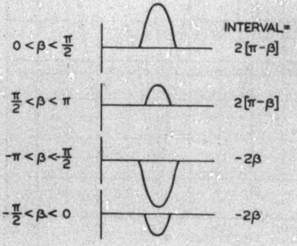


Figure 4. Types of current pulse during an excursion in the saturated region

$$\begin{aligned} Q_{k+1} &= \frac{2E}{L}[\sin\beta + (\pi - \beta)\cos\beta] - Q_k \qquad && \beta > 0 \\ &= \frac{2E}{L}[\sin\beta - \beta\cos\beta] - Q_k && \beta < 0 \end{aligned} \tag{32}$$

IV. Subharmonic Oscillation of the First Kind

One type of oscillation which may occur involves one excursion in the saturated region with positive current, one excursion down the ψ axis, one excursion in the saturated region with negative current and finally one excursion up the ψ axis to the beginning of a new cycle. Due to symmetry the charges must have the same magnitude but opposite signs in two successive excursions along the ψ axis. Such a cycle is indicated in figure 5 in which T_k has been chosen as the instant of time at which operation starts on the saturated portion of the characteristic with *positive* current. The angle β is therefore restricted to positive values.

The time interval for one complete cycle is equal to

$$T = 4(\pi - \beta) + 2\delta \qquad \beta > 0 \tag{33}$$

where δ is the time for one traverse of the ψ axis. Since for a subharmonic the period T must be given by

$$T = 2s_1\pi$$

we have

$$\delta = 2\beta + (s_1 - 2)\pi \qquad \beta > 0 \tag{34}$$

Further since the oscillation is assumed symmetrical, the angle of entrance into the negative saturated region must be an odd multiple of $\pi + \beta$. That is

$$\beta + 2(\pi - \beta) + \delta = \beta + (2n+1)\pi$$

where $n = 1, 2, 3, \ldots$. Comparing this expression with (34) it is seen that s_1 must be an odd integer. Oscillations of this kind are all *odd* subharmonics. Equation (32) gives the value of Q_k in terms of the angle β if we impose the condition $Q_{k+1} = Q_k$ (since $-Q_k$ was taken as the charge during travel up the ψ axis),

$$Q_k = \frac{E}{L}[\sin\beta + (\pi - \beta)\cos\beta] \tag{35}$$

If the boundary conditions corresponding to one travel up the ψ axis are inserted in (21) we have

$$\psi_0 = -\psi_s \qquad \psi = \psi_s$$

$$Q_0 = -Q_k \qquad t = 2r\pi + \beta$$

$$T_0 = (2r - s_1 + 2)\pi - \beta \qquad \beta > 0$$

$$2\psi_s = E[\cos(\beta + s_1\pi) - \cos\beta] + \frac{Q_k}{C}[2\beta + (s_1 - 2)\pi] \tag{36}$$

Substituting (35) in (36), we have

$$\frac{2LC\psi_s}{E} = -2LC\cos\beta + [2\beta + (s_1 - 2)\pi][\sin\beta + (\pi - \beta)\cos\beta] \tag{37}$$

The alignment chart figure 6 gives solutions of this equation for various values of the parameters. In this chart

$$A = \frac{2LC\psi_s}{E} \qquad B = 2LC \qquad m = s_1 - 2$$

The dotted line in figure 6 gives a representative solution. In this case $A = 5.75$ and $B = 5$. For $m = 1$, we have two values of β, 10° and 102°. For $m = 3$, we have one value of β, 134°, etc.

V. Subharmonic Oscillation of the Second Kind

Another type of oscillation encountered experimentally is that shown in figure 7. This type corresponds to the route over the saturation curve outlined in the figure. In this case the charge $-Q_k$ which exists at the instant T_k, is removed due to the flow of current

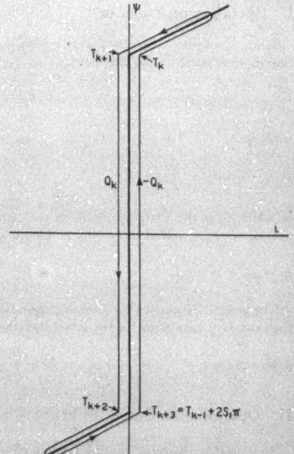


Figure 5. Path of subharmonic oscillation of the first kind

in the interval $(T_{k+1} - T_k)$, so that during the interval $(T_{k+2} - T_{k+1})$ the charge on the capacitor is zero. The value of the order s_2 of the subharmonic may be calculated as follows:

$$\text{Half period} = (T_{k+4} - T_k) = (T_{k+4} - T_{k+3}) + (T_{k+3} - T_{k+2}) + (T_{k+2} - T_{k+1}) + (T_{k+1} - T_k)$$

$$s_2\pi = 2\beta + (s_1 - 2)\pi + 2(\pi - \beta) + 2\beta + 2(\pi - \beta)$$

Hence s_2 is equal to $(s_1 + 2)$; s_2 is therefore an odd integer.

In the first type of oscillation the charge is reversed each time current flows; in the oscillation of the second kind the first pulse of current after an excursion along the ψ axis reduces the charge to zero and the second pulse charges the capacitor to a value equal and opposite to the original charge. Therefore equation 35 must be rewritten

$$Q_k = \frac{2E}{L}[\sin\beta + (\pi - \beta)\cos\beta] \tag{38}$$

Equation 36 remains unchanged. However we wish to express (36) in terms of s_2:

$$2\psi_s = E[\cos(\beta + s_2\pi) - \cos\beta] + \frac{Q_k}{C}[2\beta + (s_2 - 4)\pi] \tag{39}$$

Substituting (38) in (39) we have:

$$\frac{LC\psi_s}{E} = -LC\cos\beta + [2\beta + (s_2 - 4)\pi][\sin\beta + (\pi - \beta)\cos\beta] \tag{40}$$

Solutions for various values of the parameters are given by the alignment chart figure 6. In this chart

$$A = \frac{LC\psi_s}{E} \qquad B = LC \qquad m = s_2 - 4$$

VI. Oscillation of the Third Kind

It sometimes happens, as in figure 3, that the condenser charge accumulated during the transient causes the two portions of a cycle to be unsymmetrical. As before let T_k be the instant of time at which operation starts on the positive saturated portion of the characteristic. The charge during the interval preceding T_k was $-Q_k$. During the interval $T_{k+1} - T_k$ the negative charge is removed and a *greater* positive charge Q_{k+1} is put on the condenser. Hence we have

$$Q_k + Q_{k+1} = \frac{2E}{L}[\sin\beta + (\pi - \beta)\cos\beta] \tag{41}$$

If the two current pulses are to be the same, we have

$T_{k+2} - T_k = \beta + s'\pi$ where s' is an odd integer (42)

and also

$T_{k+4} - T_{k+2} = \beta + s''\pi$ where s'' is also an odd integer (43)

If $Q_{k+1} > Q_k$, as assumed, then s' which includes a passage along the ψ axis with charge Q_{k+1} will be smaller than s'' which includes a passage along the ψ axis with a charge of Q_k; we have

$$2(\pi - \beta) + \delta' = s'\pi$$
$$2(\pi - \beta) + \delta'' = s''\pi$$

where δ' and δ'' are the two time intervals on the ψ axis.

$$\delta' = 2\beta + (s' - 2)\pi \tag{44}$$

$$\delta'' = 2\beta + (s'' - 2)\pi \tag{45}$$

The flux linkage equations become

$$2\psi_s = -2E\cos\beta + \frac{Q_k}{C}[2\beta + (s'' - 2)\pi] \tag{46}$$

$$2\psi_s = -2E\cos\beta + \frac{Q_{k+1}}{C}[2\beta + (s' - 2)\pi] \tag{47}$$

If we let Δ be the unbalanced charge and Q be the charge which is reversed at each current pulse we have

$$Q_k = Q - \Delta$$

$$Q_{k+1} = Q + \Delta \tag{48}$$

Subtracting (46) from (47), it follows that

$$\Delta = \frac{(s'' - s')\pi Q}{2[2\beta + (s_3 - 2)\pi]} \tag{49}$$

in which $s_3 = \dfrac{s' + s''}{2}$ is the order of the subharmonic.

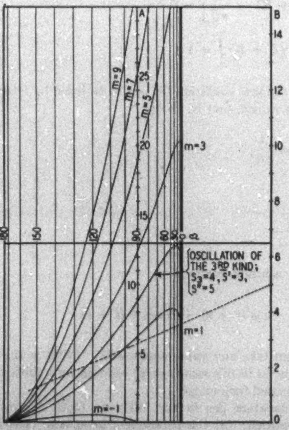
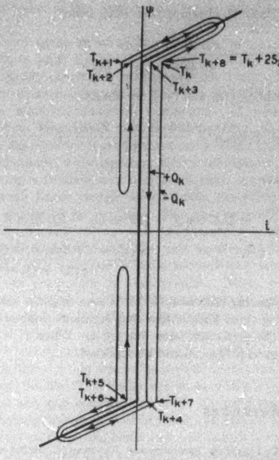


Figure 6. Alignment chart for the equation

$$A + B\cos\beta - (2\beta + m\pi)[\sin\beta + (\pi - \beta)\cos\beta] = 0$$

Figure 7. Path of subharmonic oscillation of the second kind



Q may be obtained from (41), hence

$$\Delta = \frac{(s'' - s')\pi E[\sin\beta + (\pi - \beta)\cos\beta]}{2L\,[2\beta + (s_3 - 2)\pi]} \tag{50}$$

Adding (46) and (47), we obtain

$$\psi_s + E\cos\beta = \frac{[2\beta + (s_3 - 2)\pi]Q}{2C} + \frac{(s' - s'')\pi}{4C} \tag{51}$$

Hence

$$\frac{2LC\psi_s}{E} + 2LC\cos\beta = [2\beta + (s_3 - 2)\pi][\sin\beta + (\pi - \beta)\cos\beta] - \frac{\pi^2(s'' - s')^2\,[\sin\beta + (\pi - \beta)\cos\beta]}{8[2\beta + (s_3 - 2)\pi]} \tag{52}$$

Comparing (52) with (37) it is seen that the solution for oscillations of the third kind is like that for oscillations of the first kind except for the correction term due to Δ. When s'' is equal to s' (52) reduces to (37) as should be expected.

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Discussion

Ernst Weber (Polytechnic Institute of Brooklyn, Brooklyn, N. Y.): In explaining the production of subharmonics in circuits containing iron-cored inductors the authors employ, in addition to the differential-analyzer attack, two phases of theoretical attack. The more important and most interesting one is the region-by-region method, substituting three straight lines for the nonlinear characteristic of the iron-cored inductor. Particularly happy is the concept of the "trapped" charge on the condenser which is amply confirmed by the results obtained with the differential-analyzer.

Less convincing is the attempt to show by a direct analytical attack the effect of initial conditions upon the "steady state" response of the nonlinear circuit. The circuit analyzed consists of a capacitance C in series with a linear inductance L and a nonlinear inductance of flux linkages $\psi = Ki^{1/3}$, and having an applied voltage $1.\sin(t + \theta)$, the time measured in radians. The circuit is treated as nondissipative, that is, disregarding resistances and this feature invalidates most of the reasoning. In order to show that the analysis cannot be valid, let us apply the authors' reasoning to a linear circuit of the same type. Then the flux linkages ψ can be expressed as $\psi = K.L$ and, in differentiating, can be combined with the first term Ldi/dt so that one obtains

$$(L + K)\frac{di}{dt} + \frac{1}{C}\int i\,dt = \sin(\mathrm{t} + \theta) \tag{1}$$

Assume now that the solution of (a) consists of an n'th harmonic current in addition to the current of impressed frequency,

$$i_a = A\cos(t + \theta) + B\cos n(t + \theta) \tag{2}$$

where A and B are undetermined coefficients. Substitution of (2) and equating coefficients of sines of like multiples of $(t + \theta)$ leads to the two conditions:

$$\left.\begin{aligned} B \cdot \left[n(L + K) - \frac{1}{nC}\right] &= 0 \\ A \cdot \left[\frac{1}{C} - (L + K)\right] &= 1 \end{aligned}\right\} \tag{3}$$

Obviously the first condition (3) can be satisfied by letting the term in brackets vanish, that is, choose

$$(L + K) = \frac{1}{n^2C} \tag{4}$$

giving

$$A = \frac{n^2C}{n^2 - 1}$$

so that the solution finally is

$$i_a = \frac{n^2C}{n^2 - 1}\cos(\mathrm{t} + \theta) + B\cos n\,(t + \theta) \tag{5}$$

where B can take any value whatsoever. What is the meaning of two frequencies in this steady-state solution for a linear circuit with single impressed frequency?

The explanation lies in the following facts: (1) resistance has been disregarded; (2) the parameters have been chosen to satisfy the resonance condition (4) for the n'th harmonic, making this a proper frequency of the circuit; (3) in a nondissipative circuit zero voltage can produce a current of arbitrary magnitude if resonance exists; (4) in a nondissipative circuit the application of any voltage initially different from zero will excite a steady a-c current of the proper frequency which can persist indefinitely.

Now, the authors follow in the case of their nonlinear circuit exactly the same procedure as outlined above for the linear circuit. Since they assume for the flux linkages the law $\psi = Ki^{1/3}$, the introduction of the third harmonic (sub- or super-) is natural. Their solution does, therefore, *not* give the steady-state term but includes an involuntary a-c steady transient term as well. The nonlinearity hides this fact, particularly since the amplitude of the harmonic must have a special value in order to be able to satisfy the nonlinear differential equation. The conclusion of the authors that there can exist entirely different characteristic responses to an applied a-c voltage for different switching angles is not justified. In fact, experience shows that the final steady state as seen in oscillograms is typically the same for various switching angles, though not necessarily of exactly the same form. Otherwise any practical use of the relay circuits based upon the critical features of the ferroresonant element would be incomprehensible.

I. A. Travis and C. N. Weygandt: Doctor Weber shows that in a linear circuit with an applied voltage there can exist two sustained sinusoidal components of current having different frequencies, if resistance be neglected. One of these components has the frequency of the applied voltage whereas the other has a frequency dependent on the inductance and capacitance in the circuit but *not* dependent in any way upon the initial conditions. The initial conditions determine the maximum value of this component but do not influence the frequency.

In the nonlinear circuit given in part II of the paper there can be many solutions involving many frequencies. Of these we give two, which happen to be readily expressible in trigonometric form. The essential thing is that these solutions show that in a circuit with a given capacitor and nonlinear inductor, components of current having entirely different frequencies may flow depending on the initial conditions. Thus equation 9 is a solution containing a fundamental and a third harmonic, while equation 10 is a solution for the same circuit containing a fundamental and a one-third harmonic. In a given single mesh linear circuit such as that described by Doctor Weber it is impossible to get more than one frequency in addition to that of the applied voltage and this frequency is never dependent upon the initial conditions. The only special attribute of the one-third power with reference to these two solutions is that they are the only two expressible in simple form. Many other solutions were actually obtained on the differential analyzer, which had frequencies in no way related to the number three.

The question as to whether or not the solutions (9) and (10) contain "involuntary steady transient" terms, which might die out if resistance were considered is one which the authors cannot answer. The whole question of stability of subharmonics is one with which the present investigation did not concern itself. It is, however, believed that circuits having resistance exist in which can flow currents of frequency dependent on the initial conditions, and which so far as oscillographic study is concerned have every right to the name steady state.

Recovery-Voltage Characteristics of Typical Transmission Systems and Relation to Protector-Tube Application

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Introduction

THE SUCCESSFUL operation of an arc-interrupting device requires that the insulation recovery-voltage characteristics of the device be higher than the recovery-voltage characteristics of the system. A broad understanding of the recovery-voltage problem from the application point of view, requires a knowledge of the effects caused by the range of systems, by the different types of faults and the different fault conditions encountered in practice. With this object in mind an investigation was undertaken to provide a general picture of the recovery-voltage characteristics of transmission systems.

In order to carry out this investigation consideration was given to the several possible modes of attack which include analytical, field-test, and a-c calculating-board methods. Analytical methods are of limited value for such a study because of the involved nature of the problem, the range of conditions, and number of cases which must be considered. Field tests, although providing results of value as benchmarks, are of restricted use because it is impractical by this means alone to study an adequate number of conditions with suitable range of system constants and fault conditions. Because of these limitations in analytical and field-test methods, this investigation has been carried out by the a-c calculating-board method, described by the authors in a recent paper.[1]

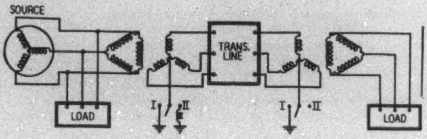

Figure 1. Schematic diagram of system selected for study

Group I—Solidly grounded at both ends. Used for curves of figure 6
Group II—Reactance grounded at sending end; ungrounded at receiving end. Used for curves of figure 7

It is proposed in this paper to present the results of general analyses made on a number of typical transmission systems subjected to faults of such a type that their clearing restores the systems to the original circuit conditions. This type of fault does not include the ordinary circuit-breaker sectionalizing operation but it does include the protector tube and many "shunt" type applications with circuit breakers or fuses whose operations do not change the main circuit condition. It is also proposed to illustrate the use of the results of such recovery-voltage studies with the application of protector tubes and to discuss the operating experience with certain typical applications. Although the usefulness of this general study will be illustrated by only one specific application the authors believe that it has a definite use in connection with other applications.

Part I

Systems Selected for Analysis

A broad perspective of the recovery-voltage problem can be obtained from the study of a representative set of systems. For this purpose three-phase, 60-cycle systems with transmission lines of the three voltage classes, namely, 34.5, 69, and 138 kv were selected. Since the recovery voltage of a system is materially affected by the length of connected line, the lengths were selected to represent the shortest that would be encountered at that particular voltage for the large majority of systems. These selections were as follows: 22.5 miles for 34.5 kv, 45 miles for 69 kv, and 90 miles for 138 kv. Although it was recognized that more than one circuit would be required to transmit the maximum amount of power a single-circuit line was used because this gives the more severe recovery-voltage conditions.

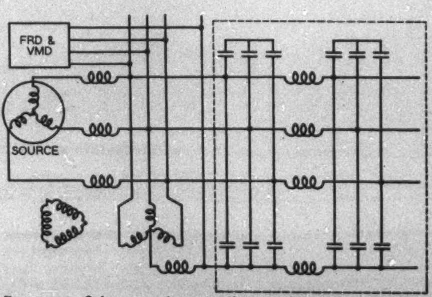

Figure 2. Schematic diagram illustrating method of system representation used on the a-c calculating board

FRD—Fault-representation device *VMD*—Voltage-measuring device

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[1] For all numbered references, see list at end of paper.

Table I. Characteristics of System Selected for Study

Voltage	34.5 Kv	69 Kv	138 Kv
Short-Circuit Amperes			
(*a*)	500	500	500
(*b*)	2,000	2,000	1,000
(*c*)	6,000	6,000	4,000
Short-Circuit Kva			
Load kva	5.0	4.0	3.0
Line length (miles)	22.5	45	90

The general features of these systems are shown schematically in figure 1. For each voltage class three different conditions were assumed, each having different values of generating capacity and load. Table I gives the symmetrical three-phase short-circuit current that would be encountered for a "bolted fault" at the sending end. Systems capable of supplying the higher short-circuit currents have in general larger connected loads. Table I gives the ratio of short circuit kilovolt-amperes to load kilovolt-amperes that was used in this study. Both no-load and loaded conditions were considered. An 80 per cent power factor load was divided equally between the generator bus and the receiver bus as shown in figure 1.

The generating and transformer capacities were proportioned to the load for the particular system and short-circuit current, and typical constants were assumed for all of the system elements. Transmission lines without ground wires were chosen. The additional complication to take care of the cases with ground wires is not warranted because the problems are similar and also the recovery voltages are less severe when ground wires are used.

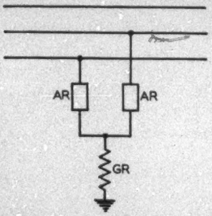


Figure 3. Connection diagram for double line-to-ground fault

AR—Arc-representation device
GR—Ground or tower-footing resistance

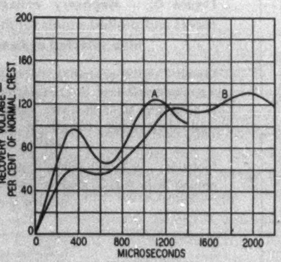


Figure 4. Examples of Cartesian plot of polar oscillograms taken during general studies

A—Single line-to-ground fault at sending end. 138-kv solidly grounded system having 1,000 amperes symmetrical three-phase short-circuit current, zero tower-footing and arc resistance, and load

B—Double line-to-ground fault at sending end. 138-kv solidly grounded system having 500 amperes symmetrical three-phase short-circuit current, 33.0 ohms tower-footing resistance, zero arc resistance, and load

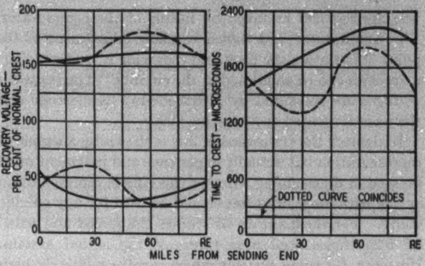


Figure 5. Recovery voltage for first and maximum crests and time to first and maximum crests as a function of location of fault on 90-mile solidly grounded 138-kv system supplying 826 amperes symmetrical three-phase short-circuit current at sending end

——— Single line-to-ground fault, no tower-footing or arc resistance
– – – – Double line-to-ground fault, 33.0 ohms tower-footing resistance

Variable Factors Considered in the A-C Board Study

In making the general analysis the factors that were varied were method of grounding, load, type of fault, fault resistance, arc resistance, and location of fault. The studies can conveniently be divided into two groups with respect to the method of grounding, namely: Group I with system solidly grounded both at sending- and receiving-end transformers, and group II with the system grounded through a reactance at the sending end only. The solidly grounded systems, of group I, were studied for both single line-to-ground and double line-to-ground faults and for fault or tower-footing resistance of 0, 25, and 100 ohms, and with and without resistance for arc representation. The effect of load was investigated by comparing the results of the above tests with results of a few tests made without load and without fault or arc resistance. The second set of tests for the reactance-grounded system, group II, was made with the addition of a neutral reactor of such magnitude as to make the zero-sequence reactance at the sending end equal to 8.5 or 34 times the zero-sequence reactance of the supply transformer.

Connection and Adjustment of the A-C Calculating Board

The method of connecting the a-c calculating board is illustrated in figure 2. The actual connection and adjustment of the board was similar to that described in the paper[1] referred to previously. The selected systems were set up in miniature on a three-phase basis and the fault was applied and removed by means of the commutator. The commutator is designed to allow adjustment of the point of initiating the fault and removing it, and arc characteristics can be simulated in the circuits. The transient voltages are measured by a cathode-ray oscilloscope and the records secured on a rotating film.

In figure 2 the transmission line is shown by a simple pi representation but actually experience and judgment must be used in determining whether this simple representation or one with more branches should be used for any specific study. For the majority of studies the forced and natural frequencies used were those of the actual system. However, for some systems which involve relatively high natural frequencies it may be advantageous to use lower frequencies and apply proper corrections. The need for such a procedure depends upon the relative losses of the miniature and the actual systems. In this particular set of investigations it was necessary to use this procedure for the lower-voltage systems. Another network representation problem was encountered with systems having relatively low source inductance. In this case the transients had relatively large amounts of energy associated with the higher-frequency components and adjustments were required to compensate for the increased attenuation.

The general procedure of making adjustments and obtaining the recovery voltage has been described in the previous paper. The manner in which fault and arc resistances were introduced in the circuit for the case of the double line-to-ground fault is illustrated in figure 3. The point of application of the fault was selected and the fault duration was modified so that each phase would clear at its earliest normal current zero. The point of application of the fault was then shifted successively until the maximum recovery voltage of either phase was obtained. It is data secured in this manner that are plotted in the curves given in a subsequent section. The scope of this study

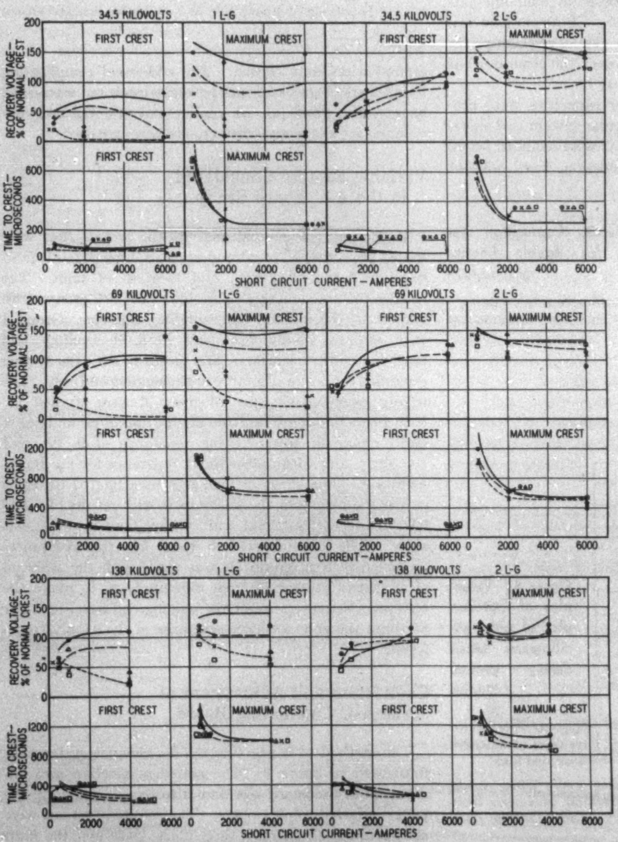


Figure 6. Recovery voltage curves and plotted points for the nine selected systems

Group I—Solid grounding. Voltage magnitude and time to crest plotted as a function of three-phase symmetrical short-circuit current at the sending end

Notation: 1-*L-G*, single line-to-ground fault; 2-*L-G*, double line-to-ground fault

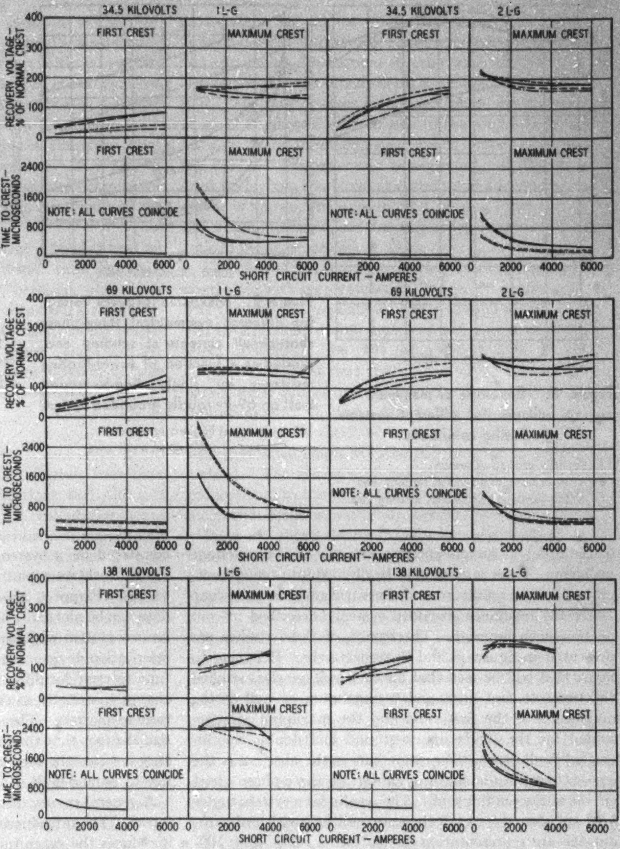
	Load	Arc Resistance	Tower-Footing or Ground Resistance (Ohms)
———	Yes	No	0
— —	Yes	Yes	0
- - -	Yes	No	100
⊙	Yes	No	0
△	Yes	No	25
X	Yes	Yes	25
□	Yes	Yes	100

Figure 7. Recovery-voltage curves

Group II—Reactance grounding. Voltage magnitude and time to crest plotted as a function of the three-phase symmetrical short-circuit current at the sending end

Notation: 1-*L*-G, single line-to-ground fault; 2-*L*-G, double line-to-ground fault

	Reactance Value	Arc Resistance
———	Low	No
- - - -	Low	Yes
— - —	High	No
— — —	High	Yes



was limited to the cases in which the faults were cleared at normal current zero without restriking of the arc.

Typical recovery-voltage curves obtained from the a-c calculating-board study are plotted in figure 4. These curves show definite first crest and maximum crest. In some cases, second and third crests were of approximately the same magnitude, and in these cases the second crests were used as they are more severe for an insulation-recovery curve that has a rising slope.

Results of A-C Calculating-Board Studies

In an investigation of this type it is not practical to consider the application of the fault at several points on the system. A preliminary study was, therefore, made to determine the effect of moving the fault along the line. Figure 5 shows the results of such a study for a 90-mile, 138-kv system capable of supplying a symmetrical three-phase fault current of 826 amperes at the sending end and with no load on the system. The recovery-voltage results for the single line-to-ground fault are for zero tower-footing resistance and the results for the double line-to-ground fault are for 33-ohm tower-footing resistance. When it is considered that both time and magnitude are of importance in studying the effect of recovery voltage on a piece of apparatus, it will be seen that although the magnitude varies as the fault is moved along the line, the time also varies. Considering the shape of insulation-recovery voltage curves for certain specific pieces of apparatus, it was concluded that the recovery-voltage conditions were about as severe for faults at the sending end as at any location along the line. Therefore, the fault at the sending end was considered to be representative and was used in the general studies.

Based on the selected systems, a series of studies was made on the a-c calculating board to determine the effect

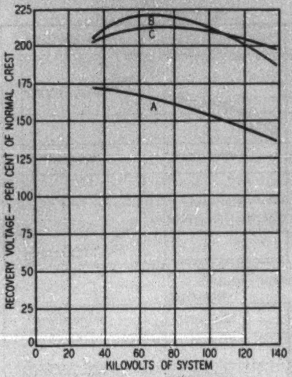


Figure 8. Envelope of maximum recovery voltages for different system-grounding conditions

A—Solidly grounded system
B—Low-reactance ground at sending end
C—High-reactance ground at sending end

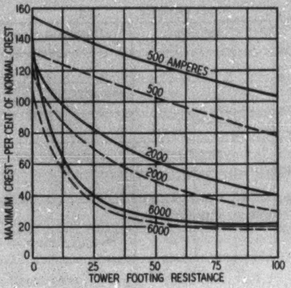


Figure 9. Maximum recovery voltage for different symmetrical three-phase short-circuit currents at sending end, plotted as a function of tower-footing resistance for single line-to-ground fault on 69-kv solidly grounded system

——— No arc resistance
– – – – With arc resistance

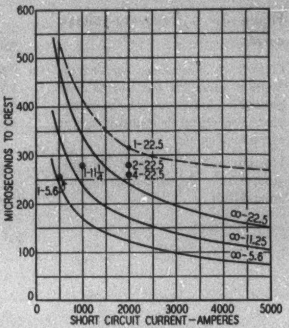


Figure 10. Recovery-voltage time to crest for 34.5 kv system plotted as a function of symmetrical three-phase short-circuit current

——— Infinite number of lines of different lengths
– – – – Finite number of lines aggregating 22.5 miles

Example: Notation 1–22.5. First figure—number of circuits; second figure—aggregate miles of line

of the different factors entering into the recovery-voltage problem. These studies may be divided into two groups; group I for the solidly grounded neutral systems, and group II for the reactance-grounded systems, described in connection with figure 1. The results of these studies are presented in figures 6 and 7, respectively. Referring to figure 6, it will be seen that for each voltage class studied the times to first crest and to maximum as well as the magnitude of the first crest and the maximum are presented for the single line-to-ground and double line-to-ground fault cases. For these curves the abscissa is the current magnitude for a symmetrical three-phase short circuit at the sending end. The results for a system under load are plotted for each current condition, with and without the arc representation device for 0-, 25,- and 100-ohm tower-footing resistance. For comparison, one study with no load and zero arc and tower-footing resistance is plotted. The arc representation device is of the same general type as described in the previous paper making use of Rectox resistors to simulate the arc characteristics of a deion protector tube. The Rectox resistor was adjusted to give approximately 20 per cent of line-to-neutral voltage at the peak of the symmetrical short-circuit current wave. This value was chosen to give the maximum effect of this factor as the minimum effect is shown by the condition of load and no arc representation device.

Figure 7 presents the results of similar studies made on the same systems except with reactance grounding. The system arrangement with respect to grounding is shown in figure 1.

The recovery-voltage data in figures 6 and 7 can be plotted in a large number of ways, depending upon the point under investigation. It is of general interest to know the maximum recovery voltage that can be encountered on a system when all the factors are varied. Figure 8 shows a plot of the envelope of such maximum values. Figure 9 shows another manner in which the data can be plotted. This example is for the 69-kv solidly ground system with load and with and without the arc representation device. It will be seen from figure 6 that the time to crest for different fault-resistance values does not change materially so that the significant factor is magnitude as plotted. This curve has been included to emphasize the fact that the introduction of tower-footing resistance reduces the recovery-voltage magnitude for a single line-to-ground fault.

A general review of the results of these studies is of interest. The introduction of load has little effect although it reduces the magnitude and decreases the time to crest slightly. The introduction of arc resistance in the solidly-grounded system reduces the magnitude for both single and double line-to-ground cases but very materially for the former. The time to crest is not influenced very much. Tower-footing resistance lowers the magnitude and the greater the resistance the larger this effect. Again the time is not affected.

Reactance grounding materially increases the magnitude. The introduction of resistance tends to lower the magnitude for both values of reactance grounding but the effect is not marked. The time to maximum crest for the single line-to-ground fault is considerably increased with the introduction of reactance.

On the solidly-grounded neutral system the magnitudes are very close for the single and double line-to-ground faults. For the reactance-grounded systems the magnitude of the double line-to-ground fault is considerably higher than for the single line-to-ground case.

When a comparison of the results of the tests on systems with line connected is made with the severity of tests for circuit breakers,[3] it is found that the magnitudes are lower. The largest difference is in the time to crest, this being 6 to 12 times as much as for the circuit-breaker test. If the aggregate length of line is increased over the values used in the studies, the spread in the time to crest will be still greater. The effect of line is, therefore, very important in the recovery-voltage problem. It is also of interest to note that the envelope of maximum magnitudes decreases for the higher system voltages as a result of the greater lengths of connected lines.

Since the system recovery voltage has a decided bearing on the economical design of the protector tube and since the laboratories for testing circuit breakers give more severe recovery voltages than found on actual systems, we believe the data presented will be very useful in the preparation of a test code for protector tubes. Such requirements might be specified as magnitude of recovery voltage and time to crest for given short-circuit currents.

Effect of Other Factors

Although a large number of different factors have been investigated for different fault conditions and different conditions of grounding in the a-c calculating-board studies, there are, nevertheless, certain other pertinent factors that enter into an application which must be considered. The analytical method of solution is useful in considering some of these factors, such as the number of circuits and their total capacitances, the latter being proportional to the aggregate length of line. In order to illustrate the effect of *total capacitance* or aggregate length of line, figure 10 has been prepared and it shows that the time to crest is very materially affected. For example, for the case of a system capable of supplying a symmetrical three-phase short-circuit current of 2,000 amperes and with the capacitances equivalent to 5.6 miles of line lumped at the sending end, the time to crest is approximately 120 microseconds whereas with the capacitances of 22.5 miles of line lumped at the sending end the time to crest is approximately 240 microseconds. It is seen that the severity increases with the decreased aggregate length of line and approaches a maximum severity with zero line, which is the case for the circuit-breaker sectionalizing application. The time to crest for a single-phase circuit is equal to $\pi \sqrt{LC}$, where L corresponds to the source inductance and C to the total capacitance assumed lumped at the sending end, and line reactance is neglected. If the time to crest for a given short-circuit current and aggregate length of line is known, the time to crest for any other length of line would increase approximately as the square root of the ratio of the aggregate line lengths.

The effect of *length of line*, that is straightaway length as contrasted with aggregate length, is brought out in figure 10. The condition of capacitance being lumped at the sending end for 22.5 miles of line is plotted and compared with the effect of a single line 22.5 miles long. This, in effect, shows the range of time that can be expected for a given aggregate length of line for anything from a single to an infinite number of lines. Points are also shown for two and four lines for the 2,000-ampere condition. It will be seen that when length is increased the time to crest is increased. The severity of recovery voltage is inversely proportional to the times to crest given in figure 10 as the magnitude remains practically constant for this case.

Figure 11 shows the results of a number of calculations[1] based on the wave theory, for the nine selected systems previously studied. These calculations are made for single line-to-ground faults at the sending end with the sending and receiving ends solidly grounded. These do not consider the effects of load, initiating transients, or tower-footing or arc resistance. They do, however, illustrate the general shape of the recovery-voltage curve and give a close check for the a-c calculating-board studies for maximum crest. In the curves of figure 11 the magnitude of the "first crest" at the shorter times is also of interest as it is probably a lower limit of the values that can be expected on the system, as the transients arising from the initiation of the fault are not included. The results of the a-c calculating board study are probably an upper limit as

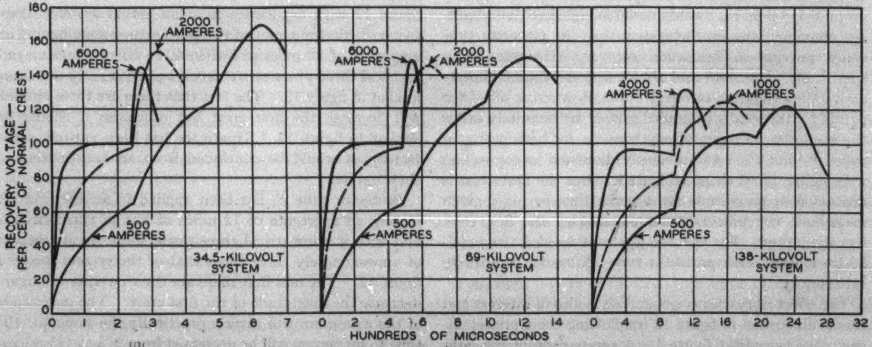


Figure 11. Calculated recovery voltages for single line-to-ground fault on solidly grounded systems

these include the effect of initiating transients and some extraneous oscillations introduced by the networks. The actual determining conditions are probably between these two limits but the relative position has not been definitely determined as it is not of immediate interest in presenting broadly the results of this series of studies.

It is of interest to note the effect on the first crest of varying the length and number of lines, as previously was done in connection with the effect on the second crest in discussing figure 10. For example, if four lines of the same length were used for, say, the 2,000-ampere case the recovery-voltage curve would approximate that of the 500-ampere case, since one may assume four fictitious systems supplying 500-ampere symmetrical short-circuit currents operating in parallel to give the 2,000 amperes.

Part II

Application of the Data

It is the purpose of this section to show broadly how the recovery-voltage data can be used in the application of apparatus to a system. Although the recovery-voltage data presented is useful in a large number of applications, the particular application of the deion protector tube will be taken as an illustration. The case of the 34.5-kv solidly grounded system with 22.5 miles of transmission line with a symmetrical three-phase short-circuit current of 2,000 amperes at the sending end has been selected. These data are plotted in the form of recovery-voltage magnitude-time curves as illustrated in figure 12. The points joined with broken lines are for the first crest and maximum overshoot taken from the general curves of figures 6 and 7. The particular fault and arc resistance condition is identified by the notation on the figure. In figure 12 there is also plotted the calculated curve taken from figure 11. There are also plotted curves P_1, P_2, and P_3, which are the insulation recovery-strength curves for three different deion protector tubes with different inner bores but the same fiber length. An examination of the curves of figure 12 shows that there are certain conditions for the solidly grounded system that would result in higher recovery-voltage characteristics on the system than the protector tube could provide in insulation recovery strength. More specifically, points *A*2 and *B*2 for first crest are definitely above the insulation-recovery curve P_3 whereas all of the points for the solidly grounded system are definitely below the insulation recovery-strength curves for both protector tubes P_1 and P_2. As previously discussed in connection with figure 11, if there are parallel lines the first crest is decreased in magnitude for a given time or for a given magnitude the time is increased, making the first crest less significant. For the case just considered, if two parallel lines were used protector tube P_3 would be entirely satisfactory.

The effect of reactance grounding is also of interest and this is illustrated in figure 12 by plotting the more significant conditions from figure 7. A comparison of the data from figures 7 and 12 shows for the single line-to-ground fault, that the interruption problem diminishes as the neutral reactance is increased. It will further be noted for the double line-to-ground faults with the higher value of reactance that the severity is increased very materially over that of the solidly grounded system to a point that it approaches protector tube P_2.

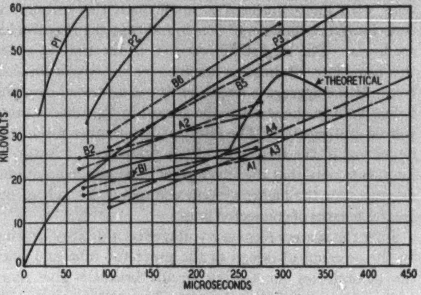


Figure 12. Comparison of system recovery voltage and insulation recovery strength for deion protector tubes for 34.5-kv solidly grounded system supplying 2,000 amperes three-phase short-circuit current at sending end

P_1, P_2, P_3—Insulation recovery strength for deion protector tubes of identical design except for increased bore for higher subnumerals

B6, B5, etc.—System-recovery voltage

Nomenclature: *A*—Single line-to-ground fault
B—Double line-to-ground fault
1—Solidly grounded system, arc resistance
2—Solidly grounded system, no arc resistance
3—Low-reactance grounded system, arc resistance
4—Low-reactance grounded system, no arc resistance
5—High-reactance grounded system, arc resistance
6—High-reactance grounded system, no arc resistance

Correlation of System Recovery Data and Operating Experience With Deion Protector Tubes

It is of interest to compare the conclusions drawn from figure 12 with experience from an actual system. Protector P_1 has been applied to a particular system having an aggregate of 22 miles of line in three circuits and a symmetrical three-phase short-circuit current nearly the same as that of figure 12. The fact that there are three circuits will decrease the first crest but otherwise it should be similar to figure 12. Experience has been entirely satisfactory as would be concluded from an examination of these curves.

Protector tube P_1 has been applied to another system having an aggregate of 12 miles of line in three circuits and with a symmetrical three-phase short-circuit current of approximately three times that of the system used for figure 12. The fact that there are three circuits will again decrease the magnitude of the first crest. The magnitude of the maximum will remain practically the same but the time to maximum will be decreased from 275 to 115 microseconds. This is because, as previously shown, the time varies as $\sqrt{LC}$. L varies inversely as the short-circuit current and C varies directly with the aggregate line length. The line length is decreased in the ratio of 12 to 22.5 which will decrease the time as the square root of this ratio or to 73 per cent; also the fact that the source inductance has decreased to one-third will decrease the time 58 per cent, making a total reduction of 42 per cent. Experience has again been entirely satisfactory as would be concluded from an examination of the curves of figure 12.

In the case of protector tubes P_1, P_2, and P_3, shown on figure 12, the designs are identical except the bore is increased for the higher subnumerals. It will be noted that the insulation recovery characteristics of the protector tube are lower as the bore increases. In the normal operation of the deion flashover protector tube successive operations tend to increase the bore. It has been recognized in the application that after a number of operations the bore will eventually erode to a diameter that will cause the protector to fail. This is illustrated by a comparison of curves P_1, P_2, and P_3. For example, if protector tube P_1 leaves the factory with a definite inner bore it is quite conceivable that after a number of operations the bore will increase to approach the characteristics of protector tube P_2 or even protector tube P_3. It is, therefore, seen that eventually the protector will fail to clear.

It is believed that there has been undue apprehension in connection with the erosion of the protector tube for the practical application. Through the co-operation of the Interstate Power Company it has been possible to secure direct measurements of this factor on the oldest extensive application of protector tubes. This experience was obtained on the 66-kv line joining Dubuque and Clinton, which consists of 47 miles of single-circuit line with flat configuration and seven miles of double-circuit line with vertical configuration. Ground wires are not used and the line is insulated with four ten-inch units, spaced five and three-fourths inches. The line was placed in service August 15, 1926, and in six years time, before the deion protector tubes were installed, there were 277 interruptions that could be traced to lightning. This is an average of 46 interruptions per year. The large number of outages per year is direct evidence that there were and are a large number of strokes to the line per year.

Deion flashover protector tubes were installed on this line in August 1932. The following is an interruption record after the installation was made:

Period	Interruptions
August 1932 to December 1, 1932	0
Year 1933	3
Year 1934	0
Year 1935	3
Year 1936	8
January 1, 1937 to August 20, 1937	2
Total	16

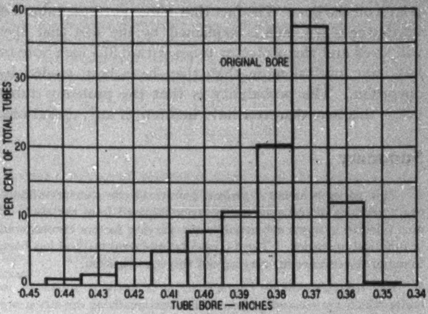


Figure 13. Erosion data obtained after five years' operation of deion protector tubes installed on the Clinton-Dubuque line of Interstate Power Company

This is an average of 3.1 interruptions per year which corresponds to a reduction of 15 to 1. An analysis of these explained 15 of the 16 interruptions. In the year 1933 two protector tubes were destroyed due to high surge current estimated at from 75,000 to 100,000 amperes. In the latter part of 1935 and the early part of 1936 there was a sudden increase in the number of outages which led to an investigation. The outstanding improvement in the performance of the lines justified at times shutting down the station at Clinton although the original application assumed it in operation. This change resulted in reducing short-circuit current at the Clinton end of the line to approximately one-third of the minimum rating of the deion flashover protector. An analysis of the outages for 1935, 1936, and 1937 indicated that all these failures occurred within a few miles of the city of Clinton. An analysis of this particular condition indicates that with the lower short-circuit currents the recovery-voltage strength of the protector tube is lower than the recovery voltage on the system with the result that the protector tubes failed to clear. In August 1937 the protector tubes on the Clinton end covering a distance of approximately 20 miles, were replaced with protector tubes having higher insulation recovery strength. No outages have since resulted on this line up to the time of writing although there have been a number of lightning storms in the vicinity of the line.

In changing the protector tubes a program was carried out by the Interstate Power Company whereby it was possible to obtain bore measurements on all of the protector tubes originally installed. This allows a detailed analysis of the increase of bore or the decrease of insulation-recovery strength as a result of a large number of operations in five years. A limit of 0.45 inches was set for the bore that would be satisfactory for the original application. The original bore of the protector tubes was 0.375 inches and they were very close to this when they left the factory. Figure 13 shows the range of bores after the five years of operation, these being given to the nearest one-hundredth of an inch. It is of interest that none of the protector tubes have reached the limit set as satisfactory for the original application. Only 2.7 per cent have reached 0.43 inches and five per cent have reached 0.42 inches. The protector tubes that were eroded the most were installed at high points on the line. The fact that some protector tubes are below the 0.375 inch is explained by the fact that fiber will swell and the protector tubes with 0.375 inch bore or less have swelled more than they have been eroded by operation. The probability is that the protector tubes below this bore diameter have had few, if any, operations.

Summary

1. This paper presents a general picture of the recovery-voltage characteristics of transmission systems obtained from the study of nine selected systems subjected to the varying factors encountered in practical operation. The a-c calculating-board method has been of major importance in carrying out this investigation.

2. The recovery-voltage characteristics of systems subjected to faults which are cleared without a line-sectionalizing operation are much less severe than that for the clearing of the last circuit breaker on a bus. This difference is so marked that advantage should be taken of it, in the design of certain pieces of apparatus.

3. The method of analysis and the data presented should be of value in the application of certain pieces of apparatus and in the preparation of test codes for specific devices.

4. This general analysis is of immediate interest in connection with the application of protector tubes, and operating experience with this device supports the analysis. Erosion of the protector tubes due to operation is such that the insulation-recovery characteristics are not changed at an impractical rate.

Acknowledgments

The authors wish to acknowledge the co-operation of Messrs. W. Wisco and H. Chambers of the Interstate Power Company in securing the valuable data on the performance of the protector tubes on their system. They also wish to acknowledge the helpful suggestions of their associates, particularly Mr. R. L. Witzke who conducted the a-c calculating-board studies and prepared the data presented.

References

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2. BREAKER PERFORMANCE STUDIED BY CATHODE RAY OSCILLOGRAMS, **R. C. Van Sickle**. ELECTRICAL ENGINEERING, volume 54, page 178.

3. TRAVELLING WAVES ON TRANSMISSION SYSTEMS, **L. V. Bewley** (a book). John Wiley and Sons, New York, 1933.

Discussion

W. J. Rudge, Jr. (General Electric Company, Pittsfield, Mass.): Messrs. Evans and Monteith have made a valuable contribution in their paper on the recovery-voltage characteristics of transmission systems. Since their studies were conducted on an a-c calculating board, it will be of considerable interest to compare the results given in their paper with results which were obtained by actual measurements in staged field tests.

In figure 8 a plot is given of recovery voltage crest in per cent of normal line-to-neutral voltage. The recovery voltage is given for both solidly grounded neutral systems and systems grounded through reactors. This curve indicates that on solidly grounded neutral 115-kv system the recovery voltage will rise to about 150 per cent of normal. We have recently completed tests on a 115-kv system where the recovery voltage measurements varied from 115 to 178 per cent of normal. We have also made tests on a 26.4-kv system which was grounded through resistance where the recovery voltage measured to ground varied between 115 and 187 per cent of normal. Similar tests on a 33-kv system with neutral grounded through resistance give recovery voltages varying between 100 per cent and 185 per cent of normal. Tests made on a 33-kv isolated neutral system give values ranging between 100 and 170 per cent of normal.

Referring to figure 11 of the Monteith-Evans paper, it may be noted that the highest values of recovery voltage occur with the lower currents on the low-voltage system and on the higher-voltage system the highest recovery voltage occurred with the larger currents.

Referring again to our actual field tests, the data on each of the four separate field tests mentioned above indicates that the highest recovery voltage occurs with the lower values of short-circuit current. Further data may, however, show that it is possible to obtain as high recovery voltage with large currents as have been obtained with the lower values. In the field tests the highest values of recovery voltage occurred with currents ranging between 500 and 1,500 amperes whereas the lower values of recovery voltage were for currents in the order of 3,000 amperes.

In giving these results, we do not intend to infer that low values of recovery voltage do not occur with low values of current, as there are many instances where the recovery voltage varied between 100 per cent and 150 per cent of normal in the low range of short-circuit currents. The data from field tests cited above show the highest values of recovery voltage occurring in the range of currents obtained.

In other words, this comparison of field data with the data obtained on the a-c calculating board, show that the results obtained from the board can be no more accurate than the basic data upon which the board is set up. It is therefore essential to have an adequate background of field tests before accurate results can be expected from the a-c board.

On the basis of the field tests, it appears that the crest recovery-voltage characteristic, when plotted against system voltage, is almost a flat line, whereas in figure 8 of the Monteith paper, the curve indicates that for the higher system voltages the maximum recovery voltage has a downward trend.

We note in figure 12 of the Evans-Monteith paper that the characteristics for the protector tubes *P*-1, *P*-2, and *P*-3 do not extend to the region near normal voltage so that when comparing the recovery characteristic of the system with recovery characteristic of the tube, one would infer that the rate at which the voltage rises at the early part of the curve has no effect on tube operation unless the voltage rose at least to the point where the tube characteristic curves begin. We would like to have the authors explain whether these curves are the limit of the data which has been taken on the tube, or whether we have correctly interpreted their results. Tests which we have made on tubes indicate that it is essential for the characteristic curve to be extended back to voltage equal to or even less than normal, as tubes may fail to clear because of too rapid rate of voltage rise, rather than long time in which the peak value of the circuit recovery voltage occurs.

R. C. Van Sickle (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): Although the data presented in this paper applies specifically to protector-tube application, it has additional interest to those associated with high-voltage circuit breakers and their application, as is suggested by the authors. The recovery-voltage characteristics of the typical circuits on which breakers are applied cover a wider range and unfortunately the upper limit is not as well fixed as in the case of flashover protectors because the minimum amount of effective capacitance is more difficult to determine. That the effects of lines and loads greatly reduce the severity of the voltage recovery characteristics has already been demonstrated by measurements taken during field tests on circuit breakers but such data are naturally limited and apply to the particular conditions existing for the test. Consequently, the data presented in this paper is a welcome addition to the general fund of information on the severity of some of the service conditions which may be imposed on circuit breakers.

The characteristics imposed on a tube at the far end of a line are the same as those on a breaker opening at the end of the line. Likewise, the characteristics imposed on a tube at the near end of a line are the same as the characteristics imposed on a circuit breaker opening one line with another line still connected to the same bus.

Since the lengths of the lines have been chosen as short as are typical for the corresponding voltage, the effects produced can be regarded as being the minimum which would normally be expected. A 138-kv circuit having a three-phase short-circuit current of 500 amperes might impose, on the circuit breaker during a single line-to-ground fault, a voltage crest of 190 per cent which occurs about 110 microseconds after current zero. This corresponds to a voltage-recovery rate of 1,020 volts per microsecond. The data in this paper shows that a parallel line would reduce the maximum voltage from 190 per cent to 135 per cent and the rate from 1,020 to 104 volts per microsecond.

Other data shows that locating the circuit breaker at a substation fed by a 90-mile 138-kv line with the neutral solidly grounded and a three-phase short-circuit current of 826 amperes, subjects it to a maximum voltage 160 per cent of normal and a voltage-recovery rate of 89 volts per microsecond. These conditions are approximately the same as those encountered with a parallel line.

These are only a few of the interesting comparisons which can be made by use of the data presented in this paper.

D. C. Prince (General Electric Company, Philadelphia, Pa.): This paper naturally divides itself into two parts; (1) methods of analysis of circuits to obtain the form of recovery-voltage characteristics; (2) the nature of the interrupting characteristics of the expulsion tube with reference to recovery voltages impressed upon it.

There are several methods already in use for determining recovery voltage, ranging all the way from straight calculation through the calculating board methods, discussed by these authors, and ending up at the other extreme with field tests by cathode ray oscillographs. With the addition of the authors to this situation there are available methods of determining recovery voltage forms for practically all conditions.

The second half of the paper is outstanding in that it represents the most exact correlation yet established between any circuit interrupting apparatus and the recovery voltage of the circuit upon which it must work. In the body of the paper, it is stated that the important elements of recovery voltage are the length of time to reach the crest value and the magnitude of that crest value. This statement is not supported by the test information shown. Referring to figure 12, it appears that the variation of strength with time for three expulsion tubes is shown in the curves P_1, P_2, and P_3. If the recovery voltage lies wholly below any one of these three curves, the corresponding design of expulsion tube may be expected to operate satisfactorily.

If, on the other hand, there is a point where a curve of recovery voltage crosses one of these lines P_1, P_2, or P_3, as the case may be, at that point the tube will break down. It would be interesting to know how these curves P_1, P_2, and P_3, were obtained since if this statement of the case is correct, this represents the most accurate correlation between the behavior of any circuit interrupting device and the recovery voltage characteristics of a system yet published.

Above and below the portions plotted in figure 12, it is presumed that waves likely to be obtained would not cross the portions of these lines not plotted, but this could be judged with considerably more assurance if the lines were extended particularly in the direction of shorter times.

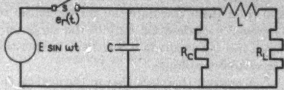
C. N. Weygandt (University of Pennsylvania, Philadelphia): I note that in the method used by the authors an elaborate switching scheme for the cathode ray oscilloscope is required in order to record the high-frequency transients, and also that when the capacitance of the actual circuit is low capacitance of leads and so forth becomes a source of trouble. I should like to suggest an equivalent circuit which may in some cases avoid these difficulties. Suppose that the circuit of figure 1 is to be studied.

The voltage across the switch s can be obtained as the time function corresponding to the operational expression:

$$e_v(t) = \frac{E}{L}\frac{p\omega}{p^2+\omega^2} \times \frac{1}{pC + \dfrac{1}{R_c} + \dfrac{1}{R_L + pL}}$$

Let us construct an equivalent circuit such that its operational impedance is the reciprocal of that of the original circuit. The inductance now has a value numerically equal to the capacitance in the original circuit and the capacitance is equal to the original value of inductance and the resistances are reciprocals of their original values. The current which flows when the switch is closed at the instant $t = 0$ is:

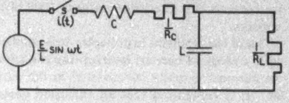
Figure 1. Original circuit



$$i(t) = \frac{E}{L}\frac{p\omega}{p^2+\omega^2} \times \frac{1}{pC + \dfrac{1}{R_c} + \dfrac{1}{R_L + pL}}$$

which is identical with the voltage across the switch in the first circuit. Moreover by reducing the frequency of the applied voltages and consistently altering the values of the parameters, the natural frequency may be reduced so that it may be recorded with an ordinary magnetic oscillograph. Also since the applied voltage in the equivalent circuit represents the interrupted current in the actual circuit it should be more easily possible to simulate interruption when the current is not zero, nonlinear arc characteristics and the like. A numerical example follows:

Figure 2. Equivalent circuit



Given an original circuit with the constants in the left-hand column, the equivalent circuit will have the constants in the right-hand column:

Original Circuit			Equivalent Circuit		
0.25	millihenry	$= L$	25	microfarads	$= L$
0.00025	microfarad	$= C$	2.5	millihenrys	$= C$
0.01	ohm	$= R_L$	10^2	ohms	$= 1/R_L$
10^b	ohms	$= R_c$	0.01	ohm	$= 1/R_c$
377	per second	$= \omega$	0.377	per second	$= \omega$
4×10^6	per second	$= \dfrac{1}{\sqrt{LC}}$	4,000	per second	$= \dfrac{1}{\sqrt{LC}}$
60	cycles per second	$= f$	0.06	cycle per second	$= f$

C. Concordia (General Electric Company, Schenectady, N. Y.): In the past few years there has been a large number of papers written on recovery voltage. One of the striking features of these papers is that there are apparently two different viewpoints about the relative importance of the various factors involved in recovery voltage calculations. One group leaves us with the impression that the important factors are the traveling waves and their reflections, the other that the important factors are the natural frequencies of the circuit. The present paper by Messrs. Evans and Monteith appears to fall in the latter class.

There need be no essential or theoretical conflict between the two points of view. On the other hand, as the traveling waves depend on the surge impedances and transmission distances, while the natural frequencies depend in practice on lumped circuit approximations to the actual system, one may arrive at different conclusions regarding a particular system by the two methods of approach. It is essential to examine carefully each case to determine what must be considered and what can safely be neglected.

One may construct a lumped circuit which possesses the same first, second, etc., natural frequencies and the same steady state characteristics as the actual circuit. In a practical case one will match the circuit frequency response characteristics only up to some limiting frequency. The initial response of such a circuit to an impulse will bear no relation to the initial response of the system.

Similarly one may determine the surge impedances of the system and from them calculate the correct initial response to an impulse. However, consideration only of the surge impedances and reflection times will not result in a correct determination of the oscillations of the recovery voltage.

It is evident therefore that in the general case both effects must be taken properly into account. In some cases, however, the reflection times are so short that the circuit behaves very nearly like its lumped circuit equivalent. In other cases the reflection times are sufficiently great so that the voltage during the period of interest may be determined entirely from the surge impedances. Still more significant is the fact that for some applications the initial response (e.g., the initial rate of use of voltage) is the most important factor, while for other applications the magnitude of the voltage swing may be the most important. In the general case again the entire recovery characteristic must be considered.

L. G. Smith (Consolidated Gas, Electric Light, and Power Company of Baltimore, Baltimore, Md.): Data of the type included in this paper are of real interest to those companies applying deion protector tubes and expulsion protective gaps. Our company has made several applications of these tubes on our 13.2-kv lines and a few isolated applications at 110 kv. Details of these installations are covered in a discussion on page 1509 of the 1937 AIEE TRANSACTIONS.

One of the problems in protector-tube applications is to obtain tubes with a range of current interrupting capacities, within which range the currents actually encountered in the particular application will lie. It is recognized that all expulsion devices have two limits on their current interrupting capacity:

1. The maximum current, which is limited by the bursting strength of the tube notwithstanding the pressure produced by the high currents.
2. The minimum current below which sufficient pressure is not developed to extinguish the arc.

It is understood that the minimum current rating of an expulsion device depends to a large extent upon the rate of rise of recovery voltage. For a given bore diameter the lower the rise of recovery voltage the lower the minimum current that can be interrupted.

Since protector tubes are applied by connecting them from each phase to ground, the ground connection usually being a common connection for all three phase tubes the maximum current interrupting capacity must be sufficient to clear a phase-to-phase fault if two tubes are discharging simultaneously. On the other hand, the minimum interrupting capacity of the tube is determined by the maximum ground resistance, if a single tube discharges. As ground resistances may at times be high and also vary over considerable ranges of values due to weather conditions, it is frequently necessary to use a so-called multiplex installation in which phase tubes of high current interrupting capacity are installed in each phase and a single ground tube installed in the common ground connection, this tube having a lower current rating. If more definite information were available concerning the probable rates of rise of recovery voltages on various systems it should be possible to extend the application of tubes and possibly eliminate the multiplex connection in many instances. Therefore, information on this point is desired by the industry.

R. D. Evans and A. C. Monteith: The principal objects of our paper are to provide (1) a general picture of the recovery-voltage characteristics of transmission systems which has been obtained with the aid of the a-c calculating board and (2) to illustrate the use of the recovery-voltage data in connection with the application of a specific arc-interrupting device, the protector tube. This is the first time that a broad picture of the recovery-voltage problem on transmission systems has been presented. Until such a picture is available and generally understood, refinement in methods of analysis and greater detail in the characteristics of the device are of secondary consideration. A review of the discussions shows a general agreement with the importance of our objectives and the interpretation of the data presented. This general viewpoint should be kept in the mind in considering the replies to the specific discussions.

We fully agree with Mr. Concordia that there is no essential or theoretical conflict between recovery-voltage calculations by traveling waves and by networks with lumped constants, provided the network constants have been suitably chosen and provided that both types of calculation have been carried to a fair degree of refinement. Both methods have been discussed and used in the paper, one method providing a check against the other. It is the particular advantage of the method using networks with lumped constants that makes possible the use of the a-c calculating-board method which provides a practical form of solution for general analyses, the results of one such study being presented in the paper under discussion. This point of view was discussed in greater detail in our paper entitled "System Recovery Voltage Determination by Analytical and A-C Calculating-Board Methods" presented at the 1937 summer convention in Milwaukee.

Mr. Van Sickle's discussion is of value in that it shows the application of the system recovery voltage data to another specific apparatus problem, the circuit-breaker problem.

Mr. Rudge has presented certain results of field tests and made comparison of measured voltage magnitudes with those of figure 8 of the paper. In this connection it is pertinent to observe that the recovery voltages of figure 8 are definitely associated with the figures 6 and 7 which incorporate time as well as magnitude. In figure 5 we have shown that higher recovery voltages than those plotted in figures 6, 7, and 8 can appear on the system but those voltages were generally associated with considerably longer time so that they were not considered as significant as the voltages plotted. It is, therefore, unfortunate that Mr. Rudge has not given the times associated with the voltage magnitudes presented in his discussion so that a detailed comparison could be made. Mr. Rudge's discussion raises the question as to whether more severe conditions are met in the field than are indicated in the paper. This question cannot be answered unless the times associated with the recovery voltage magnitudes are also presented. Results of tests given in the previous paper just referred to, together with the results of other tests, give recovery voltage magnitudes and times checking closely with the values given in the paper under discussion when these are put on a comparable basis.

Mr. Rudge also makes comparisons between the results of field tests and the data presented in the paper with respect to maximum recovery voltage as a function of fault current and as a function of system voltage. Again the comparisons are not complete unless time as well as magnitude is considered.

In the discussion Mr. Rudge implies, but does not state, that the systems tested fall within the range of systems studied on the a-c calculating board. It would be valuable if more detailed information had been given on the systems tested so that direct comparisons could be made.

Concerning field tests, we have recognized and have emphasized in both of our papers the desirability and necessity of such tests as a means for providing benchmarks. We do feel, however, that, on account of the limitations in making such tests, it is difficult if not impossible by this means alone to obtain a broad grasp of the recovery voltage problem.

Both Mr. Rudge and Mr. Prince have raised the question of the characteristics of protector tubes in the short time range. The object of figure 12 is not to provide a complete picture of the characteristics of protector tubes but instead to illustrate the use of the recovery voltage data given in the paper in connection with the application of a particular piece of apparatus, the protector tube, giving only the characteristics of tubes essential for this purpose. We quite agree with Mr. Rudge's statement that protector tubes may fail to clear at voltages less than normal crest value which have a "too rapid rate of voltage rise." The operation of a protector tube depends on the tube design and on the characteristics of the circuit. For example, tube *P*-3 would fail at less than normal crest for the particular system considered in figure 12, but tube *P*-1 would obviously give satisfactory operation for the same system recovery voltages. More data is available on the characteristics of protector tubes but this is not believed to be pertinent to the present discussion.

Mr. Prince seems to have misinterpreted our discussion on the application of the data in that he infers that we believe the pertinent point in the recovery-voltage problem is the maximum voltage crest and corresponding time. The basic factor to be considered is the relation of the complete curve of system recovery voltage and the complete curve of insulation recovery voltage. However, the complete curve is rarely needed and usually the conditions may be defined by a few critical points usually describable as first crest and maximum crest. We wish to emphasize the fact that these critical points must be given in terms of voltage magnitude and time, or some pair of related quantities, and cannot be given in terms of a single quantity such as the rate of rise of recovery voltage.

Mr. Prince inquired as to the method of obtaining insulation recovery curve of the protector tubes of figure 12. The points were obtained by measuring the voltage and time on cathode-ray oscillograms when restriking occurred. Therefore, as stated by Mr. Prince, if the system recovery voltage curve crosses the insulation recovery curve, failure will occur at the point of crossing.

Mr. Weygandt has made an interesting suggestion to replace the equivalent network used in recovery voltage studies by a "reciprocal" network in order to avoid the effects of stray capacitance. This is not, however, an important limitation of the a-c calculating-board method. This is due to the fact that it is usually possible to select an equivalent network using high values of capacitance and still obtain the desired natural frequencies. An alternative method is to use different ratios of fundamental to natural frequencies as explained in the closing discussion of our previous paper (see ELECTRICAL ENGINEERING, October 1937, page 1311). It is also important to retain the advantages inherent with the present procedure which permits obtaining in a straightforward manner the transient effects due to the application of a fault and its clearing as well as the effects of subsequent restrikes.

We agree with Mr. L. G. Smith that when a broad knowledge of the characteristics of recovery voltages of systems is obtained and when more data on the performance of protector tubes becomes available, a more scientific application of protector tubes will become possible and this in turn will eventually lead to a simplification of the application procedure.

Critical Conditions in Ferroresonance

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Introduction

THE APPLICATION of "ferroresonant" circuits to engineering problems has become increasingly important from both theoretical and practical considerations. While much progress has been made in the general study of ferroresonance, treatment is lacking in an accurate description of, perhaps, the most interesting part of the phenomena, namely, the so-called critical points at which the current suddenly changes in value. From a practical viewpoint, it is this characteristic property that makes ferroresonant circuits useful as sensitive elements in relay and control apparatus. Accordingly, there is a definite need for simple quantitative information that completely predetermines the behavior of the circuit under critical conditions and specifies the required circuit parameters in terms of known quantities.

Of the various methods employed in the study of ferroresonance, the method of graphical solutions presents itself as the most flexible and gives a somewhat better physical picture of the phenomena. On the basis of this method, a simple rule is proposed for the criteria of stability which uniquely determines the critical conditions and permits analytic development of simple formulas defining the critical quantities. An interpretation and discussion of equations bring to light several interesting aspects of the phenomena and point to inaccuracies of previous methods. Various applications of the theory are given.

Review

The ferroresonant circuit consists of elements of resistance and capacitance connected in series with an iron-core inductance. In the study of ferroresonant circuits with a-c applied voltage, rigorous mathematical treatment has been found practically impossible. With certain simplifying assumptions, however, solutions[3-7] have been given which serve in a general sense to describe the nature of the phenomena. Essentially, the methods followed in treating the series circuit (figure 1) require the determination of a functional relationship between the current and the inductance of the iron-core reactor. Under the assumption of sinusoidal variation of current, a continuous voltage characteristic is obtained from the well-known circuit equation,

$$E = I\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}\;, L = \Phi(I) \tag{1}$$

Several theoretical characteristics are given in figure 2 showing the influence of resistance and capacitance. To attach meaning to the multivalued curves thus obtained, the region in which the voltage decreases with increase of current is designated as unstable since it is physically unattainable. At the extremities of this region therefore, an abrupt change in current takes place as indicated by the dotted lines. The points of sudden transition of current are defined as critical points (1, 2).

It is of particular interest to note that the analytical method provides no means for determining the critical points except by the graphical procedure just outlined. Furthermore, it would appear that the critical points (2) associated with the collapse of current are bounded by the resistance line (IR) since the variable impedance has a minimum value equal to the resistance. This relation, however, is at most approximate and does not hold in general.

The method of graphical analysis, first introduced by Stark,[4] is advantageous because of its simplicity. In this representation, the volt-ampere characteristics of the reactor ($E_L = f(I)$) and the condenser ($E_c = I/(\omega C)$) are employed to determine graphically the variation of the resultant reactive voltage which is given by the difference of ordinates (figure 3). The results are then applied to a modified form of equation 1,

$$E = \sqrt{(IR)^2+\left(E_L-\frac{I}{\omega C}\right)^2} \tag{2}$$

and operating characteristics similar to those in figure 2 are obtained.

A refinement of this method, due to Rouelle,[19] requires the separation of the reactor voltage into active and reactive components on the basis of fundamental frequency. A predetermination of the operating characteristics shows good agreement with experimental results; but just as in the previous case, the critical points are not defined.

Method of Graphical Solutions

Historically, the method of graphical solutions represents one of the earliest attempts to explain the multivalued character of ferroresonance and was introduced by Bethenod[1] in a brief treatment on the ideal circuit. The method was later extended to the general circuit by Margand[6] whose interpretation uncovered several important facts, particularly concerning stability. There remain, however, a few considerations that need amplification in order to arrive at a better understanding of the critical conditions.

The graphical solution for the series ferroresonant circuit (figure 1) follows from two independent relations for

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1. For all numbered references, see list at end of paper.

reactor voltage; namely, the volt-ampere characteristic of the reactor,

$$E_L = f(I) \tag{3}$$

and the circuit equation (assuming sinusoidal quantities)

$$E_L = \pm\sqrt{E^2 - (IR)^2} + \frac{I}{\omega C} \tag{4}$$

In the right-hand member of equation 4, the expression under the radical represents an ellipse whose principal axes have values E and E/R, respectively. The second term is a straight line through the origin having a slope

$$\tan\gamma = \frac{1}{\omega C} \tag{5}$$

The sum of the two terms is an ellipse (a fact that may be readily ascertained) whose intersections with the reactor characteristic are the graphical solutions for current (points 1, 2, and 3 in figure 3). The representation in this form because of its component nature permits a simple graphical study of the circuit characteristics with a variation of any of the parameters. To avoid ambiguity of multiple current values, however, it is necessary to distinguish the real or stable solutions from the unstable solutions.

Stability of Current Values

In the previous methods outlined, the question of stability of certain current values arose as a result of the current-voltage relations obtained theoretically. The portion of the curve that was found experimentally nonexistent was classified as unstable, and the curves were modified accordingly to account for the characteristic discontinuity (dotted lines in figure 2). The method of graphical solutions, on the other hand, provides a means to test the stability of the individual solutions and gives, therefore, a better physical picture of the related phenomena.

It was not until Margand's treatment appeared that a reasonable explanation was given concerning the stability of the various points. He showed that points 1 and 2 (figure 3) represented stable solutions, whereas point 3 represented an unstable solution. The following reasons are the basis for this distinction.

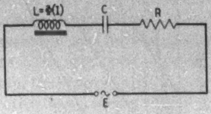
Suppose that to leave a stable point, the current suddenly changes by an amount ΔI. The effect of this change upon the circuit may be studied from the relation,

$$(RI)^2 = E^2 - \left[f(I) - \frac{I}{\omega C}\right]^2 \tag{6}$$

If the quantity $[f(I) - I/\omega C]$ increases in magnitude with a slight increase of current, then $(RI)^2$ tends to decrease in accordance with equation 6. The sense of variation of $(RI)^2$, however, is opposed to the assumed change of current; therefore, the point is indicative of a circuit condition in favor of stability. If on the contrary $[f(I) - I/\omega C]$ decreases in magnitude with increase of current, $(RI)^2$ tends to increase, indicating a sense of variation similar to that of the current. Under this condition, the current continues to increase, and the point is characterized as unstable. In a corresponding manner, the same argument could be advanced if the current were assumed to decrease slightly.

The stability of the various points may be established by observing graphically the variation in magnitude of the quantity $[f(I) - I/\omega C]$ as shown by the hatched ordinates in figure 3. Thus, point 3 corresponds to an unstable solution since $[f(I) - I/\omega C]$ decreases with increase of current. The opposite is true for points 1 and 2, hence these are stable solutions.

Figure 1. Ferroresonant circuit



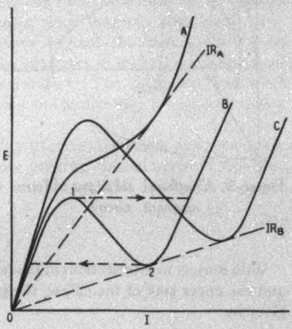


Figure 2. Voltage as a function of current

While the foregoing principle permits a distinction between stable and unstable current values, there appears to be an inconsistency concerning the limiting conditions. For example, take the case where capacitance is the only variable parameter and observe graphically the conditions for the limit of stability. At point 1 the limit of stability takes place when the reactor characteristic and the ellipse are tangent to each other, but at point 2, it occurs when the condenser line $(I/\omega C)$ intersects the reactor characteristic at the maximum value of current, E/R (figure 4). Consequently, with further increase of capacitance, point 2′ which defines a current value less than E/R should be unstable according to the above principle of stability. This, however, is contrary to experience, since it has been observed that the current may decrease considerably beyond the maximum value before instability occurs (see figure 15). Margand's explanation does not account for this effect since it is based upon the aggregate effect of the squared quantities in equation 6, thus concealing a change of sign. More specifically, the criteria for stability should be based upon the points of intersection of the two curves in such a manner that each portion of the ellipse as divided by the condenser line is properly expressed with respect to sign. As a modification, therefore, the following rule is proposed.

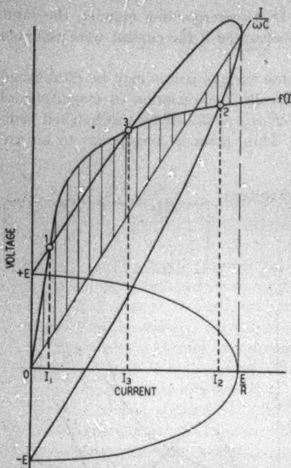


Figure 3. Graphical solutions of ferroresonant circuit

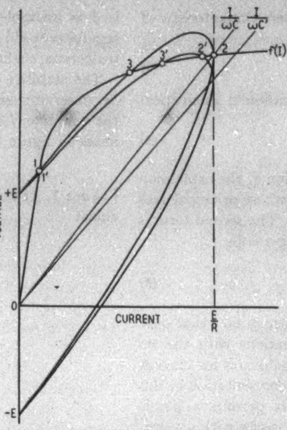


Figure 4. Influence of variation of capacity upon stability; C′ >C

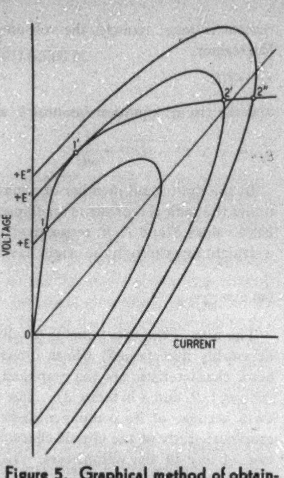


Figure 5. Graphical method of obtaining circuit characteristic, current as a function of voltage

With respect to intersections of the reactor characteristic and the upper part of the ellipse, the graphical solution is given by

$$f(I) = +\sqrt{E^2 - (IR)^2} + \frac{I}{\omega C} \qquad (7)$$

Any such point is stable if, for a slight change in current, $f(I)$ correspondingly changes at a greater rate than the right hand member of (7), taking cognizance of the sense of variation of current. Thus, for an increment of current, the ordinate of the reactor characteristic must be greater than that of the upper part of the ellipse, and less, if the current is assumed to decrease slightly.

For intersections associated with the lower part of the ellipse, the graphical solution is given by

$$f(I) = -\sqrt{E^2 - (IR)^2} + \frac{I}{\omega C} \qquad (8)$$

and the conditions for stability are reversed. In this case a point is stable if the right hand member of (8) changes at a greater rate than $f(I)$ corresponding to an assumed deviation of current.

An application of the proposed rule will show, as before, that points 1 and 2 are stable, and that point 3 is unstable. Unlike the conclusions reached from Margand's principle, these points retain the same identity throughout their entire range of existence. For example, point 2′ (figure 4) which was previously found to be unstable, is according to the rule (7) a stable point. Furthermore, the limiting conditions for the stable points are geometrically similar, namely, common tangency of ellipse and reactor characteristic (Margand, in a later paper,[12] calls attention to this fact as a "peculiarity," but without giving an adequate explanation). At this point the conditions for stability and instability simultaneously exist, and a slight variation of the parameters in the proper direction will cause a sudden change in current. Graphically, this means that points 1 and 2 are related to the critical conditions of sudden increase and decrease of current, respectively.

Circuit Characteristics

To demonstrate the method of graphical solutions in the study of circuit characteristics, a few illustrations will now be given. The current as a function of capacitance may be obtained by observing the resulting intersections of the ellipse with the reactor curve as the ellipse is rotated clockwise with increasing capacity, having always a vertical tangent at E/R. (The variation of point 3 is unnecessary to observe and is therefore excluded.) If reference is made to figure 4, it is clear that the current, defined by point 2 for small values of capacitance, first increases continuously to a maximum and then decreases until the limit of stability is reached (point of tangency), beyond which the current suddenly falls to a low value defined by point 1. With further increase of capacitance, the current decreases slowly approaching a constant value. Upon diminishing capacitance, the current retraces the same path in the region of low current values and goes beyond the previous point of collapse, since now point 1 is established and governs the value of current until the point becomes unstable. This will occur when the ellipse is tangent at the knee of the reactor curve, at which point the current suddenly increases to a value defined by point 2. Thereafter with further decrease of capacitance, the original curve is followed. The complete characteristic is similar to those shown in figure 15.

In addition to the current, the reactive voltages may be determined simultaneously by the same process on account of the component nature of the graphical solution. Furthermore, as Rüdenberg[8] has shown, the intersections associated with the upper and lower parts of the ellipse are related to lagging and leading power factor, respectively. The effect of resistance may be observed by varying only the major diameter of the ellipse. If the resistance is sufficiently large, instability is impossible, and a single continuous curve is obtained.

The variation of current as a function of voltage may be studied by observing intersections when both major and minor diameters of the ellipse change while the resistance and capacitance remain constant. With reference to figure 5, it is clear that with rising voltage, the current first increases slowly, then jumps suddenly to a high value (point 2′), and increases continuously thereafter. Upon decreasing the voltage, the current decreases until instability is reached and then falls to a low value. Curves of the same form as those in figure 2 are obtained.

The current as a function of frequency may be adequately described in a relatively simple manner if the reactor voltage is assumed to be directly proportional to fundamental frequency, that is,

$$E_L = \omega F(I) \tag{9}$$

The graphical solution then takes the form

$$F(I) = \pm \frac{1}{\omega}\sqrt{E^2 - (IR)^2} + \frac{I}{\omega^2 C} \tag{10}$$

and the procedure for determining the operating characteristics is analogous to the previous illustrations. In this case, the curve $F(I)$ is constant for all frequencies, whereas both components of the ellipse vary inversely with frequency.

The foregoing examples are indicative of the flexibility of the method of graphical solutions. Although the process of obtaining the circuit characteristics is, from a practical viewpoint, tedious, the presentation gives an unusually clear picture of ferroresonant phenomena. Intrinsically, however, the value of the method lies in the study of critical conditions.

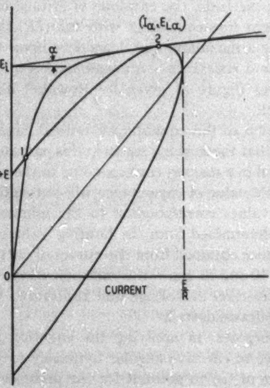


Figure 6. Graphical representation of critical conditions

Critical Quantities

On the basis of the proposed rule, the limit of stability is uniquely defined by common tangency of ellipse and reactor characteristic. In the region of low current (point 1) this condition is possible only in the neighborhood of the knee of the characteristic; hence, the range of critical current values is small. On the other hand, in the region of high current (point 2) the range of critical current values is considerably greater since tangency may occur anywhere along the curve beyond the knee. With these general concepts, the character of critical quantities may be analytically determined. The following treatment deals with point 2, but the results may be applied equally to point 1.

Since, for instability to occur, the ellipse and the reactor must have a common tangent, the intercept of this tangent on the voltage axis E_i (figure 6) may be expressed as

$$E_i = E_{L\alpha} - I_\alpha \left(\frac{dE_L}{dI}\right)_\alpha \tag{11}$$

also

$$\left(\frac{dE_L}{dI}\right)_\alpha \equiv \tan\alpha \tag{12}$$

where $E_{L\alpha}$ and I_α are the critical values of reactor voltage and current at the point of tangency, and $\tan\alpha$, the slope of the tangent.

Employing equation 4, $E_{L\alpha}$ and $(dE_L/dI)_\alpha$ may be evaluated. Thus

$$E_{L\alpha} = +\sqrt{E^2 - (I_\alpha R)^2} + \frac{I_\alpha}{\omega C} \tag{13}$$

$$\left(\frac{dE_L}{dI}\right)_\alpha = \frac{-R^2 I_\alpha}{\sqrt{E^2 - (I_\alpha R)^2}} + \frac{1}{\omega C} \tag{14}$$

Inserting (13) and (14) in (11) and solving for the current, we have

$$I_\alpha = \frac{E}{R}\sqrt{1 - \left(\frac{E}{E_i}\right)^2} \tag{15}$$

The critical capacitance is determined by means of (12), (14), and (15), hence

$$C_\alpha = \frac{1}{\omega\left(\tan\alpha + R\sqrt{\left(\frac{E_i}{E}\right)^2 - 1}\right)} \tag{16}$$

By making use of the expressions for current and capacitance the reactive voltages are obtained.

$$E_{L\alpha} = E\left\{\frac{E}{E_i} + \sqrt{1 - \left(\frac{E}{E_i}\right)^2}\left(\frac{\tan\alpha}{R} + \sqrt{\left(\frac{E_i}{E}\right)^2 - 1}\right)\right\} \tag{17}$$

$$E_{C\alpha} = E\sqrt{1 - \left(\frac{E}{E_i}\right)^2}\left(\frac{\tan\alpha}{R} + \sqrt{\left(\frac{E_i}{E}\right)^2 - 1}\right) \tag{18}$$

Within the limits of assumptions (fundamental frequency), these equations describe the behavior of the critical quantities as functions of the applied voltage, circuit resistance, and characteristic constants of a given iron-core reactor.

Discussion of Equations

A. Region of High Current

In the region of high current characterized by the saturated portion of the reactor characteristic, only the critical conditions for sudden decrease of current exist and are described graphically as the limit of stability at point 2. As this portion of the curve does not include all possible values it is evident that critical quantities related to this point can be defined only within certain limits of applied voltage and circuit resistance. By defining I_0 as the minimum critical current (theoretically, the point at which the tangent leaves the characteristic), the limits of voltage and resistance may be determined by means of equation 15 as follows:

$$I_0 = \frac{E}{R}\sqrt{1 - \left(\frac{E}{E_i}\right)^2} \qquad (19)$$

Solving for the voltage,

$$E = \frac{E_i}{\sqrt{2}}\sqrt{1 \pm \sqrt{1 - \left(\frac{2RI_0}{E_i}\right)^2}} \qquad (20)$$

for which

$$R \leq \frac{E_i}{2I_0}$$

in order to obtain a real result. Equation 20 gives two values of voltage, and the limiting case occurs for maximum resistance.

$$R_{max} = \frac{E_i}{2I_0} \qquad E_{max} = E_{min} = \frac{E_i}{\sqrt{2}} \qquad (21a)$$

$$R \longrightarrow 0 \qquad E_{max} \longrightarrow E_i,\ E_{min} \longrightarrow 0 \qquad (21b)$$

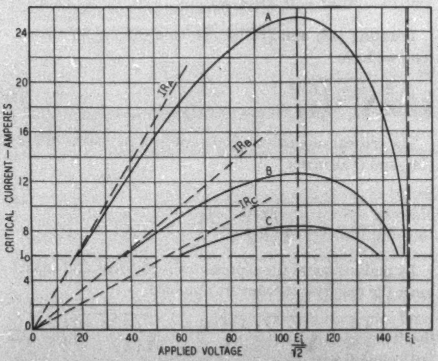


Figure 7. Critical current as a function of voltage. Circuit resistance: A, 3 ohms; B, 6 ohms; C, 9 ohms

Between these limits of applied voltage and resistance instability is possible, and the value of the critical current will be given by (15). It is to be recognized that the above limits are general, and determinate only for the linear portion of the reactor characteristic where E_i is practically constant for a wide range of current values. The maximum critical current is found in the usual manner by differentiation, that is

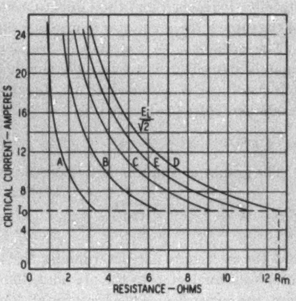
$$\frac{dI_\alpha}{dE} = \frac{1 - 2\left(\frac{E}{E_i}\right)^2}{R\sqrt{1 - \left(\frac{E}{E_i}\right)^2}} = 0$$

giving

$$E = \frac{E_i}{\sqrt{2}},\ I_{\alpha\, max} = \frac{E_i}{2R} \qquad (22)$$

Curves showing the variation of critical current as a function of applied voltage and resistance for the saturated portion of the reactor curve (figure 12), neglecting core losses, are given in figures 7 and 8. These curves give a remarkable insight into the phenomena of ferroresonance and call attention to an interesting fact. Contrary to the results of previous methods, the envelope of critical current is not a straight line coinciding with the (IR) line, but a curve having a maximum and dependent upon the values of voltage and resistance. An examination of the experimental curves (figure 9) given by Rouelle[19] illustrates this point well.

Figure 8. Critical current as a function of resistance applied voltage; A, 20 volts; B, 40 volts; C, 60 volts; D, 106.8 volts' E, 130 volts



From consideration of the equation for critical capacitance, it is evident that the working region in this particular case must be limited in a manner corresponding to the current. Since a single value of capacitance will not suffice, a set of limiting values corresponding to the minimum critical current is determined from the limiting values of voltage and resistance obtained from the curves of critical current. Figures 10 and 11 show the variation of critical capacitance as a function of voltage and resistance; the limiting curves are drawn dotted.

For practical purposes, in applying the equation for critical capacitance, an effective angular frequency (ω^1) is introduced in place of (ω) to account for the presence of

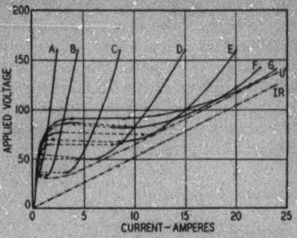


Figure 9. Voltage as a function of current: frequency, 50 cyles per second, resistance 5.2 ohms, capacitance, A, 25 microfarads; B, 40 microfarads; C, 56 microfarads; D, 90 microfarads E, 110 microfarads; F, 126 microfarads; G, 131 microfarads. U', active component of voltage (experimental results of Rouelle, R. G. E., 1934, page 728)

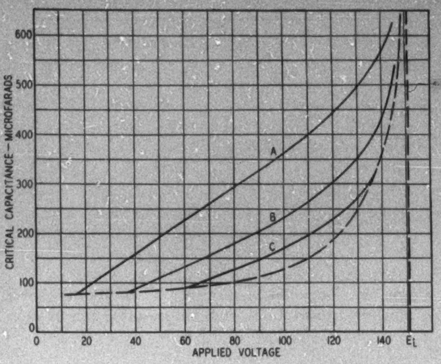


Figure 10 (right). Critical capacitance as a function of voltage. Circuit resistance: A, 3 ohms; B, 6 ohms; C, 9 ohms. Frequency, 60 cycles per second

harmonics. Although the variation of harmonic components of current in ferroresonant circuits is extremely complicated, it has been found that for values of current in the vicinity of breakdown, an effective angular frequency computed from measured values of current, capacitance, and condenser voltage ($\omega^1 = I/E_C C$) is strikingly uniform. In a circuit having a fundamental frequency of 60 cycles per second, the average over a wide range of values of current and capacitance gives

$$\frac{\omega^1}{\omega} = 1.24$$

with a maximum deviation of less than two per cent. For extremely small values of current (point 1), a ratio about nine per cent less is suitable.

Concerning the factors involved in the formulas, the evaluation of circuit resistance is, perhaps, the most important. In addition to the series resistance, consideration must be given to the apparent resistance of the reactor which takes into account the iron losses. This quantity changes both with the degree of saturation and wave form, and it is therefore necessary to have a measure of the variation if accuracy is to be attained.

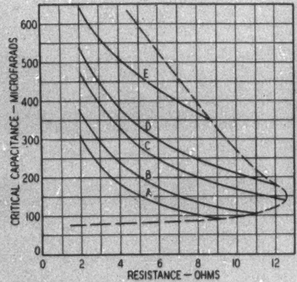


Figure 11. Critical capacitance as a function of circuit resistance. Applied voltage: A, 60 volts; B, 80 volts; C, 106.8 volts; D, 120 volts; E, 140 volts; frequency, 60 cycles per second

As an approximation, it is convenient to determine apparent resistance on the basis of fundamental frequency. The method employed, similar to that of Rouelle,[19] requires power measurements on the reactor with sinusoidal voltage applied. Since under these conditions the average power indicated by the wattmeter is related only to fundamental quantities, the apparent resistance may be defined accordingly as

$$R_A = \frac{P}{I_1^2} \tag{23}$$

where P is the average measured power and I_1, the fundamental component of current at a given value of applied voltage. The variation of apparent resistance and the corresponding fundamental voltage drop (I_1R_A) as a function of effective current are shown in figure 12. The latter quantity is important because in the application of equation 20, total resistance voltage drop required is properly expressed as the sum of the series (external) and the apparent resistance voltage drops. Since the apparent resistance is determined on the basis of fundamental frequency, it follows that the same conditions must be observed for the corresponding voltage drop.

B. Region of Low Current

In the region of low current characterized by the nonlinear portion of reactor characteristic, critical conditions for both sudden increase and decrease of current are theoretically possible. It is necessary therefore, in applying the formulas to determine in what manner the results are related to critical points 1 and 2. In general, it may be inferred from graphical analysis that for point 1, the critical current increases continuously with voltage, whereas, for point 2 a free variation is possible.

Due to the variability of the apparent resistance and the characteristic constants in the nonlinear region, it is advantageous to choose current as an independent variable in order to make a theoretical investigation of critical conditions. For this purpose, equation 20, which gives the voltage as a function of current may be employed. Ob-

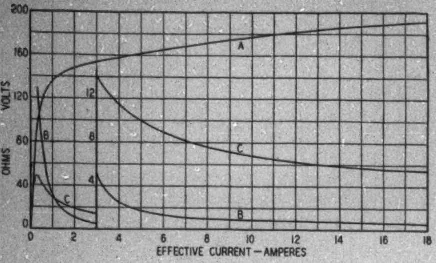


Figure 12. Properties of reactor: A, volt-ampere curve; B, apparent resistance; C, apparent resistance voltage drop (I_1R_a)

viously two values of voltage are possible for every value of current, but only one of these may be associated with point 1 if it exists, the other with point 2. The existence of a critical point 1 depends upon the value of current and resistance in such manner that the voltage given by (20) (positive sign) increases continuously with critical current. This means that only a certain set of critical current values can be defined by this point, and its range, therefore, must be determined by the limiting conditions which are fixed by minimum and maximum values of critical current. With respect to the minimum value the limits of applied voltage and resistance are bound by the same restrictions as in the previous case (21*a*), but are of an indeterminate nature. For the maximum value, these limits are likewise indefinite and can be found only by successive computation; it is the point at which the voltage ceases to increase with critical current (upper limiting value). For critical current values greater than this maximum, both values of voltage given by (20) are clearly associated with the critical point 2. Consequently, the locus of critical current with respect to voltage determined for the nonlinear region of the reactor characteristic includes two distinct portions according to the type of critical condition.

Curves showing the variation of critical current as a function of voltage, taking into account apparent resistance of the reactor (figure 12), are given in figure 13. The solid and dotted portions of the curves are related to critical points 1 and 2, respectively; the dividing line XY is the envelope of limiting values of voltage between which both types of critical conditions are possible. The corresponding curves for critical capacitance are given in figure 14.

The general form of these curves discloses several interesting properties of ferroresonant circuits. Although the abrupt changes in current have been observed to occur alternatively as a pair, the curves give evidence of a possibility of achieving only breakdown of current at low values of voltage beyond the limiting value without the corresponding sudden increase of current. Another interesting feature is that critical current in the region characterized by point 1 increases with resistance, although not appreciably except at higher voltages. Likewise, critical capacitance changes only slightly with increased resistance. In this case, however, there is a nonuniform variation which may be ascribed to the effect of apparent resistance. These families of curves, together with those previously discussed form the complete envelope and illustrate the typical variation of the critical quantities.

Application of Theory

The ferroresonant circuit presents to the engineer a wide variety of possibilities that are useful in relay and control problems. The extent to which these may be utilized is apparent from the circuit properties shown in the preceding curves. In this connection, therefore, it is intended only to suggest what might be done and to give some illustrative examples.

1. Suppose it is desired to design a ferroresonant circuit such that for a given value of series resistance the maximum change in current occurs when the circuit becomes unstable.

If, for example, the case of breakdown of current is considered, the necessary conditions are defined by (22) in which E_t is the only unknown quantity. Assuming that the critical current is related to the saturated portion of the volt-ampere curve of the reactor, the intercept value of

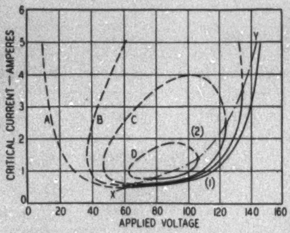


Figure 13. Variation of critical current in region of low current. Series resistance: A, 0 ohms; B, 9 ohms; C, 15 ohms; D, 25 ohms

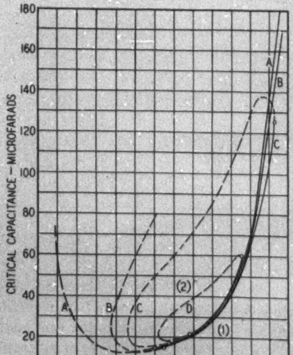


Figure 14. Variation of critical capacitance in region of low current. Series resistance: A, 0 ohms; B, 9 ohms; C, 15 ohms; D, 25 ohms; frequency 60 cyles per second

the tangent is, from figure 12, $E_t = 151$ volts. Thus, if the series resistance, $R_s = 9$ ohms

$$I_{\alpha\,max} = \frac{E_t}{2R} = \frac{151}{18} = 8.38 \text{ amperes}$$

At this current value, the apparent resistance of the reactor is, $R_A = 1$ ohm, which is to be added to the series resistance. Since E_t is about the same as before, the corrected value of current becomes

$$I_{\alpha\,max} = \frac{151}{20} = 7.55 \text{ amperes}$$

and the required applied voltage

$$E = \frac{E_t}{\sqrt{2}} = \frac{151}{\sqrt{2}} = 106.8 \text{ volts}$$

The required circuit capacitance is given by (16) in which $\omega' = 468$ radians per seconds ($f = 60$ cycles per second, $\omega'/\omega = 1.24$); further corresponding to E_t, $\tan\alpha = 2.7$ ohms. Thus,

$$C_\alpha = \frac{1}{468(2.7 + 10)} = 168 \text{ microfarads}$$

If, instead of breakdown, circuit instability with respect to the sudden increase of current were considered, the applied voltage necessary for the maximum change of current is somewhat higher than in the previous case. Evidence of this fact is given in figure 9, curve *e*. To make the computation, therefore, it is only necessary to select a few current values about the knee of the volt-ampere curve that gives by means of (20) a value of voltage about ten per cent higher than in the previous case. This solution also determines the proper characteristic constants to be used in the equation for critical capacitance.

2. Suppose that it is desired to design a ferroresonant circuit that is sensitive to small changes in voltage yet single valued in character (for example, curve *g*, figure 9). As this condition is related to the upper limiting value of voltage (figure 13), it is necessary to compute several values of voltage by means of (20) corresponding to current values selected above the knee of the volt-ampere curve. The required value is that for which the voltage becomes a maximum with respect to current.

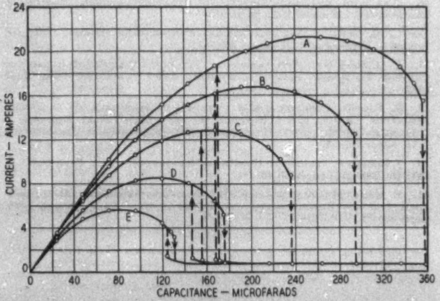


Figure 15. Experimental results; current as a function of capacity. Series resistance: A, 3.88 ohms, B, 5.02 ohms; C, 6.65 ohms; D, 9.9 ohms; E, 14.75 ohms. Applied voltage, 121 volts; frequency 60 cycles per second

For example, let $R_s = 15$ ohms. Take $I = 1.5$ amperes then from figure 12, $E_t = 127$ volts and $I_1R_A = 22$ volts; $(IR)_{\text{total}} = 22.5 + 22 = 44.5$ volts.

From (20)

$$E = \frac{127}{\sqrt{2}}\sqrt{1 + \sqrt{1 - \left(\frac{89}{127}\right)^2}} = 118.5 \text{ volts}$$

Similarly,

$I = 2$ amperes, $E = 123$ volts
$I = 2.5$ amperes, $E = 123.5$ volts
$I = 3$ amperes, $E = 122.5$ volts

Hence, the required voltage is 123.5 volts and the corresponding constants are: $E_t = 137$ volts, $\tan\alpha = 5.75$ ohms and $R_A = 7$ ohms

therefore,

$$C = \frac{1}{468\left(5.75 + 22\sqrt{\left(\frac{137}{123.5}\right)^2 - 1}\right)} = 131 \text{ microfarads}$$

3. In the general problem, we are concerned with the determination of critical current and capacitance for a given value of applied voltage and series resistance. For this case, the results of experiments on a ferroresonant circuit employing a given reactor are available (figure 15) so that a quantitative check on the theory may be obtained.

For example: $E = 121$ volts, $R_s = 6.65$ ohms (experimental values). Neglecting the apparent resistance of the reactor, a trial calculation is made using the value of intercept voltage, $E_t = 151$ volts, determined from the linear portion of the volt-ampere curve of the reactor. Thus from (15)

$$I_\alpha = \frac{121}{6.65}\sqrt{1 - \left(\frac{121}{151}\right)^2} = 10.9 \text{ amperes}$$

For this value of current, E_t is about the same as assumed above and $R_A = 0.95$, so that the total series resistance becomes 7.60 ohms. The corrected value of critical current is therefore

$$I_\alpha = \frac{121}{7.6}\sqrt{1 - \left(\frac{121}{151}\right)^2} = 9.53 \text{ amperes}$$

From test it is found (see curve *c*, figure 15), $I_\alpha = 8.9$ amperes.

Computing the critical capacitance ($f = 60$ cycles per second, $\omega' = 468$ radians per second, $\tan\alpha = 2.7$ ohms)

$$C_\alpha = \frac{1}{468\left(2.7 + 7.6\sqrt{\left(\frac{151}{121}\right)^2 - 1}\right)} = 255 \text{ microfarads}$$

From test, $C_\alpha = 238$ microfarads

The errors in both cases are seven per cent, approximately.

To find the critical current for the point of sudden increase, it is convenient to employ equation 20 and make a

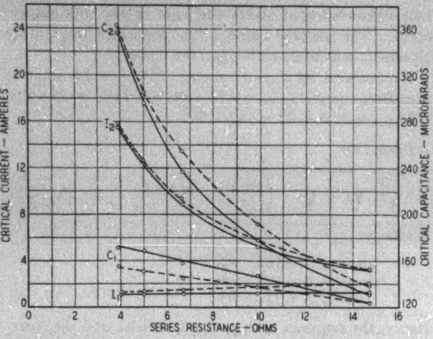


Figure 16. Comparison of measured (solid curves) and computed (dotted curves) values of critical current and capacity as a function of series resistance

few trial substitutions of current values. This is necessary since the characteristic constants change sharply with current about the knee of the volt-ampere curve of the reactor. The solution is given when the calculated voltage is equal to the applied voltage. Since the proper characteristic constants are now available, the calculation of critical capacitance follows directly.

Without going through the computations, reference is made to figure 16 in which a comparison of measured and calculated results is given. The error between these values is confined within reasonable limits and is due primarily to the presence of harmonics, particularly in the reactor voltage, which influence the value of apparent resistance.

Conclusion

The calculation of critical quantities by means of the given formulas is quite simple, although in some instances preliminary computation is required. In any case, the trial substitutions are made with respect to one quantity, and having once determined this quantity, the corresponding critical values of all the other quantities may be readily calculated. This method of obtaining critical quantities is obviously much superior to previous methods which require the determination of families of curves, as in figure 2, from which the desired information is then obtained.

In the study of critical conditions in ferroresonance, this investigation has demonstrated that the characteristic discontinuities of current are not isolated phenomena, but are directly related and occur under parallel circumstances. The principles governing stability and instability have been clearly set forth in the proposed rule for stability, and as a consequence, simple formulas defining critical quantities have been obtained. The rule for stability may also be applied in more complex problems such as the influence of superimposed d-c excitation upon ferroresonance. Critical conditions, as we have seen, are not only dependent upon the properties of the reactor, but also upon the applied voltage and resistance. These facts make for a clear understanding of ferroresonant phenomena and enable the engineer to design circuits to operate under optimum conditions.

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Overvoltages Caused by Unbalanced Short Circuits

Effect of Amortisseur Windings

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Introduction

IN CASE of unbalanced faults on power systems, the voltage on the unfaulted phase or phases may often reach very high values. There may be an overvoltage caused by the short circuit itself and a further overvoltage caused by the clearing of the fault. In the present paper only the first type will be discussed. These unfaulted phase overvoltages may vary widely with the type of fault, the system constants and arrangement, and the kinds of synchronous machines affected. However, in faults not involving ground the presence of overvoltages depends on the fact that the synchronous machines supplying the fault current are not electrically symmetric-rotor machines. Thus, these overvoltages may be largely eliminated or reduced by a properly designed amortisseur winding. Even in case of overvoltages caused by faults involving ground there is a considerable reduction with such an amortisseur winding.

Purely from the standpoint of reducing overvoltages it is evident that such rotor windings are very desirable, and it is advisable to examine carefully each particular case in order to determine how far in the direction of the ideal amortisseur it is economically desirable to go. It is the purpose of this paper to present comparative information on overvoltages caused by one type of fault on a salient-pole synchronous machine with various kinds of amortisseur windings in order to provide one basis for evaluating quantitatively the relative merits of the different types of amortisseur windings. The qualitative effects are of course already known. Mandl[1] has made an extended study of overvoltages caused by single-phase short circuits with terminal capacitance and has shown the existence of resonant points and the possibility of obtaining high voltages on the unfaulted phase of machines with electrically unsymmetrical rotors. Wagner[2] has recently made an experimental study of similar cases. Doherty and Nickle[3] have shown that the magnitude of open-phase voltage during single-phase short circuit of an open-circuited machine is a function principally of the ratio of quadrature-axis reactance to direct-axis reactance, while Park and Skeats[4] have similarly shown how this ratio affects recovery voltage. A later investigation[5] of double line-to-ground faults has given expressions for the open phase voltage for that case. The present paper takes into account more exactly than has hitherto been done the effects of amortisseur windings and of connected terminal capacitance and gives quantitative overvoltage data over a range of machine and system constants.

The practical importance of the study is brought out by lightning arrester and bus failures which could not be reasonably explained except by the high dynamic voltages described in this paper. In one case tests made subsequent to a failure have definitely shown the source of trouble to be these voltages.

In order to obtain the information reported here a detailed study of line-to-line short circuits on a synchronous machine connected to various capacitances and provided with many different designs of amortisseur circuits was made on the differential analyzer[6] at the Moore School of Electrical Engineering of the University of Pennsylvania. By means of the differential analyzer it was possible (1) to take into account most of the significant factors, the effects of which it would be practically impossible to calculate exactly and (2) to vary the machine and line characteristics easily over a wide range, which would be impossible in field tests.

A line-to-line short circuit was studied since this may result in some of the highest overvoltages, and it was not primarily the object to study the effect of various kinds of short circuits but rather the effects of various kinds of amortisseur windings.

Conclusions

As a result of this study the following conclusions have been drawn.

1. High voltages may appear on the unfaulted phase of a synchronous machine with unsymmetrical rotor during line-to-line short circuits.

2. The principal determining factor for the magnitude of overvoltages caused by line-to-line short circuits is the ratio (x_q''/x_d'') of quadrature-axis subtransient reactance to direct-axis subtransient reactance. This ratio is a function of the amortisseur characteristics.

3. The maximum possible overvoltage caused by a line-to-line short circuit on an open circuited machine is given very closely by the formula[3]

$$e_{\max} = 2\frac{x_q''}{x_d''} - 1 \tag{1}$$

4. The overvoltage may be considerably increased over the value of equation 1 by terminal capacitance, the worst condition arising when the capacitive reactance is approximately nine times the negative sequence reactance of the generator. The amount of increase of

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[1] For all numbered references, see list at end of paper.

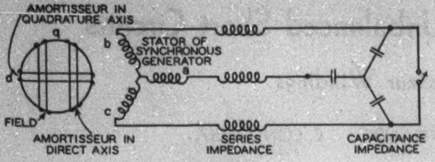

AMORTISSEUR IN QUADRATURE AXIS
q
d
b
a
c
STATOR OF SYNCHRONOUS GENERATOR
FIELD
AMORTISSEUR IN DIRECT AXIS
SERIES IMPEDANCE
CAPACITANCE IMPEDANCE

Figure 1

Table I

Case	x_d''	x_q''	$\frac{x_d''+x_q''}{2}(=x_2)$	x_q''/x_d''	e_0	e_c	$\frac{e_c-1}{e_0-1}$
1	0.439	0.456	0.447	1.04	1.08	1.6	7.5
2	0.456	0.490	0.473	1.07	1.15	2.0	6.66
3	0.478	0.544	0.511	1.14	1.28	2.85	6.6
4	0.489	0.567	0.528	1.16	1.32		
5	0.439	0.516	0.477	1.17	1.35	3.15	6.15
6	0.456	0.553	0.504	1.21	1.42	3.75	6.55
7	0.478	0.616	0.547	1.29	1.58	5.15	7.15
8	0.489	0.650	0.569	1.33	1.66	6.1	7.73
9	0.556	0.900	0.728	1.62	2.24	8.0	5.65

this maximum possible overvoltage depends on the field and armature short circuit time constants, being small when either or both of these quantities are small. For large waterwheel generators the field time constant is ordinarily very large and so may be assumed infinite with little error. On the other hand the terminal capacitance will ordinarily be that of a connected open transmission line and so will be accompanied by a somewhat increased armature circuit resistance which will tend to reduce the armature time constant and consequently the overvoltage.

5. If a salient-pole generator is to be subject to the possibility of connection to an open transmission line of relatively large capacitance or of open circuited operation the ratio x_q''/x_d'' should be kept within limits determined on the basis of permissible overvoltage.

6. The overvoltage will in general be greatly reduced by connection of the salient-pole machine to a power system.

7. The ratio x_q''/x_d'' may be determined by test from single-phase static impedance measurements. The ratio should either be measured at a current equal to the expected rms displaced single-phase short-circuit current or several points taken in order to extrapolate to this current.

Description of System

The system studied is shown in figure 1, the capacitance and series impedance being included to represent approximately the effect of a connected transformer and open transmission line.

The amortisseur circuits of the machine were represented by two windings, one in the field axis of the rotor and one in the quadrature axis, the mutual inductance of the field, direct-axis amortisseur, and direct-axis armature winding being a single quantity. The synchronous machine may therefore be represented by the equivalent circuits of figure 2, in which the machine characteristics are specified by the mutual and leakage reactances of the various windings. The field resistance was neglected, that is, constant flux linkages were assumed in the field circuit, since with the long field time constants of large synchronous generators the change of field flux linkages in the first few cycles after short circuit will be inappreciable and the short circuits will usually not last more than a few cycles. Also the effect of saturation was neglected in so far as it would tend to vary the reactance during the period of short circuit. However it is assumed that the reactances specified apply to saturation conditions corresponding to single-phase short-circuit current. The machine is assumed to conform to the theory of the ideal synchronous machine as defined by Park.[7]

The machine constants were taken as average values for salient-pole (waterwheel) generators, except for the constants of the additional rotor circuits, which were varied over a wide range. The constants (see nomenclature and figure 2 for definition of symbols) used in this study were, in per unit of the machine rating,

$x_l = 0.20$

$r_{kd} = 0.02$

$x_{ad}' = \dfrac{x_{ad}x_f}{x_{ad}+x_f} = 0.18,\ x_d' = 0.38$

$r_{kq} = 0.02$ for cases 1–4

$x_{aq} = 0.52,\ x_q = 0.72$

$r_{kq} = 0.04$ for cases 5–8

Case	x_{kd}	x_{kq}
1	0.09	0.09
2	0.14	0.14
3	0.22	0.24
4	0.29	0.29
5	0.09	0.18
6	0.14	0.26
7	0.22	0.43
8	0.29	0.56
9	∞	∞

Cases 1 to 4 apply to amortisseurs with substantially symmetric leakage reactances, while cases 5 to 8 apply to amortisseurs with quadrature-axis leakage reactance substantially twice those of the direct axis. Case 9 represents a machine with no amortisseur. In all of these cases the series impedance (figure 1) consisted of a reactance of 0.18 per unit and a resistance of 0.05 per unit, except as otherwise indicated. Also in cases 2 and 6 the amortisseur resistances were varied.

The electrical system of figures 1 and 2 may be represented by the set of simultaneous equations of appendix *A*, which were set up on the differential analyzer and solved for the unfaulted phase terminal voltage e_a.

In order fairly to compare the various types of machines, overvoltages were determined for each machine over a range of capacitive reactance, corresponding ap-

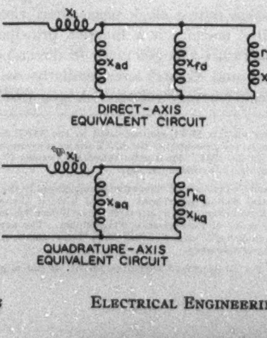


Figure 2

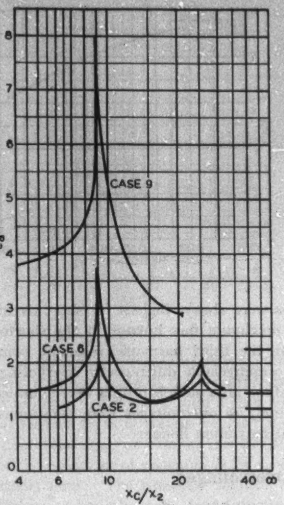


Figure 3. Overvoltage due to line-to-line fault on salient-pole synchronous generator—effect of amortisseur winding

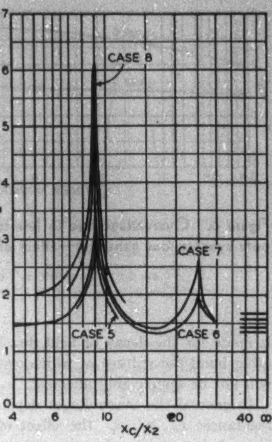


Figure 5. Overvoltage due to line-to-line fault on salient-pole synchronous generator—effect of amortisseur reactance

proximately to transmission lines of various voltages and lengths. The range covered varied somewhat but was in general about $3\,x_2 < x_c < 30\,x_2$, where x_2 is the average of the direct- and quadrature-axis subtransient reactances including the external reactance.

Results

Figures 3 to 5 show some of the results obtained. On these curves are recorded the maximum voltage reached in the first four or five cycles following short circuit, as read from the curves of instantaneous voltage versus time plotted by the differential analyzer.

It is seen that as a function of the ratio of line capacitive reactance to average generator subtransient reactance (or negative-sequence reactance) the overvoltage curves have the same characteristic shape for all machines. Voltage peaks are observed at $x_c/x_2 = 9$ and $x_c/x_2 = 25$, which correspond to series resonance of the line capacitance with the negative-sequence reactance for the third and fifth harmonics, respectively.

Figure 3 shows how the addition of the amortisseur winding of case 6 reduces the maximum voltage to about half of the value obtained with no amortisseur winding, and also the further reduction obtainable by the use of the amortisseur of case 2. This latter is particularly noticeable at the resonant point $x_c/x_2 = 9$. The overvoltages obtained when line capacitance is not considered are indicated at the extreme right of the figure. Line capacitance is seen in general to increase the voltages obtained, especially when it is great enough to approach the point of third-harmonic resonance.

Figures 4 and 5 show the effect of variation of the amortisseur leakage reactances. Figure 4 covers the range of amortisseurs with symmetric leakage reactances (cases 1–4), while figure 5 covers the range of unsymmetrical amortisseurs (cases 5–8). As would be expected, the low reactance windings are the most effective. As an indication of the relative effectiveness of the several amortisseur windings table I is given.

The subtransient reactances include the external inductive reactance. The e_0 column contains the maximum open-phase voltages obtained by the formula

$$e_0 = 2\frac{x_q''}{x_d''} - 1 \tag{1}$$

of Doherty and Nickle,[3] neglecting capacitance and assuming constant amortisseur flux linkage and short circuit at the instant of maximum flux linkage in the shorted phases. The e_c column contains the maximum voltages found by means of the differential analyzer under the worst condition of connected external capacitance $x_c/x_2 = 9$. The last column gives the ratio of maximum excess voltage $(e_c - 1)$ with capacitance to the excess voltage $(e_0 - 1)$ for an open-circuited machine. This ratio is seen to be very nearly a constant quantity since the armature time constant was not varied very much (r_a = constant = 0.05) and the field resistance was constantly zero. Since the maximum voltage e_c occurs several cycles after short circuit it will be somewhat reduced if allowance

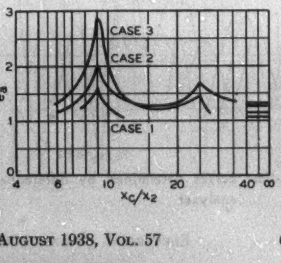


Figure 4. Overvoltage due to line-to-line fault on salient-pole synchronous generator—effect of amortisseur reactance

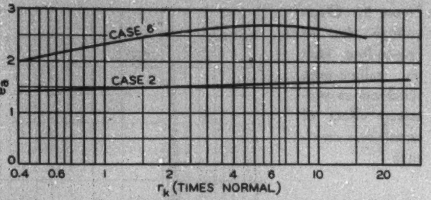


Figure 6. Overvoltage due to line-to-line fault on salient-pole synchronous generator—effect of amortisseur resistance

Case 6—x_c/x_2 = 9.9, x_q''/x_d'' = 1.21
Case 2—x_c/x_2 = 10.6, x_q''/x_d'' = 1.07

is made for the decay of field flux linkage, while on the other hand the voltage e_0, which occurs in the first cycle, will not be appreciably affected.

In general the overvoltages depend principally on the reactances x_d'', x_q'', x_c; the effect of the resistances r_{kd}, r_{kq}, r_a being of secondary importance except in the case of r_a at or near one of the resonant points (particularly at $x_c/x_2 = 9$). Figure 6 shows how at a particular value of line capacitance, and for the machines of cases 2 and 6, the overvoltage is affected by changes of the amortisseur resistances in the ratio of 32 to 1. For the cases investigated, amortisseur resistance has evidently very little effect; there is a slight increase of overvoltage with increasing amortisseur resistance.

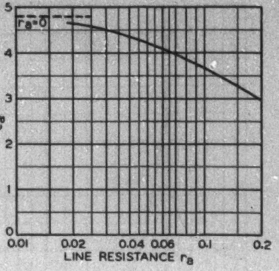
The effect of line resistance r_a in reducing the magnitude of overvoltage is seen in figure 7 for the machine of case 9 with a rather large line capacitance. The case of figure 7 is not a resonant condition, so that line resistance, unless very high, reduces the voltage only about 10 or 20 per cent. At a resonant point the line resistance has much more effect, the voltage in some cases being almost inversely proportional to resistance.

In figure 8 are given some typical voltage-time curves obtained. Inspection of these curves shows that the voltage consists principally of a term of fundamental frequency and one having the frequency $\sqrt{x_c/x_2}$, the latter corresponding to the "average natural frequency" of the series circuit formed by the unfaulted phase. That is, the frequency of the principal natural frequency term can be calculated with good accuracy by neglecting the fact that the machine has an unsymmetrical rotor and using the average of the direct- and quadrature-axis subtransient reactances.

Another point brought out by figure 8 and by the many other voltage curves taken is that with amortisseur windings the voltages in the first few cycles, if the decay of field flux linkages is neglected, are only slightly if at all higher than the values during later cycles after the transients caused by the amortisseur and stator resistances have died away. In fact, in the region where the highest overvoltages are likely to occur, i.e., near $x_c/x_2 = 9$, the voltage requires several cycles to build up to its peak value, which occurs after armature transients have vanished.

Figure 7. Overvoltage due to line-to-line fault on salient-pole synchronous generator—effect of line resistance

Case 9—x_c/x_2 = 6.86



The overvoltages were always highest for short circuit at the instant of maximum flux linkage in the shorted phases except near the third harmonic resonant point where there was no appreciable difference in overvoltage with angle. Therefore, after sufficient data were obtained to settle the question, all overvoltages were determined for short circuits at maximum linkage.

Calculation of Overvoltages

Consideration of the form of the voltage curves and of the rather small effect of rotor resistance leads one to the conclusion that for machines with amortisseur windings a good approximation to the voltage may be had by neglecting rotor resistances and also neglecting armature transients. If this is done a relatively simple method of calculation is available (see appendix *B*), the results of which show approximate agreement with those found by the differential analyzer. Figure 9 presents a comparison of calculated and analyzer results for case 6.

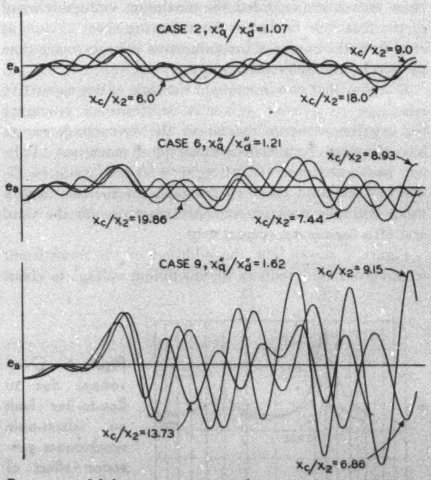


Figure 8. Voltage-time curves determined by differential analyzer

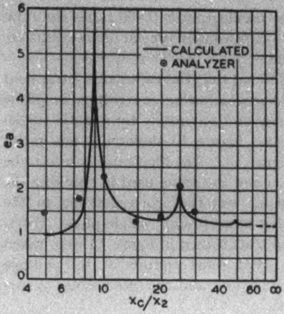


Figure 9. Comparison of calculated steady-state overvoltages with differential analyzer results

Case 6—$x_q''/x_d'' = 1.21$

In the case of the synchronous machine with no amortisseur windings (case 9) the armature transients had to be considered in order to obtain a reasonable check. Figure 10 shows a curve of voltage versus time determined by the differential analyzer compared with a step-by-step solution of the equations of appendix *B* for $r_a = 0$, $x_c = 5$, and no amortisseur. The complete solution including armature transients has also been found analytically as explained in appendix *B* and points calculated by the formula developed are shown on figure 10.

Equations 8*b* and 9*b* of appendix *B* show that the voltage is a function of the three ratios y/x, r/x, x_c/x. y/x is simply related to the ratio x_q''/x_d'' by the formula,

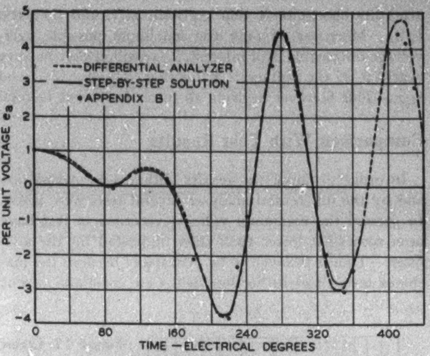


Figure 10. Overvoltage following line-to-line fault on salient-pole synchronous generator

$$\left(1 + \frac{y}{x}\right)\left(1 + \frac{x_q''}{x_d''}\right) = 2 \qquad (2)$$

while x is very nearly the negative-sequence reactance x_2. The voltage produced is thus a function of x_q''/x_d'', the

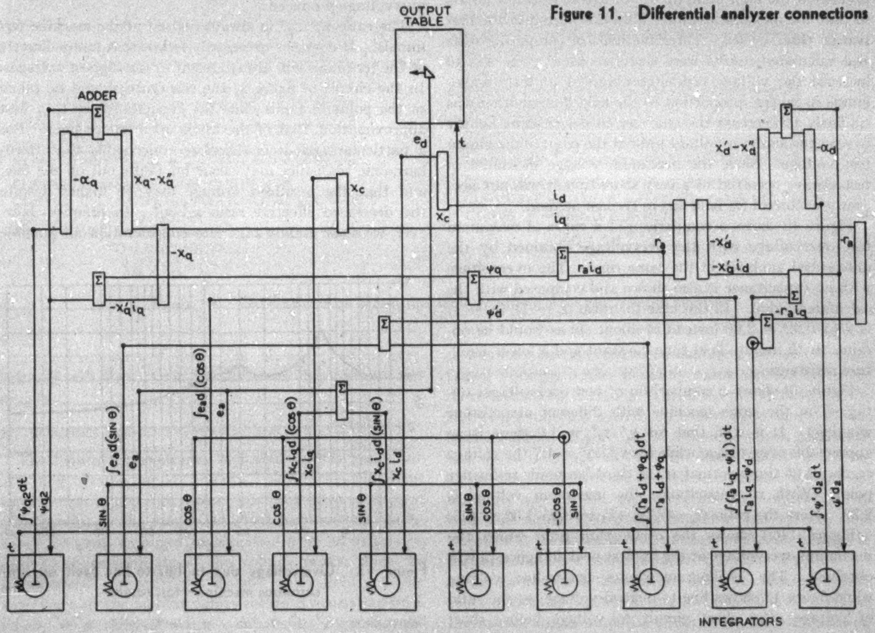

OUTPUT TABLE
ADDER
Σ
$-\alpha_q$
$x_q - x_q''$
$-x_q$
$-x_q' i_q$
x_c
e_d
t
x_c
i_d
i_q
ψ_q
ψ_d'
r_a
$r_a i_d$
$-x_d'$
$-x_q' i_d$
$x_d' - x_d''$
$-\alpha_d$
$-r_a$
$-r_a i_q$
$\int \psi_{q2} dt$
ψ_{q2}
SIN θ
$\int e_a d(\sin\theta)$
e_a
COS θ
$\int e_a d(\cos\theta)$
e_a
COS θ
$\int x_c i_q d(\cos\theta)$
$x_c i_q$
SIN θ
$\int x_c i_d d(\sin\theta)$
$x_c i_d$
SIN θ
COS θ
-COS θ
SIN θ
$\int (r_a i_d + \psi_q) dt$
$r_a i_d + \psi_q$
$\int (r_a i_q - \psi_d') dt$
$r_a i_q - \psi_d'$
$\int \psi'_{d2} dt$
ψ'_{d2}
t
INTEGRATORS

Figure 11. Differential analyzer connections

armature short-circuit time constant x_2/r, and the ratio x_c/x_2. Moreover, if only the maximum possible overvoltage obtainable is of interest, x_c/x_2 will always be very nearly 9, so the voltage is a function only of x_q''/x_d'' and x_2/r. This however neglects all rotor decrement factors.

Comparison With Test Results

In order to check the results obtained by calculation and by the differential analyzer several tests were made. In general the maximum voltages recorded in test have been somewhat lower than those indicated by the analyzer. This is believed to be principally because the machines tested had rather low field time constants (about one second open circuit). The rapid decay of field flux prevented the high build-up of voltage at resonant points so that the maximum voltage always occurred in the first two or three cycles. Three methods of comparing tests and calculated results were therefore used. One was to increase the voltage ordinates obtained in test oscillograms in inverse proportion to the field flux, another was similarly to decrease the analyzer curves or more simply to read the analyzer voltage peak at the point of maximum test voltage. Since the maximum voltage measured in test always occurred in a very short time it was not necessary to correct for field flux in the last method.

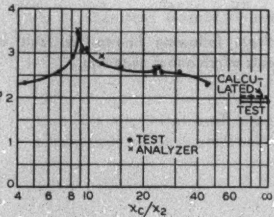


Figure 12. Overvoltage due to line-to-line fault on synchronous machine — comparison of test and analyzer results

$x_q''/x_d'' = 1.52$

Figure 12 shows a comparison of a curve of maximum test overvoltage with the overvoltage obtained by the differential analyzer in the same time. The overvoltage with no capacitance is also shown and compared with the calculated value. In this case the ratio $(e_c - 1)/(e_0 - 1)$ is $2.45/0.92 = 2.66$ instead of about six as would be obtained with infinite field time constant and a lower armature resistance.

Figure 13 shows a comparison of test overvoltages obtained on the same machine with different amortisseur windings. It is seen that for $x_q''/x_d'' = 1.0$ there is no appreciable overvoltage while for $x_q''/x_d'' = 1.7$ the voltage reaches 4.05 times normal at the third-harmonic resonance point. With no capacitance the maximum voltage is 2.32. Here the ratio $(e_c-1)/(e_0-1) = 3.05/1.32 = 2.3$.

Figure 14(c) shows the oscillogram from which the maximum overvoltage at the highest peak of figure 13 was obtained. The oscillogram shows line-to-line voltage while figure 13 shows line-to-neutral voltage, so the ratio of voltage after short circuit to voltage before short circuit read from the oscillogram is 0.866 times the ratio plotted in figure 13. A few other typical oscillograms are also given in figure 14.

In the tests it was found that the values of subtransient reactance to be used in computing overvoltage could be determined most easily and accurately by a static single-phase impedance test. Figure 15 shows the test ratios for the machine of figure 13. The high ratio is seen to be affected by saturation and the value used in checking the test overvoltage curves was that corresponding to the calculated offset root-mean-square single-phase short-circuit current. The tests shown in figure 13 were taken at reduced voltage. For short circuits at full voltage the ratio x_q''/x_d'' is about 1.5 and the overvoltage is correspondingly reduced.

Discussion

A. The Ratio x_q''/x_d''

It has been brought out above that overvoltage is principally a function of the ratio x_q''/x_d''. However, except for faults directly at the machine terminals there will be a reactance in series with the machine reactance. Therefore, in these cases a low average subtransient reactance is also desirable since for the same ratio x_q''/x_d'' at the machine terminals the effective ratio at the fault will be lower with lower machine reactance. Thus aside from their effect on the ratio as measured at the machine terminals, amortisseur windings will usually reduce the overvoltage produced.

The ratio x_q''/x_d'' is always highest at the machine terminals. It does not necessarily follow that faults directly at the terminals will always result in the highest voltages. In the circuit of figure 1, the reactance should be taken to the point of fault while the capacitance is, to a first approximation, that of the whole open line or lines. For a particular case it is therefore conceivable that third-harmonic resonance may occur for faults out on the line, and that the resultant voltage may be higher despite the decreased effective ratio x_q''/x_d''. In practice, however, we must usually take into consideration all possible

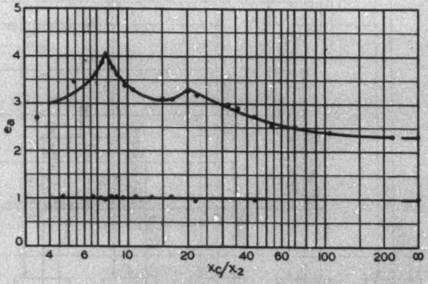


Figure 13. Overvoltage due to line-to-line fault on synchronous machine—test results

Upper curve—$x_q''/x_d'' = 1.7$ Lower curve—$x_q''/x_d'' = 1.0$

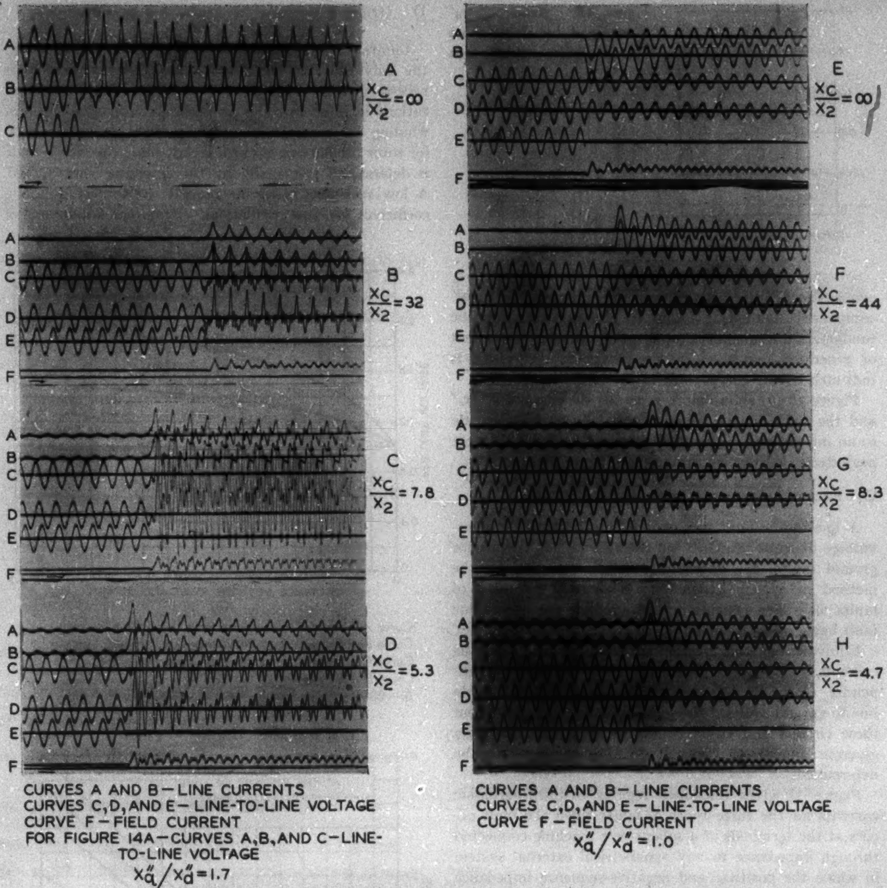


Figure 14. Oscillograms of line-to-line fault on salient-pole synchronous machine

connected amounts of line. The highest possible overvoltages will then occur for faults at the machine terminals.

B. Factors Limiting Overvoltage

The highest overvoltages indicated in this paper will not in general occur in practice. Saturation will limit the voltages to a certain extent and protective equipment will function. The saturation effect however will not be nearly as great as might be supposed from an inspection only of the magnitude of the voltage peaks, since the fundamental component is usually only slightly higher than normal, the highest values recorded consisting principally of a third-harmonic component. Lightning arresters may be expected to be damaged by the high sustained voltages. Also in many cases the machine will be connected to a system. In order to evaluate the effect of a connected system on overvoltage figure 16 has been included. Figure 16 (*a*) shows how in a particular case $x_q'' = 0.90$, $x_d'' = 0.55$, the overvoltage caused by line-to-line short circuit is reduced by the system tie reactance x_e. All the curves are for the condition of no load and equal voltages on machine and infinite bus. Curve *A* is for short circuit at maximum flux linkage in the shorted phases and approaches the value of equation 1 as the tie reactance approaches infinity. Curve *B* is for short circuit at minimum flux linkage. Curve *C* shows the voltage which would be obtained by the conventional

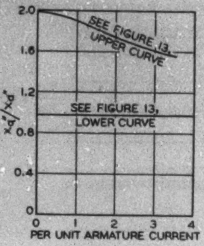


Figure 15. Ratio x_q''/x_d'' determined by single-phase static tests

symmetrical-component calculation. The symmetrical-component method is found to give substantially the fundamental frequency component of voltage. The effect of generator load angle on the maximum overvoltage is indicated by the single point at $x_e = 1.0$.

Figure 16 (*b*) shows more generally the effect of x_q''/x_d'' and the reduction in voltage caused by a reactance tie to an infinite bus. All resistances and capacitances were neglected in calculating the curves of figure 16.

C. Faults Involving Ground

A line-to-line fault was chosen here because the overvoltage depends primarily on the ratio x_q''/x_d'', while for ground faults, the overvoltage is affected also by the method of system grounding. The great majority of faults on power systems involve ground, the line-to-line fault being a less frequent occurrence.

Equivalent circuits for use in determining instantaneous currents and voltages following faults are given in appendix *C* for line-to-ground, line-to-line, and double line-to-ground faults. See figures 18, 19, and 20. From these circuits and either of the methods of calculation given in appendix *B*, overvoltages following faults can be determined.

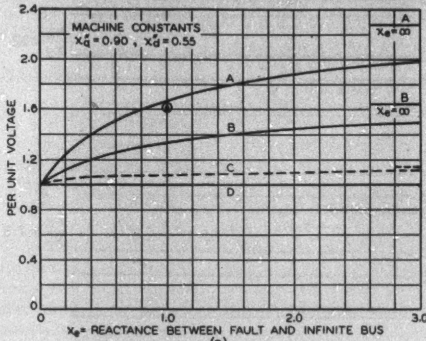
Figure 17 gives resonance regions to third-harmonic currents for the three types of faults when the fault occurs at the terminals of a salient-pole machine connected through impedance to any symmetrical external system in which the positive- and negative-sequence impedance are equal. Resistance is neglected.

Equivalent circuits representing the external system in the positive- and zero-sequence network are shown in figure 17. These circuits consist of a capacitive reactance x_c and an inductive reactance x_e in parallel. In a complex external system, x_{c1}, x_{c0}, x_{1e}, and x_{0e} of the equivalent circuits in general will not be constant for all harmonics.

Figure 17 indicates that with a line-to-ground fault and the assumed ratios of x_{0e}/x_e and x_0/x_2 the capacitance required to cause resonance on a grounded system is high. Even with open lines, a value of x_{c1}/x_2 as low as 5.4 would probably seldom be found in actual systems. For the ungrounded system, the amount of capacitance required for resonance is only about one-fourth to one-fifth as much and the ratio x_{c1}/x_2 lies in a not uncommon region for large systems with long transmission lines.

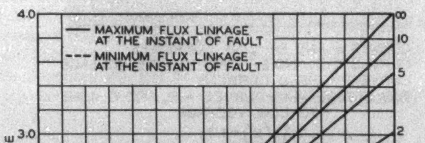
D. Other Functions of Amortisseur Windings

Limitation or reduction of overvoltage is not, of course, the only function of an amortisseur winding and it is necessary to keep the other aspects of amortisseur applications in mind in order properly to determine the best winding for a particular machine. The damping action for rotor oscillations about a steady-state operating angle is determined principally by the amortisseur resistance. A low resistance gives in general the largest damping coefficient for slow oscillations. From the standpoint of



(a)

Curve *A*—No load—fault at maximum flux linkage
Curve *B*—No load—fault at minimum flux linkage
Curve *C*—Symmetrical components—fundamental frequency only
Curve *D*—With $x_q'' = x_d''$
⊙—20-degree load angle—fault at maximum flux linkage



(b)

Figure 16. Line-to-line fault on salient-pole generator—maximum voltage at the point of fault—effect of reactance tie to system

stability the damping action of amortisseur windings is beneficial, but in this case a high-reactance winding which will limit the fault current is desirable. An amortisseur winding will also keep the field voltage on open circuit from reaching dangerous values. These functions do not involve the ratio x_q''/x_d''. However, the recovery voltage on clearing faults has been shown[4] to depend on this ratio.

Nomenclature

x_l = armature leakage reactance
x_{ad} = mutual reactance of armature direct-axis winding with field or amortisseur winding
x_f = field leakage reactance
x_{aq} = mutual reactance of armature and amortisseur quadrature-axis windings
x_d' = direct-axis transient reactance
x_q = quadrature-axis synchronous reactance
r_{kd} = direct-axis amortisseur resistance
r_{kq} = quadrature-axis amortisseur resistance
x_{kd} = direct-axis amortisseur leakage reactance
x_{kq} = quadrature-axis amortisseur leakage reactance
i_d = direct-axis armature current
i_q = quadrature-axis armature current
x_d'' = direct-axis subtransient reactance
x_q'' = quadratue-axis subtransient reactance
x_e = external reactance

$$y = \frac{x_d'' - x_q''}{2}$$

$$x = \frac{x_d'' + x_q''}{2}$$

$$x_{ad}' = \frac{x_{ad}x_f}{x_{ffd}} = x_d' - x_l$$

$x_{ffd} = x_{ad} + x_f$ = field self-reactance

x_c = capacitive reactance

$\alpha_d = \dfrac{r_{kd}}{x_{kd} + x_{ad}'}$ = direct-axis amortisseur decrement factor

$\alpha_q = \dfrac{r_{kq}}{x_{kkq}}$ = quadrature-axis amortisseur decrement factor

$x_{kkq} = x_{aq} + x_{kq}$ = quadrature-axis amortisseur self-reactance

e_a = line-to-neutral armature voltage of the unfaulted phase

$\theta = \theta_0 + t$ = angle between direct axis and phase-a axis

ψ_d', ψ_{d2}' = components of direct-axis armature flux linkage

ψ_q, ψ_{q2} = components of quadrature-axis flux linkage

Subscripts 1, 2, 0, α, β refer to positive, negative, and zero sequence, and to α and β components
Subscript e refers to external circuit

e = voltage
i = current
Z = impedance

Appendix A—Equations for Differential Analyzer

The equations as set up on the differential analyzer were:

$$\psi_d' = -x_d'i_d + \psi_{d2}' \tag{1a}$$

$$\psi_{d2}' = (x_d' - x_d'')i_d - \alpha_d\int\psi_{d2}'dt \tag{2a}$$

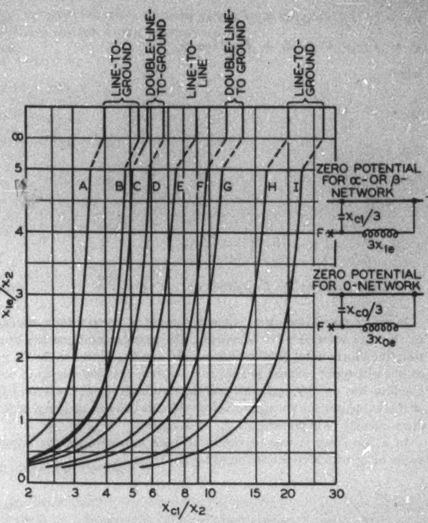


Figure 17. Third-harmonic resonance regions for various types of faults

Curve *A*—Line-to-ground fault, grounded system, $x_0 = x_2/2$; $x_{0e} = x_{1e}$; $C_0 = 0.6\ C_1$
Curve *B*—Line-to-ground fault, grounded system, $x_0 = x_2/2$; $x_{0e} = x_{1e}$; $C_0 = C_1$
Curve *C*—Double line-to-ground fault, grounded system; $x_0 = x_2/2$; $x_{0e} = x_{1e}$; $C_0 = 0.6\ C_1$
Curve *D*—Double line-to-ground fault, grounded system, $x_0 = x_2/2$; $x_{0e} = x_{1e}$; $C_0 = C_1$
Curve *E*—Line-to-line fault
Curve *F*—Double line-to-ground fault, ungrounded system, $C_0 = 0.6\ C_1$
Curve *G*—Double line-to-ground fault, ungrounded system, $C_0 = C_1$
Curve *H*—Line-to-ground fault, ungrounded system, $C_0 = 0.6\ C_1$
Curve *I*—Line-to-ground fault, ungrounded system, $C_0 = C_1$

where

$$\alpha_d = \frac{r_{kd}}{x_{kd} + x_{ad}'}$$

$$x_{ad}' = \frac{x_{ad}x_{fd}}{x_{ffd}}$$

$$\psi_{d20}' = -\left(\frac{x_d' - x_d''}{x_c}\right) - \left(1 - \frac{x_d'}{x_c}\right)\left(\frac{x_{ad}'}{x_{kd} + x_{ad}'}\right)$$

= initial value of ψ_{d2}'

$$\psi_q = -x_qi_q + \psi_{q2} \tag{3a}$$

$$\psi_q = (x_q - x_q'')i_q - \alpha_q\int\psi_{q2}dt \tag{4a}$$

where

$$\alpha_q = \frac{r_{kq}}{x_{kkq}}$$

$$\psi_{q20} = 0$$

$$\psi_d' = \int (r_a i_d + \psi_q) dt + \int e_a d(\sin\theta) \qquad (5a)$$

$$\psi_q = \int (r_a i_q - \psi_d') dt + \int e_a d\,(\cos\theta) \qquad (6a)$$

where

$$i_{d0} = -\frac{1}{x_c}$$

$$\psi_{q0} = 0$$

$$e_{a0} = -\sin\theta_0$$

$$i_{q0} = 0$$

$$\psi'_{d0} = \frac{x_d''}{x_c} + \left(1 - \frac{x_d'}{x_c}\right)\left(\frac{x_{kd}}{x_{kd} + x_{ad}'}\right)$$

$$e_a = \int x_c i_d d(\sin\theta) + \int x_c i_q d(\cos\theta) \qquad (7a)$$

It will be noted that these equations are in an integral form similar to those of reference 8. Moreover, the initial conditions are in a simplified form valid only for reasonably small r_a, since for the range of line resistance covered in this investigation the load angle δ of the machine was usually negligible and the terminal (i.e., at the terminals of the capacitance) voltage e_a before the short circuit occurred could consequently be expressed as: $e_{q0} = 1$, $e_{d0} = 0$.

In a few cases it was necessary to take into account the initial value of i_q, which could be found by the formula

$$i_{q0} = \frac{\sin\delta}{x_c} \qquad (8a)$$

where[9]

$$\tan\delta = \frac{r_a}{x_c - x_q}$$

Equations 1*a* to 7*a* may be derived from Park's[10] synchronous-machine equations, in exactly the same manner as were the equations of reference 8, and so need not be derived here. It should only be pointed out that in equation 5*a* the expression $e_a d(\sin\theta) = e_a \cos\theta\, dt = e_d\, dt$, and similarly for the similar expressions of equations 6*a* and 7*a*. This substitution was made in order to obtain the most desirable analyzer setup.

With the equations expressed as above, eight integrators are seen to be required. In addition, the two remaining integrators were used to generate the required sinusoidal functions[11] sin θ, cos θ, so the setup was entirely self contained and required no operators. Figure 11 shows a schematic diagram of the analyzer connections.

Appendix B—Calculation of Overvoltages

The equations of an ideal synchronous machine with any static terminal impedance under the conditions that (1) all rotor resistances are negligibly small, (2) the external circuit parameters are constants, (3) the external circuit is symmetric about phase *a* of the synchronous machine, and (4) the rotor speed is constant, are most simply and conveniently expressed for the present purpose as:

$$\left.\begin{aligned} e_\alpha &= -[p(x + y\cos 2\theta) + r]\, i_\alpha - py\sin 2\theta\, i_\beta = Z_\alpha(p) i_{\alpha e} \\ e_\beta &= -py\sin 2\theta\, i_\alpha - [p(x - y\cos 2\theta) + r]\, i_\beta = Z_\beta(p) i_{\beta e} \\ e_0 &= -[px_0 + r]\, i_0 = Z_0(p) i_{0e} \end{aligned}\right\} \qquad (1b)$$

where

$$\left.\begin{aligned} x &= \frac{1}{2}(x_d'' + x_q'') \\ y &= \frac{1}{2}(x_d'' - x_q'') \end{aligned}\right\} \qquad (2b)$$

r = armature circuit resistance

$Z_\alpha(p)$, $Z_\beta(p)$, $Z_0(p)$ = operational impedances of the external connected circuit to the α, β, and 0 components of current, respectively

i_α, i_β, i_0 are components of the stator currents defined in terms of the phase currents by the equations:

$$\left.\begin{aligned} i_\alpha &= \frac{2}{3} i_a - \frac{1}{3} i_b - \frac{1}{3} i_c \\ i_\beta &= \frac{1}{\sqrt{3}} i_b - \frac{1}{\sqrt{3}} i_c \\ i_0 &= \frac{1}{3} i_a + \frac{1}{3} i_b + \frac{1}{3} i_c \end{aligned}\right\} \qquad (3b)$$

$i_{\alpha e}$, $i_{\beta e}$, i_{0e} are the similar currents flowing in the external circuit. The currents i_α, i_β may also be expressed in terms of the direct- and quadrature- axis components of current by the equations:

$$\left.\begin{aligned} i_\alpha &= i_d\cos\theta - i_q\sin\theta \\ i_\beta &= i_d\sin\theta + i_q\cos\theta \end{aligned}\right\} \qquad (4b)$$

i_α is the component of armature current which produces a field in the axis of phase *a* of the synchronous machine and may be regarded as a current which flows through phase *a* and splits into two equal parts which flow through phases *b* and *c* in parallel. i_β is the component of armature current which produces a field in the common axis of phases *b* and *c* and may be regarded as a current flowing only through phases *b* and *c* in series. e_α, e_β, e_0 are components of armature voltage defined by voltage equations similar to (3*b*) and (4*b*). In the special case of a machine without an amortisseur winding the subtransient reactances x_d'', x_q'' are replaced by x_d', x_q, respectively.

Equations 1*b* are most easily derived from the equations of a synchronous machine (with no external impedance) in terms of direct- and quadrature-axis quantities.[10] These are, for zero rotor resistances,

$$\left.\begin{aligned} e_d &= -(p\,x_d'' + r)\, i_d + x_q'' i_q \\ e_q &= -x_d'' i_d - (px_q'' + r)\, i_q \end{aligned}\right\} \qquad (5b)$$

Now if equations 4*b* and the similar voltage equations are applied there are equations 1*b* except for the $Z_\alpha(p)$, $Z_\beta(p)$ terms. It is now evident from a consideration of the circuit and the physical interpretations of i_α, i_β, $i_{\alpha e}$, $i_{\beta c}$, e_α, e_β that if a static network is connected to the machine terminals the complete equations 1*b* are obtained. Moreover, if the network is completely symmetric, there is:

$$Z_\alpha(p) = Z_\beta(p) = Z_1(p) = Z_2(p) \qquad (6b)$$

where $Z_1(p)$, $Z_2(p)$ are the positive- and negative-sequence operational impedances of the external circuit.

For a line-to-line fault, as shown in figure 1, there is

$$Z_\alpha(p) = \frac{x_c}{p} \qquad (7b)$$

and there are no zero-sequence currents or voltages.

Before the fault let

$$e_a = -\sin(\theta_0 - \phi + t)$$

where ϕ is the load angle. Then, from (3*b*)

$$e_\alpha = -\sin(\theta_0 - \phi + t)$$

$$e_\beta = \cos(\theta_0 - \phi + t)$$

The fault is equivalent to the application of a voltage at the terminals of the machine equal and opposite to e_β. Thus when the voltage $e_\beta = -\cos(\theta_0 + t)$ is applied to equations 1*b*, we obtain the components of current caused by the fault. Further the terminal voltage e_a is given by

$$e_a = \frac{x_c}{p} i_\alpha \qquad (8b)$$

and the total resultant is obtained by adding e_α to the voltage existing before short circuit. If the speed is constant the current may be obtained as follows.

Equations 1*b* may be written

$$\left.\begin{aligned}&[p(1 + y' \cos 2\theta) + r' + Z_\alpha'(p)]\, xi_\alpha + (py' \sin 2\theta)\, xi_\beta = 0\\&(py' \sin 2\theta)\, xi_\alpha + [p(1 - y' \cos 2\theta) + r']\, xi_\beta = \cos(\theta_0 + t)\end{aligned}\right\}\quad(9\text{b})$$

where

$$y' = \frac{y}{x}, \quad r' = \frac{r}{x}, \quad Z' = \frac{Z}{x}$$

It is difficult to obtain a simple finite operational solution to equations 9*b* because of the presence of the periodic time functions in the current coefficients. However a solution may easily be found by expanding the currents in powers of the troublesome coefficient y' about $y' = 0$. We then have

$$\left.\begin{aligned}i_\alpha &= i_{\alpha 0} + i_{\alpha 0}'y' + \frac{i_{\alpha 0}''}{2!}y'^2 + \ldots\ldots\\i_\beta &= i_{\beta 0} + i_{\beta 0}'y' + \frac{i_{\beta 0}''}{2!}y'^2 + \ldots\ldots\end{aligned}\right\}\quad(10\text{b})$$

where the primes on the currents indicate differentiation with respect to y' and the subscripts zero indicate evaluation at $y' = 0$.

To obtain $i_{\alpha 0}$, $i_{\beta 0}$ we have merely to let $y' = 0$ in equations 9*b* and then to solve the resulting two independent equations operationally. Next, to obtain $i_{\alpha 0}'$, $i_{\beta 0}'$ we must (1) differentiate (9*b*) with respect to y', (2) let $y' = 0$, (3) substitute for $i_{\alpha 0}$, $i_{\beta 0}$ their values as previously found, (4) obtain the operational solution to the resulting equation. To obtain $i_{\alpha 0}''$, $i_{\beta 0}''$ we proceed in exactly the same way by differentiating again and utilizing the previously found values of $i_{\alpha 0}'$, $i_{\beta 0}'$. For the general coefficients $i_{\alpha 0}^{n'}$, $i_{\beta 0}^{n'}$ of (10*b*) there are the equations,

$$\left.\begin{aligned}&[p + r' + Z_\alpha'(p)]\, i_{\alpha 0}^{(n-1)'} =\\&\qquad\qquad - np\,[(\cos 2\theta)\, i_{\alpha 0}^{(n-1)'} + (\sin 2\theta)\, i_{\beta 0}^{(n-1)'}]\\&[p + r']\, i_{\beta 0}^{n'} = - np\,[(\sin 2\theta)\, i_{\alpha 0}^{(n-1)'} - (\cos 2\theta)\, i_{\beta 0}^{(n-1)'}]\end{aligned}\right\}(11\text{b})$$

The currents, and consequently the terminal voltage by equation 8*b*, are now determined as infinite series of transient and steady-state terms which, however, converge rapidly because in the usual case y' is much smaller than unity. Figure 10 shows for a particular case a comparison of voltage calculated by the formulas developed above with a curve taken on the differential analyzer and with a step-by-step solution of equations 1*b*. Only the first two terms of (10*b*) were considered in calculating the points of figure 10.

If only the steady-state (as far as armature transients are concerned) voltage is desired the equations are very much simplified since then the currents and voltages appear only as a series of odd-harmonic terms and the operational impedances of equations 1*b* and 11*b* may be replaced by steady-state impedances at the proper frequency. Figure 9 shows a comparison of maximum overvoltages calculated by this simplified method with the overvoltages obtained by the differential analyzer.

A second method of procedure in solving equations 1*b* is to assume that currents and voltages of fundamental frequency can be determined with sufficient accuracy by the method of symmetrical components. It can be applied to systems of any degree of complexity. Since symmetrical components are in general use, the procedure in determining instantaneous currents and voltages following faults will be given in terms of symmetrical components, the α- and β-components being used only to pass from direct and quadrature components to symmetrical components. An advantage of using α- and β-components over positive- and negative-sequence components, is that the relations among the α and β currents and voltages may be expressed by real coefficients.

The following equations give the relations between the positive- and negative-sequence components of currents, voltages, and impedances and the α- and β-components.

$$\left.\begin{aligned}i_\alpha &= i_{a1} + i_{a2}\\i_\beta &= - j(i_{a1} - i_{a2})\\e_\alpha &= e_{a1} + e_{a2}\\e_\beta &= - j(e_{a1} - e_{a2})\\i_{a1} &= \frac{1}{2}(i_\alpha + ji_\beta)\\i_{a2} &= \frac{1}{2}(i_\alpha - ji_\beta)\\e_{a1} &= \frac{1}{2}(e_\alpha + je_\beta)\\e_{a2} &= \frac{1}{2}(e_\alpha - je_\beta)\\Z_\beta &= \frac{1}{2}(Z_1 + Z_2)\\Z_\alpha &= 2\frac{Z_1Z_2}{Z_1 + Z_2} \cong \frac{1}{2}(Z_1 + Z_2)\end{aligned}\right\}\quad(12\text{b})$$

Reference Vector

In the method of symmetrical components, sinusoidal voltages and currents of fundamental frequency are represented by vectors referred to some particular vector as reference. Thus, if the voltage e_a of phase *a* at some specified point leads the reference vector by an angle α, it is written

$$e_a = |e_a|\,\underline{/\alpha} = E_a\,\underline{/\alpha}$$

To express e_a as a function of time, let the fundamental frequency component of e_a be written

$$e_{1a} = - E_{1a}\sin(\theta + \alpha_1) = E_{1a}\underline{/\theta + \alpha_1}$$

and the *n*th harmonic component of e_a

$$e_{na} = - E_{na}\sin(n\theta + \alpha_n) = E_{na}\underline{/n\theta + \alpha_n}$$

where θ is the position of the axis of the rotor of a specified machine in electrical degrees measured from the axis of phase *a*.

The following expressions for instantaneous values of currents and voltages of any frequency will be used interchangeably,

$$- V\sin(n\theta \pm \alpha) = V\,\underline{/n\theta \pm \alpha}$$

$$V\cos(n\theta \pm \alpha) = - jV\,\underline{/n\theta \pm \alpha} = V\,\underline{/n\theta \pm \alpha - 90^\circ}$$

$$\pm jV\sin(n\theta + \alpha) = V\sin(n\theta + \alpha \pm 90^\circ) = \pm V\cos(n\theta + \alpha)$$

$$\pm jV\cos(n\theta + \alpha) = V\cos(n\theta + \alpha \pm 90^\circ) = \mp V\sin(n\theta + \alpha)$$

Let

$$\left.\begin{aligned}i_{na1} &= \text{positive-sequence } n\text{th harmonic current}\\&= - jI_{na1}\,\underline{/n\theta + \alpha_{n1}} = I_{na1}\cos(n\theta + \alpha_{n1})\\i_{na2} &= \text{negative sequence } n\text{th harmonic current}\\&= -jI_{na2}\,\underline{/n\theta + \alpha_{n2}} = I_{na2}\cos(n\theta + \alpha_{n2})\end{aligned}\right\}\quad(13\text{b})$$

where α_{n1} and α_{n2} are determined by the system impedances, type of fault, and load angle.

Generated Voltages

When $x_q'' = x_d''$, currents and voltages following faults will in the general case have a fundamental-frequency term, one or more natural-frequency terms, and a d-c term. These can be determined by

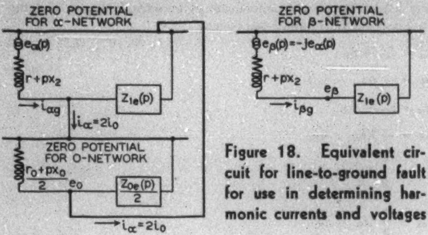


Figure 18. Equivalent circuit for line-to-ground fault for use in determining harmonic currents and voltages

applying a voltage at the fault equal and opposite to that which existed there before the fault and adding resulting changes in currents and voltages to initial values. When $x_q'' \neq x_d''$, additional voltages are generated. From (1*b*) the voltages generated by sinusoidal currents i_α and i_β of any frequency are

$$\left.\begin{aligned} e_\alpha &= -py(\cos 2\theta i_\alpha + \sin 2\theta i_\beta) \\ e_\beta &= -py(\sin 2\theta i_\alpha - \cos 2\theta i_\beta) \end{aligned}\right\} \quad (14b)$$

Replacing i_α and i_β in (14*b*) by their positive- and negative-sequence values from (12*b*) and substituting for them the expressions in (13*b*), positive- and negative-sequence voltages generated in a given machine can be determined as follows:

$$\left.\begin{aligned} e_\alpha &= -py[I_{na1}\{\cos 2\theta \cos(n\theta+\alpha_{n1}) + \\ &\qquad \sin 2\theta \sin(n\theta+\alpha_{n1})\} + I_{na2}\{\cos 2\theta \cos(n\theta+\alpha_{n2}) - \\ &\qquad\qquad \sin 2\theta \sin(n\theta+\alpha_{n2})\}] \\ &= -py[I_{na1}\cos\{(n-2)\theta+\alpha_{n1}\} + \\ &\qquad I_{na2}\cos\{(n+2)\theta+\alpha_{n2}\}] \\ &= (n-2)yI_{na1}\sin\{(n-2)\theta+\alpha_{n1}\} + \\ &\qquad (n+2)yI_{na2}\sin\{(n+2)\theta+\alpha_{n2}\} \\ e_\beta &= -py[I_{na1}\{\sin 2\theta \cos(n\theta+\alpha_{n1}) - \\ &\qquad \cos 2\theta \sin(n\theta+\alpha_{n1})\} + \\ &\qquad I_{na2}\{\sin 2\theta \cos(n\theta+\alpha_{n2}) + \cos 2\theta \sin(n\theta+\alpha_{n2})\}] \\ &= -py[-I_{na1}\sin\{(n-2)\theta+\alpha_{n1}\} + \\ &\qquad I_{na2}\sin\{(n+2)\theta+\alpha_{n2}\}] \\ &= (n-2)yI_{na1}\cos\{(n-2)\theta+\alpha_{n1}\} - \\ &\qquad (n+2)yI_{na2}\cos\{(n+2)\theta+\alpha_{n2}\} \end{aligned}\right\} \quad (15b)$$

If $n \geqq 2$

$$je_\beta = -(n-2)yI_{na1}\sin[(n-2)\theta+\alpha_{n1}] + (n+2)yI_{na2}\sin[(n+2)\theta+\alpha_{n2}] \quad (16b)$$

$$\begin{aligned} e_{a1} &= (n+2)yI_{na2}\sin[(n+2)\theta+\alpha_{n2}] \\ &= -(n+2)yI_{na2}\underline{/(n+2)\theta+\alpha_{n2}} = \\ &\qquad (n+2)\,y\underline{/2\theta-90^\circ}\,i_{na2} \end{aligned} \quad (17b)$$

$$\begin{aligned} e_{a2} &= (n-2)yI_{na1}\sin[(n-2)\theta+\alpha_{n1}] \\ &= -(n-2)yI_{na1}\underline{/(n-2)\theta+\alpha_{n1}} = \\ &\qquad (n-2)y\underline{/-2\theta-90^\circ}\,i_{na1} \end{aligned} \quad (18b)$$

If $n = 1$

$$\begin{aligned} e_{a1} &= yI_{na1}\sin(\theta-\alpha_{n1}) \\ &= -yI_{1a1}\underline{/\theta-\alpha_{11}} = y\underline{/-2\alpha_{11}-90^\circ}\,i_{1a1} \end{aligned} \quad (19b)$$

$$e_{a2} = 0$$

From (17*b*), (18*b*), (19*b*), positive- and negative-sequence voltages generated by harmonic current without decrements can be determined.

Steady State

Since zero rotor resistance has been assumed in the direct and quadrature axes, there will be no field decrements and fundamental-frequency and odd-harmonic terms of currents and voltages will have no decrements. Natural-frequency and even harmonic terms will disappear in the steady state.

Table II gives the positive- and negative-sequence odd-harmonic voltages generated by positive- and negative-sequence odd-harmonic currents.

Procedure in Determining Steady-State Values of Currents and Voltages Following Faults

Assuming zero resistance in the field for machines without amortisseur windings and also in the amortisseur for those with amortisseur windings, the procedure for determining currents and voltages in the steady state following faults is as follows:

1. Calculate currents and voltages of fundamental frequency in the system by means of symmetrical components. Since it has been assumed that fundamental-frequency currents and voltages are determined with sufficient accuracy by this method, voltages of fundamental frequency generated in machines in which $x_d'' \neq x_q''$ will be neglected.

2. From the negative-sequence current flowing in machines in which $x_d'' \neq x_q''$, determine the third-harmonic positive-sequence generated voltages from table II.

3. From third-harmonic positive-sequence generated voltages determine by symmetrical components the third-harmonic voltages and currents in the systems, assuming positive and negative impedance of the synchronous machines to third-harmonic currents to be $3x_2$. x_2 may differ slightly from the fundamental-frequency value.

4. From the negative sequence third-harmonic currents flowing in machines in which $x_d'' \neq x_q''$, determine from table II the fifth-harmonic positive-sequence generated voltages.

Proceeding in this manner, odd-harmonic currents and voltages can be determined at any point of the system.

It is also possible to calculate the transient voltage caused by odd harmonic currents and, by using the natural frequency and even harmonic currents in equations (14*b*) to calculate the generated transient voltages and from them in turn the resulting terminal overvoltage.

Appendix C—Equivalent Circuits for Various Types of Faults

Let i_a, i_b, and i_c be phase currents flowing into the fault; $Z_0(p)$ the zero-sequence impedance viewed from the fault; e_a, e_b, and e_c the phase-to-ground voltages at the fault, and e_α, e_β, and e_0 their α-, β-, and 0-components, respectively.

Line-to-Ground Fault (Phase a)

Conditions at the fault are:

$$i_b = i_c = 0;\ e_a = 0$$

From (3*b*)

$$i_\beta = 0;\ i_\alpha = 2i_0;\ e_\alpha = -e_0$$

Table II. Positive- and Negative-Sequence* Odd-Harmonic Generated Voltages

Positive-Sequence Odd-Harmonic Voltages Generated in Machines by Negative-Sequence Currents

$$e_{3a1} = 3yI_{1a2}\sin(3\theta+\alpha_{12}) = -3yI_{1a2}\underline{/3\theta+\alpha_{12}} = 3y\underline{/2\theta-90^\circ}\,i_{1a2}$$
$$e_{5a1} = 5yI_{3a2}\sin(5\theta+\alpha_{32}) = -5yI_{3a2}\underline{/5\theta+\alpha_{32}} = 5y\underline{/2\theta-90^\circ}\,i_{3a2}$$
$$e_{7a1} = 7yI_{5a2}\sin(7\theta+\alpha_{52}) = -7yI_{5a2}\underline{/7\theta+\alpha_{52}} = 7y\underline{/2\theta-90^\circ}\,i_{5a2}$$
$$e_{9a1} = 9yI_{7a2}\sin(9\theta+\alpha_{72}) = -9yI_{7a2}\underline{/9\theta+\alpha_{72}} = 9y\underline{/2\theta-90^\circ}\,i_{7a2}$$

Negative-Sequence Odd-Harmonic Voltages Generated in Machines by Positive Sequence Currents

$$e_{1a2} = yI_{3a1}\sin(\theta+\alpha_{31}) = -yI_{3a1}\underline{/\theta+\alpha_{31}} = y\underline{/-2\theta-90^\circ}\,i_{3a1}$$
$$e_{3a2} = 3yI_{5a1}\sin(3\theta+\alpha_{51}) = -3yI_{5a1}\underline{/3\theta+\alpha_{51}} = 3y\underline{/-2\theta-90^\circ}\,i_{5a1}$$
$$e_{5a2} = 5yI_{7a1}\sin(5\theta+\alpha_{71}) = -5yI_{7a1}\underline{/5\theta+\alpha_{71}} = 5y\underline{/-2\theta-90^\circ}\,i_{7a1}$$
$$e_{7a2} = 7yI_{9a1}\sin(7\theta+\alpha_{91}) = -7yI_{9a1}\underline{/7\theta+\alpha_{91}} = 7y\underline{/-2\theta-90^\circ}\,i_{9a1}$$

*α and β voltages when required can be obtained from table II and (12*b*).

It follows that

$$-e_0 = Z_0(p)i_0 = \frac{Z_0(p)}{2} i_\alpha = e_\alpha \qquad (1c)$$

From (1*c*) the equivalent circuit to replace the fault in the α-network is $Z_0(p)/2$. When $x_d'' = x_q''$, the β-network is not involved, e_β is not changed by the fault, and e_α and e_0 have only fundamental- and natural-frequency terms. In machines in which $x_d'' \neq x_q''$, fundamental-, harmonic- and other frequency voltages are generated (see appendix *B*) which affect e_α, e_β, and e_0.

At the fault, since $e_\alpha = -e_0$, e_a, e_b, and e_c are:

$$\left.\begin{aligned}
e_a &= e_\alpha + e_0 = 0 \\
e_b &= -\frac{1}{2}e_\alpha + e_0 - \frac{\sqrt{3}}{2}e_\beta = \frac{3}{2}e_0 - \\
&\qquad \frac{\sqrt{3}}{2}e_\beta = -\frac{3}{2}e_\alpha - \frac{\sqrt{3}}{2}e_\beta \\
e_c &= -\frac{1}{2}e_\alpha + e_0 + \frac{\sqrt{3}}{2}e_\beta = \frac{3}{2}e_0 + \\
&\qquad \frac{\sqrt{3}}{2}e_\beta = -\frac{3}{2}e_\alpha + \frac{\sqrt{3}}{2}e_\beta
\end{aligned}\right\} \qquad (2c)$$

Line-to-Line Fault (Phases b and c)

Conditions at the fault are:

$i_b = -i_c;\ i_a = 0;\ e_b = e_c$

From (3*b*)

$i_\alpha = i_0 = 0;\ e_\beta = 0$

For a system symmetrical before the fault, the zero-sequence network is not involved. The β-network is shorted at the point of fault. The α-network is unaffected when $x_d'' = x_q''$, but when $x_d'' \neq x_q''$ voltages are generated in the machines which affect e_α. (see appendix *B*). At the fault,

$$\left.\begin{aligned}
e_a &= e_\alpha \\
e_b &= e_c = -\frac{1}{2}e_\alpha
\end{aligned}\right\} \qquad (3c)$$

Double Line-to-Ground Fault (Phases b and c)

Conditions at the fault are:

$i_a = 0;\ e_b = e_c = 0$

From (3*b*)

$e_\beta = 0;\ e_\alpha = 2e_0;\ i_\alpha = -i_0$

It follows that

$$2e_0 = -2Z_0(p)i_0 = 2Z_0(p)i_\alpha = e_\alpha \qquad (4c)$$

The equivalent circuit to replace the fault in the α-network is therefore $2Z_0(p)$. The β-network is shorted at the point of fault. When $x_d'' = x_q''$, there is no mutual coupling between the α- and β-networks, but when $x_d'' \neq x_q''$, voltages are generated in the machines which affect e_α and e_0. (See appendix *B*.)

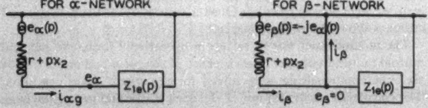


Figure 19. Equivalent circuit for line-to-line fault for use in determining harmonic currents and voltages

At the fault,

$$\left.\begin{aligned}
e_a &= e_\alpha + e_0 = 3e_0 = \frac{3}{2}e_\alpha \\
e_b &= e_c = 0
\end{aligned}\right\} \qquad (5c)$$

In figures 18, 19, 20 are shown the equivalent circuits for line-to-ground, line-to-line, and double line-to-ground faults at the terminals of a synchronous generator with series impedance connected to any symmetrical external system in which the positive- and negative-sequence impedances are equal. $Z_{1e}(p)$ and $Z_{0e}(p)$ are the operational expressions for the positive- and zero-sequence impedances of the external system. $r_0 + px_0$ is the zero-sequence impedance of the generator. The positive- and negative-sequence impedances of the generator to currents of frequencies other than fundamental is approximately $r + px_2$ except for very high frequencies. This is also the α and β impedances to them.

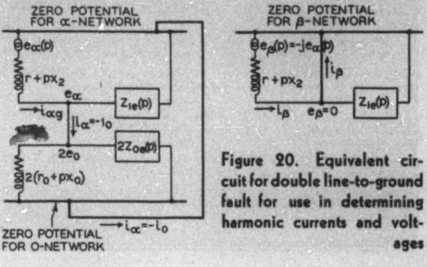


Figure 20. Equivalent circuit for double line-to-ground fault for use in determining harmonic currents and voltages

Except for voltages of fundamental frequency which are determined by the method of symmetrical components, the equivalent circuits of figures 18, 19, 20 can be used to determine all other components of current and voltage. The principal natural-frequency terms are calculated by the application of a voltage at the point of fault equal and opposite to the initial steady-state voltage. The point of application of voltages generated when $x_d'' \neq x_q''$ is taken in the machine.

Resonance to Harmonics

Harmonic voltages are generated in machines in which $x_d'' \neq x_q''$. The external impedance which will cause resonance for various types of faults can be determined from the equivalent circuits as follows:

From equation 2*c* and figure 18, resonance for a line-to-ground fault can occur either in the unfaulted β-circuit or in the α-circuit. For resonance to the *n*th harmonic in the β-circuit there is the equation:

nx_2 = negative reactance component of $Z_{1e}(p)$

For resonance in the α-circuit,

nx_2 = negative reactance component of $\dfrac{r_0 + px_0}{2}$, $\dfrac{Z_{0e}(p)}{2}$, and $Z_{1e}(p)$ in parallel.

It is thus possible to have resonance to one harmonic in the α-circuit and to another in the β-circuit.

For the line-to-line and the double line-to-ground faults resonance can only occur in the α-circuit. For the line-to-line fault it will occur when

nx_2 = negative reactance component of $Z_{1e}(p)$

For the double line-to-ground fault when

nx_2 = negative reactance component of $2(r_0 + px_0)$, $2Z_{0e}(p)$, and $Z_{1e}(p)$ in parallel.

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Discussion

D. E. Brainard (General Electric Company, Schenectady, N. Y.): The authors have pointed out that they controlled the point on the voltage wave at which the short circuits were applied. By choosing the point of maximum linkages they obtained the maximum possible overvoltage. When the usual random shots must be accepted, it is necessary to estimate the maximum overvoltage for a given machine by observing the point on the cycle at which each value of overvoltage occurs.

This procedure was applied in the case of a 26-pole, 1,250-kva 600-volt generator having a copper amortisseur winding without connected end rings. With the machine operating at 0.33, 0.66, and 1.0 per unit voltage, single phase, line-to-line, sudden short circuits were applied. The values of the ratio of the maximum voltage obtained on the open phase to the voltage before the short are plotted in figure 1. These voltages were measured by an oscillograph both from open line to neutral and from open line to shorted lines. The angle was determined from the oscillogram of the voltage between the two terminals which were shorted.

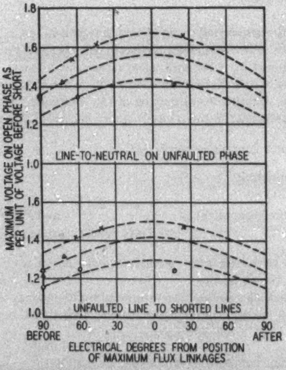


Figure 1. Open-phase voltages following single-phase line-to-line short circuits (as a function of linkages in shorted phase when shorted)

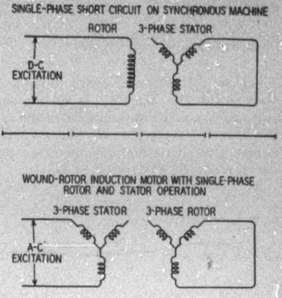
Tests on 26-pole 1,250-kva 600-volt machine having copper amortisseur winding without connected end rings

$x_q''/x_d'' = 1.33$ by static test at 1.0 per unit current

Per unit voltage before short as follows:

x = 0.33 Δ = 0.66 0 = 1.00

Figure 2


SINGLE-PHASE SHORT CIRCUIT ON SYNCHRONOUS MACHINE
ROTOR
3-PHASE STATOR
D-C EXCITATION
WOUND-ROTOR INDUCTION MOTOR WITH SINGLE-PHASE ROTOR AND STATOR OPERATION
3-PHASE STATOR
3-PHASE ROTOR
A-C EXCITATION

A curve drawn through the test points gives a maximum ratio for the line-to-neutral case of 1.68 for the short from 0.33 voltage. The short-circuit current for this case is of the order of rated current for which condition static tests show a ratio of $x_q''/x_d'' = 1.33$. The formula given in the paper gives a maximum voltage ratio of 1.66 with which the test is in substantial agreement. It is to be noted, however, that the line-to-line-voltage ratio reaches a maximum of only 1.5 for the same conditions.

J. H. Fortenbach (nonmember; General Electric Company, Schenectady, N. Y.): The problem of single-phase short circuit on a synchronous machine is analogous to the problem of a wound-rotor induction motor when operating with single-phase excitation on the stator and with only one phase short-circuited on the rotor. The chief differences are that the induction motor is a round-rotor machine and the excitation is alternating current rather than direct current. Figure 2 illustrates the analogy.

We have known for many years that operation of a wound-rotor induction motor in such a manner produced abnormal voltages across the open phases. From the various tests which were made in the past, it was found that under these conditions the voltage across the open phase of either the rotor or stator might be in the order of ten times normal.

The most recent tests which we made on a three-phase wound-rotor induction motor with single-phase rotor and stator were taken in 1931. These were made on an 8-pole 15-horsepower 900-rpm 220-volt three-phase 60-cycle motor. The motor, at first unexcited, was driven and held at a constant speed. Single-phase normal voltage was then suddenly applied across two stator lines. The oscillogram (figure 3) shows a typical result for the case when the machine was driven at 900 rpm.

By assuming that the mutual inductance between a stator and a rotor phase is proportional to the cosine of their relative angular displacement and that the rotor and stator resistances are equal to zero, we were able to calculate (for the steady state) the voltages and currents shown by the oscillogram.

The method used was to set up in operational form, the self and mutual induced voltages in each phase of the rotor and stator. The six equations resulting were solved for the steady state condition. The calculated results for one of the open stator and one of the open rotor voltages are shown by figure 4.

The circled points are the tested values taken from the oscillogram.

Figure 3

A—Open-stator voltage; one centimeter = 328 volts

B—Open-stator voltage; one centimeter = 518 volts

C—Rotor current; one centimeter = 115 amperes

D—Applied stator voltage; one centimeter = 622 volts

E—Open-rotor voltage; one centimeter = 516 volts

F—Open-rotor voltage; one centimeter = 258 volts

E. M. Hunter (General Electric Company, Schenectady, N. Y.): This study of overvoltages caused by unbalanced short circuits described by the authors is illuminating when viewed in the light of operating experiences reported on ungrounded-neutral systems. The disturbances resulting from overvoltages during ground faults on ungrounded neutral systems have been described frequently in the technical press. They have been attributed to "arcing grounds." Numerous attempts have been made to reproduce "arcing grounds" during staged tests with little success, which has led to the suspicion that the overvoltages that have occurred should be explained in some other way.

The authors' figure 17 shows the areas in which overvoltages may occur. A study of this figure indicates that the chances of obtaining the overvoltages described are greater on the ungrounded-neutral system. It would appear that at least some of the overvoltages previously attributed to "arcing grounds" have actually resulted from the phenomena described in this paper.

The authors have suggested that to reduce the overvoltages, the generators be equipped with amortisseur windings. The ground

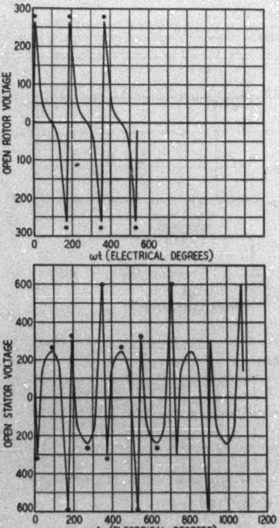


Figure 4

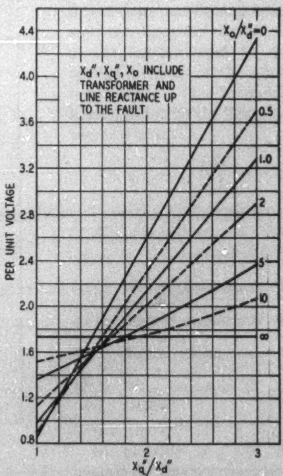


Figure 5. Line-to-ground fault—maximum voltage of the unfaulted phases in per unit of crest voltage before the fault. Constant rotor flux linkages—resistance and capacitance neglected—maximum flux linkages

fault neutralizer (Petersen coil) is also effective in reducing them. On a system with a neutralizer the zero-sequence impedance is infinite. Accordingly, during line-to-ground faults, there is no flow of positive or negative fault current. When the neutralizers are located on the system, there is little negative-sequence current flowing to produce the excitation necessary for the overvoltages. Referring again to the authors' figure 17, curves *F*, *G*, *H*, and *I* represent the areas where resonance occurs and overvoltages may be serious for single and double line-to-ground faults on ungrounded neutral systems. These areas are eliminated with ground-fault neutralizers on the system.

Edith Clarke, C. N. Weygandt and C. Concordia: Mr. Brainard's tests illustrate the effect of closing angle and show that in order to obtain consistent results, control of this angle is essential. He also finds that the ratio of line-to-line voltage after the fault to that before the fault is smaller than the corresponding line-to-neutral voltage ratio. In the paper it is shown that theoretically the relation is: (line-to-line voltage ratio) = 0.866 (line-to-neutral voltage ratio).

The results presented by Mr. Fortenbach are particularly interesting since they show such an excellent theoretical check of the observed steady-state voltage wave and since they demonstrate the essential similarity of synchronous and induction machines as far as these overvoltages are concerned.

As brought out in the paper, a line-to-line fault was studied in detail because it may result in the highest voltages and because it is the type most affected by changes in x_q''/x_d''. To show the effect of x_q''/x_d'' on voltages arising from line-to-ground faults without capacitance, figure 5 of this closure has been included. The curves of figure 5 show that for small x_q''/x_d'' the overvoltage at the fault increases with increasing x_0, while for large x_q''/x_d'' the situation is reversed and the overvoltage decreases with increasing x_0. It has been pointed out by Mr. Hunter that the effect of a ground fault neutralizer is to make $x_0 = \infty$, so that voltages at the fault would then be practically limited to the value shown in the figure for $x_0 = \infty$.

For the case studied, the open phase voltages are given by:

$$e_b = -\frac{3}{2}e_\alpha + \frac{\sqrt{3}}{2}[e_\beta + \cos(\theta - \alpha)]$$

$$e_c = -\frac{3}{2}e_\alpha - \frac{\sqrt{3}}{2}[e_\beta + \cos(\theta - \alpha)]$$

where

$$e_\alpha = -x_0\frac{[x_0 + 2x + 2y\cos 2\theta]\sin\theta - 4y\sin 2\theta(\cos\theta - \cos\theta_0)}{[x_0 + 2x + 2y\cos 2\theta]^2}$$

$$e_\beta = 2y \times \frac{(x_0+2x+2y\cos 2\theta)\sin\theta\sin 2\theta - [4y + 2(x_0+2x)\cos 2\theta](\cos\theta - \cos\theta_0)}{[x_0 + 2x + 2y\cos 2\theta]^2}$$

Resonant Nonlinear Control Circuits

By WILLIAM T. THOMSON
ASSOCIATE AIEE

Synopsis: The resonant type of nonlinear circuit containing a resistance, a condenser, and a saturable-core reactor is finding a number of applications in the various engineering fields,[1] and an analytical treatment has here been developed to add to the understanding of the behavior of such circuits. Certain features of the superposition of a d-c magnetizing force on the reactor are discussed. A method of increasing the sensitivity of this d-c controlled circuit, with a mathematical analysis for the criteria of stability and amplification is given. Finally, a specific application of the circuits discussed to a sensitive and stable power controlling device is described.

THE GENERAL characteristics of the resonant nonlinear circuit have been known for some time.[2] However, in order to determine specifically the behavior of such circuits, an analytical treatment is necessary. With such an analysis it is possible to calculate the critical resistance, the critical voltage and the critical current, and a better understanding of such circuits is gained.

The Series Circuit

For the general characteristics of the nonlinear circuit, the reader is referred to the paper on nonlinear circuits by C. G. Suits.[3] For orientation purposes, however, some of the properties of the series nonlinear circuit will be briefly reviewed. Figure 1 illustrates the typical volt-ampere relation of such circuits. The observations to be noted here are:

1. Upon reaching a certain critical voltage, the current takes an abrupt rise.
2. The steepness of this rise in the critical region is some function of the resistance.
3. If this resistance is reduced to a value less than the critical resistance, the circuit becomes unstable in the critical region and displays a hysteresis effect. In this unstable region, the slope dE/dI of the curve becomes negative. See figure 4.
4. The magnitude of the current jump is some function of the condenser size.

The peculiar volt-ampere relation found in such circuits is due to the joint action of saturation of the iron and the partial resonance of the circuit. Figure 2 is an oscillogram illustrating the cumulative action of the current jump in the unstable region.

With these points in mind, the problem will be treated analytically. Referring to the unstable curve of figure 1, there exist two points on the curve where $dE/dI = 0$. Since with a critical resistance in the circuit these two points merge into one, the value of the critical resistance can be obtained by equating the two expressions for the current when $dE/dI = 0$.

Unfortunately very little is known of the nonlinear differential equation encountered for such circuits, and therefore one must resort to the use of an equivalent sine wave and treat the problem vectorially. With such a method, it is obvious that the reactance and the apparent resistance of the reactor must be expressed in terms of the effective current. This is readily accomplished by the following vector relation:

$$X_L = \frac{1}{I}\sqrt{E_L^2 - \left(\frac{W}{I}\right)^2} \qquad (1)$$

$$R_L = \frac{W}{I^2} \qquad (2)$$

where

X_L = effective reactance of the reactor
R_L = apparent resistance of the reactor
E_L = effective voltage across the reactor
W = watts consumed by the reactor
I = effective current

In plotting X_L and R_L against the current, the curves

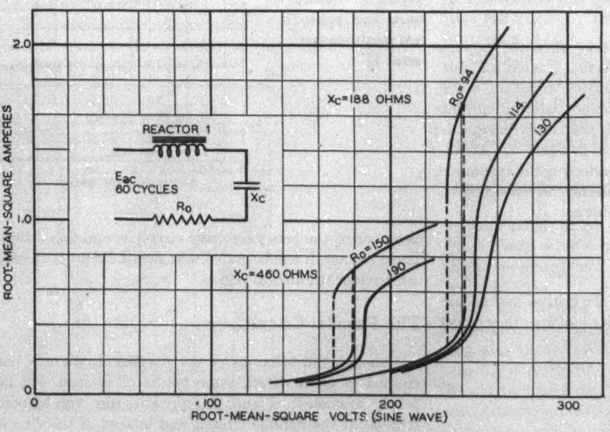


Figure 1. Characteristics of the series circuit

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1. For all numbered references, see list at end of paper.

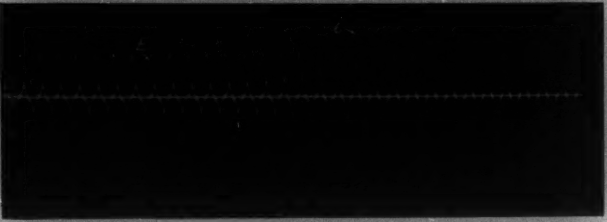


Figure 2. Current jump at critical voltage

appear as in figure 3. Since the critical condition always occurs on the negative-slope portion of the reactance curve, it is only necessary to fit an equation over this region. The equation best fitting these curves is a rectangular hyperbola with displaced axis, and R_L and X_L can be expressed as follows:

$$R_L = \frac{A}{\alpha + I} + K_1 \tag{3}$$

$$X_L = \frac{B}{\beta + I} + K_2 \tag{4}$$

where A, α, B, β, K_1 and K_2 are constants of the reactor. The constants are calculated by taking three points covering the desired interval of the curve.

Using these expressions in the voltage equation;

$$E = I\sqrt{R^2 + X^2} \tag{5}$$

where

$R = R_0 + R_L$, the sum of the series and the apparent resistance of the reactor

$X = X_L - X_c$, the difference in the reactive impedances,

equation 5 can be differentiated with respect to the current and set equal to zero.

$$R\left(R + I\frac{dR_L}{dI}\right) + X\left(X + I\frac{dX_L}{dI}\right) = 0 \tag{6}$$

The difficulty with this last equation when the expressions for X_L and R_L are substituted is that it is not solvable mathematically. However, by plotting equation 6 for various values of resistance it is found that the current for the maximum negative dE/dI is practically independent of the resistance. This is shown in figure 4. Therefore by differentiating equation 6 neglecting the resistance term and setting it to zero, the current corresponding to the critical stable voltage is expressible as:

$$I_m = -2\beta + \sqrt{\beta^2 + \frac{3\beta B}{(X_c - K_2)}} \tag{7}$$

Also, the current corresponding to the critical and release voltages for the case with a resistance less than the critical value can be expressed approximately by;

$$I_1 = -\beta + \sqrt{\frac{\beta B}{X_c - K_2 - R_0}} \tag{8}$$

$$I_2 = -\beta + \frac{B}{X_c - K_2 + 2R_0} \tag{9}$$

and the critical resistance found from equating the two equations above is;

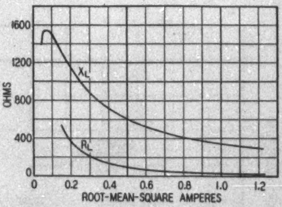
$$R_0 = -\frac{B}{8\beta} - \frac{1}{2}(X_c - K_2) + \sqrt{\left(\frac{B}{8\beta}\right)^2 + \frac{3B}{8\beta}(X_c - K_2)} \tag{10}$$

The critical stable voltage with a critical resistance in the circuit is then found by substituting R_0 and I_m in equation 5.

The equations derived in this analysis are found to be in very good agreement with the experimental values, the test being performed with four different reactors and various sizes of condensers. The tabulation of results with a comparison with the experimental values for one of the reactors is given in table I.

It is evident from these equations that the current jump increases as X_c decreases, as observed in figure 1, and when the value of the capacitance is equal to the constant K_2 of the reactor, the current rises to a very large value. Also, for this condition, the critical resistance will be equal to zero. For any resistance less than the critical value, the same analysis can be made, in which case the values of I_1 and I_2 must be used in calculating the critical and the release voltages. In summarizing the behavior of the series resonant circuit containing a nonlinear reactor, it can be stated that such a circuit is voltage sensitive in the critical region.

Figure 3. Reactance and apparent resistance (reactor 1)



The Parallel Circuit

Although the behavior of the parallel circuit was not studied in great detail, some points of interest will be briefly discussed. Figure 5 represents the volt-ampere relation of the parallel circuit, and instead of the circuit being voltage sensitive it is known to be current sensitive.

Table I (Reactor 1)

Calculated Values				Experimental Values	
X_c	R_0	I_m	E	R_0	E
26	0				
43	18	2.48	322	30	323
93	55	1.13	282	60	284
188	113	0.653	245	114	248
272	151	0.490	218	145	224
355	178	0.395	200	170	205
460	206	0.305	186	190	186

As in the series circuit, the critical region characterized by the negative slope is some function of the resistance.

One thing of interest here is that the critical voltage and current for the reactor branch of the parallel circuit are essentially the same as those for the series circuit. Starting from the expression,

$$\frac{dI}{dE} = \frac{dI}{dI_L} \cdot \frac{dI_L}{dE} \tag{11}$$

the critical condition is found when $dI/dI_L = 0$, since dI_L/dE cannot become zero for any region. Neglecting the resistance,

$$I = -I_L \left\{ \frac{X_L - X_c}{X_c} \right\} \tag{12}$$

and when

$$\frac{dI}{dI_L} = 0$$

$$I_L \frac{dX_L}{dI_L} + X_L - X_c = 0 \tag{13}$$

When the expression for X_L is substituted into equation 13, the resulting equation becomes;

$$I_L = -\beta + \sqrt{\frac{\beta B}{X_c - K_2}} \tag{14}$$

and it is seen that this is essentially the same equation as that of the series circuit.

Since the reactor voltage is known for this current, and is also equal to the condenser voltage, the line current I is found by vector addition.

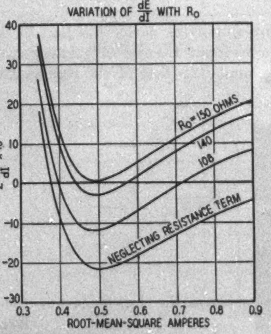
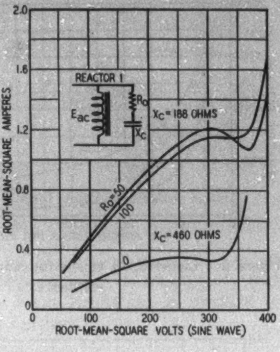


Figure 4

Figure 5. Characteristics of the parallel circuit



The Generalized Form of Reactance and Apparent Resistance

From the foregoing it is evident that in order to predict the behavior of the circuits discussed, the reactance and the apparent resistance of the reactor are necessary. The two quantities can be predicted approximately for any reactor provided certain experimental data for the iron are at hand.

The first of these is the effective permeability curve shown in figure 6. Since the reactance is

$$\frac{0.4\pi N^2 A}{l} \mu \times 10^{-8}$$

from this permeability curve, it is possible to determine the reactance of any reactor using the same iron.

In the same manner, the apparent resistance of any reactor can be obtained from the second curve of figure 6. This curve was calculated from the density-core-loss data and the a-c magnetization curve. The apparent resistance curve is obtained by multiplying the values of this last curve by $(N/l)^2$ and the number of pounds of iron used.

Although, in the method described above, the effect of the leakage flux and the air gap are assumed to be the same as that of the test reactor from which the data is obtained, the results obtained by this method agree very well with the actual curves.

The Series Circuit With D-C Saturation

Some points of interest in regards to the series resonant circuit with a d-c magnetizing force superimposed on the iron-core reactor was investigated mainly from the utility point of view.

To superimpose the d-c flux, the well-known three-legged reactor commonly used in controlling theater lighting was used. If the volt-ampere relation for such a reactor is plotted for various values of d-c saturation, it would be found that the effect of the d-c is to shift the magnetization curve to the right.[4] Also, the d-c saturation decreases the maximum of the reactance curve and

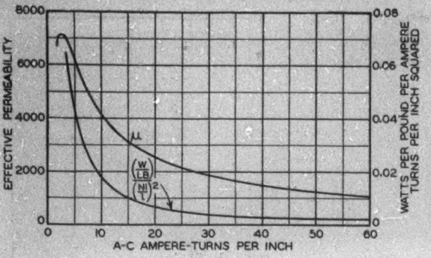


Figure 6. Generalized curves for reactance and apparent resistance

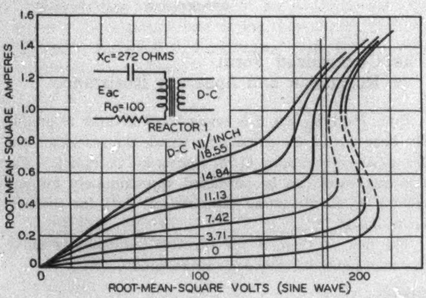


Figure 7. Series circuit with direct current superimposed

shifts it in the same direction. It will be found here that for a large value of a-c ampere-turns per inch, the reactance curves all come together and, therefore, in this region, the d-c controllability is greatly decreased. This will then determine for the d-c controlled circuit, the upper limit of the a-c ampere-turns per inch. The lower limit will obviously correspond to the peak of the reactance curve with zero d-c saturation. For the iron used in this investigation (Allegheny dynamo grade) the useful range of the a-c ampere-turns per inch is roughly between four and 30.

When the three-legged reactor is used in series with a condenser and a resistance, the volt-ampere relation will appear as in figure 7. Here it is noted that the effect of increasing the d-c saturation is to shift the curve upward and the critical voltage to the left.

When this circuit is operated with a constant voltage and the circuit controlled by the d-c coil, the locus of the a-c current will be along a vertical line such as that shown in the diagram, and it is seen that the change in the a-c current is greatest between certain values of d-c saturation as shown by curves 2 and 3. Therefore, if a very sensitive control is desired, it is possible to bring the circuit up to this sensitive level by means of another coil and operate the circuit about this region. By connecting two such circuits biased to this sensitive level in a bridge form, the initial output can be made equal to zero and the disturbing effect of the line-voltage fluctuation greatly eliminated.

When operating with a d-c bias, some difficulty is encountered when the controlling power is very small. This is due to the fact that with a d-c level of magnetization, there is a voltage induced in the control coil which may render the circuit unstable. With a d-c level of saturation, the flux variation in the two a-c legs of the reactor becomes unequal. The difference in these two fluxes must then flow through the center leg of the reactor, thereby inducing a voltage in the control coil. An analysis from the magnetization curve indicates that this unbalance of flux is mainly in the opposite direction to that of the control coil, and therefore is demagnetizing. For these reasons, it is advantageous to prevent this unbalance of flux from flowing in the center leg. This is accomplished by placing a short-circuited coil around the d-c leg.

With this short-circuited coil, an interesting difficulty was encountered. If the resistance R_0 in the circuit is reduced to a value less than the critical resistance, the circuit goes into a low-frequency oscillation. The oscillogram of figure 8 illustrates this phenomenon. It is found that the frequency of this oscillation is mainly a function of the d-c magnetizing force. This low-frequency oscillation, however, was checked by increasing the resistance to a value greater than the critical resistance of the circuit.

A Regenerative Method of Increasing the Sensitivity of the D-C Controlled Circuit

Noting that in any amplifying device if a part of the output is fed back to the input, the over-all gain is greatly increased, a scheme similar to that of the regenerative

Figure 8. Low-frequency oscillation

radio circuit can be used to increase the sensitivity of the d-c controlled circuit.

Figure 9 indicates the circuit diagram of the regenerative amplifier. The a-c output obtained across the condenser is rectified by means of the copper-oxide rectifier to supply the d-c load, the circuit being controlled by the d-c saturation. The amplification is obtained by feeding the output current through a regeneration coil wound on the same leg as the d-c control coil, the effect being to increase the total control of the circuit. With such a scheme, if the number of turns on the regeneration coil is too large, the circuit becomes unstable and therefore some criteria for the stability as well as the amplification must be developed. This is done by first assuming a straight-line relation between the input and the output current and developing the mathematics by a step-by-step process.

Starting with an input of i_0, the initial output current will be hi_0, where h is the slope of the output current for zero regeneration. This output flowing through the regeneration coil produces an additional control of $(nh/N)i_0$, which must be added to the initial control to give the effective control. Continuing with this process, the resulting equation is a geometric series,

$$I_{output} = hi_0 \left\{ 1 + \left(\frac{nh}{N}\right) + \left(\frac{nh}{N}\right)^2 + \left(\frac{nh}{N}\right)^3 + \ldots \right\} \tag{15}$$

the sum of which is given by the following equation:

$$I_{output} = hi_0 \left\{ \frac{1}{1 - \left(\frac{nh}{N}\right)} \right\} \tag{16}$$

where

n = number of turns on the regeneration coil
N = number of turns on the control coil
h = slope of the initial curve with no regeneration
i_0 = control current
I_{output} = output current with regeneration

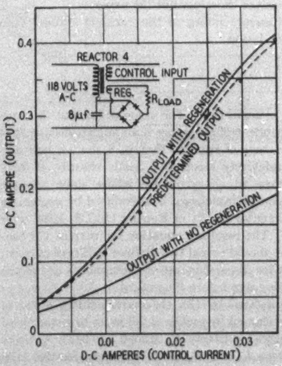


Figure 9. Regenerative amplifier

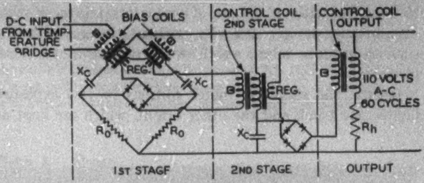


Figure 10. Control circuit of radiation meter

From this equation it is evident that the circuit is stable as long as (nh/N) remains less than one, and therefore this becomes the criterion for stability. The amplification factor is given by the bracketed term. For the case where the initial response curve deviates greatly from the linear relation, better results can be obtained by using the following equation,

$$I_{output} = I' \left\{ \frac{1}{1 - \left(\frac{nh'}{N}\right)} \right\} \tag{17}$$

where

I' = actual current for no regeneration, at the point in consideration
h' = actual slope of the curve corresponding to the point in question

To verify this equation, several predeterminations were made, one of which is shown in figure 9. In this diagram, the lower curve is the output plotted in terms of the input for no regeneration, while the upper solid curve represents the output of the same circuit with regeneration. The dotted curve is the predetermined output calculated from equation 17, and it is seen that the agreement between the theoretical and the experimental curves is very close.

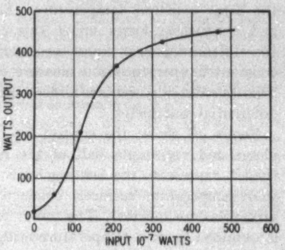


Figure 11. Output curve of the radiation meter

A Specific Application to the Radiation-Meter Control Circuit

The circuits discussed in this paper have been successfully applied to a specific engineering problem in connection with the radiation meter. The function of this meter is to measure the net radiation taking place at the earth's surface, such data having important applications in the study of plants.

These measurements must be taken continuously for a long period extending into years. It is necessary to maintain control without the use of contactors, relays, or other intermittent devices and also without the use of units, such as vacuum tubes, which would require frequent replacements.

Briefly, the radiation meter consists of two exposure plates, one black and the other bright, which are kept at the same constant temperature. Since the convection and conduction losses under this condition are equal, the differential heat input necessary to maintain this constant temperature is a measure of the net radiation. This differential power is recorded continuously by a graphic differential wattmeter.

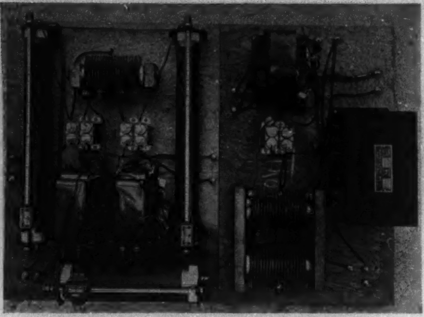


Figure 12

Figure 10 shows the control for one of the exposure plates and represents half of the radiation meter. In this R_h represents the heating loads for one of the constant-temperature surfaces, connected in series with a saturable-core reactor. The power for the control input is obtained from a temperature-bridge thermometer imbedded in the same constant-temperature surface. For a two-degree deviation from the set temperature, the temperature bridge sends a signal into the control circuit sufficient to produce maximum heating at the heater coil.

The first stage of this control circuit is connected in a bridge to overcome any effect of line voltage fluctuation, and each arm of the bridge is biased to the most sensitive level by means of the bias coils. The output of this bridge is then rectified by means of the copper-oxide rectifier and fed through the regeneration coils and the control coil of the second stage. The connection of the regeneration coils in the bridge is such that it aids the bias and the control signal in the first leg and opposes the bias in the second leg, thereby producing a maximum of unbalance. In the same manner, the output of the second stage is rectified and fed through the regeneration coil and the control coil of the final stage. Here the bridge circuit is not necessary since the control signal is sufficiently large and the effect of the line voltage fluctuation relatively small.

The amplification obtainable from such a circuit is quite striking as shown by the curve giving the over-all characteristics of the control circuit. (See figure 11.) It is seen here that an input of 450×10^{-7} watts is sufficient to control an output of approximately 450 watts, thereby resulting in a power amplification of approximately 10,000,000. Such a circuit is quite inexpensive to build, and once the control is adjusted, the circuit will require practically no replacement of parts or upkeep for service. The simplicity of this control circuit is shown by the photograph of figure 12.

In conclusion it may be stated that the greatest utility of such circuits will undoubtedly be in control circuits where large amounts of power must be continuously controlled by means of a relatively small input. As more and more becomes known of such circuits, the resonant type of nonlinear circuit will find a definite place in the various engineering fields.

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4. Direct Current Controlled Reactors, **C. V. Aggers** and **W. E. Pakala.** *The Electric Journal*, February 1937, pages 55–7.

Discussion

P. H. Odessey (Heyer Products Company, Belleville, N. J.): In Mr. Thomson's paper, the derivation of critical quantities associated with ferroresonance embodies a number of concepts that are not quite clear. In particular, the following points are noted with respect to approximations and assumptions made in deriving the expressions for current and critical resistance as defined in this paper; the numbers of equations are those used in the paper.

1. The current (7) corresponding to the "critical stable" voltage is derived from the relation

$$\frac{d^2E}{dI^2} = 0$$

whereby the resistance is neglected. The assumption that this current is practically independent of resistance is an approximation that is valid for a particular type of reactor, that is, if the a-c volt-ampere characteristic is relatively steep and bends sharply at high saturation. For the type of characteristic whose slope changes slowly with saturation considerable error is introduced by neglecting resistance. The experimental results of Rouelle (*R.G.E.* 1934) give evidence of this effect. The resulting equation for current (7) contains a radical and hence should reveal two distinct values of current. No explanation is given for discarding one of these values nor of the significance of the negative sign before the radical.

2. When current is defined by (7), the corresponding value of critical stable voltage evaluated by means of (5) must be determined by using the value of critical resistance R_0 defined by (10). As there is no interrelation between I_m and R_0 the justification for this step

is not quite clear. Thus, employing the criteria of stability,

$$\frac{dE}{dI} = 0$$

two "approximate" equations are given for the currents corresponding to the release voltages and contain a resistance term R_0. Since equation (6) was stated to be mathematically unsolvable when resistance was included, it is not quite clear how (8) and (9) were derived. As the forms of these equations indicate, their derivation does not follow as an approximation made after solving (6) by neglecting resistance.

3. Since these two equations are identical for the condition of critical stable voltage, the analytic expression for critical resistance (10) is obtained by equating (8) and (9). But since (7) already expresses the value of current corresponding to the critical stable voltage, then (8) and (9) could each be made identical with (7) and two other equations for critical resistance could be obtained. This discrepancy is not explained.

4. If, as in the presentation, the critical resistance is properly expressed, then by substituting this value (10) in the current equations (8) or (9) we should obtain equivalence with I_m defined by (7). There is, however, no assurance that this may be attained even approximately. It appears, therefore, that the critical resistance and I_m are entirely unrelated, yet must be employed together in the evaluation of critical stable voltage by means of (5).

5. In view of the assumptions and approximations, the results indicate an unusually high degree of accuracy although not consistent. For example, corresponding to a 40 per cent error in R_0 ($X_c = 43$, table I) the error in critical stable voltage is 0.3 per cent whereas for a 0.88 per cent error in R_0, the error in voltage is 1.21 per cent ($X_c = 188$). Thus, it appears that any error in R_0 does not materially affect the value of voltage. Although not given, it would have been interesting to observe the check on I_m, I_1 and I_3, and so justify some of the approximations.

W. T. Thomson: In reply to Mr. Odessey's discussion, I will endeavor to justify the assumptions made and indicate why it is possible to arrive at accurate results for the critical voltage using the more inaccurate values of critical current and resistance. Dimensions of one of the reactors for which the results were published are given together with the sample calculations (see figure 2).

Although my investigation was limited to a certain type of reactor (three-legged reactor) using a certain grade of iron, this type of reactor is extensively used for various control purposes and quite frequently with iron of similar characteristics. It seems therefore that Mr. Odessey's criticism that the method is applicable only to a very special and limited condition is not justifiable.

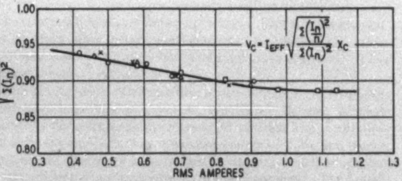
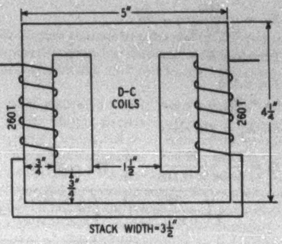


Figure 1. Reduction in voltage across condenser due to harmonics

$$V_c = I_{eff}\sqrt{\frac{\Sigma\left(\frac{I_n}{n}\right)^2}{\Sigma(I_n)^2}}\,X_c$$

The apparent resistance of the reactor defined by the expression $R_L = W/I^2$ is of course an approximation for circuits containing iron. It is agreed that for small values of current this error is not great. Let us investigate then the discrepancy for a large value of current. Assume in figure 3 that $I_1 = 0.8I$ when $I = 1.2$ amperes. R_L from the curve is 15 ohms and its value based on I_1 is $15/0.8^2 = 23.5$ ohms. The per cent error is therefore considerable but the difference in ohms is only 8.5. In the calculation of the voltage this small error in a term which is small compared to the reactive term produces practically no error when combined at right angles.

Figure 2. Dimensions of reactor 1



It may be stated that in order to obtain values for the more correct definition based on the fundamental current I_1, special equipment such as the harmonic analyzer is necessary, and for the average person to whom such equipment is not available, an approximation of this type is not out of order as long as it produces negligible errors in the final results. The writer would like to point out that Mr. Odessey in his paper ("Critical Conditions in Ferroresonance") makes an approximation which is far more serious in assuming the first term of equation 4 to be an ellipse. This term of the equation is an ellipse only when the total resistance is constant. The apparent resistance which must be considered as part of the total resistance varies greatly with the current as shown in figure 3.

Equations 8 and 9 are empirical. When $R_0 = 0$, they are the roots of equation 6 with the resistance omitted. The expression for R_0 given by equation 10 is therefore empirical and is unrelated to I_m as stated in the paper. To show that equation 10 is not a special equation limited only to a certain reactor, the following results obtained for three other three-legged reactors made with the same iron are given.

Reactor 3			Reactor 2			Reactor 4		
X_c	R_0 (Calc.)	R_0 (Exp.)	X_c	R_0 (Calc.)	R_0 (Exp.)	X_c	R_0 (Calc.)	R_0 (Exp.)
93	57	63	25	8	10	93	47	56
187	120	120	43	24	26	187	108	107
272	162	164	93	49	50	272	146	139
355	194	182	187	75	68	339	172	158
			272	87	75			

As stated before, a few ohms error one way or another when combined at quadrature with a much larger reactive term produces negligible error in the calculation of the voltage.

Equation 7 will always give a value of current in the unstable region. It is apparent from figure 1 that with a critical stable condition the current may have a wide range of values and still give the same critical voltage. Therefore even if this current varied with resistance for other types of iron, very little error would be produced in the critical voltage provided the current obtained is within the unstable region.

I fail to see any physical significance of the negative sign in front of the radical in equation 7. The equations as given in my paper apply only in as far as they fit the experimental curves for X_L

and for this case the negative value of the effective current seems meaningless.

Appendix

I. SAMPLE CALCULATION

For Reactor 1:

$$X_L = \frac{360}{0.115 + I} + 26$$

Therefore $B = 360$

$\beta = 0.115$
$K_2 = 26$

For $X_c = 272$ ohms:

$$R_0 = -\frac{360}{8 \times 0.115} - \frac{1}{2}(272 - 26) + \sqrt{\left(\frac{360}{8 \times 0.115}\right)^2 + \left(\frac{3 \times 360}{8 \times 0.115}\right)(272 - 26)} = 151 \text{ ohms}$$

$$I_m = -2 \times 0.115 + \sqrt{(0.115)^2 + \frac{3 \times 0.115 \times 360}{272 - 26}} = 0.49 \text{ ampere}$$

therefore

$X_L = 620$
$R_L = 100$

From figure 1

$$\sqrt{\frac{\Sigma\left(\frac{I_n}{n}\right)^2}{\Sigma I_n^2}} = 0.93$$

$$E = 0.49\sqrt{(100 + 151)^2 + (620 - 0.93 \times 272)^2} = 218 \text{ volts}$$

II. DATA FOR REGENERATION CURVE, FIGURE 9

Reactor 4:

Control and regeneration coils on center leg

$n = 200$
$N = 2{,}000$

therefore

$$I_{dc} = I^1\left\{\frac{1}{1 - 0.1K}\right\}$$

Discussions

of AIEE Technical Papers Published Before Discussions Were Available

ON THIS and the following 15 pages appear discussions submitted for publication, and approved by the technical committees, on previously published papers presented at the AIEE winter convention, New York, N. Y., January 24–28, 1938. Authors' closures, where they have been submitted, will be found at the end of the discussion on their respective papers.

Matrices in Engineering

Discussion and author's closure of a paper by Louis A. Pipes published on pages 1177–90 of volume 56, 1937, AIEE TRANSACTIONS (September 1937 issue of ELECTRICAL ENGINEERING) and presented for oral discussion at the basic sciences session of the winter convention, New York, N. Y., January 26, 1938.

Joseph Slepian (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): Linear systems of equations such as are continually met in practical engineering problems, present their primary data to us in the form of a rectangular array of numbers, the coefficients of the various terms in the set of equations. All the information which it is possible to obtain mathematically about the physical system represented by the equations is buried or latent in this array of numbers. Whatever new information we succeed in arriving at about the system by purely mathematical means will be reached ultimately only by successions of arithmetical operations upon the numbers of the array. Hence, we may say that this rectangular array of numbers is the appropriate manner of mathematical description or representation of the physical system for which the linear system of equations was set down. Given the sufficient description of the kind of equations for which the numbers serve as coefficients, the array gives all the information about the system with no irrelevancy. Both the position in the array of a number and its magnitude are significant in determining the result of any actual calculation.

Thus the matrix as an entity presents itself naturally and easily from the study of systems of linear equations. The various manipulations we make upon linear equations amount always to the calculation of new (usually simpler) matrices from given matrices. Thus the idea of a matrix calculus arises.

Two particular ways of calculating a new matrix from two other matrices keep recurring in practical problems. Because of their similarity to the arithmetic operations of addition and multiplication of ordinary numbers, they are called matrix addition and matrix multiplication. Thus we have a matrix algebra which is similar, except for one important property, to ordinary algebra. The beauty and utility of this matrix algebra applied to important every day problems in electrical engineering are well brought out in the very excellent paper by Professor Reed.

A finite number of applications of the operations of addition and multiplication of matrices will, of course, yield only polynomials, but Pipes in his remarkable paper shows the practical value of considering an infinite number of applications of these operations, as in a power series, thus arriving at the notion of more general functions of matrices. Then the beautiful, powerful, but little-known Cayley-Hamilton Theorem shows that these general functions of a matrix are not nearly as complicated as one might first expect, but reduce to simple sums of these functions of the latent roots of the matrix with easily expressed matrix coefficients.

I believe that this is the first time that the Cayley-Hamilton Theorem has appeared in ELECTRICAL ENGINEERING. Pipes is to be highly commended for calling this powerful tool to the attention of electrical engineers, particularly as he gets down to brass tacks in his remarkable paper, and for several well chosen examples, shows how by proper use of this theorem, numerical results are quickly and easily reached. I hope this paper will receive the wide and careful attention it deserves, and that it will stimulate the further proper application of higher mathematics to electrical engineering problems.

Louis A. Pipes: Doctor Slepian summarizes the utility and power of matrix algebra in the usual engineering problem leading to a discussion of linear dynamical system in a most concise and comprehensive manner.

There appears to be some confusion among some engineers concerning the relation of matrix algebra to tensor algebra. It should be realized that a matrix may be regarded as a tensor of the second rank with respect to a linear transformation. Since practically all the equations met in practical engineering problems are linear, linear transformations are encountered and consequently the summation convention, upper and lower indices, and other tensor machinery may be dispensed with and the classical matrix notation employed.

Matrix theory has received a great deal of attention by the mathematicians and many powerful theorems exist in the mathematical literature which have great practical importance. By the use of these theorems, the solution of polynomial equations, the calculation of normal modes, and other processes of great importance in engineering and physics may be most easily accomplished.

In conclusion, it should be pointed out that on page 1178 of the paper "Matrices in Engineering" the last word in section D should be "commutative" and not "symmetric."

The Properties of Three-Phase Systems Deduced With the Aid of Matrices

Discussion and author's closure of a paper by M. B. Reed published on pages 74–7 of this volume (February section), and presented for oral discussion at the basic sciences session of the winter convention, New York, N. Y., January 26, 1938.

Joseph Slepian: For discussion, see this page.

Irven Travis (University of Pennsylvania, Philadelphia): I agree most heartily with Mr. Reed that the matrix formulation of electric circuit problems clears away the algebraic debris and thus more clearly reveals the main line of reasoning. Mr. Reed has given a number of very interesting examples.

There is one point which I should like to amplify. Below equation 39 is the statement, "This equation cannot be used to determine E_{pg} in terms of I_{pg} because of the singularity of the transformation. Further investigation will show that no relation can be found for E_{pg} in terms of I_{pg}, and the circuit impedances or admittances; i.e., given a set of wye-connected impedances and the line currents, the phase generator voltages of the source cannot be computed." Although it is true that there are infinitely many voltages which will satisfy the system, valuable information can be obtained by transforming equation 39 to symmetrical co-ordinates.

Let the matrix S be defined as

$$S = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \qquad (1)$$

where

$$a = \epsilon^{j\frac{2\pi}{3}}$$

Multiplying both sides of equation 39 by S and inserting $S \cdot S^{-1} =$

U_1 in the product on the right hand side, we have

$$(SI_{pg}) = (ST_y YT_{Yt}S^{-1})(SE_{pg}) \qquad (2)$$

By expanding (SI_{pg}) and (SE_{pg}) it is easy to see that

$$SI_{pg} = (I_{A10}, I_{A11}, I_{A12}) \qquad (3)$$

$$SE_{pg} = (E_{100}, E_{101}, E_{102}) \qquad (4)$$

in which the added subscript (0, 1, 2) denotes a zero-, positive- or negative-sequence quantity.

Let Y, which is the inverse of 32, be denoted by

$$Y = \begin{pmatrix} y_{11} & y_{12} \\ y_{12} & y_{22} \end{pmatrix} \qquad (5)$$

then the product on the right hand side of equation 2 is

$$ST_y YT_{Yt}S^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_{11} - y_{12} + y_{22} & -(ay_{11} + 2a^2y_{12} + y_{22}) \\ 0 & -(a^2y_{11} + 2ay_{12} + y_{22}) & y_{11} - y_{12} + y_{22} \end{pmatrix}$$

Equation 2 is therefore of the form

$$I'_{pg} = Y'E'_{pg} \qquad (6)$$

where

$$I'_{pg} = (I_{A11}, I_{A12}) \qquad (7)$$

$$E'_{pg} = (E_{101}, E_{102}) \qquad (8)$$

$$Y' = \begin{pmatrix} (y_{11} - y_{12} + y_{22}) & -(ay_{11} + 2a^2y_{12} + y_{22}) \\ -(a^2y_{11} + 2ay_{12} + y_{22}) & (y_{11} - y_{12} + y_{22}) \end{pmatrix} \qquad (9)$$

The matrix Y' is not singular, hence it is possible to express the positive- and negative-sequence components of the generator phase voltages in terms of the phase currents and the circuit impedances. Thus the phase voltages are arbitrary because their zero-sequence components are arbitrary, but they cannot be assigned any values at random as might be inferred from the statement below equation 39.

M. B. Reed: As Mr. Travis points out, the generator phase voltages of a wye system are arbitrary only through the arbitrariness of the zero-sequence component of these voltages. This voltage component may have any one of a doubly-infinite set of values—any one of an infinite set of lengths, for a particular position or any one of an infinite set of positions for a particular length—and still the phase voltages will give the same set of line voltages. This result can be readily shown without recourse to symmetrical components as follows:

If the point at the center of gravity *cg* of the line-voltage triangle is designed by o'

$$\mathfrak{E}_{Lg} = \mathfrak{M}_{12}\mathfrak{E}'_{pg} = \mathfrak{M}_{12}(E_{10'}, E_{20'}, E_{30'}). \qquad (1)$$

Since $\mathfrak{M}_{12}$ is singular, this transformation is irreversible. However, if the relation

$$E_{10'} + E_{20'} + E_{30'} = 0$$

is used to eliminate $E_{30'}$ from the first two of the equations represented by (1), and if the third equation is omitted, the result is

$$\mathfrak{E} = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix} (E_{10'}, E_{20'}) = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix} \mathfrak{E}' \qquad (2)$$

and since the transformation matrix is nonsingular

$$\mathfrak{E}' = \frac{1}{3} \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \mathfrak{E} \qquad (3)$$

i.e., the voltages to the *cg* of the line-voltage triangle determine the line voltages. But from the fact that

$$\mathfrak{I} = \mathfrak{Y}\mathfrak{E} = \mathfrak{Y}\begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}\mathfrak{E}' \qquad (4)$$

evidently the currents are completely determined from the voltages to the *cg* of the line-voltage triangle. Further since

$$\mathfrak{E}_{pg} = \mathfrak{E}'_{pg} + E_{o'o}(1,1,1) \qquad (5)$$

it is evident that $E_{o'o}$, the voltage between the common point o of the generator and the *cg* of the line-voltage triangle, is the arbitrary element in the phase voltages of the generator, and further that $E_{o'o}$ has no effect on the currents in the system no matter what its value.

Corona Voltages of Typical Transformer Insulations Under Oil

Discussion and author's closure of a paper by F. J. Vogel published on pages 34–6 of this volume (January section), and presented for oral discussion at the electrical machinery session of the winter convention, New York, N. Y., January 27, 1938.

I. W. Gross: For discussion, see page 194 of this volume (April section).

V. M. Montsinger (General Electric Company, Pittsfield, Mass.): Mr. Vogel's paper is of considerable interest to me as it deals with a subject along my line of work.

I wish to discuss it from several viewpoints.

Mr. Vogel's experience with the harmful effects of air in his barrier parallels closely my own experience. As stated in my paper I found that any air at all in the structure gave misleading results and I found it necessary to assemble the sample under oil, not allowing the small coil used as the line electrode to come in contact with air after vacuum treatment and oil impregnation.

I subscribe 100 per cent to his second conclusion in that it is very important to remove all entrapped air in transformers before making commercial tests at the factory. Also if high-voltage transformers are not shipped filled with oil or are untanked for inspection, special means should be taken to remove the air. If a vacuum cannot be applied, one of the best means of getting the air out is to make a short-circuit heat run. Laboratory tests have shown that if oil is circulated by an air pocket it takes only a few hours to absorb the air. Tipping the transformer often is helpful in removing air pockets that are not adjacent to the oil in circulation.

I note that Mr. Vogel obtained an impulse ratio in the order of 2 to 2.3. My own tests do not bear this out. I have found that the ratio varies over a much wider range, depending on the construction of the sample. Based on the tests given in my paper and on the injurious corona voltage, I found a ratio of 1.75 for barrier *C* and 2.48 for barrier *D*. Barrier *C* had a spacing of 0.641 inch from the coil to ground and *D* a spacing of 2.5 inches from the coil to ground. I cannot agree, therefore, that there is a constant impulse ratio for all voltage ratings of transformers.

Mr. Vogel's statement that the dielectric strength varies approximately as the barrier thickness raised to the two-thirds power checks my experience. I have used this two-thirds-power rule for several years. I have found that the dielectric strength of oil when tested with edged electrodes or with round rods varies with the spacing raised to the two-thirds power. However, if tested in a uniform dielectric field it varies more nearly in proportion to the spacing. I believe that cable engineers have found that the strength of the paper covering varies approximately as the thickness. While cable insulation is stressed highest next to the bare part of the cable on account of its reduced periphery, the conditions are such that it corresponds fairly well with a uniform field. That is, there are no sharp corners or edges to cause voltage concentrations.

I believe it can be said, therefore, that the two-thirds-power law is not inherent in the characteristics of the material—solid or oil—but by the shape of the electrodes.

Conditions in transformers are such that most all insulations are in nonuniform fields and on this account we must generally increase the insulations faster than the test kilovolts. Roughly speaking, by the two-thirds rule, if the test is doubled, the distance or insulation must be trebled. This explains why the cost and size of transformers increases so fast with voltage.

L. H. Hill (Allis-Chalmers Manufacturing Company, Milwaukee, Wis.): It is interesting to note from Mr. Vogel's paper that as a result of his impulse tests he has reached the conclusion that transformers should be filled with oil under a vacuum.

The company with which I am associated has been filling high-voltage transformers with oil under a vacuum for about five years, and repeated tests have indicated the desirability of going to that extra trouble. In connection with filling transformers under vacuum, we have found that the method of filling the transformer tanks with oil has considerable bearing on the amount of time required to do this. If the transformer tank is filled from the bottom, we have found that it may take as long as four days for the surplus air to come out of the oil; however, if the transformer tank is filled by running the oil in at the top so that the oil goes into the transformer as a foam, by the time the transformer tank has been filled, the oil is perfectly quiet, indicating that all the air has been removed.

From a practical point of view, the increased cost of building transformer tanks so that oil could be installed in the field with the tanks under a vacuum would not be very great.

Eric A. Walker (Tufts College, Mass.): Mr. Vogel, in his paper, has undoubtedly opened a field which may well help us to discover more of the factors which lead to the breakdown of fibrous insulation under oil.

In this paper Mr. Vogel mentions the detection of corona by audible frequencies only, that is, directly by ear or by the use of a microphone.

There are at least three methods by which corona may be detected. An audible method as described by Mr. Vogel makes use of the energy in a limited frequency range. A radio-frequency detector and amplifier, used in the radio- and intermediate-frequency ranges, has met with some success. One can also find the corona voltage by power-factor measurements and ascertaining the point at which there is an abrupt rise in power factor. In a recent investigation, in which the three methods were compared, the power-factor method showed a lower critical voltage than the other two methods. In another investigation, on a system of concentric electrodes separated by several layers of fibrous insulation, it was possible to detect when each layer of occluded air became ionized by a sudden increase in power factor. Of course such a method would not give a definite point on a large transformer unless the corona became quite general.

The second consideration in this paper is its effect on operating practices. It is recognized that the first few days during which a transformer is energized are critical ones. This is probably more true of transformers which have served some time as spare transformers than of new ones. Failures during this critical period have usually been attributed to the absorption of moisture by the fibrous insulation. Although evidence indicates that the insulation does absorb moisture by breathing there is no proof that moisture alone is the only cause or even the principal cause of deterioration. Now Mr. Vogel points out that occluded gases may lower the corona voltage. This will cause "tracking" which would not take place if the gas were not present as is usually true in normal operation. This type of deterioration may well be more serious than the absorption of moisture for the damage caused by such carbonization is permanent and cannot be remedied as the absorption of moisture can be by a mere drying process.

Ultimately it will be desirable to detect the formation of corona in transformers in service, both during normal operation and during overvoltage tests. It is extremely doubtful if aural methods would be sufficient for such tests. Radio-frequency tests are difficult to use because of the normal corona in substations. A power factor test would have a wide application as a field test where the moisture content of the insulation is important as well as the voltage at which corona is initiated. Such a test also has the advantage of being a nondestructive test if applied at, or below, the operating voltage.

J. O. Fenwick (Line Material Company, South Milwaukee, Wis.): Mr. Vogel's paper on "Corona and Oil-Impregnated Insulation" describes one of several lines of investigations now being pursued in the improvement of transformer insulation, particularly against lightning surges. Elaborate studies, such as outlined in this paper, are ordinarily considered as being confined to large power transformers, but similar investigations are now being adopted in the development of distribution transformers as well. Such studies are of particular value in the design of rural-type transformers where exposure to lightning is severe and protection is not of the best. Other parallel lines of investigation on insulation are studies of power factor of the insulation, radio interference, as well as the more familiar puncture tests on 60-cycle and impulse voltages.

The corona test described by Mr. Vogel as well as the radio-noise and power-factor studies give valuable information not shown by the more conventional puncture tests. For one thing, this specimen under test need not be totally destroyed as is frequently the case with puncture tests and yet the voltage can be raised to the point at which the insulation begins to show signs of distress as manifested by the corona markings mentioned by Mr. Vogel, or by radio noise where this is used as an indication of corona. Very often a better picture of the electrostatic field is obtained and corrected measures can be applied more intelligently than is the case where the test specimen is totally destroyed.

Another result is that since corona is one of the first immediate indications of over-stressed insulation, the corona voltage is somewhat analogous to the "yield point" of a metal specimen with mechanical loading and the value of this information to the designer is obvious.

The point not emphasized by Mr. Vogel is the uncertainty of results and limited conclusions that can be drawn unless a great amount of test data is accumulated. The voltage resulting in corona and ultimate puncture of the specimen depends so much upon the shape of the electrodes, presence of sharp corners, and other such incidental factors that a seemingly comprehensive test often gives results which are rendered nearly valueless when some apparently minor change is made in the method of test. For this reason, tests made with electrodes and specimens which appear to simulate conditions in the actual transformer often give misleading results as far as numerical design data is concerned, and a large amount of testing must be done in order to obtain even a moderate amount of reliable and usable information.

The seeming inconsistency of the results of insulation studies is illustrated by a comparison of the results reported by Mr. Vogel and others obtained by our own company. Mr. Vogel reports a ratio of roughly two to one between impulse puncture and 60-cycle corona voltage. Our own tests on samples of insulation and also assembled transformers indicate that this ratio, instead of being two to one as reported by Mr. Vogel, for well-impregnated specimens, may vary as much as ten to one, for certain peculiar conditions of test. Our results do emphasize, however, the importance of thorough impregnation and the necessity for eliminating air from the test specimen if consistent results are to be obtained. The results are of considerable value in giving the designer a "feel" of the distribution of the electrostatic field, but as indicated in the foregoing, the numerical results must be used with considerable caution.

F. J. Vogel: I was greatly interested in Mr. Montsinger's discussion, inasmuch as he found that the air entrapped in insulation structures caused them to give erratic results, and in his agreement with me that it was important to remove all entrapped air in transformers before making commercial tests at the factory. It is my opinion, based on our tests, that higher values can be obtained by vacuum filling than by removing air in any other way. Our own experience has been that transformers in service as long as a year and a half may still have entrapped air which can result in a failure. Therefore, vacuum filling would seem to be essential even in the field if maximum dielectric strength and certainty were to be obtained.

I agree with Mr. Montsinger that it is possible to obtain impulse ratios over quite a range. However, I have tried to be careful to limit the application of the 2.2 ratio to a specific type of construction. My paper covered only barriers with interleaved insulation. Depending upon the interpretation of Mr. Montsinger's data for barrier *D*, somewhat lower values than 2.48 might be given. I have tried to be conservative in this to furnish values which could be expected rather than the maximum values. Barrier *C* is not of the same construction and, as stated in my discussion of his paper, I do not believe that his construction is typical of any insulation structure actually used in a transformer.

With relation to the dielectric strength of oil in uniform fields, or in fields due to edged electrodes or round rods, it is interesting to note Mr. Montsinger's findings, and I believe this is a subject which is worthy of a future paper.

The future of power-factor tests on transformers is uncertain in the writer's opinion. Data which we have to date indicate that corona even in service is unlikely. To detect corona by power-factor measurements with present-day equipment seems impossible.

The question of using voltage measurements and times only, instead of gap lengths for specifications and test purposes, seems to be evading the issue. One organization of consulting engineers specifies gaps to be installed at the terminals of their transformers. They specify that a voltage wave of given steepness be impressed upon the transformer to prove the transformer will withstand this condition. The purpose of this test undoubtedly is to prove that the transformer with the gap installed in service will be protected against failure of the transformer windings. If voltage data from one source is used to specify the voltage to be applied, and the tests on the unit are not made in the laboratory where these readings agree, it might be possible to obtain a transformer which would not withstand the operating conditions desired by the purchaser. It seems to the writer that it is better to use a direct method of test rather than to use an intermediate step where there is a possibility of error.

I believe that the previous discussions show clearly that it is possible to increase the impulse strength of commercially oil-insulated transformers by admitting the oil to the tank while the winding is still under vacuum. It should be possible to do this in the field and, with special equipment filtering the oil, should be possible without even a temporary reduction in impulse strength from transferring the oil. The per cent increase in strength, due to vacuum-filling, as compared to nonvacuum filling, is variable.

A Formula for the Reactance of the Interleaved Component of Transformers

Discussion and authors' closures of a paper by H. B. Dwight and L. S. Dzung published on pages 1368–71 of volume 56, 1937, AIEE TRANSACTIONS (November 1937 issue of ELECTRICAL ENGINEERING) and presented for oral discussion at the electrical machinery session of the winter convention, New York, N. Y., January 27, 1938.

A. N. Garin (General Electric Company, Pittsfield, Mass.): A theoretically rigorous solution of the problem of transformer reactance presents many difficulties due to the complexity of the physical structure of transformers. The factors chiefly responsible for these difficulties are: the curvature of windings, the presence of core iron, and the nonuniformity in the turn distribution in the cross section of windings. A practical engineering solution consists, therefore, first, of idealizing the structure until it is amenable to calculation by one of the theoretical methods available; second, of selecting the theoretical method best suited for the purpose and of performing the calculation; and third, of applying to the result of the theoretical calculation, whenever necessary, an empirical correction factor, which should not exceed a few per cent of the calculated value. Thus, calculation of reactance for a modern complicated transformer is not science, but an art, based on science.

The present trend in transformer design naturally results in a race between the growing complexity of windings and the increasing skill of the reactance calculator. The reactance calculator usually wins the race, and when he does not, he can use a reduced scale model of the transformer to be built and obtain the reactance by test, as described in a recent AIEE paper,[1] a procedure which may save time, and incidentally a professional reputation.

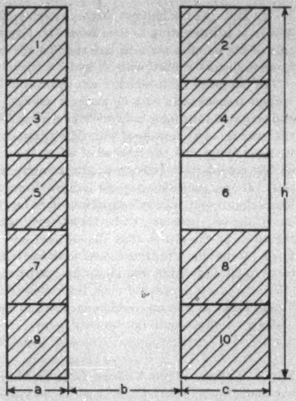


Figure 1. Cross section of windings

Mr. H. O. Stephens' method of resolving the reactance of nonuniform concentric windings into the concentric and interleaved components is a definite milestone in the art of reactance calculation and Messrs. H. B. Dwight and L. S. Dzung are to be congratulated on the very clear presentation, in the first part of their paper, of the physical phenomena justifying this method. At the risk of being accused of pedantry, I should like to call their attention, however, to an obvious oversight in their statement that I^2X is stored energy and is equal to the summation of $H^2/8\pi$. Being power, and not energy, I^2X cannot possibly be equal, but is proportional, to the summation of $H^2/8\pi$.

Having resolved the reactance of the transformer into its concentric and interleaved components, the next step is to select suitable formulas for calculating the two components separately. For the concentric component Dwight and Dzung use the well-known expression given by their equation 1. It will be observed that this formula does not take into account the curvature of windings and the presence of core iron, moreover, it assumes the effective axial length of windings to be equal to their physical length. The last two factors introduce errors of opposite signs, partly compensating one for the other, so that, as stated by the authors, the formula may be quite accurate, but only for a certain range of dimensions and proportions of the core and the windings.

For the interleaved component Dwight and Dzung use a formula based on the theory of geometric mean distances. The effects of curvature and of iron are again neglected. The reactance in question can be calculated with equal or greater ease, and with all the accuracy required for the purpose, by older and better known methods, of which that due to Rogowski[2] is perhaps the most advanced. The principal value of the second part of this paper is, therefore, in my opinion, not in the solution of this particular interleaved reactance but in the relative novelty of the method of attack. Although a rather complete study of the application of geometric mean distances to calculation of transformer reactances was published several years ago in German,[3] American technical literature seems to have neglected the subject.

Since the effect of curvature has been neglected in this paper in calculating both the concentric and the interleaved reactances, nothing would be sacrificed by performing the complete reactance calculation in terms of geometric mean distances. Figure 1 of this discussion represents the windings of the transformer investigated in the paper. Each winding is shown subdivided into five equal blocks. When operating on the 11,000-volt connection block 6 is idle.

Primary-to-secondary reactance of a transformer is given, in terms of geometric mean distances, by the following formula:

$$X_{P-S} = K \log \frac{R^2_{P-S}}{R_{P-P} R_{S-S}} \tag{1}$$

where

X_{P-S} = primary-to-secondary reactance

K = coefficient depending on frequency, the length of mean turn, the base of the log, and the choice of units

R = geometric mean distance between areas indicated by subscripts

For the present case the primary consists of blocks 1, 3, 5, 7, 9, the secondary of blocks 2, 4, 8, 10, so that equation 1 becomes:

$$X_{13579-24810} = K \log \frac{R^2_{13579-24810}}{R_{13579-13579} R_{24810-24810}}$$

$$= K \log \frac{R^{2\cdot 5}_{13579-246810} R^{0\cdot 4}_{13-24}}{R^{0\cdot 9}_{579-6810} R_{13579-13579} R^{0\cdot 5}_{24-24} R^{0\cdot 5}_{24-810}} \tag{2}$$

Equation 2 contains only geometric mean distances between rectangles and from rectangles to themselves, all of which can be calculated by the method described in the paper. The point of particular interest in this solution is that the final reactance has been obtained directly, without resolving it into concentric and interleaved components.

If it is desired to retain the resolution of reactance into concentric and interleaved components, the concentric component can be readily calculated as follows. Let the primary, consisting of blocks 1, 3, 5, 7, 9, be called A and the secondary, consisting of blocks 2, 4, 6, 8, 10, be called C, so that equation 2 becomes:

$$X_{A-C} = K \log \frac{R^2_{A-C}}{R_{A-A} R_{C-C}} \tag{3}$$

Following the method described in the appendix to the paper and denoting the space between A and C by B:

$$X_{A-C} = K \log \left[\frac{R^{\frac{(a+b+c)^2}{ac}}_{ABC-ABC} \; R^{\frac{b^2}{ac}}_{B-B}}{R^{\frac{(a+b)^2}{ac}}_{AB-AB} \; R^{\frac{(b+c)^2}{ac}}_{BC-BC} \; R_{A-A} \; R_{C-C}} \right] \tag{4}$$

Finally, using natural logs and expressing geometric mean distances in terms of semiperimeters, equation 4 can be replaced by the approximate expression:

$$X_{A-C} = K_1 \left[\frac{\frac{a}{3} + b + \frac{c}{3}}{h} - \left(\frac{\frac{a}{2} + b + \frac{c}{2}}{h} \right)^2 \right] \tag{5}$$

For a wide range of proportions the approximate expression (5) gives a value within two per cent of the exact expression (4).

An interesting side light on H. O. Stephens' method may be obtained by substituting algebraically the values of concentric and interleaved reactances in terms of geometric mean distances, the former being given by equation 4, into the H. O. Stephens' formula:

$$X_{13579-24810} = X_{13579-246810} + \frac{1}{25} X_{24810-6} \tag{6}$$

If equation 6 were exact the result of this substitution would be equation 2. Actually, however, although a very close approximation, equation 6 is not exact. The exact formula is:

$$\begin{aligned} X_{13579-24810} = {} & X_{13579-246810} + \frac{1}{20} (5X_{13579-246810} - 9X_{135-246} + 4X_{13-24}) + \\ & \frac{1}{25} X_{24810-6} + \frac{1}{100} (X_{2-4} + 2X_{2-6} - 2X_{2-8} - \\ & X_{2-10} - X_{1-3} - 2X_{1-5} + 2X_{1-7} + X_{1-9}) \end{aligned} \tag{7}$$

Considering the difficulty of obtaining accurate reactance by tests on a model of the size described in the paper, the agreement between calculated and test values is satisfactory, even if inconclusive. It is regrettable, however, that the authors do not state whether in making their tests without an iron core they had the primary and secondary windings of equal turns and connected in series opposition, thus obtaining

$$X_{1-2} = \omega(L_1 + L_2 - 2M_{1-2}) \tag{8}$$

corresponding to the basis of their calculation. If in their tests they had the secondary short-circuited they were measuring:

$$X_{1-2} = \omega\left(L_1 - \frac{M^2_{1-2}}{L_2}\right) \tag{9}$$

Although (8) and (9) are practically identical for an iron core transformer, they may be radically different for an air core transformer.

References

1. Reactance and Stray Losses of Power Transformers, H. L. Cole. AIEE Transactions, 1934, page 338.

2. Mitteilungen ueber Forschungsarbeiten, W. Rogowski, 1909.

3. Weitere Entwicklung der allgemeinen Methode zur Berechnung der Streuung von Transformatoren, G. Petrow. *Electrotechnik und Maschinenbau*, 1934.

H. B. Dwight: In connection with the interesting discussion by A. N. Garin, a distinction may be made between expressions which are so long as to be of merely academic interest, and formulas which are concise enough to be of practical use. For instance, equation 2 of the discussion contains four geometric mean distances of one rectangle from another, and if these are to be calculated by the method described in the paper, each would give rise to several self geometric mean distances, making 13 self geometric mean distances for the formula. But equation 2 of the paper gives the reactance with only four self geometric mean distances, and so is not unduly long.

The observation is correct that I^2X is proportional to stored energy, not equal to it. The coils in the model ironless transformer had a turn ratio of one to one and were connected in series. The secondary was not short-circuited.

In the thesis work upon which the paper is based, the self- and mutual inductances were calculated, as a check, by formulas for cylindrical coils. This brought the curvature of the conductors into the calculation but not the effect of the iron core. It seems probable that such long calculations will not be practical for routine computations in designing. The need at the present time appears to be a comparison of the results of the shorter calculations with measurements on a number of practical transformers whose dimensions have been accurately found, so that the percentage error to be expected from a given calculation may be known.

L. S. Dzung: Rogowski's method of attacking the problem of reactance of interleaved transformer windings, as mentioned in Mr. A. N. Garin's interesting discussion, is very general in character; but the restriction[1] on dimension ratios of his approximate formula[2] limits its application more suitable to the calculation of the reactance of an interleaved transformer rather than to that of the interleaved component of a transformer. The coil shape of the former is usually such that the axial height is small compared to the radial depth. This condition is seldom realized in the case of the interleaved component of a transformer. His more exact formula[3] is too lengthy to be applied in practical designing.

It may be noted that the curves on figure 2 of the paper and the little formula printed beside offer a very simple means to a practical designer. Equation 2 of the paper and also equation 2 and equation 4 of Garin's discussion have the disadvantage that they involve obtaining a small number from the difference between large numbers. In preparing the curves logarithms have been taken to a sufficient number of significant figures so that the final results will give engineering accuracy.

Equation 4 of Garin's discussion, which is equation 9 of G. Petrow's paper in *Elektrotechnik und Maschinenbau*, 1934, has been used in the thesis to compare the reactance of the concentric component as calculated by the geometric-mean-distance method against that by the formulae for cylindrical coils and by the usual formula, equation 1 of this paper. The results of these three calculations are respectively 5.30 per cent, 5.31 per cent, and 6.21 per cent for the transformer given in example II of the paper.

The effect of the iron core can be taken care of by the method of image as indicated also in Petrow's paper. A comparison between equation 21′ and equation 23 of Rogowski's paper shows that this effect would not be very great.

References

1. **W. Rogowski,** *Mitteilungen ueber Forschungsarbeiten,* Heft 71 (1909), Abs. II, §3, 2.

2. **W. Rogowski,** ibid., equation 23.

3. **W. Rogowski,** ibid., equations 13 and 14.

Oil Oxidation in Impregnated Paper

Discussion and authors' closure of a paper by J. B. Whitehead and T. B. Jones published on pages 1492–1501 of volume 56, 1937, AIEE TRANSACTIONS (December 1937 issue of ELECTRICAL ENGINEERING) and presented for oral discussion at the cables and research session of the winter convention, New York, N. Y., January 27, 1938.

L. E. Fogg and **R. B. Power** (Kennecott Wire and Cable Company, Phillipsdale, R. I.): About eight or ten years ago the research department of our company attempted some electrical tests on samples quite similar to those used by the authors. The results of this work were very unsatisfactory because of the nonuniform shrinkage of the paper samples. During the drying process, bulges and swills were formed in the samples to such an extent that the upper electrode was appreciably raised, even though its weight was sufficient to produce a pressure of several pounds per square inch on the sample. We would be interested to know if the authors encountered any such difficulty in their work, and how they were able to avoid it.

In connection with figure 7 of the paper, the authors suggested that a cause of the rising power factor—voltage characteristic might be the formation of some ions through atomic or electronic collisions. It is not obvious why such a factor should permit later flattening of the curves as still higher voltage stresses are applied, unless it may be assumed that there are but a limited number of atoms susceptible to such ionization. We would appreciate hearing the authors' viewpoint on this phase of the subject.

There is an apparent discrepancy in figures 22 and 23. The authors attribute to incomplete saturation both the low capacitance of figure 22 with respect to the comparable oxygen treated sample, and the initial decrease of the ionization curve of figure 23. However, this decrease apparently is brought about by more complete saturation, which at the same time should bring about an increase in the capacitance of figure 22, which increase is conspicuously absent.

The catalytic effect of metals on oxidation as well as deterioration of the oil in the absence of oxygen has been referred to at several points in the paper. However, data has been given only for brass. It seems that similar data for the metals, which the cable saturant must contact, would be of greater interest and value. We would appreciate such data as the authors might be able to supply for such metals as copper, lead, tin, and iron.

The conclusions drawn by the authors upon the effects of oxidation cannot be applied to cable practice without further investigations. All the work has been done for relatively short time intervals: 192 hours is, after all, but a brief period in the expected life of a power cable. Almost none of the curves has approached a constant value of power factor in this length of time. It seems safe to assume that these curves will continue to rise for a much longer period of time, eventually, perhaps, to a value so high that successful cable operation might be impaired. It also seems possible that there may be some further later reaction of the oxidized products with themselves or with fresh oil, which may cause a more rapid increase of power factor. Certainly, definite conclusions should not be reached until the results of the authors have been substantiated by long time tests.

Moreover, these tests have been made under constant conditions of temperature and pressure, tending to discourage any movement of the oil through the samples. If the samples were subjected to load cycles similar to those of an operating cable, so that there would be an oil migration, permitting the oxidized products to encounter fresh oil, an entirely different set of results might be obtained.

We hope that Professor Whitehead will continue his work on this phase of cable saturants, so that these factors of time and oil migration may be considered.

L. J. Berberich (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The result reported by the authors that small amounts of oxygen had practically no effect on the electrical properties of the one oil studied throws new light on the oxidation problem of cable oils. Unfortunately, however, in order to generalize this result, a range of oils of different constitution must be studied. The authors give very little information which would enable one to obtain an idea of the constitution of the oil except for the broad statement that the oil is a "highly refined paraffinic cable oil." While there is no accurate way of characterizing the constitution of oil, it has, nevertheless, been found that the "viscosity index" is of value in this connection. This leads the writer to suggest that more use should be made of this index by those studying insulating oils.

The term "viscosity index" and its use as a constitutive property of the oil was first proposed by Dean and Davis (*Chem. and Met. Eng.*, volume 36, page 619, 1929). It is independent of the value of the viscosity and permits the expression of the viscosity-temperature coefficient of the oil as a simple function of the Saybolt universal viscosities at 100 and 210 degrees Fahrenheit. The system was devised on the basis that a typical coastal oil is taken to have a viscosity index of zero and a typical Pennsylvania oil is taken to have one of 100. From empirically derived equations connecting the viscosities at 100 and 210 degrees Fahrenheit for oils from these two extremes of crudes, Dean and Davis compute data for a table. Knowing the viscosities at 100 and 210 degrees Fahrenheit, use of this table and a simple arithmetic calculation will give the viscosity index.

A number of interesting correlations of viscosity index with other properties of insulating oils have already been made. Sommerman (ELECTRICAL ENGINEERING, volume 56, page 566, 1937) has found that the amount of x-wax formed by cable oils when subjected to gaseous ionization increase with increase in viscosity index of the oil. Balsbaugh, Larsen and Oncley (1937 Report on Research Project on Electrical Insulating Oils at MIT sponsored by Utilities Coordinated Research, Inc.) have found at least an approximate relationship between ease of oxidation and viscosity index among a group of oils representing extremely wide variations in constitution. Viscosity index heads the list of physical constitutive properties which these workers suggest as useful for predicting chemical behavior. The writer also, in his work on cable oils, whether it was a study of stability to oxidation or stability to gaseous ionization, has found the use of viscosity index valuable. Hence, with this much evidence in favor of viscosity index, the writer believes this work of the authors could be better correlated with that of others if the viscosity index had been given.

G. M. L. Sommerman (American Steel and Wire Company, Worcester, Mass.): From the physical properties of the oil given in table II by Whitehead and Jones, it is calculated that the specific gravity of the oil at 60 degrees Fahrenheit is $G = 0.918$ and the viscosity at 100 degrees Fahrenheit is $V = 2{,}360$ Saybolt seconds. These data may be used to calculate the viscosity-gravity constant of the oil according to the method proposed by J. B. Hill and H. B. Coats (*J. Ind. and Eng. Chem.*, volume 20, 1928, page 641), which is as follows:

$$a = \frac{10\,G - 1.0752\,\log(V - 38)}{10 - \log\,(V - 38)}$$

For the oil in question, $a = 0.84$; for highly paraffinic oils, $a = 0.80$; for highly naphthenic oils, $a = 0.87$. This indicates that the oil is probably of midcontinent source with a viscosity index in the neighborhood of 70. It would be of interest to have oxidation data of the kind presented by Whitehead and Jones for the other types of oils.

From the data presented in this paper it is not clearly proved that the *process* of oxidation in oil (as distinct from the results of oxidation) is a source of increased power factor. First, the decrease in power factor following the degassing of aged oil may be due to the removal of volatile oxidation products, as recognized by the authors. Second, the higher "ionization factors" of the undegassed samples compared with those of the degassed ones may also be due to volatile constituents. Third, the greater rate of rise of power factor with increase in temperature for specimens containing more oxygen may be due simply to the higher power factors of the more highly oxidized oils. Power factor-temperature curves of impregnated paper similar to the authors' figure 18 were reported by the writer in a recent paper (Electrical Engineering, volume 56, 1937, page 566, figure 6). Rapidly rising power factor-temperature curves were obtained when the saturants were *degassed* deteriorated oils, and the values were practically independent of time, at least up to 24 hours. This is also in accordance with the results reported earlier by Whitehead (AIEE Transactions, volume 52, 1933, page 667).

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): We in Chicago have been making tests on 1,000-foot lengths of experimental 132-kv cable and have obtained some test data on the effects of small amounts of dissolved oxygen or air on the characteristics of oil-filled cable insulation. Apparently during either manufacture or installation a slight amount of air was entrapped in some of these cables. After these cables had been subjected to heating cycles to about 70 degrees centigrade for 10 or 20 days, measurements of power factor at 60 degrees centigrade showed no change at all from the measurements obtained when the cables were first installed. In one cable the effects of the entrapped air required one month to show increases in power factors at 60 degrees centigrade, while for the other cable a few years were required.

From this experience and other considerations, I am forced to disagree with the conclusion of Whitehead and Jones on the relative unimportance of small amounts of oxygen as affecting electrical properties. Tests on this point apparently require more than the seven days used by them.

One of the outstanding characteristics of the deteriorated experimental cables in Chicago was the development of so-called negative ionization factors. The authors' tests obviously did not last long enough to develop negative ionization factors.

It is of interest to note that the ionization factors reported by the author decreased "when oxygen and volatile products were withdrawn." This fact seems to indicate that some gaseous ionization was present, and such a conclusion is supported by the test results with nitrogen.

The radial power-factor tests were made at room temperature. We have found that samples which show interesting variations in power factor when tested at 60 degrees centigrade show in some cases insignificant variations at room temperature.

One of the helpful results of the authors' studies is that, in testing stressed oil, it is better not to remove moisture and volatile products in order to retain reliable indications of the condition of the stressed oil.

J. B. Whitehead and **T. B. Jones:** We are in agreement with the suggestion of Doctor L. J. Berberich that it would be desirable to correlate all studies of the electrical properties of oils with the respective values of the viscosity index. In the case of the oil used in our studies, we did not determine the viscosity index, nor was it furnished with other properties of the oil as given by the refiner, as set forth in table 2. We note, however, that Doctor Sommerman in his discussion has estimated the value of the viscosity index to be in the neighborhood of 70.

Mr. Wyatt raises the question of the influence of a longer period of test on variations of layer power factor due to oxidation. We agree with him that longer tests would probably accentuate layer variations of power factor. However, our results indicate that the U-shaped curves sometimes observed in new cables are probably not due to oxidation. Furthermore, we have given evidence of a quantitative evaluation of the amount of oxygen necessary to produce a layer effect within a limited time.

Replying to Messrs. Fogg and Power, we encountered no instances of buckling or nonuniform shrinkage in our samples. Throughout our tests a constant weight was kept on the upper electrode. This was done to maintain uniform pressure throughout the test. Apparently this weight was sufficient to prevent troubles of the nature mentioned. A rising power factor-voltage characteristic passing through a maximum, and followed by a descending portion has often been noted in our work. An explanation has been offered in the paper "Residual Air and Moisture in Impregnated Paper Insulation—II" (Whitehead, Kouwenhoven, and Hamburger, AIEE Transactions, volume 47, 1928). The conductivity and the number of ions in the oil have relatively low values. With increasing stress collision ionization is possible, but is not cumulative, owing to the limited length of path. Above a certain stress all of the free ions, including those caused by ionization, pass to the limits of their free paths in both positive and negative half cycles. A saturation condition results, the power factor passes through a maximum, and with further increase of voltage, conduction current is constant and capacitance current increases, causing a decrease in power factor. This picture is a suggestion only. An alternative one for the rising power factor at lower stresses is that with increasing stress large molecular aggregates surrounding the ions are stripped of their outer envelopes of neutral molecules, acquire higher velocities with resulting increased conductivity. The curves of figure 21 indicate that with increasing temperature both of these effects may be involved.

The apparent inconsistency between the power factor-voltage and power factor-time curves for nitrogen and for air is due to the difference in pressure (gas content) between the two tests. The nitrogen test was at ten centimeters and air test at one millimeter pressure. Thus in the nitrogen sample there was 100 times as much gas as in the air sample. Thus capacitance changes in the latter were too small to be detected. In this connection, it is worth noting that the constant value of capacitance of the air sample is between five per cent and ten per cent less than that of the one-millimeter oxygen sample, thus indicating a better impregnation for the latter.

We are glad to note Doctor Sommerman's computation of the viscosity index of the oil. We note also that he feels that our tests do not clearly prove that the process of oxidation is a cause for increased power factor. We agree to this, but on the other hand, we are not greatly impressed by the grounds of his doubt. What reasons has he for supposing that volatile products of themselves can cause an increase in power factor? The work of Piper in fact indicates that the addition of a wide variety of soluble oxidation products to an oil has little if any effect on power factor and loss. It seems to us that a far more probable and powerful cause is to be found in the continual process of chemical change during oxidation, and the temporary separation during this process of ions of opposite sign. If such is the case, increases in conductivity and power factor are logical.

We agree with Mr. Halperin and others that in any studies of the oxidation process long periods are desirable. He makes the mistake, however, of comparing the conditions of our tests with those of his longtime tests on completed cables. His tests contain a variety of factors and influences on power factor and stability that we have taken great care to eliminate. His succession of temperature cycles particularly, introduces the whole question of the creation of voids and the resulting gaseous ionization. Gas bubbles and voids and gaseous ionization in the ordinary sense were apparently entirely absent in our tests. In fact we find in the result of Mr. Halperin's tests a strong corroboration of our conclusion that oxygen in small amounts in impregnated paper insulation has little or no effect as a deteriorating agent. In particular we repeat that in our experiments a constant supply of oxygen was kept up, which is definitely not the case in a closed cable, and that in spite of this fact, oxygen to the extent of 0.013 per cent by volume in the oil has an effect on the oil which is not greater than that caused by contact alone with paper and with metals.

The Current-Carrying Capacity of Rubber-Insulated Conductors

Discussion and author's closure of a paper by S. J. Rosch published on pages 155–67 of this volume (March section), and presented for oral discussion at the cables and research session of the winter convention, New York, N. Y., January 27, 1938.

H. B. Dwight (Massachusetts Institute of Technology, Cambridge): In 1935 and 1936 some thesis investigations were made by C. R. Boytano, I. I. Hochberg, and B. R. Souza at Massachusetts Institute of Technology, under my supervision, on the current-carrying capacity of single-conductor, 600-volt, rubber-insulated, braid-covered conductors in three and one-half-inch fiber ducts encased in concrete. The duct line was 30 feet long, with three ducts. Two sizes of samples furnished by the Simplex Wire and Cable Company were tested, with the following results:

Over-all thermal ohms per foot of duct, from the copper to the outside of the concrete, with the copper at 40 degrees centigrade.

Size	Number in Duct	Thermal Ohms per Foot (Degrees Centigrade for One Watt per Foot of Duct)
Number 4/0	1	5.0
Number 4/0	2	3.8
Number 4/0	3	3.4
500,000 circular mils	1	4.9
500,000 circular mils	3	2.9

The thermal ohms are slightly less than the values given, for temperatures above 40 degrees centigrade for the copper.

These results for fiber duct are of course not directly comparable with those for iron conduit.

W. A. Del Mar (Habirshaw Cable and Wire Corporation, Yonkers, N. Y.): This is not merely another paper on "carrying capacity." It is the culmination of years of co-operative effort and research in which practically all who have studied the subject have had some voice. It should form the basis of future standard tables of carrying capacity and of future practice.

Mr. Rosch is to be congratulated on a fine piece of laboratory work and on his effective collaboration with other engineers in orienting his new data in relation to their data and experience.

D. W. Ver Planck (Yale University, New Haven, Conn.): The interesting paper by Mr. Rosch gives a much needed treatment of the subject of heat transfer from wires both in conduits and in the open. The study is based on the assumption that the heat transfer from the wires is entirely in the radial direction, as of course it is, for the major portion of any long uniform cable. One should not, however, lose sight of the fact that near the terminal of a cable there may be a considerable axial transfer of heat to or from the connected apparatus.

From the standpoint of this paper the case of greatest interest is where the connected apparatus tends to run hotter than the cable. This condition is encouraged by various apparatus standards which permit a maximum terminal rise of 30 degrees centigrade and in some cases even 40 degrees centigrade. Obviously, such rises above a 30 degree centigrade ambient will subject the end of the cable to temperatures from 10 degrees to 20 degrees centigrade in excess of the 50 degrees centigrade specified by Mr. Rosch as the limit for rubber insulation. It is the purpose of this discussion to indicate how great a length of the cable will be influenced by the high terminal temperature rise.

To derive the equation for axial temperature distribution let

x = distance along cable from terminal
T = temperature rise of copper at x
T_0 = temperature rise of terminal
q = heat generation per unit length
k = (radial) thermal conductance per unit length
R = (axial) thermal resistance per unit length

Upon applying the condition of heat balance to a differential length of the cable, there results the differential equation:

$$\frac{d^2t}{dx^2} - kRT = -Rq$$

Assuming that the length of cable extending from the terminal is great, the solution is:

$$T = \left(T_0 - \frac{q}{k}\right)\epsilon^{-\sqrt{kR}x} + \frac{q}{k}$$

indicating that the temperature rise decreases exponentially with distance along the cable from the terminal value T_0 to the value q/k which applies for conditions remote from the ends. A measure of the length to which the terminal temperature is propagated into the cable is $(kR)^{-1/2}$, a quantity which increases with cable size but in less than direct proportion. If the distance along the cable from one terminal to the next is not great, another solution taking account of both terminal conditions can be easily obtained.

Figure 1 of this discussion illustrates the use and checks the correctness of this analysis. A 4/0 cable in open air carrying 225 amperes is connected to a switch terminal having a rise of 35 degrees centigrade. The curve is calculated and the circled points are thermocouple measurements. The radial conductance per unit length k was deduced from Mr. Rosch's paper, table IV. The am-

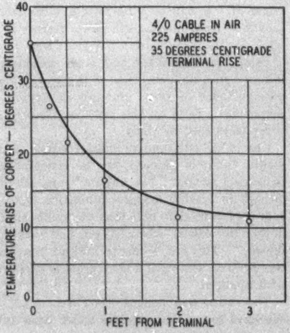


Figure 1

bient temperature was 25 degrees centigrade, and, while under these conditions the cable remote from the terminal attained a temperature of only about 36 degrees centigrade, a length of five or six inches near the terminal was subjected to temperatures in excess of the 50 degrees centigrade limit. Had the cable been worked to the limit calculated by Mr. Rosch's method, a still greater length would have been subjected to excess temperature, even though the safety factor included in curve *C* of figure 9 tends to reduce this length.

The possible overheating of the end portions of cables as a result of the high terminal rises permitted by various apparatus standards seems particularly worthy of consideration when it is remembered that these same end portions are likely to receive the greatest mechanical abuse during installation. It is hoped that the material in this discussion may help toward a better understanding of temperature conditions that sometimes exist near cable terminals.

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): Rosch and those who worked with him are to be congratulated for giving us some valuable information on rubber-insulated conductors. The data presented, however, are not convincing that code rubber will have a long life when operating at 50 degrees centigrade. In their tests at 50 degrees centigrade the physical properties decreased. Perhaps the author is relying on the load in service actually being usually less than the rated load, or on the ambient temperature being less than 30 degrees centigrade most of the time.

In connection with building wiring, it seems to me that more thought should be given by the industry to the use of better grades of rubber instead of Code rubber, especially where there are possibilities of loading near the ratings or of high ambient temperatures. I have heard of some buildings where the rubber on the circuits was even two steps better in quality than Code rubber. The additional cost for a better grade of rubber insulation is small, especially in proportion to the total installed cost of a wiring installation, while the life of a wiring installation is on the average sharply increased by the use of better rubber.

It is interesting to note in figures 3, 4, and 5 that the rubber from the number 0000 cable stood up better than the rubber from the number 14 and number 16 wires. It would be interesting to know the author's explanation.

Referring to figure 9, which shows that the surface resistivity increases for cable diameters up to one and three-fourths inches, we have in the Commonwealth Edison Company been using a similar characteristic in resistivity for about ten years. Our curve is based on test data obtained for us by Barenscher at the University of Wisconsin. The curve is convex upward and has a resistivity of 400 for zero cable diameter, which becomes a value of 1,200 at two and one-half inches cable diameter. Mention of our curve has been made in the NELA meetings and in C. A. Bauer's article presented at the Great Lakes Section of the NELA underground systems committee on April 10, 1931 (Published in "1930–31 Report of Engineering Section," Great Lakes Division, NELA, Appendix XXXV, page 237). Our values are somewhat lower for the smaller cables than shown.

It would be of interest to know why the author plotted on figure 9, apparently, data from table VII instead of from table IV of his paper, because the latter data would result in a lower resistivity and a lower curve.

In contrast to the author's proposal to use the total area of two or three cables in a duct in connection with thermal resistivity, we have tried to take some recognition of the fact that the surface in the center of the combination of cables does not radiate heat. As a result, we have worked out a term for thermal resistivity for three cables based on two and one-fourth times the surface of one cable, the corresponding term being 1.83 in the case of two cables.

R. J. Wiseman (The Okonite-Callender Cable Company, Inc., Passaic, N. J.): Every engineer who has had occasion to compute the current-carrying capacity of rubber-insulated conductors installed in conduits has been puzzled by the difficulty of checking his calculations with the National Electrical Code. The other discussers have brought out the valuable work done by Mr. Rosch and his associates for the NEMA rubber covered building wire technical committee and we cable manufacturers are very grateful for the clearing up of many of the questionable constants such as radiation from braids and conduits of various sizes. To me, the real value to the paper comes from our ability now to be more certain of our calculations and the opportunity of revising the NEC so as to properly rate cables when drawn into conduits whether or not a single cable or several cables are used. Heretofore it was difficult to convince an electrical contractor or some utility engineers that the Code at the present time does not specifically state the current loading tables are for a single cable in a conduit. We are hoping that the Code tables will be revised and that they will give loadings for both single cables and three cables in a conduit such as table XI of Mr. Rosch's paper.

S. J. Rosch: Professor Ver Planck's comments with regards to the axial transfer of heat from connected apparatus to the terminating end of a cable, are very timely and to the point. This is a feature which has received very little consideration in the past and we recommend a study of his discussion by all those who are engaged in considering the question of heating of wiring in panelboards, circuit protection and the uniform current rating of service equipment.

Mr. Halperin questions the life of Code-grade rubber insulation at 50 degrees centigrade. As stated in the text, the investigation at 50 degrees centigrade was conducted on samples of cable submitted by eleven manufacturers. Figures 4 and 5 were chosen as typical of the extremes in performance when aged at 50 degrees centigrade. A study of figure 4 indicates that the trend is definitely parallel to the horizontal axis. This was true even when the aging was extended over a period of 19 weeks. From this it was felt that 50 degrees centigrade would give satisfactory life for Code-grade rubber insulations. Although space limitations prevented giving the performance data for the other nine manufacturers' products, it can be stated that they all showed the same trend as shown in figure 4.

We sympathize with Mr. Halperin's views on the desirability of using better grades of rubber insulation than the minimum grade now required by the National Electric Code. That the tendency is definitely in that direction can be seen from the fact that the Federal Specification Bureau's Specifications *JC* 106 and *JC* 121 now require a superaging type of rubber insulation for use on all wiring in projects under Federal supervision. This same type of insulation is also being considered by suitable committees in NEMA, IPCEA, and the ASTM subcommittee on rubber insulated wire and cable specifications. The National Electric Code has recently appointed a committee to study the trends in higher grades of rubber insulation, as well as other insulations which are capable of higher operating temperatures and possibly increased current ratings.

Regarding Mr. Halperin's question as to the reason why the physical aging of the 4/0 cables showed up better than the smaller sizes, we can only assume that this was a coincidence for manufacturers *A* and *B*. Had we given the data for the other nine manufacturers, it would have been seen that in six of the cases, the 4/0 cables showed up poorer than the smaller sizes.

As regards Mr. Halperin's questions on the values of B_b, the values plotted in figure 9 are the average values for B_b given in table IV and, since they are the average, they are more truly representative. The values given in table VII are for one condition only, namely, "horizontal open," and may be found under this column in table IV. During the computation of our results, we also used the areas suggested by Mr. Halperin, but they resulted in values for B_b that could not be fitted to a curve. It can also be readily seen that to use the areas suggested by Mr. Halperin, would result in more complicated formulas. Furthermore, so long as the "total" area is used in the computation of B_b and in the ultimate calculation of current-carrying capacity, no error is introduced in the result.

We appreciate the references given by Mr. Halperin concerning the values of *B* used by his company. We calculated several values of *B* from the data presented in Mr. Bauer's paper and found that on plotting them in figure 9, they practically fitted on curve *C*, the curve adopted for standardization. What is more surprising, however, is the fact that a curve plotted from the data presented by Mr. Bauer, almost completely parallels our curve *B*. It is regrettable that the factual data on which Mr. Bauer's formula for *B* is based was not given in his paper.

The comments of Professor Dwight are of interest, even though the data is not directly comparable to that obtained with iron conduit. They do, however, afford the author an opportunity to express his deep appreciation for the very excellent work done by Mr. C. R. Boytano, mentioned in Professor Dwight's comments. Mr. Boytano was responsible for the investigation of the thermal surface resistivity under the very able supervision of Messrs. A. C. Connell and B. Jore.

We also appreciate the kind remarks of Mr. W. A. Del Mar and Doctor R. J. Wiseman. The author was extremely fortunate in being able to discuss his results with the leading cable engineers in NEMA and the IPCEA. The present paper represents the combined engineering thought and effort of the wire and cable industry and, in conclusion, the author wishes once more to express his sincere appreciation to all those whose efforts were instrumental in making this paper possible.

Mechanical Uniformity of Paper-Insulated Cables

Discussion and authors' closure of a paper by K. S. Wyatt, D. L. Smart, and J. M. Reynar published on pages 141–54 of this volume (March section) and presented for oral discussion at the cables and research session of the winter convention, New York, N. Y., January 27, 1938.

W. A. Del Mar (Habirshaw Cable and Wire Corporation, Yonkers, N. Y.): Once again we are indebted to the Detroit Edison Company engineers for useful tools to assist in cable research. They have given us the radial power factor test, the hydrophil test, and now they give us the torsional penetrometer and the styrene wafer. We manufacturers are grateful for these means to assist us in giving to the industry the best cable that science and skill can produce.

The authors are undoubtedly right in looking for improvement in cable performance by the perfecting of mechanical uniformity. This goal has been the aim of cable manufacturers for many years, as evidenced by the improvements in taping and cabling machines, the use of caterpillar take-offs, pre-spiralled sectors, humidity control, and so forth. I doubt whether the styrene wafer will add anything to the knowledge which is the basis of these developments. So far, the styrene wafer has revealed nothing of that kind which was not known to the machine designers.

It is rather in more fundamental research that the styrene wafer will prove its usefulness. It has shown, for instance, that the paper tapes are practically tangential to round strands, thereby substantiating one of the claims made for the compact type of conductor, and it has definitely revealed the voids created by load cycles, both of which features in the past have been obscured by burring of the paper in cutting sections.

The number of inherent insulation failures in cables is now very small. If totally eliminated, the effect on the total outages would scarcely be noticed. If we leave out failures due to improper design of vertical cable ends, it is probable that a large proportion of the very small residual total of inherent insulation failures would be found to be due to unavoidable defects caused by the inherent fallibility of men and machines, rather than to any general condition susceptible of discovery by means of wafers. This is not to be interpreted as meaning that, in my opinion, cables are not susceptible of improvement. We shall have thinner walls, greater dielectric strength, greater power-factor stability, better sheaths, and many other improvements. I merely say that such improvements in mechanical structure of insulation as may be made in the future, will not materially reduce outages of cable.

It is, of course, in three-conductor, and particularly sector cables, that there is the most room for mechanical improvement, but life tests show that these cables are surprisingly close in quality to the single-conductor cables. Furthermore, the most carefully made laboratory-built cables are just as variable in their performance in load cycle tests, as the ordinary factory-built cables.

Inventors, realizing this condition, have sought to extend the voltage limits of cable, not by insisting upon impossible perfection, but by the more practical plan of accepting imperfections as inevitable and devising means to make them harmless. Thus Emanueli covers a multitude of sins by using a thin oil that maintains perfect impregnation, and both Bennett and Hochstadter effect the same result by putting the insulation under pressure. It seems to me that the authors of this paper are like people who insist on geometrically plane roads for safe and comfortable motoring, while others, like myself, prefer to have good tires, knowning that occasional road bumps are inevitable.

Referring to the curves of tape tension, the authors assume that uniformly high tension is the desideratum. It is not clear why they should hold this view as no evidence is given to support it.

I have seen cables with the most visibly nonuniform tape tension, that were perfectly satisfactory, and uniformly tight cables that failed. The tension of tapes is not a matter that can be considered by itself; the best tensions are tied up with manufacturing equipment and procedures, bending tests, oil viscosity, and operating temperatures.

We shall be better equipped to study this subject with the new tools the authors have given us, but it will take time to reach sound conclusions with them.

If the wafer method is to prove of great value, it will have to be greatly simplified, preferably along lines of inspection by reflected, rather than transmitted light, so as to eliminate the time and effort expended on making thin cuts.

The authors' study of tape staggering, is very much along the lines of a study I made in February 1930. Quoting from the unpublished report of that work:

> "Twelve cases have been considered with staggerings in stages of $^1/_{32}$ inch from cable *A* with tapes staggered the maximum amount to cable *K*, with tapes staggered the minimum amount. . . . It will be noted that they differ in two respects:
>
> 1. The recurrence of superimposed channels.
> 2. The length of the leakage path from conductor to sheath assuming the channels ionized.
>
> "The recurrence of superimposed channels weakens the cable to radial stresses and a short leakage path weakens it to tangential stresses. Thus there are two potential paths of failure in parallel and on voltage-time test, the cable will fail along the weaker one."

A summary of the study of these cables follows:

The radial dielectric path is about 50 times as strong as the tangential path. Hence to compare these cables, we divide the length of the tangential path by 50 and compare it with the paper

Table I

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Overlay†		Superimposed Radial Channels*		Shortest Leakage Path**	
Cable	32d's Inch	Per Cent	Per Cent	Mils of Paper	32d's Inch	Mils ÷ 50
A	10	43	50	110	430	269
B	9	39	14	189	429	268
C	8	35	20	176	386	242
D	7	30	33	147	343	215
E	6	26	10	198	300	187
F	5	22	25	165	257	161
G	4	17	11	196	214	134
H	3	13	17	183	171	107
I	2	9	12	192	128	80
J	1	4	9	200	85	53
K	0	0	4	210	0	0

* This includes channels with edges aligned, as well as those squarely superimposed.

** Paper thickness neglected.

† Tape is assumed $^{23}/_{32}$ inch wide, with $^1/_{32}$ inch gap, so that maximum possible overlay is $^{10}/_{32}$ inch or $\frac{10 \times 100}{23} = 43^1/_2$ per cent. The per cent overlays are rounded to the nearest integer.

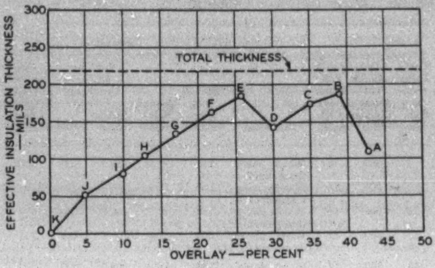


Figure 1

thickness of the radial path, as in columns 7 and 5 of the accompanying table I.

If we plot the lesser of these thicknesses (from columns 5 or 7) against overlay (column 3), we obtain the curve shown in figure 1 of this discussion from which it will be noted that there are substantially equal peaks at 26 per cent and 39 per cent overlay. The 26 per cent peak, however, gives better results due to the better bending characteristics of the cable.

Attempts were made to check curve *A* by voltage-time tests on specially made cable samples and while cables coming under the classifications of *A*, *H*, *I*, *J*, and *K* were definitely poor and cables *G* to *B*, definitely good, it was not possible to correlate the latter with the curve in figure 1, although the general trend was toward a maximum dielectric strength at *E* and *F*, rather than at *B*.

H. B. Dwight (Massachusetts Institute of Technology, Cambridge): May I ask if the mechanical tests described, particularly the test by making thin wafers showing the cross section, have been applied to large generator coils, and has information of interest been obtained?

R. J. Wiseman (The Okonite-Callender Cable Company, Inc., Passaic, N. J.): This paper again reports to us the valuable research work being carried out by the Detroit Edison Company in their desire to bring to our attention, tools or test methods which may be of help in our desire to improve the quality of our cables. However, in reporting the results of these test methods and the conclusions to be drawn from them, one must go slowly and cautiously because it is necessary to consider all the whys or wherefores that certain things are done, which ordinarily may be considered as a defect or shortcoming. For example: let us take the penetrometer and torsion tests which are supposed to indicate the degree of hardness tightness, or firmness of the insulation and the opinion is expressed that we should get a straight line plot of depth of penetration (figure 7) across the insulation. I am not so sure that we should get the same value for across the insulation. Perhaps a decreasing value as we go from the conductor outward or even an increasing value is better. We know in the taping operation that we have to adjust our tensions to prevent pinching or wrinkling of the inner tapes. Do we want a tightly wrapped cable which means a low value for penetration? If we do, we will get very little oil between the layers of the paper and this is as essential to us as oil in the paper itself is necessary if we hope to get good electrical characteristics. If too tightly wrapped we will not get good oil movement radially. If the paper is to loosely wrapped we will get drainage and voids. Therefore, we look for that wrapping which will give a well and uniformly saturated cable possessing low original ionization and high dielectric strength.

The development of the cable wafer is going to be very helpful to the cable manufacturer, as it gives him another tool for checking the mechanical setup of the cable. Here also, we must be careful about interpreting results. It does not necessarily follow that "comet's tails" unless distinctly more pronounced on one side than the other are due to the twisting of the conductor. There really is a comet's tail on both sides in the filler space. It can be due to the shape of the insulated conductor. A round conductor would show a longer comet's tail than a sector-shaped conductor. Also, it can be due to the use of too much filler material in order to get compactness and some of the filler will ride over the back of the conductor. If there is filler material between the conductors it is also due to too much filler and being forced into the space between the conductors. This is not serious as the filler is under high compression caused by the cabling operation.

Sometimes I wonder if we want the mechanical perfection that we hear about. We start with a good mechanical make-up in the factory and then as the authors say, we reel and unreel it, draw it into a duct, shape the ends around the manhole for splicing and finally it goes through the expansion and contraction due to loading. We upset the position of the papers and the oil moves but not entirely reversibly as in the case of oil-filled cables. If the tapes are wrapped tightly, we have a small amount of oil between layers. If too loose, we get drainage and poor electrical characteristics. If longitudinal movement of the cable is retarded, we get a forced radial expansion of the insulated conductors which will injure the insulation. A great deal is dependent on how a cable is handled and installed; in other words, the utility also has a responsibility where the cable leaves the factory in a good mechanical condition.

The oval type of cable overcomes some of the operating effects, but not installation. With this shape of conductor we get the paper along the minor axis stretching the necessary amount to take up the expansion of the oil and on cooling, forces the oil back again, thereby maintaining uniform saturation all the time. We also are able to hold down the internal pressure which develops during heating as well as prevent the cable from going to a high vacuum on cooling.

I am not able to go along with the idea that pretwisting of the conductors will eliminate the cabling problem. Figure 5, which is taken as an example of uniform insulation, does not have pretwisted conductors. We have made cables both ways and find it is not necessary to pretwist.

In taping, we not only look for the longest leakage path from conductor to sheath, but also the most effective use of layers of paper free of gaps. According to the table on page 151, one gets the impression that a 50-50 overlay is best. It is the worst, because we have only half the number of layers of paper effective due to the gaps lining up across the insulation. A 35-65 overlay is, on the average, about the best.

Let us not get the impression from the pictures shown in the paper that the cable manufacturers are sadly deficient in their ability to put out a quality product. If we did, the utilities would not be able to carry the loads at the voltages they do today and instead of the inherent failures of cables due to insulation being about ten per cent of the total failures, they might easily be over 50 per cent. We are constantly striving for both mechanical and electrical perfection, but there is an economic limit we can go to in view of the type of cable and how it is used. The chances of a large reduction in wall thickness are not promising in the near future.

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): I ended my study of this interesting paper with mixed feelings and, as a result, my position is somewhat between that of the authors and that of some of the discussers. As usual and as indicated in the paper, we have co-operated and we have for several months been carrying on investigations as the result of the information first brought to our attention last spring by Mr. Wyatt.

The new tools developed by the authors are being used by many manufacturers and some utilities and should be of material assistance in improving the quality of paper-insulated cables. It remains to be seen, however, whether the use of such tools will eventually make feasible the radical reductions in the thicknesses of insulation of solid-type cables as prophesied by the authors. I will have to admit that as far as belted-type three-conductor 13-kv cable and single-conductor 69-kv solid-type cable are concerned, I do not see how improvements in the mechanical uniformity of the insulation will alone result in reductions in insulation thickness such as 30 or 40 per cent as implied by the authors.

Even though the failures due to insulation in cables made in the past 10 or 12 years are small in proportion to the total failures in underground systems, I do not agree with the implication of the remarks of one of the other discussers that there is practically nothing much more to be done as far as the insulation is concerned. There is still present the economic pressure to reduce costs, and one of the important ways of accomplishing this is to reduce the insulation thickness with safety.

As to the statement in the third paragraph of the introduction that one finds little or no information in the literature concerning compactness and uniformity of layer structure of insulation, I should like to point out that there was a tremendous amount of correspondence and oral discussion in the industry on the mechanical properties of insulation some 12 or 15 years ago. As a result, the American manufacturers practically eliminated the severe wrinkles of the insulation of multiple-conductor cables with sector-shaped conductors and the improper registrations of several tapes one on top of another. From time to time they have made further improvements in mechanical uniformity and I know of two important changes started in 1936. In addition, there are still arguments as to just how the tension and compactness of insulation should be. These tools of the authors should help settle these arguments.

There seems to be an idea in the paper that the effectiveness of insulation is reduced in exact proportion to the number of gaps that are in a straight line between any two electrodes. This implication is unwarranted.

As a general proposition, improvements in mechanical uniformity should apparently be of most benefit for three-conductor solid-type shielded cable with sector-shaped conductors and rated at 25 or 35 kv, the benefits decreasing down to the least possible benefit with oil-filled single-conductor cable.

C. L. Dawes (Harvard University, Cambridge, Mass.): The authors describe a process in which the paper tapes and the fillers of impregnated-paper cables become solidly embedded in their natural positions in transparent glass-like styrene. Thin cross sections or wafers permit a visual study of the mechanical structure of the cable, showing clearly any wrinkles, voids, butt spaces and the like which may be present. This process which involves in a high degree both chemical and physical technique gives engineers another method by which to eliminate the factors which are responsible for the all too frequent cable failures. The authors should be complimented on their important contribution to the art.

From many observations of cables subject to service conditions and accelerated life tests, I am in agreement with the authors' conclusion that the presence of voids, whether due to loose wrapping of the tapes or to butt spaces, does reduce materially the dielectric strength of the cable. With the expansion of the cable such as is caused by load cycles, the compound will tend first to vacate the voids because of the lack of capillary action existing in them. A very small amount of incipient ionization immediately produces mechanical impact which further drives the compounds from the voids, thus accentuating their effect. Not only is the potential drop across the voids reduced as stated by the authors, but also longitudinal stresses along the tapes in the neighborhood of the butts are developed. This is due to the fact that the voltage across a butt space is lower than that across the adjacent tapes, causing a voltage difference along the surface of the tapes, and possibly producing longitudinal creepage currents resulting in tacking and coring. (See C. L. Dawes and P. H. Humphries, "Ionization Studies in Paper-Insulated Cables, III," AIEE TRANSACTIONS, 1930, page 766.) This effect would be accentuated where two or more butt spaces are practically superposed, such as is shown in several instances in figure 14. As the authors state, when better precision in applying the tapes is attained, reductions in the thickness of insulation become possible without diminution in dielectric strength.

Investigation has shown that in many cases the thickness of the voids must be extremely small, for the voltage gradients at which ionization begins may be four or five times the ordinary critical gradient for air at atmospheric pressure. As early as 1919 Clark and Shanklin conducted a comprehensive investigation of this phenomenon in cables (AIEE TRANSACTIONS, 1919, page 489), and Dubsky studied the matter for insulations in general (AIEE TRANSACTIONS, 1919, page 537). My own investigations of a number of cables also show such high gradients with correspondingly very thin voids. These cables were never longer than 15 feet, inclusive of guards, so that the internal pressure could not have been far from atmospheric.

Table II

Cable Identification Number	Conductor (Thousands of Circular Mils)	Insulation Thickness (64th Inch)	Insulation	Dielectric Constant	Ionization			Estimated Voids Thickness (Mils)
					Actual Kilovolts	Volts per Mil at Conductor		
						Insulation	Void	
1	500	16	Wood paper Compound	2.96	16.0	82.1	243	3.5
2	500	24	Manila paper Compound	2.96	21.0	78.9	234	3.7
12	500	16	Manila paper Compound	3.36	11.5	59.0	198	5.1
17	300	12	Wood paper Compound	3.24	7.0	47.5	154	7.1
19	300	12	Wood paper Compound	3.49	10.0	63.0	237	3.6
22	600	12	Wood paper Compound	3.20	12.5	79.6	255	3.1
24	600	12	Wood paper Oil	3.46	10.0	63.7	221	4.4

We also found that when impregnating compound was present in the voids, the gradient for initial ionization was practically double that with air alone (see Report of the Committee on Electrical Insulation, National Research Council, 1935, page 26). This increase in dielectric strength is apparently due to the presence of the vapors from the compound. There are little data on the dielectric strength of vapors under the conditions existing within impregnated-paper cable insulation. However, in J. J. Thomson's "Conduction of Electricity Through Gases," volume II, third edition, page 510, a table taken from the data of Bouty shows the dielectric strength of certain organic vapors. The data were obtained at pressures of from 0.0055 to 2 centimeters of mercury, which pressures are much lower than those ordinarily existing within cable insulation. The table does, however, show that the dielectric strength of such vapors is much greater than that of air alone, which confirms our experiments.

In the accompanying table II are a few typical data obtained with actual cables, some of which were new and others which had been in actual service. The estimated thicknesses of the voids were based on the available data for air, since no data for vapor-filled voids, particularly petroleum, of these small thicknesses and also at atmospheric pressure are available. It is very probable that many of the voids are thicker than estimated due to the presence of the vapors.

W. F. Davidson (Consolidated Edison Company of New York, Inc., Brooklyn, N. Y.): It was my good fortune to have followed, from their inception, the studies reported in this paper and I like to think that some of my words of encouragement were a factor in bringing them to a successful conclusion. Lack of factual information is a complete barrier to full engineering development. In the field of cables we have been seriously handicapped by not having exact knowledge of the inner mechanical structure of cables. Now at last it is possible for the cable designer to study the effects of various details of the manufacturing process and the user can determine the effects of installation and operating conditions; it is no longer necessary to guess as to the effects of many of these factors.

When we turn to the authors' interpretation of some of their observations, I am not able to agree with all that they have said. For cables of the 24-kv class, which form the basis for many of the comments, it seems to me that the ill effects of ionization and leakage path have been somewhat overemphasized. Care must be taken lest we allow ourselves to pay too much attention to correcting these defects at the expense of other equally important details. Much progress has been made in improving the quality of insulation but the results, as measured by cable investment, are none too encouraging. One of the most important problems facing the cable engineer during the next few years is reducing costs. To me the only hope of accomplishing this lies in finding ways and means for operating cables at higher temperatures and with a greater range of temperatures. The designer must not be unduly handicapped in his efforts to accomplish this end.

C. J. Beaver (nonmember; W. T. Glover and Company, Ltd., Manchester, England): It is agreed by the authors, and may therefore be postulated for the purpose of the following notes that

(*a*) Ionization is practically the only inherent cause of failure in a reasonably well made dielectric.

(*b*) Ionization conditions are developed in cable dielectrics by physical processes incident to the use or working of the cable.

(*c*) The physical properties of the (solid type) dielectric are deficient for the purpose of preventing such development.

It follows that under reasonable working conditions the electrical life of the dielectric is dependent on its physical properties, and consequently the main question arising from the paper is as to the relative contribution of some lack of mechanical uniformity to the sum total of deficiency in physical properties.

In a solid-type lead-sheathed dielectric subjected to cyclical temperature changes, it is evident that at any temperature below the maximum, gaseous spaces (voids) must exist in the dielectric. The electrical stressing at which ionization will commence in these spaces will vary roughly inversely as the space dimension in the direction of the electric field, for a given set of conditions such as gaseous pressure, type of gas, and so forth.

Thus it is obvious that in a radially stressed solid-type dielectric, the most vulnerable gaseous spaces (voids) will occur in the butt spaces between the edges of adjacent laps of paper, where the radial dimension of the space will be equal to that of the paper thickness.

It is difficult to conceive a dielectric so poor in a mechanical sense that larger radial dimensioned spaces can exist between radially adjacent papers.

Hence it is clear that the stress at which ionization initiates depends on the butt space conditions, and it is this stress which controls the asymptotic or long-life voltage value of the dielectric.

The importance of the length and condition of the leakage paths (zigzag and step) appears to have been highly overemphasized in the paper.

The lengths of these paths and also the physical conditions existing between radially adjacent paper surfaces, do not affect the asymptotic or long-life voltage value of the dielectric, but merely influence the shape of the time-voltage curve above the asymptotic voltage value.

The Robinson theory confirms this as quoted by the authors on page 13: "Tracking along paper surfaces, which is now coming to be accepted as the mechanism of ionization failure, after a core has formed. . . ."

After load cycle testing there will be a general slackness or lack of mechanical pressure between papers in a radial direction, owing to the action of the radial flow of impregnating oil in a zigzag manner between the paper surfaces during the temperature changes.

Although a wafer of such a cable before and after the load cycles shows this difference, the increase in ionization in the latter sample will occur chiefly in the voids in the butt spaces caused by the process of oil expansion and lead sheath distension in an initially well-impregnated dielectric.

It has been proved by the writer in a very comprehensive series of experiments during the development of the gas filled cable (Paper No. 204, "The Gas Filled Cable," C.I.G.R.E. Session 1937, Paris) that electric failure of a cable dielectric invariably occurs through, and initiates in a butt space, and is also dependent on the butt-space thickness.

Under these conditions the most important dielectric feature which should be taken care of in the manufacturing processes is the avoidance of registrations between radially adjacent butt spaces, so that the space dimensions never exceed a single paper thickness.

In this connection special lapping machines (British Patent No. 182,510) are used by a British firm in which the cable is revolved, while the papering heads carrying the paper strip spools are stationary. The paper strips pass through slots in a sheet metal framework fixed to the machine, thus ensuring at all times that each paper strip is applied to the cable at a definite longitudinal position with respect to the adjacent papers. These arrangements also provide easy means of ensuring uniform tension of application of the paper strips.

It may be mentioned that the strips are preimpregnated, so that no question arises as to "conditioning" of the paper, nor of shrinkage wrinkling or other deficiencies arising in the manner referred to in the paper.

There is one other important point which should be watched in the manufacturing process and that is, that the mechanical structure of the cable dielectric should be such that the paper strips are not torn or cracked through during bending of the cable, so that large-radial-dimensioned gaseous spaces are avoided.

In this connection it is very desirable to avoid a 50 per cent overlap owing to the tendency of the paper between two alternate butt spaces to be cracked, and to become torn in consequence.

The authors do not appear to have considered this aspect, which in actual practice—from the installation point of view—is very important.

The authors' extreme optimism for future improvements and cost reduction in solid-type high-voltage cable is in direct conflict with present-day knowledge of the limitations of such dielectrics.

No amount of control of mechanical uniformity of cable manufacture can alter the stark physical facts that in a lead-sheathed cable subjected to temperature cycles, voids or gaseous spaces equal in volume to the oil expansion must exist at the low temperatures, and that the most vulnerable spaces will form in the butt spaces where the radial thickness of void is at a maximum.

The stress rating limitation of the modern solid-type cable dielectric depends upon the following factors:

1. The radial thickness of the butt spaces in turn dependent upon the minimum thickness of paper which it is feasible to use.
2. The maximum temperature to which the cable may be heated.

The above two factors affect directly the intensity of ionization in the butt spaces and set the limit to the permissible stressing, and conversely the loading.

A comparatively slight improvement in the homogeneity or uniformity of a dielectric cannot cancel out the whole physical problem entailed by the lack of reversibility of expansion and contraction effects, leading to ionization; and in face of the well-known necessity of restricting the temperature factor at even 60 kv it would appear quixotic to pursue working voltages up to 100 kv (as mentioned by the authors) with solid-type cable.

The scientific (and commercial) solution of the problem lies in the direction of the abolition of ionization in the dielectric, for example, on the lines of the "gas filled" cable (referred to hereinbefore), in which, without external compensating devices, and without restriction of loading temperature, ionization is completely suppressed to any designed extent, for instance, up to twice working voltage.

The authors' desiderata regarding reduction of insulation thickness and increased duct loading should be much more easily attainable along this line of development in which all constructional factors are capable of being definitely designed, and behavior in use is automatically controlled by reason of the reversible physical properties of the dielectric, preventing any permanent change (leading to ionization) under any working condition of the cable.

C. H. Jolin (nonmember; Merz and McLellan, Newcastle-on-Tyne, England): The description of methods used to investigate the mechanical construction of paper-insulated cables are very interesting. The three mechanical methods, "penetrometer," "push-out," and "torsion" are somewhat crude, in particular, the push-out method where the force required, besides varying with the tightness of the paper, will also depend on the roughness of the paper surface and the consistency of the impregnating compound. The torsion method seems to be useful but it would be an advantage if an illustration could be given of the instrument used.

The wafer method is, I think, the best but it would appear necessary for this test to be carried out by skilled operators in a laboratory and under these conditions it should prove one of the most useful tests devised up to the present time. I think that the test will prove of most value to manufacturers as they will have the necessary facilities for making the tests, from the results of which they will be able to determine the effects of different methods of manufacture.

I gather from the paper that the author is in favor of the insulating space of the cable containing as large a proportion of paper to oil as is possible consistent with the cable being able to pass a reasonable bending test. I am in agreement with this for normal type of cable as it must assist in avoiding trouble due to compound expansion.

This requirement is not so important in other types of cable, for instance oil-filled cable is so designed that any gaps between papers will always be filled with oil and will, therefore, be harmless but even with this type of cable the smaller the amount of oil in the insulation, the smaller the oil reservoir capacity required to allow for oil expansion.

The gas pressure cable is in a different category as in some designs definite gaps are left between papers to allow free access of the gas under pressure to all parts of the insulation.

The author states that it should be possible to so improve the dielectric of the solid cable that it would be suitable for use at 100 kv or over.

I should like to know whether the author would recommend using solid cable at the same maximum dielectric stresses and temperature as is now used for oil-filled and gas pressure cable.

It appears to me that however well the cable is manufactured, some disturbance of the insulation will take place due to reeling and unreeling the cable on drums and handling the cable during installation. The spaces caused by these disturbances will remain in the solid-type cable whereas such spaces in the oil-filled cable and the gas pressure cable would be filled with oil or gas under pressure respectively and would thus be harmless. It would therefore appear that theoretically the oil-filled and gas pressure cables will be better than the solid type.

The illustrations accompanying the paper are interesting and show that delta-shaped-conductor three-core cable cannot be so satisfactorily insulated as a cable with circular conductors. The cable illustrated in figure 6 is so bad that it is hard to believe that any reputable cable manufacturer would turn out such a cable.

There is a point of minor interest I should like to raise with regard to figure 12. The conductor of this cable consists of part normal-size wires, together with a number of smaller sized wires in groups of three. Was there any particular reason for this method of construction?

Martin Hochstadter (Brussels, Belgium): My interest in the development published in this paper has been great for two reasons:

Firstly, because this is real pioneer work opening a new field of insight into questions of outstanding practical importance in electrical cable engineering, a field which curiously enough, has been more or less entirely neglected hitherto. Not only do I see great merit in the very discovery of such a new field and its courageous and successful attack, but also do I believe this to be of a refreshing effect on the minds of our younger generation of cable engineers. Seeing every day nothing but work on details of problems which have already been principally dealt with by others, they might be led to the conclusion that no exploration field was left to them. This paper gives evidence of white patches on the map of our technical knowledge.

Secondly, the author's problem has been occupying me during the greater part of the last ten years. Research on pressure cable was carried out at first with internal pressure, namely, with cables of such construction that the pressure medium was acting in or near the conductor. However, the pressure effect on the dielectric was found to be of rather an imperfect and unreliable nature in all such cases. After much research and speculation into the causes of such unexpected behavior it was recognized that the propagation of the pressure through the dielectric, while being rather complete and instantaneous when external pressure compresses the dielectric, is poor and uncertain when the dielectric expands under the influence of internal pressure.

Pressure in such a semisolid body is propagated from layer to layer. In the case of internal pressure this propagation consists in the elastic stretching of each individual layer. Therefore, it can only take place there where layers touch each other and are under about the same mechanical stress neutrally, that is, before the application of pressure. It is, therefore, evident that a uniform propagation of internal pressure is contingent upon uniformity of the mechanical structure of the dielectric and that the internal pressure may be stopped totally or partly at any point in the dielectric where tightly wound papers are followed by loose ones.

While at the time of which I speak the relation between propagation of internal pressure and mechanical uniformity of the dielectric was made evident, no exact measurements of the latter could be taken and it was not possible to overcome the difficulties for using internal pressure. Therefore, external pressure has been used hitherto for the pressure cable. However, the author's methods for the demonstration and exact measuring of the mechanical uniformity of the dielectric may well lead to such an improvement of the manufacturing methods that a sufficient and controllable mechanical uniformity of the dielectric be obtained for the use of internal pressure in pressure cables.

This is one of the practical possibilities which the author's methods may open in a particular field of cable making besides others for the improvement of electric cables in general.

Hugo Sonnenfeld (nonmember; Bratislava, Czechoslovakia): The present paper completes the impression experts have obtained by its predecessors. The wafer method is certainly contributing to the possibility to have a reliable look into the working of the cable, especially the insulation and the lead sheath.

Even though there are only a few pictures published in the present paper they are sufficient for showing the way to improve cable design. Compound migration being, as is well known, a very important factor, it is evident that the first step for making cable service more reliable than it is now, consists in reducing filler space as far as possible. The general practice to use sector-shaped conductors cannot be considered as the best way. This appears to be quite clear if thorough consideration is given to the wafers shown in the present paper. Filler spaces are at the minimum in the case only that the *theoretically exact sector shape* is used, such as illustrated in figure 4 of the present paper. The photograph of this wafer, however, makes it evident that the insulation is rather a nonuniform one, and this particularly due to the shape of the conductor. Not only is the conductor, on two sides, flat so that its surface does not follow the natural bulge of the paper but the unequal electrical stress on the shoulders on one side and on the sector back on the other as well as on the flat parts on third side, is in no case able to fulfill requirements of high-voltage cables. For these various reasons the theoretical exact sector shape (see figure 4) is not advisable for high-voltage purposes. In the paper the expression "extreme sector" is used for such a shape of conductor, see page 148. We should not forget that this shape of sector is the only one which brings the filler spaces to their minimum.

If, however, the sector shape deviates from the theoretical exact "extreme sector" form, we come to designs such as is illustrated in figure 5. Here the gross irregularities, discovered in figure 4, are absent, the shoulders of the conductor being less sharp and the conductor more able to conform to the natural bulge in the paper. But all this is accompanied by the serious disadvantage of filler spaces evidently becoming larger. For this latter reason such cable does not represent the best possible design.

A round conductor offers little difficulty. It is to be expected that all disadvantages illustrated by figure 4 will not be present if the same machines and the same manufacturing method are applied but with a round or oval-shaped conductor. No doubt, the circular or oval shape is in any respect the best form of the conductor. If small fillers are used, resulting in a triangular shape of the lead sheath (so-called *S.O.* type cable) the reduction of the possibility of compound migration improves the properties of the cable and the cost of the cable containing circular (or oval) conductors is brought down to the cost of the cable containing sector-shaped conductors.

Thus the application of round (or oval) conductors together with the application of the triangular shape of the lead sheath makes available the following advantages of the cable:

The volume of oil compound causing expansion is reduced.

The radial resistance to oil flow is brought down to a minimum.

The thermal resistance is equally brought down to a minimum.

The facility to wrap paper with exact regularity upon the conductor is increased.

No pretwisting is necessary.

K. S. Wyatt, D. L. Smart, and J. M. Reynar: The authors are grateful for the favorable reception accorded the methods proposed for studying the mechanical uniformity of paper-insulated cable. It appears, however, that although the tools themselves have been accepted almost without qualification, the interpretations which were made in terms of design, manufacture, and service life have furnished many points of controversy. The primary purpose of the paper was to offer new test methods for obtaining data which might be of assistance in studying the relative merits of different types of construction and in determining the causes of insulation failure. Since the value of the wafer method depends, as in the case of X-ray photography, on correct interpretation, it appeared advisable to present tentative interpretations of cable wafers as samples of what to look for in the wafers and of how irregularities might be traced to original sources. A certain amount of theory was necessarily involved in these interpretations. Space limitations necessitated a rather cursory treatment of this phase of the paper with the result that many unintended inferences were drawn by readers. In addition, it was only to be expected that, in view of the relative newness and limited use of the method, the interpretations would be questioned as were those of X-ray photographs in the early days of their use. Furthermore, the necessary omission of a large number of illustrations representing reasonably good cable wafers undoubtedly caused an erroneous impression in some minds regarding our estimate of the quality of the average modern cable.

Several discussers point to the apparent futility of producing mechanically perfect insulation in view of the amount of relatively rough handling the cable undergoes during installation. The authors direct attention to the fact that the wafer method now enables us to evaluate the effects of such treatment. Time has permitted only meager investigation of this phase of the problem but, for what they may be worth, the results obtained do not indicate any serious damage to be sustained by a mechanically well-built cable when installed by modern methods.

If it appears, as stated by several discussers, that the paper overemphasized the effects of ionization and attached too much importance to the length and condition of the leakage paths, it is because the facts revealed by a careful study of scores of cable wafers lead, first, to a critical inspection of previously held theories of the mechanism of cable failure and, finally, to a tentative revision of these theories to better fit the observed conditions. Among these conditions were three which were frequently observed and which experimental evidence and theoretical considerations indicate have an important bearing on insulation breakdown. Two of these conditions involve butt-space distribution as it affects effective solid insulation thickness and minimum leakage path, and the third involves thick interlayer spaces caused by loosely wound insulation. An explanation of the part played by these factors in insulation failures would entail much theoretical discussion and would perhaps, be out of place here. The discussion by Professor Dawes, however, may serve to partially explain the phenomena responsible. The following briefly summarizes the conclusions of the authors as a result of their initial investigations:

(*a*) Ionization is held to be the chief cause of inherent insulation failure.

(*b*) The theory of D. M. Robinson that breakdown starts at the conductor and proceeds slowly through the insulation by a coring and tracking mechanism is accepted with reservations as to the importance of voids in the middle of the insulation wall.

(*c*) Conditions conducive to ionization are present in much cable as manufactured and may be aggravated due to service or to improper handling during installation. (Initial voids, even though they be filled with oil and thus do not cause high *initial* ionization, are considered to be conducive to ionization.)

(*d*) The development, during service, of conditions couducive to ionization may be minimized by proper design and manufacture.

(*e*) The inevitable presence of butt spaces at the conductor and other inherent weaknesses in the insulation of solid-type cable are not arguments against the elimination of other defects since many of the latter add to the basic inherent weaknesses to further decrease the over-all insulation efficiency.

(*f*) As the quality of the cable insulation is made more uniform due to precision methods of manufacture, the thickness of the insulation may be reduced because it is no longer necessary to provide a factor of safety to take care of avoidable but uncontrolled weaknesses.

It is seen that we do not agree in toto with Mr. Beaver's postulates (*b*) and (*c*).

The elimination of registrations and also of interlayer gaps is bound up with the taping machinery. We are in agreement with Mr. Beaver as to the advantages of certain types of "lapping machinery," but we must point out that they possess several apparently inherent disadvantages, among which is slowness of taping. If the paper is impregnated prior to taping, the trapping of gas between adjacent layers of insulation is an additional disadvantage. The design of taping machinery plays a most important part in the quality of cable insulation and the wafer method permits a visual study of the effects of the various design factors. For best results it is the belief of the authors that taping machines should have the following characteristics:

1. Direct drive throughout with as few gears as possible, since these introduce backlash.
2. Slow, smooth, but positive starting, without slippage, and brakes which stop the machine smoothly but reasonably quickly.
3. Direct synchronization between capstan and taping heads to eliminate the possibility of backlash or slippage.
4. A tape tension control in which centrifugal effects are absent and in which the tension of the tapes is independent of the amount of tape on the pad.

This is a subject deserving of much space. Suffice it to say here that hydraulic drive and synchronization appear to the authors to offer the best solution of the problem of precision taping at reasonable cost.

Mr. Del Mar and Doctor Wiseman infer that a uniform tape tension from conductor to sheath was recommended. Such an inference was not intended and may have been due to the fact that the torsion curve shown for uniform insulation was fairly level from conductor to sheath. The only relatively uniform cable which we had examined by means of the torsion test at the time the paper was written did not have the drooping characteristic from conductor to sheath which we believe to be desirable from the standpoint of ability to withstand bending without distortion. The same discussers also question the desirability of a high tape tension with a corresponding thin oil film between tapes. We believe, and support is given to this viewpoint by the discussions of Mr. Hochstadter and Mr. Jolin, that this film should be as thin as is consistent with good bending characteristics for a number of reasons. First, less oil is present and void formation is less likely to occur as a result of differential thermal expansion. Obviously tracking in service would be expected to be retarded or possibly eliminated. Second, the thinner the oil film, with other things equal, the less the comet's tail effect produced during cabling. Third, a very thin oil film which will permit successive paper layers to come into practically direct contact permits capillary effects to work to greatest advantage to distribute oil during impregnation, to prevent imigration and drainage during operation, and to return oil to the interior of the structure as the cable cools.

Several discussers have wrongly inferred that the study of cable wafers has led us to recommend a 50-50 overlay. The fact that with such an overlay the effective solid insulation thickness is only 50 per cent, coupled with the observation that due to the superposition of alternate butt-spaces the paper between has a tendency to tear when the cable is bent, would not permit such a recommendation. No attempt has been made by the authors to evaluate the relative importance of effective solid insulation thickness and minimum leakage path as factors in insulation failure for the purpose of selecting the best overlay. For this reason the report of the study of tape staggering made by Mr. Del Mar is particularly interesting. Accepting, for the sake of this discussion, his figure of 50 for the relative strengths of the radial and tangential dielectric paths, we do not believe that figure 1 of the discussion is a correct picture of the variation in effective insulation thickness with overlay. In the first place, it seems apparent that the values given in column 6 of table I should read as follows: 430, 387, 344, 301, 258, 215, 129, 86, 43, and 0. The figures in column 7 should be changed to correspond. Furthermore, if it be assumed, as seems reasonable, that failure may proceed, not only along *either* the radial path *or* the tangential path, but along a combination of *both* types of path, neither column 5 nor column 7 represents the shortest equivalent leakage path for many overlays. For example, with an overlay of $^{9}/_{32}$ inch or 39 per cent, the radial path through a tape to the butt space in the next layer would be less than $^{1}/_{50}$th the length of the tangential path to the same layer, and failure would occur by alternate ruptures of a tape and tracking to a point opposite the nearest butt space in the next layer. The effective insulation thickness would then consist of 110 mils of paper plus, 1,302 mils of leakage path which would total 136 mils equivalent insulation thickness. This is less than the figures in either column 5 or 7 for this overlay. Similar calculations for $^{8}/_{32}$-inch and $^{6}/_{32}$-inch overlays, when combined with the changes in column 7 mentioned previously, result in the modified curve shown in figure 2. This curve is markedly different from that shown in Mr. Del Mar's figure 1 and, incidentally, appears to fit the results of his voltage-time tests more closely, although we cannot agree with Mr. Del Mar's and Mr. Beaver's use of the results of voltage-time tests as an indication of the behavior of cable in service.

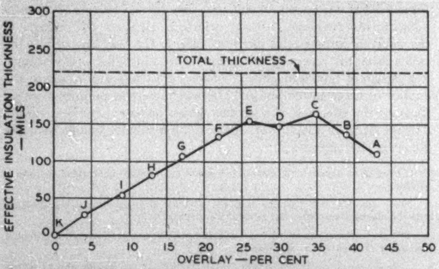


Figure 2. The effect of overlay on the effective insulation thickness

Mr. Jolin points out that difficulties are encountered in insulating sector-type cable and Mr. Sonnenfeld discusses the effect of the conductor shape on the ease with which a uniform insulation may be obtained, and on the filler-space volume. These are questions which the wafer method of examination should be of inestimable value in answering.

We agree with the discussers that much can and should be done to simplify the making of wafers. In his oral discussion, Mr. Meyerhoff described one method of simplifying the preparation of wafers from the styrenated cable sample. In this method, very thin wafers were cut with a band saw without finishing the faces on a lathe. The opacity of the sawn faces was eliminated by coating the wafer with a thin cable oil and mounting it between flat glass plates, care being taken not to wipe off all the surplus oil from the wafer. The development of a type of styrene or other material having a lower polymerization temperature and a shorter setting time would prove to be an important step. It is also suggested that the time of extraction might be shortened by permitting the styrene to trickle through the insulation continuously rather than to wash out the oil by the cyclic bath method described in the paper.

The apparatus originally used for making the penetrometer and torsion tests was admittedly crude. Since the paper was presented, however, the instruments have been considerably refined and combined into a single instrument which will be marketed by a well-known scientific instrument manufacturer. An article on the use of the compactness tester is being prepared, and it is expected that it will be published shortly in the technical literature.

In answer to Mr. Jolin, the authors are unable to give a reason for the design of the conductor shown in figure 12 of the paper. This cable was manufactured over eight years ago. It is believed that there was no intention of using the space between the strands as an oil duct and it is apparent that oil expansion would be more severe because of the large oil volume.

Mr. Dwight's question will have to be answered by others. So far as the authors know, such an application of the wafer method has not been made. The method has been used to advantage on trunk telephone cables in several countries. We believe it has a useful application in the study of many laminated dielectrics where uniformity of structure is of importance.

Our acknowledgment unintentionally omitted a word of appreciation for the support we received from engineers of other utilities. Possibly without this encouragement, this research could not have been carried to successful fruition.

System Planning and Operation for Voltage Control

By T. J. BROSNAN
MEMBER AIEE

THE two criteria of the performance of any system for the delivery of electric energy are, first, its dependability as measured by continuity of service, and second, the quality of the service as measured primarily by the ability of the system to keep the deviations from normal voltage within acceptable limits. The purpose of this paper is to consider the influence of the second of these two factors, namely voltage regulation, on the design and operation of the system.

The permissible limits of voltage regulation vary for different types of load. It is generally regarded as good practice to keep the voltage at the service switches of residential and commercial customers in an urban area within three per cent above or below the normal level. For the same type of customers in rural areas, it is economically impractical to limit the deviations to less than five per cent above or below normal voltage. Industrial customers in general do not require such close regulation.

The following sections of this paper will review the influence of these limitations on the design and operation of the various parts of the system.

General System

In a typical modern distribution system the same subtransmission circuits serve different types of substations as well as transformer vaults in the low-voltage network system.[1] Provision is made for control of the voltage at the bus of the generating or terminal station but the difference in time characteristics of the various loads and the different electrical characteristics of the distribution systems make it essential to supplement this basic regulation with other methods of voltage control.

The generating-station bus voltage is regulated to meet the requirements of the load supplied through the low-voltage network system and provision is made for independent voltage control at the substations supplying residential, small commercial, and small industrial loads. Generally the large industrial loads do not require such close voltage regulation and it is unnecessary to add supplementary voltage control in the substations supplying such loads.

Substations

Automatic substation equipment has made it economically possible to use smaller substations which can be located relatively close together and near large concentrated loads.[2] The close spacing of substations results in short distribution feeders which in turn permit automatic regulation of the substation bus voltage rather than independent regulation of the voltage of each feeder.

In regulating the voltage of the substation bus, it is desirable to introduce a "rising characteristic" with increasing load in order to offset the increasing voltage drop in the distribution system beyond the substation. The "line drop compensator" which has been used for years for this purpose with individual feeder regulators can still be used with the equipment for bus regulation. The setting of "line drop compensators" even for individual feeder regulation involves a compromise between the customers closest electrically to the substation and those farthest removed electrically. With short feeders and bus regulation the problem is no more difficult. The "line drop compensators" are adjusted to give the best average regulation for all of the customers on one small substation instead of one long feeder. The one satisfactory method of keeping the "line drop compensators" properly adjusted in either case is by periodic checking of service voltages of customers close to and remote from the substation during the heavy load periods.

The actual line-to-neutral voltages on the busses of the small automatic substations in Buffalo range from 2,420 at periods of very light load to 2,500 at the time of heavy load. In terms of service voltage this is a range of 121 volts to 125 volts.

Distribution Feeders

There may be a few feeders where the voltage along the entire length of the feeder cannot be kept within the desired limits by means of regulation of the substation bus voltage. For such cases there are several types of feeder-voltage regulating equipment available for pole-top mounting. These can be inserted in any part of the feeder where they will be of greatest value in supplementing bus regulation.

It is evident from the foregoing that it is important to keep the voltage drop in all feeders supplied from one substation within a limited range. The total drop in voltage along a feeder is a function of its length, the magnitude and distribution of its load, its voltage rating, and the size and spacing of the conductors making up the feeder. The proper balance of these factors varies for different load areas. The following combination has been found satisfactory for the urban Buffalo area.

(*a*) All feeders are rated 4,100 volts, three phase, four wire. This system has the advantage of a voltage rating that is satisfactorily high for urban conditions and yet requires only 2,400-volt distribution transformers and incidental equipment. It has its main application in urban areas where all main portions of the feeders are three phase and all single-phase branches are short.

(*b*) The main "trunk" portion of every feeder is made up of 4/0 conductors. This

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[1] For all numbered references, see list at end of paper.

uniformity in conductor size facilitates rearrangement of feeders and simplifies the transferring of load from one feeder to another under emergency conditions.

(*c*) The load on each feeder is adjusted according to the length of the feeder so as to make the load multiplied by the distance to the load center as near a constant as possible for all the feeders out of one substation. Under the conditions prevailing in the Buffalo area, the maximum normal feeder loads vary from 800 kva to 1,200 kva.

For suburban and rural areas the 4,800-volt three-phase three-wire delta-connected feeders are more satisfactory, than the 4,100-volt star-connected feeders because the single-phase branches form such a large part of rural feeders and it is very desirable to have the higher phase-to-phase voltage for these long branches.

Load balance between phases is very important on all three-phase feeders but particularly on three-phase four-wire star-connected feeders. This load balance should be maintained along the feeder as well as at the substations. The following system has been established for the control of this load balance in the Buffalo district.

A single-line diagram for each feeder indicates the phase to which each distribution transformer is connected and there is marked on this diagram the theoretical load balance at key points along the feeder. Whenever a new distribution transformer is added, the feeder diagram is used to decide the phase to which the new transformer will be connected. In this way all routine work tends to improve feeder balance. The theoretical over-all load balance of a feeder as taken from the diagram is checked periodically by the readings of the maximum-demand meters on each phase of each feeder at the substation.

Transformers, Secondaries, and Services

Any condition in a primary feeder resulting in poor voltage is serious because of the number of customers involved. However, as a result of the care that is taken in the design and operation of distribution feeders in a modern system, they are seldom the cause of poor voltage. The portion of the system that provides the real problem in voltage control is that made up of the distribution transformers, secondaries, and services because the voltage drops in these parts of the system are generally greater than in the short primary feeders in a modern distribution system.

There are so many distribution transformers, secondaries, and services in a distribution system that it is apparent each of these units cannot be given continuous individual attention. Yet, since any one unit when improperly designed, applied, or maintained may be the cause of poor voltage, it is evident that a careful set of standard designs and system of routine attention must be established to insure proper voltage control.

The importance of transformers, secondaries, and services in the voltage problem is further emphasized by a consideration of voltage complaints in the Buffalo district during 1937. Table I shows the number of cases of unsatisfactory voltage caused by each part of the distribution system.

Table I. Number of Justifiable Complaints by Cause During 1937

Portion of System Responsible for Poor Voltage	Number of Complaints	Per Cent of Total
Primary feeders	10	12.3
Transformers and secondaries	27	33.4
Services and customer wiring	30	37.0
Utilization equipment (oil burners, blower motors, flashing signs, etc.)	14	17.3
Total	81	100

The first important step to be taken in guarding against voltage troubles in the secondary portion of the distribution system is to prepare standard transformer-secondary designs for all conditions usually encountered in the field. Such standards must be based not only on proper voltage control but also on considerations of economy, ease of change when necessitated by load growth, and similar factors.

Such a set of standards prepared for the Buffalo district calls for number 2 wire for about 90 per cent of the secondaries. This means that growth of load in the great majority of cases can be accommodated by changing the size or spacing of transformers with no change of secondary wire. These standards are all based on keeping the voltage drop in the secondary within 1.5 per cent with full load on the transformer.

It is equally important to prepare standards for new service entrances to be installed by customers and corresponding service drops to be installed by the power company. This should include guides for the replacement of existing services when necessitated by load increases.

Much could be said about various methods for keeping continuous records for load so as to know when to change transformers. It is mentioned here because whatever system is adopted for field checking transformer loads should also check load balance on the three wire secondaries which is a very important factor in proper voltage control.

Utilization Equipment

Two characteristics of the new load being added to distribution systems today are disturbing to those engineers responsible for proper regulation of service voltages. The first is the low-power-factor characteristic of a large portion of the load that is being added. The second is the tendency of many of the new utilization devices to introduce sudden load changes which, if of sufficient magnitude and of frequent enough occurrence, may result in objectionable flicker. These statements refer particularly to automatic house-heating equipment, air-conditioning equipment, refrigerators, small arc welders, and similar types of apparatus.

It is becoming increasingly important, as more of this equipment is being added to the systems, that the electrical manufacturers, electrical contractors, and electric power companies co-operate to insure the proper operation of this equipment without interference with other loads and at the lowest over-all cost to the industry.

One of the most important steps in such a co-operative program should be the introduction by all electric power companies of reasonable motor-starting-current limitations based on the true capacity of the distribution systems to accommodate such starting currents without serious voltage disturbance in place of limitations based on the size of the motor.[3]

Conclusion

While voltage regulation is the most important criterion of the quality of the service rendered by an electric delivery system, very few customers purchasing electric service have any means for measuring this quality other than by comparison of present with past performance of their utilization equipment. If their lamps seem to burn out at a faster rate than normal, they conclude that the voltage is too high. If the lamps burn with less brilliance than normal or the elements of the range heat at less than normal rate, they conclude that the voltage is too low. This introduces an important psychological factor. For example, an analysis of the voltage com-

Regenerative Tension Control for Paper Winders

By H. W. ROGERS
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THE WINDER is one of the more important machines in the paper industry. In general, every paper machine has a winder intimately associated with it and there are many others used in the finishing room.

It is not strictly a part of the paper machine, but is very closely allied to it since its function is to take the reel of paper from the paper machine, trim it, slit it to the proper width, and rewind it into smooth, even density rolls of the proper size for shipment; and since it starts and stops frequently to change rolls or make splices, it must operate at speeds greatly in excess of the paper-machine speed to keep up with production.

The Winder

It consists essentially of an unwinding stand on which the roll of paper to be rewound is mounted, several free-running steel rolls over which the paper passes, a slitter, a spreader bar, and a pair of winding drums on which the roll of rewound paper rides. See figure 1.

Power is applied to the winding drums, which in many instances have been either geared or belted together in such a manner that the leading roll travels slightly faster than the trailing roll. The object of this method of drive is to wind a uniformly tight roll of proper density.

More recently there has been a definite trend toward using a separate motor on each winding roll, with provision for adjusting the relative load on each motor, thus insuring the proper winding conditions at all times for any kind of paper.

To operate successfully, and wind smooth even density rolls, there must be tension in the sheet of paper. The object of the tension and the spreader bar is two-fold: first, to remove all wrinkles from the sheet, and second, to wind a tight roll of even density. Heretofore, tension has been obtained by means of a manually adjusted band brake on the unwinding roll stand, and while it is possible to automatically control the brake, it is difficult to hold constant tension with such a device since the unwinding roll may vary in diameter as much as five to one during the operation. In a large majority of cases, this band brake is manually operated and in such instances it is impossible to hold anything that approaches constant tension, since the operator has no real indication of the tension in the sheet.

Devices of this type are, of course, in operation, and will continue on the smaller machines, possibly on the larger machines winding light-weight papers, but the trend toward wider, high-speed machines and heavier papers has so increased the power involved that the dissipation of heat in a mechanical brake is a real problem and the maintenance may become a major item of expense.

Characteristics of Paper

The situation may be more readily appreciated if we go back to fundamentals and discover some of the characteristics of paper. Figure 2 shows the relation between per cent stretch and pounds pull per inch width in per cent of ream weight, (500 sheets, 24 by 36 inches) for uncalendered kraft paper, and while this curve may vary for other grades of paper,

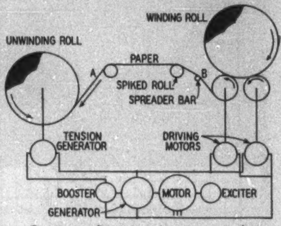

Figure 1. Diagram of a paper winder

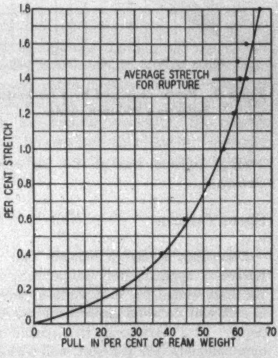

Figure 2. Characteristic stretch curve for uncalendered kraft paper

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plaints received from customers in the Buffalo district during 1937 shows that only 3.5 per cent of the high voltage complaints were justified whereas 80 per cent of the low voltage and flickering voltage complaints were justified.

There has been a very interesting example in the Buffalo district of the effect that major system changes can have on the psychological factor. During the years 1930 and 1931, the supply to all of the residential load and most of the commercial load in the Buffalo district was changed from 25 cycles to 60 cycles. A complete new distribution system was installed and the normal service voltage was raised from 115 to 120 volts as a part of the same operation. The number of voltage complaints rose steadily during this major change reaching a peak during the year 1932 at about double the number recorded in any year prior to the start of the change. There has been an appreciable decrease each year since 1932 so that the total number of complaints during 1937 was only one-quarter of the 1932 peak. The interesting feature is that during this entire period there has been very little change in the annual number of complaints found to be justified.

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2. 100,000 KW DISTRIBUTION SYSTEM AS A SINGLE PROJECT, R. T. Henry. *Electrical World*, July 25, 1931.

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it is undoubtedly characteristic of all grades of paper. It is based on some 18 samples varying in weight from 25 pounds to 70 pounds. A few samples ruptured at 1.8 per cent stretch, none ruptured at less than 1.0 per cent stretch and the average rupture was at 1.4 per cent stretch.

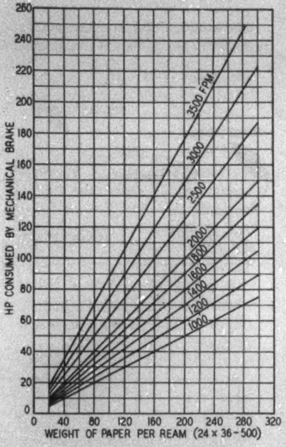

Figure 3. Power consumed by a mechanical brake on a 200-inch winder

Sheet Tension

Experience seems to indicate that a tension of approximately 4.0 per cent of the ream weight per inch width of sheet is sufficient to wind properly a roll of paper. This corresponds to a stretch of approximately 0.02 per cent, well within the rupture point, and up to the present time seems to have met all operating requirements.

Losses Due to Tension

This tension, expressed in pounds pull per inch width of sheet may not appear imposing, but when combined with a wide sheet and high speed, it presents a materially different picture. Figure 3 shows the power consumed by a mechanical brake in maintaining proper sheet tension on a 200-inch-width winder when operating at various speeds with different weights of paper. It also approximates the amount of power returned to the line on a regenerative electric tension-control system.

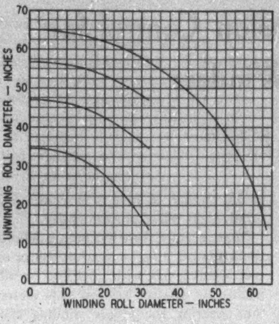

Figure 4. Corresponding diameters of winding and unwinding rolls for paper winders

Regenerative Tension Control

The idea of regenerative tension control is not new, but the enormous expansion of the paper industry, the building of many new kraft mills in the South and the more general use of this kind of paper for containers and innumerable other uses has fostered many changes and developments in this type of equipment to meet the more exacting conditions of operation.

Operating Requirements

The operation of a winder requires a smooth, easy start, continuous slow speed while paper is being threaded through the machine and for making splices, smooth acceleration from standstill to maximum speed, rapid smooth deceleration, and at times, a quick stop.

With the mechanical tension control the operator must rotate the unwinding roll by hand during the threading operation. This is not objectionable on the smaller machines, but the trend toward wider machines and much larger diameter paper rolls places a burden on the operator as some of these rolls weigh six or seven tons. Provision should therefore be made to turn these rolls over by power during the threading operation.

Furthermore, these heavy rolls of paper have an enormous amount of energy when the winder is running at full speed, and provision for quick stopping is essential.

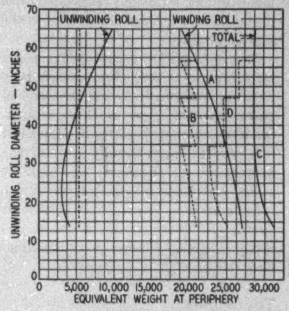

Figure 5. WR² of winder reduced to equivalent weight at the periphery

The Modern Winder Drive

One of the modern high-speed winder drives is of the Ward Leonard type with two compound-wound d-c driving motors, a d-c tension generator, a motor generator with booster unit and exciter, the necessary control and a current regulator. The control of the winder speed

Table I. Acceleration Data for a 228-Inch 3,000-Foot-Per-Minute Winder

	Total Machine			Unwinding Roll		
Diameter of Unwinding Roll (Inches)	Equivalent Weight (Pounds)*	Force to Accelerate	Horsepower Torque to Accelerate One Roll	Equivalent Weight (Pounds)	Force to Accelerate	Horsepower Torque to Accelerate
65	28,875	2,245	204	9,750	758	69
60	28,800	2,240	203.5	8,500	641	58.2
55	28,750	2,235	203	7,375	572	52
50	28,750	2,235	203	6,250	486	44.2
45	28,750	2,235	203	5,300	412	37.5
40	28,750	2,235	203	4,500	350	31.8
35	28,750	2,235	203	3,800	295	26.8
30	28,875	2,245	204	3,375	262	23.8
25	29,250	2,270	206.5	3,125	243	22.1
20	29,750	2,310	210	3,125	243	22.1
13.875	31,250	2,430	221	4,125	321	29.1

* Winding one large roll of paper from one 65-inch roll.

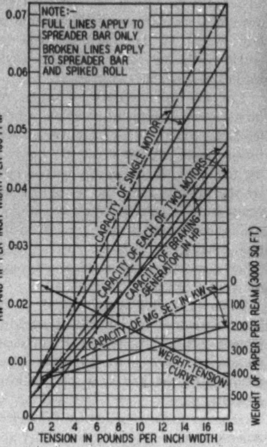

Figure 6. Power curves for selecting equipment capacity

Figure 7. 234-inch, two-drum winder driven by an adjustable generator voltage winder drive with regenerative braking tension control. Winding speed 3,000 feet per minute

is entirely by generator-field adjustment, and there is means of adjusting the load distribution between the two driving motors to insure a tightly wound roll with any grade of paper.

The Tension Generator

The braking generator is very similar to the adjustable-speed d-c motor with a speed range by field control of four or five to one depending on the maximum and minimum diameter of the unwinding roll. The four-to-one range is, however, less expensive and should be adhered to preferably. This can be accomplished very readily by increasing the core diameter of the unwinding roll a few inches, as it does not affect the maximum diameter materially, nor the amount of paper in the roll.

The Booster

The booster supplies the *IR* drop in the braking generator circuit and makes sheet tension immediately available when the winder is started up. Constant tension means constant horsepower or constant current for any given speed, but since with constant tension the horsepower varies directly with the speed, it follows that if constant current is held in the braking generator, the tension will be constant regardless of the winder speed or the diameter of the unwinding roll.

Table II. Power Data for Winders

*Power Data—With Spreader Bar Only

Tension at "A"**	Tension at "B"**	Friction Load	Tension Load	Total Load	Horsepower Returned	Main Generator Capacity	Tension Generator Capacity
0	0	0.00520	0.00000	0.00520	0.00000	0.00430	0.00000
2	2.163	0.00570	0.00606	0.01176	0.00575	0.00570	0.00517
4	4.326	0.00619	0.01210	0.01830	0.01150	0.00660	0.01035
6	6.489	0.00668	0.01820	0.02488	0.01730	0.00770	0.01550
8	8.652	0.00718	0.02420	0.03138	0.02300	0.00880	0.02070
10	10.815	0.00767	0.03030	0.03800	0.02880	0.00992	0.02600
12	12.978	0.00816	0.03640	0.04456	0.03460	0.01120	0.03120
14	15.141	0.00866	0.04240	0.05106	0.04030	0.01220	0.03630

*Power Data—With Spiked Roll and Spreader Bar

Tension at "A"**	Tension at "B"**	Friction Load	Tension Load	Total Load	Horsepower Returned	Main Generator Capacity	Tension Generator Capacity
0	0	0.00520	0.00000	0.00520	0.00000	0.00430	0.00000
2	2.489	0.00661	0.00606	0.01267	0.00575	0.00620	0.00517
4	4.979	0.00802	0.01210	0.02012	0.01150	0.00807	0.01035
6	7.469	0.00943	0.01820	0.02763	0.01730	0.01000	0.01550
8	9.958	0.01084	0.02420	0.03504	0.02300	0.01190	0.02070
10	12.611	0.01224	0.03030	0.04274	0.02880	0.01395	0.02600
12	14.938	0.01366	0.03640	0.05006	0.03460	0.01560	0.03120
14	17.429	0.01506	0.04240	0.05746	0.04030	0.01750	0.03630

* Power data is horsepower per inch width per 100 feet per minute except main generator which is kw.
** Tension is in pounds per inch width.

The Regulator

It is the function of the regulator to hold this current constant at whatever value it is set, depending upon the weight of paper being wound. Provision is made, however, for operating the braking generator as a motor during the threading operation, thus relieving the operator of considerable heavy labor.

Acceleration and Deceleration

Mention has already been made of the heavy inertia involved in large rolls of paper and since the sheet must bring the unwinding roll up to speed and involves considerable tension or pull, the regulator automatically eases off the current held by the braking generator to avoid breaking the sheet and when the winder is up to speed, it again holds the correct current. During the slowing down operation, the reverse is true; the regulator automatically holds a higher current to keep the sheet tight and prevent it from wrinkling.

Figure 8. Adjustable generator voltage winder drive with regenerative braking tension control. Two 75-horsepower 1,750-rpm 230-volt driving motors. 50–60-horsepower 250–1,000-rpm 230-volt braking generator

As an illustration we may consider a modern winder of 228-inch width designed to operate at 3,000 feet per minute on kraft papers up to 150 pounds per ream.

The equipment consists of two 125-horsepower motors on the winding drums, one 125-horsepower 185/945-rpm braking generator, and a 60-kw motor generator set with the necessary booster, constant-current regulator, and control.

The winding drums are 20 inches in diameter, the rider roll and guide rolls are $13^7/_8$ inches in diameter, and the unwinding roll is 65 inches in diameter, wound on a $13^7/_8$-inch spool.

At any given operating speed, the speed of the braking generator must vary over a range of 4.7 to 1 and as the unwinding roll decreases in size the winding roll increases as shown in figure 4 with a consequent decrease in WR^2 of the unwinding roll and an increase in WR^2 of the winding roll. In many instances the operation is such that 30- or 32-inch finished rolls are the maximum size and four of these may be wound from a 65-inch roll of paper. The operation is, therefore, intermittent with frequent starting and stopping, and, in addition to the friction and tension load the motors must accelerate this heavy inertia from standstill to top speed in about 20 seconds.

The unwinding roll, instead of being driven, is held back by the braking generator to maintain proper tension in the sheet. During accelerating this tension is composed of two elements, the pull caused by the braking generator and the pull required to overcome the inertia of the roll. Therefore, to maintain a constant tension during acceleration, the tension caused by the braking generator must be reduced by an amount equal to the pull required to overcome the inertia.

To simplify the problem involved we may consider all of these heavy inertias as so much weight concentrated at the periphery of the roll of paper as indicated in figure 5 and calculate the power required to accelerate this weight in the required time.

Curve A and C figure 5 represents a single large winding roll while curves B and D represent four 32-inch-diameter rolls wound from one 65-inch roll of paper.

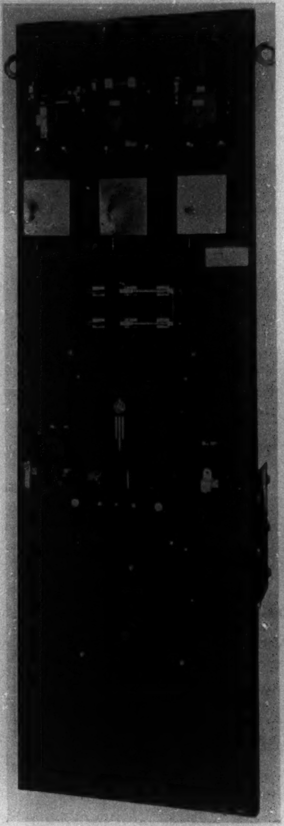

Figure 9. Main control panel for adjustable generator voltage winder drive

The tabulated data in table I show the equivalent weight of the complete winder together with the force and horsepower torque required to accelerate it in 20 seconds for various diameters of the unwinding roll. It also shows the equivalent weight of the unwinding roll together with the force and horsepower torque required to accelerate it.

The average horsepower torque required to accelerate the complete winder is 205.7 while the average horsepower torque required to accelerate the unwinding roll is 38; therefore, the tension held by the braking generator should be decreased by a corresponding amount to maintain a constant sheet tension during this operation. This is automatically taken care of by the regulator through the system of wiring connections.

When operating at top speed (3,000 feet per minute) the input to the driving motors resulting from friction and sheet tension is 212 horsepower while the power returned to the line from the braking generator is 137 horsepower leaving 75 horsepower to be supplied by the motor generator set. During acceleration the input to the motors (in terms of maximum speed) is 435 horsepower but because of the easing off in tension held by the braking generator, only 101 horsepower is returned to the line, leaving the motor generator set to supply 334 horsepower or 250 kw during this short period of time.

Figure 10. Auxiliary control panel for adjustable generator voltage winder drive

Dynamic Braking

Quick stopping is very important, especially on high-speed winders, and provision is made for electrical braking on both the winder motors and unwinding roll. This feature has a considerable bearing on the selection of motor capacities where light-weight papers are involved because the inertia of the winder and the roll of paper are likely to have a greater bearing on the motor capacity than the sheet tension under these conditions.

Power Requirements

The power required to drive a winder is made up of friction load and sheet tension and while there is some slight variation in the friction load of different winders, it is permissible to assume an average friction load and calculate the power requirements on the basis of sheet tension, assuming that the spreader bar or spiked roll or both are used in the operation. Such power data have been calculated and are herewith submitted in table II and figure 6.

An attempt has also been made to indicate the approximate tension required for sheets of various weights, based on recent experience and tests on winders now in operation. This is indicated in the curve, figure 6, and while there are scant data on this subject, and some difference of opinion, it will serve as a guide to the trade in selecting the sheet tension. The capacity of the equipment, however, is definitely determined by the sheet tension and equipments for this service should be so specified.

Conclusion

The big advantage in this type of drive is in maintaining constant sheet tension at any desired value and correctly winding rolls of uniform density. The load distribution between the motors and the sheet tension is indicated at all times and fast smooth acceleration and deceleration are available.

Regenerated power, while not of primary importance, does nevertheless make available power which otherwise must be dissipated as heat, and permits of using much smaller motor generator sets than would otherwise be possible. For example, a 228-inch winder operating at 3,000 feet per minute on a 300-pound kraft sheet would require two 200-horsepower driving motors, a 100-kw motor generator set and a 225-horsepower braking generator, which, on the basis of 50 per cent operating time would save approximately 810,000 horsepower-hours per year; power which might be used to good advantage elsewhere in a mill.

Comparison of Methods of Stopping Squirrel-Cage Induction Motors

By W. I. BENDZ

APPLICATION engineers frequently encounter the problem of choosing between the several available methods employed to stop a squirrel-cage induction motor quickly. To the best of the writer's knowledge, little has been done to present data in such a way as to make it useful to draw direct comparisons between these schemes, discuss the advantages of each, and aid in the choice of the method best suited to a particular application. Such is the purpose of this paper.

The schemes most commonly encountered in industrial equipment are: (1) Braking applied by a magnetically released, spring-set friction brake either of the shoe or disk type, or (2) Plugging, using a reversing magnetic controller usually in combination with a "plugging" or "zero-speed" switch driven from the motor shaft, (3) A scheme which is rapidly gaining favor for industrial application is the application of direct current to one or more of the motor stator phases and thereby stop the motor by what has been commonly referred to as "dynamic braking." (4) A fourth scheme discussed herewith is dynamic braking using a capacitor and resistor in which case the motor functions as an induction generator. Occasionally, an application warrants combining two braking schemes to obtain a desired function of a drive. Some suggestions in regard to this are also discussed.

Magnetically Operated Friction Brake

Probably the oldest and certainly the most common method of stopping ac-motors is by means of a spring-set magnetically released friction brake. The coil of the brake magnet is usually connected directly across two of the motor terminals so that the brake is energized (released) whenever power is applied to the motor and the brake coil de-energized (brake set) when power is disconnected from the motor. Earlier construction was such that the brake frame was separate from the motor and the two units had to be properly aligned. Co-ordinated designs now usually include the brake frame with the rear motor bracket and mount the brake wheel directly on a rear extension of the motor shaft. This design has been used for many years although a recent modification has been the introduction of disk-type brakes rather than shoe brakes. The disk brakes are neat in appearance and completely enclose all rotating parts. Since there is no difference in the function of either type of brake, the comments which follow apply equally well in both cases.

The application of "thrustor" brakes has been omitted from this paper for the reason that the writer is herein mostly

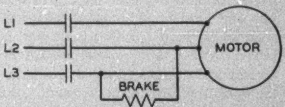

Figure 1A. Brake coil connected across motor leads

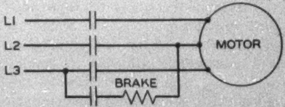

Figure 1B. Brake coil connected with separate circuit

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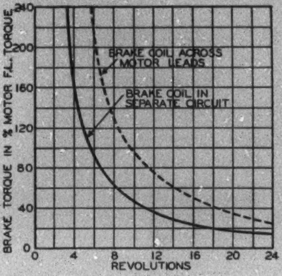

Figure 2. Comparison of brake-coil connections

concerned with motor ratings to which "thrustor" brakes are not usually adapted.

In all of the data which is presented, references to time or to the number of revolutions to stop a motor also includes the operating time of the magnetic contactor used for the motor control. This fact should be remembered when examining the shape of curves plotted against time or distance to stop.

The usual method of connecting a brake of this type is to wire the magnet coil directly across two of the motor leads, as illustrated in figure 1*A*.

The motor control disconnects the motor circuit and de-energizes the brake coil as well. This method of connecting the brake is simple and inexpensive since the brake is always beside the motor and the conduit wiring reduced to a minimum.

If the motor control switch is provided with an additional pole the connection shown in figure 1*B* can be used resulting in a considerable reduction in braking time. Any circuit which isolates one side of the brake coil from the motor can be used equally as well. The purpose of this connection is to completely deenergize the brake magnet at the same instant as the motor is disconnected from the line in order to bring about immediate application of the braking force.

The results of a test to show a comparison between these two connections, taken on the same motor and brake are plotted in figure 2. The dotted curve is the result of connecting the brake coil directly across the motor terminals as in figure 1*A* while the solid curve is the plot of performance when employing the separate brake coil connection shown in figure 1*B*. It is apparent that the difference in revolutions traversed during deceleration for the two schemes is greater at the lower value of braking torque. The explanation rests in the fact that the reduction in braking torque was obtained by weakening the spring of the brake. As the spring was made weaker the voltage appearing across the brake coil as a result of it being connected across the motor terminals caused increasing delay in the application of the brake, when this method of control was used.

The results shown in figure 2 were obtained on a three-horsepower six-pole frame *W*-254 motor equipped with a standard solenoid brake. The same tests were made on four- and eight-pole motors, both two and five horsepower. Similar results were obtained and in general it can be said that the difference between the two schemes is less for eight-pole motors and greater for four-pole motors. This is as would be expected since the higher speed motors will turn a larger number of revolutions for a given unit of time delay for the application of the brake.

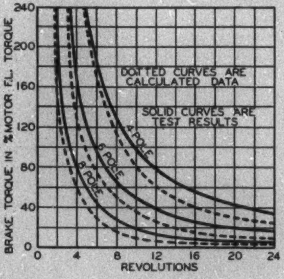

Figure 3. Comparison of calculated and test performance

The result of these tests brings about the conclusion that whenever maximum braking effect is required it is worth while to open the brake-coil circuit by means of a separate contact.

Figure 3 shows a plot of braking torque against revolutions to bring the motor to a stop for four-, six-, and eight-pole three-horsepower motors with the brake coil opened by a separate contact as in figure 1*B*. These curves may be considered as typical for motors of any horsepower rating but individual cases will depart from the performance illustrated due to variation in the ratio of torque to WR^2 for different motors. For this reason it seemed advisable to compare actual performance with theoretical calculated values from which one may consider the expected performance of any similar motor and brake.

The results of calculated performance for the motors tested in figure 3 are shown by the dotted curves. These were calculated from the braking torque, motor WR^2, and adding a factor of one-tenth second for the control and brake to operate. Tests showed that the average control and brake operated in about six cycles when the brake was adjusted for its maximum rating. A somewhat longer time was required when the brake spring (and torque) was decreased but this variation was neglected for the purpose of simplifying the discussion.

The importance of the curves plotted in figure 3 is that the performance as calculated from braking torque, motor WR^2, and control operating time checks reasonably well with actual operating conditions. Furthermore, it can be concluded that there is little to be gained by applying a brake rated much more than 100 per cent of the motor full-load torque and certainly no reason to exceed 200 per cent torque. This statement is true only when the WR^2 of the connected load is small compared with the motor. Choice of the proper brake must take this into account.

Figure 4 shows a plot of braking torque against time to stop the motor for six- and eight-pole three-horsepower motors. While other ratings were tested the results were not included in the figure because of the confusion of curves. As was brought out during the discussion of figure 3, calculated performance is reasonably accurate from which other ratings could be added to figure 4.

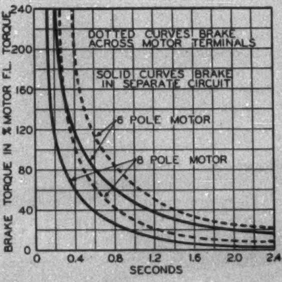

Figure 4. Comparison of brake-coil connections

The solid curves of figure 4 are the results obtained when using the separate brake coil connection shown in figure 1*B* and the dotted curves correspond to the connection of figure 1*A*.

To summarize the application of magnetic brakes the following conclusions may be drawn:

1. When the maximum braking effect is required it is worth while to select the control to provide the separate brake-coil circuit as shown in figure 1*B*.

2. Except in cases where the WR^2 of the connected load is a considerable portion of or greater than the motor inertia, there is not much gained by selecting a brake rated more than 100 per cent of the motor full load torque; no practical gain by exceeding 200 per cent torque.

3. Theoretical performance calculated by the following formulas are sufficiently close for most practical applications:

$$\text{Time to stop} = \frac{WR^2 \times \text{rpm}}{308 \times \text{brake torque}} + 0.1$$

$$\text{Revolutions to stop} = \frac{WR^2 \times (\text{rpm})^2}{3.70 \times \text{brake torque}} \times 10^{-4}$$

Where

Time is in seconds
Brake torque is in pound-feet
WR^2 is in pound-feet^2

Plugging Squirrel Cage-Motors

Plugging control to stop squirrel-cage motors is becoming more widely used for many kinds of industial applications. The reasons for this are (1) the increased capacity of feeder systems, thereby more universally eliminating the objection to the relatively high line current required for effective stopping, (2) the improvement in motor design to electrically and mechanically withstand the stress during plugging, and (3) the availability of proper and reliable control apparatus for plugging service.

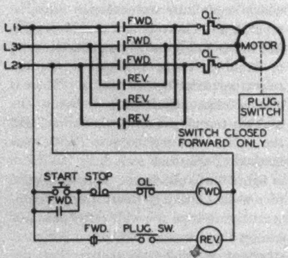

Figure 5. Schematic diagram of full-voltage plugging control

The majority of plugging applications are those for which the motor is connected across the power lines at full voltage by means of a pair of reversing contactors. A typical magnetic push-button control for this purpose is shown in figure 5.

A plugging switch, frequently referred to as a zero-speed switch, is driven from the motor shaft or from machine shafting coupled to the motor. This type of switch is designed so that its contacts close only due to rotation in one direction and are therefore open at zero speed or reverse rotation. Inspection of the diagram shows how the normally closed interlock of the "forward" contactor prevents the "reverse" contactor from being energized while the motor is running forward. As soon as the "stop" push button is depressed the circuit to the "reverse" contactor is completed through the "forward" interlock and plugging switch. The motor is then plugged to rest and at zero speed or upon slight reversal the plugging switch opens its contacts and de-energizes the "reverse" contactor.

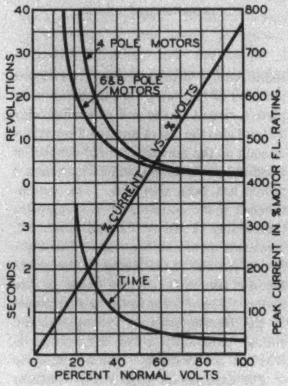

Figure 6. Performance by reduced-voltage plugging

Control is also available for reversing service in which case the plugging switch has two sets of contacts, one of which is closed by the forward rotation and the other by backward rotation. The motor is plugged to rest when the "stop" push button is depressed, no matter in which direction the motor may be running.

Figure 6 shows a plot of time and also the number of revolutions to stop against motor voltage in per cent of the normal rating. The voltage referred to is the value existing across the motor terminals at the instant of first starting to plug. The reduction in voltage below normal is obtained by connecting equal resistors in the three motor lines. Therefore, the abscissa and the right-hand ordinate are values occurring at the same instant.

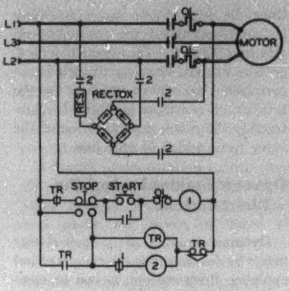

Figure 7. Schematic diagram of d-c braking control using copper-oxide rectifier

This figure also includes a plot of peak current required to plug a motor at any given per cent of normal voltage. By use of these curves the peak line current required to produce a desired rate of deceleration can be determined.

The data from which figure 6 was prepared were obtained by testing a number of motors between one-quarter and ten horsepower, four-, six- and eight- pole. Although the curves of this figure represent average values for this wide range of ratings it is important to note that individual cases did not depart from the average curve more than 10 per cent in regard to revolutions or time plotted against per cent voltage. However, there was a somewhat wider variation when considering the peak current drawn at 100 per cent normal voltage and this factor should not be overlooked when applying this data.

It is important to note that the results shown refer to a constant line voltage. If the line potential is decreased during the plugging interval the performance is changed radically. As an example, a test was made on one motor, holding the line voltage constant and adjusting series resistors to limit the plugging current to 400 per cent which resulted in 5.0 revolutions to stop the motor. The same motor was connected across a generator and feeder system of such poor regulation that the plugging current was also 400 per cent but without any series resistors. In the latter case the motor required 8.0 revolutions to stop or an increase of 60 per cent.

The amount of resistance to connect in series with the motor to limit the plugging current to any desired value can readily be calculated from the motor locked-rotor current and power factor.

Once the approximate motor reactance has been calculated the resistance can also be obtained. The required impedance is figured, knowing the maximum permissible line current and from this the total circuit resistance is obtained. Subtracting the motor resistance leaves the value to be added to each line.

Dynamic Braking by Applying Direct Current

Dynamic braking of a squirrel-cage motor by disconnecting the a-c power and applying direct current to one or more stator phases is not a new scheme but has not been widely used to date. Perhaps lack of sufficient application data and inadequacy of a source of direct current are the major factors limiting its use. There are many applications for motors three horsepower and below, particularly on machine tools, that require smooth braking and in many cases a Rectox copper-oxide rectifier is well suited to supply the necessary direct current.

One suggested wiring diagram for a unit consisting of control and Rectox is shown in figure 7. This scheme includes a time delay relay to disconnect the rectifier at the end of the braking period which is a feature that is not required in all cases.

Referring to the diagram, if the "start" push button is depressed the number 1 line contactor is energized and its interlock contact bridges the "start" push button. The motor is then running, energized by the a-c power line. If the "stop" push button is depressed the circuit to number 1 contactor is broken and the a-c power is immediately disconnected. The lower contacts of the "stop" push button complete a circuit to the coil of timing relay *TR* and also the coil of number 2 contactor as soon as the normally closed contact of number 1 seals, thereby indicating that a-c power has been removed. The instantaneous make contacts of *TR* complete the holding circuit to *TR* and number 2. The contacts of number 2 apply the alternating current to the Rectox and connect the d-c terminals to one phase of the motor. The resistor in the a-c terminals of the Rectox reduces the applied a-c voltage since only a low d-c voltage is required for braking. When the cycle of *TR* expires, its time delay contacts open thereby dropping out number 2 contactor and resetting *TR*. This, of course, disconnects the rectifier but the cycle of *TR* should be made long enough to be certain that the motor has come to rest.

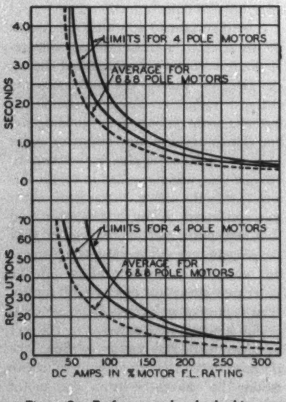

Figure 8. Performance by d-c braking

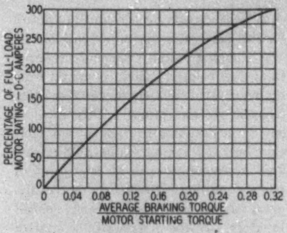

Figure 9. Required direct current for braking can be calculated by using this curve

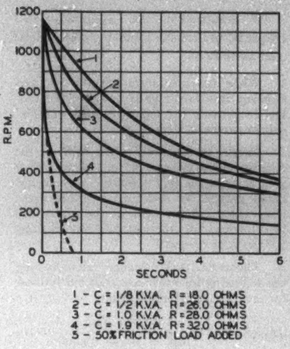

Figure 10 (right). Braking performance by capacitors and resistors

If a source of d-c power were available the Rectox rectifier could be omitted and the same control used except for the details of connecting number 2 contactor between the d-c line and the motor terminals. The timing relay would be required because in order to obtain effective braking a value of direct current must be applied which would overheat the motor if allowed to continually circulate while at rest.

The time and also number of revolutions required to stop a motor are plotted against direct current in per cent of motor full-load rating in figure 8. A large number of tests were made on motors ranging from one-quarter to ten horsepower, four, six, and eight poles. In studying the results of tests made on these ratings it was found that motors of like number of poles performed very similarly, particularly on the part of the curve having the most practical use. The minimum and maximum performance for the group of four-pole motors is plotted and designated as limits in figure 8. Limits were shown for four-pole motors only and the average for six- and eight-pole motors in order to eliminate unwarranted confusion of the curves.

The results shown by figure 8 pertain to motors not connected to a load and, of course, represent a wide range of windings and rotor inertia. In order to present data which could be used to predict the time or revolutions to stop a motor connected to a load (of which the friction and WR^2 is known) the variable factors which enter into motor design must be separated. The problem can be simplified by obtaining the average decelerating or braking torque throughout the decelerating period by test results. There is also a close relationship between the braking torque per ampere obtained and the motor starting torque. With this method of approach as a basis the curve of figure 9 was obtained. It is significant to note that although figure 9 again represents the average of a wide range of motor ratings there was no individual design of standard squirrel-cage motor that departed from the average curve more than ten per cent, which is a sufficiently close check for practical applications.

Figure 9 can be used to calculate the required direct current to obtain the desired braking effect by knowing, (1) the friction and WR^2 of the connected load, (2) the WR^2 and starting torque of the motor and preferably, (3) the stator resistance of the motor. The average decelerating torque to stop the motor and connected load is readily calculated from the formula given at the close of the section on friction brakes. Knowing the motor starting torque the ratio of

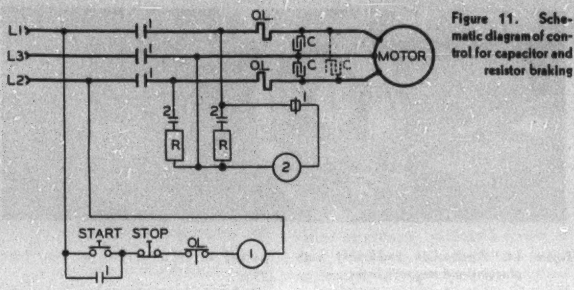

Figure 11. Schematic diagram of control for capacitor and resistor braking

average braking torque divided by motor starting torque can be obtained and figure 9 applied to get the direct current required for effective braking. If the motor stator ohmic resistance is known the resultant direct voltage can be figured and the details as to the required source of direct current easily follow.

With the exception of fractional horsepower ratings it is sufficiently accurate to assume that the motor d-c resistance is six per cent of the motor rated voltage divided by its full-load current. Fractional horsepower windings vary considerably and may depart from this rule by so much as 500 per cent. Consequently, care must be used to obtain exact winding data before proceeding with each application.

From inspection of the data presented in figure 8 and comparing this performance with figures 3, 4, and 6, it can readily be seen that it is impossible to stop a motor as quickly by means of d-c braking as by means of full voltage plugging, but the former does approach the effectiveness of a magnetic brake.

While test data show that maximum deceleration is not obtained until the direct current is increased to about 350 per cent of the motor rating, experience has shown that 250 per cent is sufficient in most cases and that the difference in braking effectiveness between these values cannot be noticed without equipment set up to make accurate measurements. There are numerous applications where satisfactory braking can be obtained by using direct current of only 150 per cent of the motor rating.

This scheme of stopping a motor is commonly referred to as "dynamic braking" but the term is not strictly true and it is not at all similar to dynamic braking of a d-c motor. While there is a decrease in braking torque as the speed is reduced, it does not fall to zero and a definite braking torque is applied with the rotor at standstill. Sufficient torque is available at low speed to prevent excessive coasting and the motor is stopped smoothly without the violent shock characteristic of a magnetic brake in particular.

Dynamic Braking, Using Capacitors and Resistors

Any induction motor will function as an induction generator provided it is operated in parallel with apparatus that will supply a leading power factor load since this is the only way by which an induction machine can be excited. Usually, synchronous machines supply the leading power factor but capacitors meet the requirement equally well if connected across the terminals of the machine.

While this method of stopping a motor does not possess many practical advantages it is sufficiently interesting to deserve brief mention.

Either two single-phase or one three-phase capacitor unit may be connected across the motor. The number 1 contactor starts the motor by depressing the "start" push button. The number 2 contactor connects the resistors across the motor terminals when the "stop" button is depressed. The coil of number 2 contactor is chosen to hold number 2 contacts closed to as low a voltage as possible. Since there is very little braking torque below one-third speed there is no point in attempting to hold number 2 contactor closed below this value.

A family of deceleration curves taken with a 2-horsepower 6-pole 220-volt 3-phase 60-cycle frame *W*-225 squirrel-cage motor are shown in figure 10. These curves are motor revolutions per minute plotted against time and therein differ from the type of curves so far presented in this paper.

Because of the inherent characteristic of dynamic braking to become less effective as the speed decreases this method of stopping a motor is useless if the motor is not coupled to a load having some friction. The dotted curve of figure 10 shows how the lower curve ($C = 1.9$ kva, $R = 32$ ohms) would be changed if the motor were connected to a drive having a frictional load equal to 50 per cent of the motor full-load torque rating. In this case the motor is brought to rest in about three-fourths second.

The only possibility for a practical application of this method would be when capacitors would be desirable for power-factor correction. For instance, a capacitor value sufficient ~~to produce~~ reasonably effective braking would result in a circuit power factor of approximately 35 per cent leading when the motor operated at about full load.

Special Applications

Examples of a few unusual applications are given below for the purpose of illustrating factors leading to the choice of one or more methods for braking a motor.

The diagram shown in figure 12 is a drive for a machine in which it was desired

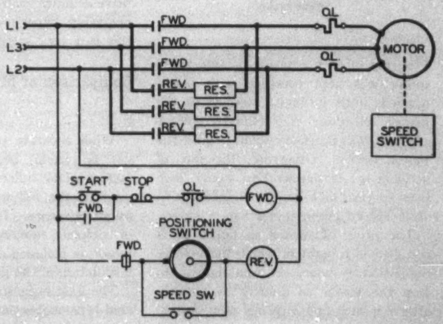

Figure 12. Schematic diagram of special reduced-voltage control

to stop the motor in a given position of a rotating shaft. Reduced voltage plugging was chosen for the application. The positioning switch was direct connected to the shaft to be positioned. Its design was such that a circuit is closed through the switch except for a few degrees of rotation. The speed switch is a centrifugal switch driven from the motor and its contacts are closed from normal down to about 15 per cent speed. Below this speed its contacts are open. The stopping function of this drive is to plug the motor by reduced voltage until the speed decreases to 15 per cent. Below this speed the motor is plugged through the positioning switch until the "dead" zone of the switch is reached. Power is then disconnected and the motor may or may not coast to a stop within the desired zone. If it coasts until the switch recloses the motor is energized through the resistors to rotate the shaft until it again reaches the "dead" zone of the positioning switch.

Reduced voltage plugging was selected for this scheme for two reasons: (1) Plugging offered a means by which the motor was kept rotating, with power applied, until it finally stopped in the proper position; (2) reduced voltage plugging was necessary so as to limit the motor torque. Otherwise, the rate of acceleration or deceleration would have been so great as to make it impossible to stop close to a desired position.

Figure 13. Application of oversized magnetic brake

The drive illustrated in figure 13 is installed in a plant in which the voltage regulation is poor. It was desired to stop the motor as quickly as possible after tripping an emergency stop switch. Poor voltage regulation would have defeated the effectiveness of full-voltage plugging. The problem was solved by using a magnetic brake rated 200 per cent of the motor torque. A two-pole master stop switch was used for the control circuit. One pole was wired into the motor contactor circuit and the other pole directly in the magnetic brake coil circuit. Consequently, when the stop switch was tripped the brake was applied by the time the contactor interrupted the motor line current, with the result that very effective braking was obtained.

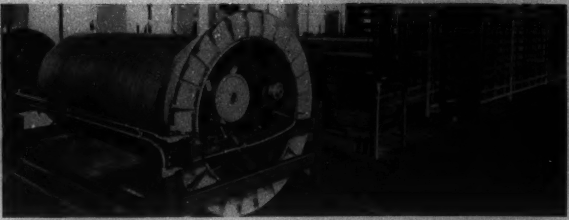

Figure 14. Application employing both plugging and magnetic brake

Figure 14 shows a drive for which the load has a WR^2 of many times the motor inertia. The drive between the motor and load is by gearing and V belts. Since several times full-load motor torque would be required to produce the desired braking it is obvious that the drive would be stressed severely if a large brake were placed on the motor shaft. It is also true that a brake on the load side would impose a peak load on the drive because of the motor WR^2. The solution chosen was to put a magnetic brake on the load and use full voltage plugging on the motor. The brake was applied so as to make the rate of deceleration of the load approximately match the motor and thereby reduce the stress on the drive as much as possible. Considerable operating experience has proved that this is a very satisfactory scheme.

Comparison of Methods

COST

While accurate cost comparisons cannot be made because of the many factors which influence the required apparatus, the following may serve as a rough reference. In each case the cost of complete equipment, exclusive of the motor, is included and allowance for the installation of the parts is made.

The least expensive method is a solenoid-type magnetic brake and is consequently taken as a 100 per cent base price.

Full-voltage plugging can usually be installed for about 135 per cent of the brake cost. Reduced voltage plugging will cost about 200 per cent.

Dynamic braking with direct current including a Rectox rectifier will be between 150 per cent and 250 per cent of the brake cost depending upon the duty cycle. If a source of direct current is available and the Rectox therefore omitted, the cost of this scheme can be reduced to about 125 per cent.

Dynamic braking using capacitors and resistors will be about 250 per cent for 220 volts and 200 per cent for 440 or 550 volts.

MAGNETIC BRAKES

Advantages. The magnetic brake is the least expensive scheme to install and while it is not the fastest method for stopping a motor it is second in this choice. No power is taken from the line during the braking period and consequently braking is independent of voltage failure.

Disadvantages. Frequently there is not sufficient space in which to mount a magnetic brake. On applications that start and stop frequently a certain amount of maintenance is always necessary. Furthermore, the person adjusting the brake must be skilled in this or else the brake operation will not be satisfactory. There is a continued consumption of power from the a-c line while the motor is running. If it is desired to manually rotate the motor when power is disconnected special arrangements must be made to either electrically or mechanically release the brake. The torque of a friction brake increases considerably at very low speed which results in a rather violent shock as the motor is brought to rest. For some applications this may be a disadvantage.

PLUGGING

Advantages. Full-voltage plugging will stop a motor quicker than any other method provided the power supply is adequate. This scheme is not very much more expensive than a magnetic brake and requires only a normal amount of maintenance. Reduced voltage plugging is a flexible method in that the rate of deceleration may be readily adjusted after installation.

Disadvantages. Plugging, even at reduced voltage, requires more than the motor full-load current taken from the line. If voltage fails the motor will coast instead of stopping quickly. A plugging switch must be mounted, driven from or otherwise coupled to the motor shaft and properly adjusted. Special care in the choice of apparatus must be considered if the application disallows a possibility of the motor rotating slightly in the reverse direction.

D-C BRAKING

Advantages. This method of stopping a motor does not require any apparatus mechanically coupled to the motor shaft nor is any maintenance required (other than is common to control apparatus). Because of this it is particularly useful on applications subject to frequent starting and stopping. The motor cannot run backwards as may occur if plugging is used and the switch improperly adjusted. The motor is free to be rotated manually at the termination of the braking period and the braking is particularly smooth. If a separate source of power is available braking will be independent of failure of the a-c supply. The separate source may be direct current or another a-c supply independent of the main a-c lines.

Disadvantages. The chief disadvantages of this scheme is its cost relative to a magnetic brake. While this scheme ranks third in its ability to stop a motor most quickly, it is often adequate for a large majority of applications. There can be no braking torque when the main a-c supply fails unless a separate source of power is available from which braking may be obtained.

DYNAMIC BRAKING WITH CAPACITORS

Advantages. This method offers the advantage of power factor correction while the motor is running and braking is independent of voltage failure of the supply line. Maintenance would be less than for any other scheme of braking the motor.

Disadvantages. Unfortunately, this scheme is a poor fourth in braking effectiveness and at the same time is about the most expensive. Consequently, it is not economically attractive as a means for braking an a-c motor.

Conclusion

In conclusion, the following suggestions should aid in selecting the best method of braking:

1. If the most important consideration is stopping the motor as quickly as possible:

(*a*) Determine if the source of supply is capable of delivering at least six times the motor full load current without dropping the voltage at the terminals of the motor more than five per cent. If the supply is adequate use a full voltage plugging controller.

(*b*) If the supply is inadequate use a magnetic brake rated 150 per cent to 200 per cent of the motor full-load torque and a control designed to open the brake coil separately from the motor winding.

2. If the most important consideration is availability of normal braking even with the failure of the main a-c supply system:

(*a*) A magnetic brake is usually employed in such cases.

(*b*) Dynamic braking using direct current is equally as good as a magnetic brake if a separate source of direct current is available. It may be worthy of consideration if the separate source is alternating current in which case a rectifier would be used.

(*c*) Dynamic braking using capacitors would meet the requirements providing a very quick stop were not necessary.

3. If the most important consideration is ability to withstand frequent operation without undue maintenance:

(*a*) A magnetic brake will require the largest amount of maintenance of any method of braking.

(*b*) Dynamic braking using direct current is the most attractive scheme both from the point of view of freedom from maintenance and least shock to the drive.

(*c*) Full voltage plugging can be used if the power supply is adequate and the motor applied so as to withstand the heating that results from frequent plugging. Some maintenance of the plugging switch and magnetic contactors will be required.

4. If the most important consideration is a smooth stop without shock to the drive:

(*a*) Dynamic braking using direct current is highly recommended for this application.

(*b*) Dynamic braking using capacitors would produce a smooth stop and may possibly deserve consideration.

5. If the most important consideration is assurance that the motor will not rotate backward:

(*a*) A magnetic brake or d-c braking are equally good in this respect as neither can result in backward rotation.

(*b*) While it would not be correct to state that plugging could not be used for such an application, it is true that this method is more subject to reversal than any other and extreme care should be used in the selection and adjustment of the plugging switch.

6. If the most important consideration is freedom of the motor to be rotated manually without the application of power:

(*a*) Dynamic braking using direct current presents no complications for such an application.

(*b*) Plugging can be used with some limitations. A friction type of plugging switch cannot be used, because manual rotation of the motor will cause the plugging switch to energize the control and apply power to the motor (unless a special control is designed to prevent this). A centrifugal type of plugging switch can be applied if the speed when rotated manually is below the minimum point at which the switch will close its contacts.

(*c*) Dynamic braking using capacitors would be applicable and may possibly deserve consideration.

The Application of Capacitors for Power-Factor Correction in Industrial Plants

By C. E. H. von SOTHEN

WHILE a paper on this subject before an AIEE meeting seems rather superfluous, perhaps a review of some of the facts to be considered in the application of capacitors to industrial loads will bring out a discussion that will be beneficial to all.

Greater interest has been shown in this subject during the past few years, both because of the wider application of power rates having some form of power-factor clause, and because of the reduction in power costs to the industrial customer, if the power factor is corrected. The utility companies generally have urged those customers who may benefit by power factor correction, to install the necessary equipment and in some cases have recommended the amount of corrective capacity to meet the conditions existing. The manufacturing companies have, of course, also made a great many surveys, each case being worked up individually to determine the amount of correction which will provide greatest return on investment.

A capacitor or static condenser consists principally of metal foil and paper insulation in a metal container and generally assembled in a rack or housing for indoor or outdoor service.

Data Required

The data required to make proper ap-

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plication generally should include the following:

1. Magnitude and power factor of the load, both maximum and average.
2. Character of the load; constant or variable?
3. Statement of power rate.
4. Is metering on high or low side of step-down transformers?
5. Are the feeders within the plant of ample capacity or are some overloaded?
6. Rating of the transformers. Are they owned by the utility company or by the industrial company?
7. Will an indoor or outdoor capacitor equipment be more satisfactory?
8. Voltage of circuit (normal and maximum).
9. Is any additional load being considered?
10. What are the atmospheric conditions at proposed capacitor location? Are they normal or are dust, corrosive fumes, and so forth, present?
11. What is the maximum ambient temperature?

Standard Types Available

It might be well to discuss briefly the various types of capacitor equipments available for industrial applications. There are four general types for use as follows:

1. The enclosed dust-tight capacitor unit for connection to individual motors or small groups of motors rated 230, 460, or 575 volts. These may also be combined in racks to make up a dust-tight equipment.
2. The small rack capacitor for indoor installation, for use at 230, 460, 575, 2,300, 4,000, and 4,600 volts, at one or several points in a plant. This design fulfills practically all industrial-plant requirements and affords maximum economy with reference to reduction of line loss.
3. The pole-type outdoor capacitor, corresponding to the indoor small rack for 2,300, 4,000, and 4,600 volts. This can be either mounted on a pole or bolted to a foundation.
4. The large rack equipment for either indoor or outdoor installation, for improving power factor of a large block of power at one point, on circuits of from 230 to 6,900 volts.

There have been a few installations of automatically controlled capacitors to maintain approximately constant power factor, reactive kilovolt-amperes, or current of some predetermined value. These depend upon some form of master relay which opens or closes oil circuit breakers or contactors controlling several blocks of capacity. The advantage is that a higher average value can be held without going too far leading at light loads. Whether such an equipment will prove economical depends, as usual, upon the power contract.

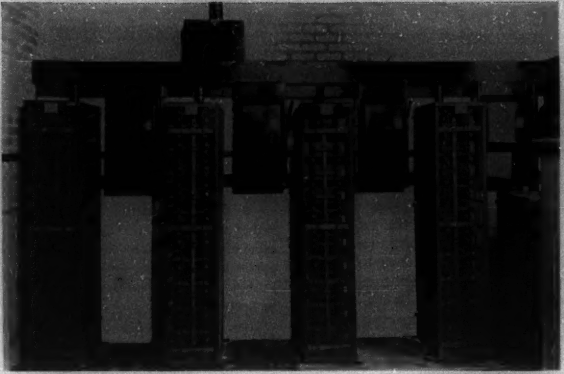

Figure 1. Four 100-kva and one 60-kva 3-phase 60-cycle 460-volt indoor small rack capacitors

The foregoing covers the types available. Now how should they be applied?

The manner in which the problem is approached to meet conditions imposed by various classes of rates may be illustrated best by a few typical examples from various parts of the country.

Calculations

Case I

The power contract contains the following rates and provisions:

Rate: (per month)
Demand charge:
  $1.25 net, per kilowatt of maximum demand for the first 500 kilowatts of demand
  $1.00 net, per kilowatt of maximum demand for all over 500 kilowatts of demand
plus
Energy charge:
  1.5¢ net, per kilowatt-hour for the first 10,000 kilowatt-hours
  0.9¢ net, per kilowatt-hour for all over 10,000 kilowatt-hours
Minimum charge:
  The demand charge.
Special provision:

(*a*) When customer's power factor is less than 80 per cent or more than 90 per cent, the billing demand shall be increased or decreased in the ratio that 85 per cent bears to the actual power factor. The power factor shall be determined monthly by computation from the registration of active and reactive watt-hour meters.

In this case power was supplied at 6,900 volts, 3 phase, 60 cycles. There was ample transformer capacity and none of the 230-volt feeders was overloaded. Therefore only primary capacitors were considered, for installation in the main switch-board room.

The power bills for one year were used as a basis for calculations. A typical month showed the following:

Actual demand..........	1,056 kw
Power factor...........	60 per cent
Reactive kilovolt-ampere hours.................	277,600
Kilowatt-hours..........	208,800

All figures were based on a rack large enough for 540 kva, but containing various numbers of units as an initial installation, starting with 495 kva. Indications were that additional capacity would be required in the future.

The 495-kva equipment would have to operate during the month in question only 560 hours to average 100 per cent power factor, but let us say 625 hours. The billed demand would be 85 per cent of 1,056 or 898. The loss on this equipment would not exceed 1.65 kw but say 2 kw which would increase the demand to 900 or a net saving in demand of 156 kw. This at $1.00 per kilowatt amounts to $156 saving in demand charge.

The loss of 1.65 kw for 625 hours at 9 mils amounts to $9.30 or say a net saving in the power bill for the month of $146.

Each month was figured similarly and the total annual saving on the power bill amounted to $1,804. The equipment had a first cost of $5,667 and an installed cost of $6,060. Carrying charges may be taken as nine per cent of $5,667 or $510. (Interest is omitted since the purpose of the calculation is to determine the return on the investment.) The net annual saving is then $1,294 or a return of 21.3 per cent on the investment of $6,060.

Other sizes were figured and the result was the recommendation of a 540-kva rack containing 360 kva in units. This showed a return 27.8 per cent on an investment of $4,700.

Of course if a 360-kva rack could have been used, the initial cost would have been lower and the return would have been greater.

Case II

The power rates in this case were as follows:

Demand—First 100 kva.....$3.00 per kilovolt-ampere
All in excess......$1.50 per kilovolt-ampere
Energy—First 200 hours use of kilowatt demand per month...........$0.007 per kilowatt-hour
All over 200 hours............$0.005 per kilowatt-hour

As the kilovolt-amperes would never be reduced below 100, all savings were at $1.50.

Power was supplied at 13,200 volts and stepped down to 480 volts. As this was a grain mill, dust-tight equipment was required. From billing data for the year 1936 it was found that the average was 611 kva demand, 455 kw, 74.5 per cent power factor, and 408 kva lagging reactive.

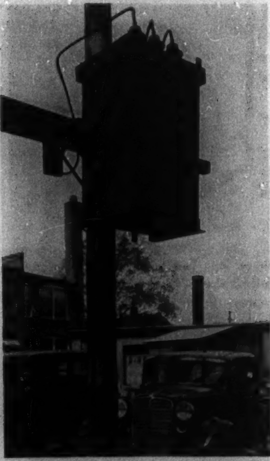

Figure 2. Forty-kva 4,000-volt 3-phase 60-cycle pole-type capacitor

As the corrective kilovolt-amperes of a capacitor varies as the square of the voltage, a 210-kva 460-volt equipment will provide 229 kva leading. This reduces the reactive to 179 and kilovolt-amperes to 489, or a saving of 122 kva per month. For 12 months at $1.50 per kilovolt-ampere, the saving in demand is $2,196. The loss on this equipment would not exceed 0.76 kw which for 16 hours per day and 300 days per year at $0.005 per kilowatt-hour would amount to $18.24 per year. The net saving on the power bill would therefore be $2,177.76 per year.

A 210-kva 460-volt dust-tight capacitor equipment had a first cost of $2,103 and estimated installed cost of $2,313. Carrying charges were taken at nine per cent or $208 so that the net annual saving would be $1,970 which represents a return of 85.1 per cent on the investment of $2,313.

Other sizes were calculated similarly with the results shown in table I.

While the 180-kva equipment shows a greater per cent return, the extra investment for the 210 kva would be paid for in two years. The same might be said for the 240 kva over the 210-kva equipment, but it was found that the 240-kva equipment would correct the power factor to 99 per cent at time of maximum demand for some months, which would mean leading power factor on average loads. The 210-kva equipment was therefore installed.

Case III

While this case involves a rather small installation, it illustrates another form of power contract which may be of interest.

Monthly Demand (Kilowatts)	Demand Charge (Dollars)
First 10 or less	30.00
Next 15	2.50 per kilowatt
Next 25	2.00 per kilowatt
Next 50	1.50 per kilowatt
Next 1,900	1.25 per kilowatt
Next 2,000	1.00 per kilowatt
All over 4,000	0.90 per kilowatt

Monthly Use (Kilowatt-hours)	Energy Charge (Cents per Kilowatt-hour)
First 1,000	2.5
Next 1,500	2.0
Next 2,500	1.5
Next 5,000	1.0
Next 190,000	0.85
Next 300,000	0.75
All over 500,000	0.65

Special Provisions. The service supplied by the company shall be taken by the consumer whenever possible at an average power factor of not less than 70 per cent lagging. If the service is taken at an average power factor of less than 70 per cent lagging, then the demand for billing purposes, in lieu of the measured demand, shall be taken as the product of the measured demand and the quotient resulting from dividing 70 per cent by the actual average lagging power factor expressed in per cent. If the service is taken at an average power factor of more than 85 per cent lagging, then the demand for billing purposes, in lieu of the measured demand, shall be taken as the product of the measured demand and the quotient resulting from dividing 85 per cent by the actual average lagging power factor expressed in per cent.

Figure 3. Three hundred-kva 460-volt 3-phase 60-cycle large rack capacitor

The average power factor for the month shall be determined by computation from the registration of an active watt-hour meter and a reactive watt-hour meter. It shall be the quotient resulting from dividing the registration of the active watt-hour meter by the square root of the sum of the square of the registration of the active watt-hour meter and the square of the registration of the reactive watt-hour meter.

As there was a detent or ratchet on the reactive watt-hour meter, nothing could be gained by operating at leading power factor during light-load periods.

In this case the primary voltage was

Table I

Capacitor Rating (Kva)	Corrected Power Factor	First Cost	Installed Cost	Net Annual Saving	Per Cent Return on Investment
150	90	$1,548	$1,703	$1,526	89.5
180	91	1,817	2,000	1,800	90.0
210	93	2,103	2,313	1,971	85.1
240	95.5	2,463	2,709	2,147	79.0

4,000 and plant voltage 230. As several circuits were rather badly overloaded, secondary capacitors, distributed over some of the branch circuits, presented definite advantages over an installation on the primary side. Furthermore, conditions were such that a primary capacitor would have to be installed outdoors, adding to its cost.

Table II

Month	Kilowatt-hours	Reactive Kilowatt-hours	Demand Read	Power Factor	Demand Billed	Penalty @ $1.25
August	25,200	38,200	124	55	158	$43.00
September	23,940	35,560	141	56	176	43.75
October	22,540	32,900	121	57	149	35.00
November	21,280	29,120	136	59	161	31.25

Billing for only four months was available, on which to base calculations, but results have indicated the assumptions to be correct. These were as shown in table II.

The plant was operating 16 hours per day and 5 days per week with the following results:

Month	Average Kilowatts	Average Reactive Kilovolt-amperes	Corrective Kilovolt-amperes Required: For 72 Per Cent Power Factor	Corrective Kilovolt-amperes Required: For 87 Per Cent Power Factor
August	71.5	108.5	39.5	68.0
September	71.3	106	34.0	66.0
October	67.0	98	34.0	60.0
November	60.5	82.8	25.0	49.0

The values of 72 per cent and 87 per cent were taken rather than 70 per cent and 85 per cent to allow some margin for changes in conditions. The first value is necessary to avoid the penalty and the second is required if a bonus is to be earned.

It is evident that a 40-kva capacitor will correct each month to 70 per cent or better. What will be its effect in November?

Forty kva for 22 days and 16 hours per day or 14,100 reactive kilovolt-ampere-hours leading. This leaves a net of 15,020 reactive kilovolt-ampere-hours lagging which combined with 21,280 results in an average power factor of 82 per cent; not sufficiently high to earn the bonus.

To determine the effect of a 70-kva equipment, consideration must be given to the fact that for a few hours each day, the power factor may be leading, and with the "detent" on the meter the result is the same as unity power factor. By checking the active and reactive meters carefully and obtaining a record of demands, it was found that during working hours, the load never dropped below 35 kw at 55 per cent power-factor or 53 reactive kilovolt-amperes. From demand-meter records for the four months it was found the power factor with a 70-kva equipment would be leading for one hour per day in August and September, two hours in October, and three hours in November and it was assumed the leading kilovolt-amperes would be 70 minus 53 or 17 kva. Therefore, the effect would be as follows for August:

$$\begin{aligned} 70 \times 22 \times 16 &= 24{,}640 \text{ reactive kva} \\ 17 \times 22 \times 1 &= -380 \\ \hline & \quad 24{,}260 \text{ reactive kva} \end{aligned}$$

$$\begin{aligned} &38{,}200 \text{ reactive kva} \\ &-24{,}260 \\ \hline &13{,}940 \text{ reactive kva} \end{aligned}$$

$$\frac{R}{KW} = \frac{13{,}940}{25{,}200} = 0.554$$

or 87.5 per cent power factor

Similar calculations for the other months gave these results:

September	88.5 per cent
October	91.5 per cent
November	96.5 per cent

The effect of the 40-kva equipment would be to avoid the penalty or a saving of $153 for four months or approximately $459 per year.

This equipment consisting of 8 5-kva 230-volt enclosed units with hangers and with fuses and dischargers would have a first cost of $728 and installed cost of $800. The net annual saving with nine per cent carrying charge would be $393 representing a return of 49.1 per cent on the investment of $800.

With the 70-kva equipment the penalty would be avoided and bonus earned each month. For example in August, the billed demand would be

$$124 \times \frac{85}{87.5} = 121$$

or a saving of 37 kw over the demand as actually billed (158 kw). For September this would be 41 kw, October 36 kw, November 41 kw, at $1.25 per kilowatt these total $193.75 or approximately $581.25 per year.

A total of 14 5-kva 230-volt enclosed units had a first cost of $1,274 and installed cost of $1,400. The net annual saving with nine per cent carrying charge would be $466 or a return of 33.4 per cent on the investment of $1,400.

Because of the greater return from the 40-kva equipment and the fact that conditions were somewhat uncertain, this equipment was recommended with the suggestion that a further study be made at a later date, with the possibility of installing additional units.

Protection

The problem of protecting capacitors and lines is similar to that on any electric system involving a relatively large number of pieces of apparatus fed from a common source. In rack and pole-type equipments the entire group of units is fed through one circuit-interrupting device, which is generally set to trip at twice normal line current. Therefore, many times normal current of an individual unit could flow into a fault within

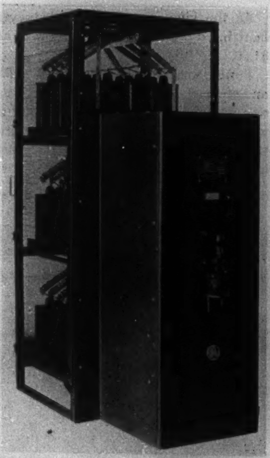

Figure 4. Two hundred forty-kva 4,600-volt 3-phase 60-cycle indoor large rack capacitor

the unit without opening the line circuit breaker. Individual fuses, however, will limit the amount of energy dissipated within a capacitor unit case by quickly clearing the fault and without interrupting power supply to the remainder of the equipment.

When a capacitor is disconnected from the line, electrical energy remains stored within the unit. All types of equipment must therefore be provided with means for draining the energy after the circuit has been opened. A discharge device should always be provided by the manufacturers as part of standard equipment.

A circuit-interrupting device of some form should in general, be used with capacitors to serve as a main switch as well as to provide overcurrent protection. As standard capacitors are designed for a permissible working voltage approximately 15 per cent in excess of rated voltage, and to provide for extra current due to possible harmonics, the main switch should have a capacity of at least 35 per cent over rated current of the capacitor.

In some cases, on 230-volt circuits, economies may be effected by using 460-volt capacitors and autotransformers, because of the lower cost per kilovolt-ampere of 460-volt capacitors. Since the capacitor is capable of handling a maximum of 135 per cent of normal kilovolt-amperes, the transformer output rating should be selected accordingly.

The possible short-circuit current at the installation should, of course, be considered when selecting the interrupting device. The circuit breaker selected must be able successfully to clear the maximum short circuit that may occur at that point.

Indoor installations of capacitors ordinarily do not require special consideration in regard to lightning protection. If the secondary circuit to the consumer presents any appreciable exposure the lightning arrester ordinarily installed to protect the consumer's apparatus will provide sufficient protection for the capacitors.

Outdoor capacitors should be protected by distribution-type lightning arresters, except where existing lightning arresters on the circuit are sufficiently close to the point of installation to provide protection.

Effects of Low Power Factor

The question of point of application of the capacitor is usually one of economics. The greatest benefit, from the electrical standpoint, can be derived from capacitors connected directly at the motor terminals, since they reduce power losses in all lines and transformers between the power source and the motor. However, such capacitors usually have a higher cost per kilovolt-ampere than equipment installed in larger blocks at the bus or on the primary feeder. The higher cost of low-voltage units must be weighed against the cost of a primary equipment plus additional copper for the low-voltage feeders, if some of these are overloaded or voltage drops are excessive. Also improvement in voltage obtained by capacitors on the low-voltage transformer secondaries, may warrant the higher first cost of a low-voltage installation.

Figure 5. Three hundred-kva 4,000-volt 3-phase 60-cycle outdoor large rack capacitor

It is often found that no more load can be added to a circuit because the wattless current has overloaded the system. By improving power factor, more load can be added at a lower over-all cost than would be incurred by installing more copper or additional circuits.

Low-power-factor conditions in a plant produce low voltage and excessive dips upon overloads and peaks. Starting and running torques of motors are reduced in proportion to the square of the voltage. Motors start slowly or not at all and may be stopped on voltage dips through action of the undervoltage device. The slip of induction motors is greater at reduced voltage, hence production suffers.

Of course, if the transformer bank is overloaded low-voltage capacitors will easily prove themselves out in comparison with additional transformer capacity.

A point which is sometimes overlooked or misunderstood when considering the effect of capacitors on an industrial load, is the fact that although the power factor of a motor is higher at full load than at part loads, the reactive kilovolt-amperes are also higher. Therefore to raise the power factor to unity requires slightly greater corrective kilovolt-amperes at full load than at part load. For example the full-load power-factor of a 10-horsepower 1,800-rpm squirrel-cage motor is 89 per cent and the lagging reactive kilovolt-amperes is 4.5 At half-load the power factor of this motor is 75 per cent and the lagging reactive kilovolt-amperes 3.9.

The thought is quite frequently expressed that, to meet certain conditions, capacitors should be installed on a motor to reduce starting current. When consideration is given to the fact that the full-voltage starting current of a normal-torque low-starting-current squirrel-cage motor is $4^1/_2$ or $5^1/_2$ times normal and its power factor at start may be 40 per cent or lower, it is readily seen that an unreasonable amount of capacity is required.

If starting current is important, the wound-rotor motor offers a better solution to the problem.

If power is generated instead of purchased, it may be necessary to improve power factor to relieve overloaded feeders or an overloaded generator. Most generators are rated on the basis of 80 per cent power factor. Therefore, unless the prime mover is oversize, there is little to be gained by raising the power factor above 80 per cent. For example, if a 500-kw 625-kva 80-per cent-power-factor generator is actually carrying 450 kw at 60 per cent power factor or 750 kva, and if the prime mover is capable of driving the generator with a 550-kw load a capacitor may at first be installed simply to bring the current within the rating of 625 kva. This would take 170 kva. Later, to provide for expansion extra units may be installed.

Assuming the additional load would also be at 60 per cent power factor, it would require 280 kva additional to reduce the total kilovolt-amperes to 625 with a 550-kw load. The corrected power factor would be 88 per cent. Whether the industrial company purchases or generates its power, a power-factor study is warranted because savings may be possible which may far exceed the cost of corrective capacity required.

Multiple Lightning Strokes—II

By K. B. McEACHRON
FELLOW AIEE

THE multiple stroke has been defined as a succession of discharges in substantially the same path, either between clouds or between cloud and ground. Multiple strokes are of considerable interest to the engineer, because of the possible effects of successive application of impulses to equipment with a very small cooling or resting period interposed. The author has undertaken studies of natural lightning in three different locations with three somewhat different methods, one of the objects being in each case the determination of characteristics of multiple strokes. One of these investigations is a detailed study of lightning strokes to the Empire State Building in New York, another in Pittsfield, Mass., employs photographic methods making use of an observatory erected for the especial purpose of studying lightning, and the third is an investigation of multiple strokes to transmission lines as measured by the use of the especially designed crater lamp oscillograph installed and operated in co-operation with the American Gas and Electric Company at Roanoke, Va.

These three investigations have all been in operation for three or more lightning seasons, but because of the large amount of data obtained from the other investigations, it will be necessary to confine the present paper to the results obtained in Roanoke, although naturally the results from the other investigations will influence the point of view.

Photographs showing the multiple character of some strokes of lightning have long been in existence, but it seemed to be important to find out how many of these effectively applied successive impulses to electrical apparatus connected to transmission lines. Fortunately, the application of the expulsion protector tubes supplied a means of determining, through the use of magnetic oscillograph elements arranged to record fault currents to ground, some of the successive discharges of a multiple stroke, since the protector tube allows but one-half cycle of power current to flow following each impulse. Of course such a method does not indicate the presence of more than one discharge in any one-half cycle, and from photographs of strokes it is known that successive discharges may be as close together as a few hundred microseconds. Recognizing this situation, during one lightning season a circuit was set up with a capacitance coupling to the transmission line, in such a manner that it was hoped to superimpose any during-half-cycle impulses, but none were found. This does not indicate that they were not present, rather that the system used was not able to record them, and if they are to be recorded other methods will have to be used. It will be readily understood that on a line not equipped with expulsion tubes the continuous flow of fault current obscures the effect of successive lightning discharges.

Table I. Occurrence of Strokes

			Multiple Strokes			Number of Storms Giving		
Year	Total Strokes	Single Discharges	Number	Per Cent	Discharges	Single Strokes Only	Multiple Strokes Only	Single and Multiple
1934	42	32	10	24	44	11	3	2
1935	21	19	2	9	4	9		2
1936	45	32	13	29	38	6	1	9
1937	76	49	27	35	90	13	2	11
Total	184	132	52	28	176	39	6	24

The crater lamp oscillograph[1] was installed in Roanoke early in 1934, and connected to read fault current to ground in each of the three phase conductors of the number 1 circuit of the double-circuit 138-kv Roanoke-Glenlyn line,[2] and the fault current in phases 1 and 2 of the Roanoke-Danville line. The sixth vibrator was used to record the potential to ground of phase 1 of the Danville line. The results obtained during the first year of operation of the oscillograph were reported to the Institute in December 1934.[3] The present paper will present the results of four lightning seasons beginning with 1934.

Analysis of Results

In analyzing the 295 records obtained in Roanoke, judgment must be exercised because a flow of current on either of the two lines being studied will draw current from the other, since they are bussed together at Roanoke. This is illustrated in the typical oscillogram of figure 1, where the stroke was to the Glenlyn line. It was assumed, however, that simultaneous strokes to the same phase conductors did not occur on the Glenlyn and the Danville lines. While such an occurrence is possible, it is too unlikely to be given weight.

Obviously, the operating data of the two lines were studied carefully, and circuit breaker operations correlated with the crater-lamp-oscillograph records. The *PM*-13 oscillograph at Glenlyn recorded the fault currents flowing in both of the Glenlyn circuits, and the records were studied with engineers of the American Gas and Electric Company. The *PM*-13 records were particularly helpful in studying the performance of the number 2 circuit of the Glenlyn line, which was on the same steel towers but which was not equipped with protector tubes. Oscillograph records which did not cause power follow current through protector tubes were neglected as being too uncertain. If included, these would add to the number of single-stroke records obtained.

Although some data from the Carolina Power and Light Company were included in the 1934 paper by the author, the data in the present paper are confined to the lines of the American Gas and Electric Company.

Frequency of Occurrence

A total of 184 strokes was recorded by the crater lamp oscillograph, of which 52 are known to be multiple. Out of 69 storm days, 39 yielded 73 single strokes; 24 days gave 59 single and 42 multiple

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1. For all numbered references, see list at end of paper.

Figure 1. Crater lamp oscillogram showing operations of expulsion protective gaps on successive lightning discharges to the Roanoke-Glenlyn line

strokes; and on 6 days only, 10 multiple strokes were recorded with no single strokes. These data seem to indicate that certain storms produce few if any multiple strokes, and other storms seem to have on the average about an equal number of single and multiple strokes.

In view of the New York investigation, it does not appear strange to find some degree of similarity between strokes in the same storm as to general characteristics. Cloud formation is probably responsible for the different characteristics of lightning when one stroke is compared to another, and it is probable that the controlling conditions lie in the cloud itself. There is evidence to indicate that the same cloud center will tend to continue to produce the same type of discharge, at least while discharging to objects within a limited area.

When considering the data presented in this paper, it is recognized that great accuracy is not possible, since a certain number of single strokes are not recorded which did not cause tube follow current and multiple strokes with time intervals less than one-half cycle are not identified. In table I the data obtained are given by years.

The importance of the multiple stroke is realized when it is considered that of the discharges recorded, 176 or 57 per cent of the total discharges were multiple, or putting it another way, 28 per cent of all the strokes produced 57 per cent of the discharges.

A brief comparison with the results from Pittsfield, using a multi-lens moving-film camera will be of interest. During the years 1935, 1936, and 1937 a total of 73 storm days were recorded. Two hundred thirty-two strokes were photographed, of which 55, or 23.7 per cent, were multiple. This percentage is lower than the true number, since the photography becomes poor at distances of several miles under weather conditions often existing. The check, however, with the Roanoke data is quite good, indicating that roughly one-fourth of the strokes over a period of years consist of more than one discharge.

Multiple Stroke Data

A summary of the data obtained is given in figure 2, where each multiple stroke is plotted against its time duration in cycles (60 cycles). The maximum time duration is still as it was reported in 1934, namely, 40 cycles. Although, if enough data were available, a somewhat longer time might be found, yet other work not yet published indicated that the probable maximum time will not much exceed one second. When examining these data, it should be realized that there are many multiple discharges which will not have sufficient magnitude to cause a second tube operation, and thus would not get in the record. Thus data, on multiple strokes close by, obtained by photographic means, may differ somewhat from that obtained from an investigation of this sort even of the same strokes. There is also another point of departure, in that another stroke could occur to the ground wire within a second of time at other points and the oscillograph could not identify them as being of the same multiple discharge. This possibility, based on other observations, can be dismissed as being highly improbable in any appreciable number of cases.

Time Interval and Number of Successive Discharges

The time interval between successive discharges is important, since it has to do with the rest period afforded insulation, whether air or solid before being restressed. In figure 3 the data of figure 2 have been replotted to show the time intervals and the frequency of their occurrence per 100 discharges. The longest time was 26 cycles between discharges, while the shortest time will of course be zero cycles when current flows in two succeeding half cycles. The maximum time recorded is of considerable interest from the point of view of deionizing time of the air under lightning conditions.

In Pittsfield, the longest time between successive strokes was 0.44 second, which is slightly over 26 cycles. It might be argued that in the case of the crater-lamp-oscillograph data, a second stroke caused the record at the 26 cycle point, but the Pittsfield photographs show discharges in substantially the same path 0.44 second later, so it appears that a time as long as 26 cycles is possible.

Concerning the minimum time, nine per cent of the discharges occurred with a time interval of one cycle. Lacking means for determining shorter times, in this particular investigation no conclusion can be drawn. However, in the Pittsfield investigation, times shorter than 0.0005 second were measured between discharges.

The number of successive discharges as shown in figure 4 vary up to 12 as a maximum. Sixteen per cent of all the strokes and 61 per cent of the multiple strokes consisted of at least 3 discharges, while 4 per cent of all strokes and 15 per cent of the multiple strokes had at least 5 discharges.

One might suppose that in general the largest number of discharges would go with the greatest stroke duration, and this is true in a very general way, but

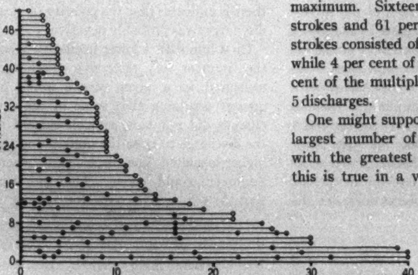

Figure 2. Distribution of multiple strokes and discharges

the data show serious exceptions since one 40-cycle stroke had 12 discharges and a 39-cycle stroke had 3, while a 6-cycle stroke had 4 discharges.

Discussion and Conclusions

This investigation indicates clearly that multiple strokes do take place to transmission line conductors and must be considered in any protection setup. Protective devices designed to protect apparatus from the effects of lightning should not include mechanical motions which take time for restoring to normal condition if proper protection is to be maintained.

In the four-year investigation, 308 discharges were recorded from 184 strokes. Fifty-two multiple strokes yielded 176 discharges. Approximately 25 per cent of all strokes were multiple. The largest number of successive discharges was 12. The longest time between the first and last discharge was, 0.66 second or 40 cycles, while the shortest time is not indicated from this investigation. Other work shows a time as short as 0.0005 second.

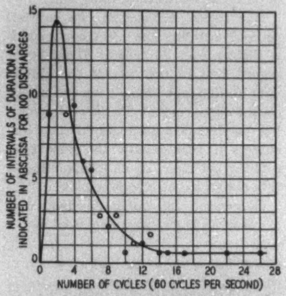

Figure 3. Time intervals between discharges in multiple lightning strokes

As to the characteristics of the successive discharges with regard to the initial discharge, little information is given, except that a study of the data seem to indicate more current is associated with the multiple stroke discharges than in general occurs with strokes having but a single discharge. There is also some tendency for the first discharge of a series to pass more current than the succeeding discharges. These observations are deduced from a study of the number of phases involved with the different discharges.

There appears to be a definite tendency for some storms to produce more multiple strokes than others. In fact as far as the records of strokes to the transmission line are concerned, more than half of the storms did not produce multiple strokes. This is probably related to the cloud structure and the disposition of charges within it.

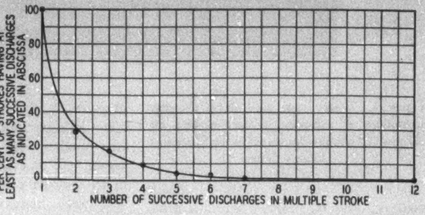

Figure 4. Number of successive discharges in multiple lightning strokes

In general, the multiple strokes which occupy the greatest time between the first and last discharges will have the greatest number of discharges, although this is not always true. The methods used could not distinguish between discharges occurring during a half cycle of protector tube follow current.

The great majority of the follow currents measured were positive in polarity indicating a negative direct stroke. This is in agreement with the work of Lewis and Foust,[4] who found that of 358 strokes recorded during the years 1934, 1935, and 1936 through tower legs of transmission lines, approximately 95 per cent were negative. Since the succeeding strokes did not show a polarity of follow current particularly different from the first discharge of the series, there seems to be no evidence which indicates a reversal of polarity as between the first and succeeding discharges, rather such evidence as there is indicates that the polarity of most discharges was probably negative.

On a line with a lower insulation level, the number of successive discharges measured in a given stroke would be greater assuming that some of the discharges did not have enough current in the discharge to raise the potential of the higher-insulated line to a point where flashover would occur. This indicates that the results given as to the number of successive discharges is probably pessimistic for low-voltage circuits where the apparatus strength may be low compared to the insulation strength of the line.

In thinking about the effects of multiple strokes upon connected apparatus, it should be remembered when the stroke takes place some distance from the station, assuming a line without overhead ground wire protection, and a line flashover occurs, only the first impulse of the series will be transmitted along the line unaffected by the tower footing resistance of the flashed structure. Succeeding discharges in the stroke will develop potentials on the line dependent upon the *IR* drop through the tower footing resistance.

Where overhead ground wires are used, and strokes do not contact the line conductors, the potential of the traveling wave, both for the first discharge and succeeding ones will be dependent upon both the footing resistance and the current in the various discharges. Under these conditions, many low-current discharges which are observed upon moving-film photographs of direct strokes would not be of serious consequence with reference to connected apparatus. It is at this point that the investigation is probably of greatest value, in that it indicates the number of successive discharges which had sufficient magnitude to cause operation of the protector tube.

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Photoelectric Weft-Straightener Control

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THE PROCESS of weaving consists of interlacing spun threads at right angles on a loom, figure 1. In the woven material thus formed, of which cotton cloth is a very common example, the lengthwise threads are designated as the "warp" and the crosswise or filling threads are designated as the "weft," figure 2.

As woven, the weft threads of fabrics are substantially at right angles to the warp threads. Numerous subsequent finishing processes involve rehandling the wet material in rope form. When finally stretched and dried, much of the cloth will not have weft and warp threads at right angles in spite of inspection and manual means for mitigating this defect. This skewed goods tends to change shape with use so that fit or hang or design does not hold up to the purchaser's expectations.

This paper describes a successful automatic means, for squaring up the goods in the final drying and stretching process, thus making it possible to raise materially the textile finishing standards in this respect.

The size of the threads and the number used per unit area varies widely. Likewise grouping, and arrangement of threads to form patterns are subject to wide variations. In general, however, it may be said thread diameters range from 0.005 inch in fine shirting to 0.050 inch in heavy duck. Also, thread counts as low as 20 per inch are used in open-weave marquisettes while in fine shirting as high as 100 per inch are frequently found. Still others use as high as 136 warp threads with 60 weft threads. Cloth is produced in a great variety of widths, ranging from 30 to 108 inches and many hundreds of feet in length.

Cloth as it comes from the loom is strong and serviceable and a limited amount of it is used as "gray goods"; another name for "unbleached cotton cloth," but most of it, some 90-odd per cent of it, is "bleached" to give it that fine white well-known appearance. The bleaching operation involves much rough handling; treatment with chemicals, subsequent washing, and in most cases starch is added to improve its appearance. Cheaper grades are also filled with talc or white clay.

In the process of finishing, the cloth must be transported from one vat to another which may be a thousand feet or more away. Transportation is usually effected by pulling the cloth in rope form from one vat to another, often over a circuitous route guided through large porcelain rings or "pot eyes," figure 3. This is harsh treatment as far as the structure of the cloth is concerned and it results in considerable distortion of the weave from the right angular relationship of the warp and weft threads so accurately woven by the loom.

Following the washing operation, the cloth is "beaten out" in a scutcher, to its original flat form, after which it passes through a starch mangle, figure 4, where the filling starch or clay is added and subsequently through drying apparatus to remove the wash water. The drying operation is accomplished with a combination of dry cans and tenter.

Dry cans consist of a series of steam heated cylinders arranged so that the cloth will pass from one to another, in contact with about three quarters of the circumference of each, figure 4. The temperature of the cans and speed of the cloth is controlled so that when leaving it contains a definite prescribed amount of moisture.

A tenter is a machine designed primarily to restore the cloth to its original width making due allowance for shrinkage, by exerting a suitable pull on the selvage edges at opposite ends of the weft threads, figure 5. Frequently tenters are enclosed in ovens and the sizing and drying operation combined either wholly or in part.

In practice, depending on the type of cloth and the finish required, the cloth may be completely dried on a tenter, or stretched to the required size on a tenter and dried on cans or partly dried on a tenter and finished on cans; but regardless of the system used, the weave distortion or skew introduced in the previous processes must be removed; e. g., the cloth must be "straightened" to make it salable.

Cloth straightness may be defined as the perpendicular relationship between weft and warp threads, also deviations from straightness may be expressed as "skew." It is very essential that cloth be straight, both in plain fabrics as well as those embodying woven patterns. Similarly cloth should be straight before being printed, otherwise, the pattern will be distorted after the cloth is cut into small pieces, such as required for curtains or wearing apparel.

The straightening operation should be performed on the cloth while it is moist as the fibers are then pliable and susceptible of proper positioning. Straightening is accomplished by moving the selvage on one side with respect to the other, an amount necessary to correct the distortion.

Several machines are now on the market for straightening cloth which perform with a fair degree of success. One of the more common is the "swing tenter" in which an oscillating movement of one chain with respect to the other is introduced which tends to "spread" the skew over the entire area, thereby reducing the magnitude of the distortion in any particular area. Strictly speaking, this device could not be classified as a "straightener," but rather one to even out the peak skews to passable limits in cloth where true straightness is not required.

In the "canting roll" type shown in figure 6, the cloth which has been previously spread to its full width is passed over two or more rolls, the ends of the axes of which are made to approach or recede from each other, thereby slackening or stretching the selvage as may be necessary to introduce the required correction.

Still another type of straightener performs the straightening operation in the tenter by advancing or retarding one chain with respect to the other so that the perpendicular relationship is secured in the cloth as it leaves the tenter. Figure 7 shows one way of accomplishing this. A differential gear is introduced in the main drive shaft by means of which the position of the left-hand chain may be advanced or retarded with respect to the right-hand chain by adding or subtracting revolutions to the main drive shaft. The differential is operated by a small motor, which is controlled in terms of cloth skew as observed at the leaving end of the tenter.

With manual control of the motor, the operator must visualize the skew and manipulate the motor switch accordingly. Obviously, this procedure is subject to

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C. W. LaPierre and A. P. Mansfield are employed by the General Electric Company, Schenectady, N. Y.

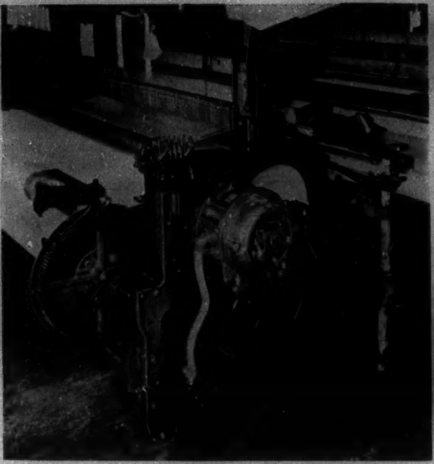


Figure 1. Loom showing cloth in process of being woven

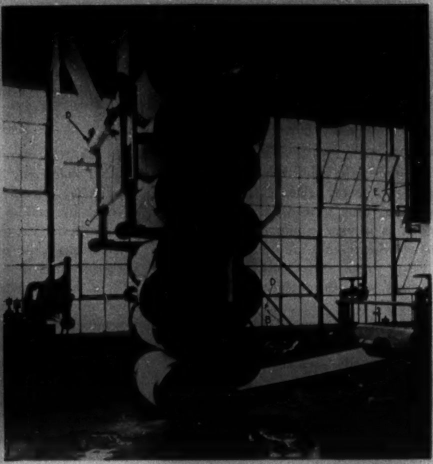


Figure 4. "Dry cans" for drying cloth, with starch mangle at right

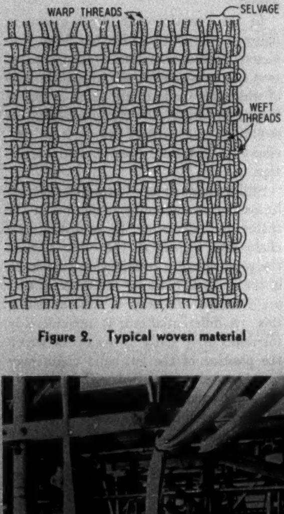


Figure 2. Typical woven material

human limitations and restricted to relatively coarse weaves and low cloth speeds, hence the development of an automatic control became necessary to meet present day standards of finishing with high cloth speeds now employed.

Automatic Control

Principles

To automatically control the weft straighteners just described, a method of detecting skew is necessary. Figure 8 illustrates the method of this paper; (*a*) represents weft threads of cloth of one direction of skew; (*b*) cloth having no skew; and (*c*) cloth having the other direction of skew. Outlined on these weft threads are two convergent paths, *A* and *B*, each traversed in the direction indicated by spots of light. In figure 8(*a*), light spot *A* crosses 15 weft threads and spot *B* nine weft threads; in figure 8(*b*) each spot crosses 12 weft threads; and in figure 8(*c*) *A* crosses nine threads and *B* 15 threads. Thus, if weft threads can be made to intercept two spots of light in this manner, a comparison of the rates at which the two spots are intercepted will indicate the direction and magnitude of the angle of skew.

In a practical case, convergent light spot paths can be obtained by moving the two light spots laterally as in figure 8(*d*).

Figure 3. Transportation in rope form through "pot eye" guide

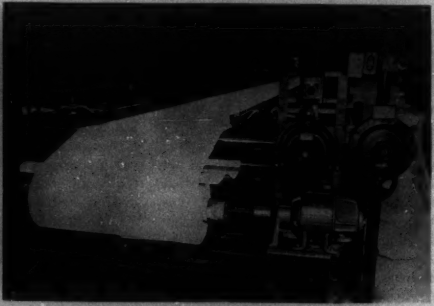


Figure 5. Tenter for restoring cloth to proper width

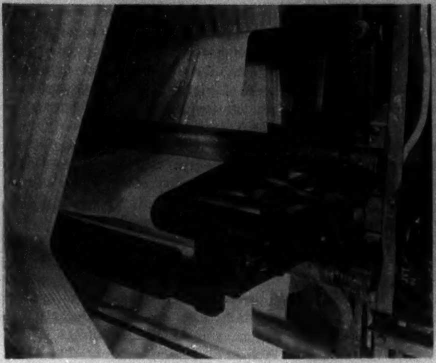


Figure 6A. Canting-roll type of straightener

Figure 6B. Canting-roll type of straightener

The cloth moving forward as the spots move inward will produce the required diagonal paths.

To carry out such a principle two things are necessary; first, suitable scanning detectors which will give an impulse for each weft thread interception in the manner just described; and second, a control circuit which will compare the impulse rates from the two scanning detectors and operate the straightener motor when necessary to make the two detectors generate impulses at the same rate.

SCANNING DETECTOR UNIT

A device for accomplishing the first of these requirements is illustrated in figure 9. A pair of these, one near each edge of the cloth, comprise the complete scanning system of the weft-straightening control.

Each of these scanning units comprises six parts, namely: (1) a scanning optical system; (2) scanning disk; (3) scanning table; (4) detector optical system; (5) amplifier; and (6) a good rigid framework to hold these parts together.

SCANNING OPTICAL SYSTEM

The scanning-unit optical system is shown in figure 10*A* and *B*. The straight filament of the lamp is focused on a slit which in turn is focused on the cloth be means of cylindrical lenses at the end of the optical system housing. The line of light so formed by these lenses is approximately 0.0035 inch by two inches.

The final spot after passing through the scanning disk is 0.0035 inch by 0.063 inch. Its dimensions are determined by the maximum number of threads per inch and their size. Thus it must be thin enough to be completely intercepted by one weft thread. It must be wide enough to span a number of warp threads; otherwise, the warp threads will generate a frequency too. On the other hand, the spot cannot be too wide or at large angles of skew, the weft threads will cross the spot diagonally and not intercept it.

These dimensions are important and are subject to precise calculation for any given set of conditions.

SCANNING DISK

The scanning disk is illustrated in figure 11. It is a simple circular duraluminum disk with 16 radial slots $^{1}/_{16}$ inch wide. The disk is driven at a speed proportional to the cloth speed by means of "Selsyns," the transmitter of which is coupled to the main tenter drive. Thus, the lateral motion of the spot bears a constant relation to the forward motion of the cloth, or in other words, the angle of the path of the light spot is constant.

SCANNING TABLE

Due to the short focal length of the cylindrical lenses, it is necessary that the cloth be held flat at the focus of the light

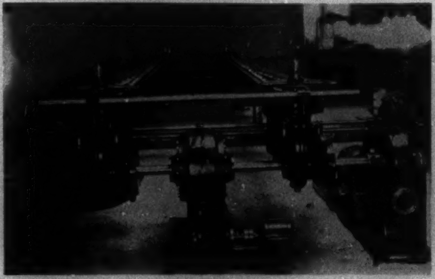


Figure 7. Tenter differential straightener

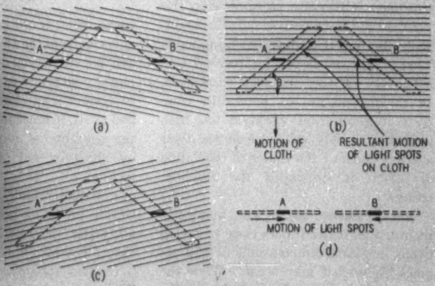


Figure 8. Principles of skew detector

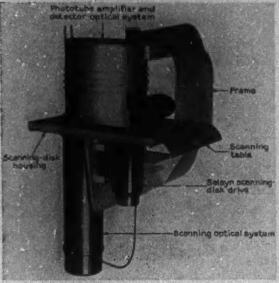

Phototube amplifier and detector optical system
Frame
Scanning table
Scanning-disk housing
Selsyn scanning-disk drive
Scanning optical system

Figure 9. Scanning unit

spot. This is accomplished by passing the cloth over a smooth flat table having a window over the light spot, figure 12. This table is mounted about $^1/_4$ inch above the normal cloth level so the cloth is drawn smoothly taut across it.

DETECTOR OPTICAL SYSTEM

After passing through the cloth, the spot of light strikes a large spherical lens in the phototube amplifier housing where it is focused on a phototube, figure 13.

PHOTOTUBE AMPLIFIER

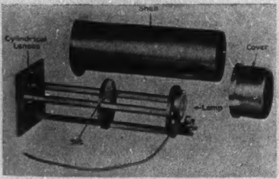
The interruption of the light by the weft threads causes impulses in the output of the phototube. These impulses are amplified by the phototube amplifier before the control transmission unit, figure 14.

The frequency range of the amplifier is from approximately 200 to 12,000 cycles. The gain increases somewhat with frequency to compensate for circuit losses. The primary reason for locating an amplifier in the phototube housing is to change the high-impedance phototube circuit into a low impedance output circuit suitable for transmission to the control unit. Otherwise, an amplifier at this point would have been avoided as excessive vibration is frequently encountered. This vibration has required careful consideration of the amplifier suspension. In the present device, practically complete freedom from vibration has been gained by flexibly hanging from the cover a heavy mass at the center of oscillation of which is suspended the amplifier chassis.

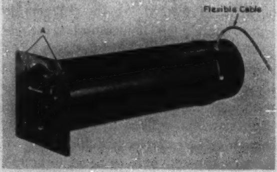
The scanning units are frequently mounted on tenter frames close to drying ovens in which temperatures of 250 degrees Fahrenheit may occur. While the temperature outside is much lower, the phototubes have a maximum allowable temperature of 150 degrees Fahrenheit so in some cases it is necessary to cool the phototube-amplifier housing. This is accomplished by passing cooling water through a few turns of copper tubing soldered to the housing. Very little water is required as the housing should not be cool enough to condense moisture and drip it on the cloth.

CONTROL UNIT

The design of the phototube detector and amplifier as described has proved to be subject to engineering calculation. The design of the control unit, on the other hand, has required a considerable amount of experience to insure proper functioning of the equipment. The control unit performs three major functions: first, it compares the impulse frequencies from the two scanning detectors; and second, it operates the straightener motor contactors in the proper direction to correct any skew which may exist; and third, it must so adjust its control of the straightener motor that it will not overshoot in making the necessary correction.



A—Covers removed



B—Assembled

Figure 10. Scanning-unit optical system

The unit which accomplishes this is shown in figure 15*A*, *B*, *C*, and the schematic diagram of the circuits involved is shown in figure 16.

In brief, the operation is as follows: The output of the two scanning detectors is amplified further and passed to the frequency comparator which compares the impulse frequency from detector *A* with that of detector *B*. If either frequency is higher, a d-c potential appears on capacitor C_1 of polarity dependent upon which is greater, frequency *A* or frequency *B*. The potential, which is a measure of magnitude and direction of skew, then passes through the antiovershoot network and is impressed upon the relay tube which in turn operates relays and they in turn operate small contactors and finally the straightening motor contactor.

FREQUENCY COMPARATOR

Starting with the input amplifier *A* of figure 16, each impulse produces at the output transformer secondaries a full cycle of alternating current. The initial half cycle of this impulse makes tube A_1 conducting and thereby charges capacitor C_{2A} positive. The second half cycle of the impulse shuts off tube A_1 and makes tube A_2 conducting so that the charge accumulated by C_{2A} is passed to C_1. Thus, each impulse from *A* first charges C_{2A} positively to approximately one-half the supply voltage and then dumps this charge into C_1 on the next half cycle.

Similarly, the first half cycle of impulse from *B* charges C_{2B} negatively to approximately one-half the supply voltage and then dumps this charge into C_1 on the next half cycle.

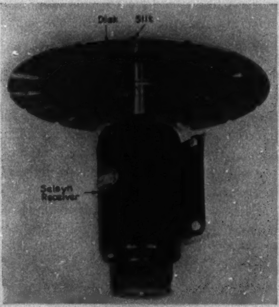


Figure 11. Scanning disk

Thus, if impulses arrive from *A* faster than from *B*, C_1 will accumulate a positive charge, and if they arrive faster from *B* than from *A*, C_1 will accumulate a negative charge. The charge across C_1 is filtered by resistors R_3 and appears across C_3 as the potential *E* which is a measure of the magnitude and direction of the skew.

Overshoot Prevention

Actual experience in operating automatic weft-straightening controls has shown that care has to be exercised to prevent the correction from overshooting. Consequently, the potential E cannot be used directly on a polarized relay to operate the straightening motor.

The network preceding the relay tube prevents overshoot by doing two things: first, it throws the straightener motor on and off in such a manner as to cause the correcting rate to be somewhat proportional to the amount of skew; second, if there is a considerable skew which decreases suddenly due to the cloth coming through more nearly straight, the network will shut off the straightening motor and hold it off until the skew stops decreasing.

The network is composed of simple resistors and capacitors coupled to contacts on the motor relay contactors.

This action may be explained by the circuit diagram. Under the conditions of figure 16, the straightener motor is not operated. If a skew comes along, C_3 takes on a charge E, which throws one of the grids of the relay tube negative so that its relay drops out and closes the motor contactor When this happens, the corresponding contact in the anti-overshoot circuit shifts to "in" placing the capacitor C_4 in series with the corresponding grid. This produces no immediate change because the capacitor C_4 has zero voltage. However, as C_4 becomes charged, the negative voltage on the grid decreases until the relay drops out, the straightener motor stops and the contact returns to "out." Capacitor C_4 is now charged and will discharge through R_3 tending to hold the relay-tube grid positive and keep the motor stopped until C_4 is discharged. If there is still a considerable skew, however, the skew control potential E will exceed the effect of the discharging C_4 and the relay will drop out right away and start the motor again. If E is small, a considerable time will elapse before the grid goes negative enough to start the motor. Thus, the circuit starts and stops the straightening motor frequently and the amount of time the motor runs as compared with the amount of off time is a function of the magnitude of the skew. For large angles of skew, the straightening motor will run most of the time and be off but a small part of the time. On the other hand, for small angles of skew, the motor will be off most of the time and will run only a small part of the time.

This explanation covers more or less the static operation of the circuit. In any particular case, the skew-control potential E is likely to be changing continuously. These changes are transmitted through the capacitor C_4 and if sufficiently large will cause the relay circuit to operate on the direction of the change rather than upon the magnitude of the skew. Thus, if E is increasing, the capacitor C_4 will start the motor running more quickly than would have been the case if E had not changed during the sequence of events. Thus, the circuit anticipates to a considerable extent the coming skew condition of the cloth and controls the straightening motor accordingly.

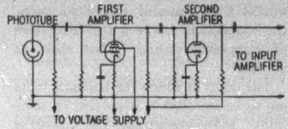


Figure 14. Schematic diagram of phototube amplifier

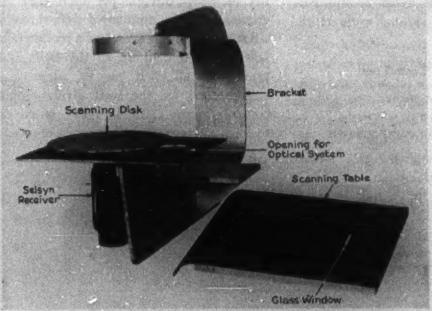


Figure 12. Scanning table with window and main bracket

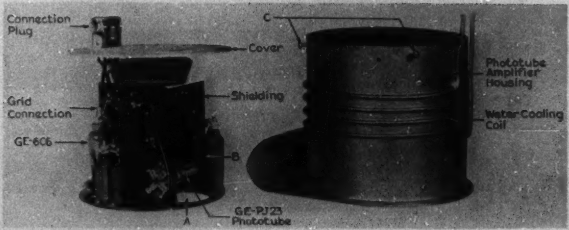


Figure 13 (below). Scanning-unit detector

Relay Circuit

The relay circuit comprises a twin triode with a relay in each plate circuit. The relays are normally closed and are interlocked so that the motor contactors will be closed only if one of the relays opens and the other remains closed.

Control Contactor Circuit

The relays cannot handle the motor contactors directly so some control contactors are also necessary. These control contactors carry the antiovershoot contacts described above.

Safety Circuit

The circuit of figure 16 performs one additional function. In case a large skew-control potential appears, it is probably due to a fault. The cloth may have torn or pulled out of the tenter on one side so that only one detector is functioning or a tube may have failed, the angle of skew may be too great for the detectors to handle, etc. Under these conditions, the safety circuit relay will drop out, opening the motor circuit, light a red lamp or ring a signal, or both.

Associated Equipment

In addition to the scanning and control unit, the other electrical equipment required for automatic weft straightening includes the straightener motor with

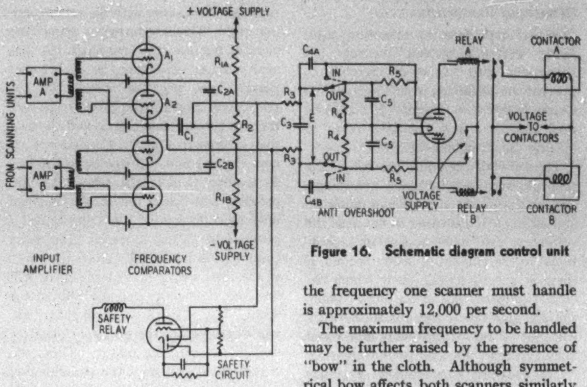

+ VOLTAGE SUPPLY
FROM SCANNING UNITS
AMP A
AMP B
A1
A2
R1A
R1B
C2A
C2B
C1
R2
- VOLTAGE SUPPLY
INPUT AMPLIFIER
FREQUENCY COMPARATORS
SAFETY RELAY
SAFETY CIRCUIT
C4A
C4B
R3
C3
E
IN
OUT
R4
R5
C5
ANTI OVERSHOOT
VOLTAGE SUPPLY
RELAY A
RELAY B
CONTACTOR A
CONTACTOR B
VOLTAGE TO CONTACTORS

Figure 16. Schematic diagram control unit

brake and contactors. These devices must withstand frequent operation and reversals.

Installation

The installation of the equipment is simple, as the complete scanning unit is mounted on one bracket which bolts to the tenter rail, figure 9. The control unit is bolted to a wall or panel by four bolts, figure 15*C*.

Because of the high frequencies transmitted from scanning to control units, the connections between the two pieces of equipment should be as short as possible.

The point at which the scanning units are to be installed is important in some cases. If the cloth under the scanner is completely dry and not pliable, the fact that it is straightened at that point does not mean it will remain straight after it leaves the tenter, because it will tend to return to the condition under which it was dried.

Cloth speed is important because it determines, together with thread count, the maximum frequency of the impulses to be handled. The equipment is now rated at a maximum of 140 yards per minute. At this speed with 80 threads per inch, the thread frequency is 6,720 per second. With a skew of approximately four inches in 40-inch-wide cloth, the frequency one scanner must handle is approximately 12,000 per second.

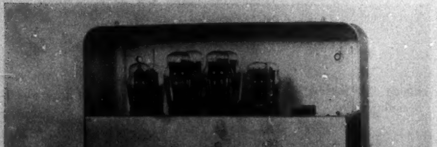
The maximum frequency to be handled may be further raised by the presence of "bow" in the cloth. Although symmetrical bow affects both scanners similarly and consequently does not cause operation of the straightener, bow may, in special cases push the frequency beyond the range of the amplifier, or it may be of such a sharp angle that the weft threads pass the light spot at such an angle that they do not intercept it. Figure 17 illustrates typical forms of skew and bow.

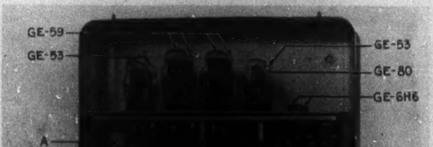
Cloth Limitations

Most cloth is of the type illustrated in figure 2, although in much of it the number of warp threads per inch may be greater than the number of weft threads. All such cloth, however, has a rectangular texture when held up to the light and practically all of it can be adequately straightened by the weft straightening control. Some cloth, however, is woven to form patterns or the threads may not

Figure 15. Control unit

A (left)—Completely assembled
B (lower left)—Main cover removed
C (lower right)—Chassis cover removed


Inside Cover
Connection Plugs
Skew Control


GE-59
GE-53
A
Chassis
GE6R7
Connection plugs
GE-53
GE-80
GE-6H6
B

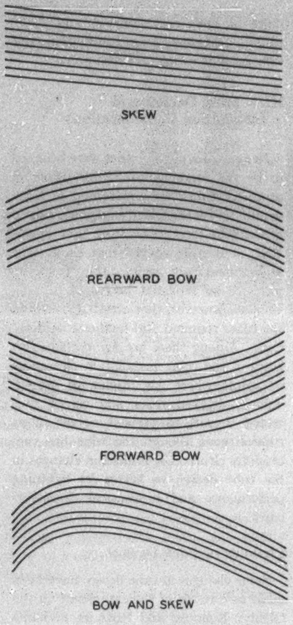


Figure 17. Typical forms of skew and bow

interlace adjacent threads. In such cases, each kind of cloth must be inspected closely to determine whether it can be handled by the automatic scanning unit. In many cases, cloth has actually been straightened which appeared hopeless at first inspection. Loosely woven marquisettes with large tufts have worked very well, even though each tuft blocks off a large part of the optical system.

The optical system was originally designed to handle 100 by 100 thread count which was thought to be about as fine as would be found. Experience now shows, however, that cloth with many more warp threads than weft threads is more difficult to handle than the finest square weaves. For instance, in some types of broadcloth with a weave of 60 by 136, the large number of small warp threads has little or no bending effect upon the relatively large weft threads and consequently the warp threads mat together and in some cases leave practically no interstices.

Another factor decreasing the amount of light is the amount of sizing applied to the cloth. In the cheaper grades of cloth, the interstices appear completely filled. However, the sizing must be at least slightly transparent, because in all cases so far encountered, successful straightening has been accomplished after some experimenting with proper location of the scanning unit.

As might be inferred, a color has no effect upon the operation of the control. The dye is a part of the thread fiber and does not fill the interstices.

Maintenance

In any equipment such as this, the question of maintenance is important. Controls have been in operation over a period of several months without attention and this is to be expected. The equipment contains, however, a number of electronic tubes having a life rating of 1,000 hours although experience has shown this rating to be conservative. The two lamps will ordinarily last several hundred hours, but this will depend to some extent upon the amount of vibration of the particular machine to which they are connected.

The electrical circuit in these controls is necessarily somewhat complicated for ordinary servicing. To facilitate such servicing, the phototube amplifier units and the control units are interchangeable, and in case of failure, a new unit can be substituted in a very few minutes. All connections are made by plugs so changes of units require no wiring.

Application

Several commercial installations of the automatic weft-straightening control have been made in conjunction with the differential-gear-type straightener, typical of which, is that at the Danvers Bleachery, Peabody, Mass. Cotton sheeting after being bleached and washed is finally sized in a twenty-foot tenter where it is automatically straightened, just prior to entering the dry cans. Figure 18 shows the arrangement of the scanning and control units, and figure 19 the complete tenter with dry cans in the background.

Acknowledgments

Any development of this nature is of necessity the result of co-operation of many individuals. The authors wish to acknowledge particularly the contributions of H. A. MacKnight of the Winsor and Jerauld Manufacturing Company who saw the need and participated in the application of the device; also the contributions of T. M. Berry and E. F. Travis of the General Electric Company for many of the essentials that have made the equipment so successful in the field.

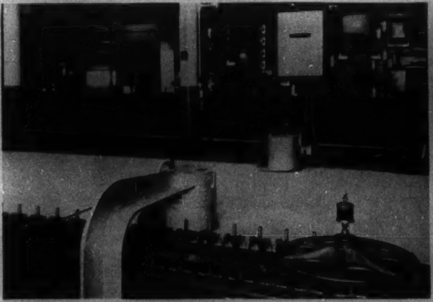


Figure 18. Close-up of Danvers Bleachery Installation

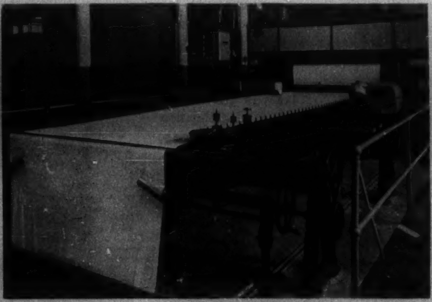


Figure 19. Installation at Danvers Bleachery

Protector-Tube Application and Performance on 132-Kv Transmission Lines—II

By PHILIP SPORN
FELLOW AIEE

I. W. GROSS
ASSOCIATE AIEE

I. Introduction

THE USE of protector tubes (previously referred to as expulsion protective tubes or "deion" gaps) during the past five years or so as devices for protecting transmission lines and equipment against lightning has gained considerable headway, particularly in the protection of high-voltage transmission lines. A number of applications have been made in the medium voltage transmission field, a few in the low-voltage field, and a number of scattered installations for protection of equipment and cables.

Since the protector tube was first proposed[1] as a protective device, considerable experience has been gained in making practical use of this device in the field. The development of any new type of equipment usually consists of a period of experimental work in the laboratory followed by field tests and then more or less limited operating experience before the merits of the device are fully proved. This situation has been true of the protector tube, which is now well past the first two stages of its development. As a matter of fact, the application and operating performance of the protector tube have found quite frequent space in the technical literature during the past few years.[2,3,7,9–13]

The performance of protector tubes on two 132-kv lines was described by the authors in a paper[3] presented two years ago. Since that time three additional 132-kv lines of the interconnected transmission system have been equipped with protector tubes. It is the purpose of this paper to now discuss the tube application to these lines, to give their operating performance, and to summarize the experience which has been obtained on the application and performance of protector tubes on some 250 miles of 132-kv lines equipped with these devices.

II. System Analyzed

The lines on which protector tubes were installed are 132-kv steel-tower lines of the operating companies of the American Gas and Electric Company. These lines are constructed with one ground wire and use standard suspension insulators. The circuits are in vertical configuration. All tower structures are of the double-circuit type. The Roanoke-Fieldale, Fieldale-Danville, and Turner South Point lines are single-circuit. The Glenlyn-Roanoke line is double-circuit, one circuit only being equipped with protector tubes. The Philo-Newcomerstown and Newcomerstown-Canton lines were formerly a continuous line from Philo to Canton, but during the period when equipped with tubes, they have operated as separate line sections, although still located on the same double-circuit tower structures which carry the still-existing Philo-Canton number 1 line. More complete information on the characteristics of the above lines has been presented previously.[4]

In table I is given pertinent information on these lines as regards protector-tube application. It will be noted that the tube-equipped lines comprise 255 miles of line, 1,080 towers, and 3,240 tubes. Dates of tube installation and tube current ratings are also given.

III. Tube Design and Installation Considerations

In an earlier paper,[3] what were believed to be the fundamental requirements of protector tubes were presented in some detail. With the added experience gained in the past three years from tube operations, it is still believed that these same requirements are sound; they therefore will not be repeated here. It has developed, however, that certain features of the tubes required still further consideration. Among these are aging characteristics of the tube, installation clearances at points where gas discharges are involved, delayed relay settings of protected circuits to prevent circuit interruption even though the tube has successfully cleared the fault, and changes in the tube design to better its lightning performance and mechanical characteristics.

CHANGES IN TUBE DESIGN

Some changes in tube design have been made as a result of the experience on the Glenlyn-Roanoke and Roanoke-Fieldale-Danville lines previously reported.[3] Such a change was the result of an attempt to provide a better voltage grading along the surface of the tube. One method of accomplishing this is shown in figure 1 where grading shields have been applied to the tubes. This additional grading was provided on the Roanoke-Fieldale and Fieldale-Danville lines prior to the 1937 lightning season.

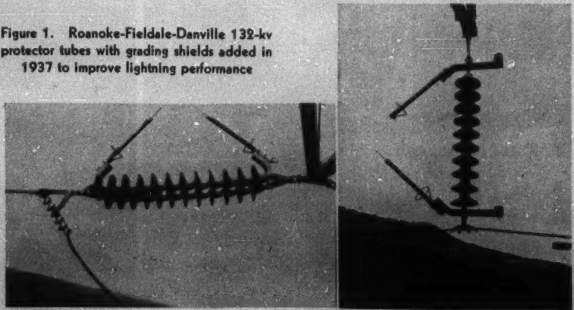


Figure 1. Roanoke-Fieldale-Danville 132-kv protector tubes with grading shields added in 1937 to improve lightning performance

Another method used to improve grading along the tube was the use of a resistance material built into the tube itself. This type of construction was used on the more recent tubes installed on the

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PHILIP SPORN is vice-president in charge of engineering of the American Gas and Electric Service Corporation, New York, N. Y., and I. W. GROSS is electrical research engineer for the same organization.

1. For all numbered references, see list at end of paper.

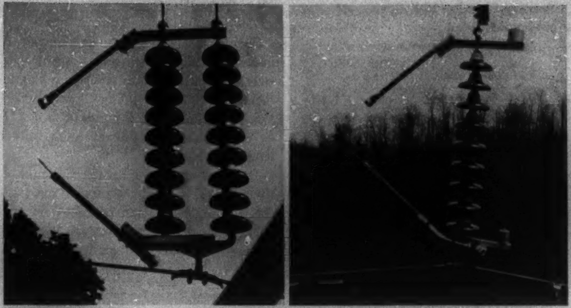

Figure 2. Protector tubes—V type—on Turner-South Point 132-kv line. Note double angle vent at air-gap end of ground tubes, and at bottom end of line tube on single-suspension assembly. Double-suspension assembly has L vent at bottom end of line tube. These types used on Philo-Canton and Turner-South Point lines

Philo-Newcomerstown, Newcomerstown-Canton, Turner-South Point, and most of the new tubes installed on the Glenlyn-Roanoke.

A second change made in the new tubes installed on the Philo-Canton, Newcomerstown-Canton, and Turner-South Point lines is shown in figure 2. This change deals largely with the venting of the gases. It will be noted that gases which formerly vented along the axis of the tube straight into the external gap have now been diverted with a Y fitting at the free end of the tube and so designed as to throw the gases clear of any conductor or grounded structure in the direct path of the discharge. Similarly, venting at the rigid end of the tube has been changed to divert the gases by an L fitting to throw them away from the conductor or other hardware. This change in design has been of a precautionary nature rather than on account of any extensive trouble developed in the initial design.

RELAY SETTINGS

Since the protector tube normally clears a fault in approximately a half cycle, precautions must be taken in the relay setup to insure that high-speed relays do not initiate the tripping circuit during the period when the tube is carrying current. In the tube installations on the Philo-Newcomerstown and Newcomerstown-Canton lines, it was recognized that the relay setup was such that in heavy short circuits near Philo, there was a possibility of the trip circuit being set in action even with normal tube operation. However, to check the limit of low time to which these relays could be set, they were permitted to operate for two seasons with a minimum time of approximately one to $1^1/_2$ cycles. During the last two years, eight relay operations have taken place with perfect tube performance thus indicating that relay time is too short. Three of these occurrences took place in 1936 and five in 1937. However, no service interruptions were experienced. Positive indication that this low time setting of the relay was the cause of the outage was evidenced by the fact that the circuit did not trip at the opposite end.

Results of tests, not reported in detail here, indicate that successful tube operation has occurred even in three cycles. This indicates that a minimum relay time of something in the order of two to three cycles may be desirable on a circuit where tubes are installed. Another alternative may be the retention of one-cycle relaying and the use of ultra rapid reclosing.

PROTECTOR-TUBE CURRENT RATINGS

To successfully interrupt short-circuit currents on the Philo-Canton line protector tubes, it was necessary to develop a tube of higher current rating than previously available. The design work, which was carried on prior to the instal-

Table I. Summary of 132-Kv Protector-Tube Installations

Line Designation	Length (Miles)	Number Towers	Number Tubes	Date Installed	Calculated Fault Amps* Max.	Calculated Fault Amps* Min.	Tube Rating (Amperes*) Max.	Tube Rating (Amperes*) Min.	Vents
Glenlyn-Roanoke	65	270	810	7/ 1/33	1960 3990	660 1250	2500 5000	600 900	Angle†
Roanoke-Fieldale	37	164	492	4/10/34	1960	660	2500	600	Straight
Fieldale-Danville	31	137	411	4/10/34	1350	700	2500	600	Straight
Philo-Newcomerstown	37	140	420	9/ 1/35	9400	1400	10000	1300	Angle
Newcomerstown-Canton	37	164	492	9/ 1/35	5100	1400	7500	1200	Angle
Turner-South Point	48	205	615	4/15/36	3160 5830	1000 1980	5000 7500	900 1200	Angle
Total	255	1080	3240						

† Straight vents prior to 1937.   * Root-mean-square.

Table II. Protector Tube Operations,* Glenlyn-Roanoke 132-Kv Line
(Data From Physical Inspection)

		Fiber Tubes				Composite Tubes	
	Year	1933	1934	1935	1936	1937	Total
3-phase installations in service		270	270	270	270	270	
Equivalent single-phase installations in service		810	810	810	810	810	
Normal tube operations:	Top phase	1	14	10	35	30	90
	Middle phase	1	9	6	16	17	49
	Bottom phase	1	8	3	14	4	31
	Top and middle phases	2	8	1	10	6	27
	Top and bottom phases	1	5	3	8	5	22
	Middle and bottom phases	0	3	2	6	2	13
	Top, middle, and bottom phases	2	8	1	8	7	26
Total tubes		16	87	34	137	98	372
Total towers		9	55	27	97	71	259
Tube flashovers:	Top phase	0	5	2	9	2	18
	Middle phase	0	4	5	10	0	19
	Bottom phase	0	1	0	5	1	7
	Top and middle phases	0	0	1	0	0	1
	Top and bottom phases	0	0	0	0	0	0
	Middle and bottom phases	0	1	0	1	0	2
	Top, middle, and bottom phases	0	0	0	1	0	1
Total tubes		0	12	9	29	3	53†
Total towers		0	11	8	26	3	48

* Exclusive of tube target indications due to wind or vibration.
† Including two cases of tube flashover to tower.

Table III. Protector-Tube Operations,* Roanoke-Fieldale 132-Kv Line
(Data From Physical Inspection)

	Composite Tubes			With Shields	
Year	1934	1935	1936	1937	Total
3-phase installations in service	164	164	164	164	
Equivalent single-phase installations in service	492	492	492	492	
Normal tube operations: Top phase	12	2	20	15	49
Middle phase	9	6	11	20	46
Bottom phase	13	8	15	20	56
Top and middle phases	5	0	8	7	20
Top and bottom phases	1	0	2	12	15
Middle and bottom phases	3	1	8	6	18
Top, middle, and bottom phases	11	5	14	17	47
Total tubes	85	33	124	156	398
Total towers	54	22	78	97	251
Tube flashovers: Top phase	0	3	1	2	6
Middle phase	0	0	3	4	7
Bottom phase	1	0	2	3	6
Top and middle phases	0	0	0	0	0
Top and bottom phases	0	0	0	0	0
Middle and bottom phases	0	0	0	1	1
Top, middle, and bottom phases	0	0	0	0	0
Total tubes	1	3	6	11	21
Total towers	1	3	6	10	20

* Exclusive of tube target indications due to wind or vibration.

Table IV. Protector-Tube Operations,* Fieldale-Danville 132-Kv Line
(Data From Physical Inspection)

	Composite Tubes			With Shields	
Year	1934	1935	1936	1937	Total
3-phase installations in service	137	137	137	137	
Equivalent single-phase installations in service	411	411	411	411	
Normal tube operations: Top phase	14	0	15	14	43
Middle phase	8	0	7	15	30
Bottom phase	8	0	13	8	29
Top and middle phases	4	0	1	5	10
Top and bottom phases	3	0	2	3	8
Middle and bottom phases	7	0	1	4	12
Top, middle, and bottom phases	4	0	9	8	21
Total tubes	70	0	70	85	225
Total towers	48	0	48	57	153
Tube flashovers: Top phase	0	0	0	0	0
Middle phase	0	0	0	0	0
Bottom phase	0	0	0	0	0
Top and middle phases	2	0	0	0	2
Top and bottom phases	0	0	0	0	0
Middle and bottom phases	0	0	0	0	0
Top, middle, and bottom phases	0	0	0	0	0
Total tubes	4	0	0	0	4
Total towers	2	0	0	0	2

* Exclusive of tube target indications due to wind or vibration.

lation of tubes for the Turner-South Point and Philo-Newcomerstown-Canton lines, resulted in a tube with a maximum 60-cycle root-mean-square rating of 10,000 amperes. The field test of such a tube successfully discharging 17,800 crest amperes is shown in figure 3. The current record of such a tube discharging is given in the oscillogram of figure 4. Similar tests were reported previously on protector tubes before they were actually installed on the Glenlyn-Roanoke and Roanoke-Fieldale-Danville lines.[2,3]

TYPES OF TUBE INSTALLATIONS

The installations previously reported[3] utilized protector tubes mounted on the insulator assemblies with three types of construction. In one, the tubes on the insulator strings were of the so-called V type, in the other, of the parallel type, and in the third, of the 30-degree type but mounted on the structure. In the two more recent installations made on the Turner-South Point and Philo-Canton-Newcomerstown lines, the types of installations have all been of the V type. This procedure was followed due to the fact that the 30-degree type is not particularly desirable from a maintenance point of view as it clutters the tower structure rather badly. Further experience with the parallel and V types had up to that time shown no particular superiority of either, and since the V type was somewhat simpler to install, the decision was made to make the installation on this basis.

IV. Tube Performance

Performance of the tubes themselves under lightning conditions during the time they have been in service up to and including 1937 (excluding Turner-South Point for 1937 only) is given in tables II to VI, inclusive, for the six line sections listed in table I.

Before commenting on the contents of these tables, it should be pointed out that the initial installation of tubes on the Glenlyn-Roanoke line made in 1933 was replaced in 1937 with an improved de-

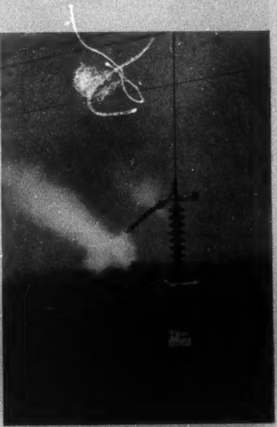

Figure 3. Field test of 1,300/10,000-ampere, 132-kv protector tube discharging satisfactorily 17,800 crest amperes in one-half cycle

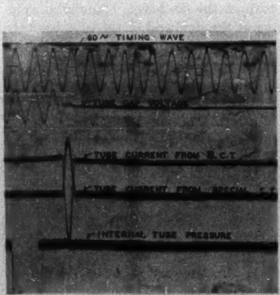

Figure 4. Protector-tube test successfully interrupting 17,150 crest amperes in one-half cycle. This type of tube now in service on Philo-Canton 132-kv line. (Field test November 15–17, 1934—internal tube pressure measured as approximately one ton per square inch)

Table V. Protector-Tube Operations,* Philo-Newcomerstown 132-Kv Line
(Data From Physical Inspection)

	Composite Tubes			
Year	1935	1936	1937	Total
3-phase installations in service	140	140	140	
Equivalent single-phase installations in service	420	420	420	
Normal tube operations: Top phase	0	20	17	37
Middle phase	0	3	9	12
Bottom phase	0	1	2	3
Top and middle phases	0	1	4	5
Top and bottom phases	0	0	2	2
Middle and bottom phases	0	1	0	1
Top, middle, and bottom phases	0	2	4	6
Total tubes	0	34	52	86
Total towers	0	28	38	66
Tube flashovers: Top phase	0	1	0	1
Middle phase	0	0	0	0
Bottom phase	0	0	0	0
Top and middle phases	0	0	0	0
Top and bottom phases	0	0	0	0
Middle and bottom phases	0	0	0	0
Top, middle, and bottom phases	0	0	0	0
Total tubes	0	1	0	1
Total towers	0	1	0	1

* Exclusive of tube target indications due to wind or vibration.

Table VI. Protector-Tube Operations,* Newcomerstown-Canton and Turner-South Point 132-Kv Lines
(Data From Physical Inspection)

	Newcomerstown-Canton, Composite Tubes				Turner-South Point
Year	1935	1936	1937	Total	1936
3-phase installations in service	164	164	164		205
Equivalent single-phase installations in service	492	492	492		615
Normal tube operations: Top phase	0	13	22	35	14
Middle phase	0	8	10	18	6
Bottom phase	0	1	5	6	6
Top and middle phases	1	6	3	10	5
Top and bottom phases	0	2	1	3	2
Middle and bottom phases	0	2	0	2	0
Top, middle, and bottom phases	0	2	3	5	4
Total tubes	2	48	54	102	37
Total towers	1	34	44	79	52
Tube flashovers: Top phase	0	0	0	0	0
Middle phase	0	0	0	0	0
Bottom phase	1	0	0	1	0
Top and middle phases	0	0	0	0	0
Top and bottom phases	0	0	0	0	0
Middle and bottom phases	0	0	0	0	0
Top, middle, and bottom phases	0	0	0	0	0
Total tubes	1	0	0	1	0
Total towers	1	0	0	1	0

* Exclusive of tube target indications due to wind or vibration.

Table VII. Performance of Flashed-Over Protector Tubes

Tube Operation	Glenlyn-Roanoke	Roanoke-Fieldale	Fieldale-Danville	Total
Tubes with one flashover	51	20	4	75
Tubes with two flashovers	1	1	0	2
Tubes with: one normal operation before flashover	7	4	0	11
two normal operations before flashover	0	6	0	6
one normal operation after flashover	5	2	2	9
two normal operations after flashover	0	0	0	0

sign of tube. The initial tubes were all fiber and had reached the end of their useful life at the end of four years. Since it was found that the fiber, when exposed without protection to the weather, had experienced serious deterioration, the new tube installation on this line, as well as all other tubes now in service on the 132-kv system, have the improved waterproof Textolite covering. In the analysis that follows, therefore, it should be borne in mind that the performance of the Glenlyn-Roanoke line during 1936, when the all-fiber tubes were reaching the end of their useful life, is somewhat worse than would be expected from more modern design tubes even in only reasonably good condition. The data in the tables definitely indicate that tube deterioration on the original tubes had become serious by 1936.

Of the 810 tubes in service on the Glenlyn-Roanoke line during the past five years, there has been a total of 372 successful tube operations, 51 tube flashovers, and 2 tube failures where internal pressure burst the tube (table X). On a percentage basis this gives 87.1 per cent successful tube operations, 12.4 per cent flashovers and 0.5 per cent failures. However, if the high flashover record of 1936, when the all-fiber tubes had badly deteriorated is omitted, the record shows 93 per cent successful operations, 6 per cent flashovers and 0.5 per cent failures.

Of the 492 tubes in operation on the Roanoke-Fieldale line (table III) 398 normal operations occurred during the four-year period. There were 21 tube flashovers and one tube failure. This performance can be stated on a percentage basis as 94.8 per cent successful operations, 5 per cent tube flashovers, and 0.24 per cent tube failures. On the Fieldale-Danville line where 411 tubes were installed 98 per cent of the tube operations were successful and two per cent resulted in tube flashovers. There were no tube failures on this line in four years' operation. As the tube type, rating, age, and method of installation were identical on the two above lines (Roanoke-Fieldale and Fieldale-Danville) the difference in tube performance can be accounted for only on the basis of frequency and severity of lightning conditions encountered.

The record of tube operations on the Philo-Newcomerstown, Newcomerstown-Canton, and Turner-South Point lines (tables V and VI) was much better. For these three lines there have been two flashed-over tubes in addition to 123 successful operations, and two ruptured tubes. This gives 97 per cent successful operations, 1.5 per cent tube flashovers and 1.5 per cent tube failures.

It will be noted in general that a very much better performance is indicated on the Fieldale-Danville, Philo-Newcomerstown, Newcomerstown-Canton, and Turner-South Point lines than was obtained on the initial installation of tubes on the Glenlyn-Roanoke line where the all-fiber tubes were used. It is believed that this better performance is due in part not only to the comparatively short time the tubes have been in service (two to three years) but also to the improvements in design on the newer tubes worked out in the period elapsed between the instal-

Table VIII. 132-Kv Protector Tubes Disrupted in Service (1933 to 1937, Inclusive)

Ref.	Line	Year	Phase	Tube Location	Line Tripout	Remarks
8R	G-R	8/ 8/37	Middle	Line	No	Tube ruptured. Top and bottom phase tubes operated O.K. Tower current 10,300 amperes. Ground-wire current 17,700 amperes.
1R	G-R	6/17/37	Middle	Ground	No	Tube ruptured. Severe lightning. No other tubes operated. No tower current.
113	R-F	9/10/36	Middle	Line	?	Tube ruptured. Top phase tube operated O.K. Bottom phase tube flashed over.
246	N-C	8/21/36	Top	Ground	No	Tube blown off. Fiber threads stripped.
6	P-C	8/25/36	Top	Line	No	End of tube blown off.

Table IX. 132-Kv Protector Tubes Having Repeated Operations (Normal and Flashover)
(Observed by Target Indication)

Number of Tube Operations	Glenlyn-Roanoke		Roanoke-Fieldale		Fieldale-Danville		Total Number	Average Per Cent
	Number	Per Cent	Number	Per Cent	Number	Per Cent		
2	63	12.4	79	16.0	42	10.2	184	12.9
3	13	1.6	14	2.9	2	0.5	29	1.7
4	8	1.0	1	0.2	0	0	9	0.4
5	0	0	0	0	0	0	0	0
Years operation	5		4		4			

lation on the Glenlyn-Roanoke line and these installations.

An analysis of the performance of the flashed-over tubes, made in table VII shows that out of 75 tubes which flashed over, 11 or 13.7 per cent experienced only one normal operation before flashover. Six or eight per cent experienced two normal operations before flashover. Further analysis shows that nine tubes or 12 per cent underwent one normal operation before external flashover had occurred. The above operations are, of course, those noted by target indication and may have actually been somewhat greater due to the possible occurrence of multiple strokes.

Tubes Blown up in Service

When initially applying protective tubes for line protection, the possibility of tube failure due to exceedingly high lightning currents and to 60-cycle currents either above or below the tube rating was considered. As pointed out in table I, the 60-cycle maximum and minimum short-circuit currents of each line were calculated, and tubes selected to cover the range of 60-cycle currents expected. It is gratifying, as will be seen from table VIII, that the record of exploded or blown-up tubes for all lines during the entire time when tubes have been in operation, totals only five, and that of these five tube failures, four resulted in no line tripout, and there is no absolute certainty that a line tripped out even in the fifth case. The experience cited, therefore, seems to indicate that the tubes have been applied within their actual rating, and that the tubes have the ability to safely interrupt 60-cycle currents that can reasonably be demanded of them. Trouble from excessive lightning stroke currents disrupting a tube and producing a line outage appears, from records so far obtained, to be negligible.

Repeated Operation of Tubes

The number of times a tube may be called upon to interrupt currents in service must be considered in estimating the life of the tube. To throw some light on this situation, multiple operations recorded on the tubes from physical inspection of targets in the field have been analyzed and are given in table IX. This record shows that during the five-year period of operation studied here, 184 tubes have operated twice, 29 tubes three times, 9 tubes four times, and none five times or more.

As pointed out under "Multiple Strokes" discussed later, additional duty

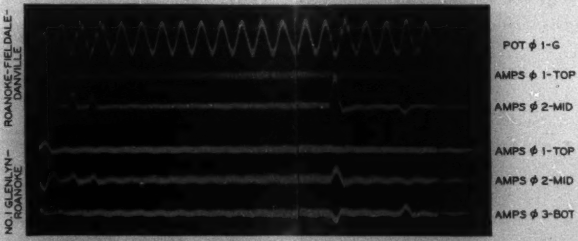

Figure 5. Multiple lightning stroke (five strokes in eighteen cycles) cleared successfully by protector tube. (Oscillogram number 20—June 18, 1937—1:05 p.m.—stroke to Roanoke-Fieldale-Danville 132-kv line.) Note reversal of line polarity at instant of strokes

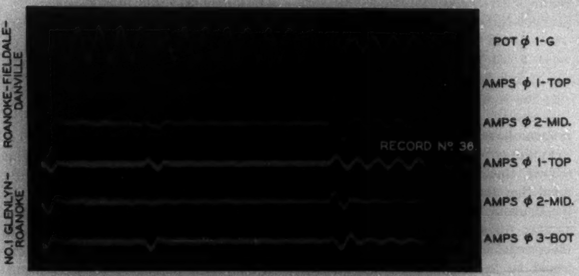

Figure 6. Multiple lightning stroke (three strokes in 13$^{1}/_{2}$ cycles) which developed into line fault (Oscillogram number 36—July 10, 1937—4:45$^{1}/_{2}$ p.m.). Stroke to Roanoke-Fieldale-Danville 132-kv line

Table X. Line Outages Caused by Lightning on Tube-Equipped 132-Kv Lines Before and After Tube Installation

	Glenlyn-Roanoke	Roanoke-Fieldale*	Fieldale-Danville*	Philo-Newcomerstown**	Newcomerstown-Canton**	Turner-South Point
Length of line (miles)	65	37	31	37	37	48
Years in service without tubes	6	7	7	11	11	1
Years in service with tubes	5	4	4	2	2	2
Lightning outages:						
1927	16	9.5	7.9	1	1	—
1928	4	7.6	6.3	3.5	3.5	—
1929	23	12.1	10.1	3	3	—
1930	10	17.5	14.5	1.5	1.5	—
1931	22	13.3	11.3	3.5	3.5	—
1932	4	4.5	3.8	1.5	1.5	—
1933	0†	9.7	8.1	2.5	2.5	—
1934	9	1	2	6	6	—
1935	5	3	1	7	6	2††
1936	28	7	1	2	1	1¶
1937	4	12	0	1	2	0
Outages per 100 miles of line per year without tubes	20.0	29.0	29.0	8.86	8.56	4.16
With tubes	14.2	15.0	3.2	4.1	4.1	1.04
Ratio: with tubes to no tubes	0.71	0.52	0.11	0.46	0.47	0.25
Ratio: average of six lines	—	—	0.42	—	—	—

* Outages prorated 1927–1933 when part of Roanoke-Roxboro line. ** Outages prorated 1927–1935 when part of Philo-Canton line. † Three outages occurred before tubes were installed July 1, 1933. †† In service seven months. ¶ Tripped at one end only; relays set for three cycles.

Table XI. Lightning Outages on Two-Circuit Lines With and Without Tubes on One Line

	Philo-Newcomerstown-Canton			Glenlyn-Roanoke		
Year	Tube Circuit (Total)	Nontube Circuit (Total)	Both Circuits (Only)	Tube Circuit (Total)	Nontube Circuit (Total)	Both Circuits (Only)
1927	2	3	1	16	13	3
1928	7	7	2	4	5	2
1929	6	7	2	23	17	10
1930	3	4	3	10	6	5
1931	7	6	5	22	23	15
1932	3	4	2	4	4	2
1933*	5	5	4	3†	5	2†
1934	12	21	8	9	8	1
1935**	13	13	6	5	4	2
1936	3	4	0	28	19	8
1937	3	9	0	4	6	0
Tripouts per 100 miles of line per year: without tubes	8.7	10.5	5.0	20.0	17.2	9.4
with tubes	4.1	8.8	0	14.2	13.0	3.4
Ratio: with tubes to no tubes	0.48	0.84	0	0.71	0.76	0.36

* Tubes installed on Glenlyn-Roanoke line July 1, 1933. ** Tubes installed on Philo-Newcomerstown-Canton line September 1, 1935. (One tripout on tube line after this.) † These tripouts occurred before tubes were installed.

is placed on the tube by the multiple stroke. The above data give, however, fairer indications of how many times lightning may be expected to occur at a given location than they indicate how many times a particular tube may be called upon to operate. For example, if a tube is subjected to the effect of four lightning strokes in a five-year interval, and these are multiple strokes each time and the multiple stroke has, say seven successive discharges, the tube would, of course, be subject to 28 discharges during the five-year interval. From the fact that only five tubes have been completely disrupted by internal pressure in the five-year experience cited herein, and four of these tubes failed (perhaps under conditions of direct lightning stroke) it seems quite evident that our experience to date does not indicate that sufficient internal erosion can be expected in the tubes, in four years at least, to become a serious hazard to their life or performance.

V. Line Outage Record

While the performance of the tube itself, that is, its ability to interrupt 60-cycle or lightning currents without damage to itself or without external flashover is important, it must not be forgotten that the ultimate goal of the tube is protection of the line from flashover or tripout. It is therefore of paramount interest to study the record of the tube-equipped lines both before and after the tubes were applied.

The outage record of the six line sections previously referred to is given in table X from 1933 (or date of line installations) to 1937, inclusive. The record of outages on a 100-mile-per-year basis both before tubes were installed and after are given in the lower part of the table. It will be noted that the line outages range from ten per cent to 70 per cent of the outages on the same lines prior to tube installation. Such a comparison, of course, should include an evaluation of the lightning severity and frequency during the period under comparison, two factors which are difficult to evaluate. In general, however, it will be noted that the reduction in outages made averages well over 50 per cent when tubes are used.

While the performance of the protector tube is to prevent line outages, it is also true that when installed on one circuit of a double-circuit line where one circuit only is sufficient to carry the load momentarily during an interruption, the number of double-circuit outages is a major basis on which to judge the tube performance. It will be noted in table XI that in the two-circuit lines involved (Philo-Newcomerstown-Canton and Glenlyn-Roanoke) the two-circuit outages have, for a two-year period, been entirely eliminated on the Philo-Newcomerstown line and have been reduced to 36 per cent on the Glenlyn-Roanoke line. Excluding 1936 when the Glenlyn-Roanoke tubes had deteriorated, the double-circuit outages on this line averaged 1.25 per year for a four-year period or 13.3 per cent of the average double-circuit outages before tubes were installed. Another point of interest in this table is that the outages on the nontube-equipped line in each case have likewise been reduced in the order of 16 to 24 per cent. While the periods analyzed are comparatively short, particularly in the case of the Philo-Newcomerstown-Canton line and it may therefore not be representative of what can be expected over a long period of time, it appears, however, that the tube-equipped line has reduced the number of outages on the nontube line. It is quite reasonable to expect this as well as a reduction in double-circuit outages.

VI. Multiple Strokes

During the time that the protector tubes were in service on the Glenlyn-Roanoke line, an investigation on this line jointly conducted with the General

Electric Company has produced some interesting data on the lightning conditions under which protector gaps are required to operate. A crater-lamp oscillograph[6] was installed at Roanoke for the past four years and many records of the tube performance were obtained thereby. The summarized data on multiple lightning strokes as obtained by the crater-lamp oscillograph in conjunction with protector-gap operations is shown in table XII. It will be noted that successive strokes in one lightning discharge range from two or seven. The polarity of the conductor when line fault is initiated, that is, when lightning presumably strikes, might be expected to be predominantly positive on the basis that some 90 per cent or more of observed lightning strokes to line are of negative polarity. The record in table XII shows that of 300 faults, including single and multistroke faults, in 59 per cent of the cases, the faulty conductor is positive, in 34.5 per cent they are negative, and in 6.5 per cent of the cases, the conductor is near zero potential. Even with the multiple stroke there is no consistency shown in the stroke occurring successively to the conductor or to the conductors which are positive. The conclusion may be drawn therefore although the tendency to be struck is much greater when the line is positive than when it is negative, nevertheless when lightning is ready to strike the line, it often strikes irrespective of the 60-cycle polarity of the line wires.

VII. Conclusions

From the data presented above based on five years of operation in service, and other records and related information too voluminous to present and discuss here, the following conclusions seem justified:

1. Protector tubes, when applied within their rating, appear able to operate without mechanical rupture under all but a few infrequent currents encountered on a transmission system. Even when rupture occurs the faulted line may not and generally will not trip out.

2. Internal tube erosion, as an agent in reducing the current interruption capacity of the tube has not, in five years' operation, appeared as a factor limiting the tube life.

3. The multiple lightning stroke is a factor tending to shorten the tube life of the tube. However, rapidly recurring discharges in a multiple stroke have been successfully interrupted by the tube (three discharges in three cycles have been interrupted satisfactorily).

4. In applying tubes for line protection considerable care should be given to adequate disposal of the discharge gases. Two cases were found where the tubes had apparently flashed to the tower as a result of improper dispositions of gases. Several cases of slightly pitted conductors have also been observed where the gases were expelled directly onto the line conductors.

5. The operation of tube life is still an incompletely solved problem. From a weighing of the present knowledge concerning the theory and performance of tubes it would appear that life depends largely on the weathering qualities of the exterior of the tube covering or surface.

6. The weathering qualities of the tube, and their relation to the tube protective features need further study and investigation. It is suggested that the rate of system recovery voltage may be closely related to this aspect of the problem.

7. Relays on tube-protected lines should either have a definite minimum time to

Table XII. Polarity of Phase Potential at Occurrence of Fault

Multiple Strokes to Roanoke-Danville Circuit

	1934 · 1935 · 1936	Total +	Total −	Total 0
Record number	40.. 43.. 48.. 49.. 51.. 57..72.. 73 ... 9... 19 .. 40.. 45			
Number of discharges	2.. 2.. 4.. 3.. 2.. 3.. 2.. 7 ... 2... 7 .. 2.. 4	+	−	0
Phase 1	−+.. 0−..+++.. ++..+−.. ++.. +..0 0 ...− ...+−− −−+..−−..−−−−			
Phase 2	+ ..++.. +..+ + −+..− −.....++ + ++...+−...−++x++ ..++..++++			
Phase 3	− .. −..+.. + ++ ...−+...+ + −− ..++..−+++			

	1937	Total +	Total −	Total 0
Record number	6 9 20 25 36 41 43 44 47 48 59 83 90 92			
Number of discharges	4 4 5 3 3 2 3 3 2 −+ 5 2 4 2 3			
Phase 1	+−−0 − + − −00 −+0 − − + + + −+ + + −+ ++−	25	26	7
Phase 2	++ ++?0 −++++ +++ − − + ++ +00 + +−−− −− ++− − +−+	44	16	3
Phase 3	+ − + +−− +−+ ++ −++ 0 −+−++ + + +− −+	23	15	1

Multiple Strokes to Glenlyn Circuit Number 1

	1936 · 1937	Total +	Total −	Total 0
Record number	10.. 28.. 52.. 54.. 58..74... 31.. 54			
Number of discharges	2.. 3.. 3.. 4.. 2.. 2... 3.. 3			
Phase 1	−−.. 0........0 ++.. +..+............ −+	5	3	2
Phase 2	+..+ −..+−..−0+ ..+−..+............++−	8	5	1
Phase 3	− ..++........−++..+ 0	5	2	1

Single Strokes to Danville Circuit

	1934																			1935								
Record number	1	2	4	7	14	30	31	46	50	53	54	55	56	58	59	60	61	67	79	11	14	19	23	24	25	31	32	34
Phase 1	+	+	+	0	−				+	+	+		+	−	−	+	+	−	+	+	0	+			+			
Phase 2				+	−	+	+	−		+			−	+	−			+		−	+	+	−	+			+	+
Phase 3								+	−	−		+					−				−	−	+		+	+		−

	1936					1937													Total +	Total −	Total 0
Record number	18	20	21	27	33	5	15	16	24	27	28	46	50	63	67	79	89	96			
Phase 1	−	−	+	+	+	+				−	+	−	−	+	+		+	+	22	9	2
Phase 2	+	+	−		+		−	−	−			+				+	−		16	11	
Phase 3	−	−		+			+		+							−	+		9	9	

Single Strokes to Glenlyn Circuit Number 1

	1934												1935		1936		1937			Total +	Total −	Total 0
Record number	3	8	10	12	13	23	27	28	38	44	62	64	12	26	6	37	4	8	19			
Phase 1		+		0		0						+	+			+		+		5		2
Phase 2			−			+		0	+	−	+			−	−	+	+		+	6	4	1
Phase 3	+		+		+		+	−			+					−				5	2	
																				178	102	20

+ Indicates conductor with positive potential when fault begins. − Indicates conductor with negative potential when fault begins. 0 Indicates conductor at zero potential when fault begins. ? Indicates fault with phase indeterminate.

initiate the oil circuit breaker trip circuit of not less than two to three cycles, or as an alternative, when using one cycle relaying, breakers should utilize ultrarapid reclosing.

8. When properly applied the tube protects the line insulators against lightning flashover. No flashovers of insulator strings protected by tubes has so far been observed.

9. Protector tubes have reduced line outages over 50 per cent on the average and show a reduction as high as 89 per cent in the case of one line.

10. Reductions in service outages on two-circuit lines of over 85 per cent have been obtained by equipping only one line with tubes (omitting the one year's experience, 1936—on the Glenlyn-Roanoke line when the initial design tube had become defective).

11. On two-circuit lines, with one line tube-equipped, outages on the nontube line have been reduced by from 15 to 25 per cent.

Acknowledgments

The authors wish to acknowledge the assistance of G. D. Lippert of the American Gas and Electric Service Corporation in compiling and correlating data in the paper, and the co-operation of the staffs of the Appalachian Electric Power Company and The Ohio Power Company in supplying field records of tube operation.

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Discussion

E. J. Allen (General Electric Company, Pittsfield, Mass.): The authors are to be commended for a most comprehensive summary of 132-kv protector-tube operation. The large amount of operating experience accumulated and presented in this paper represents a total of over 10,000 tube-years of service.

Figure 1 of the paper shows the grading shields applied to the protector tubes on both the Roanoke-Fieldale and Fieldale-Danville lines just prior to the 1937 lightning season, as a means of reducing external tube flashovers due to lightning. Table III indicates 11 tube flashovers in the 1937 lightning season, whereas there were 6 in 1936 before the shields were added. It is therefore of interest to analyze this relationship.

Analysis shows that there are two types of protector-tube mountings on these lines; one the V mounting and the other the parallel mounting. These mountings were described in a previous paper[1] by the authors. With grading rings on the parallel-mounted tubes in 1937, no external flashovers occurred except in one questionable instance. Out of 450 parallel-mounted tubes equipped with shields in 1937, 113 normal operations were recorded. Most of the parallel-mounted tubes, or 73.5 per cent, are located on the Fieldale-Danville section, where no external flashovers were reported.

The 11 external flashovers listed in table III for the Roanoke-Fieldale line, however, occurred entirely on the V-mounted tubes. Eighty-two per cent of the V-mounted tubes are located on the Roanoke-Fieldale line, where the external flashovers were reported.

Analysis of reported flashovers shows that 8 out of the 11 external flashovers classified in table III were apparently not external tube flashovers. In all 8 cases where this was believed to have occurred, the single suspension V-mounting, as shown in figure 1, was installed. Targets located on the lower (line end) tubes had operated in every case, and hence the tubes must have discharged internally. The flashover occurring subsequently was probably due to discharge gases from the upper tube improperly contacting the line. The design of tube shown in figure 2 with gas deflector readily overcomes this difficulty. On this basis there would have occurred not more than three external tube flashovers on the Roanoke-Fieldale line in 1937, where grading rings are installed. As table III will show, this is a 50 per cent reduction as compared with the 1936 season, although in 1937 the tubes were called upon to operate 36 per cent more frequently.

The field data on repeated operations and multiple strokes presented in the paper, indicate that the chances of any individual tube having a large number of repeated operations is remote. Except in possibly a limited number of cases where unusually high line exposure to lightning exists, this is in agreement with conclusion 2 of the paper.

Moreover, on lines having an unusually high impulse spark-over level, advantage can be taken of the surplus margin of protection offered by the tube in order to reduce the number of tube operations to a minimum. This is accomplished by increasing the protector tube series gap setting above normal, so that the impulse spark-over is also increased.[2] Care should be taken, however, to allow not less than 20 per cent margin of impulse protection to the line insulation. By this procedure, the lesser magnitude surges would not cause unnecessary tube operations, but at the same time, the protection level of the tube would be adequate to prevent insulator flashovers or outages. For the first few structures out from stations, however, it is probably desirable to retain the normal series gap settings in order to minimize the magnitude of impulse waves entering the station.

As the authors point out, the principal objective of protector tubes is the protection of a line from lightning flashover and resultant line outage. Table X shows protector tubes have reduced line outages over 50 per cent on the average, with a reduction as high as 89 per cent in the case of one line. It is particularly significant that 132-kv protector tubes of modern design installed on one circuit of the Philo-Newcomerstown-Canton line, a distance of 74 miles, double circuit outages have been entirely eliminated over a two-year period. Prior to the installation of protector tubes on the one circuit, this line had an average of 3.66 double circuit outages per year over the preceding nine years.

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K. B. McEachron (General Electric Company, Pittsfield, Mass.): One might infer from the last sentence of the section entitled "Multiple Strokes" that the polarity of the line might be expected to have some control over which conductor would be struck. There seems to be no good justification for such a point of view. Rather, it is assumed that as a rule the ground wire will be struck, but which tube operates as the tower potential rises is determined to a considerable degree by conductor polarity at that instant.

Table XII shows all of the data plotted with respect to polarity of the protector-tube follow current. Considering the successive discharges of a multiple stroke as a single discharge, follow current occurred 178 times when the conductor involved was positive, 102 times when it was negative, and 20 times when the potential was recorded as zero. The per cent positive, negative, and zero are 59.8, 33.5, and 6.7.

Considering only those discharges which involved but one phase, it is found from table XII of the paper that 50 times the polarity of the follow current was positive and 12 times it was negative. This is shown in table I.

The number of successive discharges as shown in figure 4 vary up to 12 as a maxi-

Table I. Polarity of Follow Currents When One Phase Only Is Involved

Phase Involved	Polarity of Follow Current for Multiple Strokes Positive	Multiple Strokes Negative	Single Strokes Positive	Single Strokes Negative	Total per Phase
1	9	3	15	2	29
2	10	2	7	4	23
3	4	1	5	0	9
Total	23	6	27	6	

mum. Sixteen per cent of all the strokes and 61 per cent of the multiple strokes consisted of at least three discharges, while 4 per cent of all strokes and 15 per cent of the multiple strokes had at least five discharges.

Lewis and Foust[1] have found that of 358 strokes recorded during the years 1934, 1935, and 1936 through tower legs of transmission lines, approximately 95 per cent were negative. Sufficient data are now available both from the present investigation and in the laboratory,[2] so that it is quite certain that the most positive conductor will in general flashover first with a negative impulse applied to the tower or ground wire. This follows since the impulse and the system potential to ground at that instant are additive if the impulse is applied to the tower end of the insulator string.

Perhaps the fact that 95 per cent of the strokes are negative, while but 81 per cent of the follow currents with one phase involved are positive, is partly explained by the possibility of a small but unknown number of strokes to line conductors rather than to the ground wire. From table XII, it is found that phase 1 had 24 positive and 5 negative follow currents when but one phase was involved, phase 2 had 17 positive and 6 negative, while phase 3 had 9 positive and 1 negative follow currents. The top and bottom phases, which are phases 1 and 3, had 83 per cent and 90 per cent positive follow currents, while the corresponding figure for the middle phase was 74 per cent. This might indicate that the middle phase conductor was struck more frequently than the others. However, the data are not sufficiently good to draw such a conclusion definitely.

According to Bewley,[3] the coupling factor between the ground wire and the line conductors would be 0.278, 0.205, and 0.160 for the top, middle, and bottom conductor. On this basis, one would expect the bottom protector gap to operate first, since the coupling is only a little more than half that of the top conductor.

However, the data show that the top conductor (table I of this discussion) had single follow currents involving one phase only 29 times, while the middle and bottom conductors were involved 23 and 9 times, respectively. If it is assumed that negative follow currents mean strokes to the conductor when but one phase is involved, and further assumed that as many strokes occurred to the single conductor when positive as when negative, it might be inferred that the top phase conductor was struck 10 times, the middle conductor 12 times, and the lower conductor 2 times. On this basis, the ground wire was struck 27 times, considering the data of table I. This sort of analysis represents considerable speculation, and is probably pessimistic with respect to the number of times the ground wire was struck.

Another factor which ought to be mentioned is the effect of distance to ground. The ground ends of the insulators on the top phase are approximately 25 feet further from ground than the bottom conductors, which of course acts to increase the potential across the top expulsion protector tube compared with the others, and might be of importance if the applied wave were steep enough.

Number of Conductors Involved

If the number of phases which carry impulse current as a result of protector tube operation is an indication of the amount of current in the stroke, then the data given in table II seem to show that the multiple stroke is likely to contain more current than the single stroke, since of the discharges of the multiple strokes 35.7 per cent involved one phase, while 39.8 per cent involved two phases, and 24.5 per cent all three phases. The corresponding percentages for the single discharges are 52.3 per cent single phase, 37.0 per cent two phases, and 10.7 per cent three phases. Only 3 out of 13 multiple strokes, in which three conductors were involved simultaneously, began with less than three tubes operating at once. This seems to indicate, based on these data, that the first discharge of the multiple stroke is likely to carry current at least equal to or greater than that in the succeeding discharges.

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E. J. Wade (General Electric Company, Pittsfield, Mass.): The paper by Messrs. Sporn and Gross is a valuable addition to the field data on the performance of protector tubes. An attempt will be made in the present discussion to show that the data seem to indicate a definite relation between the number of expulsion tube operations and tower ground resistance, and also that the number of tubes which had repeated operations is in close agreement with the number which would be predicted, based on the theory of probability, which allows an estimate to be made regarding the number of discharges through tubes due to multiple strokes.

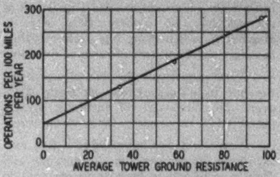

Figure 1. Effect of tower ground resistance on expulsion-tube operations

Examining first the effect of ground resistance, figure 1 of this discussion has been plotted to show the relation between average ground resistance and the number of tube operations per year for the Glenlyn-Roanoke and Roanoke-Danville lines. Since there is a definite difference in the ground resistance of the two halves of the Roanoke-Danville line, the two sections on either side of Fieldale have been considered separately.

As shown on the graph, these data may be well represented by a straight line passing through approximately 50 operations per 100 miles per year at zero tower resistance. If this interpretation is accepted, it means that the difference in the number of tube operations on the two halves of the Roanoke-Danville line is due to variation in ground resistance and not to a difference in lightning severity as suggested in the paper. Where the resistance is higher there is more tendency for adjacent tubes to operate and less stroke current is required to cause operation at the tower which is struck. The point of intersection with the ordinate at zero tower resistance may be an indication of the number of strokes which contact the line conductors. A similar analysis was made with regard to strokes contacting the line conductors and the relation between ground resistance and insulator flashovers in a discussion by S. M. Zubair (Electrical Engineering, volume 55, pages 277–9).

Considering the number of tubes which may be expected to operate more than once, and assuming that any tube on the circuit is equally likely to be struck, regardless of whether or not it has previously operated, then the law of independent trials will apply. This is closely approximated by Poisson's law which may be written:

$$P_m(n) = \frac{\epsilon^{-mp}(mp)^n}{n!}$$ the chance which a given tube has of being hit n times in m trials

p = chance that a particular tube will operate on any one stroke

$$= \frac{1}{\text{number tubes installed}}$$

Table II. Probability Calculations

Line	Number Tubes Installed		Tube Operations One	Two	Three	Four	Period (Years)
Glenlyn-Roanoke	810	Calculated	251	66	11.5	1.5	5
		Field	228	63	13	8	5
Fieldale-Danville	411	Calculated	130	35.5	6.5	0.9	4
		Field	139	42	2	0	4
Roanoke-Fieldale	492	Calculated	178	71	19.2	3.9	4
		Field	215	79	14	1	4

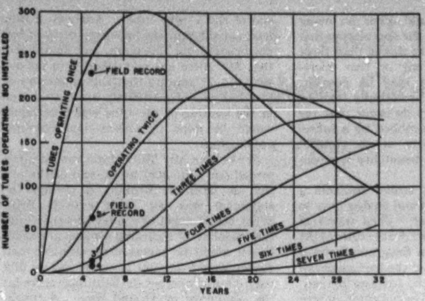

Figure 2. Probability calculation for expulsion tubes operating on Glenlyn-Roanoke line (neglecting multiple strokes)

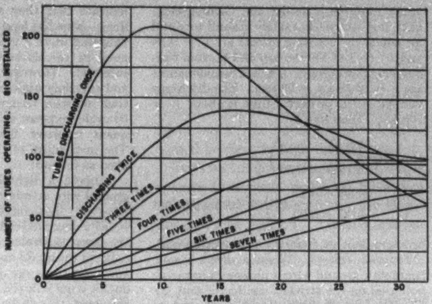

Figure 3. Probability calculation for expulsion tubes operating on Glenlyn-Roanoke line (including multiple strokes)

n = number of times each tube operates

m = total number of operations of tubes for the period being considered

Applying this to the Roanoke lines using the data from tables II, III, and IV of the paper, the results shown in table II of this discussion are obtained. The good agreement which is shown with the field data indicates that tube operations follow rather closely the probability law. The reason for the increasing discrepancy as the number of operations per tube increases is probably due to a few places on the lines which have unusually high exposure. This calculation is shown in figure 2 of this discussion carried out to a period of 32 years. It will be seen that the number of tubes which had six or seven repeated operations is very small even after a long period.

These calculations may be taken as an indication of the number of tubes which would operate on a line passing over level country where the storms are equally distributed. Of course this represents the most optimistic conditions as any line would have some degree of unequal exposure. However, the calculated values check fairly well with the data from the Roanoke lines which are over mountainous territory which indicates that no great discrepancies may be expected.

It is of interest to extend this calculation to include the number of discharges through tubes due to multiple strokes. Mr. McEachron's paper shows that approximately 30 per cent of the strokes over a period of years consist of more than one discharge, 14 per cent of two discharges, 8 per cent of three, 4 per cent of four, 2 per cent of five, and the remaining 2 per cent of six or more.

The probability that a tube will have a certain number of discharges during a given time is the sum of the probabilities of each of the various combinations of single or multiple strokes which can produce the assumed number of discharges.

For instance, three discharges may be obtained by three combinations, namely,

Three "single" strokes,
One "single" and one "double" stroke,
One "triple" stroke.

Similarly, for four discharges there are five combinations, for five discharges seven combinations, etc.

The probability of obtaining each of these combinations is found by taking the product of the probabilities for their components together with the probability of not obtaining any additional discharges.

The results of this calculation are shown by figure 3 and it appears that the chance of an excessive number of tube operations under conditions of equal exposure either due to successive hits or to multiple strokes is very small, since on the average the operations will be rather evenly distributed over the total number of tubes. However, there may be a few cases of tubes having an excessive number of operations due either to multiple strokes or to high ground resistance or to locations of excessive exposure.

As a result of this study, excessive erosion due to a large number of tube operations, is very unlikely except in a very limited number of cases. This agrees with conclusion 2 of the paper.

L. V. Bewley (General Electric Company, Pittsfield, Mass.): The data presented in this paper show that the protector tubes on the top conductors operate more frequently than those on the bottom conductor, in spite of the fact that the line is equipped with a ground wire, and therefore the coupling is materially more for the top than for the bottom conductor, being 0.30 and 0.17 per cent respectively. Such behavior also appears to be typical of insulator flashovers on lines equipped with ground wires.[1] There are possibly four explanations for such behavior:

Inadequate Shielding. By *shielding* is meant ground wires sufficiently high and properly placed so as to intercept the lightning stroke and prevent termination on the line conductors. If the ground wire or wires are not adequate in this respect, a certain percentage of the lightning strokes will evade the ground wires and contact a line conductor. If this occurs, it is the top or middle conductor most likely to be struck. Zubair[2] has plotted flashovers as a function of tower footing resistance, and by extrapolating the curve to zero footing resistance has found the number of flashovers independent of footing resistance. He assigns, as the reason for these flashovers, direct strokes to the line conductors; and arrives at a "shielding efficiency." In the case of the Glenlyn-Roanoke line the shielding efficiency is 0.77, that is 22 per cent of the strokes contacted the line conductor.[1] Of course some of these flashovers independent of footing resistance may have been due to midspan flashover rather than to inadequate shielding.

Tower Gradient. McEachron has suggested in his discussion that tower gradients may overcome the difference in coupling effect and impose flashover on the top conductor. Calculation shows, however, that a rather high gradient would be necessary to accomplish such a result. Let:

E = instantaneous voltage at top of tower and on ground wire

x = vertical distance from top of tower to top conductor

y = vertical distance between top and bottom conductor

C = coupling factor with top conductor

C' = coupling factor with bottom conductor

G = gradient down tower in kilovolts per foot

Then the voltages across the top and bottom insulators are respectively:

$$V = E - Gx - CE = E(1 - C) - Gx \quad (1)$$

$$V' = E - G(x + y) - C'E = E(1 - C') - G(x + y) \quad (2)$$

The difference in voltage is

$$V' - V = E(C - C') - Gy \quad (3)$$

Solving (1) for E and substituting in (3) there results

$$V' - V = (V + Gx)\left(\frac{C - C'}{1 - C}\right) - Gy \quad (4)$$

The gradient necessary to overcome the differential in coupling is that for which $(V' - V) = 0$, and (4) gives

$$G = V\left(\frac{C - C'}{1 - C}\right) \div \left(y - \frac{C - C'}{1 - C}x\right) \quad (5)$$

where V is now the insulator (or protector tube) flashover kilovolts of the top conductor.

For example, consider the Glenlyn-Roanoke line which has a single ground wire and for which the coupling factors are $C = 0.30$ and $C' = 0.17$ for the top and bottom conductors respectively. The vertical distances are $x = 10$ and $y = 26$ feet. The discharge voltage of the protector tube is about $V = 700$ kv. Then by (5)

$$G = 700\left(\frac{0.30 - 0.17}{1.00 - 0.30}\right) \div \left(26 - \frac{0.30 - 0.17}{1.00 - 0.30}10\right) = 5.4 \text{ kv per foot}$$

or a gradient of 5,400 kv per microsecond.

Induced Voltage. When a direct stroke contacts a ground wire it imposes a negative potential on the stricken conductor and, by coupling, on all other neighboring conductors. But the potentials existing on these conductors due to the release of their bound charges by the lightning discharge are positive. This positive potential may or may not be greater on the top than on the bottom conductor, depending upon the heights of the conductors and their coupling factors. In the case of the Glenlyn-Roanoke line the positive voltage is greater on the top conductor by 30 per cent. Now let[4]

$+e$ = voltage on top conductor due to bound charge

$+e'$ = voltage on bottom conductor due to bound charge

Then the voltages across the insulators for the top and bottom conductors respectively are, as in (1) and (2)

$$V = E(1 - C) - Gx + e \qquad (6)$$

$$V' = E(1 - C') - G(x + y) + e' \qquad (7)$$

and the difference in the voltages is

$$V' - V = E(C - C') - Gy - (e - e') \qquad (8)$$

Thus the bound charge voltages tend to neutralize the differential in coupling if, as in this case, $e > e'$.

"Stolen Flashovers." Operating experience has failed to show concrete evidence of midspan flashovers, although calculations indicate that they should occur on many lines. This has lead to the speculation that lightning wave fronts are slower than those used in calculations (one or two microseconds). However, it may be possible, and certainly appears theoretically feasible, that midspan flashovers do occur, but the 60-cycle midspan arc is robbed by a subsequent insulator flashover at the tower.[3] The mechanism of this possibility is envisaged as follows:

Suppose a lightning stroke contacts a ground wire at midspan and flashes over to the (top) line conductor. Then waves of equal voltage move toward the tower. At the tower the voltage on the ground wire is reduced to the *RI* drop of the footing resistance, but the surge on the stricken line conductor is not relieved. The insulator, or protector gap, then flashes over to the tower. But the insulator arc is much shorter than the midspan arc, and the latter therefore extinguishes, leaving the 60-cycle power arc across the insulator. Consequently the "evidence" exonerates the midspan; although the midspan flashover really precipitated the insulator flashover, and an insulator flashover would not necessarily have occurred if one had not taken place at midspan.

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3. FLASHOVERS ON TRANSMISSION LINES, **L. V. Bewley.** ELECTRICAL ENGINEERING, April 1936.

4. CRITIQUE OF GROUND WIRE THEORY, **L. V. Bewley.** AIEE TRANSACTIONS, volume 49, March 1931.

Philip Sporn and I. W. Gross: As pointed out by Mr. Allen, of the 11 external flashovers of tubes on the Roanoke-Fieldale line, only three appeared to be what might be called complete tube assembly flashovers, the other eight resulting from expelled gases contacting the line conductor and thus short-circuiting one tube only. This condition was not experienced prior to the installation of the grading shields on the gap end of the V-type tubes. Analysis of tube faults at the time the paper was written had not brought out this point, and we are glad that Mr. Allen has mentioned it in his discussion. It appears that this situation can be taken care of adequately by an alteration in the venting of the tube which certainly would be done in present-day new-tube application.

We believe Mr. McEachron has misinterpreted our statement at the end of the section on "Multiple Strokes," or perhaps the statement was not made entirely clear. What we intended to convey was that since the majority of lightning strokes are negative it might be expected the positive conductor would develop power flashover first regardless of whether the lightning stroke hit the conductor or ground wire. A breakdown of the data in table XII, however, does not indicate that this always is true as there are several cases where the faulted conductor is of negative polarity.

The question of coupling, of course, is distinctly in the picture, and the discussion of Messrs. McEachron and Bewley on this point is an interesting speculation of what may be expected to occur. In breaking down the data of figure 12, however, it does not appear that the faults in every condition can be justified on this basis alone.

The phenomenon of the midspan stroke has always been one which has appeared quite possible in theory but its actual occurrence has heretofore, at any rate not been substantiated by field experience as determined from physical damage to the line. Mr. Bewley's explanation of how this action would take place is most interesting. A large mass of data has been collected during recent years which, if thoroughly analyzed, might throw some further light on this subject.

Regarding Mr. Wade's analysis of the probability of tube operations over a period of time, it is interesting to note how closely his calculations check with field experience. As mentioned in our paper, we do not feel, as a result of our own experience in the field, that erosion of the tube is a factor in determining its life even considering the multiple nature of the lightning stroke.

Corona Voltages of Typical Transformer Insulations Under Oil—II

By F. J. VOGEL
ASSOCIATE AIEE

SOME 13 years ago, Doctor L. Dreyfus[1] published a paper on insulation design of transformers. In this paper, he assumed that electrical breakdown adjacent to square edges was a function of the stress and the distance over which this stress acted; in other words, that it was a function of the voltage gradient along a line of force for some indeterminate distance. By means of conformal representation, he was able to determine the field shape at the corners of structures representative of those in transformers. He could then evaluate the strength of these various structures. His paper was principally mathematical, and it is the purpose of this paper to call attention to his work, to restate some of his assumptions and the conclusions that he reached, and to furnish some experimental data.

In the discussion of the previous paper on this same subject, questions as to fundamental data on the dielectric strength of oil were raised. Data on this subject are included and it is shown that in practical transformer structures, Doctor Dreyfus' assumptions seem to be correct. The relationship is shown that the breakdown of barriers involving the use of square corners theoretically follows the two-thirds power of the barrier thickness. It is also shown that, due to Dr. Dreyfus, quantitative relationships existing between different arrangements can be found and it is thought that they furnish a powerful tool for practical insulation design.

The simplest arrangement of edges is that consisting of two edges opposite each other with various angles of opening α and β as shown in figure 1.

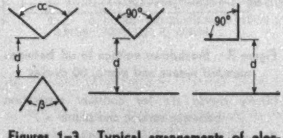
It was found in Doctor Dreyfus' paper, for the assumptions made, that the breakdown strength would vary according to various powers of the distance d. For example, for β equals 180 degrees and α equals 90 degrees (shown in figure 2) the strength should vary as $d^{2/3}$. If β equals 180 degrees and α equals 0 degrees, the strength should vary as $d^{1/2}$. If both equal 180 degrees, the strength was shown to vary directly as d. In addition, Doctor Dreyfus showed that for the case shown in figure 3, the breakdown strength should vary as $d^{2/3}$. The mathematical solution and method of conformal representation can be found by reference to Doctor Dreyfus' paper or a textbook.[5]

Experimental Data for Air and Oil

It is always of interest to check such mathematical calculations with experimental data to see how closely they agree, and to see whether the assumptions are sound. One such check would be to see if the dielectric strength between plane surfaces does vary directly with the distance. In the case of air, data have been published by Schumann,[2] and checks have been made by extrapolating the data for spheres and rounded surfaces in air (figures 4 and 5). These show that the dielectric strength of air between plane surfaces does not vary directly with the distance, but rather as the 0.92 power of the distance.

Cornered electrodes in air have been studied and the results for square-cornered electrodes are shown on figure 6. These results indicate that the dielectric strength for air between a square corner and plane does vary as the two-thirds power of the distance in agreement with the theory.

With the above data for air in mind, it is of interest to consider similar data for the dielectric strength of oil. Figure 7 shows data taken on bare rounded electrodes under oil. By extrapolating these curves, it is possible to arrive at the probable dielectric strength of oil between planes as varying as the three-fourths power of the separation, as shown on figure 8. These data were taken in oil at room temperature, and are minimum ten-minute hold values, found by five successive tests at the same voltage. To agree with the theory, the variation should be directly with the distance.



Figures 1–3. Typical arrangements of electrodes with corners

Data for conditions approaching square corners to a plane under oil have been published by Bellaschi,[3] and his results are shown in figure 9. The strength apparently varies about as the one-half power of the separation instead of as the two-thirds power.

The conclusion might now be reached that the assumptions made in Doctor Dreyfus' paper would not hold for oil alone. However, it is to be noted that transformers do not generally use oil alone, but various combinations of oil and insulating materials.

Experimental Data for Insulation Combinations in Oil

In the case of insulation barriers, much data have been previously published. For example, in the previous paper,[4] 60-cycle and impulse data for the $2\frac{3}{8}$-inch and $4\frac{3}{8}$-inch barriers were published. In these tests, the oil distance from the electrodes to the first or adjacent angle was varied for both thicknesses. It might be expected that the strength of the oil, say for a $\frac{3}{8}$-inch space between the electrode and first angle, would be

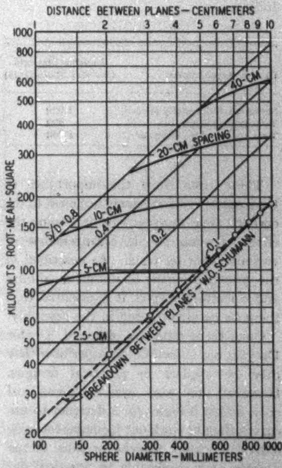


Figure 4. Breakdown voltage in air of sphere gaps with varying diameters at constant spacing

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1. For all numbered references, see list at end of paper.

constant regardless of the barrier thickness. If so, one of the assumptions in Doctor Dreyfus' theory would be fulfilled, and the corona strength should vary with the two-thirds power of the thickness of the whole barrier; this was found to be the case.[4] It is of interest that the corona strength of the barrier as a whole decreased with increased space between the electrode and the first angle. The following conclusions can be drawn:

1. The dielectric strength of oil depends greatly on the length of the oil space. It is greatly increased in terms of volts-per-inch by decrease in the jump distance.
2. The work of Doctor Dreyfus holds in actual transformer structures and the dielectric strength varies as the two-thirds power of the thickness when the arrangement and allocation of square corners remains the same.

Applications

Once it has been shown that Doctor Dreyfus' assumptions are valid, a very powerful method of analysis becomes possible. Experimental work on barriers can be performed to determine the effect of various arrangements of insulation adjacent to the corner. For example impulse tests were made on a $4\frac{3}{8}$-inch thick barrier under oil with seven interleaved angles, and also with only two interleaved angles. Also similar tests were made with the two angles close to the electrode. These barriers were *not* vacuum filled with oil. The impulse strengths of these arrangements were as follows:

Barrier Arrangement	Impulse Strength (Several Shots Only)
Seven angles, equally spaced	1,020
Two angles, separated	990
Two angles, close together	1,200

These tests show the importance of the insulation arrangement adjacent to the corner. The designer can determine the effect of such simple arrangements of various kinds by tests. As described in the previous paper, to obtain maximum and consistent results, it is essential that there be no air included in the assembly.

Once these data are obtained, Doctor Dreyfus has shown in his paper how they may be applied in practical designs. However, his work was mathematical only, and it is easier for a designer to use curves than to work out formulas for each individual case. Curves for some of the simpler cases applicable to practical design are shown in figure 10.

How these data and figures can be used practically for 60-cycle design can be demonstrated as follows:

Let us assume a barrier with an interleaved angle of one-eighth inch thick fullerboard not over three-eighths inch from a square edge. This approximately square edge is to be formed by a copper strap insulated with 0.120-inch thickness of paper. Approximate data for such barriers are shown on figure 11.

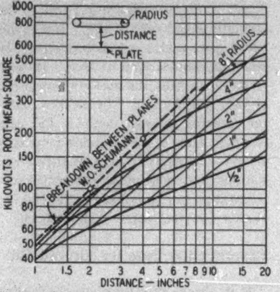

Figure 5. Corona voltage in air between rounded disks and plane, or between rounded surfaces, cylinders, etc., and plane at large separations

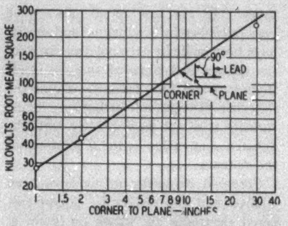

Figure 6. Breakdown voltage in air with square-cornered electrodes

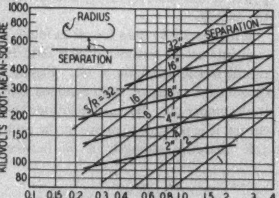

Figure 7. Breakdown voltage in oil between rounded surface and plane, 60 cycles

Heavy curves are for constant separation between surface and plane

Diagonal lines represent constant ratio between separation and radius

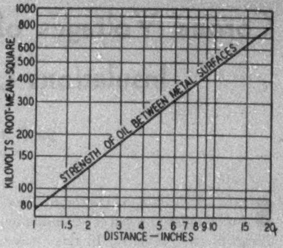

Figure 8. Breakdown voltage in oil between plane surfaces

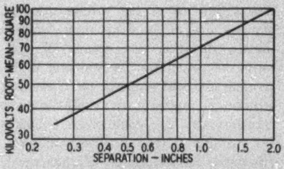

Figure 9. Breakdown voltage in oil with square-cornered electrodes

If we wish to determine safe insulation clearances, for the corner only, for a core type transformer, and for conditions where the space between windings and hence the reactance, is to be kept a minimum, we may refer to figure 10, case I. If we make H equal to about 2 m, it is seen that we can use the fundamental data from figure 11 multiplied by 0.93. It appears useless to make H, the distance to the yoke, much more than three times the space between windings. However, it may be desirable, for shell-type transformers, to keep the space between windings and to the iron the same. To meet 277-kv tests, with *no* factor of safety, a clearance of $2\frac{7}{8}$ inches for core type, or 3.6 inches for shell type transformers might be used. These values, of course, should not be used in actual design but are given for illustration of the method only.

Let us suppose that we desire to make a core-type transformer with a thin high-voltage coil as used in some types of series transformers. If the coil were to consist of a single layer, say one-half inch thick, we might apply the data from figure 10, case IV. If we assumed three inches for m, the ratio h/m would be $^1/_6$ or 0.167. From figure 10, the factor is 0.88. From figure 11, a three-inch separation would result in 315 kv

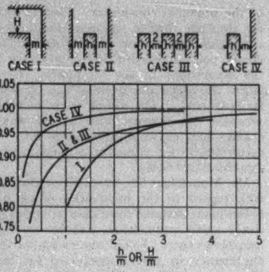

Figure 10. Effect of field form on insulation breakdown in oil of square-cornered electrodes

$V = LV_0$ for cases I, II, and IV
$V = 2LV_0$ for case III
where V_0 equals breakdown in oil between sharp corner and flat plate

strength. Multiplying by the factor 0.88 would give 277 kv strength as in the previous cases.

It is clearly seen that factors for various conditions can be determined either experimentally or mathematically. Thorough tests to determine the dielectric strength on one simple typical corner construction will then furnish very complete information for design purposes. The use of this method is not confined to 60-cycle design alone, since it can be shown that it is equally well applicable to impulse-strength calculations.

Conclusions

The work of Doctor Dreyfus and the tests reported in this paper both show that the arrangement of the insulation at the corners of transformer windings is of great importance to the designer, and that to obtain the greatest strength the area under greatest stress must obviously be free from air. If this condition is obtained, consistent test agreement with the theory for square corners can be obtained, and so-called "form factors" can be developed for design purposes. Experimental data for one reference condition can be derived which will be useful in general applications, and with "form factors" it is unnecessary to make countless other tests for every variation in design proportions.

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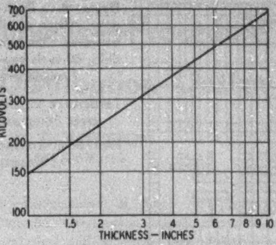

Figure 11. Breakdown and corona voltage in a typical interleaved barrier in oil with insulated electrode

3. DIELECTRIC STRENGTH OF TRANSFORMER INSULATION, P. L. Bellaschi and W. L. Teague. AIEE TRANSACTIONS, 1937, page 164.

4. CORONA VOLTAGES OF TYPICAL TRANSFORMER INSULATIONS UNDER OIL, F. J .Vogel. ELECTRICAL ENGINEERING, January 1938, page 34.

5. THEORY OF FUNCTIONS AS APPLIED TO ENGINEERING PROBLEMS (a book), R. Rothe, F. Ollendorff, and K. Pohlhausen. Technology Press, 1933.

Discussion

V. M. Montsinger (General Electric Company, Pittsfield, Mass.): Mr. Vogel has presented some valuable fundamental data on the laws of the breakdown voltage versus spacing of air and oil. I shall confine my remarks to the dielectric characteristics of oil, as air is seldom used as an insulating medium in power transformers other than around the outside porcelain of the bushings.

My experience and data have borne out quite well the laws derived by Dreyfus, namely, that for nonuniform fields the breakdown voltage of oil varies approximately as the spacing raised to the $^3/_4$ power, and for uniform fields the breakdown voltage varies approximately as the first power of the distance. My work was independent of Dreyfus' work.

Figure 1 of this discussion shows that with 12-inch vertical disks having one-inch radius on the edges, the breakdown voltage of oil varies as the 0.92 power of the distance. These electrodes produced a fairly uniform field. I am at a loss to know why Mr. Vogel's values shown in his figure 8, giving breakdown in oil between plane surfaces, are so much lower than my values. I note that Mr. Vogel's breakdown voltage varies as the 0.77 power of the distance which does not agree either with Dreyfus' prediction, or with my data (0.92 power).

Figure 1 also shows to what extent the breakdown varies for a given distance with and without pressboard in series with the oil. For example, the highest breakdown is obtained with a thin sheet of pressboard adjacent to each electrode in a vertical position. The lowest breakdown occurs with these electrodes (bare) in a horizontal position. The insertion of $^3/_{16}$-inch pressboard either half-way between the electrodes, or against one electrode only, has very little effect on the breakdown as obtained with oil alone in the gap.

Figure 2 of this discussion shows the effect of the position of the two ten-inch electrodes on the breakdown of oil. These curves show that when the electrodes were horizontal, the breakdown varied as the $^3/_4$ power of the distance. With the electrodes

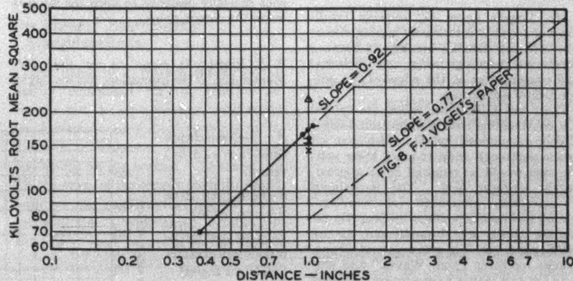

Figure 1. One-minute 60-cycle tests; breakdown voltage of oil between 12-inch disks in vertical position (unless otherwise specified)

Temperature 25 degrees centigrade
One-inch radius—oil strength varied from 28 to 33 kv
(One-inch disks, 0.1-inch gap)

		Spacing (Inches)	Number of Tests Made
△	$^1/_{32}$-inch pressboard + $^{15}/_{16}$ inch oil + $^1/_{32}$-inch pressboard	1	3
o	Oil	$^3/_8$ and 1	14 and 25
o-	$^{13}/_{32}$ inch oil + $^3/_{16}$-inch pressboard + $^{13}/_{32}$ inch oil	1	1
-o	$^3/_{16}$-inch pressboard + $^{13}/_{16}$ inch oil	1	2
x	Oil (electrodes horizontal)	1	12

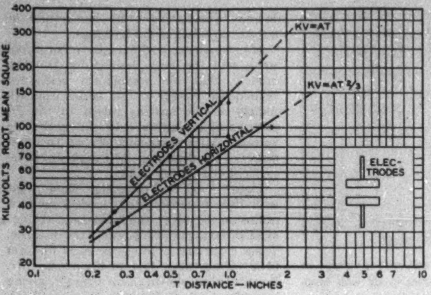

Figure 2. Breakdown voltage of oil between ten-inch disks, 60-cycle one-minute tests

Temperature 25 degrees centigrade

Each point average of five tests

vertical, the breakdown varied directly with the distance. The difference was due, no doubt, to the fact that impurities and gaseous bubbles have a better chance to bridge the gap between horizontal electrodes than between vertical electrodes. Other tests, I believe, have shown that with deaerated pure oil there is no difference in the breakdown between vertical and horizontal electrodes. This is a factor which was not taken into consideration by Dreyfus' work, and it shows that breakdown will not always follow predictions under all conditions.

Figure 3 of this discussion shows the effect of the shape of the electrodes on the breakdown voltage of oil. Briefly these curves show that:

1. **The highest breakdown is obtained with large vertical disks having well rounded edges.**
2. **The lowest breakdown is obtained with a needle point and a plane.**
3. **With the exception of the upper curve *A*, the breakdown varies very closely with the spacing raised to the $^2/_3$ power (slope of lines = $^2/_3$ on log log paper).**

Mr. Vogel's figure 11, showing breakdown of a typical interleaved barrier, checks very well my data on breakdown between interleaved windings where the dielectric field concentrates on the edges of the outside coils of each group.

These low breakdown values are typical of a construction having unprotected edges of coils surrounded by oil, which give off corona and break down at much lower voltage than would be required in an approximately uniform field. The importance of protecting electrodes (or windings) against low corona voltage by properly insulating them, was brought out in my AIEE paper on "Co-ordination of Transformers for Steep Front Waves," and presented at last winter's convention. Briefly, with a square edge line end electrode, corona occurred as low as 60 to 65 per cent of the single impulse voltage application strength, while with a proper design of the line end electrode the breakdown point was not only increased but corona did not occur until approximately 80 per cent of the single-shot kilovolt strength was reached.

With ideal electrodes, distances far below those shown in figure 11 are possible. For example, a 138-kv oil-filled cable uses only approximately $^9/_{16}$ inch of paper between the cable and lead sheath. In contrast with this, figure 11 of Mr. Vogel's paper shows that in an interleaved design a distance of 2.5 inches will fail at 277 kv which is the test voltage of a 138 kv transformer. In other words, the distance used must be (for a reasonable factor of safety) from six to eight or ten times that used in oil-filled cables. While the minimum distance as used in cables can probably never be obtained in transformers, on account of less ideal electrodes, it is possible with concentric windings to come nearer to the ideal flux distribution than the arrangement shown in figure 11.

From the standpoint of obtaining uniform dielectric fields between windings, the concentric type of winding offers many advantages to the designer, especially in cases where the line connection is brought out from the middle of the stack and the ends of the stack are grounded. There are no sharp corners to be protected and the dielectric field between windings is quite uniform.

With this type of winding even when the line coils are on the ends of the stack, there are never more than four corners where the dielectric field must be improved and these can easily be taken care of by the buffer coils specially designed to keep the stresses below the corona point.

F. J. Vogel: It seems to me that Mr. Montsinger, in his discussion, has lost sight of the principal conclusion. That is, that for similarly shaped square-cornered electrodes, in combination with insulation, the corona point and dielectric strength vary as the dimensions to the $^2/_3$ power. Even where the edges are rounded slightly, similar laws hold. The use of "form factors" provides the designer with a tool which enables him to vary proportions with security and with a minimum of fundamental experimental testing.

Mr. Montsinger has drawn his conclusions with regard to the breakdown of oil between plane surfaces from data at two relatively small separations, and by average breakdown tests. I think the reason for the discrepancy in values mentioned by Mr. Montsinger is due to the method of test. The tests reported in my paper were the minimum of five ten-minute hold tests instead of average breakdown tests. This explains both the difference in values and in the variation of dielectric strength with distance. It is of interest that for small separations, the breakdown voltage is not greatly affected by the use and location of fullerboard barriers. This is not the case with larger separations or if corona voltage instead of breakdown voltage is taken into consideration.

Figure 3 combines on one sheet data on many different arrangements. They are of interest in not being consistent. Oil, not deaerated, nor carefully processed, varies greatly in dielectric strength. Curve *B* exceeds curve *A* at small separations. They should merge together if the breakdown voltage between planes varies directly as the distance. Either the values from *B* for small separations are too high, or the values from *A* for small separations are too low. The dielectric strength of oil between planes does not vary directly as the distance between them if the latter is true.

Curve *E*, figure 3, is for an electrode condition similar to figure 9 of my paper, and it is noted that Mr. Montsinger's data are lower than mine for small separations and approximately the same for two-inch separation.

In the application of these data to design, it has been found that the corona value

A—Twelve-inch disk (one-inch radius) vertical

B—Four-inch disk ($^1/_2$-inch radius)

C—One and three-eighths inch diameter rods (right angles)

D—Four-inch disk (square edges)

E—Square-edge flange and plane

F—Two needle points

G—Needle and plane

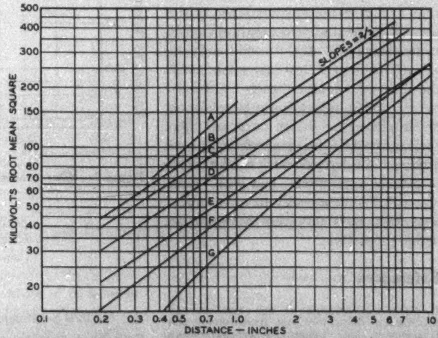

Figure 3. Sixty-cycle breakdown voltage of oil with different electrodes, one-minute tests

Twenty to 25 degrees centigrade

Oil strength ranged from 25 to 30 kv (one-inch electrodes, 0.1-inch gap) and breakdown value can be made practically equal. At any rate, the ratio between corona voltages, impulse and 60 cycle, can be made the same as the ratio between breakdown voltages, impulse and 60 cycle. It is easily seen that if there is a definite ratio, that voltage values for impulse tests can be established from the present 60-cycle test values and should be consistent. The value of 80 per cent as applied to the ratio between corona and single-shot kilovolt strength is not representative of "proper design."

The comparison of concentric types of winding with interleaved types is unfortunate, since the number of square corners is just the same (four) on the usual core type as in the shell-type winding with two high-voltage groups. However, no merit is seen in the number or lack of number of such corners since one or 20 can be insulated in the same way. Theoretically, some reduction in space between high-voltage and low-voltage windings can be made where the high-voltage lead is brought out at the center of the coil stack, but the outside shield and its insulation required to obtain a satisfactory voltage distribution also takes space and offers difficulties avoided by other types of design.

Systematic Voltage Surveys—Procedure and Application to Distribution Design

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WITH the development of systems of distribution supplying light, heat, and power from common mains, the problem of maintaining proper voltage at consumers' services has become extremely difficult. The efforts of distribution engineers in solving system voltage regulation problems have extended in many directions. To facilitate these solutions, a need has been felt by many distribution engineers for some means of clarifying the approach to the voltage-regulation problem. The systematic voltage survey, by presenting a comprehensive view of service voltage conditions, has been found, by utilities which make such surveys periodically, to serve this purpose effectively. This paper presents a procedure for conducting systematic voltage surveys, based on the practice of various electric utilities, and a brief discussion of the application of voltage-survey data.

Distribution systems are not static, but are in a continual process of development. In the course of rearrangements occasioned by load growth, some primary and secondary mains become too long, or overloaded, or distribution transformers become overloaded, with resultant adverse effect on regulation standards. To maintain voltage within system regulation standards under these conditions, the distribution engineer must have some method of ascertaining the status of service voltage conditions over the entire system. Because of the variable factors involved, it is impractical, if not impossible, to calculate with any degree of accuracy the value of voltage at consumers' services, involving as this may, approximations of load on primary laterals, distribution transformers, secondaries, and services. The systematic voltage survey is an effective means of obtaining this information, and has proved to be a material aid to the distribution engineer in providing good voltage service.

The procedure involved in making a voltage survey necessarily varies among different utility systems in accordance with individual system requirements. Data received from various companies regarding voltage survey practice indicate, however, that there is considerable uniformity in the consideration of certain fundamental aspects.

In considering plans for making a voltage survey for the first time, engineers are concerned with certain phases which may be divided into the following categories, namely,

(*a*) Preliminary engineering considerations.
(*b*) Preparation of data for field test forces.
(*c*) Procedure for field testers.
(*d*) Recording of tests, classification and tabulation of results.
(*e*) Analysis and application of data.

Preliminary Engineering Considerations

The value of a voltage survey is primarily dependent on the relative accuracy and the completeness with which the information obtained presents an indication of the true status of voltage conditions on the system. To insure the effectiveness of the survey from the standpoint of the final engineering use to be made of the data obtained, careful consideration must be given to the type of readings desired, location and number of readings to be taken, time of taking readings, etc.

TYPES OF READINGS

There are two basically different types of voltage surveys—those taken by means of graphic instruments, and those taken by means of indicating instruments. Although the cost of a voltage survey is on the whole increased by the use of graphic instruments, the added cost is offset by the fact that a complete record of voltage variations and fluctuations over a 24-hour period is available, which in many cases adds greatly to the understanding of the regulation problem. On the other hand, the use of indicating instruments speeds up a voltage survey and allows the use of a less highly trained personnel. The element of completeness is taken care of by increasing the number of readings taken. In this connection it may develop that a large number of indicating readings thoroughly covering an area will prove of greater value than a limited number of graphic records taken at a few points only, as the few graphic records may fail to disclose the extent of the poor voltage conditions. As a rule, a large number of indicating readings taken in an area should not fail to disclose low spots and high spots; and where conditions are suspicious, they may be supplemented by the more thorough graphic charts. This is usually the practice of companies which standardize on indicating readings.

The use of graphic instruments permits a check on the average voltage supplied to a consumer, which is important as it is desirable that the average voltage supplied by a system shall match the rated voltage of equipment for best service to the consumer. Nevertheless, most systems have set up *limits* of voltage service and their experience has been that if the greater part of the service voltages are kept within these limits, as indicated by spot readings, the average voltage supplied to the services will not deviate greatly from the system nominal voltage.

LOCATION AND NUMBER OF READINGS

In arranging the details of survey procedure, a most important point to be de-

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cided is the location at which readings should be taken. The voltage which influences the operation of equipment is that which is directly supplied to the terminals of the equipment; however, it is not practical, for example, to measure the voltage at the lamp sockets. The next best point, therefore, is considered to be at the consumer's service switch.

Practical considerations preclude taking voltage readings at all services. The number of readings taken should be sufficient, however, to represent a fair sampling in order that the results may be indicative of the conditions existing. To facilitate subsequent corrective measures, readings should be taken at a sufficient number of service points to disclose whether subnormal voltages are caused by primary circuits, secondary mains, or services. On radial systems readings should be taken at the consumer nearest each transformer and at a consumer at the end of at least the longest secondary branch. Where a secondary has several branches of unequal loading or a complicated configuration, judgment may dictate that more than one branch reading be taken. Special conditions may warrant taking additional readings at various points along the secondary.

A careful analysis of secondary layouts, in establishing a voltage-survey procedure, will do much to eliminate unnecessary readings. Where considerable growth is taking place on a secondary system, a certain amount of secondary rearrangement is made from year to year and successive voltage surveys must be made to conform. On the other hand, it has been found good policy in making successive surveys to choose locations in a consistent manner, and where no rearrangement has been made, to choose approximately the same locations as before in order that yearly comparisons of results may be more closely indicative of actual change.

On network systems many of the same points hold. As network secondary grids are fairly uniform in arrangement, a reading at the first consumer off the transformer and at one or two intermediate points between transformers generally suffices. Where the network grid is incomplete, additional readings on long spurs may be desirable.

NUMBER OF READINGS AT INDIVIDUAL LOCATIONS

The decision must be made as to whether or not all possible line-to-neutral readings should be taken at each location. Practice naturally differs, but with indicating instruments, particularly, the additional time and expense involved in taking more than a single line-to-neutral voltage would appear to be justified by the information obtained regarding voltage unbalance. For this reason, where a choice of consumer locations is available, three-wire services are to be preferred to two-wire services. In the case of three-phase secondaries the readings should be taken, wherever possible, on a three-phase service. It may be desirable to read the line-to-line voltages as well as the line-to-neutral values—the former for reference and check purposes, the latter for comparative records.

TIME AT WHICH READINGS ARE TAKEN

In a district which is predominantly residential, the peak load will usually occur at some hour during the evening in the winter. If a district is on the average an industrial district, the peak load will usually occur at some hour during the day and may be either in the summer or in the winter. There may be some residential load in an industrial district and some industrial or commercial load in a residential district, but if these loads make up only a small portion of the entire area, the expense of surveying them separately may not be justifiable. In general, the readings should be taken at the time the peak load is occurring in the district.

The time of the year and the extent of time over which the survey will be conducted will be determined ordinarily by the load characteristics of the system and the personnel available for carrying on the field work. Common practice is to schedule the ordinary lighting-load survey over the winter months and the survey on industrial loads and summer resorts over the summer period. This allows a year-round distribution of work on the field test forces. It has been found convenient by some systems to schedule daytime industrial-load readings as well as night lighting-load readings over the "winter peak" period to fit in better with the routine work of the field forces and relieve special personnel requirements.

Where seasonal variation of load over the "winter peak" period is limited, it may be feasible to schedule the voltage survey for any time during the period from October to April. If the load variation is considerable, it still may be desirable to take readings over a long period and correct voltage readings for variation of system demand. Such voltage readings can be corrected to allow for the difference between the feeder load at the time the readings were taken and at the time of feeder peak.

Generally speaking, readings in residental areas will be taken on week-day nights from 6:00 to 10:00 p.m. It may be desirable to take readings in store-load districts on Saturday nights. In industrial districts readings may be taken between 9:00 a.m. and 4:30 p.m. with allowance for the noon-day load dip. Where there is any doubt, an analysis of feeder load characteristics will indicate the desirable time for a particular area. Corrections for "off time" readings are not generally made, unless they are found on the "border line."

Preparation of Data for Field Test Forces

With the question of type of data decided, thought must next be given to the procedure for obtaining the data.

Prior to the beginning of field operations a certain amount of engineering study will generally be found necessary. The territory should be divided into the districts by which it is desired to keep records. Whether the territory consists of a closely built up urban area or an extended system with natural divisions, it will usually be found advantageous to proceed with the survey in terms of such districts as best fit the system, such as by towns, substation districts, or arbitrary divisions used in connection with existing operating records. For convenience it may prove advantageous to subdivide the major divisions to correspond to the grouping of secondary circuit maps. Tabulation of the readings by districts, adopted on this basis, will facilitate the analysis of voltage conditions and help in the consideration of plans for system reinforcements. Further tabulations can be made according to types of load, for example, residential-commercial and industrial readings.

Some form of indication must be devised to indicate to the field forces where and when readings are to be taken. This can be done by using secondary maps as the direct medium of designating the locations at which readings are to be taken, as illustrated in figure 1. These secondary plates are designated by plate numbers to which the survey records are indexed.

A procedure for using these plates in connection with indicating readings is as follows: A volume, consisting perhaps of approximately 100 blueprint plates covering a district, can be inspected by one or more engineers familiar with the secondary system to determine the lo-

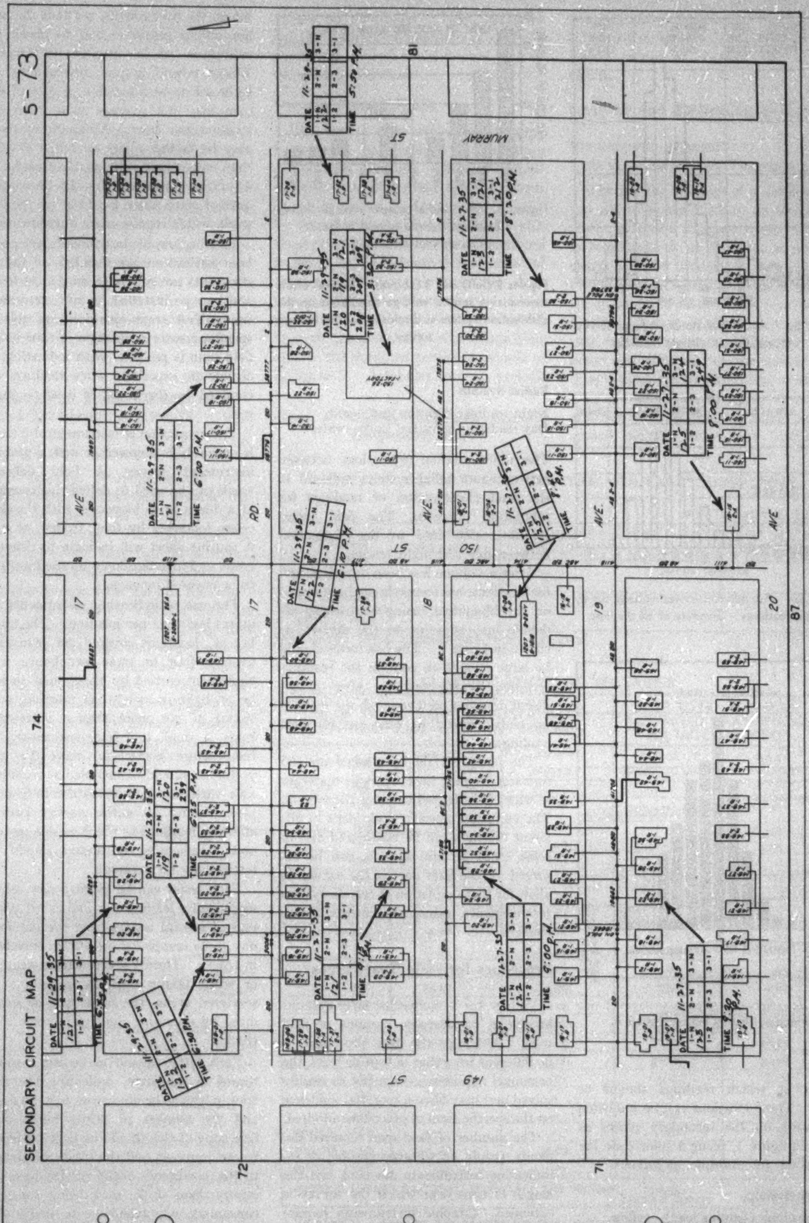

SECONDARY CIRCUIT MAP
5-73
MURRAY
ST
AVE
RD
AVE
AVE
AVE
ST
ST
17
17
18
19
20
150
149
74
72
71
81
87
DATE
TIME
11-29-35
11-27-35
6:00 P.M.
6:10 P.M.
6:25 P.M.
6:30 P.M.
5:50 P.M.
8:20 P.M.
8:40 P.M.
9:00 P.M.
9:30 P.M.
1-N
2-N
3-N
1-2
2-3
3-1

Figure 1. Secondary-circuit maps adapted for voltage-survey use. Test locations shown and indicating readings recorded directly on secondary blueprints

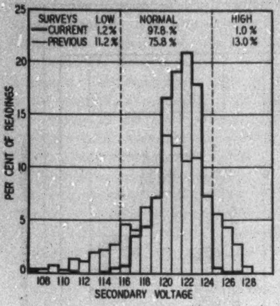


Figure 2a. A-c radial-system voltage survey, Verona district. Night readings

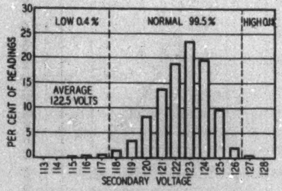


Figure 2b. A-c network-system voltage survey Night readings. Summary of all districts

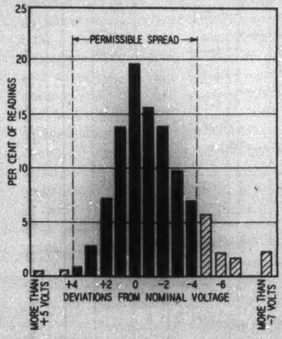


Figure 2c. Service entrance voltages during principal lighting hours, district A

cations at which readings should be taken. These locations can be indicated by arrows on the secondary plates as shown in figure 1, using a color code for the arrows, for example, as follows:

Network System

Night readings (lighting load)—yellow
Day readings (industrial load)—green

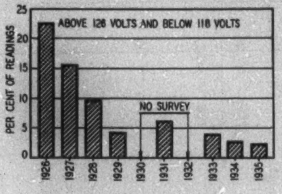


Figure 3. A-c radial-system voltage survey Readings outside normal range Summary of all districts

Figures 2 (left) and 3 (above). Typical charts summarizing service voltage conditions on the distribution system as disclosed by the voltage survey

Radial System

Night readings (lighting load)—red
Day readings (industrial load)—white

The differentiation by colors between network and radial systems will aid in the final classification of readings for tabulation purposes. The field forces are interested only in the distinction between day and night readings.

After a volume has been "marked up" for locations, box forms should be stamped on the blueprints, using contrasting indelible ink, adjacent to the arrows, as shown in figure 1. The box forms should be large enough to contain the readings recorded by the testers. After a sufficient area has been covered, the marked up plates should be sent out for field readings.

The plates should be marked up and forwarded to the field forces in the order in which it is desired to cover the system. The variation between peak loads in different districts can be considered and in some cases certain districts can be reserved for the later part of the survey to allow for the completion of certain system arrangements, substation cut-ins, feeder changes, etc.

Procedure for Field Test Forces

Voltage surveys covering large systems involve a considerable amount of field work. Although the test procedure is no different from that of routine tests, the personnel requirements are not so readily solved and may have a material influence on the specific form of procedure adopted.

The number of field men required depends mainly on whether graphic or indicating instruments are used and the length of time over which the survey is extended. Graphic instruments require the use of more experienced testers to set up the instruments, see that the pens are inking properly, and to check the calibration of the graphic instruments. Where several graphic instruments are to be set up at a location, a crew of two men may not average more than one location per hour. Although locations may be farther apart on radial systems than on network systems, the time factor is partially balanced by the preponderance of three-phase locations on the network, which require more instruments.

In using graphic instruments, the peak-hour limitations are avoided, as the instruments are generally set for 24 hours and can be installed at any convenient time. Test crews therefore are able to spend a greater proportion of time in the field than is possible with indicating instruments, especially when readings are confined to the hours of 6:00 to 10:00 p.m.

Transportation of instruments by truck is desirable in connection with a graphic instrument survey. A light delivery truck can be used to deliver instruments at a dozen or so locations, with the field crews following by foot, trolley, or bus. A routing sheet will indicate to delivery crews and installation crews the locations to be covered by each.

The use of indicating instruments requires less time per location. The number of locations covered per man may average four or more per hour. The equipment carried by these men should be as light in weight as possible, consisting of not more than a voltmeter, leads, a small tool bag, and maps. A temperature correction curve for the voltmeter should be included as considerable variation in temperature may take place during an entire survey period, affecting readings as much as one or two volts. Indicating voltmeters should be tested daily for accuracy.

Each tester can be given a new set of maps daily, leaving the unfinished readings of several testers from the previous day to be completed by testers traveling in trucks. During severe winter weather or when taking readings in remote or scattered areas, the work can be expedited by operating multiman crews from trucks.

Each tester should make out a route record in duplicate, indicating the district in which the days work is to be done and the number of prints taken out. One copy of this should be kept for office record purposes and the other forwarded to the company's night service force to inform them of the work being done, as consumers may telephone to verify the authenticity of the tester. At the end of the day the tester should check off on the office record copy the number of plates covered, the number of readings taken on each plate, total number of readings taken, and any irregularities, however slight.

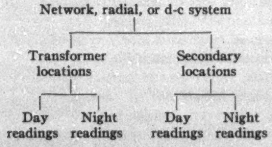
The readings recorded on the prints should be rechecked by a clerical force to see if the readings appear normal. Unusual readings should be called to the attention of the engineer in charge of the voltage survey so that an investigation of conditions can be made without delay.

Where the scale of the secondary circuit maps is such that readings cannot be entered directly thereon, this procedure can be modified to specify addresses and to record test readings on standard test forms.

Recording of Tests, Classification and Tabulation of Results

Tabulation of Readings

The readings obtained by the field forces, whether indicating or graphic, will usually be referred to distribution engineers for analysis. The data must be tabulated before they can be classified. In the case of graphic voltage charts, it is desirable to record the low and high readings for each location together with the average voltage. It is desirable that transformer readings and end-of-secondary readings be tabulated separately so that the results may be weighted according to the number of readings of each type. A number of different classifications, for each district, may be necessary, as follows:



Plotting Readings on Maps

A useful summarized record of the results of a voltage survey may be obtained by plotting the readings on a map of the territory, drawn to some small scale. Where locations are too close together to permit plotting each reading, the average may be shown, particularly if the variation is not great. Such a map is of interest to executives and to the distribution engineers in obtaining a general view of system voltage conditions. Voltage-regulation conditions on individual feeders are advantageously revealed by plotting on detailed maps of each feeder the first consumer readings of each transformer on the feeder. These maps assist the distribution engineers in studying regulation on individual feeders and on the system as a whole.

Voltage Survey Reports

The results of a systematic voltage survey may be presented in some form of periodic report. Various graphic methods are illustrated by the charts shown in figures 2*a*, 2*b*, and 2*c*. It may be desirable to contrast the results with those of the previous survey, as illustrated in figure 2*a*, to indicate improvement of system conditions. A separate chart for each district and for each type of load is usually desirable. It is also interesting to show the percentage of readings lying outside the accepted service standards as in figure 3. A tabular form of periodic report is shown in figure 4. The readings tabulated in figure 4 are the highest and lowest values as recorded on graphic charts.

Analysis and Application of Data

Engineering Analysis of Voltage Survey Readings

Whether the readings of a voltage survey are indicated directly on secondary plates, as shown in figure 1, are recorded on test forms, or are available on graphic charts filed by feeders or districts, the results should be analyzed thoroughly by the engineers in charge of the distribution system. It may be desirable to investigate further by means of graphic voltmeter doubtful high- or low-voltage conditions disclosed by indicating readings. This procedure can be extended to high and low readings one volt within the accepted service limits. All conditions

Figure 4. Monthly comparative report of voltage-survey results

MONTHLY REPORT – A.C. LIGHTING VOLTAGE SURVEY

MONTH–APRIL 1936

PERIOD	CHARTS OBTAINED	SECONDARIES SURVEYED	% OF TIME ELAPSED	% OF SECONDARIES SURVEYED
THIS MONTH				
TOTAL SINCE BEGINNING SURVEY SEPT. 10, 1934				

ESTIMATED TOTAL SECONDARIES TO BE SURVEYED

CLASSIFICATION OF VOLTAGES

% OF TOTAL LOWEST SUSTAINED SECONDARY END VOLTAGE RECORDS			% OF TOTAL HIGHEST SUSTAINED TRANSFORMER VOLTAGE RECORDS	
VOLTAGE	PRESENT SURVEY TO DATE	PAST SURVEY	PRESENT SURVEY TO DATE	PAST SURVEY
BELOW 105	.01	.03	–	–
AT 105	.01	.03	–	–
" 106	.01	.04	–	–
" 107	.05	.09	–	–
" 108	.11	.24	–	–
" 109	.32	.50	–	–
" 110	1.13	1.71	–	–
" 111	2.83	3.64	–	–
" 112	6.67	7.74	–	–
" 113	13.20	14.20	.01	.01
" 114	21.15	20.80	.09	.03
" 115	26.50	26.30	1.06	1.00
" 116	19.00	16.85	7.36	7.50
" 117	7.30	6.38	26.70	28.00
" 118	1.50	1.28	39.50	38.50
" 119	.18	.16	21.70	20.56
" 120	.03	.01	3.12	3.88
" 121	–	–	.38	.40
" 122	–	–	.06	.06
ABOVE 122	–	–	.02	.05
BELOW 112	4.47	6.28	–	–
ABOVE 119	.03	.01	3.58	4.39
AVERAGE LOWEST SUSTAINED SECONDARY END VOLTAGE			114.31 PRESENT SURVEY	114.32 PAST SURVEY

REMARKS: 4.54% OF THE SECONDARY END VOLTAGE RECORDS FOR THIS MONTH WERE BELOW 112 VOLTS.

finally determined to be above or below the accepted standards should be studied carefully to determine whether the subnormal condition is due to primary circuits, secondary mains, or service wires.

System Reinforcements Based on Survey Results

A voltage survey will disclose cases of low voltage due merely to inadequate service wires. These facilities may have been installed in accordance with old standards, inadequate for present-day loads. Such conditions are easily corrected.

Cases of unbalanced voltage will be revealed if all line-to-neutral readings are taken at each location. Some of these conditions can be corrected with little trouble by rebalancing secondary load.

Experience indicates that a high percentage of poor voltage conditions is due to the secondary mains. These conditions are indicated by analysis of the voltage survey data and may be remedied by replacing existing secondary with larger wire, by hanging additional transformers to cut down the length of secondary feed, or in some cases merely by rearrangement of existing secondaries.

In some cases low-voltage conditions can be improved by raising feeder substation voltage, provided this does not lead to overvoltage conditions elsewhere. In other cases high voltage may require lowering substation voltage.

Low-voltage conditions may be found to be due to inadequate primary circuits. Where this occurs, redesign of primary circuits may be necessary. Additional voltage readings and load readings at various points on the circuit may be necessary in such cases to provide engineers with full information. The results of the voltage survey may justify the introduction of new feeders or new substation capacity.

Where general low-voltage conditions exist over an area of relatively high load density, engineering and economic considerations may warrant changeover to underground network supply. The extension of underground network on a step-by-step basis in urban territories where all distribution is planned ultimately to be underground, is facilitated by periodic voltage surveys to disclose which areas first exceed the regulation limits of existing overhead facilities.

Conclusion

The readings of the systematic voltage survey, appropriately taken and carefully analyzed, provide the engineering justification for redesign of the distribution system to correct existing unsatisfactory voltage conditions. The periodic survey data, by revealing trends of regulation, make it possible, in planning system changes, to anticipate unsatisfactory conditions before service voltages actually fall outside the accepted standards of good service. The broad view obtained of system voltage conditions facilitates the consideration of various plans proposed, and is a helpful guide in considering the necessity for general system reinforcement and improvement projects.

It is not anticipated that a survey such as outlined in this paper need be made every year. As voltage conditions are improved, it may be found that a complete survey made every other year, or a policy of surveying a half or a third of the system each year may suffice.

Voltage Regulation and Control in the Development of a Rural Distribution System

By G. H. LANDIS
MEMBER AIEE

Synopsis: The rapid growth of rural electrification and the importance of maximum service continuity commensurate with economical expenditures has given impetus to the study of means to provide a satisfactory solution. The development of a rural network system with its sources of feed to supply this system is described. Selection of voltage rating for rural lines is often dependent upon operating conditions peculiar to the area served, and must be considered integrally with voltage regulation. Methods applicable for voltage regulation and control are reviewed with particular reference as to their application for rural line use. The economic application of regulating devices for voltage control of a rural load area is illustrated by a specific example and comparison with other means available. Methods used for obtaining a constant check of voltage conditions throughout a rural area are described and the results obtained by these means are given in a special case.

The Rural Problem

THE rapid growth of rural electrification in the past few years has stimulated activity in new methods and approach to the problem of providing adequate capacity for the present and future demands which will be made for this service. Of necessity, due to the low customer concentration, the distribution lines serving the rural areas must be constructed at a minimum cost commensurate with existing or near future usage of such service. In the United States, this has resulted almost universally in the supply of these areas by means of single-phase lines, the construction and voltage of the lines varying with the conditions encountered in the areas served, and varying with the economic, engineering, and operating conceptions of the individual utilities.

The rural area in which the Central Hudson Gas and Electric Corporation operates is probably typical of a major portion of the northern Middle Atlantic and southern New England States. The area cannot be considered strictly as a farm area. Essentially, the rural territory is a region populated by two general classes of people; first, the farmers who primarily specialize in the production of special perishable products for consumption of the large urban centers; second, small commercial and industrial enterprises in the rural centers, combined with those who have sought freedom from the city confines by temporary or permanent residence in the rural area. In the first classification, the following types of farming will dominate with their special applications of electric load:

Poultry, requiring electric service for incubators, brooders, water heating.

Stock and dairy, with milk coolers, pasteurizers, water heating, ventilating, milking machines, pumping, and general utility.

Truck and fruit, with soil heating, minor irrigation, fruit grading, insect control, spraying, pumping, cold storage.

General, with pumping, general utility.

In addition to these, the domestic usages of cooking, refrigeration, water heating, and pumping must be provided.

In the second classification, the following groups of people must be served:

Hamlets, with the country store, garage, and occasional small industrial load.

Small residential customers migrating from local urban centers.

Large residences and estates, generally drawn from the large cities.

Vacation residences, road houses and stands, boarding houses, and hotels, most of which are summer loads, but a small increasing amount is becoming either winter and summer loads, or year-round loads.

Religious, educational, and governmental institutions.

The first classification of customers requires an adequate and dependable service supply, for, in essence, they are small industrial establishments, and their livelihood is dependent upon uninterrupted service. As an illustration of the importance of continuous service, one poultry farm sells fancy stock eggs at prices as high as 40 cents each, ships quantities of these abroad for hatching, and incubates as many as 50,000 eggs at one time. An hour's interruption of the rural service to this customer may destroy a large percentage of his incubator supply.

The second classification of customers, being in a large measure people from urban territories, demand essentially the same grade of service here as that to which they have been accustomed, especially since electric service becomes more essential to their comfort and living than before.

The necessity of a high degree of continuity combined with adequate voltage control is therefore essential to the rural population. For this type of service, freedom from interruptions must be considered along with voltage control, and the choice of voltage level is in great part predicated on the ability to maintain service with the greatest freedom from interruptions.

The Rural System of the Central Hudson Gas and Electric Corporation

The franchise area supplied by the Central Hudson Gas and Electric Corporation is shown in figure 1, roughly extending 75 miles north and south along the Hudson River between New York and Albany, and 55 miles east and west, a small portion to the east bordering the state of Connecticut. The territory covers approximately 2,600 square miles and has over a quarter of a million inhabitants. The three largest cities, Poughkeepsie, Newburgh, and Kingston along the Hudson River, account for a total population of less than 100,000. Numerous villages are located throughout the area. Scenically magnificent in the mountainous regions of the Catskills and Shawangunk Mountain Ranges, beautiful for its well-kept farms and orchards, and within close driving distance of the great metropolitan areas, it attracts many thousands of vacationists.

The general layout of the Central Hudson electrical system is shown in figure 1. The company, having developed only a small amount of generating capacity (25,000 kw in water power, and 12,000 kw in steam) is dependent upon the supply of power over strategically located transmission interconnections. A well-located transmission system at 66 kv ties its main load areas into an integrated system, supplemented by secondary transmission feeders and tie lines

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Paper number 38-47, recommended by the AIEE committee on power transmission and distribution and presented at the AIEE North Eastern District meeting, Lenox, Mass., May 18–20, 1938. Manuscript submitted March 18, 1938; made available for preprinting April 25, 1938.

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1. For all numbered references, see list at end of paper.

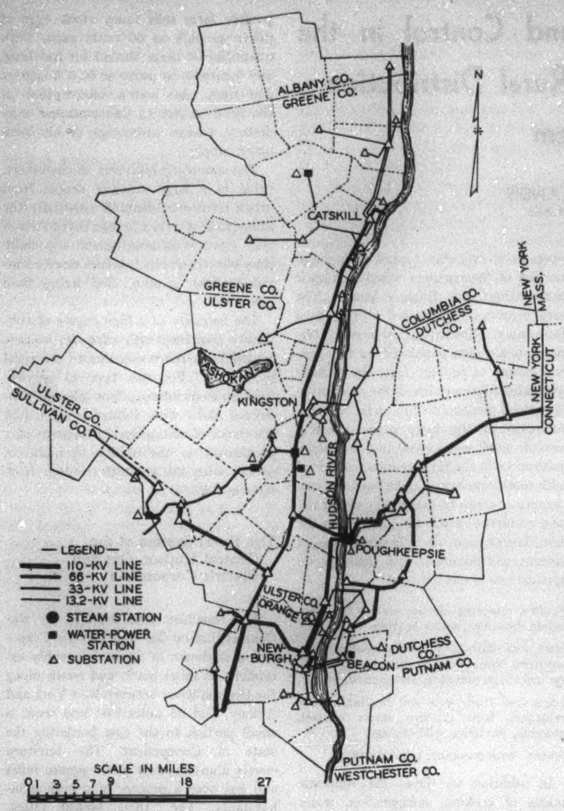

ALBANY CO.
GREENE CO.
N
CATSKILL
GREENE CO.
ULSTER CO.
COLUMBIA CO.
DUTCHESS CO.
NEW YORK
MASS.
NEW YORK
CONNECTICUT
ASHOKAN
KINGSTON
ULSTER CO.
SULLIVAN CO.
HUDSON RIVER
POUGHKEEPSIE
— LEGEND —
110-KV LINE
66-KV LINE
33-KV LINE
13.2-KV LINE
● STEAM STATION
■ WATER-POWER STATION
△ SUBSTATION
ULSTER CO.
ORANGE CO.
NEWBURGH
BEACON
DUTCHESS CO.
PUTNAM CO.
PUTNAM CO.
WESTCHESTER CO.
SCALE IN MILES
0 1 3 5 7 9 18 27

Figure 1. Franchise area showing transmission lines, steam and water-power stations, and substations

at 33 and 13.8 kv, thereby forming a fairly well-defined grid over the entire area.

Among the pioneers in recognizing the necessity, desirability, and value of rural electrification, the company initiated its rural-line program in 1924 and has maintained an aggressive and liberal policy for the development of rural loads. An accelerated program during the last two years has brought about virtual saturation of the area with rural distribution lines, these lines now being within reach of 98 per cent of the rural population. A total of over 2,700 miles of rural lines has been built in the rural construction program, to which have been attached some 6,000 farm, 25,000 residential, 5,000 commercial and small industrial customers. The usage per rural customer averages 1,650 kilowatt-hours annually, but may be expected to increase rapidly since customers on recently constructed lines first make use of electricity primarily for lighting and later take full advantage of a liberal policy on the purchase of electrical appliances.

A typical rural area showing the coverage of highways in a rural section is shown in figure 2. Practically all roads not supplied by distribution lines have a load density of less than three customers per mile.

Power supply to the rural areas has been planned upon the projection of a number of small automatic substations upon the grid formed by the network of rural lines. Theoretically, such an area would appear as shown in figure 3, resembling in many respects a primary network grid. However, several distinct factors differentiate the plan from a primary network. Three-phase tie lines are generally provided, or are anticipated between substations, these being constructed if, and when such tie lines are economically justified. This construction consists of polyphasing existing single-phase lines, such single-phase lines later to form tie lines having been built of greater conductivity size in anticipation of their future tie-line application. At the present time plans do not contemplate the operation of the tie lines closed, these being operated as radial feeders from each substation. Their main purpose is to provide facilities for emergency feed into an area in the event of a substation failure, or in case of the necessity of taking a substation out of service for routine maintenance. Routine maintenance is necessarily scheduled for off-peak periods. In case of a substation failure during peak periods, lower voltage than normally considered fully adequate would be experienced, but service could at least be maintained. This latter condition has not yet been experienced on our system, and should be relatively infrequent. Future development may show the possibility of operating the rural network as an integrated network, as recently accomplished in a section of suburban Philadelphia.[1,2] Figure 4 shows a portion of such a development showing the location of stations and tie lines.

The locations of substations to supply this rural network are necessarily influenced by a number of factors, as follows:

1. Load density of the area to be supplied.

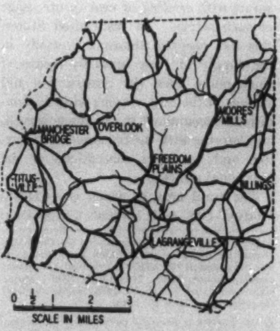

MANCHESTER BRIDGE
OVERLOOK
MOORES MILLS
FREEDOM PLAINS
TITUSVILLE
ILLINGS
LAGRANGEVILLE
0 ½ 1 2 3
SCALE IN MILES

Figure 2. Typical rural area showing coverage of highways

2. Proximity to existing transmission lines. Wherever possible, advantage is taken of existing lines and switching structures in such lines. Substations are tapped through suitable protective equipment to lines of any voltage available, 13.8-, 33-, and 66-kv equipment having been used.

3. Proximity to villages or industrial loads.

Various types of substations have been utilized, the type depending to a great extent upon the following factors:

1. Probable load growth in the immediate area.

2. Possibility of load shift as area is developed.

3. Availability of existing equipment, which may be transferred from some other substation due to necessary changes in that location.

During the early periods of development, when the number of rural lines in an area were limited, a number of two- or three-pole platform structures both with and without regulators were installed. These have mostly been replaced by more adequate equipments, and at the present time there appears to be little need for the use of this type of structure, since it is usually possible to pick up an immediate load of 400 to 500 kw by the time a substation is required in a rural area.

New substations are now installed with a capacity of 750 to 1,000 kva with the possibility of increasing these ratings 25 per cent with air-blast equipment. They are tapped to the transmission line through a manual three-pole disconnect switch and protected on the high-voltage side by means of fuses. Voltage regulation is provided in new transformers through tap-changing equipment. Where existing transformers are available, regulation is supplied by means of step regulators, pole-type or outdoor substation-type induction regulators, or indoor induction regulators rebuilt for outdoor service, the type used depending upon the equipment available.

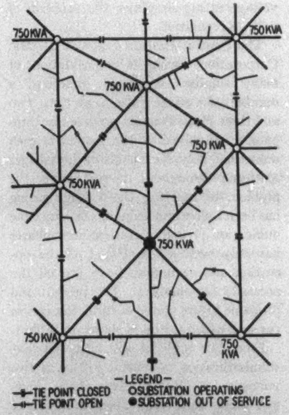

750KVA
750KVA
750 KVA
750 KVA
750KVA
750 KVA
750KVA
750 KVA
—LEGEND—
TIE POINT CLOSED
TIE POINT OPEN
O SUBSTATION OPERATING
● SUBSTATION OUT OF SERVICE

Figure 3. Theoretical rural network showing polyphase tie lines between substations, and operation with one substation out of service

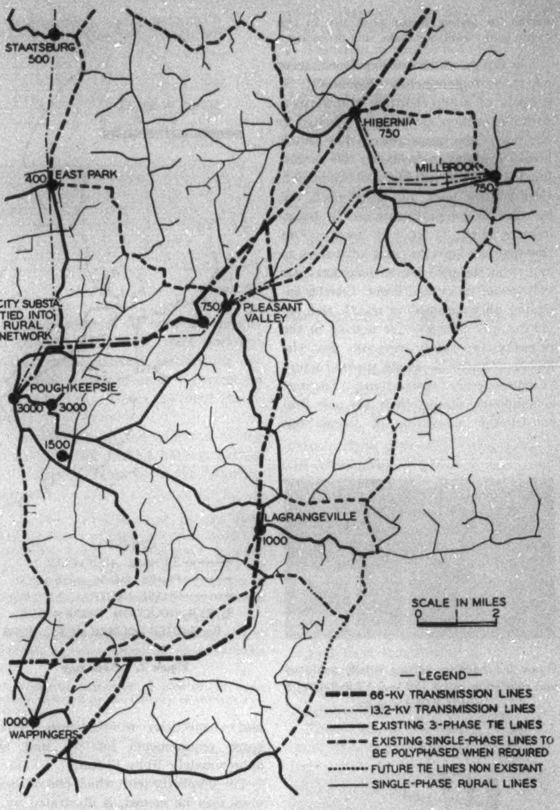

STAATSBURG
500
400 EAST PARK
HIBERNIA
750
MILLBROOK
750
CITY SUBSTA. TIED INTO RURAL NETWORK
750
PLEASANT VALLEY
POUGHKEEPSIE
3000
3000
1500
LAGRANGEVILLE
1000
1000
WAPPINGERS
SCALE IN MILES
0
2
—LEGEND—
66-KV TRANSMISSION LINES
13.2-KV TRANSMISSION LINES
EXISTING 3-PHASE TIE LINES
EXISTING SINGLE-PHASE LINES TO BE POLYPHASED WHEN REQUIRED
FUTURE TIE LINES NON EXISTANT
SINGLE-PHASE RURAL LINES

Figure 4. Development of rural network showing substation and tie lines

Where new equipment is required, a unit-type substation has been adopted as standard equipment. The first unit was installed in 1931, and has been followed by the installation of three additional units. The 1938 program calls for the installation of three units, one to be ready for service by the end of May.

The essential requisites for a unit substation are as follows:

1. The total installed cost must be low, comparing favorably with the cost of the conventional structure substation.

2. The unit must provide voltage regulation to compensate for transmission and primary voltage drops.

3. It must be inconspicuous, require a minimum of space, and present a minimum hazard from public liability standpoint.

4. The installation cost must be low, thereby providing maximum salvage in the event it becomes necessary to shift the location due to a change in load distribution or development.

5. It must be easily and safely maintained. In the event of breakdown of any piece of apparatus, particularly the transformer, the layout must allow for its ready removal and replacement.

6. The design must provide economical

means for adding feeder positions, in the event load conditions make this necessary.

The design which was developed meets all these requirements. Essentially, it consists of a tap-changing transformer, with one or more switch houses bolted to one side of the transformer. The switch house contains a reclosing oil circuit breaker and load-ratio-control equipment. High-voltage fuses and disconnects are installed in compartments on the transformers rated 23 kv and lower. Our practice has been to bring all cables to and from the pole line underground, for voltages of 23 kv and lower, thereby enclosing all equipment in metal, and obviating any necessity for fencing in the property for safety reasons. For the higher voltages, a simple superstructure permits overhead connections. The unit is completely factory built, the only field construction consisting of foundations and connections to the lines. Floor space requirements for the unit are approximately 10 by 13 feet.[3]

Figure 5. Medium-voltage totally enclosed unit substation, rated 750 kva, 13.8–2.4/4.16 wye/4.8/8.3 wye kv

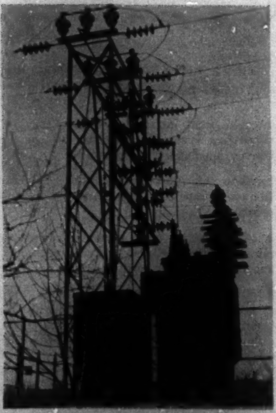


Figure 6. High-voltage unit substation with superstructure for high-voltage control, rated 1,000 kva, 66–2.4/4.16 wye/4.8/8.3 kv

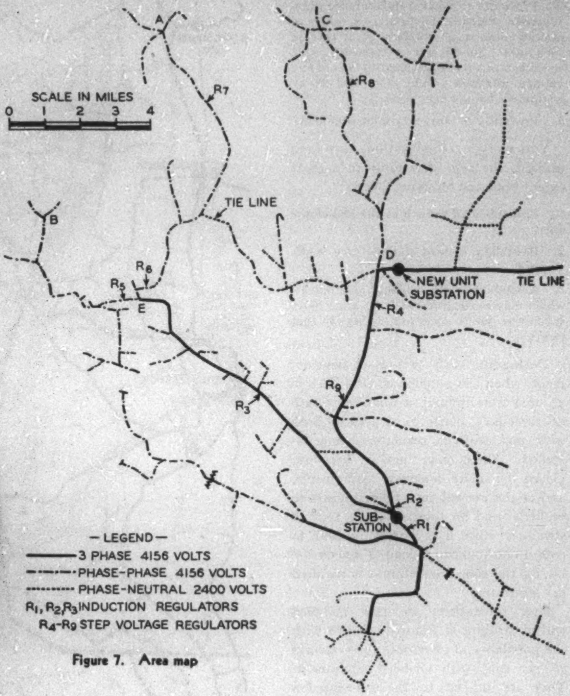


Figure 7. Area map

The simplicity with which one of these units may be moved, is illustrated by a case in which it was necessary to move a 750-kv unit complete with feeder equipment, approximately four miles. The unit was available for operation within two working days, including a change in internal connections from 4,800 volts delta to 4,160 volts wye.

Figure 5 shows a typical unit substation rated 13.8 kv, 750 kva, with all live parts metal clad. Figure 6 illustrates a 66-kv 1,000-kva unit with exposed high-voltage switching on a steel superstructure. The air-break switch on top of the structure and the capacitor on the side have no relation to the unit structure, being automatic sectionalizing equipment for the 66-kv transmission line.

Voltage Rating of Rural Lines

The selection of voltage rating for rural lines is dependent on many factors, and the economics of a voltage rating in one part of the country may well be reversed in others. Obviously, a high voltage rating simplifies the problem of voltage regulation.

The Central Hudson Gas and Electric Corporation, being a consolidation of many smaller companies, inherited a number of voltage ratings at both two and three phase from its predecessor companies. Involved as the system is with numerous polyphase loads in the villages scattered throughout its rural area, the problem of unifying the voltage rating has been a slow and arduous one, since the unification of voltage ratings and phases has only been accomplished as the operation and economics have proved the necessity for changes. The present and probable future status of rural voltages in per cent of total load is shown in table I.

From this table, it will be noted that standardization in general will be at two voltages; 4,160-volt, three-phase, four-wire mains with single-phase branches at 2,400 volts phase to neutral and 4,000 volts phase to phase, 4,800 volts single and three phase. A limited area will, in

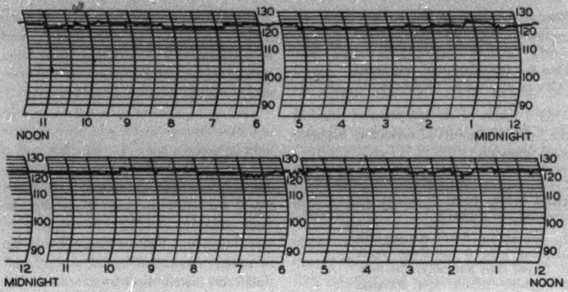


Figure 8. Typical 24-hour chart—primary voltage at point A

all probability, remain at 8,320 volts, three phase, four wire, with branches at 4,800 volts phase to neutral, or at 2,400 volts phase to neutral, the reduction in voltage being obtained through 2:1 autotransformers.

It will be noted that the company's experience has covered a wide range of voltages. Beginning initially with 2,400-volt two- and three-phase systems, conversions were made first to 2,400/4,160 wye operations, and at a later date to some 4,800-volt systems. A trial installation at 4,800/8,320 wye was also made. This has been successfully operated, but was found to have definite operating disadvantages for conditions encountered in our particular area. For sparsely settled areas where clearances from trees can be readily obtained, and where economic joint usage with communication facilities can be satisfactorily negotiated, there appears to be a distinct field for its application.

Consideration of the factors involved in the analysis of our rural problem has led us to the conclusion that the most satisfactory voltage for rural distribution in our area is 2,400/4,160 wye volts, three phase, four wire with common-neutral multiple-grounded construction. However, since some of our areas are operating at 4,800 volts, delta, there does not appear to be sufficient justification for conversion of such areas to the common neutral system. Hence the dual standard.

Table I. Distribution of Rural Voltage Ratings

Voltage Rating	Present		Ultimate
	Connected Kilovolt-amperes	Per Cent of Total	Per Cent of Total
2.4 kv, three phase delta	3,571	6.4	...
2.4 kv, two phase	4,721	8.5	...
4.2 kv, three phase wye	32,196	58.3	72.0
4.8 kv, three phase delta	11,977	21.8	25.0
6.2 kv, three phase wye	1,460	2.6	...
8.3 kv, three phase wye	1,333	2.4	3.0
Total	55,258	100.0	100.0

The decision on the adoption of the common neutral 2,400/4,160 wye system was primarily based upon the following factors:

1. Spacing of substations in the fully developed plan will provide rural feeders of reasonable length for adequate primary regulation, especially with the judicious use of step regulators and shunt capacitors in providing economical and satisfactory means of voltage control.

2. Advantages of a common-neutral system in obtaining better operating characteristics in:

(*a*) Lightning protection of distribution transformers as shown by the analysis of outages from this cause.

(*b*) Low ground resistance of the neutral wire through multiple grounding.

(*c*) Simplification of all line and transformer devices.

(*d*) Improvement in sectionalizing and branch fuse operation.

3. Safety in live-line maintenance, due to voltage being well within the safe operating range of standard rubber goods.

4. Minimum of trouble due to tree interference. This factor is particularly important in this region, due to the difficulty of obtaining permission for ample clearances from trees in a territory prolific in its abundance of large and old trees. The common-neutral system also permits the economical use of cable construction through the use of self-supporting cable or cable supported on a neutral conductor messenger where tree conditions prevent successful operation of tree wire.

5. Economic analysis indicates that this system can be used at costs comparable with systems of higher voltages.

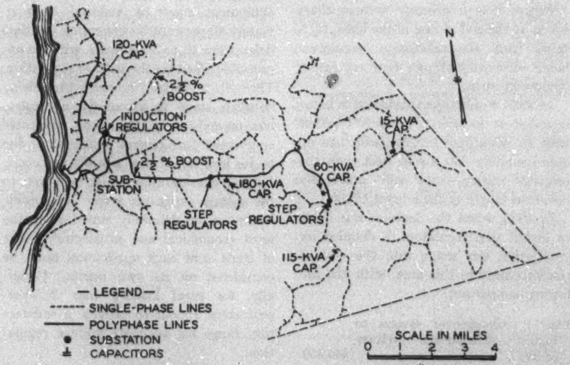
Development of a Rural Area

In the development of a rural area, it is by no means unusual to attach rural lines of extreme length to a substation supply source. During the initial period, the loading on these lines may be extremely light, gradually increasing with the attachment of additional customers and the further utilization of the service. The development of low-cost regulating equipment has materially increased the time limit before additional reinforcement becomes essential. The use in particular of step regulators and automatic boosters may lengthen the period of adequate regulation on a rural circuit for years. One of the big advantages of this class of equipment is that the cost of installation is a small percentage of the total cost and a large part of the cost in equipment is salvageable.

Thus it may be economical to use regulating devices on the lines to furnish temporary relief for any one of the following reasons:

(*a*) To allow further study to be made to

Figure 9. Circuit map



determine the most desirable and economical permanent solution.

(*b*) To allow further development of the area supplied before definite plans are put into effect which may be affected by these developments.

(*c*) To delay the expenditure necessary to accomplish the ultimate plan until economically warranted.

These points are, of course, interrelated and whichever may be the primary reason for the use of regulating devices, the other advantages will be gained.

A description of such a development in one of our rural areas has been presented previously[4] but it may be of interest to trace the development through its successive stages.

Figure 7 shows the area under discussion. Originally supplied at 2,400 volts single phase, lines were extended and supply reinforced through the following stages:

Polyphase of main stems at 2,400 volts (1924–26)..................	peak load 250 kw
Conversion to 4,000 volts (1927)..........	peak load 325 kw
Addition of two pole-type induction regulators and several fixed boosters (1929–34)..................	peak load 700 kw
Relocation of existing regulators and additions of six step regulators, resulting in four step regulators in series on one feeder (1935)...............	peak load 750 kw

The addition of step regulators in 1935 was estimated to provide regulation for the system through 1937 for an estimated peak of 950 kw. Actually, with the installation of an additional polyphase step regulator installation on one of the branches, adequate voltage was maintained with an actual peak of 1,100 kw.

Figure 8 is a primary voltage chart taken at the end of one of the lines, $19^1/_2$ miles from the substation, regulation being obtained through four regulating devices in series.

In 1938 a unit-type substation is being installed at location *D*, which together with an additional tie line, will take up approximately 500 kw of load from the original station, and will practically halve the length of the longest circuits.

In 1935, when the decision was made to install step regulators, a detailed investigation was made into the cost of reinforcement to this area, with the following comparison:

Plan 1. Changeover system to higher distribution voltage (8 kv)........................	$45,350.
Plan 2. Additional 13.2-kv substation, and necessary transformer and feeder extension.....	40,500.
Plan 3. Installation of step regulators and feeder reinforcement.	15,700.

As a result of the adoption of plan 3, the following points of economic interest were obtained:

1. Investment not salvageable or useful when plan 2 would be essential for capacity.............	$3,000.
2. Saving in fixed charges by adopting plan 3 for three years at 12 per cent annually, less non-salvageable items, less increase in losses due to adoption of plan 3...........................	4,400.

The adoption of the step regulator plan was not only the most economical but also postponed the necessity of making a much larger immediate investment, and gave three additional years for the region to develop so that advantage could be taken of changes in load distribution in the area.

Voltage Control and Regulating Devices

No attempt will be made here to present the technical details of operation, or the theoretical limits of applications, as this subject has been fully covered in numerous papers previously.[5–7] A brief outline of the devices available, and their adaptability to rural-line application appears in order.

For strictly rural load, consisting mainly of a multiplicity of comparatively small individual loading devices, the use of high-speed devices for following small voltage fluctuations appears to be unnecessary. A number of tests on the effect of time delay conducted on our regulating devices indicate that the number of operations of tap-changing equipment may be reduced approximately 80 per cent by increasing the time delay from 10 to 30 seconds, with no appreciable effect on the voltage regulation. The step regulator, in which time delay between steps is inherently necessary, has proved to be the most adaptable equipment for general application due to its low initial and installation costs. Careful consideration must be given to the voltage range per step. No specific limits can be definitely stated as to the most economical and satisfactory range of steps, and each application must be considered on its own merits. Generally, for rural loading four $2^1/_2$-per-cent steps appear to provide a satisfactory range for adequate voltage regulation.

The automatic step booster has a very limited application for rural loading, and its application is usually dependent upon comparatively large changes of load connected for appreciable periods of time.

The fixed booster, though obviously not a regulating device, still has a definite field in voltage control. By their use, a line can be controlled to have practically flat regulation at minimum load conditions, and such use has practically the same effect as obtained by increase in size of copper. The amount of boost can be readily controlled by the use of multiple-ratio distribution transformers, and if a standard make transformer rated 2,400/3,600/4,156/4,800–120/240 volts is used, voltage boosts of 2.5, 2.88, 3.33, 5.0, 5.76, 6.66, and 10 per cent may be obtained on a 2,400-volt primary line.

The application of shunt capacitors for voltage control on rural lines has little justification, except where it is possible to make use of the advantages obtained by the capacitor in releasing substation transformer or transmission-line capacity, or in system power-factor correction. The low installed cost of distribution capacitors compares favorably with the installed cost of reasonably large rotating equipment, and where a certain fixed amount of capacity may be economically used at all times, distribution capacitors with their much higher efficiency, may readily be justified. The amount of voltage control supplied by these devices being fixed within narrow limits (two to three per cent on rural lines), they must

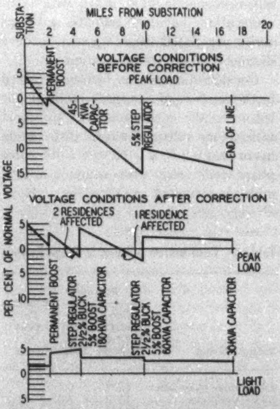


Figure 10. Application of voltage-corrective measures following graphic instrument survey of rural-line loading and regulation

be applied with the same caution as the fixed booster.

The series capacitor can rarely be applied for rural-line regulation. Depending upon the elimination of the reactance drop by insertion in the line of a capacitive reactance equal to the inductive reactance of the line, its purpose is defeated by the large resistance component of the average rural line. Elimination of the reactance drop is insufficient as shown by table II.

By overcompensating, part or all the drop due to resistance can be balanced out at any power factor other than unity. At high power factors, this requires an unreasonable amount of capacity. The series capacitor may be compared to an instantaneous regulator set to maintain a fixed voltage at some load center (assuming the supply voltage constant). It is subject to limitations where customers are distributed along the line in that any decrease in drop effected for the last customer on the line is accompanied by a corresponding increase for the first customer. This can be overcome by splitting the capacity up in small-size units, but only at prohibitive cost.

Except in those cases where instantaneous regulation is required to overcome a flicker condition or where lines are used to feed a load at some distance with no intervening load, some other one or combination of devices will give more satisfactory or economical results than the series capacitor.

Voltage Surveys

In order to keep a constant control of voltage conditions throughout the territory, a program was started several years ago of installing permanent voltage-recording stations attached to the primary lines through individual potential transformers. Completion of this program is expected during 1938. The location of each of these test stations is carefully checked to provide a close control and indicator of the voltage and load trend.

In addition, a plan of conducting periodic surveys of all load areas has been instituted. This survey covers the procurement of a complete series of simultaneous graphic records of voltages over the entire area, coupled with graphic ammeters at pertinent points for loading and load balance, and graphic wattmeters and reactive-volt-ampere meters for circuit loading. These charts, assembled and pasted together, show simultaneous readings all over the circuit, and readily reveal the necessity for load balance, reinforcement, correction for regulator settings, and improvements which may be effected through the use of additional boosters, step regulators, and shunt capacitors. This recording is continued until corrective measures have been applied and reasonable assurance is obtained from graphic records that voltage conditions will continue to be satisfactory. In conducting such a survey 20 to 30 graphic voltmeters are utilized, these instruments being calibrated in the field after warming up in location, a high-grade voltmeter with low temperature error being used for this calibration.

An illustration of the effectiveness of this field survey is shown in an area which had built up very rapidly in rural extensions, and which had not received a previous check of the type described. Figure 9 shows the area in question and figure 10 shows the voltage conditions before and after voltage conditions were corrected. In this case four additional step regulators and 165 kva in shunt capacitors, provided adequate relief from the serious voltage difficulties encountered. Further improvement to this area, contemplated several years previously, will be provided when the fringes of this area can be transferred to a new unit substation to be installed in 1938.

Conclusions

The necessity of reliable rural lines with adequately controlled and regulated voltage has been shown to be essential for the type of service requirements. Voltage rating is dependent upon many factors, and reliability and freedom from interruptions must be considered along with voltage rating. With the equipment now available, voltage can be accurately regulated over substantially long lines at reasonable cost, and frequently major expenditures for reinforcement can be deferred for long periods by the proper application of regulating equipment. The field for application of step regulators is large and due to the low installation cost may be justified in many cases for temporary relief. The application of shunt capacitors is more restricted on rural lines where resistance drop may be a large portion of the total drop, and cannot be justified for voltage regulation alone, but may be used where interrelated savings in substation capacity or system power-factor improvement aid in the justification of their use. Series capacitors appear to have little application for general rural use, and their justification is dependent on special applications usually in conjunction with motor loads.

Table II. Voltage Drop on Four-Kv Three-Phase Line

Copper Wire Size	Volts Per Ampere Per Mile at 85 Per Cent Power Factor: Due to Resistance	Due to Reactance	Total
2	.66	.38	1.04
4	1.17	.39	1.56
6	1.67	.40	2.07

Rural electrification, being still in its infancy with respect to utilization, requires continual and careful study and checking of load growth, circuit balance, and voltage regulation to the end of producing greater satisfaction among its users, increased revenues from adequate regulation, and stimulating the further utilization of the service available.

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Voltage-Regulating-Equipment Characteristics as a Guide to Application

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Synopsis: In addition to being based on economic factors, the application of voltage-regulating equipment to systems should take into account its characteristics and inherent limitations. These characteristics include operating speed, sensitivity, suitability for parallel operation and for frequent voltage-correcting action, how affected by the load power factor and by line constants, ability to compensate for supply-voltage variations, effect on wave form, etc. The majority of systems need both generator voltage regulators and suitable feeder regulators, if maximum ease, efficiency, and flexibility of system operation are desired. Proper choice and adjustment of the voltage-responsive controlling elements of such regulating equipment will insure harmonious functioning of the regulators throughout the system, without hunting between regulators of different construction.

THE selection of voltage-regulating equipment involves two classes of problems. The first of these is technical, and concerns itself with the characteristics of the various available classes of regulating equipment with their corresponding limitations. The second problem is economic and is concerned with finding the least expensive solution to the problem of maintaining adequately constant voltage. The economic problem predominates in the application of voltage-regulating equipment to the distribution system,[1] as distinct from the generating and transmitting portion of the system. As far as the technical characteristics of voltage-regulating equipment are concerned, it might be felt that these are too well recognized to require further consideration at this time. However, instances frequently occur of misapplication of regulating equipment due to disregard of the inherent limitations of the types of regulating equipment concerned, usually because insufficient consideration has been given to the analysis of the causes of the poor voltage conditions which it is desired to correct.

As here used, the term "voltage-regulating equipment" signifies not merely the voltage regulator itself but also the associated apparatus such as for example, a synchronous condenser when controlled by an automatic voltage regulator.

Only equipment that functions automatically is here considered.

As above defined, voltage-regulating equipment is subjected to supply-voltage variations due to transient conditions such as those caused by faults and also to those produced by slow or rapid load changes. (By load changes are meant not only changes in the amount of steady load connected, but also noncyclic, abrupt changes in load impedance, such as may result from the starting of large motors connected with the circuit and from similar causes.) Voltage-regulating-equipment performance during fault conditions and its effect on stability[2,3,4] will not be discussed here, nor will the subject of voltage variations due to cyclic changes be discussed at this time.

Progress in industrial and domestic electrification has gradually imposed added requirements upon the quality of electric service, such as the present-day insistence on maintenance of sufficiently constant frequency for electric-clock operation. Fortunately, however, this tendency toward more exacting electric-service requirements is to some extent compensated for by the improvement of apparatus. For instance, the advent of adequate generator voltage regulators made it possible to discontinue insisting upon small natural voltage regulation as a generator design requirement. This change makes it possible for generator designers to emphasize other more important characteristics of their machines. It is believed that by proper application of voltage-regulating equipment to a system, the range of taps required in transformers can be kept within economical limits, thereby reducing the investment in what would otherwise be special power transformers. Also maximum freedom in system operation is afforded so that the voltages at different points on the system can be held at the most appropriate values from the standpoint of minimizing transmission line losses and keeping the power factor within the desired limits in interconnecting lines.

Classification of Voltage-Regulating Equipment

Voltage-regulating equipment may be classified on the basis of the manner in which it functions, as shown in table I.

As used in this tabulation, the term "generator voltage regulator" means, according to a proposed standard definition, "a regulator which functions to maintain the voltage of a synchronous generator, condenser, motor, or of a d-c generator, at a predetermined value, or vary it according to a predetermined plan."

The question of whether extensive use should be made of generator voltage regulators in preference to the use of feeder voltage-regulating devices sometimes arises, but except in special cases of systems of very limited extent, a definite choice between the two types of equipment should rarely be necessary. Generator voltage regulators are, by virtue of their construction, usually better suited to rapid and frequent operation than are feeder voltage regulators, if maintenance of the latter is to be kept within reasonable limits. Generator voltage regulators are, however, electrically farthest away from individual

Table I

(*A*) Voltage-correcting equipment (corrects for supply-voltage variations)	
(*a*) Direct (series) correction	
Varying excitation (generator voltage regulator)	(1)
Varying voltage ratio	
I. With major change in voltage level (Load-ratio-control transformer)	(2)
II. Without major change in voltage level	
A. Smooth changes	
(Station-type induction regulator)	(3)
(Branch-feeder induction regulator)	(4)
B. Changes by steps	
(Regulating transformer)	(5)
(Station-type step regulator)	(6)
(Branch-feeder step regulator)	(7)
(Branch-feeder booster)	(8)
(*b*) Indirect (shunt) correction	
(Synchronous condenser with voltage regulator)	(9)
(Shunt capacitors with voltage-regulating relay)	(10)
(Shunt reactors, in blocks, with voltage-regulating relay)	(11)
(Shunt reactors with saturating windings)	(12)
(*B*) Voltage-drop compensating equipment (does not correct for supply-voltage variations)	
(*a*) Direct (series) compensation	
(Series capacitors)	(13)
(Series reactors with saturating windings)	(14)

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[1] For all numbered references, see list at end of paper.

loads responsible for fluctuations in the voltage. Thus there appears to be little relationship between the speeds of operation of these devices and the duty ordinarily imposed on them. This dilemma can be avoided by proper adjustment of the sensitivity of the voltage-responsive control elements of the generator and feeder voltage-regulating equipment, respectively. In general, it may be said that:

1. The need for rapid correction of the voltage in case of major system load changes makes generator voltage regulators essential for most systems.
2. The value of generator voltage regulators in limiting overvoltage in case of sudden loss of load is now well recognized.[5]
3. The impracticability of applying line-drop compensating schemes to most systems to permit generator voltage regulators to hold constant voltage at a remote distribution center makes feeder regulators desirable.
4. The diversity of feeder lengths and loading makes the application of individual feeder regulator equipment of appropriate type essential to good voltage conditions at the load.

It is hoped that a brief discussion of the comparative characteristics of each of the voltage-regulating equipment types mentioned in table I will be of assistance to anyone faced with the problem of choosing between the available means for solving this problem.

Generator Voltage Regulators (1)

These regulators are best suited to holding constant voltage at the low-voltage or high-voltage generating station bus. Although it is possible to use line-drop-compensating schemes to hold constant voltage at the remote end of a transmission line, there are very few cases in which as much line-drop compensation as this is used. The basic reason for this is that changing load and generating conditions on a system make it desirable to hold different voltage levels at different points on the system at various times of the day. Compensating for all of the transmission-line voltage drop makes it difficult to control the division of reactive power between different generating stations and is comparable to attempting to operate two generators, each under voltage-regulator control, connected to the same bus without having the usual provision for eliminating excessive reactive-current interchange between the two generators.[6] The use of motor-operated voltage-adjusting rheostats for generator voltage regulators in unattended stations makes it possible to adjust the voltage levels held at such stations by means of supervisory control just as readily as the kilowatt load on units in such unattended stations can be adjusted.

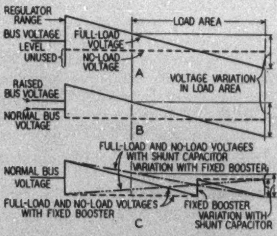
REGULATOR RANGE
BUS VOLTAGE LEVEL
UNUSED
FULL-LOAD VOLTAGE
NO-LOAD VOLTAGE
LOAD AREA
A
VOLTAGE VARIATION IN LOAD AREA
RAISED BUS VOLTAGE
NORMAL BUS VOLTAGE
B
FULL-LOAD AND NO-LOAD VOLTAGES WITH SHUNT CAPACITOR
VARIATION WITH FIXED BOOSTER
NORMAL BUS VOLTAGE
FULL-LOAD AND NO-LOAD VOLTAGES WITH FIXED BOOSTER
FIXED BOOSTER
VARIATION WITH SHUNT CAPACITOR
C

Figure 1. The induction regulator inherently has a lower as well as a raise range. Full use can be made of the total range by the proper selection of substation bus voltage (B) or by the use of a fixed boost in the feeder circuit (C). Shunt capacitors produce a voltage rise, and in this case if the voltage rise produced by the capacitor equals the lower range of the regulator, full use can be made of the full regulator range with minimum voltage variation in the load area

In special cases, current limiting relays have been used in conjunction with generator voltage regulators, but it should be recognized that any such supplementary equipment may impair the normal regulator performance during transient conditions. Some use has also been made of power-factor regulators, especially when applied to generators of relatively small size located electrically close to much larger stations, with a view to avoiding attempts on the part of the small machines to change the voltage level of the larger systems and in so doing overload themselves. The use of such power-factor regulators would seem to be extremely limited and ordinarily requires supplementing by the use of a conventional voltage regulator in order to take control away from the power-factor regulator in case of faults on the system. The low power factor of most faults would otherwise cause the power-factor regulator to reduce the generator excitation, which is just the opposite to what is required under these conditions.

The use of generator voltage regulators relieves the station operator of the need for frequent adjustment of the excitation; if there are no generator voltage regulators, the only alternative is to leave sufficient excitation on the machine to insure reasonably stable operation under any anticipated load condition. This additional excitation will result in added losses and heating of the machine.

Although generator voltage regulators are, in general, rapid in action, it should be realized that even if there were no time delay at all in the operation of the regulator, it could not possibly cure light flicker in the case of voltage fluctuation sufficiently widespread to appear at the generator terminals. The first drop in voltage occurs instantaneously, but can only be corrected at a finite rate in accordance with the time constant of the a-c machine. Other methods of mitigating light flicker should be considered instead.[7]

Load-Ratio-Control Transformers (2)

Although originally chiefly used in transmission-line loop circuits and in tie-lines interconnecting different systems, increasing recognition is being given to the suitability of load-ratio-control equipment for maintaining approximately constant voltage at a bus whenever this can be accomplished by ratio change. In general, the load-ratio-control equipment is not intended for very frequent operation, in the interest of reducing maintenance on the tap-changing equipment. Accordingly, the automatic relay equipment ordinarily includes time-delay relays to insure that the departure from normal voltage must persist for a definite time before any attempt is made to correct it.

Station-Type Induction Regulator (3)

The characteristics of the direct or series regulating device used for feeder regulation are a combination of the inherent characteristics of the equipment itself and the control with governs its operation. The design, adjustment and therefore the characteristics of the control are largely determined by the limitations of the device.

In the direct or series devices, the ratio of incoming to outgoing voltage is varied by changing the ratio of $\phi_1 N_1/\phi_2 N_2$. This is accomplished by changing the turn ratio N_1/N_2 in the case of step regulators and load-ratio-control transformers or by changing the flux ratio ϕ_1/ϕ_2 in the case of induction regulators.

The induction regulator is controlled by the same voltage-regulating relay as a load-ratio-control transformer but its response is instantaneous rather than delayed because of the fact that it can stand the duty of making frequent voltage corrections.

The characteristics of the regulated

voltage and therefore of the device itself are determined by the setting of the voltage-regulating relay and the line-drop compensator. The voltage-regulating relay is sensitive and responds to changes in voltages of only a fraction of a per cent. To take full advantage, as far as regulated voltage is concerned, of the sensitivity of the voltage-regulating relay would result in almost continuous operation of the induction regulator due to the continual variation in the incoming supply of voltage. It has been found practical to adjust the voltage-regulating relay used with induction regulators to initiate corrections when the voltage variation exceeds one per cent from the desired level. This constitutes a total band width of two per cent. Corrections would be small and very frequent except for the fact that the voltage-regulating relay is equipped with holding coils which, when the correction is initiated, hold the contacts closed until the voltage is corrected to approximately the center of the band. This holding adjustment of the voltage-regulating relay for induction-regulator control differs from that for step regulators in that with step regulators only sufficient holding is used to insure successful contact operation.

The induction regulator responds immediately when the control initiates a voltage correction, with the result that, except for instantaneous fluctuations, the regulated voltage is kept within the band setting. This is an important characteristic of induction regulators and should be evaluated on the basis that any decrease in variation of regulated voltage permits an increase in the allowable drop in the load area for certain given maximum and minimum voltages on customers' premises.

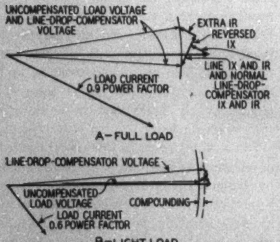
UNCOMPENSATED LOAD VOLTAGE AND LINE-DROP-COMPENSATOR VOLTAGE
EXTRA IR
REVERSED IX
LINE IX AND IR AND NORMAL LINE-DROP-COMPENSATOR IX AND IR
LOAD CURRENT 0.9 POWER FACTOR
A—FULL LOAD
LINE-DROP-COMPENSATOR VOLTAGE
UNCOMPENSATED LOAD VOLTAGE
COMPOUNDING
LOAD CURRENT 0.6 POWER FACTOR
B—LIGHT LOAD

Figure 2. **Reducing or even reversing the reactance compensation is an effective method of stabilizing regulators operating in parallel where reactance constitutes a large part of the impedance of the circulating path and the circulating current is therefore largely reactive. Increasing the resistance setting to offset the reduced reactance setting will have a compounding effect as shown if the power factor is higher at full load**

Single-phase induction regulators are basically different from three-phase induction regulators in the manner of varying voltage. In the single-phase induction regulator the voltage induced in the series winding varies from a maximum value to zero and is always in phase with the excitation of the shunt winding. The fact that no voltage is induced in the series winding of a regulator at a certain position of the rotor makes it possible to by-pass the series winding when switching the regulator in or out of service. The fact that the induced voltage is in phase makes parallel operation feasible. The three-phase regulator, on the other hand, varies voltage by changing the phase of a constant series voltage through 180 degrees. By-passing three-phase regulators requires paralleling with another similar three-phase regulator set in the same relative position, or the use of impedance to limit the circulating current when the series winding is short circuited. Except in the maximum raise or lower position, the three-phase induction regulator displaces the phase of the regulated voltage and parallel operation requires the use of some means for keeping the rotors of the regulators in the same relative position. This has been accomplished by mechanical connection or by the use of Selsyns.

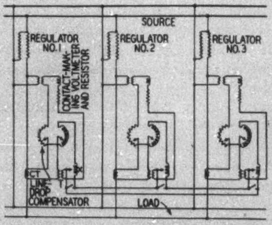

Figure 3. **The use of paralleling reactors (X) inserted in the contact-making voltmeter circuits by means of insulating transformers (T) is an effective method of paralleling regulators in the same station. This scheme also requires a low-power-factor circulating current for successful operation, but it has the advantage of permitting the line-drop compensator settings to match the characteristics of the feeder circuit**

One method of eliminating phase displacement in three-phase regulators that is used occasionally is the interconnection of two three-phase regulators in such a way that the phase displacement of one is compensated for by an equal and opposite phase displacement in the other.

It is inherent in the design of the induction regulator to have a lower as well as a raise range. Full use can be made of the total lower and raise range by properly selecting the substation bus voltage with the aid of transformer taps or by the use of a fixed boost in the feeder circuit as shown by *B* and *C*, figure 1. The installation of shunt capacitors for power-factor correction produces a voltage rise in the feeder circuit as shown by figure 1 *C*, and like a fixed boost enables the full use of the lower and raise range of the regulator without raising the substation bus voltage, providing the voltage rise produced by the capacitor equals the lower range of the regulator.

Branch-Feeder Induction Regulator (4)

The small single-phase induction regulator (12 and 24 kva, 2,400 and 4,800 volts) especially designed for pole mounting has the same general characteristics as the station-type induction regulator. Utilizing the same control, it is capable of the same precise voltage regulation. This characteristic makes it suitable for use on main circuits serving important loads. Being designed to supplement the station induction regulator on long heavily loaded branches or to compensate for voltage drop in the load area, the branch-feeder induction regulator operates more slowly than the station regulator. In case of simultaneous operation, the station regulator will make its corrections very rapidly in comparison to the branch induction regulator and eliminate any unnecessary operation on the part of the latter.

The operation of the branch induction regulator is sufficiently slow to permit the omission of the brake. This important simplification points to low maintenance requirements, which is an important characteristic of the device.

Regulating Transformers (5) and Station-Type Step Regulators (6)

The characteristics of a regulating transformer are identical with those of a load-ratio-control transformer as far as correction of voltage is concerned. It is a question of transformer design as to whether load-ratio-control features should be provided in the power transformer used for producing the major change in voltage levels, or, alternatively, to build the main transformer as a fixed-

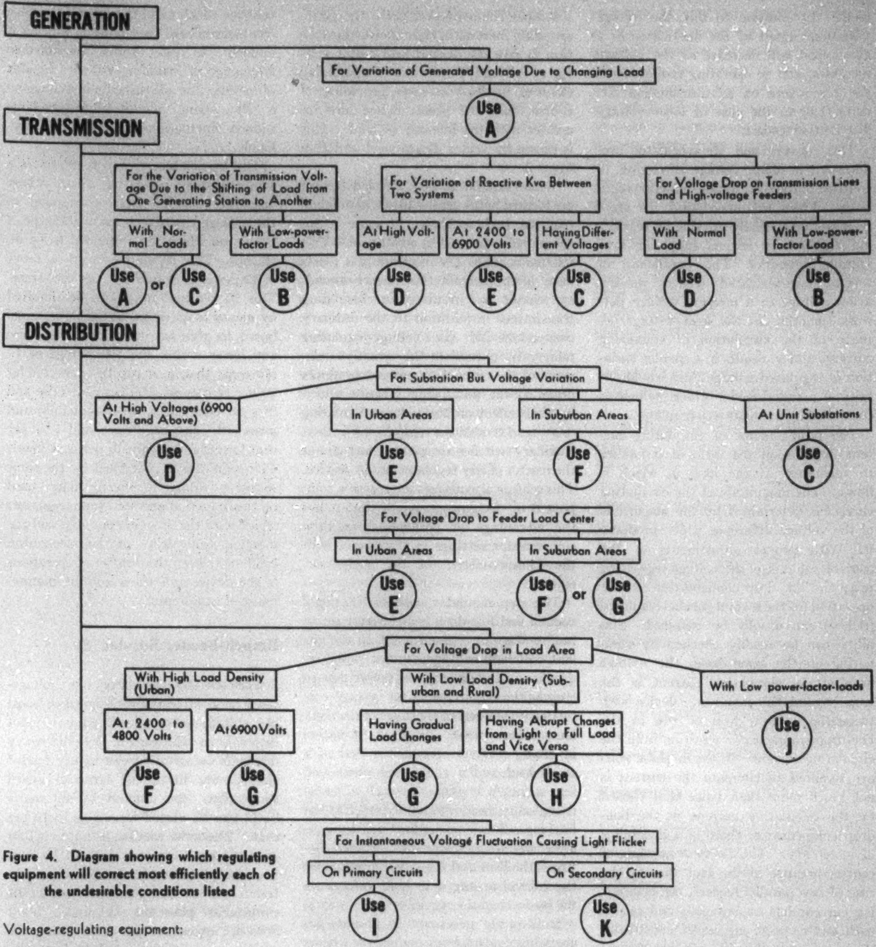
GENERATION
For Variation of Generated Voltage Due to Changing Load
Use A
TRANSMISSION
For the Variation of Transmission Voltage Due to the Shifting of Load from One Generating Station to Another
With Normal Loads
Use A
or
Use C
With Low-power-factor Loads
Use B
For Variation of Reactive Kva Between Two Systems
At High Voltage
Use D
At 2400 to 6900 Volts
Use E
Having Different Voltages
Use C
For Voltage Drop on Transmission Lines and High-voltage Feeders
With Normal Load
Use D
With Low-power-factor Load
Use B
DISTRIBUTION
For Substation Bus Voltage Variation
At High Voltages (6900 Volts and Above)
Use D
In Urban Areas
Use E
In Suburban Areas
Use F
At Unit Substations
Use C
For Voltage Drop to Feeder Load Center
In Urban Areas
Use E
In Suburban Areas
Use F
or
Use G
For Voltage Drop in Load Area
With High Load Density (Urban)
At 2400 to 4800 Volts
Use F
At 6900 Volts
Use G
With Low Load Density (Suburban and Rural)
Having Gradual Load Changes
Use G
Having Abrupt Changes from Light to Full Load and Vice Versa
Use H
With Low power-factor-loads
Use J
For Instantaneous Voltage Fluctuation Causing Light Flicker
On Primary Circuits
Use I
On Secondary Circuits
Use K

Figure 4. Diagram showing which regulating equipment will correct most efficiently each of the undesirable conditions listed

Voltage-regulating equipment:

A—Generator voltage regulators
B—Synchronous condensers
C—Transformers with tap changing under load
D—Station-type step regulators
E—Station-type induction regulators
F—Branch-feeder induction regulators
G—Branch-feeder step regulators
H—Branch-feeder boosters
I—Series capacitor
J—Shunt capacitor
K—Autotransformer

ratio transformer, all regulating to be performed by load-ratio-control equipment in a regulating transformer. The latter type of transformer is also applicable where there is no question of any major change in voltage level but where a variable voltage ratio between two circuits of the same nominal voltage is desired for voltage regulating purposes. Changing the voltage ratio of the regulating transformer and the station-type step regulator, as previously pointed out, is accomplished by changing the turn ratio. This, of course, involves a switching operation of the tap-changing mechanism. Recognizing contact wear and mechanism life as functions of the number of operations, all manufacturers of step-type regulators recommend delayed response to reduce the number of operations and so obtain reasonable maintenance expense. Increasing the band width of the control likewise reduces the number of operations. Either of these expedients to reduce the number of operations results in increasing the variation of regulated voltage.

In the regulation of higher-voltage circuits (6,900 volts and up) the lower cost of the step-type regulator usually is more important than the closer regulation obtainable with induction-type equipment. In addition to this, the voltage variation caused by the fluctuation of a given load will decrease as the voltage increases, and so operating requirements are less severe on subtransmission circuits than in the case of lower-voltage distribution feeders.

The conventional step-regulator produces an in-phase voltage correction, in contrast to the three-phase induction regulator. From this standpoint the step-regulator is especially suitable for regulating high-voltage feeders supplying low-voltage networks. The paralleling of several step-regulated feeders serving a low-voltage or a medium-voltage network smooths out the load voltage because of the circulation of equalizing currents and so results in a smaller variation of regulated voltage than would obtain on a radial feeder whose voltage is controlled by a similar step regulator.

The power factor of circulating current depends on the ratio of resistance to reactance in the loop in which it flows. The magnitude of the circulating current is determined by the magnitude of the voltage difference which produces it. With proper adjustments of the control—that is, of the voltage-regulating relay and line-drop compensator—stable operation (of the several parallel regulated feeders) can usually be obtained. Stability can be readily checked by comparing on the same base, the voltage drop of the circulating current in the loop beyond the regulating device with twice the voltage drop or rise in the line-drop compensator resulting from the circulating current. If the in-phase voltage required to circulate the current is not much more than twice that caused by the circulating current in the line-drop compensator, there is a possibility of instability. The factor 2 appears, of course, because of the fact that in the case of two parallel feeders, the circulating current has an outgoing and return path and as only one feeder constitutes the outgoing path and only one the return path, the circulating current would affect only two line-drop compensators. Obviously the greater the uncompensated impedance in the circulating path, the more stable will be the operation.

In cases where stable operation is difficult to obtain, several possible expedients may be considered. The simplest of these is to decrease or reverse the reactance element in the line-drop compensator. This is effective where reactance constitutes a large part of the impedance of the circulating path and the circulating current is therefore largely reactive. Decreasing or reversing the reactance compensation (with the corresponding increase in resistance compensation to give the desired load-center voltage at full load) results in a compounding effect as the load increases, providing, of course, that the power factor also increases with the increase in load. This is shown by vector diagrams *A* and *B* in figure 2.

Where regulators of paralleled feeders are located in the same station, paralleling reactors, such as shown in figure 3, can occasionally be used to advantage. This scheme requires the insertion of a reactance in the current transformer secondary circuit by means of an insulating transformer in addition to the ordinary connections of the voltage-regulating relay. In operation the reactive element of the circulating current produces drops in the paralleling reactors which by their effect on the voltage-regulating relay tend to hold the regulators together. Auxiliary switches are used to short-circuit the reactor of any regulator not in service. This scheme also utilizes the reactive component of circulating current, but it has the advantage of permitting line-drop compensator settings in accordance with the characteristics of the feeder circuits.

The step regulator with conventional control and line-drop compensator set to hold voltage at a given point on the circuit will hold voltage at that point regardless of the direction of power flow so long as the requirements for voltage correction do not exceed therange of the regulator. The most frequent situation involving reversal of power is that of a small load and a generating plant tied into a larger system. In such a set-up it is usually desired to hold voltage at the load regardless of power flow and the regulator would therefore be installed between the load and the larger system and the control arranged to hold voltage on the load side of the regulator.

It is rarely necessary to provide for changing control from one side of a regulator to the other. This is required only when the reversal of power is caused by a complete reversal of the principal load and the principal source of power. In installations of this type the voltage-regulating relay is usually switched from one line-drop compensator to another so that different settings may be used for the two different operating arrangements.

Branch-Feeder Step Regulator (7)

There is but little justification for installing a device having operating characteristics which excel the inherent voltage characteristics of the circuit on which it is applied. In other words, the practical advantage of installing a device capable of making fine adjustments in voltage on a "shoe string" circuit subject to large sudden fluctuations in voltage is negligible.

The branch-feeder step regulator is a supplementary regulating device whose operating characteristics correspond to the voltage characteristics of the type of circuit on which it is intended to be installed. This regulator covers a range of 10 per cent in four $2^1/_2$ per cent steps. This 10 per cent range can be allocated by means of an easily accessible terminal board to give any combination of raise and lower within the total range of 10 per cent; that is, it can be connected for 10 per cent raise, $7^1/_2$ per cent raise and $2^1/_2$ per cent lower, 5 per cent raise and lower, $2^1/_2$ per cent raise and $7^1/_2$ per cent lower, and finally 10 per cent lower. This regulator is controlled by the same sensitive voltage-regulating relay used on the induction and step-type regulators which with the $2^1/_2$ per cent step and the inherent time delay of the mechanism combine to keep the number of operations of the device and the resultant maintenance at a minimum.

Branch-Feeder Booster (8)

The one-step regulator has voltage-sensitive control and is designed to boost the voltage automatically at infrequent intervals to take care of periodic heavy demands on an otherwise lightly loaded circuit. By means of terminal board connections, the amount of automatic boost can be varied from $2^1/_2$ to 10 per cent. The series winding is tapped in four $2^1/_2$ per cent sections which can be connected to give a continuous boost in addition to the automatic boost as long as the continuous plus the automatic boost does not exceed 10 per cent. Also, the winding can be connected to lower instead of raise the voltage or to provide any combination of continuous raise or lower plus an automatic raise or lower within the range of the series winding.

A one-step device allows the use of a two-position control instead of the three-position voltage-regulating relay (raise, lower, and balanced) required for multistep regulators. The single-step booster uses a simple resonant relay as the voltage-sensitive element. The basis of this relay is a nonlinear circuit consisting of a resistor, a reactor, and a capacitor connected in series with an auxiliary relay coil connected across the capacitor. The nonlinear characteristics of the circuit permit an accurate and permanent adjustment of the pick-up and drop-out of the auxiliary relay.

Synchronous Condenser With Voltage Regulator (9)

The chief application of this means of regulating the voltage is to situations requiring power-factor correction. Such correction, in the case of transmission systems, may involve under-excited operation of the synchronous condenser. Modern types of generator voltage regulators are available for providing continuous automatic control of the excitation of the condenser throughout its leading and lagging range. Obviously, such equipment is suitable for rapidly correcting deviations from the required voltage level.

Shunt Capacitors Controlled by a Voltage-Regulating Relay (10)

Automatically switched shunt capacitors have found limited application in Europe[8] as a substitute for the voltage-regulator-controlled synchronous condenser where the absence of rotating machinery with the associated maintenance and control problem, as well as the losses involved, have favored this solution. This scheme is in general limited to adjusting the amount of connected capacity in a few steps and because of the duty on the switchgear involved the scheme is not well adapted to rapid and frequent correction of voltage conditions. In case of loss of load, further switching problems arise which have apparently been successfully solved in Europe only because of the longer switching times that are considered satisfactory there as compared with American practice.

The recent introduction in the United States of a low-capacity low-cost equipment to switch shunt capacitors installed on feeder circuits has opened a new field for this method of voltage control.

In some instances the voltage rise imparted to a circuit by a shunt capacitor, installed for power-factor correction, exceeds desirable limits under light load conditions. This objectionable feature can be eliminated by disconnecting all or part of the capacitors with this automatic switch. A conventional voltage-regulating relay controls the switch which has sufficient inherent time delay to prevent unnecessary switching of the capacitors with momentary transient voltage fluctuations.

If more than one step is desired, a corresponding number of these switches, all controlled by the same voltage-regulating relay, can be used. The use of a multistep equipment located at the extremity of a feeder circuit provides an effective means for compensating for more or less voltage drop throughout the entire load cycle.

Shunt Reactors Controlled by a Voltage-Regulating Relay (11)

The use of such reactors has been proposed, especially where the charging current of high-voltage transmission lines would otherwise require the connection of synchronous condensers having excessive lagging capacity. However, this scheme is subject to some of the same disadvantages as the use of shunt capacitors controlled automatically in blocks and has the further disadvantage of being useless if correction of low power factor is required.

Shunt Reactors With Saturating Windings Automatically Controlled (12)

Although not subject to the disadvantage of changing the voltage in large steps as is the case when shunt reactors are switched in and out of circuit, shunt reactors having saturating windings are limited to locations where correction of low power factor will not be required and also cannot be used to correct lighting flicker conditions, since the time constant of the d-c saturating circuit precludes instantaneous response to voltage changes. Furthermore, unless special construction is used, it is not possible to eliminate objectionable harmonics, due to the presence of high saturation in the reactor core, that will introduce new problems from the standpoint of inductive coordination. Most of the special construction features used or proposed to eliminate such harmonics are definitely limited to certain system connections not usually encountered in the United States.

Series Capacitors (13)

When used to correct voltage conditions, it must be realized that at best, series capacitors cannot correct for variations in the supply voltage of the circuit but can only correct for the inductive voltage drop. To be effective at all, the ratio of reactance to resistance in the circuit must be high, and care must be used in their application to circuits to which certain types of load are connected.[9] Series capacitors are relatively expensive because they must be entirely at line potential and also include special protective means for short-circuit conditions on the feeder or transmission circuit to which they are applied. In general, it will not be found advisable to attempt to over-compensate for the reactance drop in an attempt to correct for part of the resistance drop in the circuit. The existence of tapped loads along the feeder will frequently make it inadvisable to consider their application. Although unsuited for correcting light flicker due to sudden loads at high power factor, they offer probably the most effective single corrective means for rapid voltage fluctuation due to low-power-factor load increments.

Series Reactor With Saturating Windings (14)

Although the freedom from moving parts and absence of need for special short-circuit protection for the reactor itself at one time made this device look promising for feeder-regulating purposes, it has not as yet found general application. It necessarily has a bad effect on the circuit power factor, and is too slow in action to mitigate light flicker appreciably.

Summary

As an example of how the commoner types of voltage-regulating equipment may be applied to a power system primarily on the basis of their characteristics, figure 4 has been prepared.

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A D-C Transformer

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THE COMPANY with which the authors are connected undertook the design and construction of a device using a mechanical commutator which would transform direct current from one voltage to another.

The two major commutator problems are current collection and commutation. It is well known that it is difficult to solve these problems especially if certain limits are exceeded. In the case of current collections, the speed of moving contacts cannot be indefinitely increased.

In the case of commutation the chief limitation is in the voltage per bar. Designers of commutator machines have established limits of a few hundred volts in the case of small current and limits of about 60 volts for machines of any considerable output. To obtain practical designs of commutators for high-voltage circuits it is necessary to increase the voltage per bar to very large values, usually approaching the breakdown voltage of the air or other medium surrounding the bar. As sparking may ionize the air and initiate arc-over, it is necessary to have very nearly perfect commutation and current collection.

Accepting this requirement of design, it is necessary to devise circuits which will give very good commutation of current and voltage under all conditions of service. Such circuits may be arranged to rectify or invert alternating or direct current or to transform direct current from one voltage to another. If rectification or inversion is attempted, two great difficulties are encountered. First, that of obtaining perfect synchronization of the commutator with the alternating potential and, second, the balancing of the current and voltages of the a-c system against those of the d-c system. If the currents from any finite number of phases of alternating current are rectified and totaled, the resultant current is pulsating and consequently will not perfectly balance a continuous current.

These difficulties are absent in the case of transformation of direct current, using two commutators carried on one shaft and connected respectively to the primary and secondary of a suitable transformer. Synchronization is perfect and inherent and if the number of commutator bars per circuit is the same on the primary and secondary sides of the apparatus, a true balance of current and voltage is possible. The work described in this paper was based on this type of apparatus.

In the course of the experiments, many types of commutators and circuits were tested, the earlier tests resulted in the arrangement shown in the circuit diagram of figure 1. Typical oscillograms are shown in figure 2. Figure 3 is a photograph of the test arrangment.

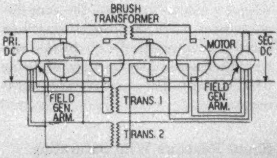

Figure 1. D-c transformer circuit with synchronous excitation and commutation and with brush transformer

This circuit employs synchronous condensers to supply the transformer excitation and to commutate the load current. This earlier work covered a period from 1924 to 1926. At that time we were not aware of any work having been done with this type of circuit. Subsequently a patent[1] came to our attention. Within the last few years the work of others[2] on this same type of circuit has been published. Since these publications explain the principle of operation we need discuss only the brush transformer which is a novel feature and not included in other circuits.

The most desirable wave shape of transformer voltage, from the standpoint of the commutator, is a trapezoidal shape. This would ordinarily necessitate a special and difficult design of synchronous condenser to supply the excitation, but, by adding the brush transformer to the circuit, a standard sine-wave generator may be used, as the brush transformer absorbs the difference between the supplied and the required voltage wave shapes.

Since the apparatus is essentially a transformer, it was felt that the measure of its success would be its ability to meet the same tests as a normal power transformer which includes short circuit, sudden application and removal of voltage with and without load, double-voltage test, etc.

The apparatus shown in figure 2 was operated at loads up to approximately 30 kw, 1,300 volts for periods up to 12 hours. Nevertheless, it was concluded that this type of apparatus could not be made to successfully withstand the tests specified above for the following reasons:

(1) It is extremely difficult, and one might say practically impossible from a practical and economical standpoint, to design synchronous condensers of any appreciable size that would respond fast enough to prevent flashing over the apparatus on sudden large voltage or current changes.

(2) In the type of commutator (open-circuit type) required, the brush rides alternately on conducting and insulating segments. Because of the difference in the material of the segments, unequal wear results, producing sparking and ultimate failure. Also the insulating segments become dirty and necessitate providing very long creepage paths. To minimize these troubles, very frequent servicing of the commutator would be required.

Figure 2. Wave forms obtained with circuit of figure 1

(3) The difficulties outlined above and other difficulties, such as minute sparking, that are negligible at small currents and voltages are greatly magnified at higher currents and voltages and cause frequent flashover.

Further experiments to find a more satisfactory circuit resulted finally in the circuit shown in figure 4 and photograph,

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1. For all numbered references, see list at end of paper.

figure 5. In this circuit static condensers are substituted for synchronous condensers, the brush transformer is eliminated and the commutator is changed to the type (short circuit) in which the brushes ride solely on conducting segments with an air gap as insulation between the segments.

Most of the subsequent tests were made with the circuit shown in figure 4. A large number of tests were made at loads of 100 to 200 kw, 3,000 volts direct current. Since the only previous disclosure of this apparatus has been by patent,[3] it will be discussed in detail.

Referring to figures 4 and 5, there are seen to be two commutators mounted on a driving shaft with a motor. A minimum of six bars per commutator, connected through suitable slip rings to the primary and secondary windings, respectively, of a three-phase transformer are required. For mechanical reasons, a larger number of commutator bars, which must be a multiple of six, is desirable. The commutators shown in photograph, figure 5, have 18 bars, of which three are connected to each slip ring.

The transformer is of normal construction with reactance and losses such as are acceptable for commercial a-c use. Lower core density than usual was used, for reasons to be explained later. The frequency depends on the rotational speed of the commutator and may consequently be selected to suit the particular design.

Figure 3. Apparatus used for tests of circuit of figure 1

The motor has a suitable regulating device to enable it to maintain constant speed under all conditions.

Voltage Commutation

Three excitation capacitors are connected diametrically on the six-phase primary winding of the transformer. They may equally well be connected to the secondary as the magnetizing ampere-turns may be supplied through either winding.

Assume that the commutators are rotated at constant speed, that d-c voltage is applied to the brushes of the primary commutator and that the three-phase transformer is of core-type construction. The voltage generated in each phase will be of the form shown in figure 6, provided the correct size of excitation capacitors are used and provided the timing of the commutator as determined by brush width, bar width, and slot width is correct.

To explain the generation of the voltage, reference should be made to the simplified diagram of figure 7*a*. As the commutator revolves the two active bars come under the brushes and full d-c voltage is applied to the winding and capacitor. This continues for the period 1-2 in figure 7*b*. The d-c line is then disconnected by the commutator until connected in the opposite direction for the period 3-4.

When the connection is made at 1, sufficient current will flow into the capacitor to charge it up to the potential of the d-c lines. At the same time current will begin to flow through the winding and will increase at such a rate as to generate a counter electromotive force in the winding equal to the d-c line voltage in accordance with equation:

$$e = L\frac{di}{dt}$$

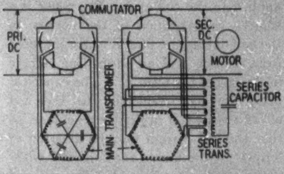


Figure 4. D-c transformer circuit with static-capacitor excitation and commutation

Figure 5. Apparatus used for tests of circuit figure 4

When point 2 is reached and the coil and capacitor are disconnected from the line, this current in the coil will be forced to flow through the capacitor. The result is that the capacitor is rapidly discharged and an oscillatory current set up in the loop formed by the coil and capacitor. The resultant oscillatory voltage across the coil and capacitor will have the form shown at E_1 in figure 7*b*. It will oscillate until the commutator reconnects the d-c line at 3, when it will be absorbed by the line, and d-c voltage applied to the coil in the opposite direction.

The frequency and amplitude of the oscillatory voltage are determined by the relative inductance, resistance, and capacitance of the circuit. These factors are under the control of the designer, as are the relative periods of time 1-2 and 2-3. Consequently, by varying these constants, and more particularly, by selecting the correct capacitor for a given transformer, the oscillation may be made to take the form E_2 of figure 7*b*. Here the crest of the first half cycle of the oscillation coincides in time and magnitude with the d-c line voltage which is applied to the coil at 3. The result is a smooth commutation of the voltage of the coil during the time it is disconnected from the line, and, an alternating voltage in the coil of the form E_2. The capacitor being directly connected to the coil offers a circuit of negligible reactance, so that the coil current may be diverted at 2 from the d-c line to the capacitor without causing a spark. At 3 the coil and capacitor have a voltage equal to line voltage and are consequently reconnected without sparking.

In the case of the six-phase system the interaction of three phases results in the voltages shown in figure 6. The lower trace of the oscillogram, figure 8, shows such a wave form of voltage.

It will be noted that commutation of the voltage is obtained by matching the frequency of an oscillation with the speed of a commutator. Obviously, in order to maintain this condition the speed of the commutator must remain constant. Moreover the inductance and capacitance of the circuit must also be constant, as otherwise the frequency of oscillation will change. This precludes the use of transformer core densities approaching saturation, as a small change in applied voltage will result in a great change in inductance in such a transformer. The iron should be worked at appreciably lower densities than those customary in power transformers. This necessitates a reasonably high frequency if economical design is to result, and that in turn requires high commutator speeds—this being one of the difficulties of the design of such apparatus. The commutation of rapidly changing voltages offers other problems which will be described later, together with their solution.

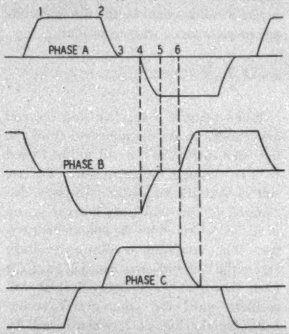
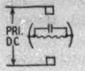


Figure 6. Phase voltages obtained with circuit of figure 4

Load Commutation

Now assume that a load is connected to the secondary terminals of the circuit shown in figure 4. Direct current will flow from the brushes to the load and must pass through the transformer windings. Corresponding currents will flow in the primary windings and will be drawn from the primary d-c line.



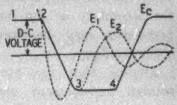


Figure 7a (left). Schematic diagram of excitation of circuit

Figure 7b (right). Wave form of voltages

The wave form of current in the transformer winding will be as shown in figure 9. During the period when the brushes are not short-circuiting any bars, all phases will be in series on each side of the polygon and $^1/_2$ d-c load current will flow through each of them. This condition exists during period 2-3, figure 9. The currents in the secondary commutator (figure 4) must also pass through the windings of the single-phase series transformer and a corresponding current will be induced in its secondary winding. This current will charge the series capacitor and raise the potential across its terminals. It may be noted that all the currents in the primary windings of the series transformer are not additive, four being in one direction and two reversed, the algebraic sum being effective in inducing current in the secondary.

At time 3, figure 9, the commutator will have moved so that two bars of phase 1 are simultaneously under each brush and both windings of one phase of the transformer are short-circuited. Also, these short circuits each include two windings of the series transformer. The voltage of the charged series capacitor is, therefore, available to circulate current in the loop formed. At the moment the short circuit is caused by the brush, one-half d-c line current is flowing in each winding. The current set up by the capacitor voltage tends to decrease this current. Due to the leakage reactance of the transformer windings in which the current is flowing, the current not only decreases to zero but passes through zero and tends to oscillate at a frequency determined by the constants of the circuit. By suitably designing the commutator and transformers and selecting the correct capacitance, this oscillation is made such as to cause the current, at approximately the crest of the first half wave, to coincide in time with the clearing of the short circuit by the further rotation of the commutator and in value with one-half of the d-c line current. The result is that current is diverted to the d-c line without sparking. The dash curve C_1, shown for phase 2, figure 9 indicates in more detail the nature of this commutation of load current.

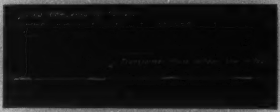


Figure 8. Oscillograms of commutation current and phase voltage of circuit of figure 4

The current in the series transformer secondary and in the capacitor is shown in the top trace of figure 9. It is seen to be the algebraic sum of the three-phase currents and to be of triple frequency. The top trace of the oscillogram, figure 8, shows this current.

As in the case of voltage commutation previously described, the success of the scheme depends on the maintenance of a fixed frequency and of fixed constants in the circuit.

The commutating circuit forms a closed loop during the commutating period and commutation proceeds independently of occurrences in other parts of the apparatus, giving a final value of current exactly equal and opposite to the initial value. Consequently, if the d-c line current changes during this period due to changes in load, the final current will not be equal to the new value of the d-c current and arcing at the brushes will occur. This is the primary cause of difficulty in commutating during load transients and short circuits. The means used to mitigate these difficulties will be discussed later.

The charging of the capacitor during the period between commutations is necessary to store up sufficient energy to supply the resistance losses of the circuit during commutation. Too short a charging period for the capacitor will result in the current failing to reach the correct value at the correct time, regardless of the value of the capacitance provided.

As a result and as may be seen from C_1, figure 9, commutation of the current begins at a definite rate, as indicated by a tangent to the oscillation at this point, and an appreciable voltage exists between the brush and each approaching bar.

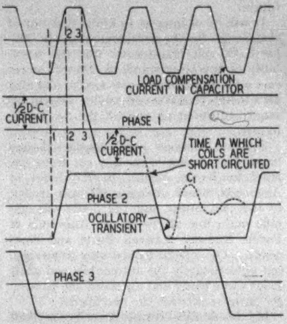

1
2 3
LOAD COMPENSATION CURRENT IN CAPACITOR
½D-C CURRENT
PHASE 1
1
2
3
½D-C CURRENT
TIME AT WHICH COILS ARE SHORT CIRCUITED
C_1
PHASE 2
OCILLATORY TRANSIENT
PHASE 3

Figure 9. Current wave shapes obtained with circuit of figure 4

The result is a tendency to burn the leading edges of both due to the sudden rush of current. This was found to be a real difficulty in the operation of the apparatus.

Commutator Design

The design of commutators intended to operate with thousands of volts between bars takes us into somewhat unexplored fields of engineering. The conventional commutator teaches little except that true roundness, and great strength and rigidity to maintain it, are of great importance. When working with high-voltage commutators it is soon found that this quality is of even greater importance and must in no way be sacrificed to insulation and other factors.

A different type of commutator from figure 5 is shown in figure 10. The largest commutator tested is shown in figure 11. Due to difficulties resulting from thermal expansion of the bars, this commutator did not operate as satisfactorily as hoped. It is of interest, however, that this commutator, together with the necessary transformers, capacitors and other parts, was used for many months as a source of 3,000 volts direct current for experiments on other models. When run under load for some hours it was observed that the leading edges of the bars and brushes would blacken no matter how much care was taken to keep the commutator smooth and round. This blackening of bars and brushes on the leading edges was due to the abrupt beginning of commutation, as previously described. The voltage of the charged series condenser causes current to flow from brush to approaching bar at a rapidly increasing rate as soon as contact is established at any point. The result is a slight spark which eats away the copper and carbon.

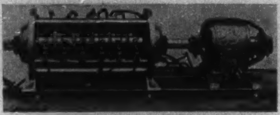


Figure 10. 100-kw 3,000-volt commutator

The commutator shown in figure 5, which was built for 25 kw, 3,000 volts was by far the most successful from the mechanical standpoint. The bars were encased in mica and clamped into deep grooves in a metal wheel. Being clamped at one point only, they could expand freely endways when heated in operation. On the heavy steel shaft is mounted a machine-steel disk. In the periphery of the disk are milled "V" shaped slots. The wedge shaped commutator bars are moulded into mica and set into these slots. The bars project from the slots on both sides of the disk. On one side were attached the slip rings and on the other the molybdenum commutator segments. Thus the whole structure was carried on the one steel disk. To obtain the necessary strength and rigidity a heavy steel retaining ring was placed over the disk and bars, and steel fillers inserted between the ring and the mica insulation on the bars. These fillers were pressed down onto the bars by set screws threaded through the ring.

The timing of the commutator is determined by the relative width of bar, of slot, and of brush. However, these factors cannot be varied outside of certain limits. For instance, a brush too narrow will not ride smoothly over the bars and slots, and slots too wide in proportion to the bar width will, similarly, lead to uneven action. As the timing is determined in part by the width of brush, it is essential that the latter be "worn in" over its whole surface and be free from broken edges.

The most unusual feature of the structure is the molybdenum segment on each bar. These were used to avoid the blackening of the leading edges of the bars. This blackening results from the voltage of commutation between brush and bar and the consequent immediate rise of current at contact. The minute spark attending this condition evidently vaporizes copper and thereby makes pittings in it. The high melting point of molybdenum, together with its slowness to oxidize prevent this to an extent within the ability of the brush to keep it polished.

However, the leading edge of carbon or graphite brushes would still blacken and get rough. In spite of the high melting and evaporating points of carbon, its poor heat conductivity, together with its readiness to oxidize, evidently caused this result. Finally a few layers of molybdenum gauze were placed against both leading and trailing faces of the brush and cemented inside a thin casing of horn fiber. The graphitized carbon of the brush was found to lubricate the molybdenum gauze sufficiently to avoid severe wear of the commutator, while the gauze resisted the destructive action of the current. With this arrangement and carefully worn in brushes many tests were run without showing any apparent deterioration of brush or bar.

Commutation With Varying Supply Voltage

Earlier tests had indicated that voltage could be thrown directly onto a "dead" machine provided it was running at normal speed, but this had never been confirmed by applying as much as 3,000 volts with a large generating capacity back of it.

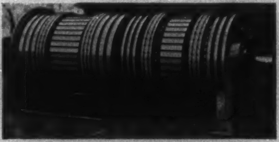


Figure 11. 300-kw 3,000-volt commutator

The principal difficulty to be anticipated here is that of commutating the transient magnetizing current that flows for a few cycles when a transformer is thrown onto the line. This transient current reaches very large values if the flux density in the iron is normally high. The low densities required for the d-c transformer, as already mentioned, bring this current down to values of the order of twice the normal magnetizing current, although the very worst condition, which is that of strong residual flux in the core and switching at the zero point of the wave, will result in somewhat more than this.

An arrangement of reactor, capacitor, and resistor, connected between the line and the commutator, when correctly proportioned, gave perfect results, no arc-overs occurring during a large number of voltage applications. If load were connected to the equipment, however, the reactor core would be saturated by the d-c load current. To avoid this it was provided with two windings having the same ratio of turns as the d-c transformer, and connected in both primary and secondary d-c circuits, but in opposite directions so that the load currents opposed, and hence had no effect on the core. The d-c energizing currents flowing from either or both d-c lines do not oppose and hence encounter the full inductance of the reactor, and are effective to reduce transient voltages on the commutators. A capacitor and resistor may be connected between the reactor and commutator on the primary side only, or on both primary and secondary if the equipment is to be energized from either side. The effect is to apply the voltage gradually during the first few cycles and so reduce the transient magnetizing currents.

Commutation With Varying Load Current

By having a suitable reactor connected in series with the equipment, variations in load current will set up counter electromotive forces in the reactor which oppose the change in current and cushion the effect on the commutators. By such means the equipment is made to function satisfactorily with a varying load.

To avoid damage from flashover, the equipment may be protected with high-speed circuit breakers. There being no mechanically stored energy in the apparatus which could flow into a flashover, such protection will be much more effective than in the case of machinery with a rotating armature or field structure.

Conclusions

It is felt that the circuit constants and difficulties in mechanical construction have been fairly well explored in connection with this investigation and the data obtained should be of value in connection with new projects. The growing need of high- and low-voltage direct current in many fields of electrical engineering should result in additional applications being undertaken.

References

1. United States patent 1,206,662.

2. (*a*). Synchronous Mechanical Rectifier-Inverter, S. S. Seyfert, AIEE Transactions, volume 52, June 1933, pages 397–407. Also AIEE *Journal*, May 1936, pages 548–53.

(*b*). New Systems for Producing Electromagnetically High Voltage Direct Current, E. Alm. *Proc.* Int. Conference on Large H. T. Systems, 1935, volume 3, paper number 347.

3. United States patent 1,659,110.

Discussion

J. J. Linebaugh (General Electric Company, Schenectady, N. Y.): [**Editor's Note:** This discussion covers also "New Types of D-C Transformers," C. C. Herskind, Electrical Engineering (AIEE Transactions), volume 56, November 1937, pages 1372–8.] It is very interesting to have the two papers describing two such radically different ways of accomplishing d-c transformation for consideration at the same time.

Engineers have long dreamed of a simple way to transform direct current from one voltage to another voltage similar to the well-known a-c transformer and numerous attempts have been made to solve this difficult problem. Most attempts have been along the lines of a commutator driven by a synchronous motor of some kind connected to operate as an inverter and as a rectifier.

The two papers on this subject—one using straight mechanical means and the other mercury-arc rectifier vacuum tubes to build up a unit which would accomplish the desired conversion—outline a very serious attempt to solve this problem so that a commercial d-c transformer would be available.

Messrs. Lenox and DeBlieux outline the line of attack followed, frank discussion of difficulties encountered, and results obtained. It is the most ambitious attempt known to the writer to solve this problem and should be of interest to many engineers and especially to any one working on the development of such a transformer in the future.

It will be of interest to know that one of the reasons for the development of the 300-kw 3,000-volt transformer was the possibility of operating standard 600-volt street cars on a standard 600-volt system and also on a 2,400- or 3,000-volt trolley system by simply throwing on and off the d-c transformer.

Several locations were considered, one being the routing of such a car from the hotel in Butte, Mont., to the hotel in Anaconda, Mont., using a 600-volt trolley in the two towns and the 2,400-volt 28-mile main line of the Butte, Anaconda & Pacific Railway between Butte and Anaconda. The scheme looked very promising, but unfortunately the automobile and good roads came along and absorbed the passenger business between the two towns.

Mr. M. A. Edwards has described a small inverter built along the lines of the unit described by Messrs. Lennox and DeBlieux using improvements and material made available since that time. This application has real merit and, if the demand for a d-c transformer becomes great enough undoubtedly the authors have opened the way to the development of a successful transformer of the nontube type.

Mr. Herskind, on the other hand, describes connections, circuits, characteristics, etc., using mercury-arc rectifier tubes to take care of commutation difficulties and sequence of circuits instead of a mechanical commutator. The circuits tried out, giving either constant current or constant voltage output, open up many possibilities for future application.

While the units under test were not as large as the 300-kw d-c transformer, they were large enough to demonstrate the practicability of such a unit.

The size of this type of d-c transformer is dependent on capacity and reliability of mercury-arc rectifier tubes.

Perhaps the d-c transformer of the future will be a combination of the two lines of attack covered by these interesting papers.

It is believed a d-c transformer can be made available for commercial use if demand is great enough to warrant the cost of development.

Instruments for the New Telephone Sets

By W. C. JONES
MEMBER AIEE

Synopsis: Transmitters and receivers for use at subscribers' telephone stations have been designed which not only materially improve transmission but also simplify manufacture and facilitate maintenance. This paper discusses these improvements and describes some of the new design technique employed in their development.

AS A RESULT of continuous development work on transmitters and receivers for use at subscribers' telephone stations, new instruments have been designed which not only materially improve transmission but also embody features which simplify manufacture and facilitate maintenance. These instruments are now being produced for use in handsets, desk stands, and wall sets in the Bell System.[1]

In many respects these instruments represent outstanding advances in transmission instrument design and performance. It is the purpose of this paper to discuss these improvements and to describe some of the new design technique employed in their development. The data presented will be confined almost entirely to physical measurements which serve to define the performance characteristics of the instruments. The interpretation of these data in terms of their relationship to the characteristics of associated apparatus and their over-all reaction on transmission in the telephone plant is covered by a companion paper dealing with the transmission features of the new sets.[2]

Handset Applications

The new transmitter unit with an adapter was first introduced in 1934 as a replacement for the earlier type of handset transmitter.[3] There are now about five million of these transmitters in use in the plant of the Bell System. While experience has shown that they effect an outstanding improvement in performance they do not take full advantage of the possibilities of the unit type of construction from the standpoint of simplification, owing to the fact that a number of additional parts are required to mount the unit on the existing type of handset handle. The advantages of the unit type of instrument have been realized in a new design of handset introduced during 1937, about a million of which have been produced. This handset is shown with the new combined set in the photograph, figure 1, and in cross section on figure 2.

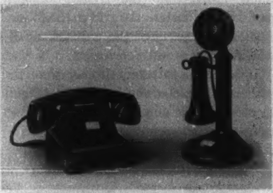


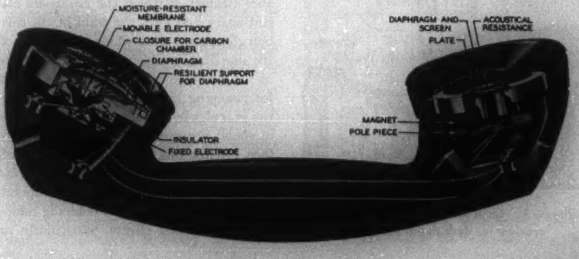
Figure 1. Handset and desk stand equipped with the new instruments

In designing this handset every effort has been made to obtain the maximum degree of simplicity consistent with the electrical requirements involved and at the same time to secure an attractive design which harmonizes with the other station apparatus on the subscriber's premises. Only three phenol plastic parts are employed; namely, the handle and the transmitter and receiver caps. In designing these parts particular attention has been paid to providing adequate cross sections at the points of maximum stress and to distributing the weight so as to reduce to a minimum the breaking moments which are developed when the handset is dropped. The transmitter and receiver caps serve the dual purpose of holding the units in place and providing mechanical protection. In addition they thoroughly insulate the user from all the metal parts which are included in the electrical circuit. Both caps have smooth surfaces which can be readily cleaned. As will be pointed out later, the grid of the receiver cap also has a transmission function and plays an important part in determining the response in the upper frequency range. Spring contacts are provided to facilitate the assembly of the units in the handle. This operation is further facilitated by the fact that specific alignment of the units and the caps relative to the handle is unnecessary. The spacing between the transmitter and receiver is such that the handset can be used with the existing type of desk mounting as well as with the new combined set.

Desk-Stand and Wall-Set Applications

The photograph, figure 1, also shows the new transmitter and receiver unit

Figure 2. Cross section of the handset



Paper number 38-81, recommended by the AIEE committee on communication and presented at the AIEE summer convention, Washington, D. C., June 20–24, 1938. Manuscript submitted April 18, 1938; made available for preprinting May 27, 1938.

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[1] For all numbered references, see list at end of paper.

adapted to desk-stand and wall-set use. Cross sections of these instruments are shown on figure 3. The faceplate, mouthpiece, and protective grid of the transmitter are combined in one phenol plastic part which is so designed as to reduce cavity resonance to a minimum and provide response characteristics essentially the same as those of the handset transmitter. On the other hand, the mouthpiece is sufficiently prominent to encourage the user to talk directly into it and in this way reduce the losses which often result when flush-type faceplates are employed with desk-stand and wall-set instruments. A phenol plastic part, equipped with contact springs, holds the unit tightly in the faceplate and provides electrical connections.

As in the handset the unit of the receiver is held in place by the cap. Springs are provided in the shell for bringing out the electrical connections. A metal insert adds sufficient weight to meet the switch hook requirements of the existing sets. The phenol plastic parts of both the receiver and transmitter are so designed as to thoroughly insulate the units and minimize breakage.

Transmitter Unit

The new transmitter unit is of the "direct action" type, that is, one in which the movable electrode serves the dual purpose of contact and pressure surface. As is shown by figure 2, this electrode is mounted at the center of a diaphragm of thin aluminum alloy formed into a shallow cone and ribbed to add rigidity. "Books" of thin impregnated paper mounted in a recess in a die-cast frame provide a resilient support for the edge of the diaphragm. The fixed electrode is held in place in the frame by a threaded ring and is insulated from the frame by a phenol fiber washer and a ceramic insulator which also forms one of the surfaces of the carbon chamber. The active surfaces of both electrodes are gold plated. A silk annulus clamped at its outer edge between the ceramic insulator and the frame and its inner edge between the movable electrode and the diaphragm forms a resilient closure for the carbon chamber. Electrical connection between the movable electrode and the frame is provided by means of metal strips of low stiffness. Provision is made for machine filling the carbon chamber through a hole in the fixed electrode and closing this hole by means of a cap which crimps over a projecting shoulder. The exposed surfaces of the cap and the threaded ring are silver plated and form the contact surfaces for the electrical connections. A moisture-resistant membrane protects the internal parts of the unit from the effects of condensed moisture from the breath. This membrane is clamped at its outer edge between a protective grid of perforated metal and the frame. A thin metal ferrule fastens the grid to the frame. The exposed parts of the unit are anodically finished to resist corrosion.

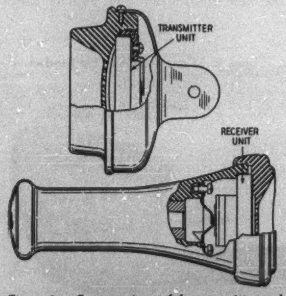

Figure 3. Cross sections of the transmitter and receiver for desk stands and wall sets

In addition to being simpler than the earlier transmitter and hence less difficult to produce, the new transmitter unit has characteristics such that:

1. Its performance is less affected by angular position.
2. There is less aging under the conditions encountered in service.
3. The electrical output is higher and the response more uniform.
4. The modulation products resulting from nonlinearity are materially reduced.

Effect of Angular Position

In order to insure good contact between the carbon granules and the diaphragm in the positions in which the handset is most likely to be held in service, the carbon chamber of the earlier transmitter was placed in front instead of the conventional location in back of the diaphragm.[3] The positional characteristics of this transmitter were further improved by the use of a "barrier" type of variable resistance element in which the electrodes are stationary and form the walls of the carbon chamber, and in which the surface of the diaphragm in contact with the granules is insulated and serves only as means for changing the contact forces between the granules in response to the variations in sound pressure at the diaphragm surface. While this transmitter represented a distinct advance in handset performance from a transmission standpoint and was quite effective in reducing undesirable positional effects, particularly in the "horizontal face-up" position, it was somewhat complicated mechanically and involved the problem of providing a closure between the diaphragm and the adjacent electrode which would be sufficiently resilient to meet the transmission requirements and at the same time prevent carbon leakage. In addition, there was some degradation in quality when it was held in the "horizontal face-down" position where the carbon granules tended to fall away from the diaphragm. While this condition occurred only infrequently in service, it was one which it was considered desirable to eliminate if this could be accomplished without making the structure mechanically complex or difficult to manufacture or maintain. A tendency also was observed in the field for the resistance to increase sufficiently under certain conditions to react adversely on the operation of the associated signaling apparatus. Owing to the inherently small areas of the sound passages leading to the diaphragm the moisture condensed from the breath could not be excluded by a membrane without complicating the structure and adding sufficient mechanical impedance to impair transmission.

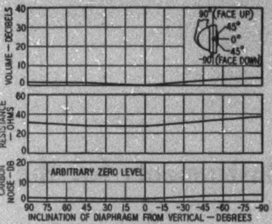

Figure 4. Positional characteristics of the transmitter

Following the introductory work on the barrier transmitter, an intensive study of the direct-action type of carbon element was made to determine whether the limitations of the earlier structures of this type, which arose from the nonfluid character of the carbon, could be overcome. This study resulted in the transmitter unit shown on figure 2. This unit eliminates the undesirable features of the inverted type without sacrificing its desirable characteristics.

The electrode surfaces of the new transmitter unit are so proportioned and so spaced relative to each other that the important current paths shift their locations in the carbon mass with changes in angular position in a manner such that the mean effective pressures in the paths and the lengths of the paths result in sub-

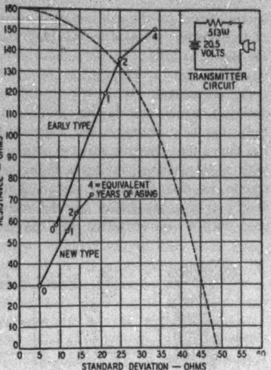

Figure 5. Limiting values of transmitter resistance

stantially constant resistance in all positions. Furthermore, the components of the axial motion of the diaphragm effective in changing the contact forces in the paths are also such as to produce essentially constant modulation. Not only is the total resistance of the paths between the electrodes substantially constant, but this resistance also is uniformly distributed between the individual contacts with the result that at no time does the contact potential rise to a value sufficiently high to produce objectionable carbon noise or "burning." These features result in resistance, volume efficiency and carbon noise characteristics which, as is shown by figure 4, are essentially independent of angular position.

Another and perhaps a more exacting criterion of the adequacy of a transmitter from the standpoint of its ability to function satisfactorily in all positions is the extent to which its transmission characteristics at normal speech intensities are adversely affected when immediately preceded by loud speech. If a poorly designed transmitter is held in a position such that the carbon granules tend to fall away from the movable electrode when this test is applied, the nonfluid action of the carbon will prevent the re-establishment of contact with the electrode surface with the result that volume losses of as much as 20 decibels and a serious degradation in quality take place. Furthermore, these losses persist until the transmitter is jarred or moved about sufficiently to change the configuration of the granules. On the other hand, if the effect of the frictional forces within the granular mass has been taken fully into account in the design of the carbon element, these forces will not react in a manner such as to prevent good contact with the electrode following the large amplitude produced by loud speech and uniform volume and good quality will obtain at all times. The new transmitter meets this test with a substantial margin.

Carbon leakage is prevented in the new instrument without impairment of transmission by the resilient silk closure for the carbon container previously mentioned.

AGING

Transmitter design has advanced to a stage where heating at the points of contact in the carbon element need no longer be an important source of aging. Therefore, such aging of the granular material as occurs in a well-designed instrument is limited almost entirely to that resulting from changes in the properties of the granules caused by abrasion of their surfaces when the transmitter is subjected to mechanical shocks such as occur when the handset is placed on the mounting. As in the case of the earlier transmitter, the new transmitter is machine filled[3] with the result that the motion of the granules and the resultant surface abrasion is reduced to a minimum.

The changes in resistance due to the residual aging have little adverse effect in so far as volume is concerned. In fact, the constants of most of the circuits in which the transmitter is used are such that an increase in resistance adds to rather than decreases the electrical output because of the greater amount of power supplied to the transmitter from the central office battery.

On the other hand, an increase in resistance, though small, may prove to be important in certain circuits where a critical relationship between transmitter resistance and the performance of associated apparatus exists. Under these conditions variations in transmitter resistance may result in failure of the associated apparatus to perform satisfactorily if certain limiting values of resistance are exceeded. In determining the limits to be placed on these values account must be taken of all the variables in the circuit in which the transmitter is connected. Obviously combinations of variables of this nature cannot be dealt with on the basis of averages alone but must include some measure of their range, such, for example, as the standard deviation.[4] The available data indicate that average transmitter resistances and standard deviations which lie within the area bounded by the dotted curve, figure 5, will have no adverse effect on circuit operation in the Bell System plant. This curve is based on certain marginal circuits of which there are a number in everyday use. An important transmitter resistance in determining the performance of associated apparatus in these circuits is the resistance during the period when the call is being established. This resistance is referred to as the signaling resistance. As is shown by the solid curves the signaling resistance of the earlier type of transmitter after artificial aging by an amount considered to be the equivalent of four years in service falls outside the acceptable area. On the other hand the resistance of the new type when aged and measured under identical conditions falls well within the limiting curve and hence not only requires less frequent replacement but also permits greater freedom in circuit design and plant layout.

Moisture condensed from the breath is an important factor in determining the life of a transmitter. A protective membrane is provided in the new transmitter unit which not only is highly moisture resistant but also results in no appreciable transmission impairment. The characteristics of the material employed in this membrane are such that it is not affected by the aging conditions encountered in service such, for example, as the alkaline reaction of water after it has been in contact with phenol plastic parts or tobacco ashes. The exposed metal parts are finished to resist the corrosive action of these agents.

RESPONSE

Reducing the transmitter to an equivalent electrical circuit provides a useful means for analyzing its performance and determining the extent to which the individual parts contribute to its response.

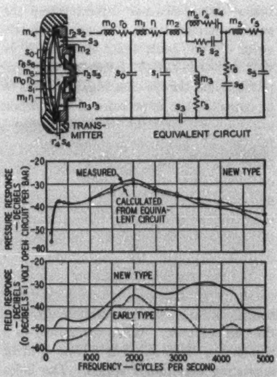

Figure 6. Pressure and field-response characteristics of the transmitter

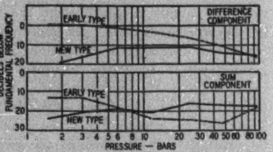

Figure 7. Sum and difference components as a function of the intensity of the fundamental frequencies

Such a circuit for the new unit is shown on figure 6.

While the diaphragm can be represented as a lumped mass for frequencies in the region below 3,500 cycles per second, it is necessary to consider it as being composed of three masses coupled by stiffnesses in order to adequately represent its performance at higher frequencies. These masses consist of the central portion m_5, the ribbed intermediate portion m_2 and the outer portion m_4. The central portion includes the mass of the movable electrode and is coupled to the ribbed portion by the stiffness s_6 which in turn is coupled to the outer portion by the stiffness s_2. The paper books which support the edge of the diaphragm have a stiffness s_4 and a resistance r_4. Their mass is included in the mass of the outer portion of the diaphragm m_4. The internal resistances of the portions which form the coupling stiffnesses s_2 and s_6 are represented by r_2 and, r_6 respectively. A hole is provided in the diaphragm to permit rapid equalization of low-frequency pressures of high intensity and prevent damage to the diaphragm and other parts. The mass and resistance of this hole m_3r_3 are so chosen that their effect on response is confined to frequencies below 300 cycles per second where the station circuit itself is relatively inefficient. The controlling stiffness s_3 is that of the cavity between the diaphragm and the die-cast frame. As is to be expected the impedance of the carbon granules is a function of amplitude and frequency. However, for the purpose of this type of analysis, their impedance characteristics can be represented to a first approximation by a constant stiffness and resistance r_5s_5. The mass of the carbon is lumped with that of the central portion of the diaphragm. The grid of the transmitter unit proper is provided for mechanical protection only and has holes large enough to have no reaction on response. When assembled in a handset or desk stand a second grid of insulating material is added. The holes of this grid have a mass and resistance, m_0r_0, which must be taken into account in arriving at an over-all picture of the factors affecting response. These holes are coupled to the moisture-resistant membrane m_1r_1, by means of the stiffness s_0 of the enclosed cavity. The cavity stiffness s_1 couples the membrance to the diaphragm.

There are two types of response-frequency measurements in general use; namely, pressure-response measurements in which a constant sound pressure is maintained at the face of the transmitter throughout the frequency range covered by the test, and field-response measurements in which a sound field of constant intensity is established at each frequency before inserting the test transmitter. Pressure response is used principally for purposes of analysis whereas field response usually affords a better measure of the performance of the transmitter under the conditions of actual use.

The pressure response of the new transmitter measured with a constant sound pressure at the grid, and the response computed from the equivalent circuit are shown on figure 6. The transmitter used in this test was artificially aged by an amount equivalent to two years of service in order to simulate more nearly plant conditions. While the computed curve departs slightly from the measured curve at certain frequencies, due to the inadequacy of some of the basic assumptions, such as those which were made relative to the impedance of the granular carbon, the agreement in general is so good as to provide a powerful tool for predetermining the response characteristics of transmitters under development and a useful method for evaluating the reaction of one element of the transmitter on another. Reducing the transmitter to an equivalent electrical circuit also has proved invaluable in determining the causes of variations in transmitter performance observed during manufacture.

As previously mentioned, the response characteristics of a transmitter under conditions of actual use may differ from those obtained with a constant pressure at its face. In general this is due to the fact that the diffraction effect of the transmitter as an obstacle in the sound field has not been taken into account. While not as readily predetermined as the pressure response, the field response can be easily measured by inserting the transmitter in a sound field of constant intensity. An artificial voice[8] is used in this measurement as a sound source and the electrical input is adjusted to maintain a constant sound pressure at the guard ring at each frequency before inserting the instrument under test. Response curves for the new and early types of transmitter obtained in this way also are shown in figure 6. Both show rising characteristics in the region of 2,000 cycles per second. While it is feasible to design a transmitter having a substantially flat field characteristic, experience has shown that the transmission obtained with a transmitter having a rising response in this frequency region more nearly approaches direct speech,[2] when used with a representative line and the new receiver, than that obtained with one having a more uniform field response.

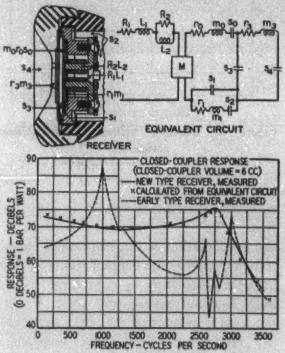
$m_0r_0s_0$
s_4
r_3m_3
s_3
s_2
R_2L_2
R_1L_1
r_1m_1
s_1
RECEIVER
R_1
L_1
R_2
L_2
M
r_0
m_0
s_0
r_3
m_3
s_3
s_4
s_1
r_1
m_1
s_2
EQUIVALENT CIRCUIT
CLOSED-COUPLER RESPONSE (CLOSED-COUPLER VOLUME = 6 CC)
—NEW TYPE RECEIVER, MEASURED
×CALCULATED FROM EQUIVALENT CIRCUIT
---EARLY TYPE RECEIVER, MEASURED
RESPONSE — DECIBELS (0 DECIBELS = 1 BAR PER WATT)
90
80
70
60
50
40
0
500
1000
1500
2000
2500
3000
3500
FREQUENCY – CYCLES PER SECOND

Figure 8. Closed-coupler response characteristics of the receiver

NONLINEAR DISTORTION

A substantial reduction in nonlinearity has been effected in the new transmitter unit.

It is a well-known fact that the slope of the input-output curve of most transmitters is not unity even for sound intensities other than those at which overloading occurs. However, this departure from nonlinearity does not entirely account for the modulation products developed when two frequencies are impressed on the transmitter. Measurements of the sum and difference components in the output of the new transmitter and the earlier transmitter for frequencies of 1,500 and 1,700 cycles per second are shown on figure 7. It will be noted that whereas the difference component produced by the earlier instrument is equal to the fundamental within the speech range, the sum and difference components are both ten decibels or more below the fundamental at all the intensities measured in the case of the new unit.

A tendency for the difference component to be considerably more pronounced than the sum component in the case of the earlier transmitter is characteristic of all of the commercial instruments measured during the development of the new unit. As previously mentioned, this cannot be fully explained by a nonlinear relationship between input and output. The available data indicate that it is due to the manner in which the resistance changes cyclically when pressure waves of two frequencies are impressed simultaneously on the transmitter. Similar effects also have been observed in the microphonic action of carbon contacts themselves. Hence it is not unlikely that this is a fundamental characteristic of the carbon itself. If this proves to be true the extent to which an improvement can be effected in the performance of the transmitter will depend upon the ultimate control which can be exerted over the basic properties of the granular material.

Receiver Unit

The new receiver unit is of the bipolar permanent-magnet type. The magnetic circuit consists of pole pieces of 45 per cent Permalloy, two straight bar magnets of Remalloy and a Permendur diaphragm.[6] The magnets are welded to the pole-pieces to form a unit which is mounted on projecting lugs on the die-cast frame. The coils are wound with enamel insulated wire interleaved with cellulose acetate. The pole tips project through a phenol fiber plate which is fastened at the edge to the frame to form a cavity in back of the diaphragm. This cavity is connected to the recess in the handset handle or receiver shell by a hole in the plate. A disk of specially prepared silk covers this hole and provides the required amount of acoustical resistance. The silk fabric is so woven that it does not change in resistance with wetting and drying. The front of the unit is protected by a perforated metal grid which is assembled to the frame by means of a thin ferrule. A disk of impregnated silk is mounted between the grid and the frame to form a screen which prevents the transfer of foreign material from the front to the back of the diaphragm when the receiver is dropped. The grid and ferrule are anodically finished to resist corrosion. The spring contact surfaces are silver plated.

Prior to the introduction of this unit the receivers in general use for telephone purposes in this country and abroad employed simple resonant diaphragms and as a result had response characteristics which were characterized by prominent resonance peaks. As a rule the peak due to the first overtone, as well as that due to the fundamental resonance, fell within the important frequency range. This resonance not only introduced frequency distortion but increased the intensity with which circuit disturbances such as clicks were reproduced. Furthermore, the diaphragms of these receivers were rigidly clamped between surfaces which differed in temperature coefficients of expansion and heat capacities from those of the diaphragm with the result that the performance of the receiver was erratic and at a given time was dependent upon the temperature changes to which it had been subjected.

The new receiver is so designed that:

1. All prominent resonances within the important frequency range have been eliminated and the response within this range materially improved.

2. The effect of changes in temperature has been eliminated.

3. These improvements have been accomplished without sacrificing simplicity of design or introducing features which complicate manufacture of the receiver or increase the maintenance required.

RESPONSE

An equivalent electrical circuit for the receiver and a typical closed coupler response curve are shown on figure 8. Referring to this figure it will be noted that there are two meshes in the circuit which contain mass, stiffness and resistance and which control the motion of the diaphragm. One of these meshes consists of the acoustical resistance m_1r_1 coupled to the diaphragm $m_0s_0r_0$ by the stiffness s_1 of the cavity between the diaphragm and the plate which surrounds the pole tips. Included in this mesh is the stiffness s_2 of the cavity in the handset handle or receiver shell. The other mesh is composed of a cap grid m_3r_3 and the load s_4 coupled to the diaphragm by means of the cavity stiffness s_3. The grid of the receiver unit proper is provided for mechanical protection only and has holes large enough to have no reaction on response. The mass of the resilient screen is small and is lumped with the diaphragm mass m_0. The electrical portion of the circuit consisting of the winding R_1L_1 and the equivalent eddy current circuit R_2L_2 is coupled to the mechanical and acoustical portion by means of the force factor M.[7]

The response computed from the equivalent circuit for a number of frequencies is included on figure 8. The agreement between this curve and the measured curve is excellent and makes it possible to predetermine the response of the receiver with a high degree of accuracy, and to evaluate the effect on the over-all response of the receiver of changes in the constants of the component parts. This type of analysis also has been invaluable as an aid in establishing the causes of variations in response which have been observed during the development and production of the receiver. A measured response curve of a receiver of the earlier type has been added to figure 8 for convenience of reference. The improvement in uniformity and range of response is obvious. It will be noted that large gains have been effected for frequencies in the range from 1,500 to 3,000 cycles per second.

The response of the receiver to a square-topped wave affords an excellent measure of frequency distortion. Oscillographic records of the output of typical receivers of the new and earlier types are shown on figure 9 for a frequency of approximately 50 cycles per second. The distorting effect of diaphragm resonance is so obvious as to require no comment beyond pointing out that for accurate reproduction of square waves uniform response for an infinite frequency range is required and that the slight rounding of the corners of the wave as reproduced by the new receiver is due primarily to the falling off of its response above 3,000 cycles per second.

The substantially uniform response of the new receiver also renders clicks and other surges much less objectionable. This is due to the fact that the ear does not respond to the peak value of an oscillatory transient alone but integrates the oscillation over an interval at the beginning of the surge, hence the higher the damping the less objectionable the click.

The nonlinear distortion produced by a receiver of the new type is negligible in its reaction on transmission, the harmonics

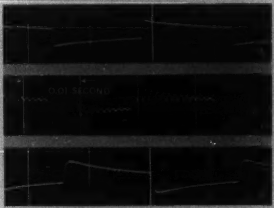

Figure 9. Response of the receiver to square waves

A—Measuring circuit—no receiver
B—Early type receiver
C—New type receiver

in the output being 35 decibels or more below the fundamental.

Magnetic Circuit

Inasmuch as the magnetic properties of the diaphragm, as well as its mechanical properties, must be considered in arriving at the preferred dimensions, it was necessary in designing the new receiver to develop criteria which could be applied in determining the optimum relationships between these factors. This study led to the use of the ratio of the force factor to the effective mass of the diaphragm for this purpose. For given magnetic materials in the pole pieces and the diaphragm and a given air-gap length, there is a pole-face area and diaphragm thickness for which this ratio is a maximum. Typical data illustrating this relationship are shown on figure 10. The available magnetic materials were studied using this technique and a decision was reached to use Permendur in the diaphragm and 45 per cent Permalloy in the pole pieces.

There is a value of polarizing flux for which the force factor of the given magnetic circuit is a maximum. The rate at which the force factor falls off above and below this optimum value of flux is a function of the magnetic characteristics of the materials employed, the length of the air gaps, etc. Without exception the more efficient magnetic circuits have been found to be the most critical as regards polarizing flux. Hence, if wide variations in the efficiency of the product receivers are to be avoided and serious losses due to subsequent demagnetization in service prevented, means must be provided not only for bringing the flux in each receiver to the optimum value, but also for insuring that it remain at this value during the life of the instrument. In order to accomplish this result the magnets of the new receiver are so designed as to overpolarize the magnetic circuit when they are fully magnetized. Equipment is provided for demagnetizing each receiver to its optimum flux value during the assembly process. Receivers which are not sufficiently overpolarized before demagnetization to resist further demagnetization under service conditions are rejected.

Temperature Effects

The diaphragm of the new receiver is held in place by the force developed by the polarizing flux and hence it is free to expand and contract independently of its seating surface. This feature renders the performance of the receiver independent of the changes in temperature to which it has been subjected. The force due to the polarizing flux is sufficiently high to prevent rattling at input intensities many times those of loud speech.

Coupling

Although station circuits can be designed which under ideal conditions result in no sidetone, this objective is never fully realized under actual plant conditions, with the result that a part of the electrical output from the transmitter always reaches the local receiver. Whether the electrical coupling between the transmitter and receiver as evidenced by the residual sidetone is of importance from the standpoint of sustained oscillation or "howling," depends upon the degree of mechanical and acoustical coupling between the instruments. Handset and instrument design has advanced to a stage where mechanical coupling need no longer be a problem. On the other hand, as the response of the instruments is improved, the acoustical coupling may become an important item in determining the howling margin. This margin is so large under the conditions where the new handset is being used for transmission purposes that there is no tendency for oscillation or distortion to occur. However, if the handset is placed face downward on a desk or table, an air column is created which resonates in the region of 2,500 cycles per second. Inasmuch as this is the region where a substantial improvement in the response of the receiver has been effected, a marked reduction in howling margin results. While there is still sufficient margin to meet all of the requirements of field use, this situation serves to emphasize the fact that such factors as acoustic coupling may limit the transmission improvements which can be effected under a given set of operating conditions.

Effective Transmission

The extent to which the better performance of the new instruments is effective in improving the grade of transmission afforded the telephone user is a complex matter and one which is influenced by such factors as the characteristics of the circuits with which the instruments are associated at a given time, the amount of noise present at the transmitting and receiving stations, the reaction of sidetone on the loudness with which the user speaks, the distance between his lips and the face of the transmitter, the tightness with which he holds the receiver to his ear, etc. Many of these factors are beyond the control of the engineer responsible for the design of the transmitter and receiver and hence can be evaluated, in so far as their reaction on transmission is concerned, only by tests made under the conditions of actual use.

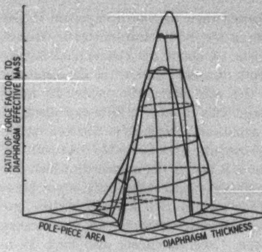

Figure 10. Force-to-mass ratio as a function of diaphragm thickness and pole-piece area

A method has been devised which makes it possible to rate the over-all effect of these factors on transmission in a way representative of the results obtained by the subscribers in their normal use of the instruments.[8] Numerous tests employing this method of rating were made during the development of the new transmitter and receiver to make certain that the course followed in their development would insure the best possible performance under service conditions. Similar tests were also made of the designs selected for production. These tests show that in many respects the new instruments represent outstanding advances in transmission instrument design and performance.

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Combined Thyratron and Tachometer Speed Control of Small Motors

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MUCH has been done in applying thyratrons to motors.[1–4] This paper is restricted to cases where the speed of the motor is measured by an electric tachometer whose output is used to control the motor input. Since this control is made fast by the use of thyratrons, it is possible to hold the speed of the motor constant under difficult conditions or to vary it rapidly in exact accordance with requirements. Three applications of control of this general type are described together with the methods used. The first application is to a null-type high-speed recorder, the second to a high-speed controller, and the third to a motor generator set held in synchronism with a source of frequency normally constant and at low power level.

High-Speed Recorder

One of the principal problems in designing a high-speed recorder is to move the pen or stylus to its final position within the allowed time without overshooting. If the quantity being measured, after making an abrupt change, holds constant at its new value until the chart paper has advanced sufficiently for the pen to draw a short section of straight line, then, any overshooting of the pen if present can be recognized as such and the record can be interpreted without serious error. However, the measured quantity may not be constant at its new value, in which case overshooting is very objectionable because it is impossible to distinguish between overshooting and the true maximum or minimum value of the measured quantity.

In the case of a null-type instrument, such as a slide-wire recorder, inertia is generally large, which fact tends to accentuate overshooting. Inertia is generally large because of certain accessories, and allowance for such accessories, as control disks, slide-wire transmitters, Selsyn transmitters, etc. By avoiding these accessories and redesigning with lighter parts the overshooting can be reduced,[5] but a very good way to eliminate overshooting entirely and at the same time have rugged parts and retain the accessories is to reduce the speed as the balance point is approached.

One way of reducing speed is to reduce motor input, but in general motor input is a poor index of motor speed because of the effect of inertia load, variable friction load, and the limitations of motor characteristics.[6,7] Ways of reducing the speed more definitely have been investigated and it has been found to be most satisfactory to use an electric tachometer to measure speed and control it.

With the use of the tachometer, the motor requirements are simplified. It need only be capable of meeting the maximum torque demand and should, of course, have minimum inertia if maximum speed of recording is desired. A series commutator motor serves very well. The field turns are reduced somewhat below standard d-c practice. This reduces field inductance and gives better performance on the pulsating output of the thyratrons.

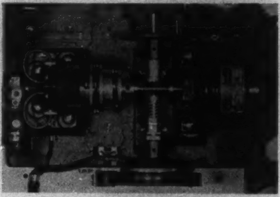

Figure 1. Motor and coupled tachometer

The d-c tachometer for controlling the motor speed is flexibly coupled to an extension of the motor shaft

Figure 2. Combined motor tachometer unit

The d-c tachometer for controlling the motor speed is built into a common frame with the motor

The tachometer is nothing more than a d-c generator with a permanent-magnet field. Figure 1 shows it as a separate unit coupled to the motor through a coupling designed to have maximum torsional stiffness. Figure 2 shows it built integral with the motor.

The motor receives its power through one or the other of a pair of thyratrons, for forward or reverse torque, as required. The connections are indicated schematically in figure 3. This figure shows the recorder as a d-c potentiometer of low range ($2^{1}/_{2}$ millivolts).[8] The current through the slide-wire potentiometer is set at a fixed value so that each and every position of the slide-wire contact corresponds to a known voltage and, since the pen moves with the contact as it is driven to the balance point by the motor, the pen position indicates the value of the measured electromotive force.

Any unbalance is detected by a vacuum-tube amplifier. This is not a d-c amplifier as might be supposed. In general d-c amplifiers are subject to a drift in zero. Instead an a-c amplifier is used which can have slight drifts in sensitivity but no drift in zero.

The a-c signal for the amplifier is derived from the unbalanced d-c voltage by the action of a carbon-microphone modulator driven at approximately constant amplitude at line frequency. This form of detector is fast and free from the effects of vibration.

In operation, if there is a sudden decrease in the electromotive force being measured, there will be an unbalanced d-c voltage in this circuit. A current will flow, the microphone will modulate the current, that is, it will produce an a-c component. The a-c component in passing through the primary winding of the low-level transformer will develop an a-c voltage across it. The secondary winding will apply a stepped-up a-c voltage to the amplifier. The amplifier output is connected to a push-pull transformer which in turn is connected to the grids of the thyratrons. The thyratron plates are supplied with an a-c voltage having a definite phase relation to the microphone drive. In one or the other of these thyratrons, the grid will be positive when the plate is positive and this tube will pass current, thus urging the motor to drive the pen downscale.

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[1] For all numbered references, see list at end of paper.

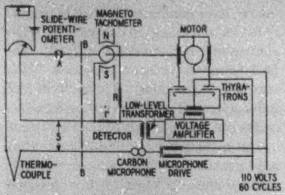

Figure 3. Simplified diagram of potentiometer recorder

As shown, the circuit is arranged for recording the voltage of a thermocouple. When used as a controller the part to the right of *B-B* is unchanged except for the addition of an inductance in series with *R*

If, instead of a sudden decrease, there is a sudden increase of the measured electromotive force the unbalanced direct current will be in the opposite direction, the phase of the alternating current applied to the amplifier will be reversed, so that the grid of the other thyratron will be positive when its plate is positive. It will pass current and urge the motor to drive the pen upscale.

If there is no unbalance there will be no direct current through the microphone, there will be no a-c component, and no motor current.

From this it can be seen that, if the recorder is balanced, it will stay balanced, and if it is unbalanced, it will be urged toward the balanced position (with full torque).

If the recorder consisted of nothing more than this, it would overshoot (and possibly oscillate) due to inertia of the parts, hence the use of the tachometer.

The tachometer voltage is attenuated by a pair of resistors connected as shown. This voltage is then introduced into the circuit of the measured electromotive force so that in operation, the speed of the motor is controlled to maintain the sum of all three voltages in this circuit at zero. In other words, the tachometer voltage is controlled to be equal to the difference between the measured electromotive force and the slide-wire electromotive force. Hence the speed of rebalance tends to be proportional to the remaining unbalance. In this way, the speed is reduced as the balance point is approached so that there can be no overshooting no matter how abrupt nor how large may be the change in the measured quantity.

It is of interest to consider a system of this general type in the case where the only limitations are a fixed torque and a fixed inertia. In this case, the quickest conceivable way to get the system balanced is to throw on full accelerating torque and leave it on up to a certain critical time, at which time the torque should be reversed. If the time be correctly chosen for reversal the system will just come to rest at the balance point. This time can be picked by using a tachometer with a voltage output proportional to the square of the speed. The method is effective for any magnitude of unbalance and for any position of the balance point. This is evident when one considers that the kinetic energy in the motor (and parts that move with it) increases in proportion to the square of the speed and that this energy must never be allowed to exceed that which can be absorbed subsequently by the braking of the motor. The energy which can be absorbed decreases directly with the remaining distance to the balance point.

At every instant the voltage from the square-law tachometer indicates the kinetic energy and the remaining unbalanced voltage indicates the energy which can be absorbed by braking, so, as soon as the former voltage equals the latter, the accelerating torque is turned off and the braking torque is turned on.

In practice, it has been found that the nonlinear current-voltage curve of a copper–copper oxide rectifier[8] when used with a linear tachometer gives a very satisfactory square-law tachometer as indicated by the resulting performance shown in figure 5. It may be noticed that the point of maximum speed does not occur exactly half way toward balance either in distance or in time as it would in the limiting case. The greater time given to deceleration is the result of a margin of safety in the adjustment of the tachometer voltage in order that overshooting may not occur under the worst combination of adverse conditions.

High-Speed Controller

Those familiar with problems of automatic control will recognize in the recorder system just described some features and elements well suited to automatic control. Refer to figure 3 and consider the system to the right of *B-B* as a controller. Disregard the part of figure 3 to the left of *B-B* and consider instead that there is here substituted the machine or system to be controlled. S is an electromotive force which indicates the deviation from normal of the controlled variable such as speed or temperature. A is the position of the control motor shaft and also the position of the rheostat or valve or whatever is involved, but which, in any case, if changed, will sooner or later affect the deviation voltage S.

From the description of the recorder already given (and with a linear tachometer) it is evident that the shaft will be moved at a speed proportional to the deviation voltage S. That is:

$$\frac{dA}{dt} = K_1S \qquad (1)$$

where t is time and K_1 is a constant. This well-known type of control,[9] which is quite satisfactory for the recorder, is likely to be quite unsatisfactory for controlling a machine or system which in itself has delay or time lag. Quite often if set up to operate with speed sufficient to be useful, it will hunt or cycle.

If the effect of the tachometer is reduced to a negligible amount by increasing R or decreasing r (figure 3) and if at the same time a slide-wire potentiometer with its output connected in series with the deviation voltage S is placed upon the control motor shaft, then the position of the shaft will be controlled to be proportional to the deviation S. That is:

$$A = K_2S \qquad (2)$$

This well-known type of control[9] is not likely to hunt. It does not, however, reduce to zero the deviation from normal

Figure 4. High-speed potentiometer recorder

The upper case contains the motor, tachometer, slide-wire, slide-wire rheostat, pen, and chart while the lower case contains the microphone, low-level transformer, voltage amplifier, and thyratrons unless equilibrium just happens to be reached for the normal position of the control motor shaft, A.

By omitting the slide-wire and placing in series with the tachometer, inductance as well as resistance we get the well-known type of control[9,10]

$$\frac{dA}{dt} = K_1S + K_3\frac{dS}{dt} \qquad (3)$$

or

$$\frac{dA}{dt} = K_4\left(RS + L\frac{dS}{dt}\right)$$

where R is the resistance in ohms and L is the inductance in henries in the tachometer circuit. L is not shown in figure 3. K_4 is revolutions per second per volt in the tachometer divided by r (the coupling resistor) in ohms. S is the deviation in volts and A the position of the shaft in revolutions. t is time in seconds.

By choosing proper constants this control becomes as stable and fast as type (2) and reduces the deviation to zero as does type (1).

It is of interest that the complete control system uses no elements which are not used in the recorder with the exception of the inductance L. The motor tachometer unit figure 2 is mounted on the rheostat or valve of the system being controlled. This type of controller is of value for processes which require sensitivity and speed in their control, but which in themselves have too great a time lag to permit the use of a simple type of control. The magnitude of the process time lag that can be handled satisfactorily with this type of control is limited by the circuit elements available for use in the tachometer circuit in the controller. Principally, the limitation is in the time constant of these circuit elements.

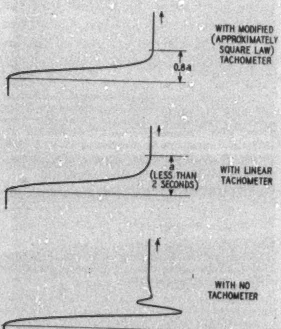

Figure 5. Recorder rebalancing curves

These curves show how overshooting can be eliminated and how the rebalancing time can be reduced by the proper use of a tachometer in a recorder

Synchronized Source of A-C Power

The problem here is to provide an a-c source of several hundred watts synchronized with a signal capable of supplying only a small fraction of a watt. The signal frequency is normally held constant at 60 cycles per second. One possibility is the use of a thyratron inverter, but in some applications a controlled motor-generator seems better suited, because tube capacity requirements are less, the output voltage can be made independent of input voltage variations, and tube failure causes only a loss of frequency control rather than a complete loss of voltage.

The generator is necessarily an a-c machine. The motor is a d-c machine, shunt (or compound) wound, and requires a d-c source of supply. The control of the speed of the shaft is the means whereby the frequency of the generator output is controlled. The speed is controlled by the action of a thyratron on the motor field. For the purpose of control the speed function is obtained by comparing the frequency of the generated a-c wave with the voltage wave of the standard signal. Hence in this application the a-c generator itself serves as the equivalent of a tachometer.

The thyratron is supplied with alternating voltage on its plate from the a-c generator (self-excited) and with alternating voltage on its grid from the controlling signal. In operation, if the machine slows down the plate voltage increases its lag with respect to the grid voltage, more current is passed which weakens the motor field until the speed is returned to normal.

If the cause of the slowing down remains, the field must continue in its somewhat weakened condition as a result of more thyratron current which in turn is maintained by a greater phase lag of the output voltage of the a-c generator with respect to the signal voltage. The control therefore sets motor field proportional to the relative angular *position* of the generator rotor.

When this system was still in the experimental stage and consisted of no more than that described above, it was found that full load variations could be taken care of readily, but that the d-c input voltage had to be maintained within rather close limits. This was not surprising because of the inherent characteristics of a shunt motor with field saturation of the usual amount. Changes in load require only small changes in field (even less if there is compounding), but changes in d-c input voltage require very considerable changes in field. A wide range of d-c input voltage, therefore, dictates that the maximum effect of the thyratron on the field be large. Since this large effect must reduce to zero as the advance in the rotor increases to only one-half of a cycle (180 electrical degrees), the phase control can be said to be "stiff."

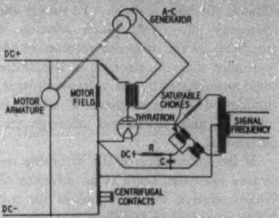

Figure 6. Simplified diagram of synchronized motor generator

This diagram shows, in addition to the basic control, how the voltage across the motor field is integrated by the *RC* circuit and how the result is used to modify the control

Stiffening the phase control increases the tendency to hunt; hence there is needed some means for counteracting this tendency so that the phase control can be made stiff enough to permit a satisfactory range of d-c input voltage.

The immediate effect of reducing the field on a shunt motor is to reduce the counter electromotive force, increase the armature current, and hence increase the torque. The simple control, as described so far, therefore makes the torque (angular *acceleration* of the rotor) proportional to the relative angular *position* of the rotor. Because it takes no account of the relative angular *velocity* of the rotor the oscillations are not definitely damped, hence the difficulty with hunting. It is not simple to take account of this relative *velocity* by measuring it directly, so instead it was decided to *integrate* the acceleration and to use the result to modify the control as will be explained. Since the *integral of acceleration* is *velocity*, this arrangement gives the desired damping effect and so allows a wide range of d-c input voltage.

A simplified circuit is given in figure 6. Normally the thyratron component of the field is determined by the phase relation between the alternator electromotive force and the signal electromotive force.

This is the basic control. To supply the damping effect this phase relation is modified by the saturable reactors in accordance with the strength of the motor field as indicated by the voltage across it, integrated by capacitance C which is charged and discharged through resistance R.

If the inductance of the saturable reactors decreases there is an advance of phase relative to the signal of the a-c voltage applied to the thyratron (grid to cathode). The saturable reactors are each standard permalloy core transformers and two are used with one winding reversed in order to eliminate transformer action.[11] Each core is saturated equally by a direct current determined by the d-c resistance of the reactors and the voltage across capacitor C. Theory and practice indicate that this current does not interfere with the functioning of C provided C is sufficiently large (50 microfarads).

In operation, when there is a shift in load or d-c input voltage, there is in general an angular shift of the rotor (relative to the signal). The rotor, as observed by a stroboscope, generally overshoots the final angle a few degrees, but after a few damped swings settles definitely into position. No cycles are lost so synchronism is maintained.

If at any time the d-c input voltage is momentarily interrupted or if the machine is overloaded, then synchronism will be lost. However, if conditions within the normal working range are restored before the motor starter drops out, the machine will resynchronize. This is accomplished by the assistance of the centrifugal contacts as shown in figure 6. These contacts play no part as long as the machine is running within a per cent or so of normal speed, one being closed and one being open at this speed. Their action in synchronizing is to cut in and out field resistance thus bringing the machine to within a per cent or so of synchronous speed from either the high or low side. From either of these speeds the machine can readily pull itself into synchronism.

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Lightning Protection of 22-Kv Substations

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ASSOCIATE AIEE

Synopsis: This paper outlines laboratory and field tests with a surge generator and cathode-ray oscillograph which led to the co-ordination of insulation and lightning-arrester performance in substations on the 22-kv subtransmission system of the Duquesne Light Company.

A basic level of 150 kv for station insulation was selected in 1932, and methods of obtaining this level are presented together with the operating record for five years. The index representing the station equipment faults in per cent of total system faults during thunderstorms has decreased 86 per cent following system-wide improvement programs modernizing insulation and arresters.

Recent papers on basic impulse insulation levels,[1] lightning arrester application,[2] and the general co-ordination problem[3,4] summarize a vast amount of work which has been done to place station lightning protection on a sound theoretical basis for the use of system designers and protection engineers.

Since 1932 the Duquesne Light Company has been actively engaged in an insulation co-ordination and lightning-arrester modernization program guided by this theory and based upon data obtained through laboratory and field tests with a 750-kv surge generator and cathode-ray oscillograph.

Because of the problems encountered in the protection of 22-kv station equipment, and the choice, in 1932, of 150 kv as a basic impulse insulation level, it is felt that a description of practice and results obtained would be timely.

The Problem

In the years 1930 and 1931 severe lightning seasons caused the number of substation equipment failures to reach such proportions that some measures to reduce them became imperative. It was already quite clear that any effective measures must be based upon accurate and voluminous data on the characteristics of equipment and protective devices.

The wide variety of insulation in use, together with little knowledge of the actual performance of various types of arrester, made it impossible to hope for an effective improvement program based upon data furnished by electrical equipment manufacturers. It was believed that only a comprehensive and fundamental attack would suffice, and in January 1932, a surge generator and cathode-ray oscillograph were installed in the high-voltage laboratory of the company.

Laboratory Tests

Kilovolt-time breakdown curves have been obtained for almost every insulation element of the system which is susceptible of such a measure. If a curve was not feasible, as in the case of transformers, minimum breakdown values have been ascertained which serve as approximate guides when considered with other data.

The characteristics of lightning arresters have been extensively studied in relation to the breakdown curves of the equipment to be protected.

Figure 1 illustrates typical laboratory tests to obtain these data. The arrester performance shown by the volt-time and volt-ampere oscillograms is representative of approximately 60 per cent of the arrester protection in service prior to 1932,

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[1] For all numbered references, see list at end of paper.

and reference to this performance will be made in the discussion which follows.

An important addition to these somewhat standardized tests was the devising and testing of methods to increase the strength of system elements which could not be replaced except at prohibitive cost.

Field Tests

The portability of the surge generator and cathode-ray oscillograph made it possible to test substations as a whole to determine points of maximum voltage caused by reflections, and to detect unsuspected weak insulation. Lightning arresters in service were tested—sometimes with startling results. In still other tests, bus and disconnect-switch insulators were flashed over, with no regard to the shape of the applied surge, and the results compared with those predicted by laboratory impulse breakdown curves. The upper curve of figure 2*a* shows a kilovolt time-lag curve of a two-unit pedestal insulator obtained in the laboratory. The lower curve is a replot of oscillogram 55-7, with zero time corresponding to the crest value of the 60-cycle breakdown, and is the record of the flashover of the same type of pedestal insulator installed on the bus of a transmission substation.

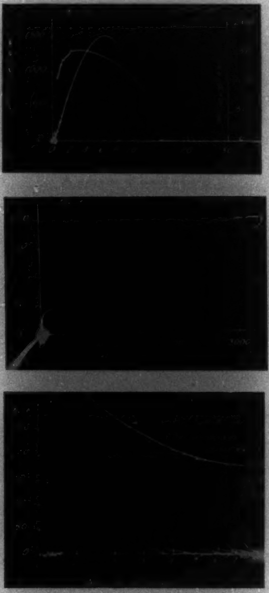

Figure 1. Typical laboratory tests
Oscillogram C45-3-4—25-kv arrester
Oscillogram C19-7—34.5-kv disconnect switch

Figure 2*b* shows a volt-time oscillogram of an arrester taken immediately after de-energizing it for field tests.

Application of Data to System Improvement

When a cross section of the system insulation had been obtained, it became apparent that much of the indoor insulation was not protected by the arresters installed, at a moderate discharge current. Circuit breakers, disconnect switches, bus insulators, transformer bushings, and even some current transformer bushings and windings were found to have minimum impulse strength in the range from 100 kv to 150 kv.

A survey of all 22-kv indoor stations revealed that a basic level of 150 kv could be established by rebuilding certain types of circuit-breaker bushings, replacing bus and disconnect-switch insulators of low strength, and eliminating some obsolete equipment entirely.

Fortunately, the most widely used indoor disconnecting switch had a minimum breakdown of 150 kv, and this value was chosen as the minimum for all acceptable insulation. In some substations, special measures were required to bring the strength above the minimum set.

Figure 3 illustrates the relation between the arrester and typical kilovolt-time curves of insulation. Although curves are shown for a disconnect switch and three different circuit-breaker bushings, many other insulation elements would practically duplicate these curves.

Three types of circuit breaker were equipped with a bushing composed of an outer corrugated porcelain tube, an inner "Micarta" tube extending above and below the procelain, and a copper stud of much smaller diameter. The air space between the stud and tube was easily overstressed, and puncture of the Micarta tube followed by air flashover from the top of the porcelain to the circuit breaker operating mechanism was common. This defect was remedied by filling the air space with another close-fitting Micarta tube and increasing the length of the external portion of both tubes.

Other circuit-breaker bushings could neither be lengthened nor replaced, except at an unwarranted cost. In such cases the connection to the bushing was heavily taped, great care being exercised to exclude air pockets. Tests indicated that this expedient increased the impulse strength well above the basic level.

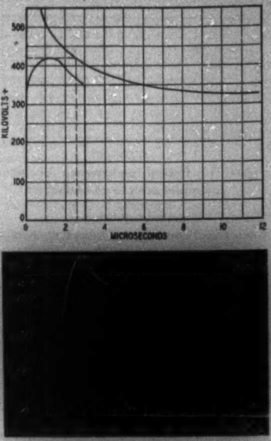

Figure 2a. Comparison of field test flashover with laboratory curve

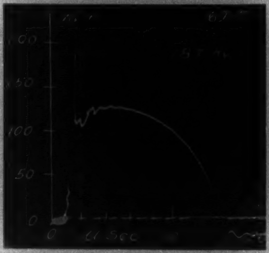

Figure 2b. Field test on 25-kv arrester, first impulse

A great many transformer bushings possessed sufficient strength external to the transformer tank, but were very weak internally because of low air or oil surface clearances between the lead and the tank. Other bushings were adequate internally but were very weak externally. To solve this problem and reduce bushing stocks, a one-piece porcelain bushing was developed having adequate impulse strength externally and which extended below the oil level in all transformers. By means of flange adapters one bushing was made applicable to transformers of different manufacture.

In one station, subject to numerous bus failures, the entire bus of bare copper was removed and replaced with 22-kv varnished-cambric insulated wire. To avoid taped joints within the small com-

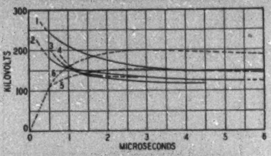

Figure 3. Typical performance of indoor equipment prior to co-ordination

Curve 1—Disconnect switch
Curves 2, 3, 4—Oil-circuit-breaker bushings
Curve 5—Arrester, 1,500 amperes in six microseconds
Curve 6—Arrester, 3,250 amperes in six microseconds

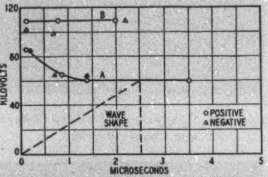

Figure 4. Breakdown of 25-kv line-type arresters

partment which would lower the impulse strength, the bus was run to each disconnect switch and then back to the compartment. The switch was at the basic level of 150 kv, and the bus much higher.

Figure 3 shows clearly that the basic level of 150 kv could not reliably be protected by the arrester whose performance is indicated. Further, insulation levels as high as 200 kv were endangered when the arrester discharge current exceeded 3,000 amperes. A general replacement program was necessary to establish system-wide arrester performance under the basic level.

Several designs of station and line-type arrester were critically tested, and from the results it was concluded that the protection provided by station type arresters was not sufficiently greater than that provided by line-type units to justify the greatly increased cost.

Two line-type units were decided upon and the replacement program begun. Figure 4 shows positive and negative break-down data on the two arresters *A* and *B*. Figure 5 shows a high current test on one of five impedance blocks used in the 25-kv arrester *A*.

Figure 6 illustrates the co-ordination of equipment insulation and the arrester performance for both light and heavy discharges. It is apparent that the maximum impedance drop of arrester *A* depends primarily on the rate of increase of discharge current, while that of arrester *B* depends upon the maximum magnitude of the discharge current. Laboratory tests and field experience indicate that the solid line curves are near the maximum voltages which the insulation of the arresters themselves will support, for the arrester is destroyed before curve 1 is reached.

In an attempt to obtain some idea of the winding strength of 22-kv transformers, a 667-kva single-phase 22-kv to 2.3-kv transformer was tested to breakdown with the results given in figure 7. Oscillogram 98-1 shows three full waves applied without a winding breakdown. Oscillogram 98-4 shows two winding breakdowns. The first winding breakdown occurred at an applied voltage of 210 kv on another test intermediate between those shown. The rarity of transformer failures makes it probable that the actual strength of most of the transformers is near 200 kv.

Since the outset of the co-ordination program, 408 circuit-breaker bushings, 388 transformer bushings, and 342 new arrester units have been installed. Numerous changes of other equipment and insulation have also been necessary to achieve impulse strength equal to or above the basic level.

System Performance After Improvements

Two methods were adopted to maintain as close a check as possible on the efficacy of the co-ordination program. The first of these was a systematic investigation of each individual station fault caused by lightning, and accurate recording of all line, cable, and station equipment faults during thunderstorms. The second method involved the selection of 13 stations notable for frequent lightning failures prior to the improvements, and the installation of lightning-arrester operation counters in these stations. This group is well scattered about the system and is thought to be a fairly representative sample. The operation recorder developed was energized by the current through the arrester, and it counted each distinct discharge.

Figure 8 gives index numbers for the total 22-kv line, cable, and station equipment faults; the station equipment faults alone; the station faults in per cent of the total faults; and the number of arrester operations recorded in the sample group of stations. To facilitate comparison of the first two data, the average of the 1930 and 1931 figures has been taken as 100. For the arrester operations, the average number of operations per year has been taken as 100. The correspondence between arrester operations and total faults merely reflects the fact that the number of surges which are virtually certain to exceed the basic station kilovolt level is proportional to the number of surges exceeding the line insulation at the point of origin. This conclusion is believed justified by the further facts that the average length of overhead line between stations is approximately two miles, and the line insulation ranges from a minimum of about 300 kv to over 1,000 kv. In this connection it is also well to remember that direct strokes to the line causing flashover to ground within a few spans of the station will result in voltages limited by the stroke current and the structure ground resistance, rather than by the structure insulation alone.

Although the impulse testing of equipment and arresters was begun in 1932, a number of arrester replacements and insulation improvements put into effect in 1931 were confirmed by the tests, and their influence on system performance is

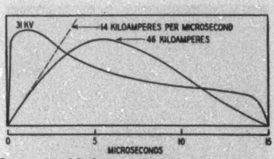

Figure 5. High-current test on one of five blocks used in arrester A

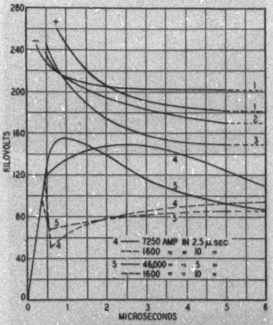

Figure 6. Typical co-ordination of indoor equipment after improvements

Curve 1—Transformer bushing
Curve 2—Oil-circuit-breaker bushing
Curve 3—Disconnect switch
Curve 4—Arrester *B*
Curve 5—Arrester *A*

shown in figure 8. Despite the fact that a slight increase in the actual number of station faults occurred in 1931, the weighed index giving these faults in per cent of total shows a decided drop.

In 1934 and 1935 the work was largely directed toward 66-kv transmission substation problems and 4-kv equipment protection, and this divergence is strikingly revealed by the index numbers. Again, further 22-kv improvements were put into effect late in 1935, and the 1936 record was still better.

Conclusions

The widely varying conditions encountered on subtransmission systems make it difficult to generalize the results of this program in any valid manner. The co-ordination of insulation and arresters has been in accordance with the present recommended basic level of 150 kv. Considerations of cost and space limitations dictated the choice of this level at the beginning of the co-ordination program in 1932, and the result of five years' operating experience are given in this paper. For systems having insulation and arrester performance whose relation to the basic level is comparable to that of figures 1 and 3, and whose exposure to surges is of the same degree as that cited, one may anticipate that similar improvement in performance will accompany co-ordination based upon detailed knowledge of insulation and arrester characteristics.

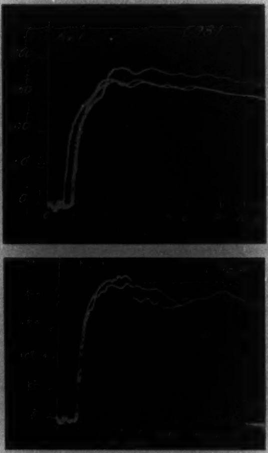

Figure 7. Winding breakdown tests on a 667-kva 22-kv to 2.3-kv transformer

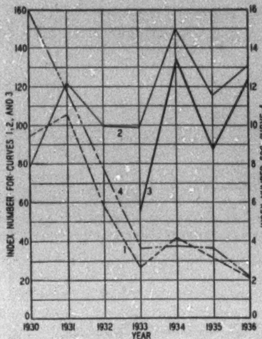

Figure 8. Lightning faults on 22-kv system

Curve 1—Station equipment faults
Curve 2—Total 22-kv faults
Curve 3—Station arrester operations
Curve 4—Station faults as per cent of total faults

Acknowledgments

The author wishes to express his appreciation of the liberal company policy which has made possible a fundamental attack on the lightning problem.

Particular acknowledgment is made of the work of Mr. D. B. Perrin who has been closely associated with the author throughout this work.

Mr. H. H. Marsh, Jr. has furthered the program by his interest in and appreciation of many problems of detail, and has been especially helpful in the preparation of the paper.

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Discussion

E. J. Allen (General Electric Company, Pittsfield, Mass.): This paper illustrates the practical benefits resulting from a comprehensive program of insulation co-ordination and protection. One of the most significant of these benefits is the great reduction in station faults caused by lightning in per cent of the total faults, after the co-ordination program was well under way in 1933. Figure 8 of the paper shows that although the total faults caused by lightning on the 22-kv system were higher in 1934 and 1936 than any other year since 1929, the percentage of station faults due to lightning has been steadily decreasing.

The choice of 150 kv as a basic impulse insulation level in 1932 for station equipment on this 22-kv system is the same as that more recently published by the EEI-NEMA Joint Committee.[1] As this paper concerns line-type lightning arresters, it is of timely interest to note the line-type lightning-arrester performance data[2] presented by the lightning arrester subcommittee. This report shows that the average impulse protective characteristics afforded by new designs of 25-kv line-type lightning arresters is less than 100 kv crest—representing over 50 per cent margin of protection to the 150-kv basic impulse insulation level.

Figure 1 of this discussion shows volt-time and ampere-time oscillograms taken on modern 25-kv Thyrite line-type lightning arresters at discharge currents of 1,790 and 5,600 crest amperes.

Figure 2 shows the general appearance of this type of arrester; employing unit type construction and containing Thyrite-shunted gaps and the main Thyrite valve element disks in the same housing. The impulse tests shown in figure 1 were obtained from a 25-kv arrester of the type shown second from the left-hand side of figure 2.

Calculations indicate that the maximum impulse wave of a 1,000-kv crest passed by the line insulation described by the author results in a lightning-arrester discharge current approximately 3,800 amperes at the line terminals, assuming 500-ohm line surge impedance. This seems to indicate that lightning-arrester discharge currents in service, except for possibly chopped waves resulting from direct lightning strokes contacting the line at a point close to the station, are well within the range of discharge currents up to 5,000 amperes, as set forth in the report of the lightning arrester subcommittee. Moreover, Messrs. McEachron and McMorris[3] have summarized the data in regard to lightning-discharge currents through lightning arresters located on primary distribution circuits rated 2,400 to 24,000 volts. These data show that 90 per cent of the discharge currents measured through lightning arresters are 5,000 amperes or less. These data apparently show that approximately 25 years may be expected to elapse for each discharge of at least 5,000 amperes through any one arrester.

Previous experience, together with the results shown in this paper, demonstrates the benefits resulting from a fundamental attack on the lightning problem. It is uneconomical and impractical to build into the connected apparatus sufficient insulation strength to withstand unmodified lightning voltage in service. Hence, lightning arresters affording ample margin of protection to apparatus insulation are the established economical basis of system co-ordination and protection.

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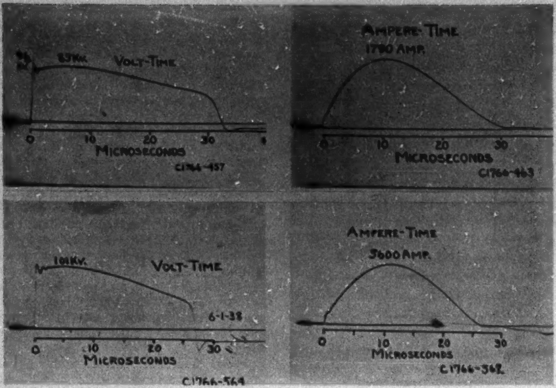

Figure 1. Impulse protective characteristics, Thyrite line-type lightning arrester, maximum rated 25 kv

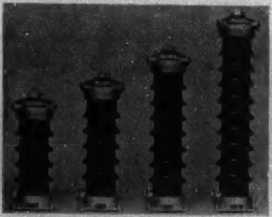

Figure 2. Thyrite line-type lightning arresters, maximum rated 20, 25, 30, and 37 kv, respectively

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P. L. Bellaschi (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): Mr. Whitehead's paper presents a fine example of the application of insulation co-ordination, where it was thoroughly planned and well carried out to provide improved protection to 22-kv substations. At the time this work was initiated, impulse data on installed apparatus were rather meager. The wisdom, therefore, in using an impulse generator from the start is fully apparent from the results that have been achieved.

A point of particular interest is that Mr. Whitehead, starting out in 1932 in his co-ordination work, arrived after certain tests and analysis, to the conclusion that 150 kv for the 22-kv substations was a sound impulse insulation level to aim for. This is also the figure now prescribed by the EEI-NEMA Joint Committee on System Insulation Co-ordination. The coincidence is not, however, accidental since the processes in arriving at the levels are basically the same.

Figure 6 of the paper which shows the apparatus voltage-time curves (1, 2, and 3) of the substation apparatus above the established level and the characteristics of the protective devices below the level (curves 4 and 5) tell a very significant story of the technical and economic problems that had to be overcome to establish the 150-kv level. They also illustrate the principles in general on which the present insulation levels have been arrived and are based.

As stated in the paper, the insulation of the 22-kv lines ranges from 300 kv to over 1,000 kv. Presumably these low-voltage lines are not equipped with overhead ground wire. It is now quite well established from the field investigations here and abroad that the induced voltages seldom exceed 300 kv and from this consideration it then appears that the larger percentage of the impulse voltages entering the substation originate from direct strokes to the line.

Since there are two miles of line between two stations on the average, half of the strokes will travel one-half mile before entering a substation and therefore their severity would be attenuated somewhat. That the author has given consideration to direct strokes is clearly apparent from the arrester characteristics for high current discharges taken into consideration in figure 6. It would be of interest to have an expression of view from the author as to what extent direct strokes in the station or within one or two spans of the station represent the per cent of substation faults shown in figure 8 and what further measures are being considered to protect the substations against direct strokes.

J. T. Lusignan, Jr. (Ohio Brass Company, Mansfield): The author has described a fine example of a rather thorough insulation co-ordination program. The fact that it was possible to bring the surge generator and oscillograph to the substations in order to check the transient voltages there and the actual insulation provided to handle them is surely a fortunate circumstance. It is interesting to find, at least from the one example he shows (figure 2*a*), how well his measured insulator voltage values agree with the laboratory curve.

I find no mention of polarity in so far as flashover values are concerned. In insulation co-ordination the question often arises as to whether co-ordination should be on the basis of positive values only or both positive and negative values. I believe that field records show most severe direct strokes to be of negative polarity although many of the lower voltage lightning strokes are positive. If the stroke occurs to the conductor a surge of the same polarity obviously is imposed upon the insulation. A stroke to the ground wire or other nearby grounded object, of course, induces a surge of the opposite polarity upon the conductors. I am wondering what consideration Mr. Whitehead has given to this. For 22-kv insulators the positive and negative flashover values are often practically equal whether base or cap mounted. However, I am wondering whether there are other insulation breakdown points in a station which will exhibit sufficient polarity characteristics to warrant the latter being considered.

E. R. Whitehead: Mr. Allen presents performance data on the newly developed Thyrite distribution-type arrester. This line, of selected unit construction, appears to offer new possibilities for economy and flexibility in equipment protection.

The characteristics of the modern distribution-type arrester viewed in relation to the basic impulse levels and the McEachron-McMorris current distribution curves should provide a sound basis for their application.

Mr. Bellaschi raises a point of considerable interest. Prior to the co-ordination program it was not uncommon for weak station equipment to flash over during thunderstorms without an accompanying line fault. A combination of poor co-ordination and induced surges seems to explain this type of fault.

During the period covered by the paper, there has been a considerable increase in the use of wood crossarm braces and guy insulators. As a consequence most substation faults now correlate with direct-stroke line damage such as splintered braces, arms, or guys located within a very few spans of the station. The large number of stations involved in relation to the very small number of substation faults precludes any general use of direct-stroke protection on the 22-kv system. Consideration is now being given to auxiliary current-limiting schemes such as expulsion tubes or gaps one span from the station at the most exposed locations.

Doctor Lusignan asks what consideration has been given to polarity in the co-ordination studies. At the outset of the program it was essential to cover as many system elements as possible in a short time in order that early application of the data might be made. Positive-polarity time-lag curves were found to be lower than negative-polarity curves for most of the equipment, and therefore the earlier work was largely done with the former. In the later stages of the program both polarities were used.

Tests on Oil-Impregnated Paper—III

Fluid Flow

By HUBERT H. RACE
MEMBER AIEE

I. Introduction

IN TWO previous papers[1,2] we have described some of the results of life-test studies of miniature cable specimens started in 1934. One of the general observations resulting from a large number of tests is that all specimens made with high-density paper and standard-viscosity oil or specimens made with standard-density paper and high-viscosity oil have had short life at 30 kv whereas many specimens made with standard viscosity impregnant but otherwise identical have had very long life under the same conditions.

In attempting to explain these results we made the hypothesis that with either high-density paper or high-viscosity impregnant, the rate of flow of the impregnant through the paper during the cooling portion of the daily heat cycle was not great enough to prevent void formation. This would then result in ionization and early failure. We had all the data necessary to check this hypothesis except the flow characteristics of oil through paper. A search of the literature revealed little pertinent information although a considerable amount of work has been reported on the permeability of paper and other materials to air. A summary of this work and a bibliography of 31 references is given by Carson.[8] It is characteristic of all the work done on paper that the pressure head on the paper does not exceed a few centimeters of mercury and in Carson's work was limited to five to ten centimeters of water. This is due, of course, to trouble from deformation of unsupported sheets of paper at the higher pressures and also to certain convenient methods utilizing low heads. Goldberg performed some experiments of this type but neglected to take account of the expansion of the air as it passes through the paper. He noticed deviations at higher pressures and attributed these to turbulence, which is probably incorrect since we have used much higher rates of flow without turbulence. It also appears that his apparatus is subject to leakage, which can cause large errors.

A number of workers have investigated the flow of liquids through porous media.[5–16] Some of these have found anomalous results, which our preliminary work indicates was caused by the fact that the liquids used were not gas free. When the liquid contains gas, part of it comes out of solution as the pressure decreases, causing gas pockets in the paper, which decrease the rate of flow. This agrees with the work of Muskat[12] on mixtures of gas and oil flowing through sands although the latter work indicates other effects besides the formation of gas pockets. The only work we have found related to the flow of oil through paper is that of Whitehead and Greenfield,[17] relying on capillarity to produce the flow. However, we are not so much concerned with the flow in the plane of the paper as in the flow through it under hydraulic pressure.

Apparently few comparative studies have been made with different fluids on the same porous medium. Bull and Wronski[11] have made a comparative study with seven alcohols, carbon tetrachloride, and water in cellulose, glass, and carbon diaphragms with the interesting conclusion that the viscosity of the liquid was not the only property of the liquid important in determining the flow. No experiments at all have been reported, so far as we can determine, which give direct comparison between gas and liquid flow.

This paper gives the results of such a direct comparison together with studies of oil flow under varying conditions and the application of these data in determining limiting conditions causing void formation in cables.

II. Flow Equations

The flow of fluids through porous media can be expressed by Darcy's law (see reference 12),

$$\nu = (-K/\eta)\nabla p \qquad (1)$$

where ν is the vector rate of flow of the fluid, p the fluid pressure, η the viscosity of the fluid, and K a coefficient which has been called the "permeability" or flow constant of the porous medium. K may accordingly be defined as the volume of a homogeneous fluid of unit viscosity passing through a unit area in unit time under the influence of unit pressure gradient.

A. Flow of Oil Through Paper

In our experiments flat sheets of paper were used and the lines of flow through the test area A were everywhere normal to the paper. Consequently, the general flow equation 1 may be written in the one-dimensional form:

$$v_x = (-K/\eta)\, dp/dx \qquad (2)$$

In this case $v_x = Q/A$ where Q is the rate of flow of oil in cubic centimeters per second through the test area (A in square centimeters). Therefore,

$$Q = -(KA/\eta)(dp/dx) \qquad (3)$$

For steady state, with constant pressure difference and constant time rate of flow

$$dp/dx = -(p_1 - p_2)/NL \qquad (4)$$

where p_1 and p_2 are the inlet and outlet pressures in dynes per square centimeter, N is the number of layers of paper, and L is the thickness of one layer of paper in centimeters. Then

$$K = (Q/A)(\eta NL)/(p_1 - p_2) \qquad (5)$$

(η is in poises, that is, dynes times seconds divided by square centimeters, and K has the dimensions of an area).

B. Flow of Gases Through Paper

In dealing with the flow of gases through porous media, we may still use Darcy's law, and since the geometrical

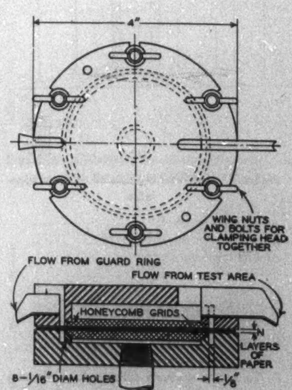

Figure 1. Oil-flow head

Paper number 38-61, recommended by the AIEE committee on power transmission and distribution and presented at the AIEE summer convention, Washington, D. C., June 20–24, 1938. Manuscript submitted March 12, 1938; made available for preprinting May 13, 1938.

Hubert H. Race is a research engineer for the General Electric Company, Schenectady, N. Y.

[1] For all numbered references, see list at end of paper.

features of our gas- and oil-flow apparatus are the same, equation 3 will also apply to gas flow. Due to the expansion of the gas as it passes to a region of lower pressure, it is convenient to express this relation in terms of the mass of gas flowing per second which we will call M. Then since

$$M = \rho Q \qquad (6)$$

where ρ is the density, we have from equation 3

$$M = -(KA\rho/\eta)(dp/dx) \qquad (7)$$

For gas flow dp/dx varies with x so that (7) will have to be integrated. In order to do this we assume Boyle's law to hold so that ρ/p is a constant, the temperature being assumed constant. The ratio may be taken at any condition (a), say at one atmosphere as a matter of convenience. Thus

$$\int_0^{NL} M dx = -\frac{KA}{\eta}\left(\frac{\rho_a}{p_a}\right)\int_{p_1}^{p_2} p dp$$

Integrating and rearranging we have

$$M = (KA/2\eta NL)(\rho_a/p_a)(p_1^2 - p_2^2) \qquad (8)$$

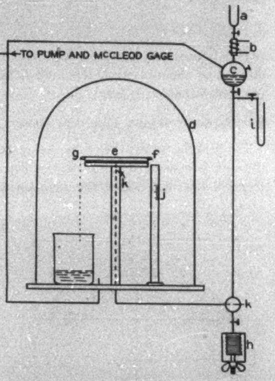

Figure 2. Essential features of oil-flow apparatus

a—Filling reservoir
b—Heater for degasifying oil
c—Oil level during test
d—Bell jar
e—Head
f—Overflow for test area
g—Overflow for guard ring
h—Sylphon bellows
i—Mercury manometer
j—Calibrated graduate cylinder for collecting oil from test area of paper
k—Thermocouple leads

The determination of M can be made by direct measurement or by using a calibrated capillary. We have used the latter by allowing the gas to flow directly through the capillary after it leaves the measuring section of the paper. Consequently M is the same for the paper and the capillary. M may be determined for the capillary by the method described in Barr's "Monograph of Viscosity."[18] Although a radial component flow exists in the case of a gas flowing through a capillary, it has been found that Poiseuille's law of flow, which holds when the flow is strictly laminar, can be used if the rate of change of density with length is not too great.[18] This condition is realized in a long capillary of a fairly large diameter. Poiseuille's law is:

$$Q = -(\pi r^4/8\eta)(dp/dl) \qquad (9)$$

where r is the radius of the capillary, dl is an element of length, η the viscosity of the gas. As in equation 7 we replace Q by M/ρ

$$M = -(\pi r^4/8\eta)(\rho/p)p(dp/dl) \qquad (10)$$

and assume that the gas expands according to Boyle's law in passing through the capillary. Integrating (10)

$$\int_0^l M dl = -\frac{\pi r^4}{8\eta}\left(\frac{\rho_a}{p_a}\right)\int_{p_2}^{p_a} p dp$$

$$M = (\pi r^4/16\eta l)(\rho_a/p_a)(p_2^2 - p_a^2) \qquad (11)$$

Here η has been assumed to be independent of pressure as it is within wide limits (see reference 18 and also volume V of International Critical Tables, page 2). p_2 is the same as in (8) and p_a is the outlet pressure of the capillary (atmospheric in our apparatus). (ρ/p) is the same as in (8).

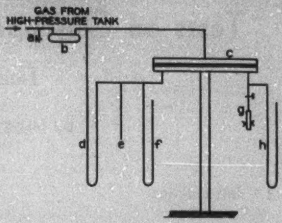

Figure 3. Essential features of gas-flow apparatus

a—Stop cock for regulating pressure
b—Phosphorus-pentoxide drying tube
c—Head
d—Differential manometer for measuring head across test area of paper
e—Capillary
f—Manometer for measuring head across capillary
g—Stop and pinch cock arrangement for regulating head across guard area of paper
h—Manometer for guard area of paper

Equation 11 holds fairly well for rough measurements, but for accurate work it must be corrected for kinetic energy of the gas, end effects, slip and so on (see reference 18). The kinetic-energy correction is by far the largest correction; therefore, neglecting the others, equation 11 becomes

$$M = (\pi r^4/16\eta l)(\rho_a/p_a)(p_2^2 - p_a^2) - (1.1\, M^2/8\pi\eta l) \qquad (12)$$

K can now be calculated by eliminating M from equations 8 and 12.

Before this can be carried out numerically it is necessary to calibrate the capillary, that is, determine the factor $(\pi r^4/8l)$; l can be measured directly (33.8 centimeters in our case), but r, the radius, is not determined so easily. The usual practice is to partially fill the capillary with a thread of mercury, measure its length, weigh it, and from the density, weight, and length, determine its volume and from that the radius r. One may also test the uniformity of the bore by observing the length of the thread at various positions along the capillary.

Table I. Properties of Papers M and N

Property	M	N	Ratio M/N
Type	Kraft	Kraft	
Apparent density	0.75	1.10	0.68
Tensile strength (pounds per one-inch strip)			
Lengthwise	90	81	1.11
Crosswise	36	32	1.12
Per cent elongation			
Lengthwise	2.9	2.2	1.32
Crosswise	7.8	9.9	0.79
Tearing strength (grams, Elmendorf)			
Lengthwise	189	229	0.82
Crosswise	235	293	0.80
Per cent ash	0.56	0.52	
Alkalinity (milligrams of sulphuric acid per gram)	0.118	0.090	
Air resistance (seconds per mil, Gurley)	21	318	0.066 (Ratio N/M = 15.1)
Permeability K ($\times 10^{-12}$)			
Gas (N_2)	12.5	0.92	13.6
Liquid (oil A)	12.5	0.59	21.2

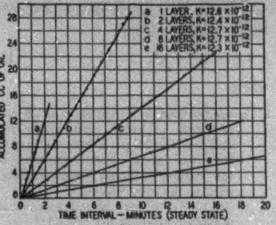

Figure 4. Oil-flow data on standard-density paper (M) for various numbers of layers

Our best determination of r^4 by this method was

$$r^4 = 70.8 \times 10^{-8} \text{ centimeters}^4$$

In using a capillary, care must be taken not to use such high rates of flow that turbulence occurs. The condition that the flow is not turbulent is that $R = \rho V d/\eta \leq 2{,}000$ (Reynolds' criterion. See International Critical Tables, volume V, page 2; also reference 19). Here ρ is the density, V linear velocity, η viscosity, d bore of capillary. For our capillary and our largest rate of flow R is about 35 so that we are working well below turbulent flow.

The question of turbulence may also be raised in connection with the paper. Here the bore diameters are all much smaller than that of the capillary and the linear velocity is also less than for the capillary. This means that R will be less than for the capillary and that turbulence cannot occur in the paper if it does not in the capillary.

In the calculations below, we use $(\pi r^4/16l) = 41.2 \times 10^{-10}$, $l = 33.8$ centimeters, and wherever $(p_a^2 - p_b^2)/p$ occurs multiply by $980.3 \times 13.53 = 1.328 \times 10^4$ (13.53 is the density of mercury to four places from 24.5 to 28.5 degrees centigrade) so that it is not necessary to convert pressures from centimeters of mercury to dynes per square centimeter. $A = 52.4$ centimeters. Then equation 12 becomes

$$M = 5.46 \times 10^{-4}(\rho_a/p_a)(p_1^2 - p_a^2) - 1.297 \times 10^{-3} M^2/\eta \quad (13)$$

and (8) is

$$K = 2.88 \times 10^{-6}\, NLM\eta\rho_a/p_a(p_1^2 - p_2^2) \quad (14)$$

III. Apparatus for Measuring Gas and Liquid Flow

The same geometrical arrangement has been used in both the gas-flow and oil-flow experiments. In both cases we wished to obtain data with pressure differences up to one atmosphere so that it was necessary to use supporting grids to prevent deformation of the paper under pressure. Although this introduces some uncertainty in the calculated area of flow, we assume that this does not affect the relative values of the gas and oil flow permeabilities. In all our calculations we have used the total area covered by the test section of the supporting grid.

In both the oil- and gas-flow experiments the paper was tightly clamped between the two halves of a device which we have called a head. Both heads are essentially the same, and the design of the oil-flow head is shown in figure 1. The test area is that area occupied by the honeycomb grids and it is the rate of flow through this area that comprises Q. In order to assure flow normal to the test area, a guard ring was provided in each case which had the same pressure drop across it as the test area. If the leakage through the edges is not too great this is an entirely satisfactory arrangement. Excessive leakage may be controlled by coating the edges of the paper with a plastic material which does not penetrate the test section. We used an air drying plastic Glyptal quite satisfactorily.

Figure 2 is a schematic diagram of the oil-flow apparatus. The apparatus is assembled as shown and thoroughly evacuated for several hours at a pressure of about one micron or less. The oil is introduced into the filling reservoir *a*, is allowed to trickle slowly through the heater *b* and into the bulb. This degasifies the oil. The oil was allowed to flow in at such a rate that the pressure of the system never exceeded about 25 microns. During the filling period the sylphon bellows was fully extended by means of the wing nut. The system was filled with oil up to a level *c*. The stopcock below *c* was then closed and the sylphon *h* released by loosening the wing nut. This puts approximately one atmosphere hydraulic pressure on the high-pressure side of the paper. The lower pressure is about 0.5 centimeter of oil plus the gas pressure, which was kept at about 0.1 micron throughout the tests. The oil from the test area flowed from *f* into the graduate *j*. The rate of flow was measured with an accurate stop watch and carefully calibrated graduate cylinder. In practice the manometer *i* was below the head so that the column of oil between the upper mercury level and the level of the paper had to be taken into account in calculating the pressure p_2.

The gas-flow head differs from figure 1 only in that pipe connections to the rest of the setup were used instead of overflow spouts. The arrangement of the gas-flow apparatus is shown schematically in figure 3; *h* is the manometer measuring the pressure above atmospheric of the guard area on the low-pressure side. Since the pressure on the top side of the paper is the same for both the guard area and the test area, the pressure drop across the guard area may be made equal to that across the test area by making the manometer reading Δh_1, of the guard manometer equal the reading Δh_2 of manometer *f*. This is done by bleeding off gas at *g*, which can be done only if there is not too much leakage at the edges of the paper.

If Δh_3 is the reading on manometer *d*, the pressures are as follows: $p_2 = p_a + \Delta h_2$, $p_1 = p_a + \Delta h_2 + \Delta h_3$. K can then be calculated from equations 12 and 8.

The gas for the gas-flow experiments was supplied by a tank of nitrogen at about 1,500 pounds per square inch pressure with a suitable reducing valve.

IV. Flow Data and Values of K

A. Oil Flow

The physical properties of the two grades of cable paper used are given in table I. The data and calculations for the oil-flow experiments are shown in table II. The slopes of the curves shown in figures 4, 5, and 6 are equal to Q for each of the indicated cases. Two or more runs on the same thickness of paper will not necessarily give the same rate of flow Q due to differences in viscosity and density of oil, which vary with temperature, and also to the variation of atmospheric pressure from run to run. This is illustrated in curves (*a*) and (*b*) of figure

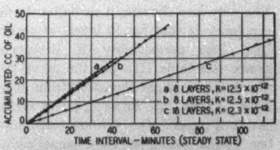

Figure 5. Oil-flow data on standard-density paper (M) for various numbers of layers

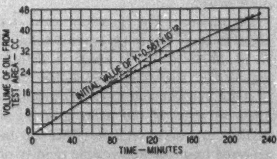

Figure 6. Oil-flow data, one sheet high-density cable paper (N)

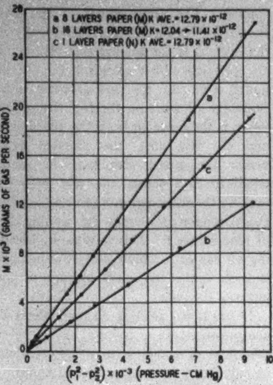

Figure 7. Flow of N_2 through cable paper

5, which give the same value for K although the slopes of the curves are somewhat different. For the same reason the slope of the curve for eight layers will probably not be exactly twice that for four layers [(*c*) and (*d*) of figure 4].

The value of K in table II for paper (N) was calculated for the first seven points of the curve (figure 6), which are sensibly a straight line ($K = 0.587 \times 10^{-12}$). From then on the rate of flow decreased, indicating the possibility of some slow swelling or rearrangement of the fibers in this highly calendered paper.

B. GAS FLOW

The data for these tests are in table III and figure 7 shows plots of the mass rate of flow M versus $(p_1^2 - p_2^2)$, which should be straight lines if K is a constant. The only one showing a deviation from constancy is that for 16 layers of paper (M). The permeability K decreases slowly (see table III), but the curve (*b*) shows only a slight deviation from linearity. Curve (*a*), figure 7, is for eight sheets of paper (M). Oil-flow test number 6 was made on these same eight sheets after taking the gas-flow data. Similarly, curve (*c*) is for the same sheet of paper (N) that was used later in the oil-flow test.

Discussion. The gas-flow data for eight sheets of paper (M) give a value of $K = 12.79 \times 10^{-12}$, which is slightly greater than the value from oil-flow data. It is not unreasonable to expect this inasmuch as some of the pores may be so small as to prevent oil flow. This may be expected from the appreciable rigidity of very thin layers of liquids. We are inclined to believe that the lower value of K obtained for 16 layers and the regular variations are not real but may be due to leakage through the edges. This is possible for a thick test specimen even when the guard ring has the same pressure drop as the test area. Moreover, examination of the formulas indicates that the calculated value of K will be reduced by such leaks.

In the high-density paper the resistance to gas flow was roughly 14 times greater than in the low-density paper whereas the resistance to oil flow was initially 21 times greater than in the low-density paper. (See table I.)

While the average density of the high-density paper (N) is only about 40 per cent greater than for the standard density paper (M), the process of manufacture of the former gives it surface layers of very closely packed fibers so that the major resistance to fluid flow is in these surface layers, thus explaining the relatively large effect on flow characteristics.

It should be noted that the above relations and results are for steady-state conditions only and are not expected to hold either when the paper is only partially saturated or when the liquid contains gases in solution or materials that will be adsorbed or filtered out by the paper, thus decreasing the effective capillary area.

V. Application of Flow Data to Conditions for Void Formation in Gas-Free Cables

A. ASSUMPTIONS

In studying the application of these data to the problem of void formation in cables we shall assume the following conditions:

1. The partial pressure of gas in the paper and impregnant is negligible, that is, the cable is virtually gas free.
2. The core dose not act as an oil reservoir so that during cooling all the impregnant required to keep the paper full must flow in from the outside.
3. The flow is radial, no longitudinal flow in the paper being considered.
4. Contraction of the paper fibers themselves is negligible.
5. A sufficient reservoir of impregnant is maintained on the outside circumference of the paper.

B. HYDRAULIC PRESSURE NECESSARY TO KEEP PAPER FULL

Let figure 8 represent the cross section of a cable in which we wish to determine the hydraulic pressure necessary to keep the interstices of the paper full of impregnant during cooling by uniform radial flow from the outside.

Let q = the total flow per centimeter length into the shaded cylinder to supply the interstices. Then

$$q = \pi\sigma sC(r^2 - r_1^2) \qquad (15)$$

where

σ = the coefficient of expansion of the impregnant in cubic centimeters per cubic centimeter per degree centigrade.
s = the space factor, that is, the ratio of the volume occupied by the impregnant to the total volume.
C = the rate of change of temperature in degrees centigrade per second.
r = radius in centimeters.

Then if V_r = the radial velocity of flow we have

$$V_r = -q/2\pi r = -\sigma sC[(r/2) - (r_1^2/2r)] \qquad (16)$$

Also, Darcy's law stated in radial coordinates gives

$$V_r = -(K/\eta)(dp/dr) \qquad (17)$$

Then eliminating V_r we have

$$(dp/dr) = (\sigma sC\eta/K)[(r/2) - (r_1^2/2r)] \qquad (18)$$

which when integrated gives

$$p = (\sigma sC\eta/K) \times [(r^2/4) - (r_1^2/2)\log_e r + \text{constant}] \qquad (19)$$

Assuming $p = 0$ at $r = r_1$, we obtain the following expression for the total pressure necessary to keep the interstices in the paper full between the radii r_1 and r_2.

$$p = (\sigma sC\eta/4K) \times [r_2^2 - r_1^2 - 2r_1^2\log_e(r_2/r_1)] \qquad (20)$$

C. APPLICATION TO MINIATURE CABLE SPECIMENS

All but two of the factors in equation 20 are already known:

$\sigma = 8 \times 10^{-4}$ cubic centimeters per cubic centimeter per degree centigrade
$s = 0.5$
$K = 12 \times 10^{-12}$ for paper (M)
$r_1 = {}^3/_{16}$ inch = 0.476 centimeter
$r_2 = r_1 + 0.065$ inch = 0.641 centimeter

The remaining two factors must be considered together since we wish to consider the most severe condition which occurs when their product is a maximum. This

Table II. Oil-Flow Data

Paper	Average Thickness in Mils	Number of Layers	K × 10¹²
M	6.5	1	12.6
M		2	12.4
M		4	12.7
M		8	12.5
M		8	12.7
M		8	12.5
M		16	12.3
N	5	1	0.587

Table III. Gas-Flow Data

Paper	Average Thickness in Mils	Number of Layers	p_a	p_2	p_1	$M \times 10^9$	$K \times 10^{12}$
M	6.5	8	76.15	77.28	79.06	.805	12.79
M			76.15	77.87	80.49	1.242	13.37
M			76.15	79.14	83.63	2.132	13.02
M			76.11	82.73	92.17	4.75	12.80
M			76.42	85.15	97.31	6.32	12.69
M			75.03	83.05	94.33	5.64	12.68
M			75.03	86.09	100.87	7.83	12.72
M			75.03	89.99	108.93	10.62	12.67
M			75.03	94.58	117.95	13.96	12.63
M			75.47	101.54	130.95	18.98	12.66
M			75.47	112.12	148.47	27.00	12.67
M	6.5	16	76.24	77.41	81.12	.811	12.04
M			76.27	77.93	83.18	1.167	12.03
M			76.28	79.76	90.35	2.44	11.85
M			76.28	81.61	97.23	3.74	11.70
M			76.28	84.16	106.17	5.56	11.65
M			76.28	84.15	105.99	5.56	11.74
M			76.28	88.11	118.76	8.42	11.60
M			76.26	93.43	134.57	12.26	11.41
N	5	1	74.99	78.91	87.25	2.72	.887
N			74.98	81.54	94.48	4.67	.909
N			74.99	84.54	102.26	6.80	.926
N			74.97	87.83	109.84	9.12	.942
N			74.99	91.54	118.42	11.80	.940
N			74.99	96.09	128.64	15.18	.926
N			74.99	101.34	139.47	19.04	.925

maximum product can be obtained graphically as shown in figure 9. Curve (1) gives the daily cooling curve taken by means of a thermocouple inserted in a glass tube extending into the oil space surrounding one of the specimens. Since the rate of cooling is slow we shall assume that the temperature is the same throughout the specimen. Curve (2) gives the rate of cooling, that is, the first derivative of curve (1). Curve (3) shows the viscosity change in oil (0) as the specimen cools. Curve (4) is the product of curves (2) and (3). As the specimen cools it is seen that the viscosity increases faster than the rate of temperature change decreases so that the maximum value of $(C\eta)$ occurs as the specimen approaches room temperature. Thus in equation 20 we shall use $C\eta = 11 \times 10^{-3}$. Then $p = 4.59 \times 10^{3}$ dynes per square centimeter = 0.0045 atmosphere (one dyne per square centimeter = 0.987×10^{-6} atmosphere).

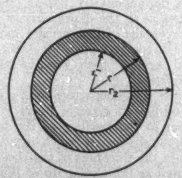

Figure 8. Cross section of a cable in which the space between radii r_1 and r_2 is occupied by paper + impregnant

In our miniature cable tests one atmosphere hydraulic pressure was maintained on the liquid surrounding the specimens so that void formation resulting from inability of the impregnant to flow in sufficient volume to keep the interstices in the paper full *was not* the primary cause of failure.

D. Application to Commercial Cable

To determine the order of magnitude of p for a commercial cable we shall assume the following data for a 69-kv solid-type cable recently installed in this country.

r_1 = 1.65 centimeters
r_2 = 3.40 centimeters
s = 0.5
σ = 8×10^{-4} cubic centimeters per cubic centimeter per degree centigrade
K = 24×10^{-12} (We have conducted a number of preliminary experiments to determine the average radial flow characteristics through and around standard density paper tapes of a machine insulated cable sample and found the rate of flow to be roughly double that through flat sheets.)
C = 11×10^{-3} (assuming the rate of temperature change in service to be the same as for our miniature specimen)

Then $p = 2.25 \times 10^5$ dynes per square centimeter = 0.222 atmosphere.

During cooling of a commercial cable, Halperin[20] shows that the hydraulic pressure in the impregnant at the sheath may drop as low as 0.16 atmosphere. Under such conditions *the hydraulic pressure would not be sufficient to keep the interstices in the paper full and void formation by this mechanism might be a significant factor in initiating ionization leading to ultimate failure.*

VI. Conclusions

1. Satisfactory equipment has been developed for absolute gas- and liquid-flow measurements through multiple layers of paper.

2. Using theoretical equations for gas and liquid flow in porous media the rate of flow of a gas-free liquid through standard-density cable paper can be calculated from air-flow data.

3. For super-calendered paper, the resistance to flow is much greater than would be indicated by the increase in average density, indicating the formation of a skin having high flow resistance on the surfaces of the paper sheet. Also the resistance to oil flow is greater than would be calculated from air-flow data and cannot be predicted from the latter, as is possible for standard-density paper. This indicates that some of the pores in the surface layers of super-calendered paper are so small that effects of absorption, adsorption, and the like on the surfaces of the cellulose fibers are no longer negligible.

4. These studies have shown that in our miniature cable samples, the resistance to flow was not great enough to cause void formation in the interstices of the paper insulation during cooling. The observed short life of specimens containing super-calendered paper or high-viscosity oil must have resulted from some other cause such as higher stress on the impregnant because of the higher dielectric constant of the paper.

5. In commercial solid-type cables using high-viscosity impregnants, on the other hand, calculations indicate that the hydraulic pressure available is not sufficient to keep the interstices of the paper full and

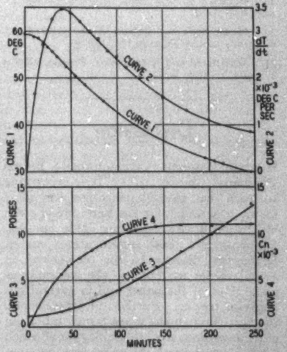

Figure 9. Determination of maximum value of $c\eta$

void formation is inevitable. This is the major argument for the use of low-viscosity impregnants and the "oil filled" principle in high-voltage cables.

Acknowledgment

The writer is happy to express appreciation to R. S. Kent, R. C. Retherford, and S. Godet for assistance in obtaining the information presented in this paper, and to H. Poritsky for checking the mathematical relations.

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Discussion

Herman Halperin (Commonwealth Edison Company, Chicago, Ill.): It is interesting to learn that gaseous ionization was not a factor responsible for the short life of some of Doctor Race's miniature cable samples. Unfortunately these investigations cannot be applied directly to the phenomena in actual cables as most of the assumptions under V-*A* of the paper will in general not apply in service. Further, in actual cables the flow through the tapes is only a portion of the total flow, a considerable part of which takes place through the spiral gaps between turns of tape. This means that the coefficient of friction of cable insulation for radial oil flow would be lower than for the author's samples. This is recognized in a parenthetical statement in part V-*D* of the paper.

Our tests on cables of the solid and oil-filled types indicate values of 4,000 to 18,000 pounds per square inch per inch thickness of insulation per cubic inch of oil per hour for oil having a viscosity of 1,000 centipoises. The value assumed by the author for 69-kv solid-type cable is 25,000 (same units), corresponding to the value of $K = 24 \times 10^{-13}$. For superdense cable insulation we find the radial coefficient of friction to be about ten times higher than for ordinary insulation, instead of 21 times as found by the author for his samples having superposed layers of paper. These lower coefficients of friction tend to reduce the probability of void formation.

In actual cables, the rate of temperature change during cooling will in general not be the same as in the laboratory models. Consideration must be given to the temperature gradient in the comparatively thick insulation of cables. The calculations on the possibility of void formation in commercial cables are over-simplified, particularly when applied to cables with thick insulation. The complete solution for pressure distribution in cable insulation was given by Miller and Wollaston in their paper "Thermal Transients and Oil Demands in Cables," AIEE Transactions, volume 52, March 1933, page 98. By the use of this method, we have found that negative pressures will not form in oil-filled cable, except possibly under extremely severe conditions. In solid-type cables having thick insulation and high-viscosity impregnant, negative pressures may easily occur due to insufficient hydrostatic pressure. It does not follow that this cause will of itself result in voids in the insulation.

Doctor Race considers the hydrostatic pressure to be the principal factor controlling the return of compound into the insulation after expansion. It seems that the surface tension and cohesion of the compound, the capillary forces of the paper, and the adhesion between compound and paper should also have been considered. These properties are important factors in void formation.

W. A. Del Mar (Habirshaw Cable and Wire Corporation, Yonkers, N. Y.): Doctor Race has done a nice piece of work in this paper in picking out of the complicated performance of a solid-type paper cable one or two separable items which are susceptible of theoretical treatment and partial verification by experiment. One must be careful, however, as indeed Doctor Race has been, not to draw conclusions of too general a character. Thus, it might appear from Doctor Race's paper that the essential defect of a solid-type cable is the radial transfer of oil from the interior to the exterior of the insulation, leaving voids in volume equivalent to the oil forced out of the insulation.

I do not believe that this is the whole story but that to a greater extent it is the entire insulation, paper and oil, that are expanded in volume during heating, leaving a larger volume of insulation to be filled by the same amount of oil.

Impregnated paper, unlike unimpregnated paper, will take a permanent set with practically any tensile stress. It will not retain all the stretch caused by a given tensile stress when that stress is withdrawn, but it will retain a considerable proportion, usually about 15 per cent, and that will account for the increased gross volume of insulation and, as this stretch is accompanied by a corresponding reduction in tape thickness, the void space will be further augmented.

Thus we see that while Doctor Race has picked one of the variables and given it an interesting theoretical treatment, there are other variables to be considered. Doctor Race, I know, appreciates this thoroughly but I have misgivings about his application of the formulas to a 69-kv cable, fearing that someone may take it too seriously as a practical tool in cable design.

R. J. Wiseman (The Okonite-Callender Cable Company, Inc., Passaic, N. J.): I am very much interested in Doctor Race's paper as it discusses the physical properties of impregnated paper insulation. We have had many technical papers dealing with the electrical characteristics, but unless we know how the oil moves through the paper, the effect of the fiber content of the paper on the oil movement, and the pressures necessary to cause the oil to flow, we do not get a complete picture of what takes place in a paper-insulated cable. Several years ago we started a similar research, but had to postpone active work due to the urgency of other researches. However, Doctor Race has gone into the subject much more accurately than we had planned.

This paper gives us a better appreciation of the relative effect of using low- and high-weight-density papers. We know in manufacture that it requires a longer time to dry and then impregnate high-density papers. This is confirmed by Doctor Race's work. In view of the much easier flow of oil through normal-density papers, the question arises as to the desirability of using a large percentage of high-density paper in a cable. Of course, this will have to be balanced against their relative electrical characteristics.

I wonder if the departure from a straight line for oil flow as given in figure 6 is partially due to the swelling of the fibers of the high-density paper which results in an increase in the thickness of the sheet under test as well as the closing up of the spaces between the fibers suggested by Doctor Race.

The attempts of Doctor Race to apply his test data to actual cables is to be commended as it puts his work on a practical basis. The difficulty of getting close agreement with actual cables is due to several factors, such as: the view expressed by Mr. Halperin regarding flow between the layers of the paper longitudinally and along the spiral gaps; the tightness of wrapping of the layers and the difference in temperature, that is, the temperature gradient between the conductor and the sheath. Doctor Race assumes the temperature is constant for any given instant of time. For very thin walls this is true, but for heavy walls it is not. The matter of the effect of tightness of wrapping could have been studied by having the flat sheets Doctor Race tested, subjected to various degrees of compression and then determine the oil flow. However, we have a better appreciation of how much pressure is needed to maintain saturation of a cable.

It is surprising that the pressure required is not very great and naturally, one may ask why do cables fail due to migration of oil and the formation of voids. In the so-called oil-filled type of cable we use a very thin oil and the pressures that we apply at joints are of such value that we can feed oil into all parts of the cable as rapidly as is necessary to maintain a positive pressure and we, therefore, do not get voids or insulation failures. It is the ideal cable. Of course, we have another solution in the high-pressure cables which use higher-viscosity oils, but always maintain saturation because the pressure will force the oil along the cable and into the insulation. However, we have many miles of solid-type cable and we will have many more miles in the years to come. We, therefore, should consider how we can improve the solid type of cable. At the present time we use a tight-fitting lead sheath on our cables which prevents easy flow of oil from the joint along the cable between the inside of the lead sheath and insulation. We do not have any follow-up of oil with moderate pressure to feed into the insulation as the cable cools. It is true that we experience stretched lead sheaths due to the expansion of the oil in localized points along the cable. We do not want this; rather we want to be able to permit easy flow of oil back and forth along the cable. Again, we use varnished cambric wrapped joints because of the shorter time to make a joint as compared to a paper wrapped joint and we seal off the strands of the conductor from the joint compound. We believe this also is wrong. We have studied both of these factors in our laboratories and have concluded that a porous type of joint which will permit oil to flow into and out of the strands of a conductor is the proper kind to use, particularly for high-voltage cables. This, with the loose lead sheath, will reduce the high pressures produced in a cable during the heating portion of the load cycle and prevent a high vacuum to occur during the cooling portion of the load cycle. We have shown by our tests that the life of a cable is increased thereby. Doctor Race has shown us that it does not take much pressure to maintain saturation in the insulation, provided we can supply the oil when needed.

H. H. Race: In general I agree with the ideas raised in the discussion of this paper. However, there are a few points on which I should like to emphasize a different interpretation.

It seems to me that the first sentence of Mr. Halperin's discussion is a misinterpretation of our data. In conclusion IV, I stated that "the resistance to flow was not great enough to cause void formation" in our miniature cable samples. However, this does not mean that *ionization* was not the cause of failure, in fact, in most cases I believe it was and have already submitted part IV of this series of papers entitled "Mechanism of Breakdown" giving the evidence.

The last sentence of paragraph 3 of Mr. Halperin's discussion also needs clarification. When a "solid type" cable cools no voids will form and the internal hydraulic pressure will remain positive so long as the lead envelope collapses or changes its shape so as to keep the total volume within the envelope equal to the volume of the solid and liquid materials within this envelope. However, lead has little true elasticity and therefore it undergoes permanent deformation easily, and follows such volume changes very imperfectly. When during cooling the sheath reaches a point beyond which it no longer follows the shrinkage of the materials within it, *voids inevitably form.* The gas pressure in these voids may be anywhere from less than one micron corresponding to the vapor pressure of the impregnant to a fairly high pressure, depending upon the temperature and how much residual moisture, air, or other gases are present. *The "solid type" cable in which no reservoirs are provided cannot be completely free from voids or gas pockets during cooling.* The electrical gradient at which ionization will start in such gas spaces depends upon a number of factors such as gas pressure, dimensions of the gas space, proximity to an electrode and condition of the intervening solid dielectric. *A properly designed gas-free liquid-impregnated cable with sufficient reservoir capacity and liquid pressure is the only type in which voids or gas pockets cannot form during cooling.*

Regarding Mr. Halperin's last paragraph, the factors he mentions certainly deserve consideration. As far as I know, however, their importance has not yet been measured quantitatively. As indicated by Mr. Delmar, the object of this paper was to give quantitative mathematical and experimental study to one factor, namely, viscous flow, which seemed most important.

The last paragraph of Mr. Wiseman's discussion advocates a partial attempt to make an "oil filled" cable out of a "solid type" cable by providing a loose sheath and porous joints so as to provide easy longitudinal flow of oil. To complete the system reservoirs must also be supplied containing sufficient low viscosity oil to keep the system full and stop joints to prevent building up excess pressures at low spots in the system. Even with these modifications a solid type cable cannot be maintained free from voids at all times. Therefore, why not use a properly designed "oil filled" system in the beginning?

Application of Large Phase-Shifting Transformer on an Interconnected System Loop

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MEMBER AIEE

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MEMBER AIEE

A GROUP of 132-kv and 66-kv transmission systems form a loop about 300 miles in circumference and connect many important generating stations and a number of load points in the western part of Pennsylvania and the eastern portion of Ohio.

When these systems are operated interconnected at all tying-in points except one 66-kv tie north of Pittsburgh, the power exchange can be readily controlled. When this tie is closed, however, a complete ring is formed with multiple paths and an uncontrollable circulating power flows around the loop. Under such conditions, controlling the division of load over the paths and the interchange between companies requires phase-shifting equipment.

The phase-shifting transformer which is described in this paper was installed at the above mentioned point in order to make it possible to provide an emergency supply of power. Under ordinary operating conditions, this tie is open and there is no transfer of power in either direction. When, however, an unusual emergency situation develops on either system, which cannot be handled by the generating capacity associated with that system, the phase-shifting transformer can be put into operation by closing the tie and emergency power can be transferred from one system to the other.

This paper describes the phase-angle problem, the system studies made to determine the type and capacity of regulating equipment needed for the tie line, including the results of interconnected system tests, and operating performance after the installation of new equipment.

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W. J. Lyman is electrical planning engineer with the Duquesne Light Company, Pittsburgh, Pa.; J. R. North is general technical engineer with the Commonwealth and Southern Corporation, Jackson, Mich.

Acknowledgment is made to the Ohio Edison Company, Ohio Power Company, Ohio Public Service Company, Pennsylvania Power Company, and the West Penn Power Company for their cooperation during the field tests; and also, to Messrs. W. C. Bryson and V. E. Hill of the Duquesne Light Company for their valuable assistance.

Characteristics of Interconnected Systems

The interconnected systems and their geographical layout are shown by figure 1. Large generating stations are located in the Pittsburgh area at Brunots Island, Reed. Colfax, and Springdale; in the Youngstown area at Toronto, New Castle, Lowellville, and Warren; and in the Canton-Akron area at Windsor, Akron, and Cleveland. The 66-kv tie line between Valley and Ellwood is normally operated open at Valley, and the closing of this tie in an emergency is the subject of this study.

A 30,000-kva quadrature regulator or phase shifter having a range of 5.7 degrees plus or minus is located at Salt Spring substation, Youngstown, in the 132-kv Warren circuit. At Colfax there is a 36,000-kva voltage regulator in the 132-kv tie line to Springdale. The kilowatt interchange over this tie line is controlled automatically by regulating the Colfax generator output.

The reactance of the component parts of the loop system is of considerable importance in this phase-angle study. A convenient graphical method was developed for illustrating the magnitude of the reactance and the interrelation of the various sections. This is shown on figure 2 with the relative electrical distances between the stations drawn to scale in terms of the reactance of the lines, transformers, bus reactors, etc. The lengths of the solid lines represent ohmic values at 132 kv. Where several paths are in parallel, the equivalent net reactance has been calculated and is indicated by the length of the dashed lines. It will be noted that the total net reactance around the loop is 465 ohms at 132 kv.

Phase Angle and Load Transfer

The flow of power current through a reactance results in a phase-angle difference between the voltages at both ends, as shown on figure 3*A*. The *IX* drop is nearly in quadrature with the line voltage. Conversely, the insertion of a quadrature voltage in a reactive circuit will cause the flow of power current. Reactive current creates an *IX* drop in phase with the line voltage (figure 3 *B*) and so produces no angular difference. Thus, quadrature voltage controls kilowatt power flow and in-phase voltage controls reactive flow (modified slightly by line resistances).

Due to the manner in which the loads and generating capacities are distributed around the loop shown in figure 2, the voltage drops in the various circuits do not cancel out, and there is usually a relatively large phase-angle difference between the voltages at the open point. If the 66-kv tie at Valley substation is closed, as much as 12,000 kw will circulate around the entire ring, passing through all the interconnection points. This uncontrollable circulating power would naturally upset normal interchange even for those companies not directly interested. It would also increase the load on several important lines and transformers, possibly necessitating increased capacity at certain points.

Hence, if the Duquesne Light Company and the Pennsylvania Power Company desire to interchange power, some of it will pass through the direct 66-kv tie and some around the opposite direction through the other systems, with no means of controlling the division over the various circuits. A phase-shifting transformer connected in the 66-kv tie at Valley would introduce a controlled

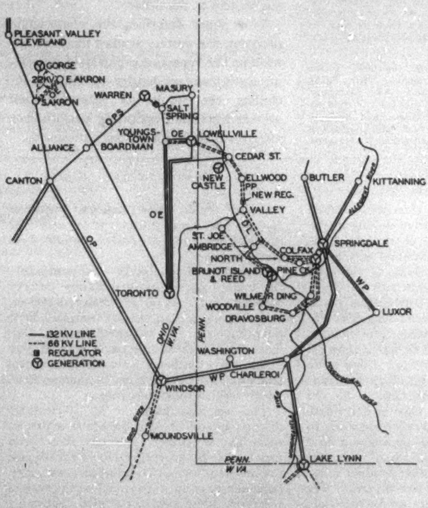
PLEASANT VALLEY CLEVELAND
GORGE
22KV
E. AKRON
S. AKRON
WARREN
MASURY
SALT SPRING
OPS
YOUNGSTOWN
OE
LOWELLVILLE
ALLIANCE
BOARDMAN
CEDAR ST.
CANTON
NEW CASTLE
ELLWOOD
PP
NEW REG.
VALLEY
BUTLER
KITTANNING
OE
OP
ST. JOE
AMBRIDGE
NORTH
COLFAX
SPRINGDALE
BRUNOT ISLAND & REED
PINE CK.
WILMERDING
WOODVILLE
DRAVOSBURG
WP
LUXOR
TORONTO
OHIO W.VA.
PENN.
WASHINGTON
WP
CHARLEROI
WINDSOR
MOUNDSVILLE
PENN. W.VA.
LAKE LYNN
— 132 KV LINE
---- 66 KV LINE
■ REGULATOR
⊗ GENERATION

Figure 1. Pennsylvania–Ohio interconnected systems

DL—Duquesne Light Company
OE—Ohio Edison Company
OP—Ohio Power Company
OPS—Ohio Public Service Company
PP—Pennsylvania Power Company
WP—West Penn Power Company

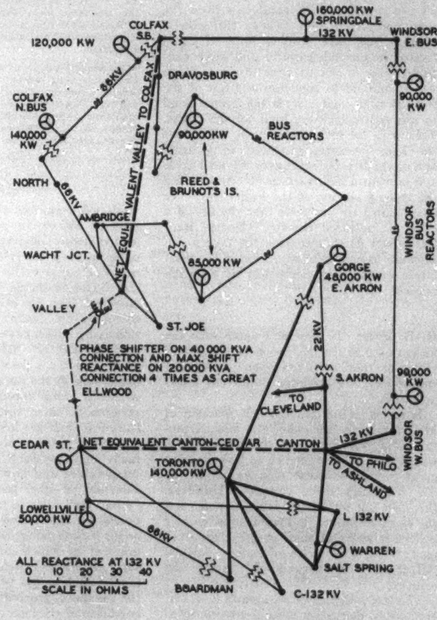
COLFAX S.B.
120,000 KW
160,000 KW SPRINGDALE 132 KV
WINDSOR E. BUS
66KV
DRAVOSBURG
COLFAX N. BUS
140,000 KW
NET EQUIVALENT VALLEY TO COLFAX
90,000 KW
90,000 KW
BUS REACTORS
NORTH
66KV
REED & BRUNOTS IS.
WINDSOR BUS REACTORS
AMBRIDGE
WACHT JCT.
85,000 KW
GORGE 48,000 KW E. AKRON
VALLEY
ST. JOE
22KV
PHASE SHIFTER ON 40 000 KVA CONNECTION AND MAX. SHIFT REACTANCE ON 20 000 KVA CONNECTION 4 TIMES AS GREAT
S. AKRON
90,000 KW
ELLWOOD
TO CLEVELAND
CEDAR ST.
NET EQUIVALENT CANTON-CEDAR
CANTON
132 KV
TO PHILO
TO ASHLAND
WINDSOR W. BUS
TORONTO 140,000 KW
LOWELLVILLE 50,000 KW
L 132 KV
66KV
WARREN
SALT SPRING
ALL REACTANCE AT 132 KV
0 10 20 30 40
SCALE IN OHMS
BOARDMAN
C-132 KV

Figure 2. Equivalent reactance interconnected systems

Table I. Check of Phase Angles Around the Loop

October 1936 Tests

Circuit	Ohms per Circuit at 132 Kv	Phase Shift (Deg. per Megawatt)	Test No. 2—Normal				Test No. 4—Load Shift				Test 8a	
			10:40 A.M.		10:45 A.M.		3:50 P.M.		4:00 P.M.		3:15 P.M.	
			Megawatts	Degrees	Megawatts	Degrees	Megawatts	Degrees	Megawatts	Degrees	Megawatts	Degrees
Valley-Wacht Junction	33.9	0.111	12.0	1.33	8.5	0.94	8.5	0.94	8.5	0.94	9.0	1.00
Wacht Junction-North	48	0.157	25.4	3.99	22.2	3.49	15.8	2.48	18.1	2.85	23.1	3.63
North-Colfax	30.8	0.130	21.4	2.78	19.9	2.59	16.7	2.17	18.2	2.36	20.2	2.63
Colfax number 1 transformer	17	0.056	43.0	2.41	41.0	2.30	36.5	2.05	30.5	1.71	46.0	2.58
Colfax number 1 reactor	30.2	0.128	16.5	2.11	12.5	1.60	9.5	1.22	3.0	0.39	17.0	2.18
Colfax number 2 reactor	39.2	0.128	5.8	0.74	0.5	0.06	−4.5	−0.58	−9.5	−1.22	9.0	1.15
Colfax number 2 transformer	17	0.056	−31	−1.74	−30.5	−1.71	−22.0	−1.23	−22.0	−1.23	−30.0	−1.68
Springdale regulator and line	50.7	0.166	−3.8	−0.63	−13.4	−2.22	0	0	0	0	3.8	0.63
Springdale-Charleroi	76.5	0.087	−8.0	−0.70	−12.0	−1.04	−10	−0.87	−9.8	−0.85	−17.0	−1.48
Charleroi-Windsor	35.4	0.116	21.5	2.50	19	2.20	16	1.86	20.0	2.32	24.5	2.85
Number 4 transformer at Windsor	53.2	0.174	22.6	3.94	20	3.48	16	2.79	20.0	3.48	24.5	4.26
Number 4 reactor at Windsor	87	0.285			−5.5	−1.57						
Reactors number 2–4	0.174	0.571			−5.5	−3.15						
Number 1 transformer at Windsor	29.2	0.096	−11.0	−1.06	−15.5	−1.49	−5.5	−0.53	−3.5	−0.34	−1.0	−0.10
Windsor-Canton number 2	46	0.151	−11	−1.66	−15.5	−2.34	−5.5	−0.83	−3.5	−0.53	−1.0	−0.15
Canton-Akron	21.5	0.071	−33.5	−2.38	−36.0	−2.55	−36.5	−2.60	−21.3	−1.51	−22.5	−0.160
South Akron transformer	26	0.085	−27.5	−2.33	−31	−2.63	−29.0	−2.46	−13.8	−1.17	−28.7	−2.44
East number 2 line	0.154	0.505	− 6.0	−3.03	−4.6	−2.32	−4.0	−2.02	−7.0	−3.54	−4.5	−2.27
Akron number 1 line and transformer	79	0.259	42.0	10.90	40.0	10.37	38.0	9.87	46.5	12.07	53.7	13.94
Z-71 line and transformer	124	0.406	−6.7	−2.71	−8.9	−3.61	−5.9	−2.40	−6.5	−2.64	−6.2	−2.51
Y-53 line	34.2	0.112	−1.9	−0.21	−5.7	−0.64	−2.1	−0.24	−1.6	−0.18	−2.1	−0.24
Ellwood-Valley number 2	36.5	0.120	0		−9.6	−1.15	0		0		0	
Valley regulator	26.0	0.085	0		−9.6	−0.82	0		0		0	
Total angle—calculated				14.2		−0.21		9.62		12.9		22.4
Total angle—measured at Valley				14.0		0		7.0		13.5		28.0

quadrature voltage to compensate for the inherent angular difference and would permit satisfactory closed-ring operation when desired.

Field Tests—1928

The character of the problem of operating the 66-kv Valley interconnection closed simultaneously with the Colfax-Springdale interconnection was recognized many years ago, and the earliest effort to analyze the problem was a specially conducted series of tests in 1928. These tests consisted primarily of taking simultaneous readings on transformers and lines around the entire 300-mile loop, as it was at that time, and reading the actual phase difference between the voltages appearing across the open switch at Valley substation. These tests were described in an AIEE paper.[1]

Field Tests—1936

In 1936, it was decided to increase the interconnection capacity, largely as a result of the March 1936 flood. One of the increases proposed was a large capacity, 66-kv phase shifter in the Valley-Ellwood City tie, in order to permit parallel operation. In order to obtain accurate and up-to-date information from which to determine the required range of voltage and phase angle for the proposed regulator, another series of tests was run in the fall of 1936, similar to those performed in 1928.

These tests were conducted during the heavy load period, and arrangements were made to measure the voltages, the kilowatt and the kilovolt-ampere loads on the circuits around the loop, with the 66-kv tie at Valley open and closed, and with different divisions of load between the power generating stations. The test procedure was to take readings regularly every five minutes and to open and close the tie switch at a scheduled time between readings. In this way, data were secured with the Valley tie open and then again, five minutes later, with the tie closed, and vice versa.

Graphic records and indicating meter readings were obtained at the various points. At Valley substation, graphic records were secured of the kilowatt interchange, reactive kilovolt-ampere interchange, and the phase angle. The phase-angle record was secured on a wattmeter which had its potential coil energized from the Pennsylvania Power Company voltage through a metering transformer which shifted the voltage 90 degrees. The current coil was supplied by a constant current from the Duquesne Light Company system through a resistance load box. For diagram of connections, see reference 1. With this connection, the zero center wattmeter would read zero when the Duquesne Light Company and the Pennsylvania Power Company voltages were in phase. The readings were calibrated by use of a recording voltmeter and ammeter.

During the week following the tests, graphic charts were obtained of the voltages and phase angles. Figure 4*A* shows the phase angle variations and figure 4*B* illustrates the characteristic periodic oscillation of about five degrees occurring about 18 times per minute. This is probably the result of synchronizing power swings following the natural period of the combined system.

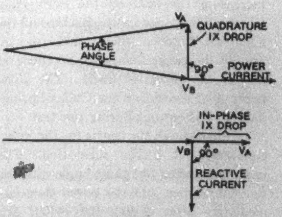

Figure 3. Relation between phase shift and power flow

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[1] For all numbered references, see list at end of paper.

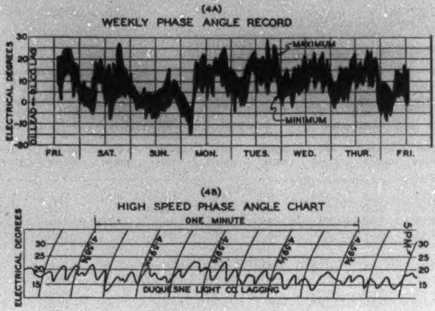


Figure 4. Phase-angle record

Summary of Test Results

One of the principal objectives of the test was to establish definitely the practicability of calculating phase angles for any given set of load conditions. The load readings were, therefore, used to calculate the phase angle and compare with the actual measured angle, and the results are shown on table I. The correspondence is very close, considering that the readings are taken from meters which are continuously fluctuating over a wide range.

Some typical power-flow and phase-angle test readings, as assembled and coordinated with the results, are shown on figures 5 and 6. It is interesting to study figure 5 and note that when the Valley tie is closed, the amount of circulating power passing through it actually appears at every point throughout the 300-mile loop. It may be seen passing through the Springdale interconnection, increasing the load on the Springdale-Charleroi lines, decreasing the load on the Charleroi-Windsor lines, and so on around the entire loop, with no effect on any of the generator loads. The automatic load control on the Colfax-Springdale tie was cut out during this test.

Figure 6 shows the charts for the interconnection at Valley substation. The relation between the phase angle with the tie switch open and the power flow immediately after closing the switch can readily be seen. During these tests the reactive kilovolt-amperes flowed in a direction opposite the power. This was due to the fact that the ratio of the original tie line autotransformer was not changed to transfer reactive power. Referring to figure 3*A*, the resistance in the system causes the current to lag behind the quadrature voltage by an angle less than 90 degrees. The circulating current, therefore, leads the line voltage.

On the 11:14 a.m. test (figure 6), 11,500 kw flowed through the Valley-Ellwood tie, and an equal amount was transferred from Colfax to Springdale. The automatic tie-line load control at Colfax was then cut in and set for zero interchange. By the time Colfax had backed off enough load to bring the Springdale tie down to zero, the Valley tie load had built up, as shown, to 22,000 kw. Efforts to control the phase angle by shifting as much as 20,000 kw between various plants did not result in changes of more than five degrees.

The normal open-circuit phase-angle difference between Valley and the Ellwood City lines during the load period 8 a.m. to 8 p.m. on week days varied between zero and plus 22 degrees (Ellwood "fast").

There is a rather definite relation between the phase angle across the open switch and the power which flows immediately on closing the switch. This ratio, as determined by test, runs from about 800 kw per degree at small angles down to 500 kw per degree for very large angles. This value may be checked analytically by taking the entire reactance around the loop, together with the voltage difference corresponding to a one degree phase angle. The result is as follows:

Reactance around loop = 465 ohms (at 132 kv)

Approximate impedance around loop = 472 ohms

Voltage corresponding to one degree =

$$V \sin 1^\circ = \frac{0.0175 \times 132{,}000}{\sqrt{3}} = 1{,}333$$

$$I = \frac{1333}{472} = 2.82 \text{ amperes}$$

$$Kw = \sqrt{3}\,EI\,(PF) = \sqrt{3}\,(132{,}000)\,(2.82) \times \frac{465}{472} = 635 \text{ kw per degree}$$

These tests demonstrated that a phase shifter is necessary in order to use the Valley-Ellwood and the Colfax-Springdale ties simultaneously without splitting the Duquesne Light Company system into separate parts. The Valley location gives the most economical installation.

Load and Phase Angle Requirements

The range of phase-angle control required to attain the desired results is made up of two component parts: (*a*) the natural phase angle which may exist at any time due to the load and generating conditions and the resulting net power flow in each individual section of the entire loop, and (*b*) the quadrature voltage necessary to send the desired amount of power through the tie line in the proper direction. With a phase shifter capable of meeting these requirements, it would be possible to interchange power in the desired amount between the two adjacent systems without affecting in any way the generation, transmission line, or tie-line loads on the other systems involved in the loop.

The amount of phase shift needed to neutralize the natural angle involved a study of the characteristic load variations on the transmission lines and the effect of new generating capacity to be added in the future.

The limits set up for power interchange during an emergency were 40,000 kva from the Pennsylvania Power Company to the Duquesne Light Company, or 20,000 kva from the Duquesne Light Company to the Pennsylvania Power Company. These amounts are consistent with the requirements of each system and its ability to furnish power at that location.

Natural Phase Angle Between Systems

Obviously, it is not sufficient to know only what the normal phase angle will be in order to determine the phase shifter required. This phase difference varies over a wide range, depending on variations in the loads on the lines and transformers occasioned by changes in load, generation schedules, and emergency conditions. In general, the line loadings around the 300-mile ring tend to cancel out, and if the location of the loads and generation were entirely symmetrical, there would be no phase difference at Valley.

It is known, however, that load on lines feeding in one direction might possibly increase at the same time those feeding in the opposite direction would decrease. This would greatly increase (or decrease) the phase difference at Valley. It is entirely possible to secure extremely large angles in this manner, but if this occurrence is to be infrequent, it would not be economical to provide for it. Consequently, the proper solution cannot be reached by providing for the worst possible condition.

In order to reduce this problem to more definite and tangible terms, the probability theory was applied to the basic information to determine the likelihood that the angle will exceed any given amount in either direction. This analysis has been limited to the period from 8 a.m. to 8 p.m. on week days. The phase difference during light loads is much less.

The first step was to study the actual angular difference which existed and which could be measured at Valley substation. The combined result of many readings taken over several months is shown on figure 7. The angle varied from 0 to 22 degrees, with 11 degrees to 13 degrees occurring most frequently.

PROBABLE VARIATION

Most variable conditions which center about an average figure, but deviate from this average with a frequency which decreases as the deviation increases, can usually be represented by the standard "bell" shaped distribution or probability curve. The experience curve on figure 7 was plotted so as to show the percentage of the total number of readings of any particular angle. This curve has the characteristic "bell" shape and can be represented very closely, as shown by the series of crosses, with a standard curve.

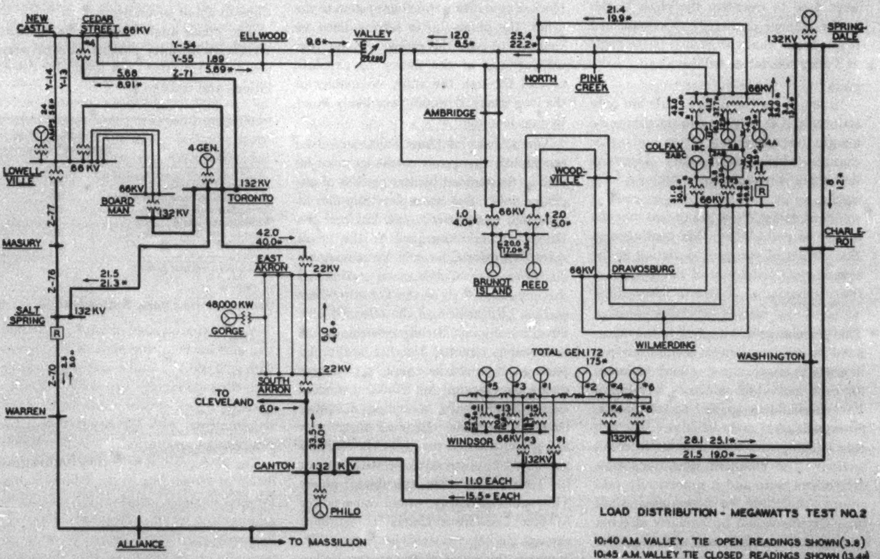
The hourly loads of each individual line or transformer, when plotted as a frequency distribution curve, will follow this general "bell" shape, having a certain average value, and varying above and below this average or mean by an amount depending on how widely the load fluctuates (8 a.m. to 8 p.m. load period only being considered here). If the curve for each line and transformer were available, they could be converted into phase angles (by using the load and impedance) and combined to obtain a probability curve for the phase angle at Valley. The measure of the magnitude of the variation of a probability curve is called the standard deviation which is the root-mean-square of the deviations of all the individual points from the mean. It can be shown from the characteristics of the standard probability curve that the chances are two to one (2/3) that the load will not deviate from the mean by more than the standard deviation. Furthermore, the chance that the load will exceed any given amount may be determined from the probability curve or standard tables.

A study was made, therefore, of actual line load variations and representative probability curves obtained. The average value of the standard deviation for these curves was found to be 15 per cent of the average load during the 8 a.m. to 8 p.m. load period. This means that for only one-third of the time is the load more than 115 per cent or less than 85 per cent of the average.

If there are several lines and transformers in a loop, each carrying a load independent of the others, and varying approximately according to the standard probability curve, the net phase shift of the whole group will also be a probability or "bell" shaped curve. The "S.D." (standard deviation or spread) of this total curve will be the root-mean-square of the "S.D.'s" of the individual circuits.

Load changes due to a general increase in system loads will not greatly affect the angle since they will tend to balance out, and it is only the independent variations due to local conditions which affect the problem. It was found by examination of simultaneous loads around the loop that 30 per cent of the variation is common to all circuits, so that the effective standard deviation is only (100 per cent–30 per cent) × 15 per cent, or about

Figure 5. Load distribution—megawatts


NEW CASTLE
CEDAR STREET 66KV
ELLWOOD
VALLEY
9.6* →
Y-54
Y-55 1.89
5.69* →
5.68
Z-71
← 8.91*
Y-14
Y-13
24 AMPS 58*
LOWELLVILLE
66 KV
4 GEN.
66KV
BOARD MAN
132KV
132KV
TORONTO
Z-77
MASURY
42.0
40.0* →
EAST AKRON
22KV
Z-76
21.5
← 21.3*
SALT SPRING
132KV
R
48,000 KW
GORGE
6.0
4.6*
22KV
SOUTH AKRON
Z-70
2.5
5.0*
TO CLEVELAND
6.0* →
WARREN
33.5
36.0*
CANTON
132 KV
PHILO
ALLIANCE
TO MASSILLON
← 12.0
← 8.5*
25.4
22.2*
21.4
← 19.9*
NORTH
PINE CREEK
AMBRIDGE
66KV
COLFAX
1BC
2ABC
3B
4A
R
66KV
132KV
SPRINGDALE
CHARLEROI
8
12*
WOODVILLE
1.0
4.0*
66KV
2.0
5.0*
20.0
17.0*
BRUNOT ISLAND
REED
66KV
DRAVOSBURG
WILMERDING
WASHINGTON
TOTAL GEN.172
175*
#5
#3
#1
#2
#4
#6
#11
#13
#15
#4
#6
WINDSOR
66KV
#3
#1
132KV
132KV
← 11.0 EACH
← 15.5* EACH
28.1 25.1* →
21.5 19.0* →

LOAD DISTRIBUTION - MEGAWATTS TEST NO.2

10:40 A.M. VALLEY TIE OPEN READINGS SHOWN(3.8)
10:45 A.M. VALLEY TIE CLOSED READINGS SHOWN (13.4*)

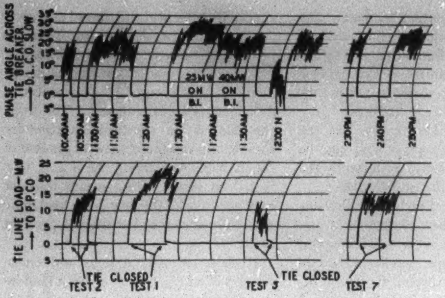


Figure 6. Relation between phase angle with tie open and power flow with tie closed

11 per cent, for individual circuits. Using this figure of 11 per cent applied to all circuits around the loop gives a phase angle variation curve for Valley with a standard deviation of 3.7 degrees. This compares favorably with the 3.8-degree standard deviation of the average curve on figure 7, taken from actual readings.

Future Phase Angle Conditions

The next step was to secure estimates by the various companies included in the transmission ring of the expected normal loads on their lines and transformers during heavy load periods after the installation of the scheduled generators at Reed, Springdale, Windsor (two 60,000-kw units), and New Castle. These loads were used to calculate the phase angles in the manner previously described, and it was found that the normal phase angle at Valley substation will be about 7.5 degrees.

Inasmuch as the future loads are only estimates, it is necessary to provide some margin for estimating errors and other changes which may occur. Assuming that there is one chance in ten of a forecasting error exceeding 25 per cent, a standard deviation of 15 per cent must be added to provide for this contingency. The combined standard deviation is the square root of the sum of the squares of the individual standard deviations, $\sqrt{15^2 + 11^2}$, which gives 18.6 per cent. This percentage was applied to the calculated future phase angles around the loop in order to obtain the standard deviation for each individual section. A value of 100 per cent was applied to the Colfax-Springdale tie and Windsor reactors, over which power may flow in either direction. The standard deviations thus determined were added numerically into groups, according to companies, since these circuits would be similarly affected by load shifts within each system. The root-mean-square of these latter figures gave the standard deviation for the angle at Valley as 6.4 degrees.

The resulting curve at the bottom of figure 7 shows, for example, that the angle will exceed 20 degrees about two per cent of the time, and will be less than −5 degrees about three per cent of the time. This curve gives a definite basis for applying a regulator with a fixed minimum range to compensate for an angle which has no fixed limits and which may vary over a very wide range.

Power Transfer

After setting the phase shifter to compensate for the natural angular difference, the tie line can be closed and no power will flow. If, then, it is desired to shift a block of generation from one system to the other, the phase-shifter setting must be changed to pass the power without permitting any of the power to circulate around through the other companies in the loop where it would very likely upset tie-line loadings.

The amount of phase shift needed to accomplish this power exchange may be readily determined by the process of superposition. For example: Suppose it is desired to transfer 20,000 kw from the Duquesne Light Company to the Pennsylvania Power Company by picking up 20,000 kw at the Colfax power station and dropping 20,000 kw at the Toronto power station. Regardless of the magnitude or direction of loads already existing on the intervening circuits, the net additional phase shift would be equal to that produced by transmitting 20,000 kw from Colfax to Toronto, assuming no other loads on the lines. In order to transfer this power without the phase shifter, Colfax would have to advance with respect to Toronto by this calculated angle. This, quite obviously, would cause power to flow also from Colfax to Toronto through the other systems in the ring. If, however, the phase shifter in the Valley tie is advanced by an amount equal to the calculated angle, Colfax and Toronto will not change their relative angular positions, and the only power flow will be the 20,000 kw from the Duquesne Light Company to the Pennsylvania Power Company through the Valley-Ellwood tie.

The phase angle produced by a given kilowatt load is not greatly affected by the power factor of the load on a system composed of transformers, reactors, and large capacity overhead lines, with consequent low ratios of resistance to reactance. It can be calculated approximately by the formula:

$$\text{Angle (degrees)} = \frac{(KW)\ (\text{Reactance})\ 0.057}{(KV)^2}$$

$$\text{Note: } 0.057 = \frac{1}{1000 \sin 1^\circ}$$

The phase shift corresponding to the requirement of transferring 20,000 kw to the Pennsylvania Power Company, as described above, was found by calculation to be eight degrees (exclusive of *IZ* drop in phase shifter). Since the natural angle may be 20 degrees (Pennsylvania Power Company leading Duquesne Light Company), a total net range of 28 degrees would be necessary in order to be able to furnish 20,000 kw under all reasonable conditions. If the power factor of the interchange were 90 per cent lag, the above value of eight degrees would reduce to about seven degrees.

The phase angle required for shifting 20,000 kw at unity power factor between various points has been found by calculations and tests to be:

	Net Phase Shift Required* (Degrees)
Colfax and Toronto	8.0
Brunots Island and New Castle	6.5
Springdale and Windsor	3.5
Windsor and Philo	2.5
Toronto and New Castle	3.0
Colfax and Brunots Island	1.5

* Excluding regulator drop.

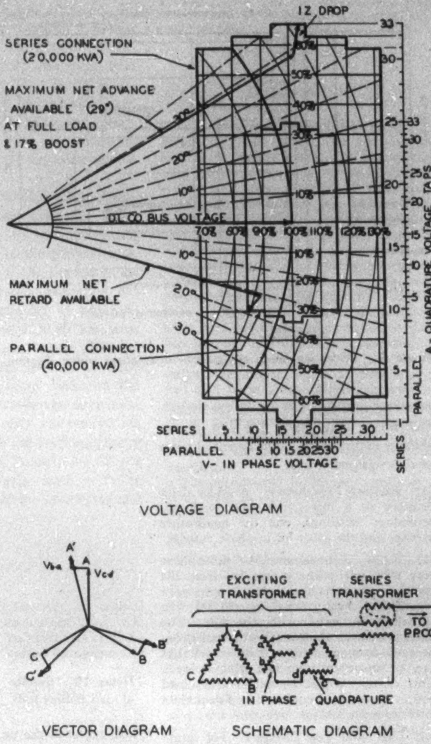
Selection of Phase-Angle Range

A regulator capable of handling 40,000 kva and having a net shift of 28 degrees with a 20,000-kva load would be very large and correspondingly expensive. Consequently, a close scrutiny was made of the requirements, with the result that a series-parallel arrangement was developed.

The 28 degrees is necessary for the condition of 20,000 kva to the Pennsylvania Power Company, which is the maximum requirement set up. However, since the Pennsylvania Power Company voltage is normally in advance of the Duquesne

Light Company voltage, a smaller phase shift is adequate to transfer power to the Duquesne Light Company. By the simple expedient of dividing into two parts the windings which carry the line current and providing a series-parallel switch, it was possible to reduce the rating of the regulator to 20,000 kva and 28 degrees net (33 degrees no load) on the series connection. By changing to the parallel connection, the carrying capacity becomes 40,000 kva, and the available phase shift of 15 degrees net (18 degrees no load) is adequate to transfer full load to the Duquesne Light Company under all reasonable conditions.

Figure 8. Regulator diagrams



VOLTAGE DIAGRAM

VECTOR DIAGRAM

SCHEMATIC DIAGRAM

The phase shift requirement for either company to take power through the Valley tie is materially reduced (about five degrees) if the company is at the same time taking power from other interconnected systems. This would be the normal condition during an emergency. Should conditions arise when the phase shift required is greater than the regulator capacity, the result is not serious. For a five degree shortage, the Valley tie will carry 3,500 kw less than desired and the other tie 3,500 kw more than the desired amount.

Load Increments

With 20,000 kw at unity power factor transferred as described above, the total phase shift would be 8 degrees through the systems, plus 5 degrees in the regulator itself, or 13 degrees. The load shift per degree is, therefore, computed to be about 1,500 kw when shifting between Colfax and Toronto. As shown by previous tests, and verified by calculation, an angle of one degree will send only 600–800 kw around the entire 300-mile loop. In this connection, the automatic load controller on the West Penn Power Company tie line at Colfax must be taken into account. A sudden insertion of one degree, such as by changing taps on the phase shifter, would circulate about 800 kw around the entire loop. This power, which would also flow through the West Penn tie at Colfax, would operate the tie line load controller, which would change the Colfax generation to return the tie line load to its original value. This would further increase the load change through the phase shifter after a brief interval to about 1,500 kw, corresponding to a transfer between the adjacent systems only. If the regulator steps were two degree each, the kilowatt change per step would be 3,000, which is not considered excessive, and is less than the normal power oscillations which occur continuously.

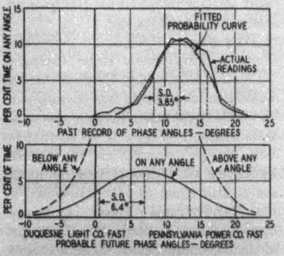


Figure 7. Natural phase-angle variation between Duquesne Light Company and Pennsylvania Power Company at Valley (no load on tie) Monday to Friday inclusive—8:00 a.m. to 8:00 p.m.

Voltage Regulating Requirements

It is also necessary to provide an adequate range of voltage control in the regulating equipment. The normal voltage on the Duquesne bus at Valley is 63–65 kv. With a load transfer to the Pennsylvania Power Company of 20,000 kw at 0.8 power factor, the Duquesne bus would have a voltage regulation of 4.2 per cent and the regulator would, therefore, have to raise the voltage from approximately 61 kv to the required value of 71 kv, or 16 per cent net boost.

The normal voltage on the Pennsylvania Power bus at Valley is 65–68 kv. When supplying 40,000 kva at 0.9 power factor to the Duquesne Light Company, the voltage would be 60.5 kv. The desired voltage at Valley under this condition is 65.5, or a net boost of eight per cent (buck toward Pennsylvania Power Company).

Description of New Regulator

The single-line diagram on figure 8 shows the schematic arrangement of con-

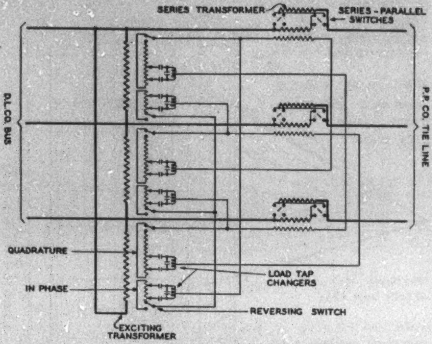


Figure 9. 66-kv regulating equipment—three-phase wiring diagram

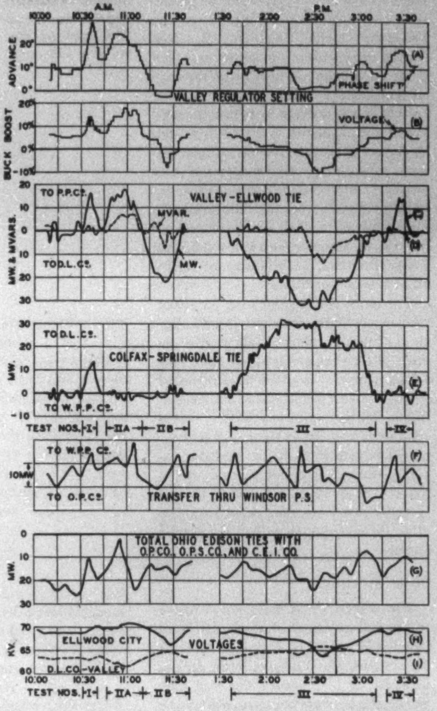


Figure 10. Results of load shifting tests

nections, and figure 9 shows the simplified wiring diagram of the new phase-shifting transformer. The essential parts of this equipment are:

(*a*) *Exciting Transformer.* A 65-kv delta primary on a three-phase core, with two secondary windings, one for quadrature voltage and the other for in-phase voltage.

(*b*) *Series Transformer.* A three-phase core with one primary supplied from the exciting transformer and double secondary for 20,000 kva (series) or 40,000 kva (parallel) in series with the line. The change between the series and parallel connections is made with a hand wheel that can be operated from the ground level. This change cannot be made under load and, therefore, requires a five-minute interruption.

(*c*) *Two Load Tap Changers.* For quadrature and in-phase voltages, each with 33 steps, 16 on either side of neutral.

(*d*) *Two Preventive Autotransformers* for tap changing under load.

(*e*) *Fans* for increased rating. Rating 15,000/30,000 kva self-cooled and 20,000/40,000 with fans.

The voltage diagram on figure 8 shows the range of phase shift and voltage control available. In order to secure the net shift of 28 degrees on the 20,000-kva connection, a quadrature voltage of 67.5 per cent was needed. This would give 33.75 per cent quadrature voltage on the parallel connection. In order to utilize this and obtain an eight per cent boost when feeding toward the Duquesne Light Company (eight per cent buck toward the Pennsylvania Power Company) a 16 per cent buck was required as shown on the diagram. This determined the in-phase voltage required, although the resulting 32 per cent on the series connection is not all needed.

The quadrature tap changer can be made to operate automatically from a watt-meter in the tie line, in order to hold the load within any desired range.

Results of Tests on Completed Installation

Arrangements were made to perform some load-shifting tests as soon as the regulator was available for service, in order to verify its ability to perform the functions for which it was designed. The load charts reproduced on figure 10 show the power interchange over the tie lines between the various systems making up the loop. The tests may be described briefly as follows:

Test I—20,000-Kva Connection, Power to Pennsylvania Power Company. With the automatic load control on the Colfax-Springdale tie inoperative, 15,000 kw were sent to the Pennsylvania Power Company by the phase-shifter control. The 15,000 kw flowed around the entire loop with practically no effect on generator output. Under similar conditions, during a previous test (before the phase shifter was installed) it had been impossible even by adjusting generator output and switching, to transfer any power directly from Valley to Ellwood substations.

The power flow per degree of phase shift was found to be 790 kw per degree. This is higher than the theoretical average value of 635 kw per degree due possibly to some unlooked for generator adjustments around the loop.

Test II-A—20,000-Kva Connection, Power to Pennsylvania Power Company. This was similar to test I, except that the automatic load control was operating on the Colfax-Springdale tie as it was in all tests except I. Figure 10 shows that no load change occurred on the Colfax-Springdale tie, Colfax picking up 15,000 kw of load equal to that transferred to the Pennsylvania Power Company. Curve *G*, however, shows that a large part of the 15,000 kw passed through Canton and South Akron and to the neighboring companies on the west.

The load shift at Valley proved to be 1,220 kw per degree, instead of 1,500 kw per degree, as had been calculated for a transfer from Colfax to Toronto, obviously because the load picked up at Colfax was dropped by power stations beyond the Ohio Edison Company system.

The 15,000 kw were transferred on regulator quadrature voltage tap *A*-28. Two more steps to tap *A*-30 would have increased the load to about 20,000 kw. The three remaining steps would have transferred the 20,000 kw even if the Pennsylvania bus voltage had been 18 degrees ahead of the Duquesne bus voltage (tie open) instead of 12 degrees as it was during this test. Furthermore, if all the load decrease had been made at the Ohio Edison Company plants. there is no doubt that the 20,000 kw transfer could have been made in the face of a natural phase difference of 20 degrees, thus demonstrating that the phase-angle range provided is neither too small nor unnecessarily large to accomplish the established requirements.

Test II-B—20,000-Kva Connection, Power to Duquesne Light Company. About 20,000 kw were sent from the Pennsylvania Power Company to the Duquesne Light Company. In this case Colfax dropped 20,000 kw and most of it was picked up on the Ohio Edison Company plants, only about 5,000 kw coming through its ties to the west. The power shift in this case proved to be 1,530 kw per degree, which corresponds very closely with the predicted figure of 1,500 kw per degree for transfer between Colfax and Toronto. The method of analyzing this part of the problem is thus fully substantiated.

On this test, reactive power was transferred in order to adjust the power factor to the desired value. Each two per cent in-phase step transferred about 1,700 kilovars. Each 4.2 per cent quadrature step transfers about 3,000 kw between Duquesne Light Company and Ohio Edison Company. The greater effectiveness of the in-phase voltage is due to the fact that the only impedance to be overcome is between points where the voltage (magnitude) is held constant, which points are closer together than the generating stations between which power is being transferred. Synchronous motors and condensers also tend to maintain the voltage magnitude, but not the phase relation.

Test III—40,000-Kva Connection, Power to Duquesne Light Company. This test was run to demonstrate the ease with which desired amounts of power may be obtained by the Duquesne Light Company from both the West Penn Company and the Pennsylvania Power Company. A total of 64,000 kw was obtained, divided nearly equally between the two interconnections. With system conditions as they were at the time, very little phase shift was needed, but during emergencies such as would require this large amount of power, the phase-angle conditions would vary over a considerable range and most of the 15 degrees available would be needed.

Test IV—40,000-Kva Connection, Power to Pennsylvania Power Company. This test was performed to see how much power could be sent to the Pennsylvania Power Company with the limited range available on the parallel connection and the natural angle existing at the time (12 degrees). About 14,000 kw were transferred, using the full range of 18 degrees (no load). This average of 2,300 kw per degree is greater than on the series connection because the impedance of the regulator is only one-quarter as great. This reduces the total impedance between Colfax and Toronto by 33 per cent, which corresponds closely with the relation between 1,535 kw per degree and 2,300 kw per degree.

Conclusions

1. A regulating transformer to control phase angle as well as voltage magnitude is necessary in order to utilize successfully the tie line between Valley substation and Ellwood City.

2. The methods outlined herein for analyzing the characteristics of the interconnected system are entirely practical and accurate for determining the size and range of the in-phase and quadrature voltage control required.

3. The phase-shifting and voltage-regulating equipment, as installed, performed satisfactorily and gave positive and independent control of the kilowatt and reactive flow through the tie line under the desired conditions.

4. The use of a series parallel (20,000–40,000 kva) arrangement permitted a more economical solution of this problem where special load carrying and phase-shifting requirements had to be met.

5. The method outlined herein for expressing the future phase-angle variations by use of the probability theory is believed to be applicable to a variety of other problems of a similar nature.

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2. Interconnected Power Systems, Philip Sporn. Electrical Engineering, January 1938, page 16.

3. Economic Statistics (a book), Crum and Patton. A. W. Shaw Company, Chicago and New York City, page 199.

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Discussion

Robert Treat (General Electric Company, Schenectady, N. Y.): The considerable number of studies made, and of inquiries received for quotations on phase-shifting transformers over a period of years indicates that this subject has intrigued the imaginations of many utility engineers. The small percentage of such inquiries which have resulted in orders, however, indicates that the cost of solving the problem by this means has generally been considered disproportionate to the expected benefits. It would appear that most of the problems have been solved in a less expensive, perhaps also in a less satisfactory, manner.

This paper is of interest in that it describes one case in which a really competent and penetrating analysis showed the installation of a phase-shifting transformer to be economically justified. Operating experience appears to have confirmed the wisdom of the decision to proceed with the installation.

The information in this paper will be of great value to other engineers who are confronted with similar problems.

Philip Sporn (American Gas and Electric Service Corporation, New York, N. Y.): The type of transmission loop, described by Messrs. Lyman and North in this paper, is distinctly a product of the interconnection idea, as carried out by the various companies involved in this loop, including particularly the company with which I am associated. This problem of loop operation and inability to maintain complete control of power flow over individual portions of such loops is one that we are confronted with continually.

The authors have described a very thorough method of analysis of their particular problem, from the standpoint of both computations and field observations. The graphical representation of the actual and equivalent loop reactances as shown in figure 2 of the authors' paper is novel and helpful, and the application of probability theory to the selection of the range of control which can be economically justified, is also a sensible and practical means of approach. In fact, starting out on the premise that fairly complete control of power flow is necessary on an interconnection of this type in order to obtain the advantages of the interconnection, the authors have attacked and solved the problem in a sound and clever manner.

However, in the light of the experience we have gained in operating interconnected systems, I should like to raise a question as to the premise on which this entire study was based. At the outset it is stated that the interconnection is to be used only in case of an unusual emergency situation to provide an emergency source of power. I should like to ask why, as an emergency proposition, it would not have been possible to have taken care of the emergency by simply closing the switch at Valley, regardless of the particular phase-angle situation

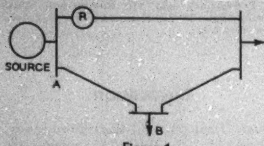

Figure 1

existing at the time, and allowing the power to flow in whatever direction and amount it chooses. Even though power did not flow in the direction and amount desired at that time, nevertheless, continuity of service would be maintained, and voltages would be upheld by the necessary exchange of reactive kilovolt-amperes, independent from the flow of power. After all, it is generally true in networks that power will flow over the path of least resistance, so that the simple closing of the switch without any attempt to force power to flow in any particular direction will generally result in the least over-all system losses from generating stations to loads. Of course, there may exist limitations in some cases, such as small capacity transformers in the loop which might be overloaded, but from the values of circulating load mentioned in the paper, it is my impression that this would not be the case on this particular loop.

For a permanent operating setup, the closing of a tie such as this, without phase-angle control, would undoubtedly introduce complications into the billing of the various groups physically connected in the loop. While this may appear insurmountable for permanent operation, it has been our experience that in most cases a solution is possible and that a great deal of effort to work out the problem on that basis is justified before resorting to phase-angle control. In other words, we believe that it is far better to let the power flow naturally and carry out such adjustment as may be called for in the billing arrangements rather than to resort to devices like phase-angle-control equipment which not only are a major expense in themselves but obviously decrease the reliability of the interconnections and in many cases serve only as a "bottle neck." In the particular case described, since it was established purely for taking care of emergency situations and was not intended for permanent operation, I would more than ever question whether the phase-angle-control solution is economically sound.

T. G. LeClair (Commonwealth Edison Company, Chicago, Ill.): The phase-shifter application described in the paper is an unusual one because the phase-angle range is so much greater than would normally be expected.

The discussion by Mr. Sporn might infer that phase shifters are not economically sound because in a closed network the power will follow the path of minimum reactance and, theoretically, the path of minimum loss. However, there are two conditions where phase shifters are of great economic value and where the general statement does not apply. One is the condition of two parallel lines in which the ratio of carrying capacity to impedance is not the same. For example, in Chicago we have two 66,000-volt cable circuits with nearly equal impedance, but with one having nearly twice the load carrying capacity of the other. A phase shifter is essential for this application.

The other necessary application for phase shifters is in large loops involving more than one company where the power flow must be controlled to meet arbitrary contractual agreements. In some cases, where one party to an interconnection is not co-operative, it may be cheaper to install a phase shifter than to pay penalties under a contract for not having maintained the correct line loading.

S. B. Griscom (nonmember; Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The authors are to be commended for the thoroughness with which they analyzed the problem, and for their clarity in presenting the steps by which the required equipment ratings and characteristics were determined. The use of the probability theory to insure adequate but not excessive phase-angle range is particularly noteworthy.

Referring to figure 2, it is interesting to note that a very considerable portion of the total loop reactance is attributable to the bus reactors and voltage transformations at Colfax, Brunots Island, and Windsor. This would offhand lead to the conclusion that these connections are responsible for a large part of the required phase-angle shift. However, a review of table I shows that this is not the case, as under the expected loading conditions, the transfer of power through these elements is comparatively small. It is also very interesting to note that of the total quadrature range of the regulator, the major part is required to compensate for the cumulative phase angles generated around the loop, as compared to the nominal phase angles tabulated under "Power Transfer," required to shift 20,000 kw between various points.

The writer would like the authors' opinion as to the probable needs of the central station industry for combined phase-angle and voltage control for fields now commonly using step-type voltage regulators of nominal capacity, say 50- to 500-kva regulator size. For example, it seems to the writer that there must be frequent cases where load is fed as shown by figure 1 of this discussion. Load *C* is fed by two lines, one of which is tapped at *B*. Under these circumstances, the limiting load that can be delivered to *C* is reached when the line from *A* to *B* is fully loaded, whereas the direct line from *A* to *C* may be well under its maximum rating. A regulator at *R* will serve to force the necessary increment of load over the direct line to *C*, relieving the load on the line from *A* to *B*, and thus increasing the total load that may be served at *C*. In the simple case as illustrated, probably the emergency condition of one line out of service fixes the ultimate limit, but with other standby feeds to *C*, it would seem that dividing the load between the principal circuits in proportion to their capacity, a small investment in regulators would permit fuller utilization of a larger investment in lines.

It is possible to combine both phase-shift and voltage-ratio changes with a single regulating mechanism, provided that a fixed relation between them is permissible. In addition to the well-known 60-degree relationship, between the line and increment voltage, any other desired value may be obtained by proper design of the regulator.

The authors were able to make extensive tests on the interconnected system to obtain confirming tests of these calculations. These were supplemented by calculations projected to cover expected future conditions. While not particularly complicated, such calculations may prove quite arduous, particularly if a number of the lines have considerable resistance, and there are a large number of possible future conditions to be examined. In such cases, the work would be very much facilitated for the use of the a-c calculating board, such as the one used by the company with which the writer is connected. This board has been used on over 200 system studies since its installation eight years ago.

W. J. Lyman and J. R. North: The authors are indeed grateful for the interest shown in this subject by the discussions presented. It is worthy of note that no exceptions were taken to the method of approach and the manner in which the problem was solved. Particular notice was taken of the favorable reception accorded the use of probability analysis. This seems to reinforce other indications that the probability theory is coming to be more generally accepted as a method of approach with certain types of power system problems.

Mr. Griscom raised the question of the probable use of phase shifters in the future to balance loads on parallel circuits. The authors feel that this is a distinct possibility, and as described in Mr. LeClair's very interesting comments, the idea has already been applied. It would seem that, particularly in metropolitan areas where additional rights-of-way are becoming more difficult to obtain, there may be more pressure brought to bear to secure the maximum possible use of existing facilities. The example cited by Mr. Griscom is quite common, and in a few cases at least, reactors have been used to secure better load balance. However, where voltage regulation is a problem, the phase shifter might be justified by the amount of voltage control which could be obtained. In our opinion, such applications would, in general, involve larger sizes than those mentioned by Mr. Griscom. Mr. Treat's comments on the inquiries received on phase-shifting equipment may bear out the contention that more interest is being shown in this method of control and that it is at least being considered for the solution of a wider variety of problems.

Mr. Sporn's discussion of the economic aspects of the problem is quite pertinent. Naturally, the expenditure must be justified in one way or another. In this particular case it was largely as a result of the 1936 flood that it was decided to increase the interconnection capacity. Due to the great distances and the number of parties involved in the transmission loop it would not be possible to use the full rated capacity of both large interconnections without some form of power control. The phase shifter thus actually increases the effective interconnection capacity. Inasmuch as a new regulating transformer was required in any event to increase the tie line capacity, the phase-angle control was obtained for a reasonable incremental investment.

Field Testing of Bushings and Transformer Insulation by the Power-Factor Method

By I. W. GROSS
ASSOCIATE AIEE

I. Introduction

THE IMPORTANCE of maintaining the insulation on all parts of an electric system in good operating condition is fundamental with any power company today, particularly on such equipment as high- and medium-voltage bushings and transformers. The maintenance of such insulation requires some sort of periodic test to determine its condition, an exact location of faults, and requirements for reconditioning or replacing faulty parts.

Until the introduction of the power-factor method of insulation testing, the methods available were generally unreliable, uncertain, or of a type which destroyed the insulation if faulty or perhaps weakened it by excess voltage, even if normally good. The introduction by the Doble Engineering Company of a power-factor test set for field use some eight years ago made possible the practical testing of bushing insulation under service conditions. The further development of this test set, together with extension of its application to testing transformers and other equipment, and refinement in measuring technique and interpreting of results, has produced a highly satisfactory test device for determining the condition of high-voltage insulation in the field.

This successful development of field power-factor testing of insulation to its present state has been possible only through the closest co-operation between the manufacturer and users of the test set in the interchange of ideas, field records and technical data. Without such a procedure, progress would have been much impeded and the present usefulness of power-factor test application restricted. The equipment manufacturer has also lent his aid to this work, and in fact today at least two bushing manufacturers are power-factor testing new bushings as a routine test procedure, and one manufacturer has started power-factor testing high-voltage transformer windings.

Although the goal of complete elimination of all bushing, oil circuit breaker, and transformer failures in service probably never will be reached, the power-factor test method now available and in use by a large number of power companies has proved to be a useful tool in detecting faulty insulation prior to equipment failure in service. Reduced failures of equipment in service, decrease in cost of maintaining such equipment, and better service supplied by electric systems to their customers are the objectives of the periodic testing program of field insulation testing which has been in effect for the past five years on the system of the company with which the author is associated.

Figure 1. Light delivery truck, specially fitted to carry portable insulation test set and all accessories. A field test is being made on a 33-kv oil circuit breaker

II. Scope of This Paper

Power-factor testing of oil circuit breakers and their bushing insulation was tried out on the properties of the American Gas and Electric Company in 1929. The results were so satisfactory that a periodic test schedule was started on the same equipment in 1932 and extended to transformer insulation in 1934, and gradually to other miscellaneous equipment as the test work progressed. The results of much of this earlier work have been previously reported[1,3] in 1934.

Herein will be given:

1. An outline of our present test procedure.
2. General description of test methods used.
3. Our present test standards for judging insulation condition.
4. Summary of fault conditions found in bushings, power and instrument transformers, and oil circuit breakers, for the years 1932 to October 1937, inclusive, including some records on cork gaskets.
5. Discussion of results obtained.
6. Conclusions based on the results of some $5^{1}/_{2}$ years of actual field tests by the power factor method on equipment on the American Gas and Electric Company system.

It is outside the scope of this paper to discuss in detail bushing design as contributory to service faults, or to describe procedure for servicing equipment found faulty by test.

III. General Test Procedure

The test set, which has previously been described,[1] consists essentially of a 10,000-volt test transformer with means for adjusting the voltage and an instrument case containing wattmeter, voltmeter, and ammeter for obtaining readings of watts, voltage, and current. The power factor is readily obtainable in most cases

Figure 2. Rear view of test car showing instrument cases on top shelf, test transformer with control panel, oil test cup, and tool box on second shelf; and high-voltage test cable coiled in cylindrical pan on car floor

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1. For all numbered references, see list at end of paper.

by one slide-rule setting. Other miscellaneous equipment, such as standard condenser, oil cup, test leads, guard circuit, safety switches, etc., comprise the balance of the equipment. All major test equipment is specially designed, and carefully shielded—an important feature required for use in the field.

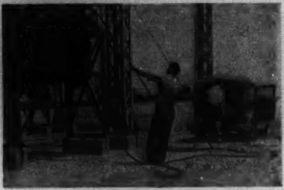

Figure 3. Field test of oil circuit breaker. The test man reads his instruments without removing them or the test transformer from the test car. The other man, standing on the ground, connects the test lead to the bushing with an insulated switch stick

In actual operation, the test equipment is carried in a light delivery truck as shown in figure 1. A view of the rear of the truck, with full door opening, is shown in figure 2. The interior of the truck has been specially fitted with shelves and partitions to hold the test equipment, not only rigidly in place during transit, but without being moved when testing is in progress. Figure 2 shows the instrument case on the top shelf, the test transformer, oil cup, and tool box on the middle shelf, and the 60-foot test lead coiled in a cylindrical container on the floor. A typical field test on an oil circuit breaker is shown in progress in figure 3.

The test schedule was first set up on a yearly basis. As the test work was extended each year to other equipment not contemplated when the schedule was first arranged, equipment has been tested every 18 months to two years instead of yearly as initially planned. At the present time we have three test sets in continuous use throughout the year.

All fault conditions found are immediately brought to the attention of the person in direct charge of maintenance, and major faults are recommended for immediate correction. Minor faults are pointed out, but are usually left until regular maintenance periods, or until next periodic test, according to the apparent importance of the fault and service conditions.

Copies of all field test records giving classification of faults according to severity, and recommending servicing procedure are promptly distributed to the person in charge of operation, to the Doble Engineering Company, and to the engineering department of the American Gas and Electric Service Corporation. Although the field-test men make the initial recommendations for servicing faulty equipment, subsequent triple review of the test data and recommendations are possible by the above procedure. Be it said to the credit of the test men, rarely has any exception been taken to their initial recommendations given at the time the tests were made in the field.

From the very start of this test program it was believed that the best results would be obtained by placing the test set in the hands of a technically qualified man, specially trained in this type of work, making tests continuously, and held highly responsible for the progress made and the recommendations given. Our experience bears out this belief. Such a man often detects fault conditions in the field which can easily be overlooked by adherence to an arbitrary set of power factor, watt loss, current, or other limits. Such limits are, however, essential as a basis of evaluating faults, although supplemental tests or inspections have frequently detected faults which would not have otherwise been found.

IV. Detailed Test Methods

In applying test voltage 10 kv is used unless this is above the safe limit of the equipment being tested or unless the charging current exceeds the range of the measuring instruments. If such conditions exist, the applied voltage is reduced to 5.0, 2.5, or 1 kv as required.

Oil Circuit Breakers

The method of testing oil circuit breakers has previously[1] been described in detail. It consists essentially in testing each bushing separately, with the breaker open, and each tank separately (two bushings plus internal cross connection) with the breaker closed. It is usually unnecessary to disconnect the external leads to the bushing unless a fault condition is found. Evaluation of watts, power factor, and in some cases current (See V—"Classification of Faults" and table III) enable the classification of the insulation condition.

Transformers

Testing the insulation of transformers by the power-factor method differs somewhat from the procedure followed in oil circuit breakers, first because physical separation of bushing from the internal insulation (winding) is usually not possible without opening the transformer tank (which involves extra labor), and second because it is often desirable to test the insulation of both windings of a transformer as well as their interwinding insulation. Also, another important feature that has to be considered is the relative large magnitude of watts loss and current in winding insulation as compared with those in bushings. This can easily obscure the fault in a transformer bushing unless it is disconnected and tested separately, or other means of test and analysis are used.

With instrument transformers such special test means and analysis have been

Table I. Test Procedure on Instrument Transformers
(See Figure 4)

Test Number	Meter Reading	Connections: Test To	Connections: To Ground	Connections: To Shield
1	$W_H + W_B + H_1 + H_2$	$H_1 + H_2$	L_1 L_2	
2	$H_1 + X^1$	H_1	L_1 or L_2	H_2
3	$H_2 + X^1$	H_2	L_1 or L_2	H_1

Table II. Test Procedure on Power Transformers
(See Figure 4)

Test Number	Meter Reading	Connections: Test To	Connections: To Ground	Connections: To Shield
1	$W_H + W_B$	H_1 and H_2	L_1 and L_2	
2	W_H	H_1 and H_2		L_1 and L_2
3	$W_L + W_B$	L_1 and L_2	H_1 and H_2	
4	W_L	L_1 and L_2		H_1 and H_2

successfully used to segregate winding faults from those in bushings. The procedure we have used is as follows: Referring to figure 4*A*, which represents a typical transformer, test voltage is applied simultaneously to the high voltage terminals H_1 and H_2, one terminal only of the low-voltage winding being grounded. A second test is now made with voltage on H_1, and H_2 connected to shield, other connections remaining the same. The third test is a repeat of test number two, with connections H_1 and H_2 reversed. The first test shows the over-all insulation condition of the transformer; the second and third tests (called cross checks) give, by comparison, the condition of each bushing. These three tests are outlined in table I. Where serious faults are indicated the bushings are separated from the winding and all tested separately. The insulation of the low voltage coil to ground is not tested as practical considerations have not yet indicated this is necessary.

In testing power transformers the loss in secondary winding insulation to ground must be measured and all winding losses evaluated. This has been done by the four tests outlined in table II. By proper combination of measured values the losses in, and power factor of, all three insulation paths can be obtained. The determination of the condition of the bushing insulation is not quite as simple here as with the instrument transformer on account of the relatively higher loss in the high-capacitance transformer windings. Several methods are available and have been used for determining the condition of the bushing alone without disconnecting it from the winding.

First, a comparison of the relative power factors of high, low, and inter-winding insulation which shows abnormally high loss insulation to ground usually indicates one or more faulty bushings connected to high-loss winding. Second, if the bushing is of the draw-lead type, the lead can be lowered inside the transformer tank by means of an insulating rod, and the bushing tested separately. A third method (hot-collar test) described below has also been successfully applied as an individual bushing test in some cases.

Hot-Collar Bushing Test

This test, which has recently come into use, is applied to bushings only, and consists of grounding the normally "hot" terminal of the bushing and applying the 10-kv test potential successively along the bushing at recesses between porcelain corrugations, and measuring the watts loss. Contact with the porcelain is made either through tinfoil wrapped around the entire porcelain circumference, or through a circular flat spring clamp which clips between the porcelain watersheds. This clip is energized on test, hence the name hot collar.

By this test a considerable number of faulty bushings have been detected, which upon examination were found to have such faults as cracks in the porcelain, porous porcelain, water (as much as a one-half cupful) inside the bushing at the top, and distributed moisture in the interior of the bushing. The test has always shown faults detected by the power factor test, and also faults such as the above, some of which the power factor test had not picked out.

Solid Insulation

Solid insulation such as wooden switch sticks, hot line sticks, and fibrous insulation such as insulating cylinders in bushings are being tested regularly by the power-factor test set. The standard test used is 10 kv between tinfoil or other metallic wrapping placed at three-inch intervals along the stick or cylinder, and insulation rating based on watts loss. While the technical justification why such a test is sound, is not at first clear, involving as it does creepage paths along the stressed surface of the insulation as well as the direct insulation path through it, it is a fact, however, that it is a practical and effective method of locating insulation defects in wood members and fibrous insulation.

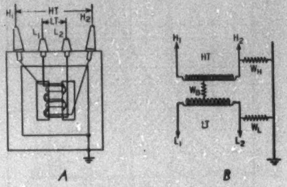

Figure 4. Insulation diagram of transformer

A—Typical transformer
B—Equivalent insulation diagram
HT—High-voltage side
LT—Low-voltage side
H_1 and H_2—High-voltage bushings and losses
L_1 and L_2—Low-voltage bushings and losses
W_H—Watts loss, high-voltage insulation to ground
W_L—Watts loss, low-voltage insulation to ground
W_B—Watts loss, insulation between high-voltage and low-voltage
X—Watts loss, $W_{B'} + W_{H'}$, where prime (′) equals the loss with one end of high-voltage grounded

Allocation of Trouble

Tests of this type are extremely helpful in definitely locating trouble, determining the cause, and applying corrective measures, once a piece of equipment such as a bushing or transformer has been found defective. The mere knowledge obtained by a test that shows equipment is faulty without allocating the trouble is only one step in maintaining electrical equipment in good operating condition. The tracing of the trouble to its source is just as important as finding a faulty piece of equipment, for the type and location of the trouble determines the nature of servicing required, even at times to the desirability of scrapping parts of or the entire equipment. This insight into insulation, the power-factor

Table III. Basic Rating for Insulation Quality From Power-Factor Field Tests
(Corrected to 20 Degrees Centigrade)

Apparatus	Type	By Per Cent Power Factor			Remarks
		Good (G)	Deteriorated (D)	Remove (R)	
Bushings	Comp. filled	Below 4.0	4.0 to 6	Above 6	
	Oil filled	Below 3.5	3.5 to 5	Above 5	Fibrous barrier
	Oil filled	Below 4.0	4.0 to 6	Above 6	Porcelain barrier
	Condenser	Below 3.5	3.5 to 5	Above 5	1-piece porcelain
	Condenser	Below 3.5	3.5 to 6	Above 6	Multi. shed
	Semicond	Below 4.0	4.0 to 7	Above 7	Also bulk type
Transformer-winding	C. T	Below 3.5	3.5 to 6	Above 6	With bushings
	C. T	Below 3.5	3.5 to 7	Above 7	Winding only
	Power and P. T	Below 3.5	3.5 to 5	Above 5	
		By Watts Loss			
Oil circuit breakers	Most	Below 0.2	0.2 to 0.5	Above 0.5	Loss in tank
	33 Kv	Below 0.15	0.15 to 0.3	Above 0.3	Loss in tank
Herkolite cylinder	3-inch	Below 0.1	0.1 to 0.5	Above 0.5	Between bands
Wood bushings	3-inch	Below 0.1	0.1 to 0.5	Above 0.5	Between bands
	Comp. and cond.	Below 0.2	0.2 to 0.5	Above 0.5	Hot-collar test

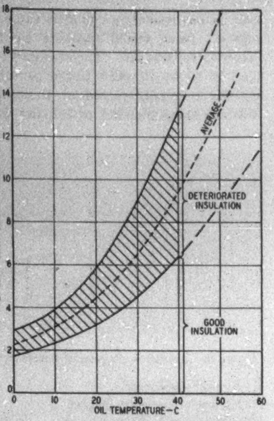

Figure 5. Power factor versus temperature correction curve for transformer winding (from field tests)

test set, with its direct readings of current and watts has been able to do with a high degree of certainty.

Table V. Power-Transformer-Bushing Faults Segregated as to Type; Summary of Four Years of Field Testing
(January 1934 to October 1937)

Type of Bushing*	Test	Number of Bushings Tested	Defective		Remove		Average—3 Tests	
			Number	Per Cent	Number	Per Cent	Per Cent D	Per Cent R
A	First	1,182	31	2.6	46	3.9	2.5	2.8
	Second	772	15	1.9	12	1.6		
	Third	134	7	5.2	0	..		
B	First	875	77	8.8	58	6.6	8.7	5.0
	Second	447	28	6.3	8	1.8		
	Third	128	22	17.2	9	7.0		
C	First	2,515	100	4.0	141	5.6	4.0	5.5
	Second	1,399	52	3.7	76	5.4		
	Third	175	10	5.7	6	3.4		
D	First	2,132	22	1.0	26	1.2	0.8	0.9
	Second	1,071	5	0.5	6	0.6		
	Third	252	0		0			
E	First	1,025	19	1.9	11	1.1	2.0	1.3
	Second	722	13	1.8	11	1.5		
	Third	145	5	3.4	3	2.1		
Total	First	7,729	249	3.2	282	3.7	3.1	3.2
	Second	4,411	113	2.6	113	2.6		
	Third	834	44	5.3	18	2.2		

* Types of bushings refer to different basic principles of bushing design.

V. Classification of Faults

From the time the insulation in a piece of equipment leaves the factory in good condition until the end of its useful life (usually a period of years) it undergoes deterioration to a degree dependent on many factors such as duty in service, the elements, field servicing procedure, and faulty design. In the field testing of such insulation, by the methods described above, one of the immediate objectives is to determine, if possible, when the deterioration has become serious enough to warrant attention. The fault may be just starting and not serious but can be economically cured without allowing it to develop into a service failure. Again it may be so serious that immediate attention or even prompt removal from service is necessary.

Following this line of reasoning we have set up three classes of insulation as determined from test results. These are:

1. Good (*G*). No attention required.
2. Deteriorated (*D*). Not as good as new but not requiring special servicing until the next periodic test is made to check its condition.
3. Remove (*R*). Remove from service and recondition or scrap as conditions warrant.

Table IV. Bushing Faults; Summary of $5^1/_2$ Years of Field Testing
(May 1932 to October 1937)

Test	Number of Bushings Tested	Defective		Remove	
		Number	Per Cent	Number	Per Cent
A. Power-Transformer Bushings (All kv ratings)					
First	7,729	249	3.3	282	3.7
Second	4,411	113	2.5	113	2.5
Third	834	44	5.3	18	2.2
Total	12,974	406	3.1 average	413	3.2 average
B. Power-Transformer Bushings (All kv ratings, excluding 15-kv and below)					
First	4,152	198	4.8	205	4.9
Second	2,477	84	3.4	57	2.3
Third	443	35	7.9	15	3.4
Total	7,072	317	4.5 average	277	3.9 average
C. Instrument-Transformer Bushings					
First	2,581	74	2.9	97	3.8
Second	2,145	70	3.3	73	3.4
Third	724	43	5.9	41	5.7
Total	5,450	187	3.4 average	211	3.9 average
D. Oil-Circuit-Breaker Bushings					
First	8,076	329	4.1	187	2.3
Second	7,302	287	3.9	114	1.6
Third	6,564	401	6.1	164	2.5
Fourth	1,758	83	4.7	38	2.2
Total	23,700	1,100	4.7 average	503	2.1 average
E. Spare Bushings (All kv ratings and types)					
First	1,084	76	7.0	86	7.9
Second	1,419	168	11.8	117	8.3
Third	1,089	143	13.1	110	10.1
Total	3,592	387	10.6 average	313	8.8 average

Conforming to this procedure for rating insulation we have set up for use in our company "the basic rating for insulation quality" (table III) for the different types of equipment tested. Power-factor limits are given in the upper part of the table and watt-loss values in the lower part. In general, the power factor of good (G) insulation is shown 3.5 to 4% or less; deteriorated (*D*) insulation 3.5 to seven per cent; and remove (*R*) insulation above five to seven per cent.

Similarly, by the watts loss check the corresponding values range as follows: (*G*) 0.1 to 0.2 watt; (*D*) 0.1 to 0.5 watt; and (*R*) above 0.3 to 0.5 watt, again depending on the type of equipment tested.

As the power factor of insulation changes considerably with temperature the above power factor values are corrected to 20 degrees centigrade. A typical temperature power-factor correction curve for transformer insulation is shown in figure 5. The shaded area represents *D* insulation at various temperatures, which permits of ready allocation of insulation-test results obtained in the field. This set of curves was obtained from a

Table VI. Power-Transformer-Bushing Faults Segregated as to Voltage Rating, Summary of Four Years of Field Testing

(January 1934 to October 1937)

Bushing Kv	Test	Number of Bushings Tested	Defective		Remove		Average for 3 Tests	
			Number	Per Cent	Number	Per Cent	Per Cent D	Per Cent R
132	First	278	2	0.7	2	0.7	1.04	0.35
	Second	209	0		0			
	Third	88	4	4.5	0			
88	First	108	10	9.3	9	8.3	7.7	6.6
	Second	58	3	5.2	2	3.5		
	Third	0						
66	First	719	14	1.9	22	3.1	1.8	2.2
	Second	529	8	1.5	5	0.9		
	Third	0						
44	First	744	80	10.8	64	8.6	10.7	8.0
	Second	156	16	10.2	8	5.1		
	Third	0						
33	First	1,490	63	4.2	78	5.2	4.2	4.5
	Second	856	17	2.0	28	3.3		
	Third	336	31	9.2	15	4.5		
22	First	909	29	3.2	29	3.2	4.4	3.0
	Second	569	40	6.3	14	2.5		
	Third	0						
15 and less	First	3,477	51	1.5	78	2.2	1.5	2.3
	Second	2,034	29	1.4	56	2.7		
	Third	301	9	2.3	3	0.8		
Total	First	7,725	249	3.2	282	3.7	3.1	3.2
	Second	4,411	113	2.5	113	2.5		
	Third	815	44	5.4	18	2.2		

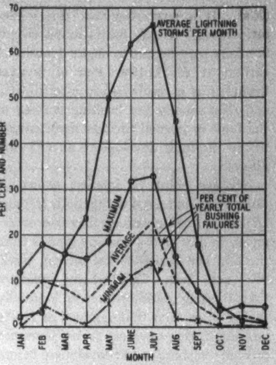

Figure 6. Bushing failures versus lightning frequency, 1932–37

number of tests on actual transformers in service tested at several different oil temperatures, and represents the envelope of power factors of transformers rated (*D*).

In applying in the field the basic insulation ratings given in table III, considerable latitude is given the field-test engineer in assigning ratings where unusual or climatic conditions warrant. Important equipment, or comparatively wide variation in test values between similar pieces of equipment cause him, in some cases, to lower the limits given in the table. Again moist weather conditions and the like will sometimes cause him to boost the tabulated values slightly. Since the test engineer has been specially trained for this work, and is thoroughly acquainted with the problem, we believe the latitude permitted him in applying ratings is more than justified, and is far better than hewing close to an inflexible set of limits. Actual experience has borne out this policy.

VI. Test Results

The field test results summarized and discussed below include some $5^1/_2$ years of testing oil circuit breakers and four years of testing on transformers. Most of the breakers have had three periodic tests and some four tests; the instrument transformers two such tests, and the third is now being made; and the power transformers have had one test, some two, and a few three tests. In the following tables the designation "first," "second," and "third" test refer to the above periodic routine tests. For purposes of clarity the data have been generally segregated into and discussed under three groups: bushings, transformer windings, and internal faults for oil circuit breakers.

BUSHING FAULTS

The summary of faults found in making over 45,000 individual tests on bushings is given in table IV. Bushings in power transformers average, for the three periodic tests, 3.2 per cent rated for removal (*R*); in instrument transformers 3.9 per cent; in oil circuit breakers (for four tests) 2.1 per cent; and in spare bushings 8.8 per cent. Bushings rated defective (*D*), but not sufficiently bad to be removed from service average for the same conditions and locations 3.1 per cent, 3.4 per cent, 4.7 per cent, and 10.6 per cent, respectively. One point of interest in the foregoing figures is the fact that (*D*) and (*R*) faults are roughly of the same order of magnitude except for oil-circuit-breaker bushings where the (*D*) faults are slightly over twice the

Table VII. Power-Transformer-Winding Faults Segregated as to Kilovolts Rating, Summary of Four Years of Field Testing

(January 1934 to October 1937)

Transformer Kv	Test	Number of Transformers Tested	Defective		Remove		Average for 3 Tests	
			Number	Per Cent	Number	Per Cent	Per Cent D	Per Cent R
132	First	179	7	3.9	1	0.56	4.5	0.27
	Second	133	7	5.3	0	0		
	Third	65	3	4.6	0	0		
88	First	52	8	15.2	0	0	16.9	0
	Second	25	5	20.0	0	0		
	Third	0						
66	First	296	26	8.8	5	1.7	8.8	2.3
	Second	227	20	8.8	7	3.1		
	Third	0						
44	First	338	66	19.5	9	2.6	19.1	2 2
	Second	70	12	17.1	0	0		
	Third	0						
33	First	634	107	17.0	28	4.5	19.5	5.0
	Second	344	76	22.0	21	6.1		
	Third	131	34	26.0	7	5.3		
22	First	326	66	20.2	9	2.8	22.0	2.9
	Second	189	47	24.8	6	3.2		
	Third	0						
15 and below	First	83	11	13.3	3	3.6	14.5	3.2
	Second	39	6	15.4	1	2.6		
	Third	13	1	7.7	0	0		
Total	First	1,908	291	15.2	55	2.9	16.1	3.1
	Second	1,027	173	17.0	35	3.4		
	Third	209	39	18.6	7	3.4		

(*R*'s). It is suggested that this higher ratio of (*D*) to (*R*) faults in oil circuit breakers may be due to slight deposits of carbon at the bottom end of breaker bushings as a result of arc interruption, and that usually the bushing is freed of the fault in normal breaker maintenance before the fault develops to an (*R*) condition.

Another point of interest shown in table IV is that total faults, both (*D*) and (*R*), do not progressively increase or decrease in any of the bushing groups from the first test period to the second, third, or fourth test. This clearly indicates that bushing deterioration is gradually taking place, and that removal of faulty bushing during one test period does not preclude bushing faults developing before the next test period some $1^1/_2$ years or so later. In fact bushings rated (*R*) on the second, third, and fourth tests have often been found (*G*) or good on the previous test; all (*R*) bushings have not progressed from a previously found (*D*) fault.

The high percentage of faulty bushings in the spare group is due to two causes: first, the fact that some bushings known to be faulty and ready for the process of reservicing were included in this group; and second, normal storage methods generally used for spare bushings is not particularly helpful in retaining such bushings in first class conditions.

A further analysis of power transformer bushing faults according to bushing type is given in table V. The data represent some 13,000 tests on bushings. The (*D*) conditions range from 0.8 per cent to 8.7 per cent and the (*R*) faults from 0.9 per cent to 5.5 per cent on the average for all three periodic tests. Here again no well-defined tendency is shown for the faults to decrease, stay constant, or increase with each successive test, in any particular type of bushing. It should be definitely pointed out that the data presented in this table should not be construed or interpreted as a quality index of bushings manufactured today. It does indicate, however, the frequency and severity of faults on all bushings as a group on the system involved, and shows clearly where maintenance work should be concentrated to reduce bushing failures.

Table VIII. Internal Faults in Equipment
(May 1932 to October 1937)

Test	Number of Transformers Tested	Defective		Remove		Average for 3 Tests	
		Number	Per Cent	Number	Per Cent	Per Cent D	Per Cent R
Winding Faults—Power Transformers							
First	1,908	291	15.2	55	2.9	16.0	3.1
Second	1,027	173	17.0	35	3.4		
Third	209	39	18.6	7	3.4		
Winding Faults—Instrument Transformers							
First	1,502	451	30.1	113	7.5	35.0	7.8
Second	1,283	497	38.6	98	7.6		
Third	407	163	40.1	36	8.9		

Tank Faults—Oil Circuit Breakers

Test	Number OCB's Tested*	Defective		Remove		Average for 4 Tests	
		Number	Per Cent	Number	Per Cent	Per Cent D	Per Cent R
First	1,346	304	7.5	65	1.6	6.2	1.7
Second	1,217	178	4.9	57	1.6		
Third	1,097	210	6.4	67	2.1		
Fourth	293	41	4.7	10	1.1		

* Multiply by three to obtain number of tanks.

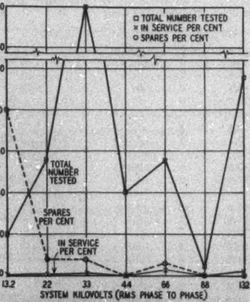

Figure 7. Effect of transformer kilovolts on power-transformer-winding faults (R) conservator and nonconservator type; 110 banks (440 transformers)

The power-transformer-bushing faults have further been segregated as to voltage class in table VI. The high-voltage 132-kv bushings show the lowest percentage of faults, the 66-kv and 13.2-kv class next lowest and the medium-voltage group the highest. The rather high percentage of faults shown for 88-kv bushings is due largely to the fact that most of these bushings are of old and obsolete designs.

Bushing Failures

It is unfortunate that accurate records are not available to show the decrease in bushing failures over the system before and after the test program was adopted. However, from the rather scattered records available from 1932 to 1937, it is interesting to note how closely the frequency of bushing failure and lightning storms coincide throughout the year. Such a correlation is shown in figure 6. Except for the first three months of the year failures and storms practically increase and decrease together. The relatively high rate of bushing failures in the first three months has been attributed to weather conditions wherein maximum temperature changes exist, frequently associated with the presence of moisture which is an ideal combination of circumstances to open up weak bushing gaskets and permit the entrance of moisture into the bushing. In fact the major number of bushing failures appear to result from the presence of moisture within the bushing.

Internal and Winding Faults

Faults found in windings in some 2,000 tests of power transformers are shown in summary form in table VII. The faulty conditions have been grouped under transformer voltage rating and further broken down to show the trend of faults from one test period to the next. Like the transformer bushings, the winding faults are lowest in the high-voltage windings, lower but still high in the low-voltage class, and highest in the medium-voltage ratings. While the (*R*) faults are in the same general order as bushings, it should be noted the (*D*) faults are much higher, ranging from 4.5 to 22.0 per cent. Again, there is no characteristic trend to prevent the occurrence of faults in other transformers of the group from one test period to another, although the faulty conditions found have been removed prior to the next test.

Internal faults in instrument transformers and in oil-circuit-breaker tanks are shown in table VIII, together with a condensed summary of power transformer winding faults given in more detail in table VII. One important fact, which correlates with the failure operating record, is the high percentage of (*R*) and (*D*) faults shown in instrument transformer windings, namely 7.8 and 35.0 per cent average for three tests.

Faults in oil circuit breakers show low (*R*) conditions and fairly high (*D*) percentages. This may be explained in part, at least, by the more frequent servicing of oil circuit breakers than transformers. Although the (*D*) faults are high they are apparently removed by sufficiently frequent periodic servicing before they develop into (*R*) faults.

Transformers in Service and Spares

An analysis of power-transformer-winding faults on the basis of transformer voltage and age is shown in figures 7 and 8. These data were taken from conservator and free-breathing single-phase transformers operated in banks of three units with one spare. So long as the transformers are in service both types indicate relatively few (*R*) conditions irrespective of voltage rating; but the (*R*) conditions slowly increase with transformer age.

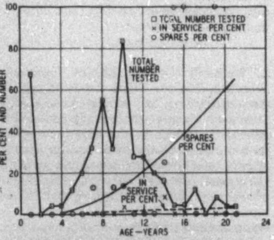

Figure 8. Effect of transformer age on power-transformer-winding faults (R) conservator and nonconservator type; 110 banks (440 transformers)

In the spare units, however, a high percentage of (*R*) conditions is indicated in the 13.2-kv class which drops abruptly in the 22-kv class and then slowly decreases as the voltage increases. On the basis of age, the spare units show a large increase in percentage of (*R*) faults after some six years, showing some 25 to 50 per cent faulty units in the age range from 15 to 20 years. Most of these spare units have from the start been held as spares and only put in service as occasion required.

Faulty Gaskets

Faulty and deteriorated gaskets have generally been recognized as one of the major sources of bushing troubles. Some information has been collected in the field on this gasket problem along with the power-factor test work and is presented here.

Figure 9 shows the condition of some 376 bushing gaskets physically inspected in the field on compound filled bushings. These have been rated (*G*), (*D*), and (*R*) with the same understanding as to terminology as applied to bushing faults. It will be noted that after some five years, the gaskets begin to grow faulty at an alarming rate. At the end of some 15 years, 60 per cent of the gaskets are in an (*R*) condition and 30 per cent in a (*D*) condition. Such a record as this leads us to believe that the older type of cork, at least, should not be depended upon in compound filled bushings much longer than five to seven years at the most.

Summary of Power-Factor Tests on Oil Circuit Breakers and Transformers

A composite summary of oil circuit breakers and transformers tested and the average number of faults found during the $5^1/_2$ years of testing is given in table IX. Several interesting points are brought out in the tabulation: First it is shown that there are 1.52 times as many faulty (*R*) conditions in power-transformer bushings as in oil-circuit-breaker bushings, and second, instrument-transformer bushings have 1.84 times as many similar faults. The (*D*) faults are shown, on the same basis, as 0.66 and 0.72 times those in oil circuit breakers, respectively.

An analysis of internal faults (*R*) shows ratios of 1.95 and 4.8 for power-transformer and instrument-transformer windings, respectively, compared with oil-circuit-breaker-tank faults. The corresponding ratios for (*D*) faults are 2.6 and 5.6. A study of this table IX indicates clearly where the highest percentage of faults exist.

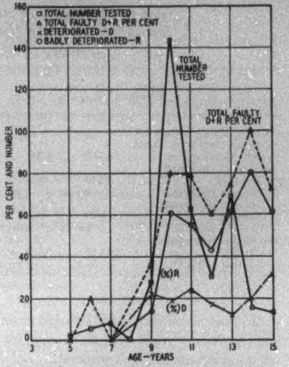

Figure 9. Faulty cork gaskets in compound-filled bushings found by physical inspection; 376 gaskets inspected

Table IX. Summary of All Equipment Power-Factor Tested
(May 1932 to October 1937)

Equipment Tested	Number of Tests	Defective		Remove	
		Number	Per Cent	Number	Per Cent
Oil Circuit Breakers	3,950				
Tanks	11,850	733	6.2	199	1.6
Bushings	23,700	1,100	4.7	503	2.1
Power Transformers	3,144				
Windings*	3,144	503	16.0	98	3.1
Bushings	12,974	406	3.1	413	3.2
Spares in storage**	410	125	30.5	82	20.0
Instrument Transformers	3,192				
Windings*	3,192	1,111	34.8	247	7.7
Bushings	5,450	187	3.4	211	3.9
Spares in storage**	242	51	21.0	48	19.8

* All windings of a transformer classed as *one winding.*
** Rating based on over-all condition—winding and bushings.

Fault Distribution by Companies

As the data of fault conditions of insulation presented above were obtained from six different companies, operating in different sections of the country, under different climating conditions, and various requirements, the question may well be asked: "How do the fault conditions compare between the different companies?" A comparison of results on this basis is given in table X. Of the some 60,000 tests reported above, the (*D*) faults ranged from 3.0 to 8.3 per cent, averaging 6.7 per cent; and the (*R*) faults ranged from 0.8 to 3.5 per cent, averaging 2.8 per cent.

It is therefore evident that faulty insulation conditions, both (*D*) and (*R*) faults run in the same general order in all six companies except company (*F*). In this one case it is known that new equipment and a large concentration of the tested equipment in power stations, where maintenance is inherently closely watched, seems to account for the lower fault record.

Table X. Summary of Power-Factor Tests in All Companies
(Including Oil Circuit Breakers, Transformers, and Bushings)
(May 1932 to October 1937)

Company	A	B	C	D	E	F	Total
Total Tests—bushings, oil circuit breakers, tanks and transformers	18,911	6,347	2,925	8,618	19,829	3,685	60,315
Number of equipment:							
Rated *D*	1,580	508	173	694	976	110	4,041
Per cent rated *D*	8.3	8.0	5.9	8.1	4.9	3.0	6.7
Number of equipment:							
Rated *R*	613	155	101	250	523	29	1,671
Per cent rated *R*	3.2	2.4	3.5	2.9	2.6	0.8	2.8
Number of equipment:							
Rated *G*	16,718	5,684	2,651	7,674	18,330	3,546	54.603
Per cent rated *G*	88.5	89.6	90.6	89.0	92.5	96.2	90.5

Equipment found good (*G*) ranged from 89.0 to 96.2 per cent with an average for all six companies of 90.5 per cent.

VIII. Summary and Conclusions

From the data presented above, and other related testing work carried on in the field during the past six years, the following conclusions appear justified:

1. The power-factor test method of determining the insulation quality of high-voltage equipment in the field has proved a most useful and practical tool in permitting a higher grade of apparatus maintenance and in reducing service failure of such equipment.

2. The type of power-factor test set which has been used has proved an accurate, rugged, and reliable instrument in use. The particular features of this set which permit of direct reading of current and watts loss have been particularly helpful in locating faults in equipment shown defective by the over-all power factor determination.

3. It is considered highly desirable, for best results, that the test set be placed in the hands of a thoroughly trained experienced test man who is familiar with the insulation problem, and one who has considerable knowledge of equipment design, maintenance and operation.

4. Insulation faults develop in all types of equipment such as bushings, transformers, wood insulation, and oil regardless of equipment design as it exists today. These faults may develop critically within test period intervals of 18 months to two years, as shown by the experience cited above, and appear, in a few cases, to develop to a lesser degree in a shorter period of time. Periodic testing, therefore, is necessary if a high standard of insulation is to be maintained.

5. Critical insulation faults found on successive tests are not always developments of (*D*) conditions found on the previous test; they often appear in equipment that previously tested good (*G*).

6. Faulty insulation is less frequent in new equipment, but may be expected to develop to a degree warranting some concern within a period not longer than some five years, after which constant periodic checking, or testing, seems desirable if critical faults are to be minimized.

7. Most faulty equipment so far found can be reconditioned with a reasonable amount of labor and expense, and many costly failures thus avoided, if prompt attention is given to (*R*) faults.

8. The classification of insulation faults on a power factor and/or watts loss basis as given in table III above, and servicing them on this basis has proved a reasonable and practical method of greatly reducing service failures on the equipment tested.

9. Insulation faults requiring prompt attention, which develop under service conditions in oil circuit breakers and transformers, as found by our experience, range, on the average over several years, as follows:

Oil circuit breakers, including bushings	1.6 to 2.1 per cent
Power - transformer windings, including bushings	3.1 to 3.2 per cent
Instrument transformer windings, including bushings	3.9 to 7.7 per cent
Bushings in the above three items	2.1 to 3.9 per cent

10. The economic value of this test work in system maintenance is unquestionable, although it is difficult to express the savings effected in money value done.

IX. Acknowledgments

The author wishes to acknowledge the careful and painstaking field work of Mr. E. W. Whitmer of the American Gas and Electric Service Corporation, and of Mr. F. D. Brook of the Appalachian Electric Power Company in obtaining the field records, and their help in preparing the summarized data in the paper.

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Discussion

H. L. Cole (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): Mr. Gross has made a valuable contribution to the art of power-factor testing in presenting the large amount of data given in the paper. This is particularly true with respect to tests on terminal bushings. The value of power-factor testing of bushings is well established.

In connection with the table of "basic rating for insulation quality," it is noted that a correction is made to bring the measured values to an equivalent 20-degree-centigrade figure. It is thus recognized that temperature plays an important part in the rating of the insulation.

For several years we have recommended lower power-factor limits for bushings operating in hot oil, as on power and distribution transformers. For instrument transformers and breakers, somewhat higher limits are safe. I believe that a basic rating should make a distinction between bushings operating in hot oil and those in oil at ambient temperature.

L. Wetherill (General Electric Company, Pittsfield, Mass.): This paper brings out the advantages of having power-factor tests made by men who are well qualified to exercise judgment in the interpretation of data obtained in the field, making it possible to take immediate action and avoiding the necessity for rigid adherence to a system of arbitrary limits. We believe that the acceptance of this viewpoint is one of the reasons why satisfactory results have been obtained from field power-factor tests on bushings, and doubtless it will prove even more beneficial in connection with other types of apparatus in which the designs are less standardized.

A question is raised regarding the theoretical justification for power-factor tests made on fibrous materials, such as insulating cylinders, between electrodes placed three inches apart on the outside surface. Our experience has been that measurements of this nature offer a particularly sensitive method of detecting the presence of moisture. It would seem that a very small trace of moisture present in fibrous or laminated materials tends to produce a marked change in electrical loss characteristics when the stress is applied parallel to the laminations. If we were dealing with large masses of insulation it would be found that the power-factor measurement obtained with the voltage applied parallel to the laminations would be the most sensitive method of detecting small traces of moisture. In materials such as cylinders, it is rather difficult to make a power-factor test in which the stress is entirely parallel to the laminations, since a test between the ends of a cylinder would permit the stray capacitance to mask the power factor of the cylinder. The method of test used by Mr. Gross, namely, the application of voltage between the electrodes placed three inches apart on the outside of the cylinder, appears to be the most practical method of making a test in which the stress is applied principally parallel to the laminations. It would seem, therefore, that there is adequate theoretical justification for the use of this test in addition to the support which has been afforded by practical experience.

The curves of figure 6 showing the correlation between failures and lightning storms is quite interesting, but the disagreement of the two curves during the early part of the year suggests that it might be well to examine the correlation between failures and humidity or precipitation. We have known for some time that moisture and lightning were the chief causes of failure, but the curves of figure 6 seem to suggest that both factors in combination are involved in the majority of cases.

Mr. Gross states that the data in his paper are not representative of the bushings which are now being manufactured. A lapse of five to seven years is required before field-test data and experience obtained on a given design begin to acquire great significance. I believe, however, that evidence obtained from developmental tests gives us excellent reason for believing that bushings as now manufactured, will in most cases be found markedly superior to those which were manufactured in past years before the present extensive body of field experience was accumulated.

L. B. Chubbuck (Canadian Westinghouse Company, Hamilton, Ont.): Our company apply routine power-factor tests to all new high-voltage bushings and have carried out extensive field power-factor tests across Canada. We fully agree with Mr. Gross as to the value of field testing, and have found may cases of apparatus showing poor power factor that after cleaning or other minor repairs returned to good operating condition.

During this discussion the point was raised as to whether power-factor tests on high-voltage apparatus should not be made at operating voltage, rather than at approximately 10,000 volts. We have equipped a power-factor bridge with an external condenser of 150 micromicrofarads designed for 100 kv at atmospheric pressure, and up to 187 kv with increased pressure. Due to ionization losses we find power factors obtained with the high-voltage bridge usually run considerably higher than with the standard bridge.

V. M. Montsinger (General Electric Company, Pittsfield, Mass.): In making use of field power-factor data obtained on transformers Mr. Gross is exploring a field in which the complexity and difficulties will undoubtedly be considerably greater and more numerous than those which were encountered when the use of field power-factor measurements on bushings and circuit breakers was being developed. The greater variety of types and designs of transformers will necessarily make the interpretation of the data a task which will call for a high degree of judgment, and it appears that Mr. Gross is making the right approach in this matter.

It would appear that the usefulness of field power-factor measurements on transformers can only be established by the creation of a considerable quantity of operating experiences and test data judiciously interpreted. The information given in this paper should prove significant and suggestive, both to operators and to manufacturers. We would like to raise a number of additional questions which Mr. Gross may be in a position to answer.

1. In tables VII, VIII, and IX data are shown regarding the number (or percentage) of power-transformer windings which were deemed to be in a condition requiring removal from service. It would be desirable to know whether or not subsequent examination verified the presence of definitely undesirable conditions in approximately all of these cases or if there was an appreciable number of cases in which subsequent examination disclosed no defect.

2. In cases where definite defects in transformers windings were found after removal from service, is it possible to indicate the approximate proportion of cases of undesirable conditions falling into the following classes?

A. Presence of water in insulating oil.
B. Presence of water in solid insulating materials.
C. Deterioration of oil.
D. Deterioration of solid insulating materials.
E. Failure or breakdown.

3. Were there any other major causes of defects not covered by the previous question?

4. In the case of transformers removed from service, was it found possible to carry out a reconditioning permitting them to be restored to operation, and if so, what type of reconditioning was used?

5. In how many cases were repairs necessary other than reconditioning mentioned in (4)?

6. Table IX indicates a higher percentage of transformers for which removal was recommended under spares than under active units. Were the types of unsatisfactory conditions found in the spare units essentially the same as those found in the active units, or was there a greater preponderance of a different type of unsatisfactory condition in the spare units as compared with the active units? Were spares modern transformers or old transformers?

It is, of course, well known that high power factor does not always mean low dielectric strength of insulating materials. For any given material or configuration there are, however, normal power-factor characteristics, and any wide deviation from these normal characteristics is indicative of some physical change which usually deserves investigation, and quite possibly correction. Increases in power factor may be caused by chemical contamination, oxidation of oil, or by absorption of moisture. For many types of apparatus it would appear that operating experience has shown that appreciable increases in power factor are usually related to the third of the three causes listed above. This would appear to be a sound justification for the use of field power-factor testing, when, as recommended by Mr. Gross, adequate judgment is exercised in interpretation of the data.

It would be interesting to know if Mr. Gross during his investigation has had an opportunity to obtain any relative power factors on a given transformer or transformers. Our experience in measuring insulation resistance of transformer windings leads us to believe that, when working with insulation structures having such a variety of configurations, relative power factors may be more valuable than actual values.

We are in sympathy with any method which aids in detecting incipient faults in transformers. At the present time power factor measurement appears to give promise, although it will need to be developed with considerable care in order to avoid the possibility of drawing false inferences by comparison of readings obtained on units of different design and characteristics.

Harold Goodwin, Jr. (Wyncote, Pa.): The paper is very valuable in making known the results of a large number of tests on particular types of insulation by a particular method. The factors obtained can be used in further practical applications of the method. But the mere presentation of the results raises in the mind the questions: "Why?" "How does the current flow to give the results indicated by the instruments?" The oral discussion and the contributions by some specialists on cable also indicated these questions. The cable specialists might join in the search very diligently as they may get some help for their problems from the solution of the bushing problem, just as I hope the bushing specialists may get some help from the telephone engineers, as I shall point out below.

Compound-filled bushings which have failed in service or which have tested defective and been removed and tested to destruction have, from my observation, generally indicated one basic type of failure: the compound has tended to separate from the porcelain rain shield, leaving a fissure that is likely to be somewhat conducting due to dampness or carbonization. But until final failure is closely approached there is not a complete path from one pole to the other. There is rarely a direct short path at the point of maximum stress or elsewhere. If this is so: what causes the increase in watts loss and power factor in the bushing with the incipient defect?

A theory developed by the writer in 1930 to explain the observed results was, very briefly, this: The moist fissure between the compound and the porcelain acts as the plate of a condenser with the conductor the other plate. Therefore the bushing draws more current than a good bushing. But why has the power factor also increased? This is due to the resistance of the condenser plate itself, the moist fissure. The energy is not then lost *throughout* the bushing insulation but only in the fissure. The current flowing to the far part of the fissure must flow through the resistance of the near part of the fissure connecting it to a terminal.

The probability of the correctness of this theory was shortly confirmed by the presentation to the Institute of a paper on telephone insulators showing exact studies of a similar phenomenon. This paper by L. T. Wilson of the American Telephone and Telegraph Company is entitled: "A Study of Telephone Line Insulators," and can be found on page 1536 *et sec.* of the TRANSACTIONS for 1930. The paper should be studied by anyone interested in this subject. I shall not try to review it here but want to point out that the loss here considered is just one of seven discussed in this paper, and—this is most interesting—some of the other losses seem to have a close resemblance to losses that are occurring in oil-filled bushings and which cannot be accounted for by the theory set forth above. This telephone-insulator paper is also recommended to the cable specialists as similar theory may be helpful in explaining some things that are now rather obscure.

A most interesting thing about the theory set out above and the further results of the telephone insulator study is the indication that the power-factor method detects a failure or defect by observation of factors that are incidental to the development of the final fault rather than by measurement of the degree of final failure which may yet be almost infinitesimal. In cases where this is so, it is logical to conclude that the apparatus may be much more safely reconditioned without reinsulation than if the final fault had proceeded to measurable proportions.

Since no exact work, similar to that on the telephone insulators, has to my knowledge been done on the subjects of this paper, bushings and transformer windings, I shall not go to further length to expound this theory, but present it for those particularly interested to take up, test and present to us in complete and verified form.

G. B. Tebo (Hydro-Electric Power Commission of Ontario, Toronto, Ont., Canada): Mr. Gross has added materially to the already extensive records of power-factor tests on insulation, and has reached conclusions similar to those previously reported. The grand total of 60,000 tests appears to lend some weight to these conclusions but the absence of any data to show reduction in failures due to power-factor tests is rather disappointing.

There still appears a need for more critical discussion of the significance of power factor in rating the serviceability of an insulation. It can be shown that a bushing may be on the verge of failure without appreciable increase in energy loss or power factor; also that a substantial increase in power factor can be obtained without any increased hazard. In table III Mr. Gross recommends removal of any bushing having more than 0.5 watt loss at 10 kv. It is evident that a loss of 0.5 watt, per se, is not hazardous in a bushing weighing several hundred pounds. Some distinction should be made between energy losses uniformly distributed throughout a bushing, and those which cause serious disturbance in potential gradient and consequent overstress on the remaining good insulation.

The "hot collar" test referred to, was used by the writer in 1936, and as Mr. Gross points out, it is distinctly superior to the over-all power-factor test. Such faults as cracked and porous porcelains, entrapped water, etc., may be located by a "step-by-step" test but may not affect the over-all power factor appreciably.

The advantages claimed for the "hot collar" test are still restricted by the low test voltage (10 kv), the complex testing equipment and the necessity for interruptions on all equipment tested. Mr. Gross stresses the need for employing specialists in power-factor testing and interpretation of results. Such requirements may be related to the complexity of the testing technique.

The potential-gradient test (described in *Electrical World*, May 8, 1937) was developed to utilize the advantages of a step-by-step procedure and at the same time to provide a simplicity of equipment and technique comparable with that used in live-line insulator testing. During the past two years, the potential gradient test has been applied to about 2,500 bushings operating at 10 kv to 220 kv. Annual failures in this group have been reduced from an average of 15 in 1935 and 1936 to two in 1937 and none in 1938. Both bushings which failed in 1937 were abnormal when tested, but failed before removal.

Having thus practically eliminated failures of bushings by means of an annual live-line test, there appears little justification for tests to disclose defective lift rods, inferior oil, etc., which faults are detectable by the present visual inspection and oil testing schedules.

Considering the wide variations in power factor of the various material used for transformer-winding insulation, an attempt to rate the serviceability of the over-all insulation in terms of power factor seems rather arbitrary. It would be interesting to know how many of the 345 transformers showing power factors calling for "removal" were actually removed, and what faults subsequent inspection disclosed.

E. J. Rutan (Consolidated Edison Company of New York, Inc., New York, N. Y.): A study of the tables in this paper indicates that the limiting values of power factor selected for the different pieces of equipment can only be arrived at by the accumulation of considerable data. From several of the discussions which have already been presented, it would seem desirable to call attention to the fact that these tests have, in general, been made at less than rated voltage. The author's results do show when using the limits selected improvements in the condition of equipment classed as defective have resulted from overhauling. It does not follow, however, that the tests as at present conducted are giving the best clue to possible defects.

I think we can justify this statement from experience in dielectric tests of cable and other apparatus. In cable testing, the existence of ionization causes a very material difference in the power-factor readings obtained. In the Consolidated Edison Company over the past few years, we have been doing considerable work on certain pieces of high-voltage apparatus and insulators to detect what we have called, "the voltage of corona initiation." These measurements were started following an operating failure and to our surprise indicated that some equipment showed evidence of corona discharge at below normal operating voltages. Since then, this test has been included as part of the regular acceptance for certain types of insulation.

In view of the fact that in the power-factor measurement for the detection of possible faults we are looking for abnormal conditions, it is reasonable to expect that as faults develop there might be corona conditions at operating voltages. These same units, when new, might not show corona or ionization until several times normal voltage had been applied. It would seem, therefore, that measurements made at normal voltage would be more likely to disclose the development of undesirable faults. It might also be expected that the power-factor limits distinguishing good from defective equipment would be better defined.

In the past, the writer's organization has made a few power-factor measurements and efforts were always made to test at rated voltage. It is recognized, however, that there are difficulties encountered which at the present time have not been overcome. This, however, should not deter those who are regularly making these measurements from attempting to obtain data at rated voltage. Until this field has been explored, it is the writer's opinion that we will not fully realize the benefits of this type of measurement for diagnosing insulation troubles.

F. J. Vogel (Westinghouse Electric and Manufacturing Company, Sharon, Pa.): Mr. Gross' paper furnishes data showing the results of several years' experience in field testing and is of great value both to operators and manufacturers. These results show shortcomings in materials used and in design practices. They also show the desirability of using checks to keep apparatus in good operating condition and avoid failures and expensive repairs and loss of service in the field.

There are several factors in power-factor testing of transformer insulation which I would like to mention. The company with which I am associated made power-factor tests for a period of a year or so in the factory on power transformers. The purpose of this was to accumulate data as to what power factors might be expected on new equipment. Due to the careful processing and drying out in the factory it might be expected that the results of the power-factor tests would be fairly uniform and to be, therefore, of little significance. This was actually found to be the case. For this reason the use of power-factor tests has been discontinued on transformers. It might also be pointed out that the power factor varied somewhat with different designs in such a way that the method is not suitable for factory testing.

It might be desirable to call attention to the fact that in service high power factors are not always caused by moisture. Oxidation and oil deterioration lead to high power factors in the oil even though dry. It is of interest that high power factor due to this cause does not result in any loss of dielectric strength. This has been demonstrated by tests on sample barriers ranging from very low power factors to 40 or 50 per cent power factors. Both 60-cycle and impulse tests showed no change with power factor.

It is of great interest that spare units not excited may deteriorate due to moisture. This might be expected particularly in transformers breathing to the outside atmosphere whether directly or in combination with an expansion tank. Where oil has been heated slightly above atmospheric temperature it has been found that it will keep fairly dry. Oil, however, which may become colder than the atmosphere and is in direct contact with the atmosphere can condense moisture and gradually become quite wet. Even transformers which breath in air through a calcium chloride breather may become quite wet due to the oxidation of the oil forming water which is not easily removed.

Mr. Gross has also called attention to the deterioration of cork gaskets. This material was the best material we knew of some years ago for use as gaskets. It is of interest that materials have recently been put into use which promise to deteriorate very little and which may last indefinitely.

F. C. Doble (Doble Engineering Company, Medford Hillside, Mass.): The paper by I. W. Gross summarizing authentic and instructive information from more than five years of consistent field testing of bushing and transformer insulation by the power-factor method is a credit to the author and should be of value to those in the industry who are responsible for the maintenance of high-voltage insulation.

It is significant that a 10-kv 60-cycle test has been adequate to maintain a completely reliable operating schedule on equipment rated from 13 kv to 132 kv.

Early in the development of field testing by the power-factor method, some insulation engineers felt that a 10-kv test voltage would be entirely inadequate for testing equipment rated by voltages higher than 10 kv. Also some others felt that an off frequency even up to 1,000 cycles would be desirable in order to get away from certain difficulties due to induction from parts of the 60-cycle system other than those under test and to mitigate possible loss effects due to the action of low frequencies on composite insulation.

The paper under discussion typifies a wide experience with dielective loss tests in the field correlated with failures of insulation of all voltages up to and including 220 kv that has proved conclusively that a 10-kv 60-cycle dielectric and power-factor test is adequate for a reliable economical maintenance program.

Referring to Mr. Montsinger's discussion, I agree with his statement that the greater complexity of construction and increased variety of designs of transformers as compared to bushings makes it more difficult to interpret the results of power-factor tests into a reliable transformer-maintenance program. However, I believe that it is not too optimistic to expect that a reasonable degree of standardization of test values can be derived in view of the very simple tables that now cover the entire bushing range that was thought to be just as hopeless some years ago when field test by power factor first began.

Pertinent to Mr. Montsinger's request to the author for an explanation of the fact that spare transformers deteriorate at a much higher rate than those in service I believe that more attention should be given to the electrical characteristics of water in oil. Recent investigations indicate that it is possible for water to be present in oil in a low-loss form that is not revealed by ordinary electrical tests, such as dielectric breakdown or power factor, to a much greater degree than is shown by the literature that describe this phenomenon.

In this low-loss form the water is no apparent handicap to electrical operation. However, a drop in temperature may cause the water to change its electrical character so that it becomes high-loss water which may be a serious operating hazard. The spare transformers pass through a much wider temperature cycle than corresponding transformers in service with a resulting tendency to faster deterioration in a given time.

I. W. Gross: The numerous discussions given of this paper indicate the subject of field testing of insulation is a very live one at the present time.

The question of correcting power-factor readings to a standard temperature and the methods for so doing is one on which we have spent considerable thought and time. Mr. Cole has pointed out that he has taken this into account by using different power-factor limits on bushings, depending on the temperature of the oil in which the bushing is located. Another method on which we have been working is that of assigning a temperature to the bushing on the basis of the composite parts which are at different temperatures. For example, the bushing temperature may be rated on ambient air, copper temperature at the top of the bushing, or oil temperature of the apparatus in which the bushing is located; or in the case of oil-filled bushings it may be based on the temperature of the oil in the bushing. Undoubtedly, some further study in refinement is possible in arriving at the most desirable basis for temperature.

Mr. Wetherill has pointed out the theoretical justification for the test on fibrous insulation parallel to the fiber structure. It is quite fortunate from a practical point of view that this test along the fiber, such as along the surface of cylindrical insulating members, has proved to be an adequate test to detect the presence of moisture, as it would be decidedly burdensome and complicated to make routine tests radially. While it is true, as Mr. Wetherill points out, that the highest per cent of bushing failures (figure 6 of paper) occurs during the lightning season, it is also true that in the cases of bushing failures in the early part of the year when general weather conditions are cold, with subsequent thawing and rain, there is a rise in bushing failures which indicates to us the possibility that moisture, which had entered the bushing prior to failure, had become destructive by penetration into or through the insulating material. As pointed out in the paper deteriorated gaskets are contributory factors under severe weather conditions. Several years ago we experienced a number of bushing failures immediately following a protracted cold spell followed by thawing and rain in the early part of the year. These failures were not accompanied by lightning but appeared to be due to deterioration of the bushing by the entrance of moisture under severe weather conditions.

Mr. Montsinger and others comment that the type of transformer design may affect the interpretation of power-factor readings. This point is well taken and, as pointed out by him, undoubtedly a large number of tests on different types of transformers will be required before we can determine transformer faults with the same certainty that is now possible with bushings. However, the work which we have been carrying on during the past few years seems to indicate, with our present knowledge of the art, that there are many other factors in the detecting of transformer faults which at the moment far outweigh the difference in transformer design. This point, however, should be given further consideration.

In answer to Mr. Montsinger's question regarding the types of faults which have been found in transformers, so far as we have been able to determine, these faults have been due to the entrance of moisture into the transformer and have in most cases been successfully removed by drying the transformer by conventional, recommended methods. With practically few, if any, exceptions, we have not discovered other types of faults in such defective transformers at the time they were serviced as a result of the power-factor test.

In many cases, of course, the oil has deteriorated where it has been in service for some length of time, and on reservicing, it is always reconditioned or replaced. I do not recall that we have had any cases of seriously deteriorated insulation attributed to anything except absorbed moisture.

As brought out by many of the discussers, there is undoubtedly a great deal of research work to be done in studying the deterioration of insulation in a complicated, composite structure such as a transformer winding with its insulation of conductors, sections, cores, etc., and it is hoped that a well organized and carried on study of this problem will be made in the future.

The deviation of power factors, concerning which Mr. Montsinger asked, is very valuable in determining the condition of transformer insulation in the field, not only by comparison of insulation between transformers of the same design, manufacture, rating, etc., but also by comparison of insulation between primary and ground, secondary and ground, and interwinding insulation. We have sometimes detected faults by comparisons of this type.

Messrs. Chubbuck, Rutan, and Doble have all commented on the power-factor testing on higher voltages than 10 kv employed in our work. As pointed out by Mr. Doble, studies made so far indicate that 10 kv is adequate for the purpose. It is to be expected, however, that at higher test voltages approaching the operating voltage of the equipment, corona formation may be detected in some cases. This, however, with our present experience with practical field testing seems to be more a question of insulation design than one of equipment maintenance.

It should not be lost sight of, too, that to attempt to test field equipment which ranges all the way from 2,300 volts up to 230 kv would undoubtedly require considerably more elaborate test equipment as regards voltage supply than has generally been considered practical in the field. Unless there is some distinct advantage, which so far has not been apparent, to be gained by the use of high voltage in the order of 100 kv or so, we believe it is a better plan to retain a practical working limit of voltage until such time as research developments show that a higher voltage is essential to the satisfactory maintenance of equipment.

Mr. Goodwin's theory on the reason for failure of compound-filled bushings is quite interesting. A number of other theories have, of course, been set up, discussed and explored in the development of the present test set which we are now using.

Mr. Tebo touches on several points of interest as regards testing procedures, methods, equipment, and results obtained. In the first place, while the paper did not report numerically the reduction in failures of bushings or other equipment since power-factor testing work was started, there is abundant evidence to show that material reductions have taken place. This information, however, is not in the form of statistical, numerical data, but rather comprises the frequency of complaints from the field of equipment failures and also the verbal reports of the different properties on which testing work has been carried on. In fact, at the present time we are using three test sets in an attempt to carry out a yearly test schedule on oil circuit breakers, bushings, and transformers, and the results of testing have been so successful in improving the operating performance of equipment that we from time to time receive urgent requests from the field that the schedule be hastened to take care of testing their equipment on a more frequent basis to forestall trouble.

While, as pointed out by Mr. Tebo, it is theoretically possible to set up conditions of faults in bushings where a serious operating hazard can exist on a bushing without an appreciable increase in energy loss or power factor, still we should not lose sight of the fact that experience seems to show that actual occurrences of this kind are very infrequent. Likewise, the theoretical consideration of a $^{1}/_{2}$-watt loss at 10 kv in a bushing may exist without the bushing being a hazard on the system must be correlated with the practical experience obtained in testing bushings where losses of such magnitude are found and inherent faults in bushings have been detected. The possibility of these two conditions existing is, of course, admitted, but their probability seems remote.

Regarding the hot-collar test referred to in the paper, Mr. Tebo has apparently misunderstood our opinion of its place in test work and the importance of this type of test in detecting faults. It is not our opinion that this type of test is superior to the over-all power-factor test. It is merely a supplementary test which, when used in conjunction with the over-all power-factor test, has been useful in detecting some types of faults in particular types of bushings. This hot-collar test, so far as we know, was first described by Mr. O. E. Fawcett of the West Penn Power Company early in 1936 and was first used on our property subsequent to this.

Further, Mr. Tebo's implication that the testing technique which we are employing is complicated and therefore requires "specialists in power-factor testing and interpretation of results" is an opinion with which we quite emphatically differ. It often happens that a test device which requires merely the turning of a handle and the reading of an instrument scale falls far short of obtaining satisfactory results unless the results are understood and analyzed by competent and skilled men.

Considerable advantages result by having the tests promptly interpreted and remedial measures taken at the time of the tests. Mr. Tebo's statement that failures have been practically eliminated on some 2,500 bushings tested by the potential-gradient method warrants some comments. The absence of failures in 1938 (which is approximately only half of the current year) and the fact that the two which failed in 1937 were abnormal when tested leave the results very inconclusive. It should be borne in mind that to remove from service all bushings which are merely "abnormal" on test would place a heavy financial burden on the operating company which could not be tolerated. The degree of abnormity must be evaluated in terms of operating hazard. It is often true that an attempt to draw conclusions from incomplete data is often more harmful than beneficial.

We question the ability of any test method to eliminate all failures of bushings in service. If it should be possible to accomplish this, Mr. Tebo's statement that "there appears little justification for tests to disclose defective lift rods, inferior oil, etc.," is not in accord with operating experience in our company where we have found defective lift rods by the power-factor test and have eliminated them before breaker failure has occurred. The importance of field insulation testing lies in testing all major equipment, if possible, rather than to confine the tests to only a small part of it such as the bushings alone.

One type of fault the potential-gradient method applied to bushings cannot detect is faulty conditions of the insulation on the bushing within the apparatus, that is, below the bushing mounting flange. Faults of this type have been found by the power-factor method.

Mr. Vogel points out that a high power factor of the oil does not affect the insulation, and we presume he has in mind that if the fault should be localized in the oil (in a transformer), then servicing the oil would be a suitable means of reconditioning the transformer. This indeed is an interesting point, but it should be borne in mind that if the high power factor of the oil is due to contamination by moisture, then over a period of time that moisture may have worked into the insulation and become concentrated at some point in the insulation structure which is vital to the successful operation of the equipment.

In closing this discussion there is one broad point of view that we wish to emphasize, and that is, in field testing of insulation of the type described in the paper, we are dealing with a most complicated structure consisting of oil, paper, fibrous insulation, wood, etc. In many cases it is fortunate indeed that a power-factor test does give us an indication of the condition of the insulation as compared with the insulation of that equipment when initially new even though we do not know all the theoretical reasons why it does. Those who choose to take chances in operating a system with insulation known to be far different from what it was in its original condition must, of course, run the risk of equipment failure. Just how bad such insulation may become, and still result in satisfactory operation of the equipment is a point which is not known at the present time, but it is hoped that as our test work continues, it will be possible to obtain some valuable information on this point over a period of time.

A Self-Checking System of Supervisory Control

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Synopsis: This paper describes a new type of supervisory control system which was developed to reduce the selection time in large, multistation applications. The principles are based on the use of short and long electrical pulses and circuit designs not requiring a check-back code for assuring accuracy. It is equally applicable to a two-wire telephone line or a carrier-current channel. The design is quite flexible, permitting many alterations and additions to satisfy local conditions peculiar to its problems. The theory behind the elimination of the check or selection is explained.

MOST modern systems of supervisory control are based on the logical principle of checking a selection before performing an operation. To do this, a checking code duplicating the selecting code is returned from the substation to the dispatching office. This insures that the desired unit has been correctly selected for operation.

The self-checking system, here described, obtains the same assurance of correct selection without the necessity of returning a duplicate of the selecting code. Thus, the over-all selecting time is reduced, particularly in the larger multistation systems. The usual initial operation of station selection is also eliminated since we can look at the whole system as a single station. Each remote piece of apparatus is assigned a definite code. The resulting saving of time is important, particularly when a carrier-current channel is used, since it minimizes the time during which outside interferences can tend to disturb or mutilate the signalling sequences.

In the design of the self-checking system, two types of selecting pulses are used. They differ from each other in their length or in the time of energizing the line. The short pulse has a characteristic time of the order of one-tenth

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of a second. The long pulse is purposely lengthened to approximately three times the time element of the short one.

Each selecting code is made up of the same number of total pulses and the same number of long pulses. By this means, the possibility of incorrect selections is eliminated. The explanation of this will be apparent from the circuit descriptions to be discussed later.

In the smaller applications, there is no advantage in the self-checking system. As the number of total pulses increase, the number of possible selection combinations enlarge enormously. This is apparent from figure 1. It is for this reason that the self-checking system is economical and most beneficial where there are many operations and several substations to be controlled and supervised.

From a given number of total pulses, the maximum number of selection combinations occur when the number of long and short pulses are equal. As an example, let us consider the case where eight pulses are used. The formula for determining the total selections may be written:

$$N = \frac{T \times (T-1) \times (T-2) \ldots (S+1)}{L \text{ (factorial } L)}$$

where

N = total number of selections
T = total number of long and short pulses
L = total number of long pulses
S = total number of short pulses

In this case, should three long pulses be used, the number of shorts would necessarily be five.

$$N = \frac{8 \times 7 \times 6}{1 \times 2 \times 3} = 56$$

However, should there be four long pulses, we have

$$N = \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} = 70$$

Should five long pulses be employed, we arrive at the same total as with three.

$$N = \frac{8 \times 7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4 \times 5} = 56$$

If our system were composed of approximately 50 units, the scheme using three long pulses would be used since less total time is required.

Where ten total pulses are required, four long pulses permit selecting any one of 210 units. On the basis of 0.1 second for a short and 0.3 seconds for a long pulse, it is apparent from the foregoing that with an increase in selection time of 0.4 seconds, the size of the system may increase from 56 to 210 points. Due to the natural limitations of the number of contacts available on standard relays, less than the full theoretical number of selections is practical. The usable codes, even in the larger systems, are at least half of the theoretical total.

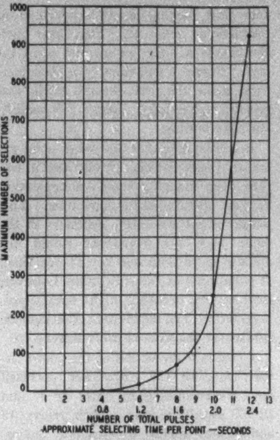

Figure 1. Curve showing the selection time in seconds for any unit out of a known maximum, given a code with a definite total number of impulses

An even number of total pulses is advisable for purposes of balanced circuit design. Also, an even number of long pulses is preferable to an odd number. In the circuit design, the first half of the total pulses is called the group selection, while the second half is known as the point selecting pulses. For example, consider the case where a total of ten pulses is used. If the total number of system points is 100, the codes would be assigned so that there would be two long pulses in each of the first and second groups of five pulses. Using a system of numbering whereby the numbers represent the long pulses, code 14–79 would mean that the first, fourth, seventh, and ninth pulses were long while the remaining ones were short.

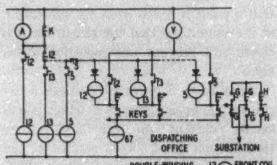

Figure 2. Code set-up circuit outlining method of establishing a definite code

The group codes would be 12–13–14–15–23–24–25–34–35–45.
The point codes would be 67–68–69–60–78–79–70–89–80–90 (the 0 denotes the tenth pulse).

This represents a ten by ten layout or 100 total points.

The codes would be:

12–67	13–67	14–67	15–67	23–67
12–68	13–68	14–68	15–68	23–68
12–69	13–69	14–69	15–69	23–69
12–60	13–60	14–60	15–60	23–60
12–78	13–78	14–78	15–78	23–78
12–79	13–79	14–79	15–79	23–79
12–70	13–70	14–70	15–70	23–70
12–89	13–89	14–89	15–89	23–89
12–80	13–80	14–80	15–80	23–80
12–90	13–90	14–90	15–90	23–90
24–67	25–67	34–67	35–67	45–67
24–68	25–68	34–68	35–68	45–68
24–69	25–69	34–69	35–69	45–69
24–60	25–60	34–60	35–60	45–60
24–78	25–78	34–78	35–78	45–78
24–79	25–79	34–79	35–79	45–79
24–70	25–70	34–70	35–70	45–70
24–89	25–89	34–89	35–89	45–89
24–80	25–80	34–80	35–80	45–80
24–90	25–90	34–90	35–90	45–90

From this, it is apparent that the selecting time of any one unit is the same as that for any other unit in the same system, since each code must contain the same number of long and short pulses.

The order of preference of reporting the codes in case of more than one attempt to start at the same instant is determined by the long pulses. The code with the first long pulse will take preference over one having a short pulse at that time. This is also true in multistation layouts where the station which is sending a short pulse locks out if some other station is sending a long pulse. All points have "storage" circuits so that there is no possibility of losing an indication under such circumstances.

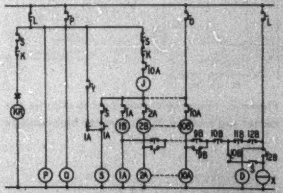

Figure 3. Simple short-pulse counting scheme used for recording the short pulses in a given code

The first application of this system was made on a pair of 287-kv power lines approximately 270 miles in length. The line circuit consists of a carrier-current channel of 60 kilocycles. Connections to the lines were so made that operation is possible even though either transmission line is open circuited or grounded.

In this case, ten total and four long pulses are used. However, the number of required operations demanded that the codes have up to three long pulses in either the group or point codes. When the group has one long pulse, the point must contain three, etc.

This application has many other features individual to the local conditions and these will be omitted in the general description to follow. There are two dispatching boards, one at Boulder Dam and the other at Los Angeles, on which all operations must report simultaneously. Should the Los Angeles board fail to receive a signal due to any cause, the dispatcher at Boulder is automatically notified. Again, interlocking is used so that the dispatcher cannot attempt a wrong operation. Thus, it is arranged so that a ground switch cannot be closed at one end of a line section if a line disconnecting switch is closed at the other end. Such are examples of special circuits which may elaborate on the fundamentals as will now be described.

Description of Scheme of Selection

Figure 2. As the dispatcher operates a point selection key (key 12–67, for instance) a circuit is completed through the operating coils of relays 12 and 67. As they close, locking circuits are formed by contacts on relay 12. They also open the circuit to other group and point relays. Figure 4 will show how their contacts make the first, second, sixth, and seventh pulses long. Relay *Y* is used to start the pulsing.

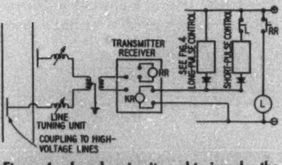

Figure 4. Impulse circuit used to impulse the carrier and method of coupling to high-voltage lines

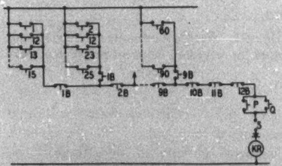

Figure 5. Long-pulse control scheme showing method of inserting the long pulses in their proper position in a given code

It will be seen that one set of point relays and only one may be operated at one time even though several point-selection keys are operated at the same time. This feature is not absolutely necessary at the office since the dispatcher would not operate more than one key at one time. It is required at the substation, however, since several breakers might open simultaneously. The office connection is made similar for the sake of uniformity.

To explain the action of the guard circuit, suppose the dispatcher operated keys corresponding to codes 12–67 and 13–67 simultaneously. Both relays 12 and 13 may operate momentarily but relay 12, being first in order, opens the positive feed to relay 13 so that only relay 12 can remain operated over its locking winding. The selection keys are thus wired in series sequence so that only the first one in the sequence is operated.

It will be noted here also that the numbers of the relays operated correspond to the numbers of the pulses which are made long. Should a code be operated which requires three long pulses in either the group or point sections (code 123–6 or 5–890, for example), the second windings of the necessary relays are connected in series by contacts of the point selection key which corresponds to that particular code.

Figure 3. When relay *Y* operates, a circuit is closed to relay *S* or sending relay. Being connected through a break contact of relay *L*, this circuit is effective only as long as relay *L* is de-energized, so that a pulse received from the substation at the same time that the circuit for *S* is closed will prevent the operation of relay *S*.

When relay *S* operates, it closes a circuit for the keying relay *KR*, which places carrier current on the high-voltage transmission lines. This results in the operation of all receiving relays (*RR*) (shown in figure 4). Neglecting for the moment the sequence of pulsing and the control of the long pulses, relay *L* is caused to pick up and drop out with the carrier-current pulses.

The pulses are counted by the counting relays 1*A* to 10*A* when relay *L* is energized and by relays 1*B* to 10*B* when relay *L* is de-energized. The first operation of relay *L* energizes relay *D* as well as 1*A*. Relay *D* is slow releasing when de-energized and does not drop out between pulses.

As long as relay *L* is operated, relay 1*B* is short-circuited and cannot operate even though the contact of 1*A* has completed its operating circuit.

When relay *L* releases after the first pulse, the shunt across relay 1*B* is opened and relay 1*B* picks up in series with the operating coil of 1*A*. As relay 1*B* operates, it transfers the *L* circuit to 2*A* so that it operates in response to the second pulse. Relay 2*B* operates as *L* drops out at the end of the second pulse. The remaining relays of the counting chain operate in the corresponding manner as the pulsing continues.

Figure 4. This shows the sequence of pulsing the carrier current. From figure 3, the keying relay *KR* was energized by the contact of the sending relay *S*. The contacts of the keying relay complete the transmitter oscillator and amplifying circuits thus applying carrier energy to the lines. All receiver relays pick up and, in turn, energize relay *L*. As relay *L* operates, it not only advances the counting chain of figure 3, but opens the keying relay circuit. As the keying relay drops out, carrier energy is removed from the transmission lines, resulting in the de-energization of all receiver relays. Thus, the counting chains in all stations step ahead simultaneously. The function of the long pulse control of the keying relay will be evident from figure 5. Even though the short pulse circuit is opened by relay *L*, a long pulse keeps the keying relay energized and carrier on the high-voltage carrier-current circuit.

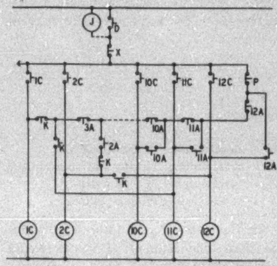

Figure 6. Long-pulse counting circuit used to record the position of the long pulses in a given code

Figure 5. For selection purposes, four of the pulses must be long. This is accomplished over long-pulse control circuit which also operates on the keying relay. To transmit code 12–67, pulses number 1, 2, 6, and 7 must be made long,

while the other pulses, that is, 3, 4, 5, 8, 9, and 10, must be short. As soon as relay *S* is energized, a circuit is completed by the make contact of relay 12 to the keying relay *KR*. Relays *P* and *Q* are normally energized through a break contact of *L*. These relays are slow to release and do not drop out immediately when relay *L* is energized. Carrier energy is thereby kept on the line and all relays *RR* and *L* remain energized. After a predetermined interval of time, relay *P*, whose circuit is opened by relay *L*, drops out and opens the circuit of slow-releasing relay *Q*. Relay *Q* also drops out after a certain time and opens the circuit of the keying relay *KR*, thus removing carrier from the line and dropping relays *RR* and *L*. Thus, each one of the four long pulses energizes the keying relay for a definite period of time.

Figure 6. As relay *P* drops on the first long pulse, it operates a long pulse register relay 1*C*. This takes place at the sending station and all of remote stations. The duration of the long pulse is increased by the drop-out time of relay *Q* in order to give the remote stations sufficient time to operate the *C* register relays and to take care of unequal dropping times of the various *P* relays in the other stations. Other long pulses are under control of relay *P* in the same manner. Thus, during the third, fourth, and fifth pulses (of code 12–67) no circuit is completed for the *KR* relay in the long pulse control circuits, and relay *P* has no opportunity to drop. Register relays 3*C*, 4*C*, and 5*C*, therefore, cannot operate. The same is true of 8*C*, 9*C*, and 10*C* of this particular code. At the end of the tenth pulse, all stations would have relays 1*C*, 2*C*, 6*C*, and 7*C* operated and no others. As relay *L* picks up on the tenth pulse, relay 10*A* opens the circuit to relay *KR*, so that no further pulses can be transmitted. (See figure 3.) Relay *D*, receiving no further pulses from line relay *L*, releases after a short interval and opens the original holding circuit for the counting and code relays. Since ten pulses have been received, these relays are held up through the winding of relay *J*. Relay *J* closes through this circuit and operates the point selection relay *B* or *F* (*B* is at the office and *F* at the substations) corresponding to the selection registered on the code relays.

Figure 7. In the present case, code relays 1*C*, 2*C*, 6*C*, and 7*C* are operated. The group selection circuit (controlled by the C relays operated during the first five pulses) is prepared after the eighth pulse by the operation of relay 8*B*. Since 1*C* and 2*C* are energized, relay 12*D* is operated. Also, since 3*C*, 4*C*, and 5*C* are not energized, the operation of relay 12*D* picks up relay *G*12.

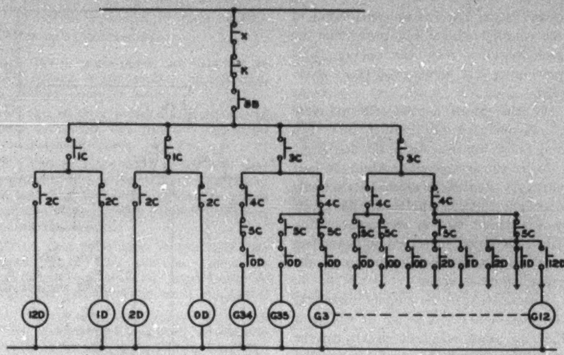

Figure 7. Group selection circuit employed to distinguish the group of a code

Figure 8. After the end of the tenth pulse and the operation of relay *J*, a circuit is prepared for the operation of the point selection circuits (controlled by the *C* relays operated during the last five pulses. Since 6*C* and 7*C* are energized, relay 67*E* picks up. The operation of relays *J*, 67*E*, and *G*12 complete the circuit for *F*12–67 since 8*C*, 9*C*, and 10*C* are de-energized. As *F*12–67 operates, it extends its operating circuit to relay *K*, which also picks up and closes a locking circuit for itself and *F*12–67. Relay *K* opens the energizing circuit for the point selection relays so no more of them can operate and also drops the counting and code relays as well as relays *J* and *S*. They are thus restored to their normal position. Relay 10*B*, as it operated at the end of the tenth pulse, opened the circuit of relays *Y* and 12, which in releasing drop relay 67. High resistance relay *A* (see figure 2) picks up and holds the selection until the key is restored to its normal position.

At this stage, the office has completed the selection of a point and is ready to receive a check code from the substation. At the substation, since no check is required on the selection, a check showing that something was received is desirable where a carrier-current channel is employed.

Circuit Design Eliminates Possibility of Wrong Selection

Before continuing with the operation of the equipment, the group and point selection circuit safeguards against a wrong selection will be amplified.

The circuit for relay *J* is completed only if relay 10*A* has been operated in response to ten pulses on the line. If less than ten pulses are sent, relay *D* will release after the pulsing ceases, but relay 10*A* will not be operated. No new holding circuit is established for the counting and long-pulse register relays, and they release immediately without attempting a selection.

If more than ten pulses are received, the eleventh pulse will operate the lockout relay *X*, which locks up until the set is released. Relay *X* opens the locking circuit of the long-pulse register relays, causing them to drop out and also stop the transmitting of impulses. No selection can then be made until the equipment is released.

Relay *J*, which can only operate if ten pulses (no more and no less) are received, closed the point-selection circuits. A point-selection relay can be operated only if four long pulses have been received and properly registered on four of the register relays 1*C* and 10*C*.

Inasmuch as the group code contains either one or two long pulses in the first five, either one or two of the *C* register relays 1*C* to 5*C* must be operated. If there is no long pulse in the first five, none of the relays 1*C* to 5*C* is picked up, and no group-selection relay (*G*1 to *G*45) can operate. Consequently, no point selection can be made. If only one long pulse is received in the first five pulses, one of the relays 1*C* to 5*C* will be closed. As relay 8*B* operates, one of the group selection relays *G*1 to *G*5 will close. Should two long pulses be received in the first five, two of the 1*C* to 5*C* relays will be energized, resulting in closing one of the group-selection relays *G*12 to *G*45.

If three or more long pulses are received in the first five, an examination of the contact circuits will prove that no group-selection relay can operate, thus preventing any subsequent point selection.

In other words, a group-selection relay can be operated only if either one or two long pulses are received in the first five.

When the group codes contain one long pulse, the point codes must contain three. Likewise, two long pulses in the group code demand two in the point codes. Thus, the point codes contain either two or three long pulses, depending on the preceding group code.

Examination of the contact circuits of the point-code relays 6*C* to 10*C* will show that no point selection circuit can be completed when no long pulse is received in the last five pulses. The same is true when only one long pulse is among the last five.

When two long pulses in the last five are received, ten point-selection circuits are prepared which lead only to those group-selection relays associated with two long-pulse group codes. Therefore, a point selection can be made only if two long-pulse group-selection and two long pulse point-selection pulses are properly received.

If three long pulses are received in the last five, ten other point-selection circuits are prepared, leading only to those group-selection relays which are associated with the one long-pulse group codes. Therefore, a point-selection relay can be operated in this case only if one long-pulse group and three long-pulse point-selection pulses are properly received.

Should four or more long pulses be in the last five, the contacts of relays 5*C* to 10*C* cannot close a point-selection relay circuit.

Summing up, it is clear that a point-selection relay can be operated only if:

(*a*) Relay *J* is operated in response to at least and not more than ten total pulses.

(*b*) One group-code and three point-code relays are operated.

or

(*c*) Two group-code and two point-code relays are operated.

(*d*) At least and not more than four long pulses are received.

All other combination of pulses will not complete a selection.

Substation Check

While no check is necessary on the accuracy of the selection at the substation, the use of carrier current as a channel makes it advisable to get some sort of a signal back to show that the carrier equipment is working. To do this, two pulses are returned which, at the same time, indicates the present position of the breaker or device to be operated. If, at that time, the device is closed, a long and short pulse code is sent to the office. If the device is in the open position, a short and long pulse code is transmitted.

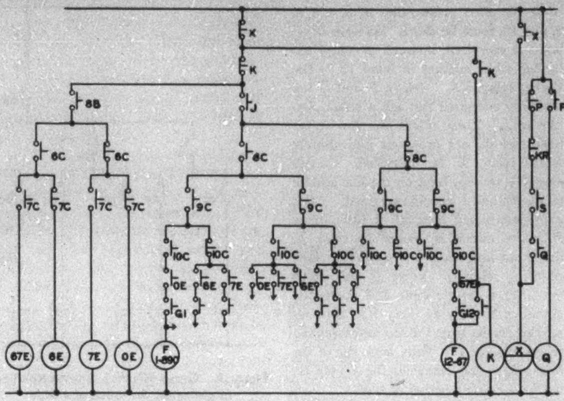

Figure 8. Point selection means used to isolate the point individual to a code

The substation sending follows the same sequence as has been described at the office except that the keying relay is opened at the end of the second pulse. The counting and code relays are restored to normal. At the office the two pulses are received registering on 11*C* or 12*C* depending on whether the selected device is closed or open. A circuit is completed only if either the first or second pulse is long. If both are short or if both are long, the contacts of 11*C* and 12*C* keep the circuit open. The relay energized in response to this signal controls the red and green lamp relay. Should the selected device be closed, the first pulse is long (and not the second) and relay 11*C* is operated. The lamp relay is energized connecting the red lamp in the circuit. Relay 12*C*, which picks up when the first pulse is short and the second one long, causes the lamp relay to drop out, changing the lamp indication from red to green.

Operation Control

At the same time, connections are made to prepare the circuits for the transmission of the operation control codes. The point-selection lamp at the office is lighted only after the reception of a correct indication code from the substation. If, at this time, the selected device should automatically change its position, the new indication code is immediately transmitted to the office.

The dispatcher may now operate the master control key which sends through a three-pulse code, depending on the position of the twist key of the selected point. The codes used are a long-short-short combination for closing and a short-short-long combination for tripping. The relays thus operated have their locking circuit through the twist key providing an antipumping feature.

After the three pulses are transmitted, further transmission is stopped by relay 3*A*. At the substation, the circuit is set up only if three pulses are received and the first or third one long. The device then operates and the changing of its auxiliary switch sets up the sending circuit to initiate transmission of the new indication code. As the new code is received at the office, the lamp relay is controlled as previously described.

In order to insure that the lamp relay has properly changed, a check code is sent from the office to the substation. A single long pulse is sent if the lamp relay is energized while a single short pulse denotes the de-energized position of the lamp relays. When one of these types of pulses is received at the substation, relay 11*C* is operated if the red lamp is lit, but will not be under the green lamp condition. In either case, if the check code from the office agrees with the position of the selected device, no further action takes place. If they do not agree, a new indication code is transmitted.

The dispatcher may keep the selected point as long as desired and repeat the closing and tripping operations at will. Meanwhile, the device is constantly supervised, and any new position is automatically indicated at the office.

Releasing the Selected Point

When the dispatcher desires to release the selected point, he restores the point-selection key. If no pulses are being received at this time, restoring the point selecting key opens relay *A*. Relay *A* is arranged to send a single release pulse.

In order to make a last check of the lamps at the office against the actual position of the selected device, the release pulse is made long if the red lamp is lit and a short pulse for the green lamp. This action is the same as previously described except all equipment resets to normal if the signals agree.

All other stations which have locked out are also automatically unlocked by the reset pulse.

Automatic Operation From the Substation

When a device changes its position in the substation, the selecting code is sent to the office in the same way as during a selection with the exception of the check code Here, both sets of signals originate at the substation, so the indication code of two pulses follows immediately after the ten-pulse selecting code as pulses eleven and twelve.

An audible alarm and flashing disagreement lamp informs the dispatcher of the change. The disagreement lamp is so named because the position of the control key and indicating lamps are not in agreement. Its chief function is to assist the operator quickly to locate the unit which has operated automatically in the remote station.

Should several devices change position simultaneously, they report to the dispatcher in sequence. The order of preference is settled by the code since a station locks out if it is receiving a long pulse while sending a short one. Also, within a station, the code with the first long pulse, where another code has a short one, will be sent through first.

Conclusion

The new self-checking system is particularly applicable to large multistation applications, especially where carrier current is used as the controlling channel.

The possible source of code mutilation may be summed up:

(*a*) One or more pulses are lost due to relay failures or line trouble. In this case, a selection cannot be made.

(*b*) One or more pulses are added due to relay failures or line trouble. Again, no selection can be made.

(*c*) Two or more short pulses may flow together to form a long pulse. This results in less than the correct number of pulses and also too many long pulses. Therefore, no selection results.

(*d*) A long pulse may be split into two short pulses. The selection cannot be completed because of too many pulses as well as an insufficient number of long pulses.

(*e*) Should the long-pulse indicating relay *P* fail to release on a long pulse, no selection results as too few long pulses are received.

(*f*) Also, if relay *P* should drop out on a short pulse, too many long pulses are received and no selection is possible.

Thus all reasonable possibilities of wrong selections are reliably prevented and no check-back is necessary.

Discussion

F. Von Voigtlander (Commonwealth and Southern Corporation, Jackson, Mich.): Increasing interest is being shown by operating companies in the application of supervisory control schemes to power systems, chiefly as a means of reducing operating costs and improving service.

In contemplating the application of supervisory control to a given power system the first consideration would be whether or not the control system was applicable. I would, therefore, like to ask the author what is the field of application of this control system?

If it was found that the installation of supervisory control could be justified on the given power system, the next question would be what does this system offer that is not available in other modern control systems?

An important consideration in the successful operation of automatic equipment is maintenance. Equipment may suffer not only from lack of maintenance, but from overzealous maintenance as well. Another question I would, therefore, like to ask is what form and extent of maintenance is required by this control system and is there any way in which the equipment would give warning of faulty maintenance before actual failure took place.

D. R. Pattison (nonmember; Pennsylvania Electric Company, Johnstown): We are large users of supervisory control equipment and have found this type of equipment very reliable provided it is not overmaintained. We are extremely interested in the general trend toward simplification and have always wondered why supervisory control systems outgrew the original audible type, which in our estimation is the simplest form of remote control.

It has been our belief that the operating companies should at all times keep their remote-control requirements along logical and practical lines and avoid frilly requirements which of necessity complicate the equipment.

The manufacturers have been very generous in this respect and have at all times attempted to meet the special requirements which in a large number of cases represent whims of individual engineers or operators.

It is felt that the self-checking system offers a simplification in that the back and forth checking operation is eliminated.

E. E. George (Tennessee Electric Power Company, Chattanooga): Operating experience with remote control of various kinds indicates that the author's proposal to eliminate the check back system is a step in the right direction.

To those who feel that the multiplicity of relays in extensive applications of supervisory control may involve operating delays due to testing and maintenance, is suggested that the methods of the communication companies be imitated. In dial switching systems a large group of relays are mounted on a base plate which has plug and jack connections so that the whole base with these relays can be instantly removed from the switchboard and replaced by a spare unit. The connector and selector switches used in the step-by-step dial system are good examples of this construction.

G. De Croce (nonmember; Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): Replying to Mr. Von Voigtlander, this system differs from other systems in that it is applicable to large multistation applications because in these instances the selection time remains a constant and the over-all equipment is materially reduced.

The best known method of anticipating trouble is to make visual inspections of the equipment. This inspection enables the maintainer to determine the cleanliness of the equipment and permits an observation as to the regularity of operation of the operating relays.

Mr. Pattison raises a point which should be generally digested by the industry at large, that is, the elimination of overspecial features and requirements. If this thought is carried out it will generally make for a reduction in cost and a simpler equipment.

Present-day supervisory control equipments have outgrown the audible type because it is generally accepted that a continuous indication of the position of remotely disposed devices is far better than to have an operator dial for a supervision and depend entirely upon his hearing to determine the exact position of the device.

Mr. George's contribution to this paper indicates the confidence in this type of equipment by those operating companies who have had previous experience with supervisory control.

The reliability of this type of equipment and its freedom from maintenance service has been demonstrated by the number of successful installations now in operation. It is felt that in view of present field experience the need for a removable jack-type relay where the relays are grouped becomes unnecessary.

Transmission Features of the New Telephone Sets

By A. H. INGLIS
ASSOCIATE AIEE

Synopsis: The new telephone instruments now being introduced by the Bell System result in an outstanding improvement in transmission performance in service. The evidence for this, as obtained by comprehensive laboratory and field tests, is presented here, together with a discussion of the factors responsible for this superior performance and of the consideration involved in its appraisal.

NEW TELEPHONE instruments are being applied in the plant of the Bell System to the desk stand, wall set, and handset, and result in markedly improved transmission performance. The new instruments are associated with the antisidetone feature which is also applied to the older sets already in plant. The selection of these particular designs from the wide choice made possible by new design technique, materials, and manufacturing methods, has been based on developments in the methods for quantitatively rating the relative merits of different designs. In general, there has been consistent effort over a period of years, to base these ratings primarily on performance in service rather than on laboratory tests.

The factors influencing service performance are so many, and so complicated in their relationship, and are in so many cases difficult or even impossible for the designer to evaluate or control, that their net effect on performance cannot be predicted with certainty by laboratory methods. Of necessity, such methods involve a limited selection of primary test conditions, and an even more limited selection from the possible combination of these conditions. This is particularly true in the rating of the transmission performance of a telephone set. Laboratory tests are essential in the study and analysis of design problems, and are invaluable similarly in interpolating, supplementing, and explaining service performance results. In determining the reaction on the user of the transmission features of possible designs, however, the field performance test has been found of first importance in deciding what particular characteristics to include in the new telephone instruments and circuits.

Important Transmission Characteristics of the New Telephone Sets

The specific transmission design features of the new instruments, are described elsewhere.[1] The purpose here, therefore, is to discuss primarily the outstanding improvements in performance resulting from the application of the new instruments and the antisidetone feature which has been available for some time.

These improvements are

1. Those due to the station circuit, which, as compared with the previous station circuit,

   a. Largely reduce the efficiency of the sidetone path between transmitter and receiver, without materially affecting the electrical efficiency of the set in transmitting or receiving. This means that sounds, either noise or speech, which are picked up by the transmitter, are reproduced in the receiver of the same set at a much lower level.

   b. Reduce the susceptiveness for certain types of party line sets to interference with reception by noise set up by power transmission systems.

2. Those due to the physical characteristics of the transmitter and receiver.

Several of these features have been available for some time, and have, of course, been introduced into the plant as they became available. The new transmitter and the antisidetone circuit, for example, have been standard for some years and have already been installed in large numbers.

Figure 1 shows both the new handset and the desk-stand forms of mounting, including all these features as integral parts of their design. The new desk-type transmitter and receiver can, of course, be used with wall sets.

The schematic drawing of figure 2 indicates the general arrangement of parts in the new station transmission circuit for either type of set.

In describing the results produced by these transmission features, and the methods employed in measuring and rating these results, it seems desirable to include some discussion of the characteristics of a telephone conversation as distinguished from a direct, face-to-face conversation, so that the various effects of the new circuits and instruments may be seen in as correct relative proportion and as generally comprehensible form as possible.

Some Elements of the Station Transmission Problem

In either a telephone or direct conversation, successful communication depends on the characteristics of the talker and of the listener, and their reactions to each other and to the character of their surroundings. In a direct conversation such, for example, as across a desk, the environment is in general the same for both talker and listener, and their ears are materially aided by their eyes. In a telephone conversation, however, not only may the surroundings of talker and listener be entirely different, but a third element, the telephone system, is added to the environment of each user, which complicates his reaction, not only to his own surroundings, but also to the other party to the conversation. Furthermore, for obvious economic reasons, the natural binaural reception of direct conversation, with its advantages in discriminating between sounds from different directions, is replaced in the telephone conversation by a monaural medium.

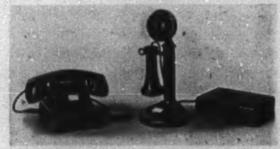

Figure 1. The new handset and desk-stand telephone instruments

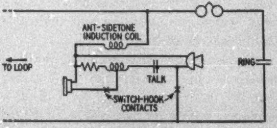

Figure 2. Schematic transmission circuit of antisidetone coil

Fundamental differences of this kind between telephone and direct conversation must be taken into account in the design of a telephone transmission system

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[1] For all numbered references, see list at end of paper.

if satisfactory results are to be obtained. For example, the talker is accustomed in a direct conversation to regulate his talking volume by what he himself hears under prevailing noise conditions (which incidentally are the same for the listener), by the ease with which he hears the other party, and by the ease with which the listener appears to hear him. By experience, under ordinary conditions, the first factor mentioned, the loudness with which the talker hears himself, probably comes to be the primary control on his talking volume.

These various factors also serve to regulate talking volumes in conversation by telephone, but their magnitudes and the relations between them differ from the condition of face-to-face air-path conversation and vary from one type of telephone connection to another. For example, the "sidetone" of the telephone set, being materially higher than the air-path sidetone, deceives the talker, not only by making him think he is talking louder than he really is, but also by apparently modifying the noise conditions under which he is talking in the pickup and amplification of room noise by his telephone transmitter. Since, in addition, the efficiency of the telephone circuit itself may be different in the two directions of transmission, the loudness heard by one party may differ more from that heard by the other than in the case of air transmission. Then, too, noise conditions may be and frequently are quite different at the two ends of the telephone circuit. Figure 3*A* shows the probability of noise of various average intensities at subscribers stations as determined by several surveys covering a large number of locations. On the assumption that any one of the stations represented by these data may with equal probability call any other one, figure 3*B* has been computed, showing the probability of noise at the two stations of a telephone connection differing by more than a certain amount. It will be noted that there is about an even chance of the noise at the two ends differing by more than 12 decibels. In view of these differences, a person's judgment of how well he is heard and understood cannot be as direct as in the case of air transmission.

In addition, the transmission over the commercial telephone system affects the quality of the received speech more than the usual room surroundings in air-path transmission. While acoustic resonance and reverberation in a room do distort speech, in the extreme case to a point where understanding may be difficult, such a condition is distinctly unusual. Equal freedom from distortion in a telephone system is a more difficult and expensive condition to obtain than in direct conversation a few feet from a listener. Something less than perfect reproduction must suffice, for the present anyway, if costs are not to be prohibitive.

All of these differences involve the acquiring by the user of a set of telephone habits which differ from those he has acquired in direct conversation. The problem of the transmission design of a practical telephone system requires, then, for a satisfactory solution, not only a determination of the proper speech levels to be delivered, and of the sidetone characteristics which will, under the conditions of a telephone conversation, give optimum results with the noise encountered, but also a decision as to what particular frequency range and characteristic to choose. Properly designed, a telephone transmission system should minimize, to the degree consistent with costs, its inherent differences from direct conversation, and make it easy for the ordinary user to get, without undue effort, results which are satisfactory to him in comparison with direct conversation.

In the earlier days of telephony, the problem presented appeared much simpler. It was, in effect, unidimensional, calling primarily for more efficient instruments and circuits; more and more power delivered to the listener's ear. While methods for the control of sidetone were not unknown, the importance of such control was not fully appreciated. Little choice was available in the quality of reproduction provided by transmitter and receiver, because of meager design knowledge.

Relatively recent and quite rapid developments in knowledge of the problems involved, in materials, methods, and measuring facilities, have now presented the necessity for a solution in essentially a three-dimensional form. These three dimensions may be described as volume, noise, and quality. The solution of the problem on this basis is obviously more difficult, and has required the development of methods for quantitatively evaluating and rating their characteristics in terms of some common yardstick.

Methods of Rating Transmission

For reasons already suggested, such a yardstick must be based on service performance—on the results obtained by actual users in the course of day-to-day telephone service.

Extensive investigation has indicated that the best comparative measure of this transmission performance in local exchange service is to be found in the time rate of the occurrence of repetitions required by subscribers for understanding telephone conversations.[2] Or, more explicitly, when two transmission conditions have the same repetition rates, all other service factors being equal, these conditions are taken to be equal with respect to transmission performance. Where two conditions are not alike it is usually possible to evaluate the difference in the repetition rates for the same users by inserting distortionless loss in the better condition until both have equal repetition rates.

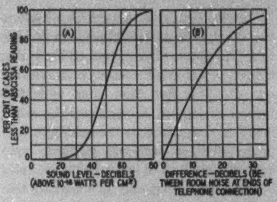


Figure 3. Noise conditions at telephone stations

Thus, by taking as a reference, a typical telephone circuit of specified make-up, the effects of various factors such as distortion, noise, attenuation, sidetone, or type of instrument, may all be expressed in the common terms of the reference circuit trunk which will give the same repetition rate.

Instead of making this adjustment in every case for the purpose of evaluating the relative performance of different test conditions for the same users, the evaluation may be made rather closely over a limited range by the following typical relation derived from repetition observations on circuits containing trunks, the losses of which were varied over a range of values.

$$db = 50 \log_{10} R_1/R_2 \text{ (see reference 3)}$$

where R_1 and R_2 are the repetition rates of two conditions under comparison, and the *db* figure is the change in the reference trunk which has the same relative effect on the repetition rate.

Such a method is, of course, somewhat cumbersome, and requires a large amount of data to iron out random variations and individual peculiarities of little general interest. But as the fundamental rating method, supplemented by laboratory test, it has been systematically used in studying the value of the antisidetone circuit and in selecting instrument characteristics.

Supplementing the repetition observations, it has been found useful in service rating to obtain data on speech levels delivered to the line for each condition observed. This has been done with the volume indicator, a vacuum-tube voltmeter so designed that the reading is approximately proportional to mean syllabic voltage.[4] The information thus obtained is useful not only in analyzing the results of service tests but in determining typical values for speech levels, necessary for laboratory tests.

Laboratory tests are of two general types, objective measurements and subjective tests. Transmission measurements cover a wide field with objectives ranging from the physical analysis and study of different designs, to the determination of over-all performance characteristics of structures and systems. It is these latter tests that we are more particularly interested in here, as most descriptive of the physical properties of importance in providing telephone transmission service.

Subjective tests in the laboratory may be said to be midway between physical measurements and field performance tests. Made under controlled and somewhat artificial conditions, they indicate quantitatively the capabilities of a telephone system in transmitting articulate speech under the particular conditions of the test. They cannot, of course, indicate the relative probability of occurrence, and hence importance, of these different conditions, nor predetermine how well the subscriber will avail himself of the capabilities provided.

Consideration of some of the results of investigations in both laboratory and field will do much to explain the rather large transmission improvement realized by the introduction of the new sets in actual service, particularly if examined with the conditions of a direct conversation as a basis of comparison.

The Station Circuit

There are two characteristics of the new station circuit of particular importance from a transmission standpoint.

REDUCTION OF SIDETONE

The first is the antisidetone induction coil through which the transmitter and receiver are coupled to the line. This coil comprises, in addition to three transformer windings, a balancing network. The circuit, made up of the four elements, transmitter, receiver, line, and network, coupled by the transformer, functions in such a manner that the transmitter and receiver are in conjugate relationship, that is, voltages produced by the transmitter are balanced out and do not affect the receiver. Theoretically, such a circuit, with pure resistance elements, can be perfectly balanced at all frequencies with complete elimination of sidetone, and at the same time be as efficient as can any transformer coupling in an invariable telephone set,[5] for the transfer of power from the transmitter to the line, and from the line to the receiver.

This type of circuit is not new in principle, and many varieties are known and have been described.[6] Many of these arrangements, for one reason or another, are not suitable for application. Some, for example, call for impedances of transmitters or receivers differing widely from those available. Certain others are not economical for common battery service, where the transmitter must receive its battery supply from the line. Still others require relatively complicated and expensive cording and switchhook arrangements. The circuit which has been chosen for general common battery subscriber station application, and shown schematically in figure 2, is not only as simple and as easily adapted to Bell System conditions as any, but permits a coil design which is economical to manufacture as well as efficient in performance. Other types of antisidetone circuit have been adopted for local battery station service and for operators' telephone sets.

The theory of operation of this antisidetone circuit has already been discussed elsewhere.[7] It is intended here to show the general purpose of the application, some of the considerations involved in the design, and the kind of results accomplished.

While in theory, complete elimination of sidetone is possible, as well as ideal efficiency of transformation, in practice, neither objective can be entirely realized. The unavoidably wide variations in line impedance looking from the set, ranging from high positive to high negative phase angle, and from a few hundred to more than a thousand ohms in magnitude, together with other practical departures from ideal conditions, necessitate a choice between a high degree of sidetone balance, and the standardization of a minimum number of coil designs. The variations in loop length and resistance, by their effect on transmitter battery supply, and consequently on transmitter resistance, furthermore cause variations in the absolute transmitting and sidetone efficiency of the terminal set, which must be taken into account in the station circuit design.

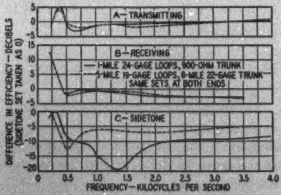


Figure 4. Circuit efficiency of antisidetone circuit versus sidetone circuit

The actual design chosen is so arranged as to favor sidetone balance on average and shorter loop conditions where transmitter battery supply is greater, with consequent higher sidetone, and to favor transmitting and receiving efficiency on longer loops where battery supply is low. Since loop losses are greater for transmitting than for receiving because of transmitter battery supply loss, the ratio of the transformer is such as to favor the transmitting efficiency of the set somewhat in comparison to the receiving efficiency. This has the advantage of raising the transmitted speech level further above line noise. The same idea, of course, was followed in the design of the sidetone set.

The resultant antisidetone circuit adopted and here discussed, as compared with the sidetone circuit previously in general use, when equipped with the same transmitter and receiver and on the same loop and trunk, reduces sidetone on the average by about ten decibels. Under the most unfavorable conditions of use, the reduction is unlikely to be less than about seven decibels compared with the corresponding sidetone connection. Under the best conditions of balance encountered the reduction may be as much as 12 decibels. On the effective basis of transmission the average net improvement in transmission which results is about six decibels.

From the electrical circuit standpoint alone, the efficiency of the antisidetone arrangement is below that of the sidetone set in the order of about one or two decibels in transmitting and in receiving, which is necessitated by the limitations of practical design and circuit conditions discussed above.

Figures 4*a* and 4*b* show for transmitting and receiving, respectively, the difference in efficiency, with respect to frequency, of the antisidetone set from the sidetone set, each with the same instruments. Two subscriber loop and trunk conditions are shown, an average loop and trunk, and a long cable connection.

Figure 4*c* shows the variation in sidetone reduction with frequency, of the new set as compared with the corresponding standard sidetone set, for the same two circuit conditions as above. The curves are indicative of the effect of variation of circuit impedance on sidetone balance, in changing not only the magnitude, but the frequency range in which the best balance occurs.

Data of this sort alone do not, of course, indicate the relative transmission performance of the two sets. The beneficial effect on the telephone user of the large reduction in sidetone must be evaluated on the same yardstick as the losses in transmitting and receiving efficiencies which, in the practical case, accompany this reduction in sidetone. McKown and Emling have shown the effect of changes of this sort on the results obtained by the ordinary telephone user, in terms of net effective transmitting and receiving loss, as determined by service observations.[8] Their data, shown in figure 5, are relative to the sidetone of a reference set. The heavy solid lines are the original experimental data, the dotted extensions to these curves being extrapolated.

In addition to the original ordinates, others are shown which are of interest. They are based on the results of loudness balance tests, and while not perhaps of great precision, do approximately indicate the relationship of the sidetone of a telephone conversation to that for direct speech, and illustrate the differences in the effects of sidetone for transmitting and for receiving.

On each curve are indicated the average sidetone value of the standard sidetone and the new antisidetone set, each, as before, with the new transmitter and receiver. There is also shown the range of sidetone for each type of set, within which practically all service conditions will fall. This indicated range takes into account not only variations in sidetone balance due to line impedance variations, but changes (with loop resistance) in battery supply to the transmitter. It should be noted that in only a few cases is the absolute sidetone of the antisidetone set on the worst sidetone conditions, as high as or higher than that of the sidetone set on the best sidetone conditions, and then only by a small amount. Furthermore, in spite of the wider variations in sidetone of the antisidetone set, these variations are over a range such that the resultant variations in effective losses are smaller than for the sidetone set.

Considering figure 5*a*, it will be noted that for either sidetone or antisidetone sets, the sidetone is louder than for natural speech sidetone, which, as noted before, makes the user think he is talking louder than he actually is. The average sidetone reduction of ten decibels for the antisidetone set results in less of this restraint on his talking level, with a resultant net effective gain in transmitting of about four decibels compared with the sidetone set.

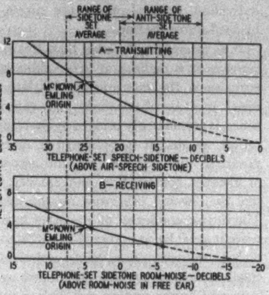


Figure 5. Effects of sidetone on user of telephone

In receiving, figure 5*b* sidetone introduces an effective loss by the reproduction in the telephone ear of room noise picked up by the transmitter. It will be noted that for the antisidetone set, the reproduced noise is in general appreciably lower than the room noise itself. Inasmuch as room noise also interferes directly with received speech in the telephone ear by leakage under the receiver cap, the contribution to the total noise of the sidetone pickup of the antisidetone set is in most cases small. This is not true for the sidetone set, where in many cases the sidetone noise may constitute the principal interfering noise. The resultant net effective gain in receiving is about two decibels compared with the sidetone set.

This is a good illustration of the type of information which can be obtained only from a field study. For example, the relationships indicated on figure 5 are dependent on how far away from the mouthpiece of the transmitter and at what level the speaker talks, and on how tightly to his ear he holds the receiver. These in turn are resultants of all the conditions of the particular telephone conversation. If incoming levels are so high as to be uncomfortable, the receiver may well be held farther away from the ear. In that event, of course, the sidetone conditions of the set become relatively less controlling. The weight, size, and shape of the instrument in his hands may similarly affect the subscriber's use of it, the results he gets, and the relative importance of various factors of telephone design.

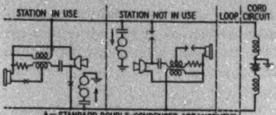
For such reasons, not only must laboratory performance tests be supplementary and subsidiary to field tests, but additional field tests must be the basis for determining the effect of any major changes in design, whether or not those changes are electrical, acoustical, or purely mechanical.

Considerations of this sort emphasize the importance of having clearly and explicitly in mind the conditions and relationships of direct conversation, as a general reference for the interpretation and explanation of the effects of telephone design on telephone conversations. The sidetone ordinates of figure 5, for example, not only suggest the difference in function of the antisidetone circuit in transmitting and receiving, but also emphasize the fact that the over-all sidetone resulting from the combination of circuit, instruments, and method of use, is the important factor rather than the sidetone circuit efficiency only. Such matters are easily lost sight of, if design is not properly co-ordinated in its correct perspective.

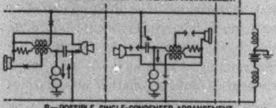
The reduction of sidetone provided by the antisidetone sets is of further advantage in two rather different ways.

In attaching a transmitter (which is an amplifier) and a receiver, to a common handle which mechanically couples the two, a condition is set up in which the gain under certain conditions may exceed the loss in the path made up of handle, air, and electrical sidetone circuit. Sustained oscillation, or howling, will then result between transmitter and receiver. Even if this point is not reached, but is approached within six decibels or so, impairment of quality results from incipient oscillation. The greater sidetone circuit loss of the antisidetone circuit provides an additional margin of safety against any such condition.

The granular carbon of the transmitter, and the design of the transmitter itself must be carefully controlled, or serious noise—transmitter "burning"—will cause noise in the receiver of the set. The mechanical and electrical wear and tear of service tends to make this transmitter noise worse. In the new transmitters this "burning" has been kept at a low inherent value throughout life. The antisidetone circuit, however, provides a margin of safety against the small amount remaining, so that with this set there is less likelihood of transmitter noise causing impairment of reception.



A—STANDARD DOUBLE-CONDENSER ARRANGEMENT



B—POSSIBLE SINGLE-CONDENSER ARRANGEMENT

Figure 6. Ringing arrangements for party-line service

REDUCED SUSCEPTIVENESS TO INTERFERENCE

It will be noted from the schematic circuit drawing figure 2 that two condensers are used in the new sets, one in the antisidetone transmission circuit, and a separate one with the ringer. In some types of party-line practice, the ringer of the set is connected for some parties from one side of the line, and for the others from the other side to ground.

Figure 6 shows schematically two such ringing arrangements during the conditions of conversation, 6*a* as used in the new sets, and 6*b* with one condenser common to transmission and ringing circuits. It will be noted that in the standard circuit adopted (figure 6*a*), if any longitudinal noise voltages exist between the central office and station grounds, there is an equal voltage drop from each side of the line to ground through the ringing paths (assuming the two ringer condenser paths to be identical). The voltage drop across the terminals of the talking set is therefore zero and no noise results.

If the arrangement of figure 6*b*, corresponding closely to previous designs, were used, however, this condition would not obtain. The condenser of the station in use being common to the transmission circuit as well as the ringing circuit, the noise voltage drop across this condenser is introduced in the transmission circuit. In addition there are other paths to ground from each side of the line through the transmission circuit which are not of equal impedance. The net result is a residual noise current through the receiver of the talking circuit.

In the actual case, the impedance of all ringers and condensers is not identical and there are often more parties connected to one side of the line than the other. Even under these relatively unfavorable conditions, however, the two-condenser arrangement adopted reduces the susceptiveness of the set to interfering noise by as much as 15 decibels. A further material reduction is realized by the high impedance of the ringer used in the new sets, so that in most cases interfering noise at grounded ringer stations will not differ materially from that at individual stations where the ringer is bridged across the line.

It is interesting to note that this improvement is realized at little additional cost, since the transmission condenser, which must be of relatively high capacitance, is permanently bridged by the transmitter, so that it is protected from exposure to any large voltages, and may be of cheaper construction and smaller in size than would otherwise be the case. The ringing condenser on the other hand, while it must be constructed to withstand higher voltages, may be of relatively small capacitance, which gives more uniform and better ringing and dialing performance.

Characteristics of Transmitter and Receiver

Since the individual design characteristics of the new transmitter and receiver are discussed elsewhere,[1] attention here will be centered on the over-all effects of these characteristics in the complete transmission system, as indicated both by laboratory and by field test. As stated before, the problem may be more or less arbitrarily separated into three correlated problems—volume, quality, and noise.

As in the case of sidetone, these problems appear, perhaps, more nearly in a proper perspective, if considered in comparison with the corresponding factors in a direct conversation. It must be remembered that telephone service does not consist in the provision of a mechanism, per se, but in the provision of facilities for conversation, to which the mechanism should be incidental, however important. Since the inherent conditions of such a conversation are quite different in many respects from those of a direct conversation with which, consciously or unconsciously, it will be compared in its overall results, the parallelism in detail should not be too exact. Departures from the conditions of direct conversation in certain respects which are relatively unavoidable, may be best compensated for by deliberate departure in certain other respects. For example, the physical absence of one party to the telephone conversation, and the monaural nature of such a conversation, may be partially compensated for by delivering to the ear of the listener a somewhat higher speech level than he is accustomed to in direct conversation. The limitation of frequency band width imposed on the telephone medium, largely for economic reasons, may be minimized in its effects if the transmission characteristics in the available band are other than a facsimile of the corresponding band in direct conversation. All such measures must be employed with knowledge of their effect on the ultimate objective, that the telephone conversation may be easy and natural.

GENERAL REQUIREMENTS

It is easily seen that for any particular over-all frequency characteristic of a telephone transmission system, there are practically an infinite number of ways in which it can be split up between transmitting and receiving characteristics. From this standpoint alone, then, there is no particular "best" transmitter or receiver frequency response. From other standpoints, however, certain general types of individual characteristics, both in frequency and efficiency, are to be preferred to others, particularly when considered in their practical application to an already existing telephone system. It has been pointed out[9] that, in general, development has been toward a telephone system where both transmitter and receiver are relatively uniform in their frequency characteristics. Induced noise appears to be so evenly distributed with frequency that such response would not appear to magnify the interference problem.

Transmitting efficiency should be as high as required to keep the speech well above induced noise but not so high as to cause excessive crosstalk into other telephone circuits. The maximum desirable received level is determined for a given telephone system by the limitations of the human ear in accepting with comfort speech levels above a certain intensity. Finally, the practical necessity of working as satisfactorily as possible in conjunction with the telephone transmitters, receivers, and sets in the existing plant during the period of transition, places a practical limitation on the amount of change that is desirable in relative levels of either transmitting or receiving.

With regard to frequency range, previous work[10] indicated the desirability of designing circuits to transmit frequencies from 200 or 300 cycles up to about 3,000 cycles. Gains in articulation and naturalness are realized by increases in this band width, but are progressively smaller for successive equal increments in frequency. A 3,000-cycle band properly used gives good transmission both in articulation and naturalness, but frequency limitation is essentially an economic one, subject to change as conditions change. Recent work on the new multiple-channel carrier systems has indicated justification in these systems for providing a somewhat wider band, from about 150 to about 3,500 cycles.[11]

OVER-ALL FREQUENCY RESPONSE

In describing the frequency characteristic of a transmission system it has become customary to refer to it as more or less "flat," where "flat" is assumed to be synonymous with "perfect" as far as the relative transmission of various frequencies is concerned. In measurements of the elements of an electrical circuit, from which this terminology came, the word is useful since, when the measurements are properly made, at any rate, the basis of comparison implied by the word "flat" is generally understood. This is also true, although probably to a more limited extent than is generally realized, when the term is applied to electroacoustic transmission systems, where free progressive, plane air waves of various frequencies are transferred to an electrical system, or vice versa, by means of microphones or loud speakers.

In the case of a telephone system, however, where a transmitter is placed close to the lips, and a receiver directly to one ear, and where the air waves are not free progressive or plane, use of the word "flat" implies a basis of comparison which is not self-evident. Much effort has been given recently to establishing an appropriate reference system, sufficiently simple in concept and ease of specification, to be useful in this connection. The result of this work has been a reference telephone system which, when spoken into, would give the listener in all respects essentially the identical sensation he receives in one ear when facing the speaker directly, with an air path one meter long between the speaker's mouth and the listener's ear, in surroundings without reverberation or noise. Such a reference transmission system has tentatively been called an "orthotelephonic" system.

The point of interest here is that when measured by any suitable objective method, the frequency characteristic of this "orthotelephonic" telephone reference system is not "flat" by a considerable amount in the ordinarily accepted usage of the term. This departure from "flatness" is caused by such factors as the frequency directive characteristic of the mouth, cavity resonance of the ear, and disturbance of the sound field by the head. The individual contribution of some of these factors is not as yet definitely determined.

Furthermore, for reasons mentioned, it is not self-evident that a practical telephone system of limited frequency range, should be "flat" with respect to the corresponding frequency band in this more or less basic orthotelephonic system which is not limited in frequency range. Having decided on the band width that is desirable and justifiable, it must still be determined, therefore, what particular frequency characteristics are preferable in this band.

In selecting from the many possible choices, the particular frequency response that seems best, several factors must be taken into account. This has been done by a study (under the conditions of actual service) of the relative results of several different experimental instrument designs, varying in frequency characteristics. The over-all frequency characteristics of the resultant choice are indicated for two typical circuit conditions in figure 7. These measurements were made with the artificial mouth and ear[12] and are plotted with reference to corresponding measurements on an orthotelephonic reference telephone system. For comparison, the results of similar tests of the earlier Bell System handset[13] are shown also.

In considering these over-all telephone-system frequency-response characteristics in the light of previous discussion, there are several points of interest:

1. The large increase in response at both higher and lower frequencies with respect to the older handset, which in itself was a notable advance in this respect over previous types. This increase amounts to ten decibels or more from about 200 to 500 cycles and from about 1,700 to 3,000 cycles. This wider frequency range gives better naturalness of reproduction.

2. The type of the response. The general uniformity and absence of any marked resonance or irregularity is obvious. For either average or long loops the entire band from about 300 to over 3,000 cycles lies within a range of 15 decibels. It will be noted, however, that, for the average condition, the response at the higher frequencies (1,500–3,000 cycles) is distinctly above that for the frequencies below 1,500 cycles. This characteristic aids materially in the understanding of the low-intensity consonant sounds. The response on the longer loops would undoubtedly be correspondingly better if the high frequencies were raised so that the over-all characteristic more nearly resembled that for the average condition shown. It should be remembered, however, that for a standard set, to be used on all loop conditions, a response designed solely for the long loop, with its large loss at high frequencies, would be distinctly "tinny" and disagreeable in quality on average or short loops.

3. The materially smaller losses in the transmitted band for the average telephone connection than for the orthotelephonic system. Even for the long loop conditions the losses are no greater than for this orthotelephonic system up to about 2,200 cycles. In other words, for a large majority of telephone calls the received speech level will be higher for the same talking level than in the case of direct conversation at one meter distance. The desirability of this in a monaural system of limited frequency range has already been indicated.

Combined Effects of Circuit and Instruments

The part played by the antisidetone circuit in permitting better utilization by the subscriber of the capabilities of the telephone system, in increasing the level of received speech, and minimizing the effect of noise is summarized by the illustrative power level diagrams in figure 8, using typical values of room and line noise and other conditions.

The data shown were obtained by objective measurements with the artificial mouth, sound level meter, volume indicator, and artificial ear. Frequency weightings appropriate to the levels involved were employed in the sound level measurements.

Figure 8*a* shows, for the antisidetone set, the losses between the talker and listener under average room noise and circuit conditions. Figure 8*b* gives corresponding information for the sidetone set. For comparison, an approximate level diagram is shown also for direct speech. The relative speech levels at the transmitter for the two telephone conditions were adjusted to give volume indicator readings on the line in accordance with the results of service observations. The upper curves in each drawing

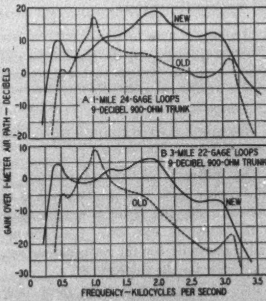
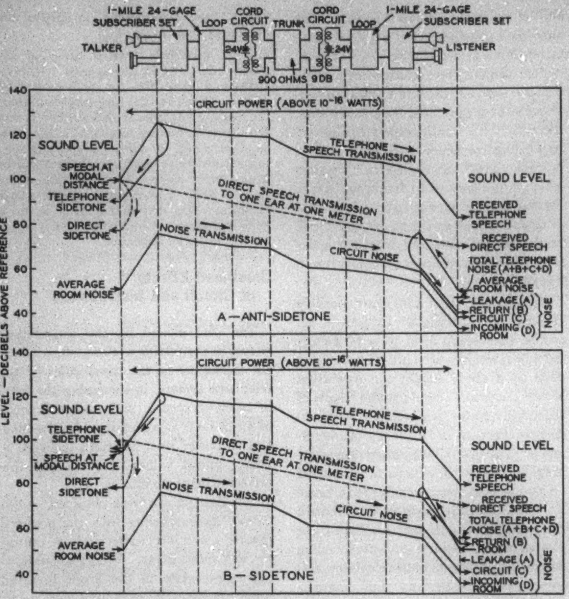


Figure 7. Over-all orthotelephonic frequency response of typical telephone connections



1-MILE 24-GAGE
SUBSCRIBER SET
LOOP
CORD CIRCUIT
TRUNK
CORD CIRCUIT
LOOP
1-MILE 24-GAGE
SUBSCRIBER SET
TALKER
LISTENER
24V
24V
900 OHMS 9 DB
CIRCUIT POWER (ABOVE 10⁻¹⁶ WATTS)
SOUND LEVEL
SPEECH AT MODAL DISTANCE
TELEPHONE SIDETONE
DIRECT SIDETONE
AVERAGE ROOM NOISE
TELEPHONE SPEECH TRANSMISSION
DIRECT SPEECH TRANSMISSION TO ONE EAR AT ONE METER
NOISE TRANSMISSION
CIRCUIT NOISE
SOUND LEVEL
RECEIVED TELEPHONE SPEECH
RECEIVED DIRECT SPEECH
TOTAL TELEPHONE NOISE (A+B+C+D)
AVERAGE ROOM NOISE
LEAKAGE (A)
RETURN (B)
CIRCUIT (C)
INCOMING ROOM (D)
NOISE
A — ANTI-SIDETONE
B — SIDETONE
LEVEL — DECIBELS ABOVE REFERENCE
140
120
100
80
60
40

Figure 8. Power-level diagrams of typical telephone connections

are for speech and the lower for noise picked up by the transmitters.

At the transmitting end, the lower sidetone of the antisidetone set results in higher talking levels, with about four decibels higher speech level at the input to the circuit. In the over-all circuit, this gain is increased by the receiving end effect of the antisidetone circuit in minimizing the effect of room noise.

The total noise in the telephone ear, as shown on the drawing, has as its principal contributing factors,

1. Leakage of noise under the receiver cap.
2. Noise picked up by the transmitter and returned via the sidetone path and receiver to the listener's ear—termed "return noise."
3. Circuit noise.
4. Room noise picked up at the far end and transmitted over the circuit.

The different relative contribution of the "return noise" for the two sets is of interest. The net result is a total noise in the telephone ear, lower than the actual room noise for the antisidetone set, and higher for the sidetone set.

For the circuit and room noise conditions shown, the ratio between received speech level and noise is about 25 decibels for the sidetone condition, and 35 decibels for the antisidetone. The corresponding ratio for air transmission to one ear under the conditions shown is the order of 20 decibels.

Results of Laboratory and Field Performance Tests

It has been seen that the new sets are superior in volume and in minimizing the disturbances of noise. The frequency measurements just discussed have indicated marked superiority also in the quality of reproduction.

One measure of the effect of this reduced distortion is by means of the articulation test. Such tests have shown that for a typical telephone system equipped with the new telephone sets, 95 per cent of the letter sounds spoken into the transmitter are correctly understood by the listener. With air transmission, 98 or 99 per cent of letter sounds are correctly understood. The difference is almost entirely due to the broader frequency band transmitted by the air path.

With the final designs of the new sets, tests have been made by the methods developed for determining "effective" transmission.[8] The results of these tests have shown that the new sets under the conditions of actual service provide a marked advance in transmission performance. The average total transmitting and receiving gain is about 15 decibels on the effective basis of transmission, as compared to sets of the sidetone type used with the older type of instruments.[13]

Conclusion

In general, it appears that the notable transmission improvement which has been achieved in the design of the new telephone sets, in their freedom from distortion, higher effective volume, lower sidetone, and general convenience in use, makes possible a closer approach to the ease of a direct conversation than has hitherto been possible commercially.

Undoubtedly, further improvements in station transmission performance will, as in the past, be forthcoming with advances in the technique of design and manufacture, and in further knowledge of the requirements of the problem.

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Noise Co-ordination of Rural Power and Telephone Systems

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THE co-ordination of telephone and power systems has been studied for about 15 years under the auspices of the Joint Subcommittee on Development and Research of the Edison Electric Institute and the Bell Telephone System. Until 1935 the noise frequency studies on distribution systems related chiefly to the types of systems employed in urban and suburban areas.[1] Between 1935 and 1937, the work was extended to include rural power and telephone systems, which differ in several respects from the systems employed in urban areas. Measurements of power circuit influence and telephone circuit noise were made on a number of representative rural systems in Nebraska, Iowa, Michigan, Ohio, Indiana, and Virginia, with the co-operation of the operating companies. These tests were supplemented by a theoretical analysis of the problem.

The investigation of the joint subcommittee indicates that it is generally practicable to secure satisfactory co-ordination of power and telephone facilities in rural areas from the noise standpoint with the types of power and telephone systems ordinarily employed when the inductive influence of the power circuits and the inductive susceptiveness of the telephone circuits are properly controlled. Such control of influence and susceptiveness includes the following:

1. Maintenance of relatively good voltage wave shape on the power system (i.e., harmonics arising in generators, transformers, and loads confined to relatively small magnitudes).
2. Suitable distribution of the loads and lengths of single-phase branches among the phases of three-phase power circuits.
3. Use of metallic telephone circuits with series and shunt unbalances to ground maintained within reasonable limits.
4. Suitable transposition of the telephone wires with respect to the power line.
5. Use of the higher impedance type of ringers where grounded ringers are required for party line service.

In particular cases, special co-ordinative measures may need to be employed in order to secure satisfactory conditions. The purpose of this paper is to discuss the factors affecting the inductive influence of rural power circuits and the inductive susceptiveness of rural telephone circuits. A summary is included of the more promising co-ordinative measures at present available for use in particular situations where such may be required. No attempt is made to present the large body of experimental and theoretical material which has been collected during this study, since this is available in other publications.[2]

The types of power distribution circuits usually encountered in rural areas are illustrated in figure 1, with the types of three-phase main lines and single-phase extensions indicated separately. A distinction is made between single-phase extensions directly connected to three-phase systems and those supplied through a step-down or step-up transformer. The latter arrangement is frequently used to provide an extension of different type or voltage from an existing system. In the systems tested during this study, these single-phase supply transformers ranged in capacity from 15 to 100 kva. In some cases, three-phase rural distribution systems serve also as lower voltage transmission systems supplying power to small towns in which a lower primary voltage is used.

The power distribution circuits in urban or suburban areas are generally relatively short and operate at high load densities. In rural areas, however, the distribution circuits tend to be longer and to operate at lighter load densities. On rural lines, therefore, harmonic charging currents and propagation effects, which are usually negligible in urban areas, may become of primary importance. The connected capacity of single-phase load transformers on the circuits tested ranged between 2 and 20 kva per mile of circuit. The lengths of the three-phase circuits tested ranged up to 65 miles, and individual single-phase extensions as long as 30 miles were encountered.

The inductive influence of a power system from a noise standpoint is determined by the magnitudes of the balanced and residual harmonic voltages and currents existing at various points. That portion of the harmonic residual current which returns in a path remote from the line conductors is often termed the "ground-return" current. The telephone influence factor (*T.I.F.*) of a power system current or voltage[3,4] in an inductive exposure is used as an index of the influence of that particular current or voltage on the noise-frequency induction in the exposed telephone circuits. It is the usual practice to express the influence of a current in terms of the product of its magnitude in amperes times its *T.I.F.*, this product being abbreviated as $I \cdot T$. Similarly the influence of a voltage is expressed as the product of its magnitude in kilovolts times its *T.I.F.*, this quantity being abbreviated as $Kv \cdot T$. These terms are used in this paper to express the influence of the balanced and ground-return currents, and balanced and residual voltages of rural power circuits.

Co-ordinated tests in exposure sections at roadway separations have shown that ground-return harmonic currents and harmonic residual voltages are generally the controlling factors in the influence of rural distribution circuits, rather than the corresponding balanced components. In exposures to the main power line, or whenever several miles of line extend beyond the exposure, the wave shape and magnitude of the *ground-return current* ($I \cdot T$) is usually the controlling factor in the over-all influence. For exposures near the end of a main line, or at any

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[1] For all numbered references, see list at end of paper.

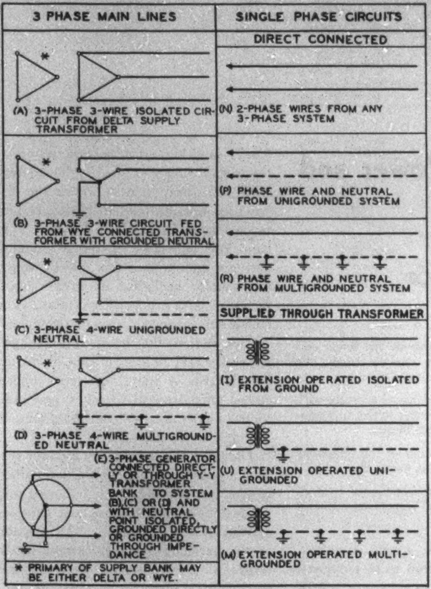

3 PHASE MAIN LINES
SINGLE PHASE CIRCUITS
DIRECT CONNECTED
(A) 3-PHASE 3-WIRE ISOLATED CIRCUIT FROM DELTA SUPPLY TRANSFORMER
(N) 2-PHASE WIRES FROM ANY 3-PHASE SYSTEM
(B) 3-PHASE 3-WIRE CIRCUIT FED FROM WYE CONNECTED TRANSFORMER WITH GROUNDED NEUTRAL
(P) PHASE WIRE AND NEUTRAL FROM UNIGROUNDED SYSTEM
(R) PHASE WIRE AND NEUTRAL FROM MULTIGROUNDED SYSTEM
(C) 3-PHASE 4-WIRE UNIGROUNDED NEUTRAL
SUPPLIED THROUGH TRANSFORMER
(I) EXTENSION OPERATED ISOLATED FROM GROUND
(D) 3-PHASE 4-WIRE MULTIGROUNDED NEUTRAL
(E) 3-PHASE GENERATOR CONNECTED DIRECTLY OR THROUGH Y-Y TRANSFORMER BANK TO SYSTEM (B),(C) OR (D) AND WITH NEUTRAL POINT ISOLATED, GROUNDED DIRECTLY OR GROUNDED THROUGH IMPEDANCE
(U) EXTENSION OPERATED UNIGROUNDED
(M) EXTENSION OPERATED MULTIGROUNDED
* PRIMARY OF SUPPLY BANK MAY BE EITHER DELTA OR WYE.

Figure 1. Types of rural power distribution systems

point along a short single-phase extension, the wave shape and magnitude of the *residual voltage* (*Kv·T*) is usually the most important factor. Therefore, these residual components are of chief interest in any discussion of the influence of rural power circuits.

Residual Harmonic Voltages

Residual voltages appear on three-phase power systems operated isolated from ground (figure 1*A*) as a result of the action of the balanced voltages on the unbalanced impedances-to-ground of the phase conductors. These impedances depend chiefly upon the length of circuit connected to each phase. Consequently, the residual voltages on a three-phase isolated system tend to be least when equal lengths of extensions are connected to each phase. Practically no residual voltage exists on a single-phase circuit supplied through a single-phase transformer and operated isolated from ground (figure 1*I*).

On three-phase systems operated with grounded neutral, residual harmonic voltages may either (1) be impressed directly upon the system, as in the case of triple harmonic voltages arising in direct-connected generators or a wye-wye transformer bank without tertiary delta, or (2) arise as a result of the action of balanced voltages upon unequal impedances-to-ground of the line conductors or connected loads. On single-phase extensions consisting of a phase wire and multigrounded neutral, the residual voltage is substantially equal to the phase-to-neutral voltage, since the neutral is at ground potential. The same is true of a unigrounded extension at the point where the neutral is grounded. However at points remote from the grounding point, the harmonic voltage from the neutral wire to ground may be comparable with the harmonic voltage from the phase wire to ground. The vector sum of the two voltages-to-ground (residual voltage) may therefore be considerably different from the voltage between the phase and neutral wires.

The range of residual *Kv·T* measured on various types of rural power circuits, in relation to the corresponding phase-to-ground *Kv·T*, is indicated in table I.

Ground-Return Harmonic Currents

The components which may contribute to the ground-return current (*I·T*) in a rural power line may be conveniently classified as:

(*a*) Harmonic charging currents.

(*b*) Harmonic components of the load current.

(*c*) Harmonic exciting current components arising in transformers and harmonics originating in connected loads.

The equivalent circuits of figure 2 illustrate the behavior of these three components on different types of single-phase lines. In the case of circuits 2*A* and 2*B*, the ground-return *I·T* is practically all

Table I. Summary of Influence of Rural Power Circuit Voltages

Type of Power Circuit	Number Cases Tested	Voltage T.I.F. Avg.	Voltage T.I.F. Min.	Voltage T.I.F. Max.	Phase-to-Ground Kv·T Avg.	Phase-to-Ground Kv·T Min.	Phase-to-Ground Kv·T Max.	Ratio Res. Kv·T / Ph-Gnd Kv·T Avg.	Ratio Res. Kv·T / Ph-Gnd Kv·T Min.	Ratio Res. Kv·T / Ph-Gnd Kv·T Max.
Three phase—11-13 kv										
Three-wire delta	1	13			87			0.14		
Four-wire-uni—from Δ-Y banks	7	17	7	36	124	55	250	1.1	0.16	2.1
Four-wire-uni—from direct connected generator	8	13	6	24	95	50	170	1.7	1.3	2.2
Four-wire-multi—from Δ-Y banks	12	21	8	62	151	60	450	0.9	0.4	1.8
Four-wire-multi—from direct connected generator	9	14	9	19	106	70	150	1.8	0.92	2.13
Single phase directly connected to three phase										
11 kv Delta	2	20	15	24	127	95	159	0.71	0.47	0.95
7.6 kv Delta	1	12			50			0.74		
Uni	4	17	14	22	128	110	165	1.03	1.0	1.08
Multi	4	17	11	27	129	85	210	1.0	1.0	1.0
2.3 kv Delta	3	26	8	40	36	11	55	0.49	0.40	0.62
Uni	3	19	16	23	46	39	56	1.0	1.0	1.0
Multi	4	21	19	23	49	46	56	1.0	1.0	1.0
Single phase supplied through transformer										
6.9–7.2 kv Isolated	1	17			56			0.006		
Uni	8	39	16	92	276	105	660	1.0	1.0	1.0
Multi	11	34	12	104	246	83	750	1.0	1.0	1.0
4.8 kv Isolated	2	37	35	38	88	85	90	0.034	0.034	0.036
Uni	3	40	13	57	191	64	280	1.02	0.95	1.06
Multi	8	53	11	172	258	50	830	1.0	1.0	1.0
2.4 kv Isolated	2	40	35	45	46	40	52	0.017	0.015	0.02
Uni	2	37	30	44	90	72	108	0.97	1.0	0.93
Multi	3	40	13	55	93	30	130	1.0	1.0	1.0

due to charging current, the load and exciting currents being confined to the line wires. The charging current is a function of the residual voltage impressed on the circuit and the impedance-to-ground, which in turn is dependent on the extent of the system. The impedance-to-ground of these systems is not affected by loads, although the voltage wave shape of the system may be modified by the presence of load, or harmonics generated in loads.

In the case of the multigrounded circuit (figure 2*C*), the ground-return $I \cdot T$ is contributed to by harmonic components of (1) ground-return charging currents, (2) load currents, and (3) transformer exciting currents.

The magnitude of the ground-return charging current component at any frequency depends upon the voltage impressed on the ground-return circuit and on the impedance-to-ground[3] of the circuit, neglecting loads, as seen from the point where the voltage is impressed. Sample curves of the calculated impedance-to-ground of a single-phase line are given in figure 3, showing the effect of frequency and circuit length. The maximum current results when the circuit length is such that series resonance (quarter wave length) occurs at the particular frequency in question. The higher the frequency, the shorter will be the circuit length for which resonance occurs.

Comparing the curves of figure 3 for the multigrounded and unigrounded condition, it appears that the impedance-to-ground of the *shorter* multigrounded circuits (i.e., less than quarter wave length) is higher than that of the comparable unigrounded circuit at any given frequency. This is one reason for the decrease in the ground-return charging current which often results when a circuit is converted from unigrounded to multigrounded operation. An additional effect results from the reactive component of the load current, which combines with the charging current at such an angle that further reduction in the ground-return current may result in the multigrounded case. In cases where the conversion results in changes in the residual voltage, or where the system is extensive (between quarter and half wave length), the change from uni- to multigrounded operation may either decrease or increase the ground-return charging current at a particular frequency.

Table III. Summary of Influence of Rural Power Circuit Currents

Type of Power Circuit	Number Cases Tested	Voltage T.I.F.			Phase I·T Product			Ratio Gnd. Ret. I·T / Phase I·T		
		Avg.	Min.	Max.	Avg.	Min.	Max.	Avg.	Min.	Max.
Three phase—11-13 kv										
Three-wire delta	4	13			220	100	320	0.55	0.16	0.80
Four-wire-uni—from Δ-Y banks	7	17	7	36	400	40	790	0.42	0.16	0.92
Four-wire-uni—from direct connected generator	9	13	6	24	430	60	650	0.79	0.34	1.17
Four-wire-multi — from Δ-Y banks	12	21	8	62	515	40	1730	0.36	0.10	0.81
Four-wire-multi — from direct connected generators	9	14	9	19	470	60	820	0.6	0.51	0.83
Single phase directly connected to three phase										
7.2–7.6 kv Delta	1	12			110			0.41		
Uni	4	17	14	22	170	45	250	0.82	0.78	0.86
Multi	4	17	11	27	170	40	250	0.62	0.58	0.64
2.3 kv Delta	3	26	8	40	175	38	400	0.09	0.07	0.11
Uni	3	19	16	23	72	7	140	0.44	0.39	0.50
Multi	4	21	19	23	90	7	175	0.51	0.43	0.62
Single phase supplied through transformer										
6.9–7.2 kv Isolated	1	17			160			0+		
Uni	9	39	16	92	106	23	350	0.75	0.28	0.94
Multi	12	34	12	104	143	30	535	0.61	0.50	0.65
4.8 kv Isolated	2	37	35	38	170	50	290	0+	0+	0+
Uni	3	40	13	57	215	80	430	0.82	0.63	0.96
Multi	10	53	11	172	146	32	560	0.60	0.50	0.70
2.4 kv Isolated	2	40	35	45	200	110	285	0+	0+	0+
Uni	2	37	30	44	245	130	360	0.30	0.28	0.31
Multi	3	40	13	55	240	85	480	0.60	0.49	0.69

Table II. Average Ground-Return I·T and 180, 300, and 420-Cycle Currents in Single-Phase Multigrounded Circuits Due to Transformer Exciting Current

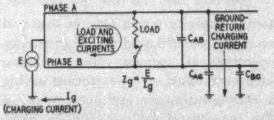
Primary Voltage	Primary Ground-Return* I·T and Mils in Ground Per Kilovolt-ampere of Connected Single-Phase Transformer			
	I·T	Mils, 180 Cycles	Mils, 300 Cycles	Mils, 420 Cycles
2,400	0.6	5.0	1.3	0.3
4,800	0.3	2.5	0.6	0.15
6,900–7,600	0.2	1.7	0.4	0.10

* These ground-return values are taken as 60 per cent of the phase values.

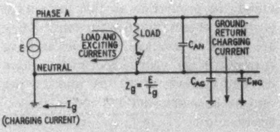
As indicated in figure 2*C*, on a *multigrounded* circuit the loads are in parallel with the capacitance-to-ground and the ground-return components of the load and transformer exciting currents, therefore, add vectorially to the ground-return harmonic charging currents. In the case of multigrounded single-phase circuits having phase and neutral wires of the same size and material and of the usual configurations, about 60 per cent of the phase current returns through the earth and the remaining 40 per cent in the neutral wire. This division holds for the charging current as well as for the load and exciting components.

In the single-phase unigrounded line, large phase displacements occur between the phase wire, neutral wire, and ground-return currents. Consequently the ground-return $I \cdot T$ expressed in per cent of the phase $I \cdot T$ may range from about 40 to well over 100 per cent. For the fundamental frequency and the lower harmonics, the percentage of phase current returning in the ground is smaller in the unigrounded than in the multigrounded circuit.

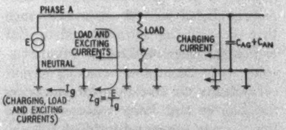
Table II summarizes the average exciting ground-return $I \cdot T$ and the individual ground-return current components at 180, 300, and 420 cycles per kilovolt-ampere of connected trans-



(*A*) Circuit consisting of two phase wires



(*B*) Circuit consisting of one phase wire and unigrounded neutral



(C) Circuit consisting of one phase wire and multigrounded neutral

Figure 2. Schematic circuits of single-phase rural power lines

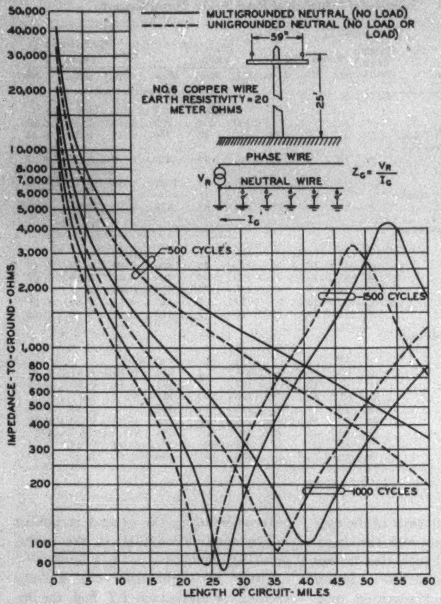


Figure 3. Impedance-to-ground of single-phase power line

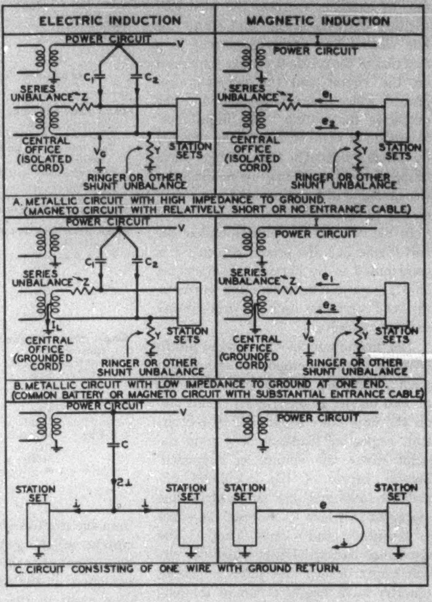

ELECTRIC INDUCTION
MAGNETIC INDUCTION
POWER CIRCUIT
V
I
C_1
C_2
SERIES UNBALANCE
Z
e_1
e_2
CENTRAL OFFICE (ISOLATED CORD)
V_G
Y
STATION SETS
RINGER OR OTHER SHUNT UNBALANCE
A. METALLIC CIRCUIT WITH HIGH IMPEDANCE TO GROUND. (MAGNETO CIRCUIT WITH RELATIVELY SHORT OR NO ENTRANCE CABLE)
I_L
CENTRAL OFFICE (GROUNDED CORD)
B. METALLIC CIRCUIT WITH LOW IMPEDANCE TO GROUND AT ONE END. (COMMON BATTERY OR MAGNETO CIRCUIT WITH SUBSTANTIAL ENTRANCE CABLE)
C
$2I$
STATION SET
e
C. CIRCUIT CONSISTING OF ONE WIRE WITH GROUND RETURN.

Figure 4. Schematic diagrams illustrating induction and susceptiveness of rural telephone circuits

former capacity which may be expected on multigrounded single-phase extensions operating at various voltages. These values are based on a transformer having a root-mean-square exciting current of five per cent of the rated full-load current, and increase or decrease approximately in proportion as this percentage is greater or less than five per cent. They are based on average harmonic exciting currents, in per cent of root-mean-square exciting current, of 40, 10, and 2.5 per cent at 180, 300, and 420 cycles, respectively.

The relative magnitudes of the three components contributing to the ground-return $I \cdot T$ of a rural line operated with multigrounded neutral are usually such that the charging currents tend to control on the longer and higher voltage circuits, while the load and exciting current components are apt to predominate on the shorter and lower voltage circuits. For a given impressed residual voltage, the ground-return charging current of a circuit operated with multigrounded neutral is usually somewhat less than that of the same circuit operated unigrounded. Consequently, where the charging current is controlling in the multigrounded case, the inductive influence tends to be less for multigrounded neutral than for isolated or unigrounded neutral operation. Conversely, where load and exciting currents become the controlling factors under the multigrounded neutral condition, the influence is usually greater for multigrounded operation.

The range of ground-return $I \cdot T$ observed on a number of rural lines of various voltages and types is indicated in table III. In this table the ground-return $I \cdot T$ is expressed in terms of the corresponding phase $I \cdot T$. The range of phase $I \cdot T$ and voltage *T.I.F.* observed in the various systems is also given for reference.

Comparison of Influence of Different Types of Systems

The relative influence under average wave shape conditions of different neutral grounding arrangements for systems of a given voltage class may be summarized as follows:

(1) Three-Phase Systems

In the *6.9 to 13.2-kv voltage range* the inductive influence of an extensive three-phase four-wire rural distribution system with metallically connected single-phase extensions is on the average, about the same for either multigrounded or unigrounded neutral operation. In this voltage range, the influence of the three-phase three-wire delta circuits tested was within the limits observed on multigrounded and unigrounded neutral four-wire systems.

In the *2.3 to 4-kv voltage range* the inductive influence of four-wire multigrounded neutral rural systems is generally somewhat higher than that of four-wire unigrounded or three-wire systems.

(2) Single-Phase Extensions

In the *7-kv class* single-phase extensions connected either metallically or through transformers have a somewhat lower influence for multigrounded neutral operation than for unigrounded operation. Extensions consisting of two-phase wires metallically connected to a three-wire system have an influence comparable to the unigrounded extension.

Extensions in the *4.8-kv class* have generally about the same influence whether operated unigrounded or multigrounded.

In the *2.3-kv class* the influence of multigrounded neutral extensions is generally greater than for those operated unigrounded.

In all voltage classes, extensions supplied

through single-phase transformers and operated isolated from ground have a very much lower influence than those operated ungrounded or multigrounded.

It is difficult to make similar comparisons between systems operating in different voltage classes because the different voltage classes tend to have somewhat different fields of application, resulting in differences in line length and load density.

Characteristics of Rural Telephone Circuits

Most of the telephone circuits used in rural areas are of the magneto type, although some common battery lines are encountered. From the noise co-ordination standpoint, there is one fundamental difference between the magneto circuit and the common battery circuit in that the former is generally ungrounded at the central office end, while the latter is grounded through the central office cord circuit. The significance of this difference will be brought out in the brief summary, given below, of the processes by which noise induction takes place. For the sake of simplicity, this discussion is confined to electrically short lines.

Schematic diagrams illustrating (*A*) an exposed metallic telephone circuit having a high impedance-to-ground and (*B*) a similarly exposed circuit operated with a low impedance-to-ground, are shown in figure 4.

Electric induction appears as a noise potential (V_G in figure 4*A*) between the exposed telephone conductors and ground. This potential is a function of the harmonic voltages on the power conductors, the mutual capacitances C_1 and C_2, and the impedance-to-ground of the telephone circuit. In the case of a telephone circuit isolated from ground, as in figure 4*A*, the noise potential to ground tends to be relatively high, the noise current flowing to ground through the capacitance-to-ground of the circuit. Where one end of the telephone circuit is grounded, as through the cord circuit on a common battery line (figure 4*B*) or through the capacitance-to-ground of an appreciable length of entrance cable, the noise potential to ground due to electric induction may become very small. In this case a noise current (I_L) flows over the two conductors in parallel toward the low-impedance ground.

Magnetic induction appears as a noise voltage (e_1 and e_2, figure 4*A*) acting in the longitudinal circuit of the telephone line. The induced voltage is a function of the harmonic currents in the power line and mutual inductance between the power and telephone conductors. The impedance of the telephone longitudinal circuit controls the magnitude of the resulting longitudinal current. Where the impedance-to-ground is high, as in figure 4*A*, the magnetically induced voltage-to-ground divides between the two ends of the circuit roughly in proportion to their impedances-to-ground. Where one end of the circuit is grounded, the entire induced voltage appears at the other end.

In the case of electric induction, differences in the capacitances C_1 and C_2 result in a difference of potential between the two telephone conductors, thus giving rise to a metallic-circuit noise current known as the "direct metallic" induction. Differences between the magnetically induced voltages e_1 and e_2 similarly result in a "direct metallic" component of magnetic induction. Telephone circuit transpositions tend to equalize the capacitances C_1 and C_2 and the voltages e_1 and e_2 and may thus greatly reduce the noise due to "direct metallic induction."

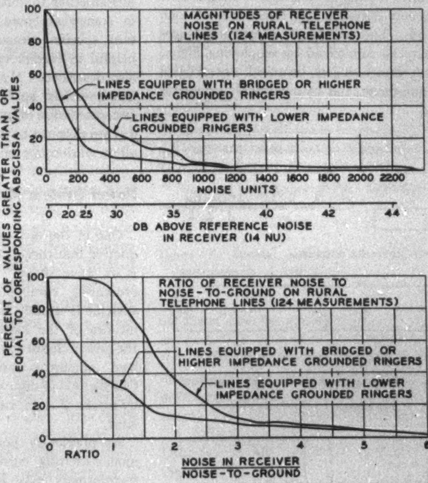


Figure 5. Noise measured on magneto telephone subscriber circuits in rural areas

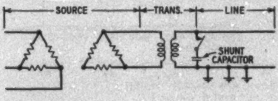
In the case of a ground-return circuit (figure 4*C*) the current-to-ground due to electric induction (*i*) flows directly through the station sets, while the voltage due to magnetic induction (*e*) acts on a longitudinal circuit which includes the impedances of the station sets. This accounts for the fact that the noise in a ground-return circuit is usually of the order of 100 times that in a reasonably well balanced metallic circuit under similar exposure conditions.

In addition to the component of noise induced directly between wires, metallic-circuit noise may also occur as a result of the action of longitudinal induction on circuit unbalances. These may be either series impedance unbalances (*Z* in figure 4) or shunt admittance unbalances (*Y* in figure 4). The more common sources of series unbalances in rural circuits are high resistance joints and certain types of switching circuits used in common battery offices. Shunt unbalances may arise as a result of contacts between line wires and trees, from faulty insulation, from the ringers associated with certain types of station sets where grounded ringing is employed, or from certain types of central office apparatus.

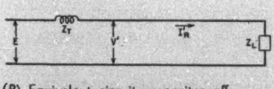
Considering first the effects of electric induction, in figure 4*A*, where the central office end of the circuit has a high impedance-to-ground, the voltage-to-ground appears across the shunt admittance (*Y*) and contributes a component of noise in the metallic circuit. In this case the effect of a series unbalance (*Z*) is small. However, where the circuit has a low impedance-to-ground, as in figure 4*B*, the voltage-to-ground due to electric induction is negligible, and the effect of the admittance unbalance (*Y*) is small. In this case, as pointed out above, the induction produces a current-to-ground and this current acts upon the series unbalance (*Z*) to produce a metallic-circuit current.

In the case of magnetic induction, the full longitudinal voltage appears across the admittance unbalance (*Y*) when the circuit is grounded at the central office (figure 4*B*). If *Y* is relatively small, the longitudinal current will be relatively small and its effect on the series unbalance (*Z*) is unimportant. If there is a low impedance-to-ground at each end of the circuit, the longitudinal current may be appreciable, and the effect of (*Z*) becomes important.

The effects of unbalances may be summarized as follows: shunt unbalances tend to be important where electric induction controls the noise in circuits operated isolated from ground, or where magnetic induction is important, particularly when the circuit is grounded at the central office and such unbalances occur near the far end of the line. Series unbalances may be important where electric induction acts on a circuit grounded at one point, or in the case where magnetic induction acts on a longitudinal circuit having a low impedance at each end. The effects of these unbalances may be minimized by employing higher impedance ringers, balanced central office equipment, and by reasonably adequate maintenance of the open-wire circuits. In some cases special measures, discussed below, may be employed.



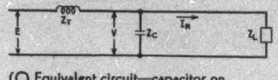
(*A*) Schematic diagram



(*B*) Equivalent circuit—capacitor off

Z_T = Impedance of source and transformer

Z_L = Impedance to residuals of line



(C) Equivalent circuit—capacitor on

Figure 6. Shunt capacitors for improving wave shape of rural power lines

The cumulative per cent curves in the upper portion of figure 5 indicate the range of noise observed on a large number of rural magneto telephone circuits exposed to various types of power circuits. No attempt is made to differentiate between the effects of the different types of power systems involved. An indication of the relative influence of these different types of system may be obtained from tables I and III. The effects of the lower impedance ringers in increasing the receiver noise are indicated in figure 5.

The ratio of receiver noise to noise-to-ground (noise current to ground through 100,000 ohms) is frequently used as an index of the balance of a telephone circuit. This quantity has been calculated for the rural circuits tested and the results are plotted in the lower portion of figure 5.

Co-ordinative Measures

Co-operative planning by those responsible for the development and operation of power and telephone systems is important in rural areas as well as in urban and suburban areas.[1] The general co-ordinative measures outlined in the introduction to this paper are discussed in somewhat more detail below. Certain specific measures which may be helpful in certain types of situation are also described. None of these measures is universally applicable. The measure or combination of measures best suited to a given case can be determined only when all the various factors are known.

Power System Measures

One of the factors controlling the inductive influence of a rural power circuit is its wave shape. The problem of co-ordinating rural power and telephone systems is usually simplified if the magnitudes of the harmonics impressed on the power circuits by either the supply or load apparatus are relatively small. One method of improving the wave shape, by means of shunt capacitors, is discussed later.

As previously pointed out, the harmonic exciting currents of single-phase transformers may be of importance in the inductive influence of multigrounded distribution circuits, particularly those operating in the lower voltage ranges. The magnitudes of these harmonic components can be controlled to a considerable extent in both the design and application of the transformers. Tests indicate that there is a rapid increase in the harmonic components of the exciting current of a transformer when the rated fundamental frequency voltage is exceeded. It is therefore desirable to avoid impressing excessive voltages on transformers, particularly when they are connected to the lower voltage multigrounded circuits.

On three-phase distribution systems, reductions may often be effected in the inductive influence if the lengths of single-phase extensions as well as the amounts of single-phase loads are balanced among the various phases in so far as practicable. Field experience has indicated, however, that where relatively large harmonics are present in the voltage wave shape of an extensive system, it is generally impracticable to secure adequate reductions in the ground-return current components of a three-phase circuit by balancing the extensions only, without resorting to reductions in the impressed harmonic voltages.

Table IV. Measured Effectiveness of Shunt Capacitors in Reducing Inductive Influence of Rural Distribution Circuits Supplied from Transformers

Case A—Six-mile multigrounded 7.2-kv single-phase extension supplied through a 25-kva 2.3–6.9-kv transformer. Capacitor connected across 6.9-kv side of transformer.

Capacitor (Kilovolt-amperes)	None	15
Voltage *T.I.F.* 2.4 kv	20	15
Voltage *T.I.F.* 7.2 kv	74	5.6
Phase *I·T* of 7.2-kv extension	390	29
Ground-return *I·T* of 7.2-kv extension	215	16

Case B—Thirty-mile multigrounded 6.9-kv single-phase extension supplied through 100-kva 2.3–6.9-kv transformer. Capacitor connected across 6.9-kv side of transformer.

Capacitor (Kilovolt-amperes)	None	15	20	35
Voltage *T.I.F.* 2.3 kv	76	36	16	15
Voltage *T.I.F.* 6.9 kv	118	32	30	16
Phase *I·T* of 6.9-kv extension	2,130	535	480	285
Ground-return *I·T* of 6.9-kv extension	1,120	285	260	163

Case C—Twelve-kv three-phase four-wire multigrounded system (total length 13 miles three phases and 27 miles one phase) fed from 150-kva 13.2–12-kv delta-wye transformer bank. Three-phase capacitors connected on 12-kv side of transformer.

Capacitor (Kilovolt-amperes per phase)	None	15	30
Average phase-phase voltage *T.I.F.* 13.2 kv	20	16	17
Average phase-neutral voltage *T.I.F.* 12 kv	58	12	9
Average phase *I·T*—12-kv line	610	90	50
Ground-return *I·T*—12-kv line	360	45	30

Power circuit transpositions afford an effective means of reducing longitudinal induction in exposed telephone circuits only if the influence is controlled by balanced rather than residual components.[5] In rural distribution exposures the induction practically always results from residual components, and in such cases transpositions are generally ineffective. Furthermore, in the case of three-phase circuits, these residuals are usually due either to the unequal lengths of single-phase extensions connected to the various phases or to direct-connected generators with grounded neutrals and consequently are not susceptible to reduction by transpositions.

Shunt Capacitors for Reducing Inductive Influence

Several situations have recently developed in the field in which the influence of the power lines was considerably increased by resonance between the leakage inductance of the supply transformer and the line capacitance at or near one of the harmonic frequencies present in the impressed voltage. In such cases it has been found practicable to improve wave-shape conditions by the application of shunt capacitors on the rural-line side of the supply transformer.

Figure 6*A* presents a schematic diagram of a single-phase extension supplied from a three-phase system through a single-phase transformer. Figure 6*B* is a simplified equivalent circuit of this arrangement, the voltage E representing the voltage at a given harmonic frequency in the wave shape of the source. Actually there are other harmonics in the system such as those generated by the supply and load transformers, or by certain types of load. However, on rural lines these frequently contribute less to the over-all influence than the supply system voltages and for simplicity have been omitted from the equivalent circuit.

In rural distribution lines, particularly under light load conditions, the impedance Z_L (figure 6*B*) is largely capacitive reactance below the quarter wave length frequency. Consequently, a condition of series resonance will exist at the frequency where the inductive reactance of the source and transformer (Z_T) is equal to the capacitive reactance of the line (Z_L), resulting in substantial increases in the voltages and currents in the rural line at harmonics in the vicinity of the resonant frequency. The connection of a suitable capacitor across the circuit, as indicated in figure 6*C*, will change the resonant point to some lower frequency, thus causing large reductions in those harmonics which were close to the resonant frequency existing before the capacitor was connected. In selecting[3] the size of capacitor for a particular situation it is important to avoid establishing the lower frequency resonant point close to one of the harmonic frequencies present in the voltage wave shape. The shunt capacitor in combination with the transformer leakage inductance also has a filtering action which tends to reduce the higher frequencies where the impedance of the capacitor is small in comparison with the impedance of the rural line.

The degree of improvement in influence effected by the application of a capacitor depends upon the particular harmonic frequencies present and upon such other factors as the relative size of the capacitor and transformer, the length of the rural line, and the magnitude of harmonic components originating in loads. The results of tests of the effectiveness of shunt capacitors in three field situations, the first two involving single-phase extensions and the third a three-phase line, are summarized in table IV. It will be noted that in all cases, before the capacitors were applied, the voltage *T.I.F.* was higher on the line than on the supply side of the transformer due to the resonant condition previously mentioned. Substantial reductions in both voltage *T.I.F.* and ground-return $I{\cdot}T$ were effected by the use of capacitors in these cases.

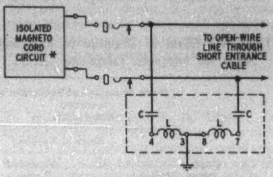
Telephone System Measures

Experience has indicated that the noise in transposed open-wire telephone circuits exposed to rural power lines at roadway separation is apt to be controlled by the action of longitudinal induction on circuit unbalances rather than by direct metallic induction. In cases where the effects of circuit unbalances have been minimized and the telephone circuit transpositions are not co-ordinated with the exposure conditions, improvements in the transposition arrangement should result in reductions in the direct metallic induction and in the receiver noise.

In general, station sets of the multiparty types on rural telephone lines are equipped with the higher impedance type of ringers, particularly where circuits are subject to noise induction. The lower impedance grounded ringer is therefore less likely to be an important factor in the co-ordination problem in rural localities than in urban areas.[1] Since unbalance of any type may contribute to the metallic circuit noise, any measures taken to improve the balance of a rural circuit will be reflected in noise reduction. In some cases the most practicable means of reducing the susceptiveness of the telephone circuits may be the application of the special measures described below.

Drainage on Ungrounded Circuits

As shown in figure 4*A*, the receiver noise on magneto lines which are ordinarily ungrounded at the central office, may be controlled by the action of voltage-to-ground from electric induction on shunt unbalances such as tree leaks or grounded ringers. In such cases substantial improvement in the noise may be secured by the application of drainage at the central office end of the circuit, with the consequent reduction of the voltage-to-ground on the circuit. Drainage is also effective in reducing the 60-cycle induced voltage which is sometimes sufficiently large to cause false ringing of subscribers' bells where grounded ringers are employed. Figure 7*A* shows schematically an arrangement of a balanced retardation coil and a pair of balanced condensers for this purpose. This combination presents a low impedance to the longitudinal circuit, and a relatively high impedance across the metallic circuit. The condensers are provided to permit satisfactory ringing and d-c testing on the circuit. Since

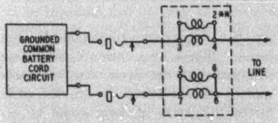

ISOLATED MAGNETO CORD CIRCUIT *
TO OPEN-WIRE LINE THROUGH SHORT ENTRANCE CABLE
C
C
L
L
4
3
8
7

(*A*) Use of drainage on ungrounded circuits (noise controlled by action of longitudinal electric induction on shunt unbalances)

L,L—Balanced retardation coil with midpoint grounded

C,C—Capacitors of approximately one microfarad capacitance

* Drainage also applicable where line is grounded through high impedance



(*B*) Use of high impedance in longitudinal circuit (noise controlled by action of longitudinal magnetic induction on series unbalances)

** Repeating coil connected to offer high longitudinal and low metallic circuit impedance

Figure 7. Special measures for reducing susceptiveness of rural telephone circuits

drainage increases the longitudinal noise current, the use of this low-impedance termination may result in increased noise in cases where important *series* unbalances are present in the telephone circuit.

Where an open-wire line enters an office through an appreciable length of cable, a degree of drainage is provided by the capacitance-to-ground of the latter. Where long entrance cables are involved and the open wire is consequently already effectively drained, little further reduction in noise would be expected from the drainage scheme described above.

Table V. Effect of Drainage in Reducing Noise on Magneto Telephone Circuits

		Noise Measured at Subscriber's Location (Noise Units)		
			Noise in Receiver	
Location	Central Office Terminal*	Noise to Ground	Bridged Ringer	Grounded Lower Impedance Ringer
A	*I*	350	70	450
	G	9	0+	0+
B	*I*	400	85	500
	G	60	65	65
C	*I*	750	210	1,400
	G	15	150	170
D	*I*	210	1,100	1,130
	G	20	160	160

* *I*—Central office end isolated from ground.
G—Central office end grounded through midpoint of balanced termination (figure 7*A*).

Although primarily intended for use on magneto circuits, this drainage device may also find application on common battery circuits where noise results from the action of longitudinal induction on unbalances in the central office. In step-by-step offices, for example, where the impedance-to-ground offered by the central office equipment may be of the order of 1,000 ohms at 1,000 cycles, advantage may be taken of the relatively low impedance of the drainage device to reduce the voltage acting on central office unbalances.

The effectiveness of this drainage device in reducing the noise on magneto circuits at several locations where electric induction controlled is demonstrated by the data summarized in table V.

High Impedance in the Longitudinal Circuit

In the case of common-battery manual circuits there may be relatively large longitudinal noise current flowing to earth through the cord-circuit ground at the central office (figure 4*B*). Where *series* unbalances are present either in the line or in the cord circuit, the component of metallic-circuit noise, resulting from the action of the longitudinal current on these unbalances, may be controlling. In cases where *magnetic* induction controls, considerable reductions in the longitudinal current and consequently in the receiver noise may be effected by inserting an inductance in the circuit, as illustrated in figure 7*B*. This device is so connected as to offer a relatively high impedance to the longitudinal circuit and a very low impedance to the metallic circuit. Increasing the longitudinal circuit impedance frequently increases the noise-to-ground on the circuit. Consequently, a net improvement in the metallic-circuit noise will be obtained by this method only where the effect of *shunt* unbalances is small.

In some cases both series unbalances in office equipment and shunt unbalances on the line may be important. In such instances, use may be made of a combination of the high longitudinal impedance, inserted at the central office, and the drainage device, discussed above, the latter being connected on the line side of the series impedance.

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2. Engineering Report 40 of Engineering Reports of Joint Subcommittee on Development and Research.

3. Engineering Report 33 of volume IV, Engineering Reports of Joint Subcommittee on Development and Research.

4. Measurement of Telephone Noise and Power Wave Shape, J. M. Barstow, P. W. Blye, and H. E. Kent. AIEE Transactions, volume 54, 1935, page 1307.

5. Engineering Report 36 of volume IV, Engineering Reports of Joint Subcommmittee on Development and Research.

Discussion

L. F. Roehmann (Anaconda Wire and Cable Company, Hastings-on-Hudson, N. Y.): The authors suggest the use of shunt capacitors as a means for lowering the inductive influence of power lines. They claim two favorable effects, namely:

1. Detuning the circuit for the harmonic for which it is in resonance, and

2. Short-circuiting other harmonics.

The measurements as shown in table IV indicate that the reductions exceed the ratio 10 to 1, a very satisfactory result.

But this result is accomplished by comparatively large and expensive capacitors (15 to 35 kva), and I wonder whether such expense be justified from the standpoint of noise co-ordination alone.

The aspect becomes much more favorable, however, if the effect on the fundamental is considered too. The capacitors should improve power factor and regulation, both desirable features especially in rural electrification.

I would like to know if the authors have studied this point, and if they can confirm my considerations.

H. E. Kent (Edison Electric Institute, New York, N. Y.): The paper by Messrs. Wahlquist and Taylor has presented the rural co-ordination problem as it affects metallic telephone circuits. I should like to refer briefly to some co-ordinative measures which can be used where ground-return telephone circuits are involved. These are predominately measures applicable to the telephone system, since the measures already described for application to the power system effect approximately the same degree of reduction in influence whether metallic or ground-return telephone circuits are involved. However, it should be borne in mind that a ground-return circuit is inherently of the order of one hundred times as sensitive to induction as is a metallic telephone circuit. Consequently, while the power system measures already described greatly simplify co-ordination with metallic telephone circuits, in many situations additional measures will be required in the co-ordination of ground-return telephone circuits.

The improvement of wave shape by means of capacitors described in the paper is obtained by preventing the higher frequency harmonics in the source of supply from reaching the rural distribution system. In one of the cases where this method was applied it was found that, after the capacitors were connected at the supply substation, various small motors supplied by the rural distribution system still introduced appreciable harmonics, the effects of which were amplified by line resonance conditions. While this was not sufficient to disturb metallic telephone circuits, it resulted in appreciable noise in the ground return circuits. Application of an additional bank of capacitors to the rural circuit at a point about 20 miles from the supply substation destroyed the resonance conditions and effected further reduction in the influence.

Capacitors provide a means of largely eliminating the higher frequency harmonics in many situations, particularly in rural areas. However, it is not feasible to also reduce the lower frequency harmonics (particularly 180, 300, and 420 cycles) by this method. Where these low frequencies control it is possible to improve conditions by reducing the sensitivity of the telephone set in this low frequency range but with no important change in sensitivity over the remainder of the voice frequency range. Two methods of accomplishing this are indicated in figure 1. The commonly used connection for a local battery telephone set is shown in figure 1*A*. This is modified in figure 1*B* by shunting the receiver with an inductance of 60 millihenries and by connecting a 0.75-microfarad condenser in series with the receiver. In figure 1*C* the modification consists of transferring the receiver from the line to a location across the transmitter winding of the induction coil, and adding a one-microfarad condenser in series with the receiver. Field tests in several locations have shown that either of these methods effects reductions in noise of somewhat better than 2 to 1 on the average when the frequencies controlling the noise are in the range from 180 to 420 cycles.

Similar results can be obtained by reducing the acoustical response of the telephone receiver at the lower frequencies. This can be done by punching a quarter-inch hole in the receiver diaphragm at a point one-third the distance across its diameter. Tests have shown that the reduction in the low-frequency noise thus obtained is approximately the same as that secured with the electrical filters. The advantage of the acoustical method lies in its simplicity. It may have some disadvantage from a maintenance standpoint since dirt can more readily enter the space back of the diaphragm.

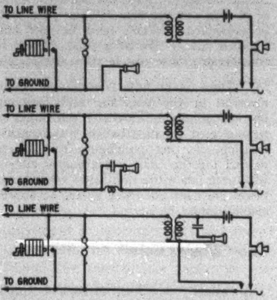

Figure 1. Schematic diagrams of commonly used and modified connections of local-battery telephone set

(*A*) Connection in common use
(*B*) Modification employing receiver filter
(*C*) Modification employing reconnection of set

Figure 2 illustrates the use of a ring-through type of repeating coil in a ground-return telephone circuit to reduce noise by, in effect, transposing the circuit without materially impairing the talking and ringing efficiency of the circuit. The coil acts as a low-impedance drain from line to ground for noise currents resulting from electric induction. For magnetically induced currents the coil acts as a high impedance in series with the line. The effectiveness of the coil depends to a considerable extent on having approximately equal induction in the line on both sides of the coil. In long exposures, particularly where subscribers are distributed through the exposure, more than one coil may be necessary.

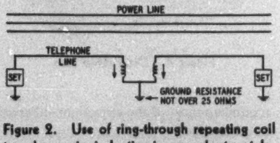

Figure 2. Use of ring-through repeating coil to reduce noise induction in ground-return telephone line

Note: Repeating coil connected as a drainage coil—that is, so as to offer low impedance to equal currents in the same direction in each winding as indicated by the arrows

In individual cases the application of capacitors for the improvement of the power system wave shape combined with a change in the frequency response of the telephone sets, or the use of repeating coils, permits the rendering of reasonably satisfactory rural telephone service over the ground-return line. However, as the load increases on the power line, and more motors are connected, it will become very difficult to maintain the extremely low influence of the power line. This, combined with greater demands made upon the telephone service, may make it impractical to render acceptable service over the ground-return line. Therefore, in applying these measures it must be recognized that in many situations they will offer only a temporary solution of the problem and that ultimately it will be necessary to "metallicize" the line. Of course, in some situations the growth of the service requirements of both the power and the telephone will be such that these measures may last for a number of years. In other situations the growth of the load will be such that continual attention will be required and "metallicizing" will necessarily follow in a very short time.

J. T. Howard (Tennessee Valley Authority, Chattanooga): The authors have touched upon two vital factors contributing to power system influence which seemingly should be given more attention for the benefit of distribution engineers. These are the ground-return harmonic components of transformer exciting currents and charging currents.

Where the rural distribution system is extensive, the voltage at the point of supply is often held 10 or 15 per cent above normal in order to obtain the desired secondary voltage at some distant point without resorting to excessive use of boosters. With distribution transformers working at high flux densities at normal voltage, this increase results in excessive magnetizing current. The net effect is an increase in the $I{\cdot}T$ product. While it is true, generally, that these components do not control on the larger and higher voltage circuits, many instances have been observed where their effect, in an extensive system, was the controlling factor in noise induction. It appears that placing more emphasis on this point would be of material benefit to distribution engineers, so that they may be guided accordingly in controlling system voltages, and to manufacturers, so that their transformer designs may be modified to allow a better tolerance for overvoltages.

It has been found that the most severe cases of noise induction result from harmonic charging currents, and that exposed communication circuits require a higher degree of maintenance than is customary, if the charging circuit effects are to be minimized. The effects of changes in impedance-to-ground do not seem to be generally understood by distribution engineers, and new systems are designed, and operating systems extended, without any consideration of this factor. In the case of systems operating with an $I{\cdot}T$ product within the range of average values, extensions may be so connected to the system that resonance occurs at a frequency with a high *T.I.F.* contribution, and the $I{\cdot}T$ product increases correspondingly. In such event, the distribution engineer is confronted with a difficult task in again co-ordinating his system with exposed communication facilities. It therefore seems desirable that the authors make available, in a concise form, the results of impedance-to-ground measurements which they have made during the past several years, classified as to voltage and configuration. In this manner, the problem of power system influence may be more readily understood.

H. W. Wahlquist and T. A. Taylor: As mentioned by Mr. Kent, the paper is confined to the rural co-ordination problem as it affects metallic telephone circuits and his discussion of several co-ordinative measures applicable to ground-return telephone circuits will no doubt be appreciated by those who are interested in this phase of the problem.

Mr. Roehmann refers to the comparatively large and expensive capacitors used in accomplishing the reductions in influence listed in table IV. While it is true that the kilovolt-amperes of the capacitors was considerable, representing about 15 to 60 per cent of the kilovolt-amperes of the supply transformer, the expense involved was found to be less than that of other co-ordinative measures of equal effectiveness. The use of such capacitors increases the 60-cycle voltage about two to 5 per cent, but has little effect on the voltage regulation of the line. In the case of a rural line having a peak load of low power factor approaching the rating of the supply transformer, the capacitors would be beneficial in reducing the load on the transformer by reducing the reactive kilovolt-ampere.

Mr. Howard feels that more emphasis should be placed on transformer exciting currents as a source of harmonics. While it is true that transformer exciting currents may control the $I{\cdot}T$ product on rural lines under light load conditions (in cases where the harmonics in the supply systems voltage are relatively small), the magnitude of the $I{\cdot}T$ product under such conditions is relatively low. On the other hand, in urban areas, transformer exciting currents have in some cases been of importance from the noise standpoint due, among other things, to the heavier load density and to different characteristics of the telephone apparatus. Mr. Howard also expresses interest in impedance-to-ground measurements on power systems. This is an important factor and has received detailed consideration. Methods have been developed for calculating this impedance and have been presented, together with measured results for a few cases, in reference 2 of the paper. Control of the *magnitude* of the impedance-to-ground by special attention to circuit layout does not appear likely to be of much practical importance because operating and service requirements will necessarily govern the system arrangement. However, as pointed out in the paper, it may be feasible to minimize the *unbalance* of the impedance-to-ground among the several phases of a three-phase system by connecting approximately equal lengths of single-phase extension to each of the three phases.

The Wound-Core Distribution Transformer

By E. D. TREANOR
MEMBER AIEE

IN THE very early days of the development of a-c transformers, various methods were tried to produce cores which would have the necessary characteristics of low loss, low magnetizing current, and suitable mechanical arrangements. It was very soon found that the most practical form of raw material was thin iron or iron-alloy sheet which could be obtained with relatively low hysteresis loss, and could be cut and assembled so as to give relatively low eddy-current loss and magnetizing current.

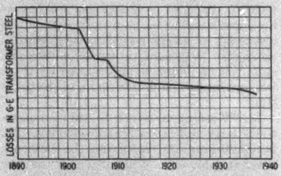

Figure 1. Improvement in hot-rolled magnetic steel

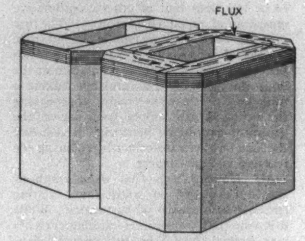

Figure 2. Shell-type core

In this form of thin sheet, magnetic steel for a-c apparatus has undergone years of intensive development from the standpoint of processing and alloying, and steady improvement has made available the silicon steel alloys of excellent characteristics which are in general use today. These are made by hot rolling in packs having maximum dimensions of about three by nine feet. A graphic history of improvement in quality is shown in figure 1.

The mechanical construction of transformer cores has almost universally continued the early practice of cutting plates or punchings from this sheet and building these pieces into massed arrangements of simple or complicated form around the coils. Characteristic examples are shown in figures 2, 3, and 4. Figure 2 shows what is usually called a shell-type core, figure 3 a "core" type, and figure 4 a modified type, which is called the distributed core.

Cores made in this general fashion have certain objectionable features.

(*a*) The pieces cannot be cut and arranged so that the flux path follows the grain orientation of the steel throughout the core and thus obtain the lowest loss possible. Reference to the sketches will show that in figure 2 and figure 3 appreciable parts of the punchings have the grain direction (shown by light lines) lying at some angle with the flux while in figure 4 practically all of the punchings show an angle between grain and flux. Some portions carry no flux at all.

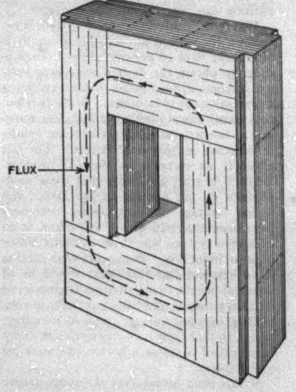

Figure 3. "Core" type core (two legs)

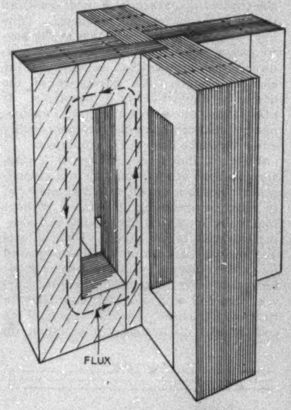

Figure 4. Distributed core

(*b*) The pieces as ordinarily laid up in one or two thicknesses per layer have a minimum of two overlapped gaps in the magnetic circuit (where L punchings are used) and a maximum of four such gaps where straight plates are used. These gaps are dependent in size upon perfection of the pieces and the skill of the workmen, and at best add a little local loss due to fringing of flux and demand an appreciable magnetomotive force, or in other terms, magnetizing current.

(*c*) Large numbers of pieces must be cut and handled through various operations, such as annealing and assembly. Substantial clamping structures must also be used to hold the structure together but without undue pressure. With the usual thickness of 14 mils, not less than 140 pieces per inch of assembled thickness must be used and handled one or two at a time. There is considerable labor cost in these operations.

(*d*) There is at best considerable waste material in the punching and handling operations. In the case of L punchings so largely used on distribution transformers, multiple dies cut punchings from sheets welded together into a continuous ribbon with even less waste than when rectangular pieces are used but there is still an appreciable waste.

The "Wound Core"

A radically new form of core of simple structure and highly efficient performance has recently been developed. As shown schematically in figure 5, it is made by winding a long ribbon of magnetic steel in a tight spiral on a mandrel into the form of a heavy-walled circular cylinder. This core construction has been designated as the "wound core." It eliminates or minimizes most of the objections inherent in other types.

(*a*) The flux runs with the grain direction throughout. All material in the core is active material.

(*b*) There is but one effective gap in the magnetic circuit and this is of very short length and large area.

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(c) Only one or two pieces of strip steel are necessary to make the average small core. These are prewound and assembled on machines at high speed.

(d) The waste in handling and fabrication is very small since each element is simply cut from large reels of continuous strip made to correct width at the mill.

A practical core incorporating these and other advantages obviously presents a distinct advance in the design and construction of transformers to which it is applicable. It is the purpose of this paper to describe its present application to distribution transformers.

As stated above, L punchings for distribution transformers are cut from reels of steel made by welding together sheets of hot-rolled steel. Distribution transformers of the wound-core construction can utilize such hot-rolled steel with improvement in characteristics and reduction in cost over existing designs, but the wound-core construction makes it possible to take full advantage of the properties of cold-rolled strip of relatively low silicon content. This strip is characterized by high permeability and very good losses at high densities, and is nonaging. It is made by high-reduction rolling on a suitable mill and mechanically is soft and ductile when compared with high silicon hot-rolled sheet. It is produced in a strip of great length.

Figures 6 and 7 show tentative comparisons of watt loss and permeability as compared with good high-silicon hot-rolled sheet. It is strongly directional in these properties, the losses at flux angles with the grain of 45 degrees to 90 degrees being from 45 per cent to 70 per cent higher than with the grain. These properties indicate the possibility of operation at higher densities than are normal with ordinary steel, particularly in a core in which the magnetic material is worked with the grain.

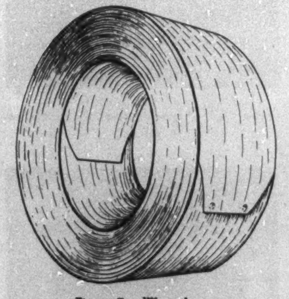

Figure 5. Wound core

Considering a core made up of single ribbon, whose grain direction runs with its length, wound into a circular cylinder and embraced by a current-carrying coil, it will be seen that the flux must follow the grain direction very closely. In such a core (figure 8), it will be seen also that each element of flux finds but one air gap in its path, and if the ribbon is tightly and uniformly wound, this gap will be of very small length and relatively great area. The result is that little magnetomotive force is consumed in the reluctance of air gaps as compared with other cores.

In the practical design of transformer cores, flux density is limited by heating or magnetizing current in large transformers and usually to still lower values by magnetizing current alone in small units. The combination of a steel having high permeability and low losses in the high density range with a core structure which reduces the number and length of air gaps promises highly effective use of material, particularly in small-sized transformers.

Designs for ratings of 1$^1/_2$, 3, and 5-kva units with primary voltages of 7,620 volts or less have been worked out and placed in production, and the promise of economy has been fully realized. The most satisfactory construction consists of two core elements as shown in figures 9 and 10, linked by a simple interleaved coil group in secondary-primary-secondary arrangement. Due to the ability to work the core at higher density than is usual for small transformers, normal characteristics can be obtained with considerable economy of material and reduced over-all cost in spite of the fact that the new steel is at present higher in cost than hot-rolled silicon steel.

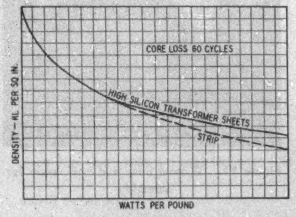

Figure 6. Watt loss of hot-rolled and new strip steel

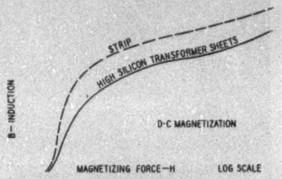

Figure 7. Permeability of hot-rolled and new strip steel

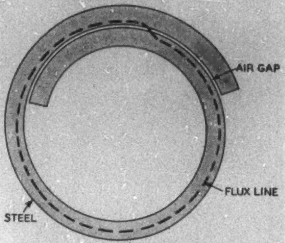

Figure 8. Path of flux in wound core

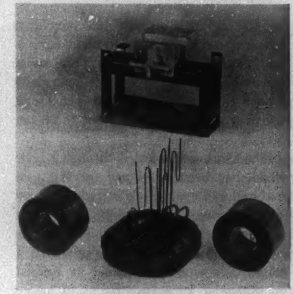

Figure 9. Core and coil elements

Core Manufacture

The steel for wound cores is produced at the mill in strips of required widths and great length. These strips are slit into multiples of desired width for various size cores and are shipped in reels of a convenient size and weight to the factory. These reels are set up in multiple before a prewinding machine which consists of an accurately sized power-driven steel mandrel. The ends of the reels are attached to the mandrel which is then rotated, drawing the strips through tension devices until the cores are built up to the desired thickness which is individually controlled by micrometer gauges. Each strip is cut and temporarily welded to hold the core in shape during anneal. The whole operation is, of course, quite rapid as compared with cutting and stack-

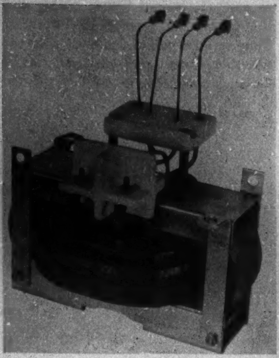

Figure 10. Core and coil unit with clamps and terminals

ing the hundreds of individual punchings which would be required to make equivalent cores of the old type. The space factor of the wound core is materially higher than with assembled punchings, reaching values of 95 per cent to 96 per cent.

The cores are annealed through a cycle which has been developed for this new material and are then delivered to the assembly room. It should be noted that the equipment necessary for putting the core into shape for assembling consists of a single core-winding machine as against an otherwise large investment in heavy high-speed punches and punching dies. The operation of gathering and stacking punchings for anneal is eliminated and several operations of grouping punchings by weight and number after anneal are eliminated.

Coils

The coils for these small transformers are of more or less normal design. They are of oval shape and short axial length. The inner and outer low-voltage coils are of shorter axial length than the high-voltage coils so that when assembled in a low-voltage-high-voltage-low-voltage group, a cross section of the group is of cruciform shape to conform to the circular window of the core. The internal insulation of the coils is similar to that which had been highly developed in past practice. The major insulation consists of mica pads between high and low-voltage windings taped on with varnished cloth and the entire group is taped and reinforced where it passes through the core. Where necessary for insulation or thermal requirements, ducts are placed between or within the coils. The corner ducts between the cruciform coil and the circular core provide satisfactory ventilation under the core and the remainder of the coil is fully exposed to the oil.

The arrangement and design of the coils is such as to give good transient voltage distribution and since the cores are applied under tension, the coils are firmly gripped and held in position against mechanical forces of short circuit. In these small units, no other mechanical bracing for the coils is essential.

Method of Assembly or Linkage of Cores With Coils

The two core units necessary for one transformer and its coil group are brought

Figure 11. Wound core and coil ready for assembly

Figure 12. First operation

Figure 13. Annealed core partially unwound

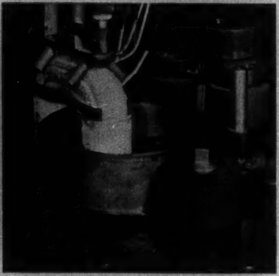

Figure 14. Annealed core completely unwound

Figure 15. Wound core in loose spiral around coil

Figure 16. Wound core tightly wound around coil

to a machine which has been developed for the operation of applying cores. The coil group is clamped between cushioned supports on this machine and one core unit is laid beside it over a short power-driven spindle (figure 11). The relative positions of the core and coil are properly adjusted and the welds which secure the outer end of the core strip broken. This outer end is then carried through the coil window in a large loop and temporarily secured on itself so as to form a circle embracing the coil leg (figure 12). A friction roller on the outside of the core and the internal spindle then rotates the core in such a fashion that its entire length is thrown out into a large cylinder embracing the coil leg (figures 13 and 14). The spindle within the core is removed and the core collapses into a loose spiral embracing the coil leg (figure 15). The temporary attachment on the outer turn is now broken and the turns of the loose spiral are drawn inward on the coil leg by power-driven friction rollers applied at its perimeter, with the result that the core is now linked with the coil leg in essentially the same condition as before the operation was begun without having been overstressed during the operation (figure 16). The other core unit is similarly applied to the other leg of the coil group. The outer ends of the strips are welded to permanently secure the cores in shape, and the core and coil unit is ready for clamping, treating, and assembly in the tank. The operations described are rapid and somewhat less dependent on the skill which is necessary to obtain good joints and compact construction in a core made of cut punchings. The machines used at this point are an added investment since cores made of punchings are usually assembled by hand labor.

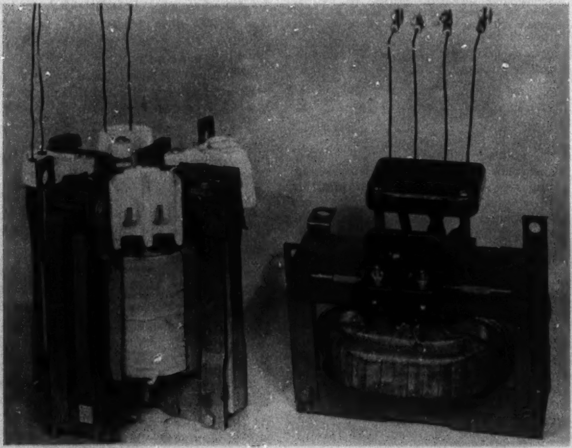

Figure 17. Transformers of distributed-core and wound-core construction

Clamps

Clamps are simple cradles of relatively light steel adapted to grip the cores securely and without distorting them. The structure of the cores is inherently solid and self-supporting.

Characteristics

In over-all characteristics, these transformers have been designed for characteristics which differ only slightly from previous values. In recognition of the importance of improving regulation, the copper loss of the wound-core transformers has been reduced and the core loss slightly increased. This revision of losses seems to be better suited to average operating conditions throughout the country than previous values. The exciting current is lower than with past transformers for reasons given earlier in this paper.

The wound-core distribution transformers, because of a marked reduction in the amount of copper and core iron used, and because of the lighter and simpler clamping structure required, are appreciably lighter in over-all weight than similarly rated units using the conventional core arrangements (figure 17).

Since the difficulty in serving small loads in areas of low density is largely economic, the development of a material and a construction which permits reduction in the price of one important element in the cost of distribution would seem to constitute a major contribution to the broader use of electricity.

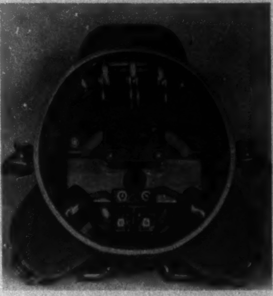

Figure 18. Wound-core unit assembled in tank

The writer wishes to acknowledge the assistance of others in the preparation of this paper, particularly of Mr. J. C. Granfield, the inventor of the wound-core construction, the method of assembly, and the machine, and Mr. M. Broverman who has closely followed the development.

Discussion

Philip Sporn (American Gas and Electric Service Corporation, New York, N. Y.): The distribution transformer, as is well known, plays a very important part in distribution economics. In some phases of distribution, for example in the problem of extension of rural service, it plays an almost predominating part. Here the transformer with its associated equipment represents from 35 per cent to 40 per cent of the average cost of a rural extension, taking for an average an extension in which from four to five transformer installations to the mile are made. Hence I think all electric utility and transmission and distribution engineers will welcome the development described by Mr. Treanor.

In reviewing the work that has been accomplished over the last decade in improving the characteristics and reducing the cost of the distribution transformer, it needs to be recalled that the present distribution-transformer design is basically a development and refinement of an idea that is

Radio Influence Characteristics of Electrical Apparatus

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Synopsis: Experience recorded in this paper with the method of measuring the radio influence factor of electrical apparatus recommended by the Joint Co-ordination Committee on Radio Reception of Edison Electric Institute, National Electrical Manufacturers Association, and Radio Manufacturers Association establishes its practicability and adequacy. This method fulfills the essential feature of all working standards in that the results are reproducible by quantitative measurements.

Various types of output meters for determining the radio influence factor of apparatus have been investigated. For practically all applications, the present type of output meter gives an indication which is sufficiently close to the audio output. This characteristic, besides the additional advantages it has in simplicity and relative inexpensiveness, well justifies its use.

Radio frequency influence factors vary in character, depending on the electrode insulation arrangement of the source, and also the dielectric medium.

Humidity and relative density of the air affect the radio influence factor and accordingly certain correction factors for the atmospheric conditions need be recognized.

The radio influence factor characteristics of insulation in air and oil, reported in the paper, establish that normally only apparatus in air need be considered for the effect of radio noise influence, as in co-ordinated apparatus design the strength of parts in oil will exceed those in air.

Experience has shown that quantitative data are essential for this type of study. Since adequate methods of measurements are available, the next step is the establishing of economical levels, which can only be obtained through the co-operation of all interested parties, by the collection of essential field data.

THE operating principles governing radiobroadcasting and reception are generally well-known. Good reception is possible from a radio receiver of suitable characteristics, provided adequate consideration is given to the receiving antenna and provided the field strength from the broadcasting station is sufficiently high. There are many factors detrimental to good radio reception. The signal may suffer distortion, undue weakening, or even complete obliteration at times from the effects in the transmitting medium and in the complex process of propagation of the signal.[1,2] Atmospheric electrical discharges and lightning superimpose their disturbances on the signal and may interfere partially or completely with reception over wide areas.[3] Another source of interference may be contributed from electric power systems and in fact from the very household electrical appliances, which, along with the radio receiver, are contributing to the conveniences and leisure in the modern home. The problems involved in securing good radio reception are therefore many and complex.

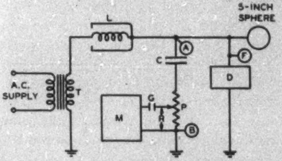

Figure 1. Recommended EEI-NEMA-RMA circuit for determination of radio-noise influence of high-voltage apparatus

T—Testing transformer
L—Radio-frequency choke, shielded, not less than 6,000 ohms reactance at broadcast frequencies
M—Radio noise meter
C—Coupling capacitor, not more than 60 ohms reactance at broadcast frequencies
G—Dummy antenna capacitor (21 micro-microfarads)
D—Device under test
P—Potentiometer or tapped resistor, 600 ohms resistance, nonreactive

This paper deals with the radio influence characteristics of electrical apparatus. As clearly defined by the Joint Co-ordination Committee on Radio Reception of EEI, NEMA, and RMA, improved broadcast reception is essentially a problem of co-ordinating the newer use of electricity and some of its older uses for light, heat, power, transportation, and communication. Accordingly after some years of experience and study this co-ordination body recommended suitable methods of measuring the radio influence factors of electrical apparatus.[4] This procedure naturally has unified effort, stimulated investigation of the problem, and it has made possible the progress which has already been achieved. The investiga-

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[1] For all numbered references, see list at end of paper.

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some 35 years old. Hence toward the last ten years the refinement and improvements that have been made have all been of a minor and detailed nature. In other words the vein in which distribution transformer designers were working was getting pretty thin. The development described by Mr. Treanor opens up a very definitely new vein. It shows very strikingly that only by using material and labor more effectively can costs be reduced in the long run. In opening up a new field for reducing both of those items Mr. Treanor and his associates have every reason to believe they have brought about a first class achievement.

It seems to me that the above is the outstanding phase of the development and that that is more important than some of the details as to how the characteristics of the particular steel that is utilized in the new design are obtained or how the ingenious arrangement for winding with so little labor the cores and placing them on the transformer has been worked out. The writer, and I feel certain many others who have been working in the field of improving distribution economics and reducing the cost of distribution of electric service, are looking forward to an intensive development of this vein and possibly to the opening up of new veins and the application of some of the economies already made possible on lower-sized transformers to larger distribution sizes where, if the need for reduction in costs is not as urgently pressing as it is in the field of smaller sizes, it is still nevertheless a major problem. There is no question that distribution costs can stand further reduction, but the distribution engineer needs material help from those who are spending all their energies in the field of designing and building the distribution equipment, the cost of which has a major influence on the whole problem of distribution economics.

E. D. Treanor: We feel with Mr. Sporn that the "vein" had been getting thin with conventional designs in spite of continued improvements in hot-rolled steel and that there is hope for a considerable extension on the new level. Initial progress may be slow but experience with the small units described inspires confidence that further effort along this line will be successful.

tion reported here fulfills further the aim and objectives of the Joint Co-ordination Committee in Radio Reception in establishing new data, findings, and principles that will be of assistance in the radio co-ordination problem.

Measurement and Testing Technique

The test circuit recommended by the joint co-ordination committee[4] for measuring the radio-noise influence factor of high-voltage apparatus is given diagrammatically in figure 1. The characteristics and adequacy of this method of measuring radio noise already have been dealt with to a considerable extent in the technical literature.[5] Its practicability has well been proved by the experience with the first installations and the design of recent installations. Two widely separated laboratories for measuring the radio-noise influence factor of high-voltage apparatus are shown, respectively, in figures 2*A* and 2*B*, and in figure 3. These installations comply essentially with the established recommendations. A brief description of the design characteristics of one installation will illustrate some of the more pertinent principles.

The physical parts of the installation which correspond to the circuit elements in figure 1 are indicated on the photographs. The possibility of appreciable or even measurable radio noise from the circuit proper was eliminated by the use of a $1^5/_8$-inch diameter rod, five-inch diameter spheres at the ends, and adequacy of all apparatus parts. Special connecting links assembled from large-diameter rods and hemispheres are employed to connect the object under test to the circuit. A 50-kv testing transformer with regulator control supplies the required voltage over the entire range. The room was shielded with metal sheets spot-welded together and solidly connected to a grid system of large strap conductors and driven rods which comprise the ground. Power to the laboratory is supplied over a several-hundred-foot cable from a small transformer station which feeds as additional load part of the lamp circuit in the large plant nearby. By these measures in shielding the room and care in the choice of power supply, background and other interference from external sources were negligible, although immediately adjacent to the radio-noise laboratory is located a 3,000,000-volt impulse generator and also a high-current generator. The attainment of freedom from external noise sources is a feature which illustrates again the advantages and practicability of the recommended method.

The capacitance (C = 0.0021 microfarad) of the coupling capacitor and the effective inductance (L = 12 microhenries) of the drop lead are tuned to 1,000 kilocycles. The ratio of the radio-frequency voltages produced by the object under test on the measuring circuit and the recorded voltages appearing across the terminating resistance is essentially unity over the entire broadcast frequency range as is shown experimentally and by calculation in figure 4. This feature is a desirable one to attain.

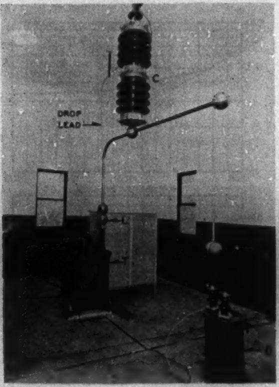

Figure 2A. View of Sharon laboratory

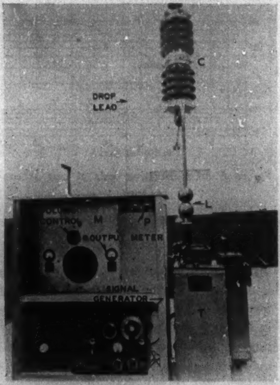

Figure 2B. View of Sharon laboratory

Calibration of the radio-noise meter is made by means of a high-frequency signal generator modulated 50 per cent at 400 cycles. The indication on the output meter from the unknown voltage is adjusted to half scale by a volume control on the noise meter. The signal generator is then switched on and its output varied until the same deflection is obtained on the output meter. All radio-noise measurements are therefore based on the signal generator and are microvolt root-mean-square values. The tests are usually made at 1,000 kilocycles which is midway in the broadcast frequency range.

Figure 3. View of East Pittsburgh laboratory

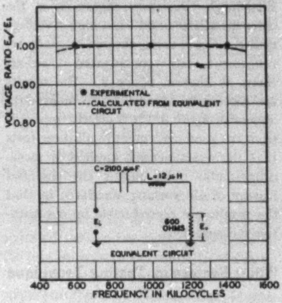

Figure 4. Characteristics of installation in figure 2

A typical set of tests, showing in figure 5, was made to correlate the measurements at the two laboratories. In test *A*, a rod to plate was employed; this electrode arrangement has served well as a "reference standard" in these and other comparative tests. Tests *B* and *C* were made on electrical equipment. The same samples of the equipment were tested first at one and then at the second laboratory. It should be noted that the laboratories are nearly a hundred miles apart. Though each conforms with the recommended method, they differ from each other in a number of details as

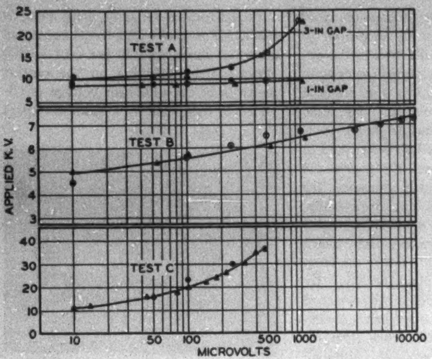

Figure 5. Correlation tests of two laboratories

shown by figures 2 and 3. The East Pittsburgh laboratory, figure 3, is the older installation and has a higher voltage rating. Again using the rod-to-plate arrangement, tests were repeated at various times at Sharon, figure 2. The results are plotted in figure 6 and similar data appear in figure 9. It is clearly apparent from the good agreement in all these tests that the recommended method of measuring radio noise fulfills the essential feature of all working standards in that the results are reproducible by quantitative measurement.

Other Factors in Testing Technique

Measurements were made on a pin-type insulator and on a rod-to-plate gap at various frequencies over the broadcast frequencies. The results are given in figure 7. These and similar tests indicate that the radio-noise influence factors decrease with increase in frequency. The experience up to the present time in testing high-voltage apparatus shows that a relatively uniform decrease in radio-noise influence factors is obtained over the entire broadcast range with no tendency in peaking of radio noise at one frequency. For this reason it appears that the practice in testing at some frequency midway in the broadcast band, such as 1,000 kilocycles, is justified.

The character of radio noise may differ considerably, depending on its generating source. Thus an important feature of the curves for the air-insulated (dry type) apparatus in figure 5 is the uniform continuity in the microvolt rise with increase in applied voltage. Physically the phenomena in these cases is one of uniformly increased radio interference factor with voltage. That the radio interference factor is sustained and uniform was evidenced also from the steady indications of the output meter. Distinctly different from this was the radio noise measured from a sphere to plate. As shown in figure 8, corona and spark-over for this electrode gap occur simultaneously up to about 3.5 sphere-diameter spacing. At the wider spacings corona appears before spark-over voltage is reached. From the visual and aural observations the corona is brusque, in the nature of streamer formation. In fact, the microvolt curves in figure 8 confirm this initial relatively intense discharge, as the lowest measurement that could be recorded is approximately 500 microvolts. The intermittent streamer discharges account also for the unsteady behavior of the output meter in these tests. A similar performance was recorded for the rod to plate above 25 kv where a discontinuity in the curve (figure 6) appears. In short, the two distinct types of corona effects referred to, well illustrate the different character of radio noises that may be encountered.

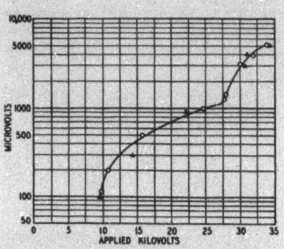

Figure 6. Comparative tests at one laboratory

Rod-to-plate gap; three-inch spacing

Other factors that need be taken into account in radio-frequency testing are the density and humidity of the air. It is recognized that breakdown and corona voltage both depend upon the relative air density and the absolute humidity. In general, the amount of correction for variation of the air density is small. Fundamentally, in radio-interference work this correction would appear to be proportional to the air density, the same as applied to voltage breakdown. Due to the fact that the effect of absolute humidity in high-voltage testing is relatively greater than the relative air density, it appeared desirable to determine its effect on radio-frequency testing.

Tests were made at one of the laboratories where by virtue of the enclosed room the absolute humidity could readily be controlled. First the tests were made at a low humidity. A certain amount of steam was then injected into the room and the air kept well stirred up until the higher humidity was uniformly attained in the room. The tests were repeated. Thus the testing at the two humidities and with identically the same test setup was completed in an hour or less. Results for the rod to plate are given in figure 9. These data have been confirmed by other tests. This investigation establishes that for the rod to plate and for similar electrode arrangements the voltage corresponding to a given radio noise value increases with the absolute humidity 1.5 to 3.0 per cent for each grain per cubic foot. On the average the increase is 2.5 per cent over the entire range in figure 9. This variation in humidity correction factor for radio noise is substantially the

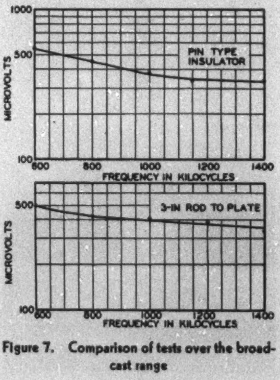

Figure 7. Comparison of tests over the broadcast range

Relative air density = 0.98
Absolute humidity = 4.4 grain per cubic foot

same as that applied to 60-cycle flashover of rod gaps and suspension insulators.[8] Below the very low values of 50 to 100 microvolts the microvolt correction for humidity becomes insignificantly small.

Comparison of Noise Meters

The Joint Co-ordination Committee on Radio Reception of EEI, NEMA, and RMA have formulated specifications for a radio noise meter. These specifications were formulated at the time when tuned-radio-frequency receivers were the most commonly used but since that time the designs of the radio receivers have changed considerably. The selectivity curves of radio receivers are a major characteristic which affects the response of radio receivers to extraneous noises. While the tendency in the past has been toward greater selectivity, the demand for higher fidelity in recent years has required the manufacturing of less selective receivers. Although the specifications for the radio-noise meter were formulated in 1931, no changes in these specifications have been found necessary.

Table I. Selectivity of Three Receivers

	Band Widths (Kilocycles) Versus Signal Ratio					
	600 Kc		1,000 Kc		1,400 Kc	
Receiver	10/1	100/1	10/1	100/1	10/1	100/1
A	12	24	18	37	31	64
B	10	17	9	15	9	19
C	12	22	21	40	27	59

In order to determine the relative response of three different radio instruments that are being used in the laboratories for measuring radio-frequency influence factors, radio-frequency voltages of different character were measured with the three instruments. Two of these instruments are of the tuned-radio-frequency type; the third is a superheterodyne receiver.

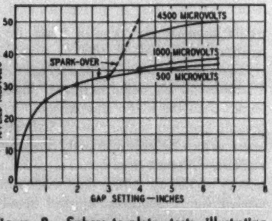

Figure 8. Sphere-to-plate tests illustrating character of radio noise

Two-centimeter sphere to plate

The curves in figure 10 show the results of these three receivers when measuring the radio influence factor from a rod to plate gap separated three inches. This type of noise has produced the greatest deviation between receivers. Figure 11 shows the results when measuring the radio-influence factor from a pin-type insulator. It should be noted that the superheterodyne receiver *B* indicates a value of noise which on the average falls between the two tuned-radio-frequency noise meters.

The selectivity curves of all receivers are shown by table I and it is interesting to note that although the superheterodyne receiver *B* is highly selective at 1,000 kilocycles, the results obtained by this instrument are relatively close to the tuned-radio-frequency instruments. The maximum deviation that has been noted in the values indicated by the three instruments has been slightly over two to one. These instruments do not always maintain the same relative position as shown in figures 10 and 11. In some cases, receiver *B* will measure less than receivers *A* and *C* but always within the deviation limits given above.

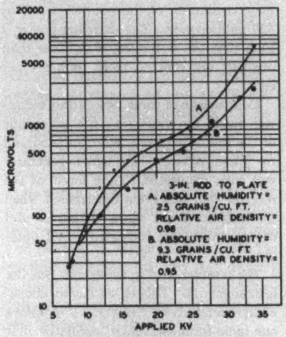

Figure 9. Effect of humidity

Instruments *A* and *C* correspond to the specifications formulated by the Joint Co-ordination Committee on Radio Reception. The output of instrument *A* is highly distorted for values of interference above 1,000 microvolts. This distortion does not exist in either the superheterodyne receiver *B* or instrument *C* up to values in the order of 20,000 to 30,000 microvolts. Repeated tests have indicated that the higher values measured by instrument *A* are in error. A potentiometer is now being used with instrument *A* so that all voltage measurements are made below 1,000 microvolts. With this arrangement consistent results are obtained between the three instruments.

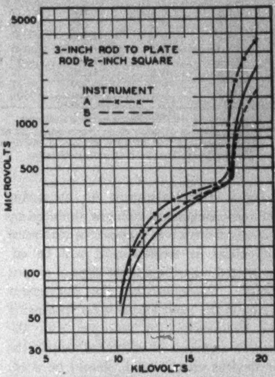

Fig. 10. Comparison of radio-frequency voltages measured by three instruments (tests on rod gap)

Relative air density = 0.99

Absolute humidity = 3.5 grains per cubic foot

Instruments *A* and *C* are battery-operated radio-noise meters. Because of the continuous service required and due to the fact that the superheterodyne receiver incorporated a loudspeaker, it was used in obtaining a portion of the data presented in this paper. It is not advocated however to substitute any radio receiver for the recommended radio noise meter unless facilities are available to determine the relative merits.

Output Meter

The joint co-ordination committee specifies that the output meter for the radio-noise meter should be a critically damped instrument having an undamped period of one-half second. The present noise meters have a Rectox-type output instrument which gives an average value of the radio interference voltage. There has been and probably will be in the future more or less criticism of the output instrument specified by the Joint Co-ordination Committee on Radio Reception. The problem of selecting such an instrument is similar to that which has been encountered by committees in standardizing on a sound level meter. Some feel that the meter should be faster, others feel that the output meter should be over-damped. Still, others advocate

Table II. Radio-Frequency Voltage and Noise Measurements

Voltage Applied Rod Gap (Kilovolts)	Output Meter** Milli-amperes	Micro-volts	Noise Level (Decibels) (40)*	(80)*	Noise Intensity (Micromicro-watts)	Noise Loudness
16.6	0.1	680	75	75	3,200	11,750
20.0	0.2	940	80.5	80	11,250	17,000
20.7	0.3	1,150	82	82	16,000	19,500
22.0	0.4	1,300	84.5	84.5	28,300	24,000
23.5	0.5	1,400	85.5	85.5	36,000	25,000
25.8	0.6	1,550	87	87	50,000	28,500
27.0	0.7	1,700	87.5	87.5	56,000	30,000
27.3	0.8	1,820	90	90	100,000	37,000

* Forty and 80 decibel equal-loudness contours.

** Volume control of noise meter (instrument *B*) at 45 on dial.
Microphone one foot directly in front of the loudspeaker.

using a crest instrument for measuring the radio-frequency voltages produced by various devices. It is apparent that some difficulties in measurement will be encountered regardless of the type of output meter. Consequently, the instrument which is the most effective for measuring the radio-frequency voltages from the largest number of electrical apparatus or devices encountered, should be used.

It was realized that the crest voltage produced by the majority of electrical devices or apparatus would not compare favorably with the loudspeaker response. Consequently, it was felt that an averaging tube voltmeter, such as is being used for monitoring broadcasting stations (figure 12) might be an adequate compromise between measurements of peak values of noise by the grid bias and measurements of average values with a highly damped and slower instrument as recommended by the joint co-ordination committee. An investigation was conducted in the laboratory to determine the performance of this average tube voltmeter (figure 12) as compared to the Rectox-type output meter. This average tube voltmeter circuit is such that peaks of duration between 40 and 90 milliseconds are indicated to 90 per cent of full value and the discharge rate adjusted so that the pointer returns from full reading to 10 per cent of zero within 600 milliseconds. The speed of the indicating meter is such that the time for one complete oscillation of the pointer is 290 to 350 milliseconds. The damping factor is such that the overswing is not less than one-half per cent or more than six per cent.

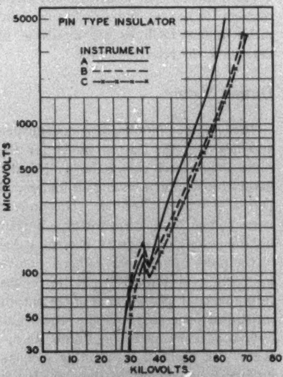

Figure 11. Comparison of radio-frequency voltages measured by three instruments (tests on insulator)

Relative air density = 1.0
Absolute humidity = 1 grain per cubic foot

Comparison of these two output meters was made by measuring the amplitude of the radio-frequency voltages produced by three devices. A signal generator was then connected to the input of the receiver and observers determined when the output of the signal generator and device under test indicated equal loudness.

The receiver used in these tests was the instrument *B*. This is, as stated above, a superheterodyne receiver. It employs one radio-frequency stage, oscillator, first detector, two intermediate amplifiers, second detector, and push-pull audio amplifier. A circuit analysis of it appears in *Radio*, November 1930. This receiver was equipped with a loud speaker. A Rectox output meter and the averaging tube voltmeter were connected in parallel and condenser coupled to the plates of the push-pull amplifier.

In making the measurements, the sensitivity of the receiver was adjusted so that the output of the noise was sufficient to give half scale deflection on the Rectox meter. The input potentiometer on the crest instrument (average tube voltmeter) was then adjusted so that maximum deflections could be read on the scale. A more exact determination of the pointer deflections was obtained by holding the edge of a card in front of the dial and adjusting it so that the needle was just visible for maximum variation. The adjustment of the input potentiometer was not changed during the time of test with any one observer. A signal generator with a 400-cycle audio component, and 50 per cent modulated output, was substituted for the noise source to determine the value of these two noise levels in microvolts.

The observer then compared by ear the relative volume obtained from the loud speaker when the noise source was connected to the receiver and when the signal generator output was substituted for the noise. The sensitivity of the receiver was held constant and the signal generator output adjusted until the two seemed to be of equal loudness.

The curves in figure 13 show the results of tests made with a 110-volt a-c universal-type motor, 33-kv pin-type insulator, and spark gap, respectively, as the source of the noise. The spark gap was used to produce a peak discharge which would simulate the type of noise from ignition systems.

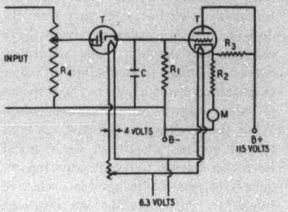

Figure 12. Circuit diagram of average tube voltmeter

R_1—40 megohms
R_2—68,000 ohms
R_3—680,000 ohms
R_4—500,000 ohms
C—0.007 microfarad
M—Rectifier-type meter
T—Tube type 76

It should be noted that the average tube voltmeter in all cases indicated higher values than the audio comparison. The present specified output meter indicated lower values by approximately the same amount. Present data indicate that the sensory build-up time, i.e., the time needed for a suddenly applied steady wave to build up to a steady loudness, is in the order of 0.2 to 0.25 seconds.[7] The average tube voltmeter will measure peaks of much shorter durations than the ear while the present output meter will only measure peaks of longer durations.

For practically all applications it appears that the present type of output meter gives an indication which is sufficiently close to the audio output of a radio receiver. This characteristic well justifies its use. It has the additional advantage of being simple and inexpensive which is highly desirable for industrial laboratory and field work.

Correlation Between Radio-Noise Measurement and Acoustical Loudness

The desirable radio noise measuring instrument is one that would indicate values which simulate the ear response. The American Standards Association has published tentative standards for acoustical noise measurement[9] and has formulated the requirements of sound level meters for measurement of noise and other sounds.[9] This instrument attempts to approach the human ear by weighing the various audio-frequency components with respect to the standard 1,000-cycle reference loudness level.

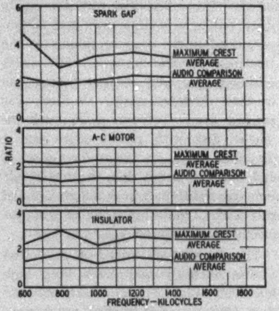

Figure 13. Comparison of audio, average, and crest values

It is interesting to note that the radio noise meter and the sound level meter primarily attempt to determine or weigh the effect of two different types of complex waves on the ear. One is used to measure the modulated high-frequency voltage while the other measures sound picked up by a microphone.

Tests were conducted to determine if any relationship existed between measuring the output of the radio set with Rectox-type output meter (instrument *B* in figures 10 and 11) and with a sound level meter which complies with the ASA standards.[10] The modulated radio-frequency voltages were produced by a rod-to-plate gap and measured by the standard radio noise method. In measuring the audio output of the radio set, the microphone of the sound level meter was placed at a distance of one foot directly in front of the loudspeaker. The scale of the output meter was zero to one. The volume control was adjusted for 0.1 on the output meter and the radio-frequency voltage and the sound level meter readings taken. The voltage on the rod gap was increased and for every 0.1 increase in deflection on the output meter, the modulated radio-frequency voltage and the sound level noise meter readings were noted.

Table II shows the results obtained with one sensitivity setting of the radio set. The radio-frequency voltages increased from 680 to 1,820 microvolts. The noise level increased from 75 to 90 decibels and the auditory sensation or loudness from 11,750 to 37,000 units. It should be noted that the microvolts increased 2.68 times and the auditory sensation 3.15.

Figure 14 shows three curves, each taken at a definite sensitivity and covering a given range of microvolts. In these three measurements the microvolts increased, respectively, 2.0, 7.0, and 4.0 times and the corresponding noise loudness increased 1.5, 5.5, and 5.2. These tests were made with the radio set at 1,000 kilocycles. Similar tests as for curve *C* but at 1,400 kilocycles gave a microvolt increase of 5.6 times against a loudness increase of 4.4 and at 600 kilocycles the microvolts increased 5.1 and the loudness 5.2.

Similar relationships in the relative increase between microvolts and loudness as shown in figure 14 were obtained with the microphone at greater distances from the loudspeaker. For instance, in one test similar to curve *C* but with the microphone placed at two feet directly in front of the loudspeaker, the microvolts increased 6.3 times while the loudness increased 6.0 times.

All these tests were made in one of the two laboratories where inherently the background noise level can readily be reduced to 50 decibels and even less so that it practically had no influence or required no compensation on the tests proper. An attempt was made to analyze the frequency components of the loudspeaker noise. It was found that the frequencies contained in this type of radio noise were too numerous and complex to permit frequency analysis.

From these tests the ratio of the increase in microvolts to the increase in auditory sensation did not vary more than 0.77 to 1.34. In a number of cases and on the average the ratio was close to one. If we assume that the response of the sound level meter simulates the human ear response, we can accordingly assume that our present output meter likewise corresponds in effect somewhat to the response of the human ear.

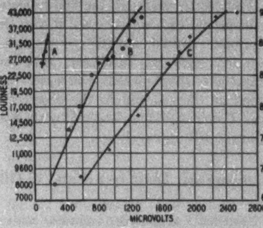

Figure 14. Correlation between radio noise measurements and acoustical loudness

Noise meter at 1,000 kilocycles
Volume dial setting *A*, *B*, *C*
Microphone one foot in front of loud-speaker
Radio-frequency voltage from three-inch rod to plate

Comparative Measurements of Corona Starting Voltage

Various means have been employed in determining the starting voltage of corona of high-voltage equipment. Recently tests were conducted to determine the effectiveness of detecting corona starting voltage by the use of a cathode ray oscilloscope, ear, eye, and the standard method of measuring radio noise.

Difficulty was encountered in attempting to use the cathode-ray oscilloscope and after considerable experimenting, an untuned radio-frequency choke coil was connected in series with the ground lead of the apparatus. A parallel resonant circuit tuned to 1,185 kilocycles was connected across this choke through a blocking condenser. The oscilloscope was then connected across this resonant circuit. By this means, the 60 cycle and low frequency harmonics were filtered out.

The tabulation below shows the threshold of corona by different methods from three pieces of apparatus. The standard method of measuring radio noise detected the lowest starting voltages.

In totally inclosed apparatus, the standard method of measuring radio noise detects corona starting voltage very much below the audible or visual methods.

Sample	Kilovolts			
	Audible	Oscilloscope	Visual	Radio
High-voltage switch	26.0	28.2	26.0	20.0
Current transformer	11.0	12.0	12.0	6.2
Circuit breaker	6.1	7.7	13.0	4.85

Tests were made on the rod-to-plate (one-quarter-inch square rod) at various spacings. Initial audible corona was detected with an insulation tube of funnel-like shape which conveyed the sound at the rod end to the observer's ear. Corona could be detected by the ear at about 100 microvolts.

Comparative Tests in Air and in Oil

Electrical apparatus assumes many forms, depending on the application and on the design. It is apparent that a complete survey of the radio-noise influence of all apparatus would require an extensive program, although a good deal of work already has been done in this direction. However, an appraisal of this problem is simplified when it is recognized that most apparatus at the present operates either in air or in oil. The corona characteristics of these two dielectrics either alone or in combination with the dielectric bodies of the apparatus proper, such as porcelain, fullerboard, and so on, are important factors in a survey of the radio noise influence of electrical apparatus. Certain principles will become apparent from a consideration of typical tests which have been made in air and in oil.

Tests in air are shown in figure 15. These characteristic curves bear out in the first place the influence of the shape of the electrode. The radio noise measurements and therefore corona appear at a lower voltage when the electrode is a plain rod and it can be seen also that the characteristic curve is essentially the same whether this is a one-quarter-inch square rod or a one-eighth-by-one-inch rectangular strap. The curves show that for the same microvolt level twice the voltage may be applied to the sharp-cornered disk as compared to the voltage on the rod. A rounded electrode such as a sphere of sufficiently large diameter presents certain advantages in reducing corona formation, provided the surface of the electrode is kept clean. In practice apparatus may be exposed to dirt, moisture, rain, and so on, with the result that large smooth surfaces are in such case more detrimentally affected than would be the sharper-edged electrodes.

A point of interest in the characteristic curves for air (figure 15) is that considerably higher microvolt values are reached by the short gaps. Thus with the one-half-inch gap spacing, for either the rod or disk-type electrode, very high radio-frequency voltages can be generated whereas for the correspondingly larger gaps sparkover occurs before these high values can be reached.

In electrical apparatus the body of a solid dielectric or a combination of dielectrics appears between the line and ground electrodes assuming one form or another which depends on the design and the application of the apparatus. If the presence of the dielectric body increases or decreases the stress on the air as compared to the stress between the electrodes in air alone, then the microvolt-voltage characteristic curves will accordingly be modified, respectively, upward or downward. Thus the presence of the porcelain body in the case of the line insulator may further raise the microvolt-voltage curve in comparison to that of the tie wire and pin electrode in air alone. By proper design and grading of the body part, the stress on the air may be reduced. In the case of a condenser-type bushing the inherent grading of the stress in the dielectric body and over its surface contributes also in reducing the stress in the adjacent surrounding air. In this case the stress on the air at the line terminal even for sharp electrode parts is much less than it would be anticipated from the stress which appears between the terminal electrodes in air alone. It is apparent then that the radio-frequency voltage from electrical apparatus in air depends essentially on the relative stress imposed on the air. Characteristic curves of the air alone as shown in figure 15 are nevertheless of fundamental guidance in appraising the general problem.

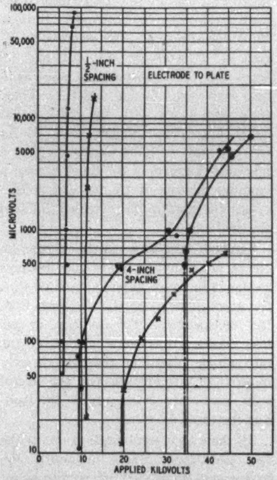

Figure 15. Radio-noise characteristics of air for various electrode arrangements

Type of electrode:

- ● One-quarter-inch square rod
- ⊡ One-eighth by one-inch rectangular strap
- × Nine-inch diameter, one-inch thick disk
- ⊙ 0.8-inch diameter sphere at end of one-half-inch diameter rod

The rod-to-plate gap was tested in oil at spacings from a fraction of an inch to an inch. The voltage was raised until breakdown of the oil gap occurred so that up to 45 kv was applied for the larger gap tested. No radio-frequency voltage could be detected up to breakdown of the gap.

Tests were made on fullerboard sheets in oil. The fullerboard was thoroughly dried out in a heating oven and oil impregnated under vacuum similar to the treatment for transformers. The sheets of fullerboard were immersed under oil in their own test tank. A sheet was pressed firmly between a 2½-inch diameter disk with square edges and a grounded plate, as shown in figure 16 and tested. Curve *A* is the test result for a one-eighth-inch thick fullerboard and curves *B*, *C*, and *D* give similar tests for 0.056-inch fullerboard.

The characteristic curves *A*, *B*, *C*, and *D* rise abruptly and reach very high microvolt values. They clearly indicate that corona in oil is in effect much more intense than in air. In similar tests that had previously been made on the strength of insulation[11] the corona at the edge of the disk was detected visually at about 25 kv and 36 kv for the 0.056-inch and one-eighth inch fullerboard sheets, respectively. It appears to the writers that the radio noise method is also an effective means to study and measure corona. When the phenomena is partly or entirely concealed, such as is in these tests, it has been found a superior method to any of the others available. To illustrate this point further, a one-eighth inch fullerboard sheet that had not been dried or treated was tested. The results are plotted in curve *E* which by comparison to curve *A* show the benefits which accrue from drying and treating the fullerboard.

In practical design even for the lower classes of distribution apparatus, clearances in oil normally exceed the amount indicated in figure 16. The mechanical requirements alone generally require larger clearances. Besides the conducting parts normally are insulated thus securing relatively better performance than indicated. Considering the fact that the corresponding air clearances would be in the order indicated in figure 15, it is clearly apparent that the radio noise in-

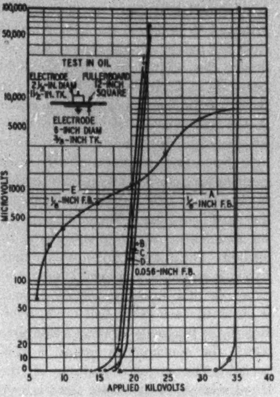

Figure 16. Radio-noise measurements of insulation in oil

fluence of electrical apparatus principally concerns the apparatus or the part of the apparatus in the air. Similar consideration of and experience with voltage apparatus of higher voltage classes likewise establish that normally only the apparatus in air need be tested for the effect of radio noise influence as the strength of the apparatus or corresponding part in oil well exceed in good design those in air.

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Discussion

C. W. Frick (General Electric Company, Schenectady, N. Y.): During the past six or seven years considerable experience has been gained with methods of determining radio influence characteristics of apparatus at several different laboratories and to some extent in field investigations. The present paper is a valuable contribution to this work. As the paper points out, the Joint Co-ordination Committee on Radio Reception of EEI, NEMA, and RMA has recommended circuits for the determination of interference influence of high-voltage and low-voltage apparatus. The quantity which is measured is called noise voltage in the committee report and is expressed in modulated microvolts, but in the paper it is called the radio influence factor. The name used in the paper has considerable merit because the quantity in question serves as an index of influence on radio reception. In this respect it is analogous to the telephone influence factor (TIF) which is a useful index of the influence of power-system wave shape on wire communication systems. On the other hand, the present name, noise voltage, has some advantage because it suggests the units in which the quantity is measured while a factor is generally thought of as a ratio, for instance power factor, form factor, space factor, and similar terms. The joint committee has not as yet adopted the name radio influence factor, and those wishing to use the term should explain its meaning as the authors have done.

The paper presents an interesting study of the effect of different types of output indicator. The present output meter is a microammeter which indicates an average of the rectified output of the radio receiver element of the noise meter. It is a critically damped instrument having an undamped period of 0.5 second as specified by the joint committee. Some engineers have suggested that an instrument which responds to noise peaks might give a better indication of the effects on radio reception. The paper compares the present output meter with one consisting of a tube voltmeter operated on a capacitance-resistance network characterized by rapid charge and slow discharge so that it tends to average the noise peaks. The authors call this an averaging tube voltmeter. Figure 13 of the paper shows that this averaging tube voltmeter gives higher readings than the present output meter for common types of noise, such as motor noise; that is, it requires a higher value of the calibrating voltage modulated 50 per cent to match the noise indication on that instrument than it does when the present output meter is used. This result would be expected because noises appear to consist of a rapid series of impulses. The curves also show that readings with the two instruments are different in ratios for different types of noise.

When the joint committee's specifications were prepared, it was realized that radio noise is quite complex and its effects on reception involve many factors including those associated with loud-speakers and with the ear. It was felt that the thing to do was to set up the best instrument that could be obtained according to the state of the art then, and to make improvements later as found necessary. Therefore they specify certain electrical characteristics for the receiver element and a certain period and damping for the output meter, based on the information then available. The paper includes an additional check on the present output meter by an ear comparison method where the noise is compared with the calibrating tone as heard in a loudspeaker. The curves of figure 13 for this audio comparison show better correlation with the present method than obtained by a similar test with the averaging tube voltmeter method. The present type of radio noise meter has been in the field for several years and has made it possible to collect useful data on common types of noise from different observers. For this purpose it seems to be better than anything else that has been suggested.

P. H. McAuley (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The authors have demonstrated the practical nature of the circuit recommended by the joint co-ordination committee. The radio influence factor of high-voltage apparatus can be checked in different laboratories at different times with different types of meter with good agreement. In addition to the East Pittsburgh laboratory described in the paper, which has been in operation since 1930, radio interference tests have been made at the Trafford laboratory from time to time. One of the 500-kv transformers is used, but the capacitor stack at present limits the voltage to about 200 kv. However, it is possible to test the highest voltage apparatus above its operating voltage to ground. At voltages above 150 kv, corona on leads and connections becomes somewhat of a problem. So far we have been able to use two-inch-diameter aluminum tubing with carefully made joints. Six-inch spheres are used at the ends. In general, a radio influence test, on 230 kv apparatus for instance, requires a careful arrangement of the complete high-voltage circuit.

For many years the laboratories have been able to make reliable tests. The crying need for years has been the establishment of levels of permissible noise for different classes of equipment. We are able to determine the radio influence characteristic curve of a piece of apparatus but we are unable to say whether it is satisfactory at its rated voltage. The establishment of levels is essentially an operating problem, because they should be based on experience with equipment now in service. It is hoped that it will soon be possible to correlate field and laboratory testing to the extent that permissible levels satisfactory to both manufacturer and user can be set.

A Position Regulator for Paper Slitters

By F. H. GULLIKSEN
MEMBER AIEE

PAPER slitters are used to trim the edges of paper webs so that a paper roll with even sides can be wound from an unevenly wound roll. When the paper has printed designs on it, the slitting must be done at a definite distance from the printed design. The slitter knives are stationary as shown in figure 1. In order to slit the paper at the same position relative to the design when the position of the paper relative to the shaft of the unwinding roll is varying, it becomes necessary to adjust the position of the unwinding roll transversely.

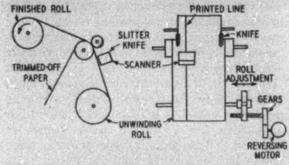

Figure 1. Schematic arrangement of paper slitter

To do this, a line is printed on the paper as part of the printed design and a phototube is arranged to scan this line. Through suitable electronic equipment and a reversing motor, the position of the unwinding roll is adjusted to keep the paper in the same position in relation to the slitter knives.

It has been customary to use a scanner with two phototubes which would scan the line printed on the paper as shown in figure 2. The phototubes were connected in a bridge circuit and the output of the bridge circuit was used to control the electronic regulator and the reversing motor.

The paper industry requires that the line printed on the paper should not be wider than one-eighth inch, which necessitates focusing the two light beams one-eighth inch apart, and obviously, the range of control with the scheme shown in figure 2 will be only one-eighth inch. If the paper, therefore, should move outside the one-eighth inch range, the regulator would lose control. It is also evident that any change in phototube or lamp characteristics would change the regulating position.

The purpose of this paper is to describe a new slitter regulator which employs only one phototube, and which widens the range of control to one inch regardless of the width of the printed line.

The Single Phototube Scanner

The new single phototube scanner shown in figure 3 consists of an 1,800-rpm synchronous motor with four lenses mounted on its shaft. A lamp is mounted as shown in figure 4 so that the lenses focus four light beams on the paper and the printed indicating line. A phototube is arranged to intercept the light reflected from the paper. A top view of the four lenses, the printed line, and the light beams L_1 to L_4, is shown in figure 5. In figure 5*A* the line is located in the center of the light-beam circle while in figure 5*B* the line is displaced to the left. The light beams rotate synchronously, each beam making one revolution in two cycles. Each time a light beam intercepts the line in the position indicated in figure 4, the illumination on the phototube is decreased. This change in phototube illumination is amplified as shown later in this paper, so that a peaked voltage impulse is obtained once every half cycle, referring to the frequency of the a-c source to which the synchronous motor is connected. The angular position of the lenses relative to the motor shaft is such that when the line is in position *A* the impulse voltage occurs at 90 degrees phase angle, whereas with the line in position 5*B* the impulse voltage location is advanced as shown in figure 6.

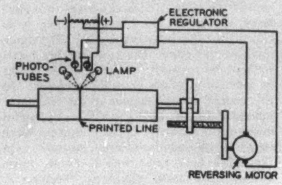

Figure 2. Slitter regulator employing two phototubes

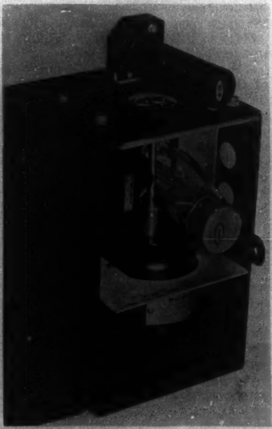

Figure 3. New scanner with one single phototube

For every position of the line on either side of the center position, an impulse is obtained, and there is a definite relationship between the angular location of the impulse voltage relative to the a-c supply voltage, and the space location of the line. This relationship is used to indicate the position of the paper.

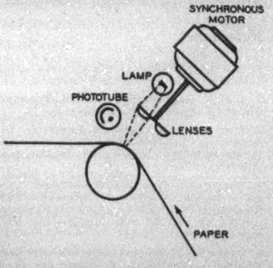

Figure 4. Schematic arrangement of single phototube scanner

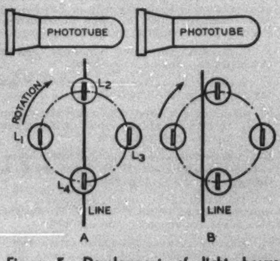

Figure 5. Development of light beams

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F. H. GULLIKSEN is an electronics engineer for the Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.

The Amplifier

A conventional amplifier with capacitor coupling in the grid circuit and transformer coupling in the anode circuit is used as shown in figure 9. The amplifier tube is normally biased to give a high anode current. When one light beam intercepts the line, and the illumination on the phototube is decreased, the grid of the amplifier tube is made highly negative and the anode current is decreased, thus giving an amplified impulse on the secondary side of the anode circuit transformer. This impulse is used to control the selective indicating circuit.

The Selective Indicating Circuit

The purpose of the indicating circuit is to detect the phase-angle location of the voltage impulse obtained from the amplifier. To make the circuit independent of variations in supply voltage and tube characteristics the circuit is arranged to respond only to phase angle.

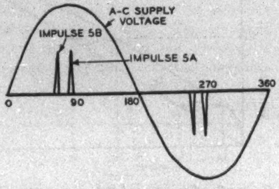

Figure 6. Location of the impulse voltage for positions 5A and 5B

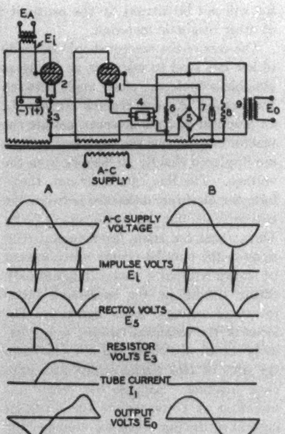

Figure 7. The selective indicating circuit

As shown in figure 7 two thyratron tubes 1 and 2 are used. The grid-control voltage for tube 2 is the amplified impulse voltage from the phototube, connected in series with a negative d-c bias voltage, and tube 2, therefore, is initiated at a point on the a-c voltage wave dependent upon the location of the impulse voltage E_i, which, as previously outlined, is a function of the space location of the line. When tube 2 becomes ionized, rectified a-c voltage appears across resistor 3, and persists during the remainder of the half cycle. The voltage across resistor 3 is connected in negative sense in the grid circuit of tube 1, so that tube 1 cannot become ionized if current is already established through resistor 3. Tube 1 has a permanent negative grid bias obtained from Rectox rectifier 5, which is supplied with a-c voltage from the 90 degrees phase-shift circuit consisting of resistor 6 and capacitor 7.

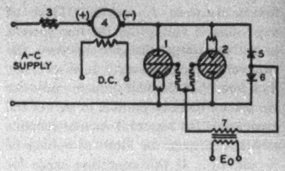

Figure 8. The motor-control circuit

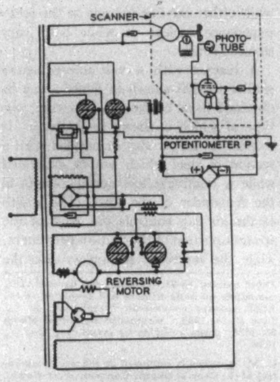

Figure 9. Schematic diagram for position regulator

Because of the interlock action of resistor 3, tube 1 does not conduct current if the impulse voltage E_i occurs ahead of the 90 degrees position as shown in the curves in figure 7*B*. If the impulse voltage occurs later than the 90 degrees position current I_1 flows as indicated in figure 7*A*.

Figure 10. A front view of the regulator cabinet

A saturable transformer 4 is connected in series with tube 1, and the secondary winding of this transformer is connected in a bridge circuit with resistor 8 so that the output voltage E_0 of transformer 9 is reversed when the primary winding of transformer 4 is energized. If, therefore, the line on the paper is moved from one side of the central position to the other, a 180 degrees reversal of voltage E_0 results. This condition is used to obtain reversal of the armature voltage for the roll-adjusting motor.

The Motor-Control Circuit

As shown in figure 8 two thyratron tubes 1 and 2 are used to supply reversible rectified a-c voltage to the armature of the roll-adjusting motor. When tube 1 is conducting current the armature voltage polarity is as shown in figure 8. Reversal of armature polarity results when tube 2 is conducting. The current through tubes 1 and 2 is controlled by means of voltage E_0. Reversal of this voltage, in the manner previously described, reverses the grid polarity of the tubes so that one tube becomes ionized and current stops flowing in the other tube. Rectox rectifiers 5 and 6 are used to refer the grid voltage to the cathode of that tube whose anode at that instant is positive relative to the cathode.

Reactor 3 is used to limit the tube

A Static Constant-Current Circuit

By C. M. SUMMERS
ASSOCIATE AIEE

Synopsis: Static constant-current circuits have been constructed by connecting an iron-core reactor, with predetermined characteristics, in parallel with a capacitor of a specified size such that the resultant input current is independent of line voltage. This parallel circuit is connected in series with the element through which the current is to be regulated. The current will remain essentially constant for large variations in line voltage; large variations in load impedance; or a variation in both quantities simultaneously as long as the variation in voltage across the parallel circuit does not exceed certain limits. This type of circuit is useful for applications where a constant heating independent of line voltage is desired; for series lighting circuits; and vacuum-tube filament circuits.

An IRON-CORE reactor and a capacitor, connected in parallel, are the essential elements of a static, a-c, constant-current circuit, providing these elements are operated within certain specified conditions. First, there must be a predetermined relation between the reactor and capacitor; second, the variation in voltage across this parallel circuit must be restricted to specified limits; and, third, the frequency must remain constant.

Theory of Constant-Current Circuit

Temporarily, assume that the reactor and capacitor in figure 1 are ideal units with their currents 180 degrees apart, and that both currents are sinusoidal. The volt-ampere curve of the capacity I_c, and of the reactor I_x, are plotted in figure 2 without regard to the phase relation between the two currents. The line current then I_0 is the numerical difference between I_c and I_x. Thus, at any value of voltage E_1 the line current is the horizontal distance between the two volt-ampere curves. It is evident, therefore, that the volt-ampere curve for the reactor must be parallel to that of the capacitor if the resultant current remains constant between the limits of voltage of E_1 and E_1'. If this condition exists for the circuit in figure 1, the current through the impedance Z will remain constant while either the line voltage or the series impedance varies; or while both vary simultaneously.

A reactor with a volt-ampere curve similar to that shown in figure 2 can be obtained by a bridge-gap iron core, one type of which is shown in figure 3. It consists of an air gap in parallel with a section of steel which can be saturated while a relatively low density exists in the remainder of the core. The length of the air gap controls the slope of the straight portion of the volt-ampere curve, while the saturated section governs the flux density, or voltage, at which the straight portion begins. This fact is illustrated in figure 4 by a series of volt-ampere curves derived from various physical dimensions of the saturated steel and the air gap. Great flexibility is obtained from this type of core because magnetic shims may be inserted in the air gap to adjust the volt-ampere curve to its desired position. Naturally, a rather high leakage flux surrounds the air gap and it is desirable, therefore, to place the air gap in such a location that the nonlinear characteristics of the reactor will not be altered by the proximity of other magnetic materials.

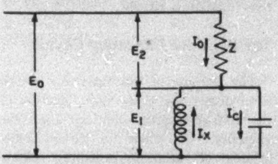

Figure 1. Parallel constant-current circuit

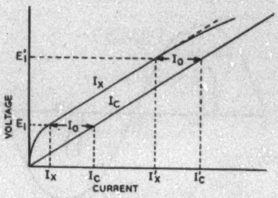

Figure 2. Current-regulating properties of nonlinear circuit

The core of the reactor should be made of low-loss steel to maintain as nearly as possible a 90-degree phase angle between the voltage E_1 and reactor current I_x. In practical designs, however, neither the reactor current nor the capacitor current are displaced exactly 90 degrees from the voltage. The line current is not, therefore, the algebraic difference between the two currents, but the vector sum of them. Thus, there are some fundamental relations in the parallel circuit which cannot be shown without a vector diagram indicating conditions for successive increments of voltage, as in figure 5. The locus of the resultant current is not exactly the arc of a circle, but it lies between the arcs of two circles which are quite close together. Suppose the vector terminating at G represents the resultant current in the line when the voltage E_1 is applied. Although the voltage may be increased as much as 100 per cent to E_1',

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peak current, particularly during the reversing period when the counter electromotive force of the motor armature reduces the effective resistance of the tube circuit. It was found that on account of the dynamic breaking action of the tubes during the reversing period, the circuit in figure 8 was particularly suited for this regulating application because idling and overtravel of the motor was practically eliminated, therefore producing very accurate and sensitive control.

Complete Schematic Diagram

Figure 9 shows the schematic diagram for the complete control circuit, and a photograph of the regulator is shown in figure 10. From figure 9 may be seen that only one single adjustment, potentiometer P, is used. The purpose of this potentiometer is to increase the sensitivity when the difference between the paper color and the line color is slight with reference to the color characteristic of the phototube. An example of colors of this characteristic is a red line on white paper. If the line color is black, blue, green or brown, the equipment will operate satisfactorily with any potentiometer adjustment, and because variations in tube characteristics do not affect the operation, the regulator will operate without any further adjustments after once being installed.

the vector of the resultant current will fall somewhere between the two circular arcs *CD* and *EF*. Beginning at the point *G*, the current increase in magnitude to the point *H;* then decreases in numerical value to the point *J;* and thereafter increases while the voltage over this range is continually increasing from E_1 to E_1'. The power factor is leading at all times but approaches unity as the voltage E_1 increases.

The Constant-Current Transformer

The simple nonlinear circuit described above is the heart of a constant-current transformer with no moving parts. To make a more complete circuit, a transformer which delivers load at unity power factor is inserted in series with the capacitor-reactor unit as shown diagrammatically by figure 6. The reactor has been converted into an autotransformer without affecting its reactance characteristics and the higher secondary voltage permits a more economical capacitor to be used on common 110-volt circuits. The vector diagram for the complete circuit is shown in figure 7 for only one value of line voltage. The resultant power factor is leading, but obviously it is more nearly unity than the power factor of the parallel branch of the circuit alone; that is, δ is less than α. The vector diagram in figure 8 shows the relation between the three voltages E_0, E_1, E_2 with a resistance load across the transformer for two values of line voltage.

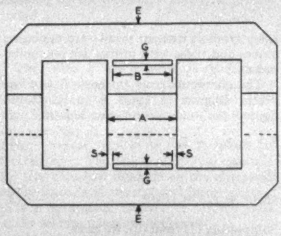

Figure 3. Bridge-gap core

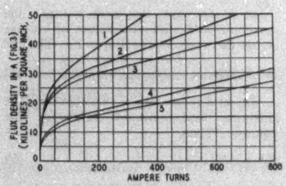

Figure 4. Calculated saturation curves for core shown in figure 3 (A = 1.375 inch)

Curve	g (Inch)	s (Inch)
(1)	0.062	0.125
(2)	0.125	0.125
(3)	0.187	0.125
(4)	0.187	0.062
(5)	0.25	0.062

The circuit shown in figure 6 will maintain a constant I_s in the load circuit for a variable line voltage, a variable load impedance or a variation of both providing the reactor and capacitor operate over a range of voltage through which the volt-ampere curves are parallel. With a constant resistance load it can be shown that the most economical design from the viewpoint of minimum capacity in microfarads occurs when the variation in voltage E_1 is $\sqrt{2}$ times the variation in line voltage. Also when the line voltage is constant and the resistance load variable the value of E_1' in figure 2 should be $\sqrt{2}$ times the value of E_1. (See also figure 9.) The load may operate at either lagging, or leading power factor, providing the limits of voltage E_1 are not exceeded.

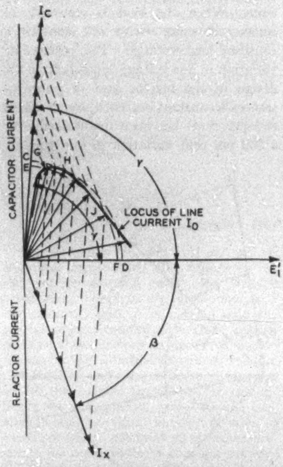

Figure 5. Successive vector diagram

Characteristics

The complete characteristics of a constant-current transformer supplying a resistance load are shown in figure 10. This unit was designed to operate on a line voltage varying between 105 and 125 volts and to deliver a constant current of one ampere at 25 volts. Between the specified limits of voltage, the output current actually varies from 1.04 amperes to 1.048 amperes.

The efficiency of the constant-current system varies between 50 per cent and 90 per cent. It is highest at the lowest value of line voltage and decreases in value as the line voltage increases. The power factor of the complete system is leading throughout the operating range for resistance load, and, like the efficiency, it is more nearly unity at the lower value of line voltage and decreases as the line voltage increases.

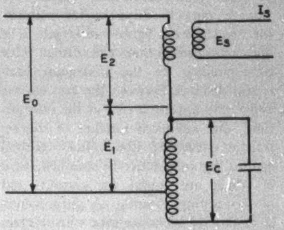

Figure 6. Constant-current-transformer circuit

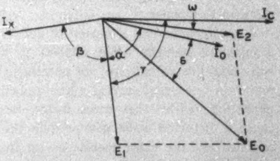

Figure 7. Vector diagram of constant-current transformer

Since reactance in the parallel circuit depends on frequency, it may be expected that the stable adjustment will be altered by changes in frequency and this is indeed the case. The volt-ampere curve of the reactor in figure 2 will swing toward the voltage axis as the frequency is increased, while that of the capacitor will swing toward the current axis as the frequency is increased. If a current-regulating circuit is designed to maintain a constant current at 60 cycles, it will gradually increase the line and load currents with increasing line voltage if the frequency is higher than 60 cycles. Conversely, if the frequency is less than 60 cycles the line and load currents will gradually decrease with increasing line voltages. Modern systems are so regulated that the variation in frequency may not exceed $^1/_8$ or $^1/_4$ of a cycle, and this is not great enough to injure the current regulating characteristics of a properly designed circuit. The characteristics of one current stabilizing unit on 59, 60, and 61 cycles are illustrated in figure 11.

Generally speaking, this particular type of nonlinear circuit has many interesting characteristics depending on the relation between the capacitor and reactor volt-ampere curves. Suppose, for example, the two curves in figure 2 intersect. The line current will contain a maximum value corresponding to the maximum horizontal distance between the two curves, and it will approach zero at the intersection. An unlimited number of characteristic curves of line voltage plotted against line current can be obtained, some of which are useful for current- and power-limiting circuits. A phenomenon of instability is encountered when certain relations exist between the series and parallel components of the circuit, but a discussion of this phenomenon is beyond the scope of this paper.

A nonlinear circuit designed to deliver a constant current when applied to a sinusoidal line voltage will not necessarily maintain a constant current when applied to voltage with other wave forms, because a distorted wave will produce approximately the same results as an increase in frequency. By properly adjusting the reactor, however, a constant-current transformer may be made to operate on distorted voltage with reasonable satisfaction. Inherently, the current stabilizing system does not produce a badly distorted wave as indicated by the oscillogram of load and current in figure 12. The reactor and capacitor currents each contain a large third harmonic but their phase relation is such that they circulate around the parallel circuit and thereby are greatly restricted from flowing in the line. Transients existing in the line voltage will disturb the regulation in the load circuit, to a certain extent as indicated in figure 13. The oscillogram shows the line voltage and output voltage on the current stabilizer when the line voltage is rapidly changed from its minimum to maximum value, which in this case was from 85 to 125 volts.

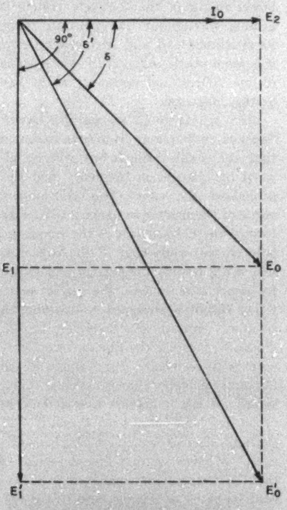

Figure 8. Vector diagram of ideal constant-current transformer for two values of line voltage

The static constant-current circuit has been applied to numerous filament-control circuits for maintaining a constant filament current on variable-voltage circuits. When the load is constant, of course, the same circuit will maintain a constant load voltage. The permissible variation in line voltage depends upon the design of the reactor and in some instances a constant current (plus or minus two per cent) has been maintained over a 500 per cent variation in line voltage. This requires a rather extreme and bulky design. For lines having a total voltage variation of 50 per cent the static constant transformer is from twice to three times the physical size of an ordinary transformer with the same rating.

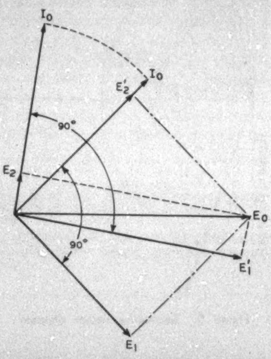

Figure 9. Vector diagram of constant-current transformer with constant line voltage and a variable resistance load

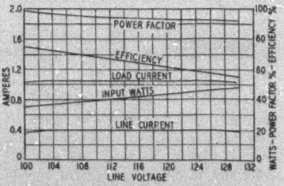

Figure 10. Characteristic curves of a 25-watt constant-current transformer

Appendix

Derivation of Formulas for Variable Line Voltage

A mathematical derivation of design formulas for a nonlinear circuit to operate on a variable line voltage and to maintain a constant current in a fixed, resistance load is given below. Sinusoidal currents and voltages are assumed and the losses in the reactor and capacitor are neglected. In all cases symbols without primes are minimum values and those with primes are maximum values.

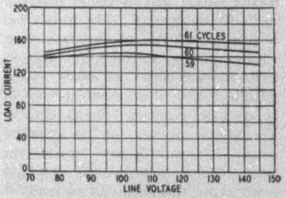

Figure 11. Frequency-regulation for a 150-watt, one-volt constant-current transformer

The circuit diagram in figure 6 and the vector diagram in figure 8 apply. From figure 8 the fundamental equations are:

$$E_1^2 + E_2^2 = E_0^2 \qquad (1)$$

and

$$(E_1^1)^2 + E_2^2 = (E_0^1)^2 \qquad (2)$$

Subtracting (1) from (2), we have

$$(E_1^1)^2 - E_1^2 = (E_0^1)^2 - E_0^2 \qquad (3)$$

Let k represent the ratio between the maximum and minimum values of voltage across the reactor-capacitor unit. That is,

$$E_1^1 = kE_1 \qquad (4)$$

Also let d represent the ratio between the maximum and minimum values of line voltage. Thus,

$$E_0^1 = dE_0 \qquad (5)$$

Substituting (4) and (5) in (3) for E_1^1 and E_0^1, respectively, and solving for E_1 we get,

$$E_1 = E_0\sqrt{\frac{d^2 - 1}{k^2 - 1}} \qquad (6)$$

From equation (1),

$$E_2 = \sqrt{E_0^2 - E_1^2} \qquad (7)$$

Substituting (6) in (7), we obtain

$$E_2 = E_0\sqrt{\frac{k^2 - d^2}{k^2 - 1}} \qquad (8)$$

The capacitor must have the highest volt-ampere rating of any element in the circuit. The current passing through it is the sum of the exciting current and load current. It is desirable, therefore, to keep the size of capacitor at a minimum, by properly adjusting k with respect to d.

The volt-ampere rating (M) of the capacitor is independent of the voltage to which it is referred. That is, from figure 6,

Figure 12. Wave form of load current of a constant-current transformer

a—Load current b—Line voltage

$$M = E_1 I_c \qquad (9)$$

Where the current is referred to the low voltage side of the autotransformer.

Allowing I_c/I_0 (see figure 2) to be represented by the symbol P equation (9) becomes

$$M = E_1 P I_0 \qquad (10)$$

The value P has a physical significance in that it represents the degree of current utilization. The capacitor is capable of supplying the current I_c in the line, but of this quantity I_x is taken to excite the nonlinear reactor which leaves I_0 available for output. The value of P is a constant for any core and depends on the sharpness of the knee, and the slope of linear portion of the saturation curve. It applies only at the minimum value considered.

The line current required (see figure 6) is

$$I_0 = \frac{W}{E_2 e} \qquad (11)$$

where

W = load watts

e = efficiency of output transformer

Then substitute (11) in (10)

$$M = \frac{E_1 P W}{E_2 e} \qquad (12)$$

and again substituting (6) and (8) in (12) we have

$$M = \frac{PW\sqrt{d^2 - 1}}{e\sqrt{k^2 - d^2}} \qquad (13)$$

Since $I_c = 2\pi f C E_1 \times 10^{-6}$ the capacity required in microfarads is

$$C = \frac{M}{2\pi f E_1^2} \times 10^6 \qquad (14)$$

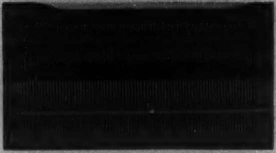

Figure 13. Effect of transients in constant-current transformer when the line voltage is rapidly changed (125 to 85 to 125 volts)

Above—Line voltage Below—Load voltage

Substituting (13) for the value of M in (14),

$$C = \frac{PW\sqrt{d^2 - 1}}{2\pi f E_1^2 e\sqrt{k^2 - d^2}} \times 10^6 \text{ microfarads} \qquad (15)$$

Equation (15) indicates the size of the capacitor necessary if placed directly across the voltage E_1. If the capacitor is placed across the secondary of the autotransformer as in figure 6, it is necessary to replace E_1^2 by E_c^2. The maximum voltage that can be impressed safely on the capacitor is a limiting feature of the design and it must be incorporated in the design formulas. If we let E_c^1 be this maximum voltage allowable, then E_1^2 in equation (15) should be replaced by E_c^1/k. Equation (15), with this transformation, becomes

$$C = \frac{PW\, k^2\sqrt{d^2 - 1}}{2\pi f (E_c^1)^2\, e\sqrt{k^2 - d^2}} \times 10^6 \text{ microfarads} \qquad (16)$$

Equations (6), (8), and (16) are general equations representing the voltage across the reactor-capacitor unit, the voltage across the primary of the transformer, and the capacity required in parallel with the reactor or autotransformer. These general equations are derived in terms of known constants, the variation in voltage existing on the line, and the variation in voltage across the parallel branch of the circuit. There are a number of values for the constant k and an equal number of sizes of capacitor which will meet all of the requirements for maintaining a constant current. When equation (16) is differentiated with respect to k, the value of k corresponding to the minimum value of capacitor can be obtained. Before the equation is differentiated, however, it can be reduced to a more simple form. For any specific design, all quantities in (16) are constant except k; and all fixed terms except d can be included in a constant term Q. Thus,

$$C = \frac{k^2 Q\sqrt{d^2 - 1}}{\sqrt{k^2 - d^2}}$$

or

$$\left(\frac{C}{Q}\right)^2 = \frac{k^4(d^2 - 1)}{k^2 - d^2} = y \qquad (17)$$

When y is a minimum, C must also be a minimum; therefore, y can be differentiated with respect to k.

We find thereby that C is minimum when

$$k = \sqrt{2}\, d \qquad (18)$$

Substituting this value of k in equations (6), (8), and (16), respectively, we obtain the design equations for the most economical conditions.

$$E_1 = E_0\sqrt{\frac{d^2 - 1}{2d^2 - 1}} \qquad (19)$$

$$E_2 = E_0\sqrt{\frac{d^2}{2d^2 - 1}} \qquad (20)$$

$$C = \frac{WPd\sqrt{d^2 - 1}}{(E_c^1)^2 \pi f e} \times 10^6 \text{ microfarads} \qquad (21)$$

Derivation of Formulas for Constant Line Voltage

The derivation of design formulas for a current-regulating circuit operating on a constant line voltage to supply a variable resistance load with a constant current is given below. The same assumption and nomenclature previously established apply to this circuit. The circuit diagram in figure 6 applies, but the vector diagram is shown in figure 9. The important difference between this circuit and the one previously analyzed is that the maximum value of reactor voltage, E_1^1, exists simultaneously with the minimum value of output transformer voltage, E_2, and vice versa. The symbol m is defined as

$$m = E_2^1/E_2$$

A procedure similar to that outlined above produces equations for minimum capacitance as follows:

$$k = \sqrt{2} \qquad (22)$$

$$E_2 = E_0\sqrt{\frac{1}{2m^2 - 1}} \qquad (23)$$

$$E_1^1 = E_0\sqrt{\frac{2(m^2 - 1)}{2m^2 - 1}} \qquad (24)$$

and

$$C = \frac{I_0 E_0 P\sqrt{m^2 - 1}}{\pi f (E_c^1)^2\sqrt{2m^2 - 1}} \times 10^6 \text{ microfarads} \qquad (25)$$

The above equations are sufficient to completely design a constant-current circuit with the exception of one factor; namely, the number of turns on the primary of the nonlinear reactor. The saturation curve for the core with flux density plotted against ampere-turns, of course, must be available; and a parallel line representing the capacitor volt-amperes must be drawn to the straight portion of the saturation curve. The horizontal distance between the two lines will represent ampere-turns and will correspond to the value in figure 2. The required line current I_0 can be determined easily for the design from the load requirement. If NI represents the horizontal distance between the saturation curve and the capacitor line in ampere-turns, and T represents the number of turns on the primary of the nonlinear reactor or autotransformer, the following equation applies:

$$T = \frac{NI}{I_0} \text{ (from curve)} \qquad (26)$$

Analysis and Design of Harmonic Generators

By FREDERICK EMMONS TERMAN
MEMBER AIEE

HARMONIC generators such as used in radio transmitters are usually designed and adjusted on the basis of experience because the only analysis of their performance that has been available is a laborious point-by-point method of attack which is much too cumbersome to be practical in the general run of problems. In the present paper a new method of analyzing the harmonic generator is presented that involves a minimum of labor and at the same time has an accuracy entirely adequate for design purposes.

Voltage and Current Relations in Harmonic Generators

The circuit of a typical harmonic generator is shown in figure 1*a*, and is similar to a class-*C* amplifier circuit except for the fact that the plate tank circuit is tuned to a harmonic of the exciting voltage. When properly adjusted the instantaneous voltages, currents, and powers follow oscillograms such as indicated in figure 1. (The assumption of a sinusoidal alternating plate voltage in figure 1 is equivalent to saying that the Q of the plate tank circuit is high enough to make the voltage drop across it negligible for all alternating components except the desired harmonic.) The grid bias is made considerably greater than cutoff and sufficient signal voltage is applied to drive the grid somewhat positive. The result is that the space current flows in pulses having a relatively large peak amplitude but lasting for only a small fraction of the cycle. Such pulses have a large harmonic content, and the tank-circuit impedance is adjusted so that the desired component builds up a crest voltage across this circuit that is only slightly less than the plate-supply potential. This makes the minimum plate potential reached during the cycle quite small, as shown. The instantaneous power supplied to the plate circuit of the harmonic generator is equal to the plate-supply potential multiplied by the instantaneous plate current, as shown in figure 1*g*. This power is divided between loss at the plate of the tube and useful output delivered to the tank circuit in proportion to the instantaneous voltage drops existing across these parts of the circuit. Similarly the power loss at the grid at any instant equals the instantaneous grid current times the instantaneous grid-cathode potential, and the power that the grid exciting voltage must supply is likewise equal at any instant to the product of grid current and exciting voltage at that moment.

Simplified Analysis of Harmonic Generators

The oscillograms of figure 1 provide a means for making an exact analysis of the performance of the harmonic generator. These oscillograms can be drawn from the characteristic curves of the tube for any assumed voltages acting on the grid and plate electrodes. The practical usefulness of the result is, however, limited by the large amount of work required to plot the various curves point by point and to planimeter them to get averages.

An approximate analysis can be made by noting that the pulses of total space current in figure 1 appear upon inspection to approximate a section of a sine-wave pulse raised to some power of the order of 3/2. In screen-grid and pentode tubes, and in high-mu triodes, the space-current pulses are in fact almost exactly sections of such a sine pulse because of the fact that in these cases the plate voltage has little or no effect upon the total space current, which is then determined by a sine wave driving voltage of fundamental frequency. The analysis of harmonic generators under such conditions has already been outlined in earlier papers,[1,2] and makes use of curves giving the ratio I_0/I_m of average current to peak space current, and the ratio I_n/I_m of harmonic component of the space current pulse to peak value of pulse, in terms of the number of electrical degrees θ during which the space current flows. Such curves are given in figure 2.

In the case of harmonic generators employing ordinary triode tubes the space-current pulse does not have a shape that is exactly a section of a sine wave raised to some power. This is because the effective voltage producing the current pulse is a combination of fundamental-frequency and harmonic voltages, which obviously cannot follow exactly a section of a simple sine wave. The extent of the discrepancy depends upon the angle of current flow, the tube characteristics, etc. A practical procedure for analyzing the ordinary triode harmonic generator can, however, be worked out by assuming as a first approximation that the space-current pulse follows a section of a simple sine wave raised to a power, and then applying a correction to this result in order to take into account the discrepancy between the assumed and actual pulse shape.

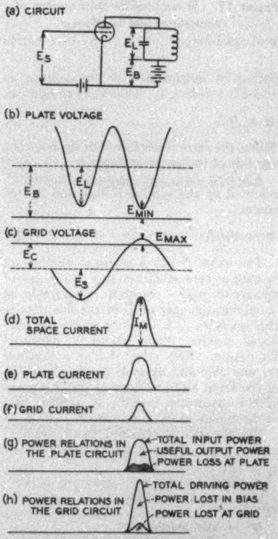

Figure 1. Circuit of typical harmonic generator, together with voltage, current, and power oscillograms

Curves giving the correction factor that must be applied to results obtained on the sine-wave section assumption are given in figure 3 for linear and square-law tube characteristics. The derivation of these curves follows.

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1. For all numbered references, see list at end of paper.

Mathematical Analysis

NOTATION

e_1 = net voltage available to produce electrostatic field in the vicinity of the cathode for the case of simple sine-wave pulse

e_h = net voltage available to produce electrostatic field in the vicinity of the cathode for harmonic generator case

$E_{\max}$ = $E_s - E_c$ = maximum instantaneous grid potential reached during cycle (see figure 1)

$E_{\min}$ = $E_B - E_L$ = minimum instantaneous plate potential reached during cycle (see figure 1)

θ = number of electrical degrees during which current flows (see figure 1) (based on fundamental frequency)

E_s = crest signal voltage (see figure 1)

E_L = $E_B - E_{\min}$ = crest alternating voltage developed between plate and cathode (see figure 1)

E_c = grid bias

E_B = d-c plate-supply potential

μ = amplification factor of tube

n = harmonic involved

E_m = $E_{\max} + \dfrac{E_{\min}}{\mu}$ = crest amplitude of e_1 or e_h

k = $(E_L/\mu)/E_m$

$$A = 1 - \cos\theta/2$$

$$B = 1 - \cos n\theta/2$$

$$C = \frac{B - A}{A} = \frac{\cos\theta/2 - \cos n\theta/2}{1 - \cos\theta/2}$$

The net effective voltage available to produce a space-current pulse when the signal voltage and the output voltage are of the *same* frequency (class-C amplifier action), is

$$e_1 = \left(E_s - \frac{E_L}{\mu}\right)\cos\omega t - E_c + \frac{E_B}{\mu} \qquad (1)$$

$$= E_m\frac{(\cos\omega t - \cos\theta/2)}{1 - \cos\theta/2} \qquad (1a)$$

This voltage produces a space current that for positive e_1 is very nearly proportional to $e_1{}^{\alpha}$, where α lies between one and two and is normally very close to 3/2. The space current is assumed to be zero when e_1 is negative.

The net effective voltage producing electrostatic field at the surface of the cathode when the alternating plate voltage E_L is a *harmonic* of the exciting voltage is

$$e_h = E_s\cos\omega t - \frac{E_L}{\mu}\cos n\omega t - E_c + \frac{E_B}{\mu} \qquad (2)$$

A comparison of equations (1) and (2) shows that e_h fails to be a section of a simple sine wave exactly to the extent that the term $(E_L/\mu)\cos n\omega t$ fails to follow a sine-wave section of fundamental frequency in the interval $\omega t = -\theta/2$ to $\omega t = \theta/2$, (i. e., the interval *abc* in figure 4). The error involved in using the curves of figure 2 to give the space-current pulses is then caused by the discrepancy between the dotted and solid curves in figure 4, where the dotted curve corresponds to a wave of fundamental frequency coinciding with the actual harmonic curve at points *a*, *b*, and *c*.

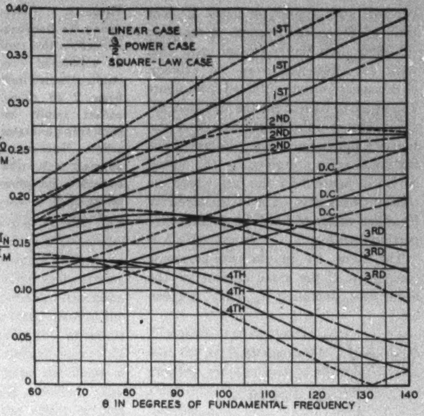


Figure 2. Curves giving the d-c and harmonic components of space-current pulses that are produced by an effective driving voltage that is a section of a simple sine wave

The magnitude of this error can be evaluated by taking the difference of the dotted and solid curves in figure 4. The equation of the solid (harmonic) curve in figure 4 is

$$e' = \frac{E_L}{\mu}\cos n\omega t \qquad (3)$$

while the equation of the dotted (fundamental) curve referred to the same axis is

$$e'' = \left(\frac{E_L}{\mu} + \beta\right)\cos\omega t - \beta \qquad (4)$$

where β represents the axis of the dotted curve as shown in the figure, and the two curves are assumed to coincide at *b*. The value of β is determined by the fact that the dotted curves must also pass through points *a* and *c*. That is

$$\frac{E_L}{\mu}\cos n\theta/2 = \left(\frac{E_L}{\mu} + \beta\right)\cos\theta/2 - \beta \qquad (5)$$

Eliminating β in (4) by means of (5) gives

$$e'' = \frac{E_L}{\mu}\left\{\left(\frac{1 - \cos n\theta/2}{1 - \cos\theta/2}\right)\cos\omega t - \frac{\cos\theta/2 - \cos n\theta/2}{1 - \cos\theta/2}\right\} \qquad (6)$$

$$= kE_m\left\{\frac{B}{A}\cos\omega t - C\right\} \qquad (7)$$

where k, E_m, A, B, and C are as defined in the table of notation.

The difference $e' - e''$ then gives the error in the net effective voltage available to produce space current that results by assuming that the effective voltage producing the space current pulses is a section of a simple sine wave. This difference is

$$D = kE_m\left\{\cos n\omega t - \frac{B}{A}\cos\omega t + C\right\} \qquad (8)$$

Here D is the amount by which the actual shape of the pulse for the case of nth harmonic generation is *less* (note negative sign of E_L in equation 2) than the effective voltage that would be acting if the pulse was determined by a simple sine-wave section of same θ and same maximum value. As a consequence, the difference term D given by equation (8) must be *subtracted* from the pulse obtained on the assumption of a sine-wave section in order to obtain the true amplitude.

In the case of a tube having a linear characteristic, the space current is proportional to the equivalent voltage producing electrostatic field in the vicinity of the cathode. Hence the error in the average or d-c component of the total space current made by assuming a section of a simple sine wave is equal to the average of D in equation (8) over the interval θ. Assuming for convenience that $E_m = 1$, then the average D_0 of D is

$$D_0 = \frac{1}{2\pi}\int_{-\theta/2}^{\theta/2} D\, d(\omega t)$$

$$= \frac{k}{\pi}\left\{\frac{\sin n\theta/2}{n} - \frac{B}{A}\sin\frac{\theta}{2} + C\frac{\theta}{2}\right\} \qquad (9)$$

The average of equation (1) as a function of the angle of flow θ has been previously derived, and is given in figure 2. The ratio of D_0 to the average of equation (1) then gives the fractional error that must be subtracted from the average or d-c space current obtained from figure 2 in order to correct for the fact that the space-current pulse is not exactly a section of a simple sine wave. Values of this fractional error are given by the dotted curves in figure 3 for the case where the constant k is unity. The fractional errors for other values of k will be directly proportional to k.

The error D_n in the nth-harmonic component of the space current for the linear case, assuming E_m unity, is given by

$$D_n = \frac{1}{\pi}\int_{-\theta/2}^{\theta/2} D \cos n\omega t \, d(\omega t)$$

$$= \frac{2k}{\pi}\left\{\frac{1}{4n}(n\theta + \sin n\theta) - \frac{B}{A}\left(\frac{\sin (n-1)\theta/2}{2(n-1)} + \frac{\sin (n+1)\,\theta/2}{2(n+1)}\right) + \frac{C}{n}\sin\frac{n\theta}{2}\right\} \quad (10)$$

The fractional error that must be subtracted from the value of nth harmonic obtained from figure 2 on the assumption of a sine-wave pulse is then the result obtained from equation (10) divided by the nth-harmonic component of the space-current pulse on the assumption of a simple sine-wave section as obtained from figure 2. Values of this ratio are given by the dotted lines of figure 3.

Consider next the case where the total space current is proportional to the square of the effective voltage producing electrostatic field near the cathode. The true value of space-current pulse is then proportional to $(e_1 - D)^2 = e_1^2 - 2e_1D + D^2$. The error involved in assuming that the pulse has a shape based upon a section of a sine wave is obviously $2e_1D + D^2$. However if the error D is not large compared with e_1, as is the case under practical conditions, then D^2 can be neglected and the error becomes very nearly $2e_1D$. The error D_0' in the average or d-c component of the total space current is hence (assuming E_m is unity)

$$D_0' = \frac{1}{2\pi}\int_{-\theta/2}^{\theta/2} 2e_1D$$

$$= \frac{1}{\pi}\int_{-\theta/2}^{\theta/2} \frac{\cos \omega t - \cos\theta/2}{1 - \cos\theta/2} D d(\omega t)$$

$$= \frac{2k}{\pi A}\left\{\frac{\sin (n-1)\,\theta/2}{2\,(n-1)} + \frac{\sin (n+1)\theta/2}{2\,(n+1)} - \frac{B}{4A}(\theta + \sin\theta) + C \sin\theta/2\right\} - \frac{2D_0 \cos\theta/2}{A} \quad (11)$$

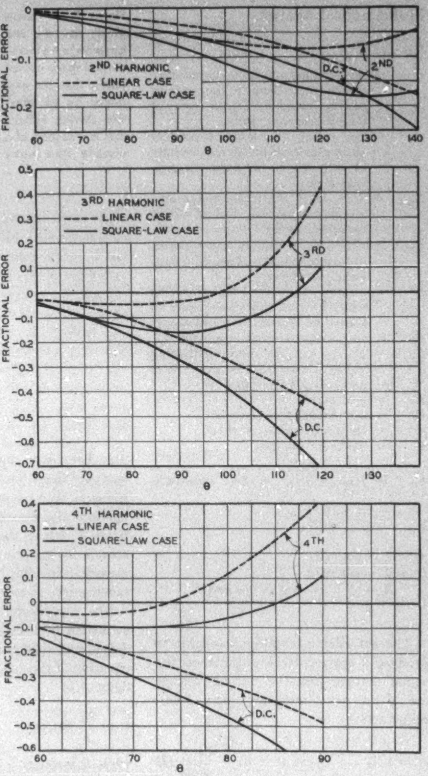
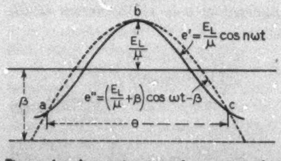


Figure 3. Curves giving the fractional corrections that must be subtracted from results obtained from figure 2 to obtain correct results for harmonic-generator action. These curves assume k = 1. The correction for other values of k is proportional to k

The ratio of this to the d-c value obtained from figure 2 for a square-law characteristic is then the fractional error involved in assuming that the pulses in the harmonic generator are based on the square of a section of a simple sine wave. Values for this fractional error are given by the solid lines in figure 3. In a similar manner the error D_n' in the nth harmonic component of the total space current that is involved in assuming that the pulse is the square of a section of a simple sine wave is



b
$\frac{E_L}{\mu}$
$e' = \frac{E_L}{\mu}\cos n\omega t$
$e'' = \left(\frac{E_L}{\mu} + \beta\right)\cos \omega t - \beta$
β
a
c
θ

Figure 4. Approximation of a section of a sine-wave section of harmonic frequency by a section of a sine wave of fundamental frequency

$$D_n' = \frac{1}{\pi}\int_{-\theta/2}^{\theta/2} 2e'D \cos n\omega t \, d(\omega t)$$

$$= \frac{2}{\pi}\int_{-\theta/2}^{\theta/2} \frac{\cos \omega t - \cos\theta/2}{1 - \cos\theta/2} D \times \cos n\omega t \, d(\omega t)$$

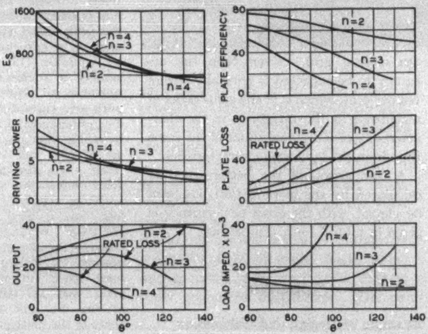
$$= \frac{2k}{\pi A}\left\{\sin\frac{\theta}{2} + \frac{\sin (2n-1)\,\theta/2}{2\,(2n-1)} + \frac{\sin (2n+1)\,\theta/2}{2\,(2n+1)} - \frac{B}{A}\left[\frac{\sin n\theta/2}{n} + \frac{\sin (n-2)\,\theta/2}{2\,(n-2)} + \frac{\sin (n+2)\,\theta/2}{2\,(n+2)}\right] + C\left[\frac{\sin (n-1)\theta/2}{(n-1)} + \frac{\sin (n+1)\theta/2}{(n+1)}\right]\right\} - \frac{2\cos\theta/2}{1 - \cos\theta/2} D_n \quad (12)$$

The ratio of the error given by equation (12) to the nth-harmonic component of the space current for the square-law characteristic as given in figure 2 is then the fractional error. This is plotted in figure 3 for various harmonics as a function of angle of current flow.

Calculation of Harmonic-Generator Performance

It is now possible to calculate the performance of the harmonic generator for any given set of operating conditions. In doing this the operating condition is defined in terms of the angle of current flow θ, maximum grid potential $E_{\max}$, minimum plate potential $E_{\min}$, grid bias, and plate-supply voltage. The first step is to determine the peak total space current I_m and peak grid current I_{gm} corresponding to the given values of $E_{\max}$ and $E_{\min}$, by using characteristic curves as supplied by the tube manufacturer. The d-c and harmonic components I_0 and I_n of the total space current for this value of I_m are then obtained from figure 2, assuming that the current pulse is a section of a sine wave raised to a power of the order of 3/2. The next step is to determine from figure 3 the fractional corrections for the d-c and harmonic components, interpolating between the dotted and solid curves according to the exponent used in connection with figure 2. These fractional corrections are then multiplied by the value of k for the case at hand, and the necessary corrections determined, and subtracted from the results of figure 2 to give the components of the actual pulse of total space current.

Figure 5. Curves showing the performance of a typical tube operated as a harmonic generator under various conditions



The division of these components between grid and plate electrodes can be determined by the fact that when secondary electron emission at the grid is not excessive (as is normally the case), the pulse of grid current can be considered as approximating a section of a sine wave of fundamental frequency that has been squared.[3] The angle of current flow θ_g for this pulse can be calculated from the equation

$$\cos \theta_g/2 = \frac{E_c}{E_s} \tag{13}$$

The d-c, fundamental-frequency, and harmonic-frequency components of the grid current can now be evaluated in terms of θ_g and the peak grid current, with the aid of figure 2. The d-c plate current is then the d-c component of the total space current minus the d-c grid current, and the harmonic component of the plate current is the harmonic component of the total space current minus the harmonic component of the grid-current pulse. (If secondary emission is large the shape of the grid current pulse is modified. Under such conditions the effect of grid current can be evaluated roughly by estimating the ratio of d-c grid current to d-c component of total space current on the basis of previous experience, and then assuming that the fundamental and harmonic components of the grid current are twice the d-c value.)

It now remains to determine the power relations. The power supplied to the plate circuit is the d-c component of the plate current multiplied by the plate-supply voltage, while the power output delivered to the tank circuit is half the harmonic component of the plate current multiplied by the crest alternating plate-cathode voltage E_L. The ratio of these two powers gives the plate efficiency, and their difference gives the dissipation at the plate. The ratio of the alternating plate-cathode voltage E_L to the harmonic component of the plate current equals the load impedance that the tank circuit must supply.

The grid bias can be calculated by the following formula derived from conventional theory:

$$\text{Grid bias} = E_c = \frac{E_B(1-\cos n\theta/2)+E_{\min}\cos n\theta/2}{\mu(1-\cos\theta/2)} + \frac{E_{\max}\cos\theta/2}{1-\cos\theta/2} \tag{14}$$

The peak exciting voltage is $E_c + E_{\max}$, and the grid driving power is this peak exciting voltage multiplied by half the fundamental-frequency component of grid current.

An example illustrating the details of the calculation procedure is given in the appendix.

Discussion of Factors Involved in Design of Harmonic Generators

The most important operating characteristics of a harmonic generator are the harmonic power delivered to the tank circuit, the power dissipation at the plate of the tube, and the driving power. The design factors available to control the performance are the angle of current flow, the minimum instantaneous potential $E_{\min}$ reached by the plate, and the maximum instantaneous potential $E_{\max}$ reached by the grid. The influence of these design factors on the performance of a harmonic generator is illustrated in figure 5. A study of this figure, together with figures 2 and 3, brings out the salient features involved in harmonic generator behavior. *First*, for a given peak space current* and harmonic, the power output will be maximum when the angle θ of current flow based on the fundamental frequency is approximately 260°/n. The maximum is not critical however, and angles of flow differing from this by as much as ±40 per cent give good output. *Second*, the plate efficiency of the harmonic generator is increased by reducing the angle of current flow to a value smaller than that giving maximum output because this causes the d-c component of the total space current, and hence the input power, to decrease faster than the output falls off. It is also to be noted that the plate efficiency of a harmonic generator will be considerably less than for the corresponding class-*C* amplifier unless the angle of flow is appreciably less than the value for maximum output. *Third*, the values of $E_{\min}$ and $E_{\max}$ not only must be such as to draw the maximum allowable space current, but should also normally have a ratio $E_{\max}/E_{\min}$ somewhat less than is customary in class-*C* amplifiers in order to keep the grid current, and hence the driving power, low in spite of the large exciting voltage required. *Fourth*, the driving power increases as the angle of current flow is reduced, and becomes excessive at angles of flow as small as 60 to 70 degrees. Because of this the driving power is as much a limiting design factor in harmonic generators as is the plate loss and power output. *Fifth*, the load impedance that the tank circuit must supply is greater for harmonic generators than for the corresponding class-*C* amplifier, and becomes greater the higher the order of harmonic. This results from the fact that for proper operation the voltage across the tank circuit is independent of the harmonic whereas the output power is less the higher the harmonic. With a given effective tank circuit Q this means that the L/C ratio should be higher for harmonic generators than class-*C* amplifiers. *Sixth*, the output power obtainable in second harmonic generation is about $^3/_4$ of the output that the same tube will develop as a class-*C* amplifier, and for higher harmonics the output is roughly inversely proportional to the order of the harmonic. *Seventh*, the fact that output goes down and driving power goes up as the order of the harmonic is increased makes it unwise to attempt harmonic generation above the third or fourth harmonic under ordinary conditions, since with fifth harmonic operation the driving power can be expected to be of the same order of magnitude as the power output.

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* The allowable peak space current is determined by the cathode emission, assuming a reasonable factor of safety. Manufacturers have not, however, established peak current ratings for their tubes. Until this is done the designer may assume that the peak current rating approximates four to five times the sum of d-c plate and grid currents obtained with recommended class-*C* telegraph operation.

Accuracy

The accuracy of the analysis that has been developed for harmonic generators is entirely adequate for ordinary design purposes. The principal assumption involved is that the total space current of the tube is proportional to $(E_g + E_p/\mu)^\alpha$, where μ and α are constants. If the values of μ and α are taken for conditions corresponding to a fairly large total space current experience[2] shows that the errors introduced by considering α and μ constant are quite small even though α and μ vary appreciably as cutoff is approached. The method of handling grid current also involves some approximation, but this is unimportant, particularly if there is no secondary emission. In any case the action of grid current is in itself only a second-order effect and therefore need not be determined with precision. The neglect of the D^3 term in obtaining equations (11) and (12) also introduces some error, particularly when θ exceeds $260°/n$, but since the values of k encountered in practice are usually less than 0.5, the error in the actual pulse of total space current from this approximation will reach five per cent only in the most unfavorable cases.

Under ordinary conditions the analysis can be expected to give calculated plate loss, output power, and plate current, correct within about five per cent unless the angle of flow is exceptionally large for the harmonic involved, or unless there is excessive secondary electron emission at the control grid. Even then the accuracy to be expected is normally within ten per cent, which is of the same order of magnitude as the variations in the characteristics of individual tubes of the same type from the published characteristics.

Table I. Agreement Between Calculated and Exact Harmonic-Generator Performance

Conditions: Type 204-*A* tube, $E_B = 1{,}500$, $E_{\max} = 250$, $E_{\min} = 375$, $I_m = 1.26$ amperes, peak grid current = 225 milliamperes, θ = 100 degrees, $n = 3$, $E_c = 751$ volts

	Approximate	Exact
D-c plate current (milliamperes)	189	196
D-c grid current (milliamperes)	27	20
Power input (watts)	284	294
Power output (watts)	105	102
Driving power (watts)	26	19
Plate loss (watts)	179	192
Plate efficiency (per cent)	37	35

A comparison of the performance as calculated by the method that has been developed, with the exact performance is given in table I for a typical case.

Appendix

Example of Design and Calculation of a Harmonic Generator; Comparison With Exact Performance

In order to show how the above analysis is applied, assume that it is desired to obtain third harmonic generation from a type 204-*A* tube operating at 1,500 volts, having $\mu = 23$, and a normal rated plate dissipation of about 200 watts. This tube is not the most satisfactory one for demonstrating the accuracy of the method of analysis that has been developed because the published characteristics of the 204-*A* tube show more secondary electron emission at the grid than do most air-cooled tubes. However, it is selected because it is one of the few tubes for which the published characteristics are sufficiently complete to permit checking of the approximate analysis by exact point-by-point calculations. Most of the so-called "complete" characteristic curves of transmitting tubes found in tube manuals are not extended to high enough plate potentials to cover the full operating range commonly used in harmonic generators.

The first step in the design is the selection of a suitable peak space current and corresponding values of $E_{\min}$ and $E_{\max}$. Reference to the data supplied by tube manufacturers indicates that $I_m = 1{,}260$ milliamperes will be satisfactory, and that this could be obtained by $E_{\min} = 375$, $E_{\max} = 250$. With this combination the peak grid current will be 225 milliamperes, which is small enough to keep the driving power low, and yet at the same time $E_{\min}$ will be small compared with the plate-supply voltage.

The next step is the selection of a value of θ. From the discussion given in the paper, combined with an examination of figure 2, it appears that $\theta = 100$ degrees would be reasonable. With this value of θ, and assuming a 3/2-power law, figure 2 then gives $I_0/I_m = 0.161$, and $I_n/I_m = 0.177$. The fractional corrections that figure 3 indicates would be applied to these for a 3/2 law and for $k = 1$ are -0.33 for the d-c term and -0.060 for the third-harmonic component. However, as k is $[(1{,}500 - 375)/23]/250 = 0.20$ the actual corrections are -0.065 and -0.012, respectively. The d-c and third-harmonic components of the total space current are then $1{,}260 \times 0.161\ (1+0.065) = 216$ milliamperes, and $1{,}260 \times 0.177\ (1 + 0.012) = 226$ milliamperes, respectively. From equation (14) the grid bias is found to be 751 volts, which makes the peak signal voltage 1,001 volts. The angle θ_g of grid-current flow is then given by equation (13) as 82.8 degrees. Assuming a square-law grid-current pulse, reference to figure 2 shows that for a peak grid current of 225 milliamperes, the d-c, fundamental-frequency, and third-harmonic components of the grid current are 27, 52, and 39 milliamperes, respectively. As the peak signal voltage is 1,001 volts the grid driving power is $1{,}001 \times 0.052/2 = 26$ watts. If grid-leak bias is used the leak resistance is $751/0.027 = 28{,}000$ ohms. Next, subtracting the d-c and third-harmonic components of the grid current from the total space current, gives 189 milliamperes, and 187 milliamperes, respectively, as the d-c and third-harmonic components of the plate current. This calls for a tank-circuit impedance of $1{,}125/0.187 = 6{,}000$ ohms. The power input to the plate is 284 watts while the third-harmonic power delivered to the tank circuit is $0.187 \times 1{,}125/2 = 105$ watts. The plate efficiency is hence 37 per cent while the plate dissipation is 179 watts. Inasmuch as this loss is slightly less than the rated value for normal operation, the assumed $E_{\min}$, $E_{\max}$, and θ represents a suitable operating condition.

The performance of this harmonic generator for the same E_B, $E_{\min}$, $E_{\max}$, and E_c has been calculated exactly by graphical integration of curves such as shown in figure 1 drawn on the basis of tube characteristics given in the tube manual. The results are given in table I, and show

The Electric Strain Gauge

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MANY new developments have grown out of the vibration recorder[1] originally described before the AIEE in 1926. One of the first of these developments resulted in apparatus for recording transient pressures electrically.[4] This has been used to measure transient pressures in switchgear apparatus, transformers, water wheels, steam turbines, gasoline engines, refrigerators, and in other places. Another is the electric gauge[8,18] which has many applications. It is used for the most accurate gauging of production parts in machine shops and inspection rooms, also it is widely used in steel mills for continuous gauging of strips as they are being rolled, for controlling their thickness, for measuring the tension in the strip[19] and the pressure applied to the mill rolls. Another application is the measurement of the thickness of paint, enamel, or other nonmagnetic coatings over steel.[13]

The electric strain gauge is an instrument for measuring mechanical strains electrically. Electrical, mechanical, and civil engineers have used it very successfully. It has proved its value in the study of stresses in railroad tracks and in bridge members as locomotives pass over them and in determining the lateral thrust produced by the axles of the locomotives.[14]

Electric-locomotive designs have been greatly influenced by the results obtained from these gauges. It is equally useful in measuring the strain in machine parts when they are subjected to stress.

Originally these developments used an oscillograph to produce records in which a 500-cycle wave was modulated, figure 5 (*a*). The envelope of the wave showed the variations in the quantity being recorded. In order to obtain accuracy the deflections were necessarily large and the number which could be recorded on a single film without overlapping each other was correspondingly small.

The record obtained from the electric strain gauge is a single-line trace of the varying strain in the part to which the gauge is attached. Several of these curves can be recorded on a film, figures 5 (*b*) and (*c*), even though the deflections are comparatively large. This makes it possible to obtain greater sensitivity without introducing difficulty in interpreting the records.

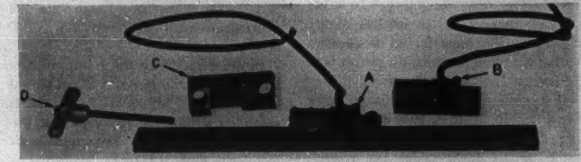

Figure 1

A—Strain-gauge head mounted on a test bar
B—Balancing unit
C—Attaching fixture

The electric strain gauge, shown in figures 1 and 2, has a circuit fundamentally the same as that of the vibration recorder.[1] The four arms of an a-c bridge circuit, figure 3, contain, respectively, a strain-gauge head, a balancing unit and two coils of a differentially connected transformer. The two differential transformer coils will have zero drop across them when the strain-gauge head and the balancing unit have about the same air gaps. The air gap of the balancing unit remains fixed while that of the strain-gauge head varies according to the strain in the piece of apparatus under investigation. The voltage drop across the differential transformer varies with the changes in the air gap of the strain-gauge head and the deflections of the oscillograph galvanometer are proportional to these.

The strain-gauge head *A*, figure 1, is shown mounted on a test bar and its balancing unit *B* is at the right. *C* is a mounting fixture used to align the gauge head while attaching it to the work. Changes in length of the test bar vary the air gap between the two parts of the magnetic circuit of the gauge head. In figure 2 is shown a four-circuit power unit for use with four strain-gauge heads and their balancing units. The four oscillograph galvanometers and the necessary 2,000-cycle excitation are also connected

Figure 2. Power unit for use with four strain-gauge heads

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[1] For all numbered references, see list at end of paper.

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an agreement entirely satisfactory for design purposes even in spite of the fact that the 204-*A* tube has considerable secondary electron emission at the control grid. This secondary emission makes the grid-current pulse more peaked than a square law, thereby reducing the d-c grid current below the calculated value while raising the d-c plate current, but without affecting the third-harmonic component of grid and plate currents appreciably. If it were not for these effects of secondary emission the agreement between calculated and exact results would be within three per cent, but even as it is the errors in calculated power output and plate loss are less than seven per cent.

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to the power unit, producing a record of four strains simultaneously. For convenience in operating in the field a 2,000-cycle generator driven by a 12-volt motor is operated from a 12-volt storage battery, figure 3. Where a power supply is available, the 2,000-cycle generator can be driven by a standard motor.

The principal difference between the old vibration recorder and the new strain gauge lies in the oscillograph galvanometer circuit. This circuit includes, in addition to the galvanometer, a copper-oxide rectifier, a 2,000-cycle filter, and a 4,000-cycle filter. The galvanometer circuit must have a low impedance for all frequencies within the range in which mechanical phenomena occur, as for example, from one to 1,000 cycles per second. Most mechanical apparatus on which strain gauges would be used will have natural frequencies well within this range.

In order to obtain relative values for the various frequencies to be recorded, it is necessary to use a galvanometer filter circuit that will give all frequencies approximately the same weighting. In other words, the output circuit impedance should be the same for all frequencies within the range in which the circuit is to be used.

A typical frequency-impedance curve is plotted in figure 4, showing that the impedance remains approximately the same up to 1,000 cycles per second, ranging from 125 to 180 ohms. Mechanical deflections in the range from 1,500 cycles to 2,200 cycles will be recorded at greatly reduced values as these fall within the resonant zone of the 2,000-cycle filter, which is designed with a high impedance to eliminate the 2,000-cycle excitation of the generator. A 4,000-cycle filter circuit is used to eliminate the 4,000-cycle ripple.

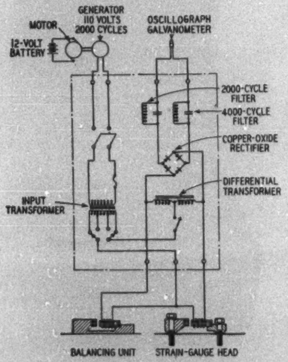

Figure 3. Schematic diagram of electric strain gauge

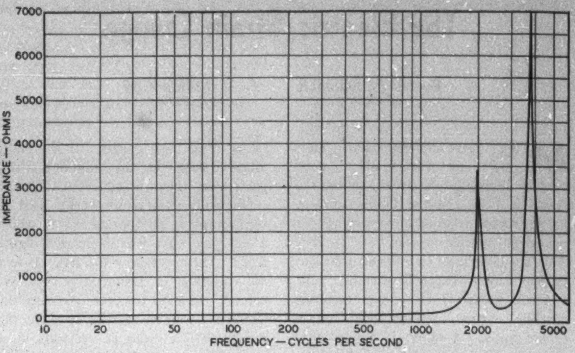

Figure 4. Frequency-impedance curve of oscillograph-galvanometer filter circuit of electric strain gauge

The two high-impedance peaks in figure 4 come at 2,000 cycles and at 3,800 cycles per second, where the circuit is in resonance for the purpose of eliminating the excitation frequencies. The impedance at these two frequencies are approximately 3,400 ohms and 7,000 ohms, respectively.

In order to clarify the results of producing a pure d-c record from an a-c excitation source, let us consider what is going on within the oscillograph-galvanometer filter circuit. The copper-oxide rectifier receives a 2,000-cycle alternating current for its input, which contains equal amplitudes for each of the individual half cycles provided there is no mechanical disturbance being investigated. When mechanical transients occur, the individual half cycles vary in amplitude in proportion to the disturbances which are being measured. In other words, the 2,000-cycle carrier wave is modulated by the mechanical phenomena. The copper-oxide rectifier receives this modulated 2,000-cycle wave and has for an output a modulated pulsating wave. The pulsations from the rectifier are eliminated from the galvanometer circuit by the 2,000-cycle and the 4,000-cycle filters, thus producing a d-c deflection.

A Fourier Series analysis of the rectifier pulsations shows a d-c component, a fundamental of 4,000 cycles, a lower frequency component of 2,000 cycles, and harmonics higher than 4,000 cycles. An oscillographic analysis of the galvanometer circuit shows that the galvanometer itself produces considerable mechanical damping as the natural frequency of the supersensitive galvanometer used is approximately 1,200 cycles per second. The 4,000-cycle filter when used alone leaves a 2,000-cycle ripple on top of the d-c deflection. This 2,000-cycle ripple comes from the rectifier and the magnetic pick-up in the circuit. The 4,000-cycle filter and the 2,000-cycle filter, together with the mechanical damping of the galvanometer, produce a d-c deflection which is practically free from alternating current.

The oscillogram, figure 5 (*b*), shows a record of the mechanical transient which occurred in a test bar when subjected to a sudden blow. Curve *B* shows this transient as recorded by the strain gauge. Curve *C* shows the corresponding 2,000-cycle transient supplied to the rectifier. Curve *A* is a 60-cycle timing wave. Strain-gauge records such as curve *B* can be easily identified even when they overlap each other. It is possible to record six such curves simultaneously when using a six-element general-purpose oscillograph. Figure 5 (*c*) shows five records which can readily be distinguished from each other at all points.

The strain-gauge head *A*, figure 1, must be calibrated very accurately in order to measure the small changes in length of the part under test. For this purpose it is mounted at *A* on a calibrating stand, figure 6, just as in actual use, with the balancing unit *B* in the second leg of the bridge circuit, figure 3, of which the remaining two legs are in the power unit, figure 2. It is customary to make this calibration in ten-thousandths of an inch (0.0001-inch steps) although less accurate calibrations are sometimes sufficient. An electric gauge *E*, figure 6, is used with its power unit *F* and indicating microammeter *G* as a calibrating

Figure 5a. A modulated 500-cycle record

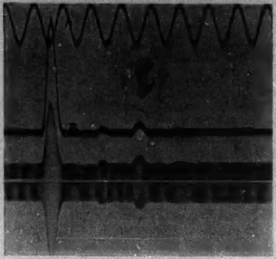

Figure 5b. Strain-gauge record

A—Timing wave, 60 cycles
B—Mechanical transient, impressed on the electric strain-gauge circuit
C—2,000-cycle a-c input to copper-oxide rectifier

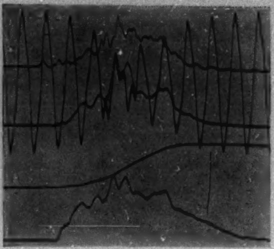

Figure 5c. Simultaneous records using electric strain gauges and vibration recorder

medium. A 2,000-cycle power supply is used for excitation and an oscillograph galvanometer for recording in order to be able to apply the calibration to rapid changes which may occur while the strain gauge is being used. The calibration is made by taking readings of oscillograph deflections corresponding to a number of definitely spaced settings of the strain-gauge head. The indicating instrument *G*, figure 6, which is used with the electric gauge *E*, gives a full-scale deflection for 1/1,000 inch movement at the strain gauge, a magnification of 5,000 to one. Provision is made on the calibrating stand for comparing the electric gauge directly with accurate thickness standards.

After calibrating the strain-gauge head on the stand, it can be moved to a test position without changing its calibration when its air gap is reset to give the same oscillograph deflection.

A typical calibration of a strain-gauge head is shown in table I. This calibration was taken with air gaps of approximately 0.010 inch in the strain-gauge head and balancing unit. It is here seen that a strain of 1/100,000 inch produces an oscillograph deflection which is easily read. Corresponding calibrations with larger air gaps can be taken if less sensitivity is desired in order to cover a wider working range.

The calibrations of table I are made statically, but they can be applied to transient measurements over the frequency range for which the instrument was designed. The variations in air gap of the strain-gauge head produce changes in impedance of that branch of the bridge circuit which are independent of their rate of change and should be recorded correctly if they are within the frequency range of the electric gauge and of the oscillograph galvanometer used. In order to obtain high current sensitivity a supersensitive oscillograph galvanometer is used for recording. This can be used up to at least 500 or 600 cycles per second without correction. A transient such as that shown in curve *B*, figure 5 (*b*), was over in $^{1}/_{30}$ second so that it was well within the working range of the oscillograph galvanometer. Errors due to change of damping of the oscillograph galvanometer with change of temperature are practically eliminated due to the fact that the comparatively low frequency of the modulated wave is recorded instead of the higher carrier frequency.

If only static or slowly varying strains are to be recorded, a carrier frequency lower than 2,000 cycles is satisfactory de-

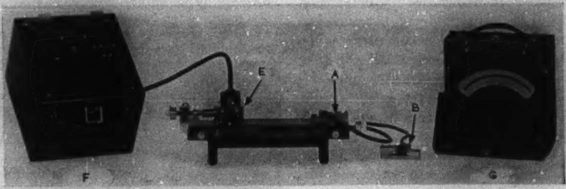

Figure 6 (above). Strain gauge calibrating equipment (power unit at left, calibrating stand and gauge heads in center, indicating instrument at right)

Figure 7 (below)

A—Travel recorder detector
B—Balancing unit

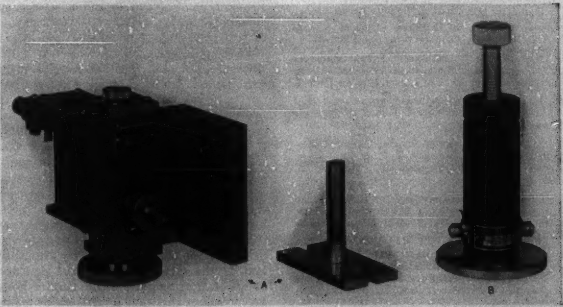

Table I. Calibration of Strain-Gauge Head

Master Gauge Blocks, Nominal Thickness (Inches)	Thickness (Inches)	Micro-amperes	Oscillograph Galvanometer Deflections (Millimeters)
0.1000	0.10000	100	11
0.1001	0.10006	121	15
0.1002	0.10019	157	22.5
0.1003	0.10028	187	28.5
0.1004	0.10036	205	32.
0.1005	0.10048	250	41.
0.1006	0.10060	288	48.5
0.1007	0.10068	312	53.5
0.1008	0.10079	349	61.
0.1009	0.10090	388	68.5
0.1010	0.10099	423	75.5

pending upon the rate of change of the stress. An indicating instrument can be substituted for the oscillograph galvanometer to show the magnitude of static strains only.

In connection with the measurement of strain, it is frequently desirable to determine the relative movement of parts. A travel-recorder detector, designed to measure deflections up to one inch, is shown in figure 7 together with its balancing unit. These are substituted for the strain-gauge head and balancing unit of figure 3, producing a record comparable with that from the strain gauge. It is used with the oscillograph when very rapid changes are to be recorded or with an indicating instrument to show slower movements.

The armature, shown in the middle of figure 7 is mounted on the moving part and the frame of the detector on the stationary part so that one moves with respect to the other. The deflections on the oscillograph film are directly proportional to the mechanical motions and are unidirectional with respect to the electrical zero.

Summary

The electric strain gauge can be used to record accurately strains as low as 0.00001 inch, whether these are static or are oscillatory at frequencies up to at least 500 cycles per second, and the single-line trace allows several measurements to be recorded simultaneously.

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Discussion

C. M. Hathaway (General Electric Company, Schenectady, N. Y.): This paper is intended by its authors to serve as a progress report to the Institute and it should be welcomed because a great deal has been accomplished since the original presentation by A. V. Mershon, one of the co-authors of the present paper, in 1926, in a paper entitled "Vibration Recorder for Electrically Measuring and Recording Small Mechanical Movements."

Although the device originally described by Mr. Mershon has received numerous modifications since that time as its application has been extended in many different directions, the same fundamental circuit still survives. This present paper described the extension of the application of the gauge to measurements at higher frequency, the addition of a filter to provide a more desirable type of record as well as a more accurate one, and the application of the gauge to the measurement of strain.

The applications of the electric gauge since its original conception have been numerous, and they are, I believe, just as interesting to engineers in all branches of our profession, so I will present a little historical sketch of its development from infancy to the present time, and describe some of its interesting and more recent applications.

The fundamentals of the electric gauge were first worked out in order to measure the motion of hydraulic gate valves under water. The principle was applied shortly thereafter to the measurement of transient pressures in oil-circuit-breaker housings and to the measurement of the displacements of rotating parts within turbine housings. About this time a refrigerator manufacturer was in need of an accurate gauge for the production measurement of compressor parts having tolerances of "plus nothing—minus 0.0003 inch." The electric gauge principle was naturally applied to this problem also and with very satisfactory results, $^1/_{10,000}$ inch being expanded to one inch on the scale of the indicating instrument. The refrigerator manufacturer was enabled to inspect his parts more accurately and rapidly than before, and actually to assort his parts for matched assembly within his small tolerance range. Special multiple gauging stands were built, permitting as many as four dimensions to be gauged simultaneously. Points of wear were tipped with tungsten carbide, diamond, or sapphire, and some of the original stands have been in continuous use since 1930 and are still in operation.

The successful operation of these gauges in the plant of the refrigerator manufacturer so impressed a gauge manufacturer that an arrangement was made for supplying the electric gauge to the trade, so that the electric gauge is now well known and is found in many manufacturing plants throughout this country and parts of Europe.

An interesting application of the electric gauge has been made to the tail-stock of the Pratt and Whitney measuring machine enabling this machine to be used for comparison work to a millionth of an inch if proper precaution is taken in regard to constancy of temperature.

Perhaps to the engineer the most interesting and useful application of the electric-gauge circuit is to the measurement of strain. Strain gauges have been built and used with good results in a number of investigations involving the measurement of stress in mechanical and structural members. Studies have been made of stresses in bridge structures during passage of high-speed locomotives, enabling the engineers to study stresses in structures too complex to permit ready calculation, and to investigate the resonant behavior of the bridge as trains crossed it at various speeds. Investigations have been made of stresses in electric locomotive members during service, and of stresses in rails and joint bars under various normal and abnormal conditions. Most of these investigations were carried on with a strain gauge weighing between four and five ounces and having a gauge length of $2^1/_4$ inches. This gauge is sensitive enough to permit measurement of stress as low as 100 pounds per square inch in steel, without the use of amplification.

A recent development in this strain gauge makes use of amplification, not to increase sensitivity but to decrease the size of the gauge head. The result is a gauge head weighing about one-half ounce and having a gauge length of an inch. This small strain gauge is particularly useful for measurement of stresses in aircraft parts and members of small section.

The electric gauge has been applied to cold-strip-steel rolling with surprising results. By the electric gauge the thickness of the finished product is continuously indicated, the pressure exerted on the rolls is measured, and the tension on the strip between stands is both indicated and automatically controlled. These applications of the electric gauge have alone increased the rolling speed from about 250 feet per minute to more than 1,300 feet per minute.

"De-ion" Air Circuit Breakers for A-C Feeder, Motor Starting, and Station Auxiliary Service

By R. C. DICKINSON
ASSOCIATE AIEE

Synopsis: The first circuit interrupters for a-c service were of the air-break type, followed very shortly by oil circuit breakers for general high-voltage service. Air "De-ion" circuit breakers introduced in 1928 provided a new type of circuit interrupter for a-c service. Recent applications of these principles result in a line of De-ion air circuit breakers for 2,500-volt and 5,000-volt ratings suitable for use in a-c feeder circuits, motor starting, and general station auxiliary service.

THE electrical industry used air-type arc-interrupting devices when it first became desirable or necessary to open the circuits between the generators and the loads. Very little was known about arc characteristics except that after an arc was stretched sufficiently, the circuit voltage would not maintain it and the circuit would be interrupted. In about 1895 the increased voltage and power requirements incidental to the extension of a-c systems led to the introduction of the oil circuit breaker on account of its compactness and the confinement of the arc. Both air and oil circuit breakers continued in use for several years, but the advantages of the oil breaker led to the disappearance of the air breaker from the higher voltage a-c fields. The older catalogues indicate the use of air circuit breakers for voltages up to and possibly higher than 66-kv as late as 1902. Figure 1 illustrates this general type of construction as applied to 2,300-volt service.

The oil circuit breaker gradually became the principal factor in high-voltage a-c switching. Nevertheless, the air circuit interrupter has continued for application to d-c service as high as 3,000 volts and for a-c service of 2,300 volts and in some cases at higher voltages. The magnetic blow-out principle was introduced as a modification of the "long arc" principle of circuit interruption and these devices continue in use for moderate power requirements.

The steady growth of the electrical industry resulted in continually increasing concentrations of power and an ever-growing necessity for continuity of service, requiring greater safety and dependability of operation, particularly on the part of the switchgear. Improvements were made in the construction of oil circuit breakers to meet these increasingly severe operating conditions and research was directed into the fundamental principles of a-c circuit interruption.

As a result of this research, the "De-ion" air circuit breaker principle was introduced in 1929[1,2]. This De-ion air-break principle was based on a new concept of arc phenomena resulting from intensive studies of the fundamental structure of matter, which work, of course, had been going on for a good many years.

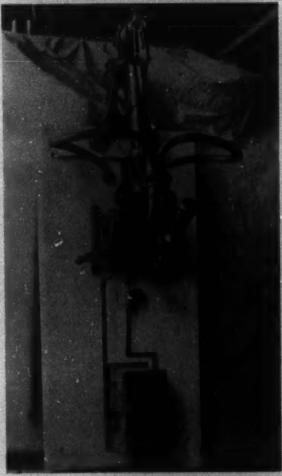

Figure 1. High-voltage 2,500-volt air circuit breaker used as late as 1902 to 1904

Because of the fundamental soundness of the De-ion theory, it then became possible to design circuit breakers for a definite performance. The inherent advantages of De-ion air circuit breakers over other types of air circuit breakers were found to be chiefly compactness, relatively high arc rupturing ability, short duration of arcing, and the control of arc products, and thus opened up new possibilities of switchgear without oil for 25,000 volts and below, and at moderate interrupting capacities.

To review briefly the theory of the De-ion air circuit breaker, its operation depends primarily upon the high initial rate of recovery of dielectric strength of a thin space at an arc cathode, this being in the order of 250 volts within the first microsecond. If a series of N arc spaces is provided, a voltage of 250 N can be interrupted on commercial circuits although a lower voltage per arc space is used in actual practice. Since the interrupting process depends primarily upon the rapid development of a very thin layer of dielectric at the arc cathode, it follows that the individual arcs in the series need not be long and in fact they may be as short as is commercially practicable. The net result is that in a 2,500-volt 100,000-kva De-ion air circuit breaker, capable of interrupting the full 2,500 volts on a single element, the length of the active parts of the interrupter is less than six inches.

In general, the duration of arcing in De-ion air circuit breakers averages from one-half to one cycle at the usual range short-circuit currents. The arc is quickly moved from the arc tips into the interrupting element and the circuit is generally interrupted at the next zero point of current.

The interrupting element of the De-ion air circuit breaker, being composed essentially of a series of copper plates separated by thin insulating spacers, completely encloses the arc and the latter is never exposed to the exterior parts of the breaker. By the proper design and choice of materials it is thus possible to control the expanding gases resulting from the arc. As a result, the De-ion air circuit breaker has been adapted to metal enclosed switchgear which compares favorably in size to corresponding oil switchgear.

The first practical design of a De-ion air circuit breaker was at 2,300 volts. This breaker was not commercialized but several were placed in trial operation on transformer and motor circuits in the East Pittsburgh plant in August 1927.

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[1] For all numbered references, see list at end of paper.

Two of these units are still in active service.

The next design of De-ion air circuit breaker was in the 15-kv class, with an interrupting rating of 20,000 amperes, equivalent to 500,000 kva. Approximately 100 of these breakers were placed in central station service between 1928 and 1932, the earliest of these units still being in operation with the original interrupting elements.

Many other interesting applications of the De-ion principle have been made for 5,000-volt, 7,500-volt, 15,000-volt, and 25,000-volt classes. However, this apparatus has in general been more costly to build than corresponding oil breakers and the great increase in oil breaker efficiency in recent years has removed to a large degree the necessity for duplication of apparatus lines.

Since De-ion grids not only became available for new oil circuit breakers, but also were applicable to circuit breakers in the customer's possession, the advantages of the De-ion air circuit breaker became less apparent. However, with the accumulation of field experience with these early De-ion air breakers, certain fields proved logical ones for the exploitation of the De-ion principle:

(*a*) Elimination of fuses from low voltage a-c and d-c switchgear has been very general in recent years and the De-ion principle as incorporated in the small line of breakers illustrated by figures 2 and 3 is typical of the apparatus now serving that field.

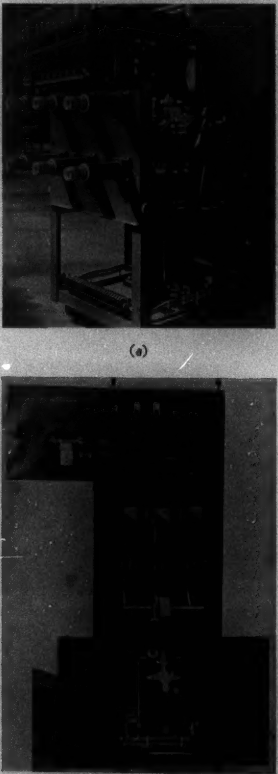
(*b*) The 6,600-volt three-phase steel-mill main-motor-drive switchgear has proved a fertile field for De-ion air breakers, because of the fact that these breakers proved their ability to handle highly repetitive service with little or no maintenance. The forward and reverse breakers for main drive motors of 5,000 horsepower at 6,600 volts, are in some cases required to operate 1,500 to 2,000 times per month on starting and full-load currents.

A line of 7.5-kv breakers was introduced in 1933 for this type of service which has since been expanded into metal enclosed equipment of both horizontal draw-out and cubicle construction. Figure 4*a* shows a 1,200-ampere 7.5-kv unit for horizontal draw-out equipment rated at 250,000 kva. This breaker is insulated for the AIEE test for 15-kv breakers. In addition to having the long life inherent in the De-ion construction, this line of breakers is also designed for extra long mechanical life. Some of these units have service records of over 150,000 operations as reported in 1937, and are still in active duty. Approximately 150 breakers of this type have been placed in service. Another form of 7,500-volt De-ion air circuit breaker for lower rupturing capacity is shown in figure 4*b*. In this assembly the complete breaker is mounted within a metal enclosure obtaining a high degree of compactness for this type of equipment.

In view of these developments the question may logically be asked, why cannot the De-ion principle be applied rather widely to the elimination of oil from power house auxiliary switchgear and similar installations? The power requirements are well within the technical limits of the De-ion development and there are certain definite improvements in the switchgear layout which can be made by the elimination of oil.

As a result of the large accumulation of field experience on De-ion air circuit breakers, there has been developed a new line of breakers for ratings up to 100,000 kva at 2.5 kv and 150,000 kva at 5 kv. These are particularly applicable to station auxiliary, motor-starting, and substation distribution-feeder service. In this field of switching equipment, space requirements and economic considerations are predominant factors.

Figure 5 illustrates a 2,000-ampere 3-pole 2.5-kv breaker unit of 100,000 kva interrupting capacity. The general design and construction of this breaker differ considerably from that of high-voltage De-ion air circuit breakers previously described before the Institute.[8] The line terminal bushings are mounted on a vertical steel base plate which forms a foundation for the entire breaker assembly. The interrupting elements or deionizing chambers are mounted vertically over the upper terminal bushings,


(*a*)

(*b*)

Figure 4. Horizontal draw-out metal-enclosed De-ion air circuit breaker for 1,200 amperes, 7.5 kv, of the type widely applied in 6,600-volt steel mill service

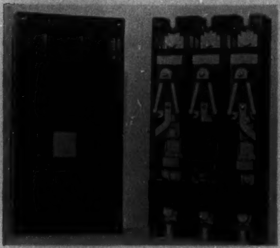


Figure 2. Low-voltage De-ion air circuit breaker, 600 volts alternating current, 250 volts direct current

Figure 3. Deionizing chamber for low-voltage De-ion breaker of figure 2

with a small post-type insulator as additional support.

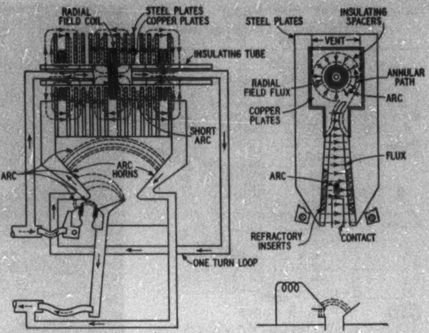


Figure 6. Illustration of the principles of construction of modern De-ion air circuit breakers showing how the arc is controlled by magnetic fields

Figure 5. Three-pole De-ion air circuit breaker unit for 2,000 amperes, 2.5-kv, for 100,000-kva interrupting capacity

The principles of construction and operation of the deionizing chamber in this new design of breaker are best understood by reference to figure 6. As in breakers previously described before the Institute, the basic principle of initiating the arc at a single contact and then breaking it into a series of short arcs between copper plates is retained. Also as in previous breakers, two magnetic fields are required, one for moving the arc from the arc tips into the series of plates and another, which in reality is a combination of magnetic fields, for rotating the short arcs over circular paths between the plates. These two magnetic fields are generally referred to as the entering or blow-in field and the radial fields. Referring again to figure 6, each copper plate is surrounded on top and sides by a steel plate of the same thickness. These pairs of plates are stacked with thin insulating spacers between, each set of spacers being so shaped as to form pathways for the individual short arcs and arc terminals and also a vent to the top of the chamber for each short arc. At intervals of a few inches in this stack of plates are located coils for supplying the radial magnetic fields. This unit assembly is clamped tightly together by stiff end plates and insulated studs.

In operation, the steel plates, which extend downward below the arc tips, act as a composite magnetic yoke which supplies magnetic flux for moving the arc through the arc box, the space between the arc tips and the lower edges of the copper plates. This yoke is energized magnetically as follows: As the arc is initially formed at the arc tips, the flux surrounding the arc current circulates through the steel plates above the arc and across the air gap between the lower ends of the steel plates below the arc. This results in a force reaction to move the arc upward toward the copper plates. The shape of the electrical circuit through the line terminal studs and contact members also assists in the initial upward movement of the arc. The impingement of the arc on the tip of the arc horn energizes the loop circuit, formed by the stud running centrally through the copper plates and returning through the two lower studs in parallel, to the arc horn immediately above the stationary arc tip. The arc horn is insulated from the arc tip sufficiently to withstand the voltage drop caused by the impedance through the loop. This is illustrated in the small diagram in figure 6. As the arc strikes the arc horn, the voltage across that portion of the arc between the contact and horn causes current immediately to flow through the loop. Due to the low impedance of the loop compared to the voltage developed by that portion of the arc in parallel with it, the entire current is quickly transferred to the loop. Since the current in the upper stud flows in the same direction as the arc current this loop then acts as an additional turn for further magnetizing the steel plates. With this strengthening of the entering field, the arc is moved upward into the narrowing throat formed by the tapered slots in the copper plates. As explained previously before the Institute, the purpose of the tapered slot in each copper plate is to increase the voltage of the single arc to effect the transition from a single arc to a multiplicity of short arcs in series.

The radial field coils are electrically connected to adjacent copper plates having extensions below the other copper plates. These coils are introduced into series relationship to the arc current shortly before the arc reaches the ends of the slots in the main copper plates, in a manner similar to that in which the entering field loop circuit was introduced. These radial fields then provide additional impetus for the final transfer of the arc, at the ends of the tapered slots, into the stack of copper plates to form a series of short arcs. Under the influence of the same radial fields, the short arcs then rotate at high velocity in the circular pathways formed by the insulating spacers until the next current zero, when the circuit is interrupted.

In the higher voltage De-ion air circuit breakers, the static shields are provided, along the interrupting element or De-ion stack, in order to divide the voltage evenly among the short arcs. Because of the smaller number of gaps and the greater compactness of the interrupting element of the 2,500- and 5,000-volt circuit breaker, satisfactory distribution of voltage is obtained without the use of external shielding.

The contact system of this breaker is interesting in that springs and full-capacity contact shunts are eliminated without sacrificing the basic advantages of the line-type butt contact. Figure 7 illustrates operation of the contacts. The stationary contacts are fixed on the ends of the two terminal studs, the upper one being a replaceable combination main and arcing contact. The movable contacts are located at the ends of a solid copper bar which forms a bridging member across the stationary contacts, the lower pair of contacts being bridged by a light, flexible shunt. The movable contact member is pivoted at the center through an elongated hole to the actuating lever, with a compression spring between the two. This spring biases the movable contact member against the stationary contacts, in the closed posi-

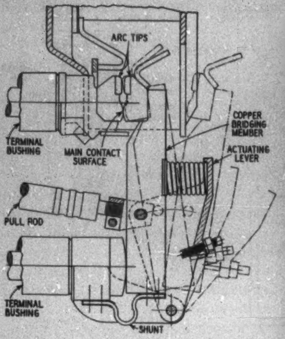


Figure 7. Contact system of the 2.5–5 kv De-ion breaker for station auxiliary and similar applications

tion, and, in opening, gives it a rolling action to cause final separation at the arc tips. These tips are of silver-tungsten, having high resistance against the burning action of the arc.

Solid silver plates in the main contact surfaces maintain low contact resistance over long service periods.

The individual pole units are isolated from each other by insulating barriers of rectangular Micarta tubing. The three-pole breaker is operated by a conventional solenoid mechanism located immediately below the breaker pole units, as shown in the illustration of the test sample, figure 8. Because of its novel arrangement, this breaker is extremely simple and compact and all parts are readily accessible for inspection or maintenance.

An experimental sample 2.5-kv breaker, shown in figure 8, consisting of a single-pole unit mounted on a three-pole frame and operated by a standard three-inch solenoid, was tested beyond its interrupting rating in the high-power laboratory at East Pittsburgh. Tests were made at from less than 50 amperes to well in excess of 25,000 amperes, the majority of tests being made with from 2,500 volts to 3,200 volts on the test generator with restored voltages from 2,000 to 2,600 volts.

Table I. Typical Data From Interrupting Tests on 2.5-Kv De-ion Air Breaker

Test Number	Interrupted Current, Root-Mean-Square Amperes	Restored Voltage, Root-Mean-Square Volts	Time From Energizing of Trip Coil to Arc Extinction (Cycles)
1	220	2,420	7.0
2	323	2,610	6.0
3	1,120	2,060	5.7
4	2,850	2,310	5.7
5	4,160	2,270	5.7
6	7,390	2,200	5.6
7	10,030	2,050	5.3
8	12,000	2,070	5.0
9	14,900	2,290	5.3
10	16,900	2,250	5.2
11	21,100	2,270	5.2
12	20,000	2,200	5.6
13	20,400	2,080	5.5
14	22,200	2,200	5.22
15	25,000	2,510	5.22

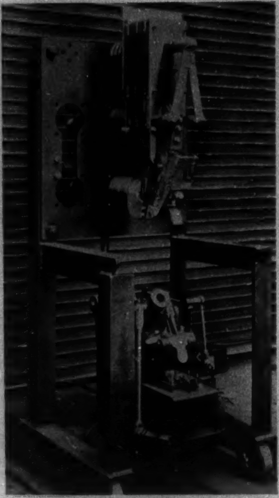


Figure 8. Single-pole experimental model of 2,500-volt De-ion air circuit breaker for 100,000-kva interrupting capacity

The first continuous series of tests consisted of 144 interruptions, stopping only at intervals to examine the breaker. As many of these tests were repeat interruptions at about the same current levels, oscillographic data from a few representative tests are shown in table I. It will be noted that the time from energizing of the tripping coil to the interruption of the circuit is consistently less than six cycles for currents down to approximately 1,000 amperes, showing a safe operating margin for the standard eight-cycle rating.

All of the above tests were made on the sample breaker as originally built and no part of the breaker was modified or replaced during the tests. These tests represent the equivalent of five years or more interrupting service on the average circuit breaker. Figure 9 shows a deionizing plate taken from the experimental breaker after 45 interruptions, 5 of which were above 20,000 amperes, 17 above 10,000 amperes, 23 above 5,000 amperes, and 44 above 1,000 amperes. This plate shows very little deterioration and was used for subsequent tests. Figure 10*a* shows a typical oscillogram made during an interruption of 25,500 amperes at 2.5 kv. Table II shows typical oscillographic data taken from another series of 19 unit-operation interrupting tests.

The experimental breaker was equipped with a deionizing chamber containing a greater number of plates and insulating spacers, designed for an interrupting rating of 20,000 amperes at 5 kv, equivalent to 150,000 kva. Similar interrupting tests were made on this unit. Table III shows typical data taken on this series of tests, with breaker interrupting times of less than six cycles. Figure 10*b* shows a typical oscillogram.

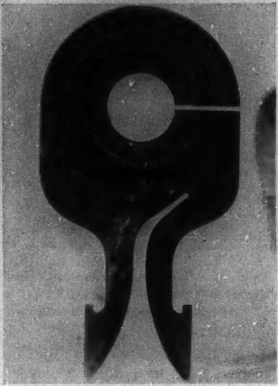


Figure 9. Deionizing plate taken from experimental breaker unit after one series of 45 short-circuit interrupting tests

In addition to the short-circuit interrupting tests, the suitability of this type of breaker for motor starting, station auxiliary, and other types of highly repetitive service was investigated by three series of endurance runs. The first two runs simulated closing the starting current and opening the full-load current of a 1,000-horse power motor. The test condition was made more severe than service condition by using air-core reactors and thus obtaining a power factor of less than ten per cent. The breaker was closed and opened 7,700 times at 235 amperes, 2,300 volts on the single-pole unit. It was then operated 7,700 times on the CO cycle at 1,350 amperes, 300 volts. This total of 15,400 operations was made without any maintenance whatever. The deionizing chamber used in these tests had previously been used in 48 interrupting tests, 20 of which were at from 9,000 to 26,000 amperes. The breaker was in good electrical and mechanical operating condition after these tests.

The third series of tests, part of which was included in the first two series, consisted of a mechanical endurance run of 35,000 operations. At the end of these tests, the breaker was still in good overall condition indicating an ample margin for many years of service.

The breaker unit, shown in figure 5, due to its compactness and small space requirements, is suitable for complete metal-clad or cubicle switchgear designs.

Figure 10a. Oscillogram showing typical interruption at 25,500 amperes—2,510 volts restored on single-pole unit

Figure 10b. Oscillogram showing typical interruption at 18,000 amperes, 4,600 volts restored on single-pole unit

Figure 11 shows the preferred horizontal draw-out metal-clad switchgear unit with the De-ion air circuit breaker. Uniformity of design is attained by making this unit adaptable for air breakers of various ratings at 2.5 or 5 kv. The insulation is of the 7.5-kv class. By comparison with the equivalent vertical-lift metal-clad unit equipped with a modern oil circuit breaker of 150,000 kva interrupting capacity shown in figure 12, it is to be noted that the space occupied by the De-ion air circuit breaker unit is approximately equal to that for the oil circuit breaker.

Table II. Typical Data From CO Interrupting Tests on 2.5-Kv De-ion Air Breaker

Test Number	Closed Root-Mean-Square Current (Amperes)	Interrupted Root-Mean-Square Amperes	Restored Root-Mean-Square Kilovolts	Time From Energizing of Trip Coil to Arc Extinction (Cycles)
1	1,080	1,050	2.48	5.4
2	3,500	2,430	2.39	5.5
3	4,700	4,180	2.28	5.3
4	11,300	9,070	2.16	5.7
5	17,400	14,400	2.05	5.7
6	17,500	14,400	2.06	5.8
7	18,000	14,700	2.08	5.8
8	40,000	18,800	1.95	5.6

In the air breaker unit the high-voltage parts are completely isolated from the low-voltage control circuits by steel barriers. The individual pole units are isolated from each other by rectangular insulating tubes as previously described. Other features of this construction, such as main disconnecting contacts, secondary contacts, insulated bus, etc., follow the most approved construction as applied to modern vertical-lift type metal-clad switchgear with oil breakers. In this unit the De-ion air breaker is horizontally movable to the "disconnect" or test position on rollers near the center of thrust of the disconnecting contacts. From the disconnect position it may be moved out of the cell by means of a small transfer truck, thus permitting convenient removal of the arc interrupting stack for examination and maintenance of the contacts.

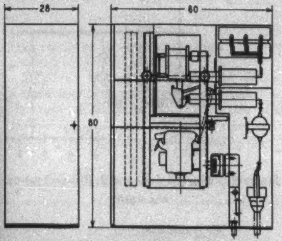


Figure 11. Horizontal draw-out metal-enclosed switchgear unit with De-ion air circuit breaker for 2.5 to 5 kv, 600 to 1,200 amperes, up to 150,000 kva interrupting rating

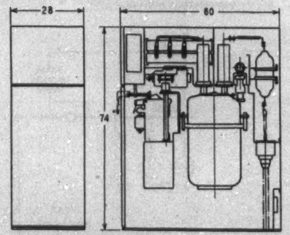


Figure 12. Modern metal-clad switchgear unit equipped with 150,000-kva oil circuit breaker

Table III. Typical Data From Interrupting Tests on Five-Kv De-ion Air Breaker

Test Number	Interrupted Current, Root-Mean-Square Amperes	Restored Voltage, Root-Mean-Square Volts	Time From Energizing of Trip Coil to Arc Extinction (Cycles)
1	1,840	4,610	5.7
2	2,910	4,560	5.8
3	3,660	4,380	5.7
4	6,490	4,300	5.7
5	8,000	4,150	5.60
6	12,900	3,890	5.6
7	13,700	3,860	5.6
8	14,900	3,610	5.7
9	16,000	4,000	5.7
10	18,000	3,600	5.8

Potential transformers are located in superstructures for both types of assemblies and in either case control switches instruments, and relays may be accommodated on panels mounted either on the front or the rear of the unit, depending upon whether the front aisle or both front and rear aisles are available.

Previous designs of De-ion air circuit breakers have found wide spread application in steel-mill, arc-furnace, and other highly repetitive services as well as in power-house feeder applications. It is expected that the completion of these new De-ion circuit breakers for lower voltage applications will extend the advantages of this type of construction to power-station auxiliary service, general motor-starting service, and substation-feeder applications.

References

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2. THEORY OF THE DE-ION CIRCUIT BREAKER, J. Slepian. AIEE TRANSACTIONS, volume 48, April 1929.

3. STRUCTURAL DEVELOPMENT OF THE DE-ION CIRCUIT BREAKER, R. C. Dickinson and B. P. Baker. AIEE TRANSACTIONS, volume 48, April 1929.

Discussion

J. A. Adams (Philadelphia Electric Company, Philadelphia, Pa.). It is particularly gratifying to see continued development of the De-ion air circuit breaker in capacities which can be used extensively in distribution substations. The Philadelphia Electric Company installed two De-ion breakers experimentally in 13.2-kv service in 1931, and in 1934 installed eight more on 4-kv circuits. These have operated successfully under both normal and short-circuit conditions. As a result of this experience it is planned to install three additional breakers in 13.2-kv service, two in 4-kv service, and three in 4.8-kv service this year.

While there may have been a great increase in oil circuit breaker efficiency during the last few years, as pointed out in this paper, it would seem that the nonoil-type breaker has a field of its own regardless of this development. This is implied in the paper by the statement that "there are certain definite improvements in the switchgear layout which can be made by the elimination of oil." A type of installation where it would be particularly advantageous to use nonoil-type breakers appears to be the unattended substations. The authors ask, "Why cannot the De-ion principle be applied rather widely to the elimination of oil from power house auxiliary switchgear and similar installations?" It is not clear why the same line of reasoning should not be extended to the main indoor switchgear. It would seem that a logical development for the nonoil-type breaker is in the higher current carrying and higher interrupting capacity ratings.

Philip Sporn (American Gas and Electric Service Corporation, New York, N. Y.): Mr. Dickinson's paper, describing the 2,500-volt and 5,000-volt "De-ion" circuit breaker development is another indication of the increasing work on the design and the increasing interest in the use of air-type breakers for certain applications, particularly for station auxiliary service. From the description of the design and tests as given in this paper, it appears that an air circuit breaker with very good characteristics and performance has been produced.

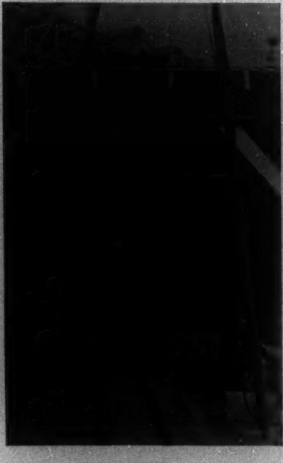

Figure 1. ITE 2,300-volt air circuit breaker set up for tests at Logan

I believe that it would be of interest to present here a brief description of a development along the same lines which was made in connection with the recently completed extension to our Logan, W. Va., plant. When this high-pressure extension to the plant was planned, the engineering and design having been started in 1935, it was felt that because of crowded conditions it would be a decided advantage in the auxiliary service for all of the new portion of the plant to use circuit breakers without oil. Such development had been under discussion for several years with the engineers of the ITE Company. Accordingly, the development of a suitable 2,300-volt air circuit breaker, having a rupturing capacity of 125,000 kva, was launched. The development was completed with the carrying through of a series of tests made at the Logan plant in April of this year.

As compared with the De-ion type of air circuit breaker described by the author, this breaker, developed by the ITE Company, is fundamentally of the magnetic blow-out type, using a special type of arc chute designed for rapid cooling of the arc.

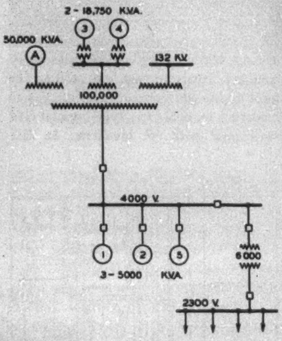

Figure 2. Normal 4-kv and 2,300-volt set-up at Logan

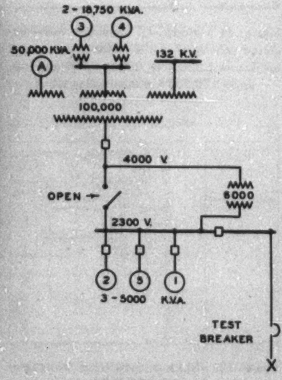

Figure 3. Special connections for 2,300-volt tests at Logan

The tests on one of these breakers at the Logan plant demonstrated very satisfactory performance. I shall attempt to describe these tests briefly with the aid of a few illustrations.

Figure 1 shows one of the 2,300-volt 1,200-ampere air circuit breakers set up in the yard at the Logan plant for the short-circuit tests. With the capacity that could be made available at the plant without interfering with the plant operation, it was not possible to obtain the full breaker rating of 125,000 kva, but by means of a special connection in which a 6,000-kva auxiliary bank was reversed from its normal direction of transformation, we were able to obtain more than 100,000 kva. The normal method of connecting this bank is shown in figure 2 from which you will note that the three 5,000-kva generators are operated at 4,000 volts in parallel with one of the windings of the large 100,000-kva bank, and with the 6,000-kva bank stepping down from 4,000 volts to 2,300. As this set-up gave a calculated short circuit of around 60,000 kva or only half of the rating of the breaker, it was decided to change the connection to that shown in figure 3 in which the three 5,000-kva generators were connected in parallel with the 2,300-volt side of bank 15 instead of the 4,000-volt bus. Although the generators operated at reduced excitation, this set-up provided a much greater short-circuit current than could be obtained in any other manner at 2,300 volts.

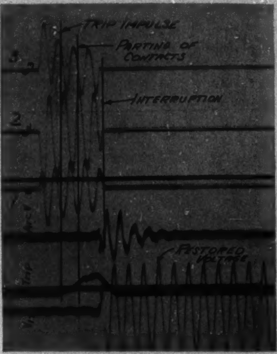

Figure 4. Oscillogram of test number 14 interrupting 22,800 amperes at 2,300 volts in three cycles

Altogether 25 shots were made on this breaker, with short-circuit current varying from approximately 3,000 amperes to a maximum of 22,800 amperes, and with voltages from 2,300 to above 4,000 volts. While time is not available to describe these tests in detail, a fairly complete record of the tests is shown in table I. You will note that, with the set-up in which the three 5,000-kva generators were connected in parallel with the 6,000-kva bank, a maximum recorded total root-mean-square current of 22,800 amperes was obtained on test 14. In order to overcome as far as

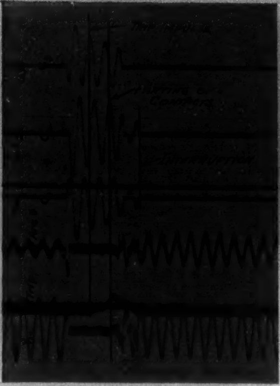

Figure 5. Oscillogram of test number 23 interrupting 15,200 amperes at 4,500 volts in 3.75 cycles

Figure 6. Contacts of ITE 2,300-volt air circuit breaker after short-circuit tests

Figure 7. ITE 2,300-volt air circuit breaker with one arc chute removed

possible the effect of reduced excitation on the value of short-circuit current, the three generators operating in parallel with the system through the 6,000 bank were loaded with reactive kilovolt-amperes up to more than half of their respective ratings by increasing the field excitation.

You will also note from table I that the circuit breaker time, that is, the time in cycles measured from the trip impulse to the actual interruption of the arc, averaged approximately four cycles or less, throughout, with the exception of a single test of 6³/₄ cycles in the third group. The extra long time in this case was undoubtedly due to an *overloaded trip mechanism.* On test number 10 immediately preceding this shot, the breaker failed to trip, entirely from this cause; while on the next shot, which was taken without making any adjustments, tripping occurred but was considerably delayed, as shown by the longer time on the oscillogram. At this point in the program the tripping mechanism was adjusted to relieve the excessive strain and, thereafter, normal tripping time was obtained throughout the remainder of the tests.

Having shown that the breaker gave satisfactory performance with the highest current that could be obtained at 2,300 volts, it was decided to test the breaker, using the three generators alone excited to 4,000 volts or above. The last two groups of tests from number 17 to number 23, inclusive, in table I represent these tests made at the higher voltage. The last two tests made at 4,300 volts gave a maximum total root-mean-square current of 16,000 amperes. It will be noted that the breaker time for all of the tests at the higher voltage was less than four cycles.

In all the tests at heavier current values, a flash was observed above the breaker at the time of interruption. That this was mainly light and not actual fire was fairly well demonstrated by a ball of cotton which was suspended above one of the arc chutes and which was not ignited until the final shot made at 4,300 volts.

Figures 4 and 5 show typical oscillograms taken during these tests. Figure 4 represents test number 14 on which a current of 22,800 amperes at 2,300 volts was interrupted in three cycles. Figure 5 shows test number 23 on which a current of 15,200 amperes was interrupted in 3.75 cycles.

Twice during the tests the arc chutes were removed and contacts examined. Each time, a moderate amount of dressing of the main contacts was done before proceeding with additional tests, but the arcing con-

Table I. Short Circuit Tests at Logan on ITE 2,300-Volt Air Circuit Breaker

Test Number	Type	Generators	Bank No. 15 6,000 Kva	Volts	Initial Current in Arc			Breaker Time (Cycles)			Equivalent Kilovolt-amperes Interrupted
					1	2	3	1	2	3	
1	O	1	Open	2,300	2,850	3,080	3,130	4.0	4.0	4.25	12,500
2	CO	1	Open	2,300	3,340	3,910	3,180	3.25	3.25	3.25	15,500
3	O	2	Open	2,300	4,300	4,910	5,080	4.0	4.0	4.25	20,000
4	CO	2	Open	2,300	6,450	1,750	5,150	2.75	2.75	2.75	28,500
5	O	None	Closed	2,390	12,400	13,300	13,500	3.75	4.0	4.0	56,000
6	O	None	Closed	No Record							
7	CO	None	Closed	2,390	9,650	12,300	11,800	2.75	2.25	2.25	51,000
8	CO	None	Closed	2,390	12,300	12,500	12,900	2.75	2.75	2.75	53,000
9	O	2	Closed	2,430	15,600	17,000	18,000	4.0	4.0	4.0	75,000
9*A*	O	2	Closed	2,470	18,200	15,200	12,900	4.0	3.75	3.75	75,000
10	O	2	Closed	No Record							
10*A*	O	2	Closed	2,430	13,500	16,500	16,800	6.25	6.75	6.25	70,000
11	CO	2	Closed	No Record							
12	CO	2	Closed	2,470	13,900	14,500	16,800	2.25	2.25	2.0	70,000
13	O	3	Closed	2,570	20,900	19,200	17,400	3.0	3.0	3.0	93,000
14	O	3	Closed	2,560	17,700	22,800	16,300	3.0	3.0	2.75	101,000
15	CO	3	Closed	No Record							
16	CO	3	Closed	2,640	16,100	22,300	17,400	2.25	2.25	2.25	99,000
17	CO	1	Open	4,000	5,900	5,920	5,460	2.75	2.75	2.75	41,000
18	O	2	Open	4,000	9,650	7,600	6,500	3.5	3.5	3.5	67,000
19	CO	2	Open	4,000	5,360	9,400	8,000	2.25	2.25	2.25	65,000
20	O	3	Open	4,000	12,850	12,200	7,850	3.25	3.25	3.25	89,000
21	CO	3	Open	4,000	10,200	13,400	10,100	2.0	2.0	2.25	93,000
22	O	3	Open	4,300	16,100	13,900	9,500	3.75	3.75	3.75	111,000
23	CO	3	Open	4,300	14,100	15,200	13,500	3.25	3.75	2.25	105,000

tacts were not touched, and the amount of burning on these contacts was considered very small. The appearance of these contacts after the heavy current shots and prior to dressing is shown in figure 6.

Figures 7 and 8 are included to give a better idea of the appearance of this breaker. Figure 7 shows the breaker with one arc chute removed, while figure 8 shows a group of these breakers as they would appear assembled for service.

Figure 8. Assembled group of ITE 2,300-volt air circuit breakers

J. R. North (The Commonwealth and Southern Corporation, Jackson, Mich.): Keen interest is being shown throughout the industry in low-voltage indoor oilless circuit breakers, particularly for station auxiliary service at 2,400 volts in large generating stations. The "De-ion" air circuit breakers described by Mr. Dickinson would seem to have considerable possibility for such applications and should be very easy to inspect and maintain.

However, for properly evaluating these air circuit breakers, we should like to inquire regarding some of the design features.

(*a*) What are the ventilation requirements from the standpoint of temperature rise and also from the standpoint of flame emission.

(*b*) What is the duty cycle upon which the interrupting capacity ratings are based and what, if any, de-rating factors need to be applied for various reclosing duty cycles.

(*c*) What is the insulation strength—60-cycle flashover, impulse flashover minimum clearance to ground, and striking distance.

(*d*) What is the five-second current-carrying rating of these breakers (root-mean-square amperes), as compared with their interrupting current rating.

M. H. Hobbs (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The series of tests given in Mr. Dickinson's paper, particularly those showing repetitive duty, show that we now have a circuit breaker which is apparently the answer to the requirements for severe service on motor and feeder circuits for station auxiliaries. A number of these have already been manufactured for this application, extending to other types of duty the operating advantages of "De-ion" breakers, which have proved so outstanding on heavy steel mill locations during the past several years.

Mr. Dickinson mentioned that the De-ion breaker is readily adaptable for mounting in metal-clad switchgear assemblies. This fact is more important now than ever before as the large majority of indoor switchgear is now this type, not only for station auxiliaries but for other general applications. It is equally important that all of the advantages of the designs of metal-clad switchgear for oil circuit breakers acquired during the years of its development should be retained in the structure for the De-ion breaker. Figures 11 and 12 show the two comparative structures, for oil and De-ion breakers respectively, and indicate comparative dimensions. It will be noted that the De-ion breaker is arranged as a horizontal draw-out unit as contrasted with the vertical draw-out arrangement for the oil breaker. This works out more satisfactorily for the circuit breaker design on account of the horizontal position of the arcing chamber and presents no handicaps as far as the switchgear is concerned. It would be just as illogical to use the vertical arrangement for this breaker as it was to use the horizontol for oil breakers having vertical stubs.

As noted from the two figures, the width and depth dimensions are identical, and the height of the De-ion unit is six inches more than the corresponding oil breaker unit. It would be possible to reduce the height except for the fact that space must be provided in the rear for potheads or similar cable terminating facilities. Accommodations for instrument panels are equivalent in that they can be mounted either in the front or rear, depending upon space facilities, that is whether only a single aisle or aisles front and rear are available.

Of course, no provision is necessary for circuit breaker gas exhaust as in the case of the oil breaker unit, although a small amount of ventilation is incorporated to assist in the small amount of gas dissipation which does occur. Porcelain is used for supporting the primary contacts, and completely insulated connections and busses are used in the housing. It is desirable to provide complete isolation of the secondary wiring from the primary compartment, and this is also provided for. Provision is made for automatic secondary contacts and for operation of the circuit breaker in the test position by means of a jumper. The usual interlocks insure that contact will not be broken on the disconnecting devices unless the circuit breaker is in the open position.

The availability of this breaker therefore enables us to have a complete switchgear unit, including bus, disconnecting devices, and instrument transformers, without the use of oil or compound in any of the compartments.

R. C. Dickinson: It is very pleasing to hear of the favor with which the 15-kv and 5-kv "De-ion" air breakers are regarded by the Philadelphia Electric Company as reported by Mr. Adams. This is typical of the high type of performance given by these breakers.

As Mr. Adams says, the non-oil type breaker has a field of its own regardless of the present state of oil breaker development. As pointed out in this paper, the widest field for the De-ion breaker, in general, has been where more frequent operation, either on overload or switching, is involved. The absence of oil deterioration and the long life of the contacts and interrupting element make this type of application most logical. However, since the De-ion air breakers are generally equipped with standard operating solenoids and interrupt the complete range of currents within their rating, they are suitable for any application where oil breakers might be applied.

In answer to a question raised by Mr. North, the De-ion air breaker is rated on the basis of the standard AIEE interrupting duty cycle and equivalent oil breaker de-rating factors are used for various reclosing duty-cycles. The De-ion breaker may be expected to require considerably less maintenance for the same duty than the ordinary oil circuit breaker.

In regard to continuous ratings, these breakers are at present supplied up to 2,000 amperes, and higher ratings will be built as occasion demands. As to higher interrupting ratings, the present ratings do not parallel oil breaker ratings in the higher kilovolt-ampere brackets. It is to be hoped that development efforts will eventually result in ratings equivalent to the higher capacity oil circuit breakers.

As to ventilation requirements, as Mr. Hobbs states in his discussion, a small amount is desirable for the dissipation of such gas as may be incident to circuit interruption but it is also desirable to assist in cooling the current carrying parts and reduce the possibility of moisture condensation on solid insulation surfaces. A small opening at the bottom of the cell door and one or two openings at the top of the cell are sufficient. The upper openings may be made drip-proof. There is no appreciable pressure built up within the cell and the gases are dissipated quite readily. Since the arc is ruptured in a completely enclosed chute, it will be obvious that no large amount of gas is set free in the cell as would be the case in an open magnetic chute construction. Years of service have indicated that the gases expelled from the breaker have caused no interference with operation nor undue maintenance. In fact, the maintenance on these breakers has been reported as unusually low.

We appreciate keenly Mr. Sporn's remarks and his interest in air circuit breakers. The new De-ion air circuit breaker has performed well on tests and over ten years of field experience on larger breakers of higher kilovolt-ampere rating, on which numerous field tests were made, indicates that these breakers will perform well in service. It is to be noted that single-phase interrupting tests were referred to in the paper. We value these tests quite highly since in most instances the actual restored voltage on the single-pole unit was as much or greater than the line-to-line

Enclosed Low-Voltage "De-ion" Air Circuit Breaker of High Interrupting Capacity

By JEROME SANDIN
ASSOCIATE AIEE

Synopsis: The demand for protection against fault currents up to 20,000 amperes at 250 volts d-c or 600 volts a-c has led to the development of air circuit breakers having the desirable features of the present line of enclosed air circuit breakers, but having higher interrupting capacity. Tests at 20,000 amperes on the circuit breaker of lower interrupting capacity showed that certain modifications were necessary. By increasing the gap between the contacts, by using contacts which resist welding, and by using a modified arc chamber and contact construction, it was found possible to make the breaker satisfactory for 20,000-ampere short circuits.

Introduction

AN INTERESTING type of enclosed "De-ion" circuit breaker was developed some years ago to meet the demands of industry for a protective device with desirable characteristics such as:

1. Small size to permit the construction of safe, compact, and efficient panel boards, switchboard, and separate enclosures.
2. Sufficient capacity to be operated repeatedly without delay in a satisfactory manner under any condition to which it may be normally subjected.
3. Time delay to permit temporary overloads and instantaneous trip to remove short circuits instantly.
4. Sufficient interrupting capacity.
5. Safe to operate and free from fire hazard.
6. Nontamperable.
7. Easy to install and operate.
8. Low in cost.

This type of circuit breaker is made in frame sizes ranging from 50 to 600 amperes at voltages up to 250 volts d-c and 600 volts a-c and has a rupturing capacity of 10,000 amperes. This construction includes a molded insulated housing, De-ion arc chambers, an operating mechanism, a removable automatic trip unit, an insulated operating handle, and end terminals.

At the time these breakers were developed, 10,000-ampere interrupting capacity was considered ample to handle almost all of the industrial lighting and power circuits of 250 volts d-c or 600 volts a-c. Since then, however, industry has been using increasingly larger blocks of electrical energy, thus making it more and more necessary to have similar compact circuit breakers to handle short circuits of greater capacity. In order to meet this demand, the largest of these available breakers, namely the 600-ampere frame size was chosen for test and modification. A short-circuit interrupting capacity of 20,000 amperes at both 250 volts d-c and 600 volts a-c was set as the goal for this development.

The circuit breaker of higher interrupting capacity is intended for installation in switchboards or separate steel boxes, and for the protection of lighting and power circuits of large buildings or industrial plants where fault currents up to 20,000 amperes must be opened.

The original small size of the breaker was retained during the development. This was important since the size of the switchboards or the building space that must be set aside for them need not be increased. Further, most of the details of the breakers of the 10,000- and 20,000-ampere interrupting capacity are interchangeable which from a manufacturing and cost standpoint is very desirable.

Limitations of the Original Circuit Breaker

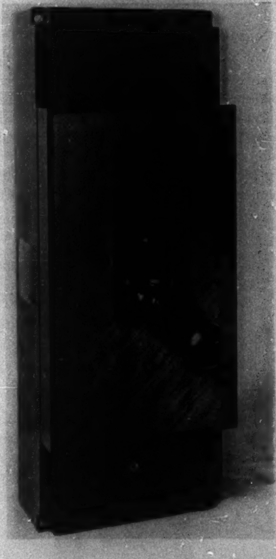

Figure 1. Two- or three-pole, 600-ampere circuit breaker of either 10,000- or 20,000-ampere interrupting capacity

The 600-ampere circuit breaker of 10,000-ampere capacity is provided with silver main and secondary contacts, silver-tungsten (30 and 70 per cent proportion) arc contacts and arc chambers of the radial field type which is described in detail under the subtitle "Arc Chambers." Externally, both the original breaker and the finally developed breaker of high interrupting capacity, are alike. Figure 1 illustrates either of them. Figure 2 is typical of this type of breaker construction.

Tests on the original circuit breaker at

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JEROME SANDIN is associated with the Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa. Acknowledgment is made to Mr. L. R. Ludwig, Mr. G. G. Grissinger, and Mr. B. P Baker for their valuable contributions to this development. Much credit should go to Mr. B. P. Baker for carrying out the initial part of the development, and to Mr. L. R. Ludwig for his very material assistance in the preparation of the paper.

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voltage rating of the three-pole breaker. On many tests the generator was excited to more than 3,000 volts open circuit, single phase, on the 2,500-volt breaker and to 6,000 volts on the 5,000-volt breaker.

Replying to Mr. North's questions regarding insulation and current-carrying capacity without getting into the commercial ratings of these breakers, we can merely state that this class of breakers for 2,500- and 5,000-volt service are insulated to meet the AIEE requirements for 7,500-volt service and their short time ratings are compatible with their interrupting ratings at rated voltage.

In closing, I wish to thank all those who have entered into this discussion for their interest and favorable remarks.

20,000 amperes showed three limitations namely:

1. The arc frequently re-established between the moving and stationary contacts after it had traveled into the arc chamber away from the contacts, indicating that the distance between the contacts was too small.
2. The silver main contacts of both stationary and moving members welded together occasionally when short circuited with the breaker initially closed, but they welded almost always when the breaker was closed on the short circuits.
3. The arc chambers were of insufficient size to effectively vent and deionize the increased volume of ionized gas.

Theories, Modifications, and Improvements

CONTACT SEPARATION

A greater gap between the contacts in the open position is required on higher short-circuit currents because of the presence of a greater amount of ionized gas between the contacts when 20,000 amperes is being interrupted.

Increased distance between the arc contacts allows the use of a larger number of plates in the arc chamber, increases the resistance of the arc, increases the discharge of ions to the plates if a larger number are used, and increases the neutralization of the ions by recombination, all of which facilitates the deionization of the gas.

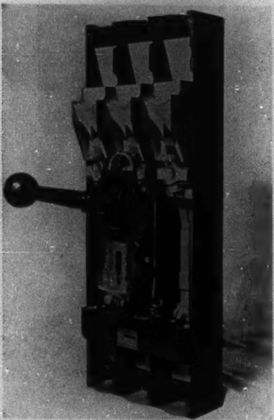


Figure 2. Three-pole, 600-ampere circuit breaker with cover removed, of 20,000-ampere interrupting capacity. Final construction

The separation between the arc contacts was increased from $^{13}/_{16}$ inch to 1 $^{1}/_{8}$ inches, or about 40 per cent. Tests up to about 36,000 amperes proved that this spacing was adequate.

CONTACTS

Silver contacts are very acceptable for current-carrying purposes since they show only a relatively small increase in contact drop after continuous duty. However, they weld together easily when subjected to high currents because of either high contact drop caused by the lessening of the contact pressure by magnetic forces, or by premature parting of the contacts before the mechanism is released by the trip unit and latch. To offset the effect of these magnetic forces it is necessary to increase the contact pressure by mechanical means or to provide a curved flexible conductor which will exert magnetic forces in such a direction as to cause increased contact pressure with increased current. If welding cannot be wholly eliminated in these ways, it is necessary to use another contact metal which is more resistant to welding.

The first modifications to the breaker included greater contact pressure and a change of the contacts from silver against silver to silver against silver-tungsten of 60 and 40 per cent proportion, to prevent the welding together of the contacts. This combination was tested at 20,000 amperes. The main contacts did not weld together when the short circuits were thrown on the breaker in the closed position, but welding occurred in about half of the tests when the breaker was closed on the short circuits.

Silver tungsten normally increases the contact drop but this was partially offset by increasing the contact pressure. The contact drop remained approximately the same for the first four short circuits and then began increasing rapidly due to the burning of the contact material.

A change in the main contacts from silver against silver-tungsten to silver-tungsten against silver-tungsten of 60 and 40 per cent proportion was tried. This combination did not entirely eliminate the possibility of welding when the breaker was closed on 20,000-ampere short circuits but the percentage was reduced.

Main contacts made of silver-molybdenum were tried in order to overcome the contact welding problem. The tests were highly satisfactory from this standpoint, for at no time did these contacts weld together on short circuit. The contact drop remained approximately the same throughout the tests. These contacts also showed considerably less signs of burning away than did the silver-tungsten contacts. However, silver-molybdenum contacts were found to have a very decided drawback. The drop across these contacts increased appreciably in a few days due to the oxidation of the molybdenum. A number of breakers without current were exposed to an ordinary atmosphere from one to three weeks, and they were tested for contact drop from time to time. Some of these breakers were in closed position and others were in open position. The drop readings across these breakers were initially between 12 and 18 millivolts at 250 amperes d-c, and the final drop values varied anywhere from 30 to 150 millivolts depending on the length of exposure. A breaker having a drop of about 50 millivolts was tested with full-load current for about 48 hours continuously, but strangely there was no increase in contact drop during this time. It was decided to abandon the use of silver-molybdenum because of the appreciable increase in contact loss under ordinary atmospheric conditions.

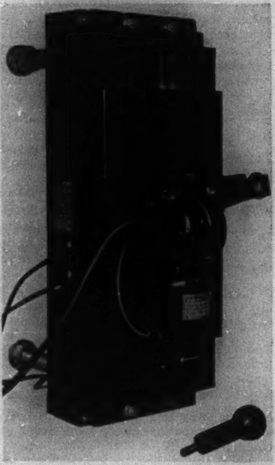


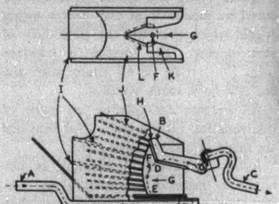
Figure 3. Two- or three-pole, 600-ampere circuit breaker of either 10,000- or 20,000-ampere interrupting capacity with motor mechanism for electrical operation

A contact construction was finally produced which included the following changes:

1. Silver-tungsten contacts of 30 and 70 per

cent proportion for all moving and stationary members to positively prevent welding at 20,000 amperes or more, with either close-open or open-close-open breaker operation. The use of this alloy reduces the burning of the contact material to a minimum.

2. Center contact members of the roll type which make contact at the heel or farthest point from the arc chamber when the breaker is closed and which roll to the toe or into the slot of the arc chamber while the breaker is opening. This center contact takes the place of the secondary and arc contacts of the old construction which derived their pressure from one spring. All of this spring pressure is now used for the one center contact thus giving less contact drop.



A—STATIONARY CONTACT MEMBER
B—MOVING CONTACT MEMBER
C—FLEXIBLE CONDUCTOR
D,E,AND F—FLUX AT D AND E TEND TO FORCE ARC F IN DIRECTION G
H—ARC HORN FACED WITH SILVER-TUNGSTEN TO REDUCE BURNING
J—STEEL PLATES
K—ARC SHIELDS
F,G,AND L—FLUX L PULLS ARC F IN DIRECTION G

Figure 4. Schematic diagram of stationary and moving arc contacts and arc chamber

3. An arcing horn at the end of the center moving contact, to elongate the arc and to more positively insure that the arc strikes all of the plates in the arc chamber. Facing the arcing horn with silver-tungsten (30 and 70 per cent) reduced the burning of the horn considerably and thereby lessened the amount of gas.

4. Moving and stationary contact members, consisting of two outside main contacts and one center arc contact, having a definite contact opening and closing sequence to reduce the burning of the contact material and to positively insure the formation of the arc between the arc contacts and not between the main contacts. This contact sequence consists of the right-hand main contact parting ahead of the left-hand main contact which is followed by the parting of the arc contacts. Since the drop is approximately the same across each of these contacts, the resistance does not rise too rapidly when the contacts are parting.

5. The direction of the flow of current through the stationary conductor to the center contact was changed to assist forcing the arc into the arc chamber. The change in current direction produces a flux condition which exerts a force on the arc in the proper direction, as shown in figure 4. The old construction produced flux, which tended to force the arc at one end away from the arc chamber.

Short-circuit tests on breakers having these modifications show that contact welding has been entirely eliminated, that contact burning has been considerably reduced, that there is less outward disturbance, and that the rupturing capacity of the breaker has been considerably increased.

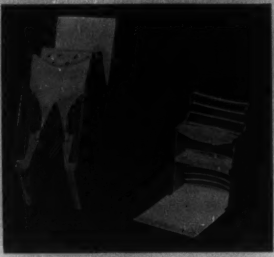


Figure 5. Arc chamber of 20,000-ampere interrupting capacity

ARC CHAMBER

The arc chamber used during the early stages of this development is of the cold-cathode type which has a radial field for the purpose of spinning the arc to keep the electrodes cool.

This arc chamber consists essentially of a series of copper plates, each fitted into and surrounded by a steel plate except for a V-shaped opening through which the arc contacts move. These plate assemblies are stacked one upon the other and insulated from each other by fiber. A flat coil is located at the center of the stack of plates and is connected to the two adjacent copper plates so that it is in series with the line after the arc is broken into a series of small arcs between the copper plates. The flat coil has a center steel core and produces a radial field from the core to the surrounding steel plates.

In addition there is a one-turn coil which is connected to the stationary arc contact and to the line terminal so that the arc contacts and the coil are in series when the main contacts part. This coil energizes the iron surrounding the copper plates and causes the arc to be moved from its point of inception up into the narrow part of the V plates. From this point the arcs are under the influence of the radial field and are moved with a high velocity in an annular path formed by the insulation between the copper plates.

When alternating current is being interrupted, these series arcs continue to rotate until the current reaches its natural zero value. The d-c arc is extinguished as soon as the current is reduced to zero, which depends on the inductance of the circuit and the amount the arc voltage exceeds the line voltage.

The arc chamber as described above was enlarged by increasing the number of copper plates from nine to 12 in order to take care of the heavier short-circuit currents. These additional plates increase the plate area which helps to cool the hot gases and assist the process of deionization. Since the spacing between the plates remains unchanged, the additional plates will permit increased venting which will help to increase the interrupting capacity.

The enlarged arc chamber was short-circuited at 20,000 amperes on both 250 volts d-c and 600 volts a-c. It interrupted all of the short circuits it was subjected to.

It was thought that by use of the plain-plate type of arc chamber, as shown by figure 5, the following advantages could be obtained at the higher short-circuit value.

1. Better insulation between the line and load sides of the breaker in open position because of the elimination of the shunt which connects the top plate of the arc chamber of the radial field type to the load side of the breaker.

2. Increased venting of the arc gases.

3. Simpler construction.

4. Better performance on direct current.

The arc chamber of this plain-plate type consists of a series of V-slotted steel plates mounted between insulated supports, and it is so placed that the slots of the steel plates extend over the arc contacts. The moving arc contact must pass through the slotted steel plates as it leaves the stationary arc contact. As this action takes place, the arc formed is magnetically driven into the steel plates and is usually broken into a series of short arcs, depending on the amount of current and whether the current is alternating or direct.

The tests on the plain-plate type of arc chamber indicated increased venting, resulting in less back pressure and less burning of the contacts, better insulation between the line and load side of the breaker, more satisfactory performance on direct current, and less heating of the arc chamber which permits less interval of time between short circuits. As a result of these tests, the plain-plate type of arc chamber was adopted for the heavy-duty breakers.

Tests on this arc chamber also indicated that an arc produced by small a-c short-circuit currents usually does not transfer to the steel plates and form short series arcs, but an arc produced by heavier a-c short-circuit currents or not too low d-c currents usually would transfer to the steel plates and form the short series arcs. The degree of venting at the top end of the steel plates determines largely whether or not the arc transfers to the steel plates. Free venting encourages arc transfer.

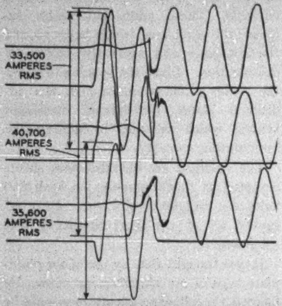


Figure 6. Oscillogram showing a three-phase interruption at 36,000 amperes, 600 volts, 60 cycles

Deionization of the arc is effected either by the ions neutralizing their charges among themselves or by discharging into the electrodes. The current and voltage a given device can interrupt depends on the length of the arc, the material and size of the electrodes, the proximity of the walls or plates to the arc, the degree of venting, and number of series arcs which determine the number of positive-ion-spaced-charge sheaths in front of the cathode. Those who wish to learn of the fundamentals of arc phenomena should refer to previous articles[2–6] which treat this subject very thoroughly.

When alternating current is being interrupted, the arc or the series arcs continue until the current reaches its natural zero value. At this time the processes of deionization transform the conducting ionized gas into an insulating gas. This prevents the arc from re-establishing by preventing the line voltage from building up current in the opposite direction. The a-c arc is extinguished at the end of the first half cycle.

The arc motion is the same for alternating and direct current, although the effect on the circuit and the way the arc is extinguished is quite different. For a given line voltage a much larger number of plates and much larger gap between the arc contacts must be used for direct current so that the total arc voltage is greater than the line voltage. This difference between the arc voltage and the line voltage is necessary to cause the current to decrease to zero.

Arc shields line both sides of the arc chamber to prevent the arc from burning or striking the lower sides of the steel plates. The burning of unprotected sides of steel plates tends to cause an excess of gas. The striking of the unprotected sides by the arc occurs mainly on low d-c currents, and hinders the arc from rising to the end of the slot, thus causing prolonged arcing.

The arc splitters are spaced to form three compartments directly above the steel plates of each arc chamber to prevent reignition of the arc gas as it is vented from the plates.

Table I. Contact-Drop Data Taken After Short-Circuit Tests

Amperes at 600 Volts, 60 Cycles, 3 Phase	Test Cycles	Millivolts Drop at 250 Amperes D-C		
		Left Pole	Center Pole	Right Pole
		*7.4	*7.8	*6.8
20,000	O			
20,000	CO	6.5	5.5	5.4
20,000	O			
20,000	CO	7.5	7.2	6.5
20,000	O			
20,000	CO	6.2	6.6	7.1
32,000	O			
32,000	CO	18.2	9.8	13.5

* Initial drop readings

The arc formed between the arc contacts is acted upon by two forces, which tend to draw it away from the contacts into the steel plates of the arc chamber as shown by figure 4. They are namely: the flux at D and E which is created by the direction of flow of the current through the arc contacts, and the magnetic field produced in the steel plates.

Description of the Final Developed Circuit Breaker

The circuit breaker is built in an enclosed insulating housing of two parts known as the base and cover. The parts are made from "Moldarta" of a composition selected to give proper mechanical strength, resistance to such heat as it may be subjected, and to resist warping and the absorption of moisture. Both base and cover are provided with deep ribs or walls to permit close assembly of the different poles and to give adequate strength.

An operating handle made of Moldarta is provided to close, open, and reset the breakers. This handle moves in a shielded slot located near the center of the cover. Position indication is obtained from the operating handle. Upper position of the handle indicates the breaker is closed, lower position of the handle indicates the breaker is opened, and mid-position of the handle indicates automatic tripping.

Quick make and quick break are assured by means of an over-center spring toggle which prevents the possibility of slow closing. The breaker is trip-free from the handle at all times after the arc tips have made contact.

All triggers and latches are made of rust-resisting nitride steel. All bearings are made of noncorrosive metal.

Automatic overload tripping is obtained by means of a thermal release consisting of a heater, a bimetal part that changes its shape with change of temperature, and a trigger which is moved by the bimetal part to unlatch the breaker. Resulting time limits are sufficient to prevent interruptions due to overloads of short duration and still give full protection against heating.

Supplementing the thermal release, there is a one-turn electromagnet which trips the breaker instantaneously on currents ranging from six to eight times the normal current rating.

Both the thermal and magnetic releases are mounted on a base, together with a trip bar and trigger. This unit is called a trip unit and is made up in different current ratings. These trip units can be easily removed from the breaker, and are interchangeable which permits a change in current rating to be made after installation.

Figure 2 shows a final developed circuit breaker with the cover removed. Figure 3 shows a complete circuit breaker of either 10,000- or 20,000-ampere interrupting capacity with a motor mechanism for electrical operation.

The circuit breakers are rated at 70 to 600 amperes, 250 volts d-c or 600 volts a-c, two-pole, and 70 to 600 amperes, 600 volts a-c, three-pole. The breakers

Table II. Weight and Size Data

Connection	Weight (Pounds)	*Over-all Size		
		Width	Length	Depth
Front	53	$8^1/_8$	22	$5^1/_8$
Rear	62	$8^1/_8$	22	**$11^1/_8$

* Operating handle not included.
** Rear-connected terminals included.

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2. For all numbered references, see list at end of paper.

as shown are for rear connection only. By adding a gas by-pass at the top front end of the cover, the breaker can be used for front connection. The gas by-pass assists the venting of the gas around the terminals. The same molded main base and cover are used for both the two- and three-pole breakers.

Since size was a very important factor in the development of this breaker, table II is given, listing the over-all size and weights of both the front- and rear-connected breaker units.

Final Test Results

Figure 6 illustrates the performance of the final breaker opening a short circuit at 36,000 amperes, 600 volts, 60 cycles, 3 phase. This particular breaker satisfactorily opened one open and CO short circuit at 20,000 amperes, one open and CO at 25,000 amperes, one open and CO at 26,000 amperes, and two open and CO short circuits at 36,000 amperes. The contacts and steel plates were burned but not excessively.

A similar breaker was tested to determine the effect of severe short circuits on the contact drop. Table I gives the short-circuit and contact-drop data. These data indicate that the contacts were still in excellent shape after six short circuits at 20,000 amperes. The increase in contact drop after the two 32,000 ampere short circuits is not enough to cause overheating or to affect the thermal trip calibrations.

Conclusion

The circuit breaker, as finally developed, is of a size which enables a compact and efficient switchboard construction, has a considerable safety margin over the 20,000-ampere rupturing capacity, and permits interchangeability of a large number of detail parts with the circuit breaker of the same normal rating but of 10,000 ampere rupturing capacity.

References

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4. Extinction of a Long A-C Arc, J. Slepian. AIEE Transactions, volume 49, 1930.

5. Extinction of A-C Arcs in Turbulent Gases, T. E. Browne, Jr. Presented at the winter convention of the AIEE, New York, N. Y., January 25–29, 1932.

Line-Type Lightning Arrester Performance Data

THIS REPORT presents characteristics of line-type arresters rated 20 to 73 kv, and is a continuation of the previous subcommittee report on "Distribution Lightning Arrester Performance Data" published in the AIEE Transactions, volume 56, 1937, pages 576–7.

Data have been presented relating to the impulse characteristics of insulation both as used on lines and in stations. Data on the rates of voltage rise and current magnitude to be encountered by protective devices in the field are being accumulated rather slowly. In order to have a basis for comparing arrester characteristics, a standard test was set up which indicates the arrester protective characteristic which falls in the range of service conditions. While the test was empirically set up, the characteristics obtained can be compared with the insulation to show the margin of protection afforded.

The present standard requires an applied impulse voltage rising at the rate of 100 kv per microsecond per 11.5 kv of rating in these voltage classes. In effect this results in an impulse sparkover of the arrester series gap in about 0.4 microsecond after the impulse voltage begins to rise. After the arrester begins to discharge the impulse current should rise to 1,500 amperes in ten microseconds and fall to half value in ten more microseconds. This current results in a voltage drop or impedance drop across the characteristic element of the arrester. As the impedance drop varies somewhat with current, characteristics also are presented for currents of 3,000 amperes and 5,000 amperes.

The lightning-arrester manufacturers have furnished the data from which the average characteristic of available arresters are plotted and tabulated. Tolerances are given to include the expected

Table I. Line-Type Arrester Characteristics (20–73 Kv)

Arrester Rating (Kv)*		Gap Breakdown (Crest Kv)	Impedance Voltage; Crest Kv When Discharging Currents of		
			1500 Amperes	3000 Amperes	5000 Amperes
20	Max	97	83	90	94
	Ave	75	69	75	78
	Min	52	55	60	63
25	Max	118	104	112	118
	Ave	95	87	94	98
	Min	64	69	75	79
30	Max	139	124	134	142
	Ave	112	104	112	117
	Min	78	83	89	94
37	Max	169	155	168	176
	Ave	135	129	140	146
	Min	98	103	112	117
40	Max	180	168	182	190
	Ave	147	140	151	158
	Min	104	112	121	127
50	Max	225	210	228	238
	Ave	180	175	188	198
	Min	131	140	150	159
60	Max	267	252	274	286
	Ave	217	210	226	238
	Min	157	168	182	190
73	Max	323	309	333	348
	Ave	265	258	278	290
	Min	195	206	222	232

Notes: Gap breakdown taken on rate of voltage rise of 100 kv per microsecond per 11.5 kv of rating. All test currents 10/20 wave. The 60-cycle sparkover potential of all arresters shown in this tabulation will not be less than 150 per cent of arrester rating.

* These ratings are maximum permissible line-to-ground root-mean-square voltages.

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Personnel of AIEE lightning arrester subcommittee: J. R. North, *chairman;* H. W. Collins, J. M. Flanigen, I. W. Gross, Herman Halperin, K. B. McEachron, J. R. McFarlin, A. M. Opsahl, A. H. Schirmer, H. K. Sels, L. G. Smith, H. R. Stewart, J. J. Torok, and H. L. Rorden.

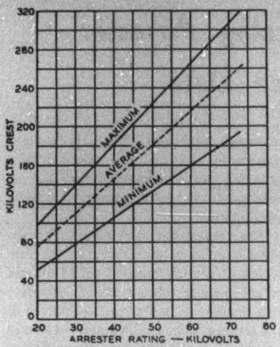

Figure 1. Impulse spark-over voltages, line-type arrester

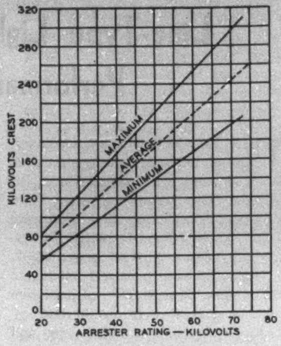

Figure 2. Impedance drop at 1,500 amperes, line-type arresters

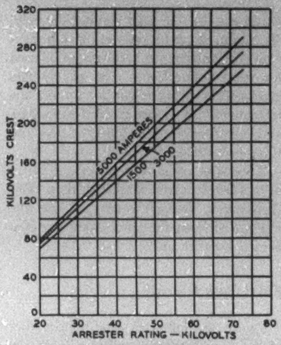

Figure 3. Average impedance drop, 1,500, 3,000, and 5,000 amperes, line-type arresters

variation from the average for the industry.

The over-all characteristic of the lightning arrester is usually given in the typical volt-time curve under specified conditions. The pertinent voltage magnitudes only are given here in order to simplify the data. By a comparison of the typical volt-time arrester characteristic with the volt-time characteristic of the insulation to be protected, a logical evaluation of the protection can be made.

System Analysis for Petersen-Coil Application

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Synopsis: Careful analysis of the characteristics of a power system has been found to be necessary before applying Petersen coils to a system which has been operating either with the neutral isolated or grounded. In these two companion papers on Petersen coils, accent has been placed on the analysis of system characteristics in order to determine whether or not the application of Petersen coils will be satisfactory.

In this paper (part I) the general features of Petersen-coil application are considered, and the fundamentals of calculating coil characteristics are given. In the companion paper (part II) by F. Von Voigtlander, the methods of performing detailed calculations on system fault currents, coil ratings, and voltages are given in an illustrative example, together with field tests to check design computations and performance of the equipment.

Part I—Operating Features and Basic Calculations—By W. C. Champe

Introduction

CONSIDERABLE interest has recently been shown in this country in the application of resonant neutral grounding reactors, or Petersen coils, to medium- and high-voltage transmission systems for the purpose of extinguishing single line-to-ground faults without the operation of circuit breakers and before they spread to phase-to-phase or three-phase short circuits. Applications have been made of Petersen coils to existing systems, most of these having been systems which were originally operated with the neutral isolated, since adequate relaying of single line-to-ground faults is ordinarily more difficult to obtain on an isolated-neutral system than on a grounded-neutral system.

A Petersen coil usually consists of a tapped inductance with iron core, connected between ground and the neutral point of a suitable transformer bank so that, in the case of an arc to ground of one conductor of the transmission system, the total reactive lagging current from the coils will be approximately equal to the residual charging current to ground of the system. The inductive reactance of the system, most of which will be in the Petersen coil or coils, will then be approximately equal to the capacitive reactance of the two unfaulted conductors to ground. If the coils are properly tuned, the fault current in a single phase-to-ground fault will be principally the in-phase component, which cannot be balanced out by the coils and ordinarily will be so low that the arc will be self-extinguishing.

Effects on System Operation

Following is an outline of the effects on system operation to be expected from the installation of Petersen coils as compared to isolated-neutral and grounded-neutral systems:

GROUND-FAULT CURRENT MAGNITUDE

1. On most grounded-neutral systems in this country, the ground-fault currents are quite large.

2. On an isolated-neutral system, the current through a single line-to-ground fault depends upon the length of transmission lines, and with an extensive system is large enough to maintain an arc.

3. On Petersen-coil systems, when properly tuned, the ground-fault current is reduced to a relatively small value, approximately the in-phase component of residual current. Damage to the equipment at the point of fault, due to current, is therefore greatly reduced. More thorough consideration of current magnitudes is given in a later part of this paper.

GROUND-FAULT ELIMINATION

1. On grounded-neutral systems, all types of ground faults ordinarily cause trip-outs.

2. On all isolated-neutral systems, unless small in extent, single-phase faults to ground are not self-extinguishing. Since these faults are difficult to relay properly, they may continue until they become two-phase or three-phase faults and the section of line is relayed out.

3. On a system to which Petersen coils can be successfully applied, practically all transitory single-phase faults are self-extinguishing before developing into two-phase or three-phase faults. Relay operations and interruptions to service, therefore, should be much reduced by the installation of Petersen coils on either a grounded-neutral or isolated-neutral system.

MAXIMUM LINE-TO-GROUND VOLTAGES

1. Grounded-neutral systems in this country usually have sufficiently low values of neutral grounding resistance or reactance that the neutral at all parts of the system does not vary greatly from ground potential. Line-to-ground voltages on the unfaulted phases, therefore, do not increase greatly during fault conditions.

2. On isolated-neutral systems, line-to-ground voltages on the unfaulted phases at the point of fault are approximately equal to line-to-line voltages, and at distant points may be 20 to 50 per cent greater. In some cases, this may result in a double line-to-ground fault.

3. With one Petersen coil, overvoltages for faults at most locations would be lower than on an isolated-neutral system, and equipment would be less likely to be damaged, since the fault duration would be reduced. The installation of additional Petersen coils would reduce the overvoltages for faults at any location. Overvoltages are discussed in more detail in a later section.

LIGHTNING-ARRESTER RATING

1. On a system having the neutral effectively grounded, these ratings can generally be approximately 80 per cent of the maximum line-to-line voltage.

2. On an isolated-neutral system, these ratings must be at least equal to line-to-line voltage.

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W. C. CHAMPE is electrical engineer in the division of engineering and construction of the City of Toledo, Ohio, and F. VON VOIGTLANDER is employed in the technical section, electrical engineering department, Commonwealth and Southern Corporation, Jackson, Mich. These papers (parts I and II) are based on a study of Petersen-coil application and operation over a period of years, and the authors wish to express acknowledgment of the suggestions given by their associates, in particular to Mr. J. R. North for his encouragement and assistance.

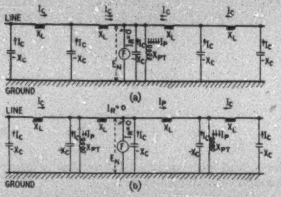

Figure 1. Zero-sequence network, showing effect of one Petersen coil (a) and two Petersen coils (b) on the distribution of fault current due to a single line-to-ground fault

X_L = Line series reactance
X_C = Line shunt capacitive reactance to ground
X_{PT} = Reactance of Petersen coil with associated transformer (per-phase value of transformer reactance X_{TR} plus three times the actual Petersen-coil reactance X_P)
X_{TR} = Reactance of transformer used with Petersen coil
X_P = Actual Petersen-coil reactance ($^1/_3$ the per-phase value)
I_C = Charging current to ground
I_P = Petersen-coil current
I_R = Residual current
F = Single line-to-ground fault
E_N = Volts, line to neutral

These zero-sequence quantities are the usual values, per phase, based on three conductors in parallel, except the Petersen-coil reactance, which is actual reactance. Actual currents at point of fault and in Petersen coils are three times the zero-sequence current

3. On a Petersen-coil system, these ratings cannot be appreciably lower than for an isolated-neutral system, provided only one Petersen coil is installed. With additional Petersen coils, it may be possible to reduce these ratings somewhat.

EFFECT ON COMMUNICATION FACILITIES

1. With a grounded-neutral system, the fundamental-frequency induced voltage resulting from single line-to-ground faults may be excessive.

2. On isolated-neutral systems, while the fundamental-frequency induction on all but very extensive systems is usually less than for grounded-neutral systems, noise-frequency induction may be severe due to the possibility of long-duration ground faults.

3. An installation of Petersen coils usually reduces the effects on communication facilities considerably, as the magnitudes of the current carried over the transmission lines during faults would in general be smaller. For certain fault and exposure locations, the residual current in an exposure may be almost zero. The number of locations at which the residual current would be small increases as the number of Petersen-coil installations on the system is increased. This is especially important upon the occurrence of a permanent fault, such as a conductor lying on the ground. The effect of additional coils in reducing line currents is shown in figure 1.

SHOCK TO THE SYSTEM

1. With a grounded-neutral system, a ground fault may result in a great shock due to the heavy current.

2. With an isolated-neutral system, a ground fault may result in a large voltage shock.

3. A properly designed Petersen-coil system results in very little shock to the system upon the occurrence of a ground fault.

The principal advantages of Petersen coils, therefore, are seen to be the clearing of most ground faults without the operation of circuit breakers or interruption to service, the reduction in damage at the point of fault and in effects on communication resulting from reduced fault currents, the reduced overvoltages as compared to an isolated-neutral system, and the reduced shock to the system caused by ground faults.

In considering the installation of Petersen coils, the following limitations of the coils should be kept firmly in mind:

1. They do not compensate for the in-phase component.

2. They do not entirely prevent system overvoltages to ground from exceeding normal phase-to-phase voltages, but only limit their extremes in magnitude and restrict their duration.

3. They require that all apparatus and equipment connected between line and ground be capable of sustaining overvoltages.

4. They have no effect on phase-to-phase faults as such.

5. They do not clear permanent ground faults, but help to mitigate their effect.

Each of the above factors must definitely be taken into account when considering Petersen coils in order to determine their efficacy.

Zero-Sequence Capacitive Reactance and Residual Charging Current to Ground

The first step in any calculations for applying Petersen coils is the determination of the zero-sequence capacitive reactance of a short line section with one conductor grounded, neglecting the effect of line series reactance. Methods of calculating this value on a per-phase-per-mile basis are given in several works.[3,6]

If the residual charging current to ground with one conductor grounded is first obtained by test or otherwise, it may be expressed on a per-mile basis. For later use in combining with other network reactances, the zero-sequence capacitive reactance to ground in ohms per phase should be obtained by dividing the volts to neutral by $^1/_3$ the charging current to ground in amperes. This charging current is the same as the charging current to ground from all three conductors connected in parallel and energized at line-to-neutral voltage. Roughly, its value for single-circuit lines of moderate or high voltage is 1.5 to 1.7 times the per-phase value of normal three-phase balanced charging current of the line. This value does not include any in-phase component or the effect of line series reactance.

Where double-circuit lines are involved, the value of residual charging current per mile per circuit must be reduced since the residual charging current to ground of one mile of double circuit line is considerably less than twice the value for one mile of single circuit line of equal spacing.

The capacitive reactance to ground is influenced by a number of factors such as the effect of: the supporting poles, substation structures, and trees and shrubbery along the right of way, and variation in sag, etc. Although the value can be calculated quite accurately, as shown in part II, it is usually advisable to check these calculations by test for any particular system if there is any question about the accuracy of the fundamental data.

A few transpositions may be necessary in the transmission lines to prevent a residual voltage of considerable proportions being maintained on the system during normal conditions, with a consequent flow of current through the Petersen coil.

In-Phase Component of Fault Current

Petersen coils do not balance out the in-phase component of current in the arc caused by losses, such as corona loss, leakage over insulators, unbalanced copper losses, and Petersen coil losses, nor do they balance out the harmonic currents in the arc. The in-phase component must be carefully checked to make sure that it will not be large enough to maintain the arc readily and prevent extinction by the Petersen coils. Fortunately, the arc will be self-extinguishing with a larger magnitude of in-phase component than of quadrature component of current, as the current and voltage will

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[3] For all numbered references, see list at end of paper.

both go through zero at the same instant.

The in-phase component varies considerably with weather conditions and is difficult to calculate accurately. Unless preliminary consideration shows that the value is small, it is desirable that tests be made on a power system to determine the magnitude of the in-phase component before Petersen coils are applied. It is also important to keep in mind that the magnitude of this component is generally reduced as the number of Petersen coils on the system is increased, as the current through the lines and the voltage distortion over the system would be reduced. An example of the magnitude of this component is shown in part II.

Methods of compensating for the in-phase component have been developed, but the complications involved tend to make their use undesirable.

System Voltages and Effect on Currents

On an isolated-neutral system the voltages at the point of fault are displaced so that ground is at one corner of the voltage triangle, the delta (line-to line) voltages remaining approximately unchanged in magnitude. As locations distant from the point of fault are considered, the voltage triangle shifts further away from ground due to the effect of charging and in-phase components of current being drawn over the series impedance of the line. The result is that voltage appears on the faulted conductor (voltage $Z'G$ of figure 2*c*), and the voltage of the conductor ahead of the faulted conductor in time sequence has a voltage to ground greater than the normal line-to-line value (voltage $Y'G$). If the transmission system is at all extensive, the effect of the series impedance is much more important than usually thought and may easily impress a voltage between an unfaulted phase and ground of 120 to 150 per cent of normal line-to-line voltage. In extreme cases, a condition of resonance between capacitive reactance to ground and series reactance of the line may be obtained with one conductor grounded.

The effect of series reactance is especially important if a long section of small-conductor line connects two capacitance centers of the system.

The reactance of a Petersen coil is not of a sufficiently low value to hold the neutral of the associated transformer at ground potential during a ground fault, and the system voltages are displaced to a condition approximating that on an isolated neutral system. If there is only one Petersen coil on the system and the fault is near the coil, voltage and current conditions on the lines will be approximately the same as on an isolated-neutral system. This results in an increase in the total charging current and, therefore, increases the capacity required in a Petersen coil. These effects can be reduced, however, by the installation of additional coils, as the charging current for any section of the system can then be balanced out by a nearby coil and large currents will not have to be drawn over the line impedance.

On some systems, particularly those with small conductors and high voltages or at high altitudes, the increase in corona current due to the increase in voltage at points distant from the ground fault may be an important factor in preventing the successful operation of the coil.

Number of Coils Required

As the fault location is moved in from the ends of the system toward a Petersen coil, the lagging current through the Petersen coil will increase appreciably; also the charging current of the system. If the Petersen coils are in tune for a fault at any one location on the system, they will be in tune for a fault at any other location, provided the system itself does not change by having lines switched on or off. This applies regardless of whether there is only one Petersen coil or several. As regards the number of grounding points, this is different than the application of grounding banks on grounded neutral systems, since then it may be necessary to place the banks close enough together to give adequate current for relays, and more than one bank is almost always required for directional selectivity.

If the scheme is used of short-circuiting the coils in case the fault should continue for an appreciable length of time, and tripping the faulted line out by ground relays, then at least two Petersen coils are frequently desirable.

Two or more Petersen coils may also be desirable for other reasons. In case the transmission system is likely to be split so that sections would otherwise be without a Petersen coil, it may be desirable to install additional coils in these sections. It will then usually be possible to set the coils on a reactance tap such that the coils will remain approximately in tune even though the system is split apart, regardless of relative size of the sections. This also avoids the possibility of considerable over-compensation from a coil in case it becomes separated from a large part of the system.

As previously mentioned, the voltage distortion in a transmission system, magnitude of total charging current, and total capacity required in Petersen coils will be somewhat reduced by the installation of additional coils and best results will be secured if they are located near the capacitance centers of the system.

System Neutral Connection

The transformer bank to which the Petersen coil is connected may be either a transformer which is used for the transmission of power or it may be a special transformer for the Petersen coil. The types of transformer connections which can be used are the same as those for system grounding; namely, wye-delta, zigzag auto, wye-wye-delta (for use also as power transformers in parallel with delta-delta connected banks), or transformers connected with one winding zigzag and the other winding delta (for operation in parallel with delta-delta transformers).

One type, known as a "Bauch" or "Bauch-Quenching" transformer, combines the grounding transformer and the adjustable reactor. The grounding transformer is a wye-delta unit. The wye winding is connected to the line and has its neutral solidly grounded. The reactor is connected across an opened corner of the delta winding and is tapped so as to be adjustable for proper tuning.

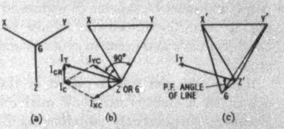

Figure 2. Effect of ground fault and charging current on system voltages, showing: (a) unfaulted line; (b) faulted line, phase Z, at point of fault; and (c) faulted line at point distant from fault

XG, YG, ZG = Line-to-ground voltages

XY, YZ, ZX = Line-to-line voltages

I_{YC}, I_{XC} = Charging current to ground (quadrature component) of unfaulted phases

I_{CR} = In-phase component of current to ground of unfaulted phases

I_T = Total current to ground of unfaulted phases

A number of factors enter into the kilovolt-ampere rating of the transformer used with a Petersen coil. If the transformer is a special one used for the coil only, it may be of the same kilovolt-ampere and time rating as the coil. The time rating should generally be at least ten minutes and possibly even continuous, depending upon the method used for isolating permanently grounded line sections. If the coil is to be short-circuited for relaying when a permanent fault occurs, the transformer should have adequate capacity to meet the grounding requirements of the transmission system. This will require at least four or five times the amperes capacity required for Petersen coil operation but the time rating will be on a much shorter basis, usually not over sixty seconds. The higher capacity but shorter time may therefore result in a transformer having a physical size not greatly different than the lower-capacity longer-time transformer required for Petersen-coil operation only without short-circuiting.

Accessory Equipment

Some means of protecting the coil from overload is necessary in case a line conductor becomes permanently grounded. The circuit to the coil can be opened, or the coil can be short-circuited to give ground relaying. Either circuit breakers or motor-operated air-break switches can be used for these purposes, and must be capable of withstanding at least line-to-neutral voltage between contacts and between contacts and ground.

If the Petersen coil is placed in the neutral of a transformer bank used for grounding purposes only, switching equipment in series with the coil can be omitted, but some means should be provided for disconnecting the transformer from the line or bus. At the lower voltages, fuses may be satisfactory, although there is an inherent difficulty in distinguishing between line faults (for which the fuses should not operate), and transformer failures (for which the fuses should operate). Use of a circuit breaker is highly desirable, but it is possible simply to use air-break switches to disconnect the transformer manually when necessary, and to treat transformer faults the same as bus faults.

The Petersen coil ordinarily must be removed from service by one of the above means when tap changing is necessary. An alarm relay and graphic ammeter connected to a current transformer in the permanently grounded lead from the coil are desirable, in addition to relays for operating the switches to short circuit or open the Petersen coil for permanent faults. The current transformer can have a comparatively low voltage rating.

Preliminary Determination of Coil Impedance and Current Rating

The fundamental factors which must be considered to determine the approximate impedance and current rating of Petersen coils are given in the following outline. The detailed methods of calculation are shown in the example of part II.

If the transmission system is not extensive the coil or coils will have a large impedance in comparison with the series impedance of the transmission lines. In this case neither the Petersen-coil current nor the charging current will vary much with faults at different locations and the effect of line impedance can be neglected. With an extensive transmission system, the Petersen-coil impedance will become smaller and the series line impedance larger. The latter then becomes an important factor.

The first step in calculating the coil ratings is to determine the value of zero-sequence capacitive reactance to ground for a short section of transmission line. The next step will depend upon the size of the transmission system to which the coils are to be applied.

COIL RATING—SHORT TRANSMISSION SYSTEMS

For the most simple case, with one coil and a short transmission system, all the capacitive reactance can be considered as lumped at the Petersen-coil location and the series reactance of the line need not be considered. Neglecting positive- and negative-sequence values for the preliminary calculations the inductive reactance of the coil *with* its associated transformer must equal the capacitive reactance of the transmission system to ground.

From this value determined for the reactance of the Petersen coil with its associated transformer, the reactance of the transformer must be subtracted, and the difference divided by three to determine the actual reactance of the Petersen coil itself. This division by three is necessary since per-phase values are ordinarily used and the Petersen coil must carry the zero-phase-sequence current from all three conductors. The current rating is approximately equal to the total residual charging current to ground (determined as previously described) and the voltage rating is equal to the actual Petersen coil ohms multiplied by this current. This voltage is slightly less than the normal line-to-neutral voltage of the system.

In the zero-phase-sequence network, the three phases of the transformer and line are connected in parallel and line-to-neutral voltage applied between them and ground. The per-phase values are, therefore, different than the total values by the factor of three. It would be possible in calculations of an approximate nature to use the total zero-sequence values, which with phase-to-neutral voltage would give the actual coil reactance directly without the necessity of dividing by three. The use of per-phase reactances on the other hand in conjunction with line-to-neutral volts gives the zero-sequence component of the fault current and this must be multiplied by three to get the total fault current. It is much better to use the per-phase values, as the refinement of considering positive- and negative-sequence reactances can easily be added since they are always per phase.

COIL RATING—EXTENSIVE TRANSMISSION SYSTEMS

If line series reactance must be taken into account, the most simple case is a single transmission line with a Petersen coil at one end. If the line is not over about 100 miles in length, the capacitance can be considered lumped at the ends or in the middle (π or T line). If π lines are used, half the capacitance is lumped at each end and expressed as a capacitive reactance. The capacitive reactance at the far end is then combined with the line series reactance (only zero-phase-sequence values being used for the approximate solution) by subtracting one from the other to obtain an equivalent reactance at the Petersen-coil location.

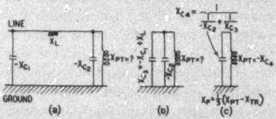

Figure 3. Method of combining line series and shunt capacitive reactances to determine Petersen-coil reactance. The network is reduced to that indicated at (c) and the actual Petersen-coil reactance is then determined by the equation shown

X_{C1} and X_{C2} are the zero-sequence capacitive reactances to ground for the two ends of the line, which are combined as indicated to determine X_{C3} and X_{C4}. See figure 1 for explanation of other symbols

(See transformation *a* to *b* of figure 3.) This value must then be combined with the capacitive reactance of the line assumed at the Petersen-coil end (reactances in parallel) to give the equivalent total capacitive reactance at the coil location. (See transformation *b* to *c*, figure 3.) The rating of the coil is then obtained in the same manner as in the most simple case where no line series reactance was considered, and is shown in the equation below part *c* of the figure.

In case of a more extensive transmission system, the line series reactances and lumped shunt capacitive reactances to ground of several short line sections (not over about 100 miles in length) are combined step by step in the same manner to obtain an equivalent total capacitive reactance for the system, referred to the Petersen-coil location or locations.

Positive- and negative-sequence values can be included in the usual manner, although if there are transformers and generators supplying power at different points of the system, the positive- and negative-sequence values usually will be sufficiently low in comparison with the zero-sequence values that they need not be considered in preliminary calculations.

OTHER FACTORS IN RATING

To provide for proper tuning when a section of the transmission system has been switched off, additional reactance is required in the coil and may be of a lower current rating. This additional reactance must be sufficient to give tuning for the minimum transmission system expected, or down to an arc current value sufficiently low that the arc will be self-extinguishing even though the charging current is not entirely balanced out by the coil current.

The questions of tap steps, tap changers and operating mechanisms, coil insulation and protection against surge voltages, and other design factors have been covered in a recent Institute paper by Mr. E. M. Hunter.[1]

Part II—System Analysis and Computations—By F. Von Voigtlander

Introduction

IN PART I of this paper entitled "Operating Features and Basic Calculations," by W. C. Champe, a discussion of the application of Petersen coils to power systems for the purpose of improving service through the prevention of outages and interruptions due to ground faults is presented. It is the purpose of part II of this paper to present in some detail the method of computation and analysis for the prediction of the behavior of a power system under the condition of single line-to-ground fault with the neutral of the system grounded through a resonant neutral grounding reactor or Petersen coil.

For purposes of discussion, a representative type of power system to which a Petersen coil could be successfully applied is set up and the solution of the required range of reactance and rating of the coil together with resultant ground-fault currents in various branches of the system is presented for two different conditions of system operation. On the basis of the most severe single line-to-ground fault conditions, with the coil applied to the system, the voltages appearing between the phase conductors and ground at the fault and at a station remote from the fault are calculated. To demonstrate the effectiveness of the Petersen coil, results of computations are also given for the fault currents and voltages which the system would experience with the neutral totally isolated. The relative accuracy of the computations and permissible approximations are discussed and an outline of field tests to corroborate design computations and to later check the performance of the Petersen coil after installation is presented.

In the consideration of the application of Petersen coils to a power system, it is assumed here that operating records on the system under consideration have shown that a sufficiently large number of service interruptions due to transitory ground faults have been experienced to warrant the expense of the application of Petersen coils to reduce troubles from this cause. If, for example, 75 per cent of all outages were due to transitory ground faults, a Petersen-coil installation which was, say, only 50 per cent effective, would still provide considerable relief. However, if only 25 per cent of all outages were due to transitory ground faults, the coil installation would have to be much more effective to be justified.

It is of importance, therefore, to carefully analyze the performance of the power system to determine in advance the range of ground fault currents, the relative importance of the in-phase components, and the range of dynamic voltages to ground that may reasonably be expected in operation. These data will serve not only as a basis for the design of the coils, but also to evaluate the probable effectiveness of their installation.

Analysis of a Simple Radial System of Moderate Capacity and Voltage

A general type of isolated-neutral power system has been taken for consideration, consisting of a simple radial network of moderate capacity and voltage, with lines of ordinary wood-pole construction without ground wires and with the conductors sufficiently transposed to keep the normal residual voltage within reasonable limits. This system has been selected as an example of a type to which a Petersen coil can be applied with reasonable assurance of satisfactory results with the possible exception of complete reduction of system overvoltages.

This system shown in figure 1 consists of 33-kv single-circuit, three-phase, 60-cycle lines using wood-pole construction of uniform configuration with number 1/0 copper conductors throughout. The system is supplied by three 1,500-kva generators through a 4,500-kva step-up transformer bank supplying a number of delta-connected transformers along 210 miles of line. All transformers are assumed to be of the two-winding type, delta-delta connected, except at the generating station, which is delta on the generator side but wye on the 33-kv side.

Since the conductors are of reasonable size and the voltage is moderate, corona is no factor and can be eliminated from consideration. Resistance of the conductors could well enough be neglected except that the in-phase component of fault current and line-to-ground voltages are to be computed. It would be expected, however, that the in-phase components of ground-fault current would be too small to sustain the ground-fault arc. The Petersen coil, therefore, would be quite effective in arc extinction if located at any point on this system, but to obtain the maximum effectiveness, particularly in the reduction of system line-to-ground overvoltages for ground faults at all locations, the coil should be located at the capacitance center of the system, which in this case would be at station *B*. However, practical considerations would probably dictate the location of the coil at the generating station *A*, where a neutral connection of adequate capacity is already available, though in this location the coil would have but limited effect in the reduction of system overvoltages for ground faults at or near the generating station.

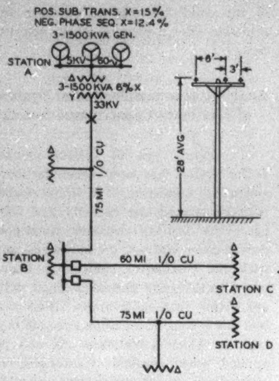

Figure 1. System diagram

Using symmetrical components, the system is divided into its component positive-, negative-, and zero-phase-sequence networks. The impedances of the generators and the transformer bank are known from manufacturer's data which are reduced to ohms on the common voltage base in the normal manner. The electrical characteristics of the transmission lines can be conveniently determined by reference to the tables and curves of Engineering Report No. 37 of the Joint Subcommittee.[3] If test data are not available, earth resistivity may be approximated from the data given in an AIEE paper by R. H. Card.[4] Whenever possible, test data in the vicinity of the system being studied should be used, as earth resistivity may vary quite widely over comparatively small geographical areas and so result in considerable variation in the zero-phase-sequence impedance of the lines. It will be taken here as 50 meter-ohms, a reasonable value for example, in the Great Lakes region. Using the formulas given in tables 2 and 5 and the curves of figures 14, 15, and 16 of Report No. 37, the positive-, negative-, and zero-phase-sequence impedances and admittances of the lines are found as shown in the appendix hereto.

PETERSEN-COIL COMPUTATIONS

Having set up the component networks, a fault location for worst conditions should be chosen. In this case, the worst location for a ground fault would be at the generating station A, as this would give the greatest residual current in the system, would require the maximum current and minimum impedance in the Petersen coil at A, and would result in maximum voltages to ground on the system, these voltages appearing at station D. It is of interest to note that the behavior of this system under these conditions is essentially that of an isolated-neutral system with the exception that only the in-phase components of current appear in the fault and in the positive- and negative-phase-sequence networks.

The required reactance of the Petersen coil can be found directly from the zero-sequence impedance of the system since under fault conditions, the coil is in parallel with this impedance and it is desired to make the reactance of the coil equal in magnitude to the quadrature component of the system zero-sequence impedance, but of opposite sign. When the value of the coil reactance is established, the resultant currents in the fault and in the coil and the component networks can readily be found. The results of these computations carried out in the appendix hereto are summarized in table I. To demonstrate the effectiveness of the Petersen coil the values obtained for the neutral totally isolated are also shown.

The required range of the Petersen coil is now established as 559 to 812 ohms with a capacity of 34 to 23 amperes, respectively. To provide for system growth, changes in operating conditions, and a reasonable factor of safety, this range of values would be extended somewhat both above and below these limits, as necessitated by local requirements. Since the coil would be required to furnish its rated current only for the duration of the fault, the basis for its time rating would depend largely on local methods for clearing permanent ground faults. Time ratings of either ten minutes, 30 minutes, or continuous are usually specified.

It is of interest to consider whether retuning the Petersen coil is likely to be essential if the branch B-C, which constitutes 28 per cent of the entire system line mileage, is switched off when the coil is tuned for the complete system. With the coil tuned for the complete system, the neutral impedance is $+j1{,}691$ ohms. With the section B-C switched off, the system zero sequence impedance becomes $43-j2{,}450$ ohms. The coil tuned for the complete system would then overcompensate the remaining lines, and a lagging current of $0.39-j10.3$ amperes would appear in the fault. However, even though the coil were not retuned, it would still effect a 57 per cent reduction in fault current and would probably result in fairly effective arc extinguishing for the majority of faults though, of course, satisfactory performance without retuning could be confirmed only by experience.

SYSTEM VOLTAGES TO GROUND

For this assumed system the Petersen coil located at the generating station would be very effective in the reduction of ground fault currents to such low values that the arcs would be self-extinguishing. It is therefore of interest to note the dynamic voltages to ground which may be expected to appear on the conductors for single line-to-ground faults on the system. Precise computation of these voltages is beyond the scope of this paper, but reasonably accurate results for most practical cases can be obtained by applying symmetrical components in the usual manner, neglecting the effects of the load and balanced currents, but including circuit resistances.

Solving for the sequence currents and their distribution in the complete system including the Petersen coil, assuming the fault to ground on phase Z at station A as before, the voltage drops in the sequence networks can be computed to the point where the system voltages are desired. The resistive components have been included to demonstrate the difference in voltages to ground of the nonfaulted phases at points remote from the fault, caused by the introduction of the quadrature component of zero-sequence voltage.

The results of these voltage computations as carried out in the appendix hereto are given in table II. The values for

Table I. System Impedances and Fault Currents

	Complete System		Section B-C Switched Off	
	Petersen Coil	Neutral Isolated	Petersen Coil	Neutral Isolated
Positive-phase-sequence impedance (ohms)	$+j50.8$	$+j50.8$	$+j50.8$	$+j50.8$
Negative-phase-sequence impedance (ohms)	$+j44.5$	$+j44.5$	$+j44.5$	$+j44.5$
Zero-phase-sequence impedance (ohms)	57,900	$49-j1{,}690$	139,700	$43-j2{,}450$
Total equivalent impedance (ohms)	57,900	$49-j1{,}595$	139,700	$43-j2{,}355$
Ground fault current (amperes)	0.99	$1.11+j35.8$	0.41	$0.44+j24.3$
Petersen-coil current (amperes)	$-j33.8$		$-j23.4$	
Petersen-coil impedance (ohms)	$+j559$		$+j812$	

the neutral totally isolated are also shown for comparison.

Figure 2 shows the vector relations of the system voltages at station *A* (the location of the fault) and at station *D* (remote from the fault). The ground fault is on phase *Z* so that its potential at the fault is reduced to ground potential or zero. The voltages of phases *X* and *Y* to ground then become equal to the line-to-line voltages. At station *D*, 150 miles away, the line-to-line voltage triangle $X'Y'Z'$ is equal to and in phase with the line-to-line voltage triangle XYZ (load and balanced components having been neglected), but $X'Y'Z'$ in this case has been displaced upwardly and to the right. At station *A*, the line-to-line-and line-to-ground-voltage triangles coincide. At station *D* the line-to-line-voltage triangle $X'Y'Z'$ is displaced from the line-to-ground voltage triangle $X'Y'G$ due to the rise and line-angle shift of zero-sequence voltage progressing along the line from the fault, so that the magnitudes of the voltages to ground at *D* no longer equal those at *A*.

These voltages given in table II are the dynamic voltages to ground sustained by the system for the duration of the fault. Coincident with the fault, tests have shown that voltage transients of the order of 100 kv crest may also be experienced. These voltages, both dynamic and transient, are *not* the result of the operation of the properly engineered modern Petersen coil, but are an attribute of operation with a flexible neutral.

The effect of the Petersen coil in reducing the dynamic voltages which may be experienced during ground faults deserves note, as by means of the coil these voltages may be limited to values safely within the ratings of equipment connected from line-to-ground, which otherwise might be damaged.

The voltages as computed for this system represent the maximum range of dynamic voltages that might be expected because the fault has been placed at the Petersen coil which is at one end of the system so that in this case it has had but limited effect in the reduction of system overvoltages. If a fault location had been chosen elsewhere on the 33-kv system, a greater reduction in these overvoltages would have been effected. The maximum reduction would be experienced for faults at station *D*, as the lagging reactive current from the Petersen coil would then flow out onto the system to balance the leading capacitance current from the lines and thereby reduce the capacitance current flowing through the system series impedance. If Petersen coils were located, for example, at both stations *A* and *D*, dynamic overvoltages to ground on this system would be eliminated, that is, they would be reduced to line-to-line voltages when one phase was grounded.

Accuracy of Computations

In making calculations of power-system fault currents and voltages, the question naturally arises as to what accuracy can reasonably be expected of the results. With the modern methods of computation and with convenient and accurate means available for calculating system impedance characteristics, it will generally be found that the greatest unknowns in the computations are the effects of the various empirical factors upon the system admittance to ground, earth resistivity, and the effects of harmonics.

Tests on a number of systems over quite a range of voltages and configurations have shown the measured values of admittances of short sections of line to be consistently greater than the calculated values. In the author's experience, these differences on well-maintained systems where complete data is available and where corona is no factor, have been found to uniformly approximate ten per cent. Where test data are not available, the calculated values of unit admittance may be increased by this amount with resultant improvement in accuracy. On high-voltage systems where corona is a factor, the calculation of the system admittance is complicated by the necessity for evaluating the capacitance-shunting resistance equivalent of the corona loss, which itself is subject to considerable variation, and by the direct effect of corona on the system capacitance. In making computations in which corona must be considered, it is well to bear in mind also that the corona loss of a system with one conductor grounded is considerably greater than the balanced corona, as has been discussed in various articles on this subject.

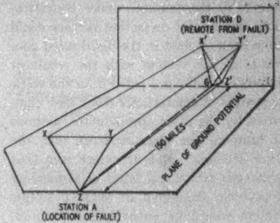

Figure 2. Voltage diagram

Table II. Voltages to Ground for Complete System

		Phase X	Phase Y	Phase Z
At station *A* (fault location)	Petersen coil	33,000	33,000	0
	Neutral isolated	34,500	34,500	0
At station *D* (remote from fault)	Petersen coil	35,000	35,800	2,780
	Neutral isolated	36,500	37,500	2,880

If test data are available on the earth resistivity in the territory of the system, the errors due to this factor can be largely eliminated. Where no test data are available, but reasonably accurate estimates can be made, resulting errors in computing zero-sequence impedance may be of the order of ten per cent to 30 per cent. In extreme cases of unsuspected high earth resistivity though, these errors may be greater. However, large errors in zero-sequence impedance will generally not result in proportional errors in fault-current computation because the impedance is usually a smaller factor than the admittance.

All computations involving residual current are subject to errors due to the effects of harmonics in that current. Errors in results due to these harmonics do not usually in practice become as serious as might be expected. However, the possibility of the system being resonant to certain harmonics should not be overlooked, as this can be predicted fairly well from the impedance characteristics of the system.

Tests have shown that when all pertinent factors have been taken into account and reasonably accurate basic data are available, the ground fault currents and voltages can be calculated to within less than ten per cent of their measured values. The in-phase component of system fault currents, however, is subject to considerably greater error, because it is usually a small portion of the total current and is displaced from it by approximately 90 degrees, and because of the difficulty of accurately evaluating leakage and corona.

Permissible Approximations

In any more or less extended analysis of this sort it is, of course, desirable to make such approximations and simplifications as may be made without introducing errors inconsistent with the expected accuracy of the computation. For instance, in the example under consideration, hyperbolic correction factors could have been applied to the distributed constants of the system with the resultant difference of about 1 per cent in the value of system characteristics from that computed by the more approximate methods of lumping the characteristics into the three sections. It is therefore seen that while the additional labor of employing hyperbolic functions in such a simple system would be entirely unwarranted, in an extensive system, it might easily be justified, particularly if the effects of loads and balanced currents were to be considered.

As shown in table I, the magnitude of fault current is but very slightly affected by the resistances in this system and obviously for calculations in which it was not necessary to determine the in-phase component, the resistances could easily be neglected. However for example, if the conductors had a high resistance-to-reactance ratio, or if considerable corona was present, the resistance might easily have become an important factor.

There is always the danger of one attempting to apply the results of approximation in simple cases to complex cases where such approximations may lead to great error. It is, therefore, always advisable to carefully consider the system involved and to observe the relative magnitudes of the various factors which it is proposed to neglect. Approximations, particularly of the rule-of-thumb type, frequently serve to magnify the errors, especially on large and extensive systems, to a point where the computations may be of little value. Considerable experience has shown it advisable to set up the component phase sequence networks completely so that the relative magnitudes of the various factors become apparent before attempting to resort to approximations.

In spite of the various doubtful factors which may come into computations of this sort, the results obtained when based on reasonably accurate system data and on well-considered choice of intangible factors, will amply bear out and justify the efforts put into the work. Although a plus or minus ten per cent error may seem large compared to the accuracy of many other electrical computations, this error becomes small when compared to the error that results from attempting to design a Petersen-coil system without making these detailed calculations.

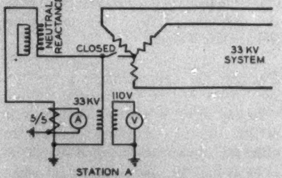

Figure 3. Diagram of connections for tuning neutral reactance

Field Tests to Corroborate Design Computation

Since the installation of a Petersen coil generally involves an appreciable investment, if there is much doubt about the accuracy of the power-system data earth resistivity, etc., field tests to check the results of calculations are easily justified. Before making any tests, however, complete computations should be made in advance, based on the best data obtainable, so that some approximation of the range of expected results can be obtained. By the use of a transformer of the required reactance and rating, a Petersen coil can be simulated if a neutral connection can be made available so that the tests can be made under conditions closely approximating expected operating conditions.

Figure 3 shows a schematic diagram of test connections for tuning a system to resonance with the neutral reactance. Tuning can be accomplished either by varying the reactance in the neutral or by varying the system admittance by switching lines on or off the system as required.

Figure 4 shows schematic diagrams for making the necessary simultaneous surge-recorder, oscillograph and indicating-instrument measurements of neutral voltage displacement, neutral current, fault current, phase angle between fault current and residual voltage, phase-to-phase voltages, phase-to-ground voltages, residual voltages, and transients during the time of the application of solid ground faults and ground fault arcs at the proposed Petersen-coil location (station *A*) and at the remote point of the system (station *D*).

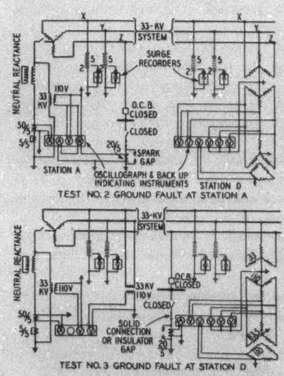

Figure 4. Diagram of system ground-fault test connections

From the results of such tests as outlined, sufficient data can be made available to accurately establish the system's electrical ground-fault characteristics so that the computations can be checked. During the ground-fault tests, simultaneous observations may be made on neighboring communication circuits and factors such as mutual coupling and earth resistivity may be determined.

Field Tests to Check Petersen-Coil Performance

Field tests to check performance of the installation after the Petersen coil has been designed, manufactured, and installed on the system are essential to check the performance of the coil and the system under the various operating conditions likely to be encountered. The setup for these tests would be quite similar to the setup for tests to determine the system's electrical characteristics as outlined in figures 3 and 4. The tests would be carried on in quite a similar manner with the possible addition of tests at various other locations and under perhaps a wider range of operating conditions. If any form of automatic clearing of permanent ground faults has been installed, the functioning of such equipment should also be observed and checked by test.

Tests with the coil tuned for the entire system but with parts of the system switched off should also be made to determine the necessity for precise tuning and to determine the stability of arcs of various current values on the system. In addition, tests can be made of bushing-type current-transformer ratios, effectiveness of relays, effectiveness of lightning arresters, rod gaps, protector tubes, etc., so that a large amount of very useful data can be obtained on the characteristics of the system during abnormal operating conditions.

With the system in reasonably good operating condition all the tests as outlined should be carried on without adversely affecting service on the system during the time of the tests. Since only single-phase ground faults are involved, barring equipment failures such as lightning-arrester failures, bushing flashovers, and similar failures, which are commonly experienced on isolated-neutral systems during conditions of single line-to-ground faults, no appreciable effect on the phase-to-phase voltages will ordinarily be noticed.

Conclusions

1. Petersen coils have a definite field of application in improving the performance reliability of a transmission system. However, there are many factors which need to be considered in determining their probable effectiveness in a given installation, including:

(*a*). The relative number of transitory single line-to-ground faults vs. faults from all causes.

(*b*). Magnitude of the in-phase component of fault current.

(*c*). Overvoltages to ground that may occur with faults at different locations.

2. The electrical circuit characteristics of the system during line-to-ground faults must be determined to properly ascertain the coil characteristics. These can be calculated reasonably accurately and substantiated by test.

3. The layout of the system has an important effect on the voltage and current conditions during ground faults. This is especially true if a long section of small conductor line connects two large capacitance centers of the system.

4. Dynamic overvoltages to ground considerably in excess of line-to-line voltages may be experienced during single line-to-ground faults even though the system is equipped with a properly tuned Petersen coil.

5. The ground-fault characteristics of a system both before and after the installation of Petersen coils may be confirmed by relatively simple tests which need not interfere with normal operating conditions. These tests should include measurements of earth resistivity, overvoltages, effectiveness of protective equipment, effect on paralleling communication systems, etc.

Appendix

System Impedances

The positive- and negative-phase-sequence networks of the assumed system are shown in figures 5 and 6 for the fault at station *A*. The portions not shown carry no current. The zero-phase-sequence-line impedances may be determined from table 2 and curves of figures 15 and 16 of Engineering Report No. 37 of the Joint Subcommittee:[3]

$Z_{11} = 0.697 + j1.46$ for 1/0 copper conductors with mean spacing of 5.45 feet and earth resistivity of 50 meter-ohms

$Z_{12} = 0.09 + j0.71$

$Z_0 = Z_{11} + 2Z_{12}$

$= 0.877 + j2.88$ ohms per mile, zero-sequence impedance

The zero-sequence line admittance may be determined from table 5 and figure 14 of Engineering Report No. 37 of the Joint Subcommittee.[3] With line conductors as above at 28 feet average height:

$$C_0 = \frac{0.179}{2\log\frac{4h}{d} + 2\left(2\log\sqrt{\frac{4h^2}{S_{12}^2}+1}\right)}$$

$= 0.00683$ microfarad per mile, zero-sequence capacitance

$= -j388{,}000$ ohms per mile, shunt impedance or $+j2.57 \times 10^{-6}$ mhos per mile, shunt admittance

Zero-sequence network is as shown in figure 7 and reduces to $Z_0 = 49 - j1{,}690$ ohms.

Petersen-Coil Computations

Complete System

Admittance corresponding to above zero-sequence impedance is $Y_0 = (17.3 + j591) \times 10^{-6}$ mho.

Neutral admittance should then $= Y_{PT} = -j591 \times 10^{-6}$ mho or $X_{PT} = +j1{,}691.5$ ohms.

Petersen-coil reactance $(X_P) = {}^1/_3(X_{PT} - X_{TR}) = {}^1/_3(+j1{,}691.5 - j14.5) = +j559$ ohms.

The zero-sequence impedance of the system including the Petersen coil then reduces to 57,900 ohms and the fault current becomes

$$I_f = \frac{33{,}000\sqrt{3}}{57{,}900 + j50.8 + j44.5}$$

Neglecting the *j* terms as insignificant,

$I_f = 0.99$ ampere

The current in the coil then is found to be $-j33.8$ amperes and the current in the zero-sequence network becomes $0.33 + j11.27$ amperes.

Section B-C Switched Off

With section *B-C* switched off the zero-sequence network becomes as shown in figure 8 which reduces to

$Z_0 = 43 - j2{,}450$ ohms

or

$Y_0 = (7.16 + j408)\ 10^{-6}$ mho

Neutral admittance should then $= Y_{PT} = -j408 \times 10^{-6}$ mhos or $X_{PT} = +j2{,}450$ ohms. Petersen-coil reactance $(X_P) = {}^1/_3(+j2{,}450 - j14.5) = +j812$ ohms.

The zero-sequence impedance of the system including the Petersen coil then reduces to 139,700 ohms and the fault current becomes 0.41 amperes, with a coil current of $-j23.4$ amperes.

With the coil tuned for the complete system but with section *B-C* switched off the zero-sequence impedance becomes $Z_0 = 213.5 + j5{,}460$ ohms, and

$$I_f = \frac{33{,}000\sqrt{3}}{213.5 + j5{,}460 + j50.8 + j44.5}$$

$= 0.39 - j10.3$ amperes

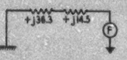

Figure 5. Positive-phase-sequence network

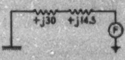

Figure 6. Negative-phase-sequence network

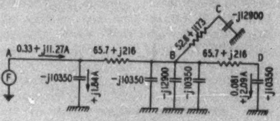

Figure 7. Zero-phase-sequence network for complete system

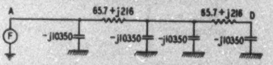

Figure 8. Zero-phase-sequence network with line section B-C switched off

Computation of Voltages

From the coil computations the current in the zero-sequence network is found to be $0.33 + j11.27$ amperes. Current distribution in the network gives $+j1.84$ amperes through the admittance at *A* and $0.081 + j2.09$ amperes through the admittance at *D*.

Voltages at the Fault (Station *A*)

Positive sequence:

$$E_1 = \frac{33{,}000}{\sqrt{3}} - (0.33 \times j50.8)$$
$$= 19{,}050 - j17 \text{ volts}$$

Negative sequence:

$$E_2 = 0 - (0.33 \times j44.5) = 0 - j15 \text{ volts}$$

Zero sequence:

$$E_0 = 0 - (+j1.84 \times -j10{,}350)$$
$$= -19{,}050 \text{ volts}$$

From symmetrical components and neglecting the above *j* voltage terms as insignificant:

$E_x = a^2E_1 + aE_2 + E_0$

$E_y = aE_1 + a^2E_2 + E_0$

$E_z = E_1 + E_2 + E_0$

where $a = -0.5 + j0.866$ and $a^2 = -0.5 - j0.866$

Phase X to Ground	Phase Y to Ground	Phase Z to Ground
$-9{,}525 - j16{,}500$	$-9{,}525 + j16{,}500$	19,050
0	0	0
−19,050	−19,050	−19,050
$-28{,}575 - j16{,}500$	$-28{,}575 + j16{,}500$	0
33,000 volts	33,000 volts	0 volt

Voltages at Station *D* for Fault at *A*

Positive- and negative-sequence voltages at *D* will be the same as at the fault (see above) as these two networks are considered as carrying no current between A and D in this case.

$$\text{Zero-sequence voltage} = 0 - [(0.081 + j2.09) \times -j10{,}350] = -21{,}700 + j840 \text{ volts}$$

$$\textbf{Phase } X \textbf{ to ground} \left\{ \begin{array}{c} -9{,}525 - j16{,}500 \\ 0 \\ -21{,}700 + j840 \\ \hline -31{,}225 - j15{,}660 \\ 35{,}000 \text{ volts} \end{array} \right.$$

$$\textbf{Phase } Y \textbf{ to ground} \left\{ \begin{array}{c} -9{,}525 + j16{,}500 \\ 0 \\ -21{,}700 + j840 \\ \hline -31{,}225 + j17{,}340 \\ 35{,}800 \text{ volts} \end{array} \right.$$

$$\textbf{Phase } Z \textbf{ to ground} \left\{ \begin{array}{c} 19{,}050 \\ 0 \\ -21{,}700 + j840 \\ \hline -2{,}650 + j840 \\ 2{,}780 \text{ volts} \end{array} \right.$$

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Discussion

H. M. Rankin and **R. E. Neidig**, nonmember (Metropolitan Edison Company, Reading, Pa.): *Part I—Operating Features and Basic Calculations*—by W. C. Champe. The author has covered the ground in a brief but clear and instructive manner. There are, however, a few points on which we wish to comment, as follows:

Lightning Arrester Rating. The author says that with multiple coil installations it may be possible to reduce the rating of lightning arrester below line-to-line voltage. We believe such practice would be questionable, as sectionalizing the system might readily leave any one section isolated with one coil, and the high impedance of the coil would maintain line-to-line voltage to ground on two conductors if the third were grounded.

Advantages of Petersen Coil. Records which we have taken with an automatic oscillograph have never indicated higher voltage to ground than 1.7 times normal line-to-neutral voltage. Our equipment is designed for voltage to ground equal to normal line-to-line voltage.

Transpositions. On existing systems, particularly some of the older ones, it may be difficult and expensive to obtain the proper number of transpositions. We have found it possible in some cases to balance one section of line against another section having a different arrangement of conductors. This, of course, can only be done providing it is possible, when one of these lines is out of service, to cut the residual current to a satisfactory value by detuning the coil still keeping within limits of satisfactory coil operation.

Number of Coils Required. It would seem that the number of coils would depend principally on the possible sectionalized operation of the system rather than on relaying requirements. We are considering the installation of two coils on a system having at present grounding transformers at four locations. The two coils would be installed in the neutrals of two of the grounding transformers and equipped with short-circuiting switches. At the other two locations, we are proposing to simply install neutral grounding switches with suitable automatic control equipment to operate similarly to the short-circuiting switches used with the Petersen coils.

Accessory Equipment. The author states that if the Petersen coil is placed in the neutral of a transformer bank used for grounding purposes only, switching equipment in series with the coil can be omitted. We believe this is somewhat questionable as it might be desirable to use the grounding transformer as such during maintenance work on the coil, in which event means of disconnecting the coil and grounding the neutral would be necessary.

In our installations, we have provided an operation counter as well as alarm relay and graphic ammeter. We have found this advantageous as the chart speed of the ammeter is not great enough to separate the lines drawn by successive operations. For instance, in a recent storm we had five operations of a Petersen coil within a period of 12 seconds. In this particular case, an operator happened to be watching the ammeter and saw the five operations of the needle, which indicated different values of current for each operation although there was only one line registered on the chart. The counter recorded all five operations.

Part II—System Analysis and Computations—by F. Von Voigtlander. Our only comments on Mr. Von Voigtlander's very instructive paper are in the nature of operating results, as follows:

The Metropolitan Edison Company has installed three Petersen coils—one at Reading, Pa., one at Middletown, Pa., on its 66-kv system, and one at Holtwood, Pa., on the system of the Pennsylvania Water & Power Company, with whom they are interconnected. These coils operate on a total of 308 miles of circuit and are tuned for 112, 95, and 101 miles, respectively. They were placed in operation in October 1937, and up to June 16, 1938, the record is briefly as follows: Total faults on 66-kv system—72. Transient faults—64, or 89 per cent of total. Transient faults cleared by Petersen coils—50, which is 78.2 per cent of all transient faults, or 69.5 per cent of all faults on the 66-kv system.

Field Tests. Staged tests were made on this system under various conditions, the worst case being with 25 per cent of the lines for which the coils were tuned disconnected but with no retuning of the coils. These tests were made by establishing an arc to ground across a string of suspension insulators by means of a fuse, and were in all cases successfully cleared within times ranging from one-half cycle to seven cycles. In recent storms, lightning flashovers have also been successfully extinguished with as much as 25 per cent of the line mileage for which the coils were tuned disconnected.

A. W. Gothberg (Public Service Electric and Gas Company, Newark, N. J.): In the paper by W. C. Champe and F. Von Voightlander on "System Analysis for Petersen Coil Application," it is stated that in considering the installation of Petersen coils it should be kept firmly in mind that they have no effect on phase-to-phase faults as such.

This limitation is a severe handicap to an effective installation of a Petersen coil on a system having an appreciable amount of wood-pole transmission. Figures have been published indicating that as many as 75 per cent of the faults on a transmission system are phase-to-phase and as many as 85 per cent of all faults are due to lightning. This clearly indicates that lightning is causing a majority of all faults and on wood-pole lines it is admitted that most of them result in phase-to-phase faults.

A paper has been published in the January 1937 issue of ELECTRICAL ENGINEERING entitled "Lightning Protection for Transmission Lines," in which a wood-pole-line design was presented which eliminated or reduced the number of phase-to-phase faults due to lightning. Although this design used lightning-protector tubes on one phase of a three-phase line, it can be made particularly applicable to a system using a Petersen coil by simply substituting a suitable gap for the lightning-protector tubes.

In brief, it would consist of making one phase of each circuit a shielding conductor by locating it so that the remaining phases will be shielded from direct lightning strokes. Lightning currents could be drained from the shielding conductor by means of suitable gaps connected to ground. This would result in a single-phase fault which could be cleared by a Petersen coil. The remaining phases could be isolated from the shielding conductor and the gap ground circuits by insulation sufficient to prevent phase-to-phase faults. On double-circuit lines, gaps could be installed on the two top shielding conductors. It is important that the same phase of each circuit be on the top arm so that in the event of a stroke involving the two top conductors, a phase-to-phase fault does not result. Methods of determining the amount of insulation required on the pole top and the frequency of the gaps along the line can be found in the paper mentioned above.

Limited experience with this type of construction has shown its ability to eliminate phase-to-phase faults.

This proposed design of wood-pole line should make the effectiveness of a Petersen coil on a wood-pole line approach or equal to the effectiveness of a Petersen coil on a steel tower line equipped with ground wires. It would overcome one of the most serious limitations in the application of a Petersen coil to a system having wood-pole transmission.

W. D. Hardaway (Public Service Company of Colorado, Denver): The authors have presented in convenient and useful form, methods for making basic calculations for the application of Petersen coils to transmission systems. They mention the possibility of unsuccessful coil operation due to corona current when applied to high-voltage systems operating with small conductors or at high altitude. In this connection it would be of interest to have stated the maximum values of corona current which are being successfully interrupted by Petersen-coil operation. On the Colorado 100-kv system, fault currents of from 20 to 25 amperes are quenched without difficulty, and North and Eaton report fault currents of from 38 to 50 amperes on the Michigan 140-kv installation (ELEC. ENG., vol. 53, page 67). It appears improbable that these values will be greatly exceeded on other systems.

The authors have called attention to the difficulty in properly relaying an isolated neutral system for single-phase faults to ground. Solidly grounding, with the addition of proper relays, will usually solve this problem. However, the application of a Petersen coil may be materially less expensive than this remedy, and has the further advantage of improving continuity of service. Also it appears that the Petersen coil eliminates the excessive harmonic overvoltages frequently present under arcing ground conditions on isolated-neutral systems.

E. M. Hunter (General Electric Company Schenectady, N. Y.): Continued interest is being shown in the Petersen coil as a service-protective device. At the last winter convention it was reported that there were seventeen coils in the United States, the majority of which have been applied in the last year or so. In the last few months seven more applications have been made, bringing the total to 24. Naturally, some application and operating experience is now available on these protective devices and this experience is the basis of the comments prepared on this paper.

IN-PHASE COMPONENT OF FAULT CURRENT

In part I of the paper it is stated "The in-phase component must be carefully checked to make sure that it will not be large enough to maintain the arc readily and prevent extinction by the Petersen coil."

I am sure it would be of interest if the authors would give their views on the magnitude of the in-phase component of fault current beyond which it would be undesirable to apply Petersen coils.

Although system analysis and operating experience on existing installations is limited, it indicates that on medium-voltage systems (66 kv and below), the in-phase component of the fault current should not exceed 20 to 25 amperes. This relatively low value of current apparently does not maintain the arc. This is likewise true on high-voltage systems (110 kv and above) unless the system has considerable corona loss. When there is corona and the system is extensive, then the in-phase component of current may be a problem, although in all of the applications made to date, successful arc extinction has been reported with in-phase components of current up to 50 amperes. It appears that the in-phase component of current need only to be considered if there is corona.

SYSTEM OVERVOLTAGES DURING GROUND FAULTS

In part II of the paper, an analysis is made of the overvoltages on a system (Figure 1, part II) with and without a Petersen coil in the neutral. It shows that with a ground fault on one of the phases at a point in the system adjacent to the station where the Petersen coil is located, the voltage to ground on the two healthy phases increases with distance from the fault point. The magnitude of these voltages is summarized in table II, part II.

Referring to this table, it is seen that the voltages with the Petersen coil increase approximately nine per cent above the system line-to-line voltage at a point 150 miles away from the fault. Since the calculations from which these voltages were derived neglects load current, they undoubtedly represent maximum values. Inductive reactive load current would, in general, have a tendency to reduce these voltages. It should be noted that they do not exceed the maximum permissible line-to-ground voltages recommended for protective equipment on such a system. Also, they are lower than if the neutral were free. These voltages would also be lower if the Petersen coil had been located at station *B* instead of a station *A*. System analysis has shown and operating experience has verified the fact that when overvoltages in remote sections of the system away from the ground fault are likely to be excessive, several coils correctly placed in the system will limit these overvoltages to reasonable values.

It is interesting to note that overvoltages at the point of fault with the system neutral isolated are much higher than if the neutral were grounded through a Petersen coil. For example, on this system of figure 1, part II, the voltages at station *D* for ground faults at station *D* have been calculated for the condition of (1) the system neutral isolated at *A*, (2) the system neutral solidly grounded at *A*, and (3) the system neutral grounded through a Petersen coil at *A*. This study has been made with all three generators in service and also with only one generator in service. The results of this study are summarized in table I.

Table I

Voltage to Ground	Neutral Isolated	Neutral Grounded	Petersen Coil
Three Generators in Service			
Phase *X*	38 kv	24 kv	33 kv
Phase *Y*	41 kv	22 kv	33 kv
Phase *Z*	0	0	0
One Generator in Service			
Phase *X*	41 kv	21 kv	33 kv
Phase *Y*	44 kv	20 kv	33 kv
Phase *Z*	0	0	0

With only one generator in service and the neutral isolated, the voltages are relatively high. This may be explained by the fact that on this system a condition of partial series resonance is obtained. It should be noted that with the Petersen coil in the neutral, the voltages on the two healthy phases do not exceed the system phase-to-phase voltage.

The authors also mentioned high transient overvoltages on the system at the time a ground fault occurs. I would like to have the authors' comment on this further and possibly show how these overvoltages occur and how they may be calculated. Experience indicates that if these overvoltages do occur, they are of a very short time duration, so short in fact that they do not record on a magnetic oscillograph. The magnitude also must well be within the limits of insulation strength of connected apparatus because there is no indication that they do any appreciable damage.

ACCURACY OF CALCULATIONS

The authors state that their system analysis gives results which can be substantiated by tests within a ten per cent error. The experience available to date on Petersen coil applications leads to the conclusion that the methods used by the authors are not necessarily that accurate.

For example, the in-phase component of current for the system analyzed in part II of the paper has been calculated by the authors to be 0.99 ampere. Experience on 11 33-kv applications of Petersen coils leads me to believe that this particular system would have an in-phase component of current between ten and 15 amperes. This would result from leakage on apparatus and also harmonics which appear to have been neglected or omitted from the authors' calculations. This current would vary with the weather, the higher value being noted during stormy periods.

With regard to the charging current in a ground fault which must be compensated by lagging current in the Petersen coil, the experience previously mentioned indicates that this system should have between 42 and 46 amperes instead of the 34 amperes given by the authors for charging current for the 210 miles of line. Possibly this discrepancy may be explained by the fact that in their calculations they have calculated only the zero sequence capacity of the transmission line alone and have made no allowance for the capacitance or apparatus, insulators, bushings, poles, etc. These other effects have a tendency to increase the charging current, making it higher than the current given by the formulas used by the authors.

As suggested by the authors, measurements to substantiate calculations of charging currents are advisable when experience is limited.

W. W. Lewis (General Electric Company, Schenectady, N. Y.): Messrs. Champe and Von Voigtlander give a very useful summary of the conditions that should be considered in the application of Petersen coils. I am especially interested in their discussion of the accuracy of computations on pages 14 and 15 of the preprint and wish to comment briefly on this feature.

If the tests are made carefully and all the conditions are known, and if the calculations are accurately made for the same conditions as existed in the tests, then the results of the tests and calculations should certainly check. However, usually it is not possible to know all of the conditions of either test or calculation sufficiently well to obtain better than an approximate check. A couple of examples will illustrate this point.

When the Public Service Company of Colorado were considering the application of a Petersen coil on their Shoshone-Denver line, we made some calculations, assuming the average spacing of the conductors to ground as the height at the towers minus two-thirds of the sag. This is probably a fair assumption for level stretches, but as much of this line is in the mountains and spans from hill-to-hill over gulches and canyons, the average height obtained from a profile by planimeter would probably be considerably greater than the height assumed. Allowance was made for the increase in capacitance due to corona and account was taken of the capacitance of the line insulators, apparatus, bushings, disconnecting-switch insulators, and transformer windings. The calculated fault current was 92 amperes capacitance current and 95 amperes including in-phase current due to corona, but neglecting harmonics, for the approximately 187 miles of line. The power company then made some tests in which the neutral was isolated and one conductor grounded. Fault current to ground and voltages to ground were measured with oscillograph, ammeter, and voltmeter. Tests were made at three different points on the system, namely, at the Denver and Shoshone ends and at an intermediate point, Leadville. Reduced to the same voltage basis of 95,000 volts, these tests showed, respectively, fault currents of 123, 101, and 137.5 amperes for the approximately 187 miles of line.

Finally, when the Petersen coil was put into operation, the Petersen-coil currents for ground faults at Leadville and Denver were measured at the tuned setting of the coil ("Test and Operation of Petersen Coil on 100-Kv System of Public Service Company of Colorado," by W. D. Hardaway and W. W. Lewis, ELECTRICAL ENGINEERING, June 1938, pages 295–302). The readings were 98 and 96 amperes, respectively. Thus the calculations checked fairly closely the measured Petersen-coil current but did not check at all well the measured ground fault current.

In another case, a single-circuit 115-kv line with two ground wires was tested. The line was opened up at both ends by disconnecting switches. Low-tension single-phase voltage was placed on the open-circuited line between the three conductors in multiple and ground. Voltage, current, and watts were read. From these readings the zero-sequence capacitance of the line was found to be 0.211 microfarad. The capacitance of the conductors was calculated, assuming an average spacing to ground equal to the height of the conductors at the towers minus two-thirds of the sag. This capacitance, added to that of the line insulators and disconnecting-switch insulators, and assuming no corona, gave a total zero-sequence capacitance of 0.172 microfarad. In this case the tested capacitance was 22 per cent higher than the calculated.

Either Mr. Hunter or I have made calculations for all of the Petersen coils that have been installed in this country and have made ground-fault tests after installation in most cases. From these tests we have worked out the empirical relations shown in figures 1 and 2, for single-circuit lines. These relations can be used as a guide in the selection of Petersen coils.

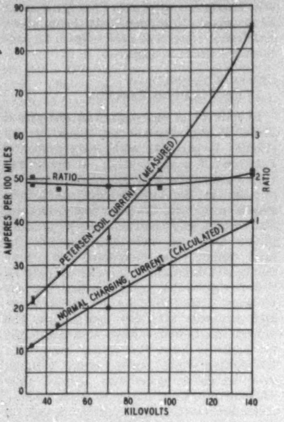


Figure 1

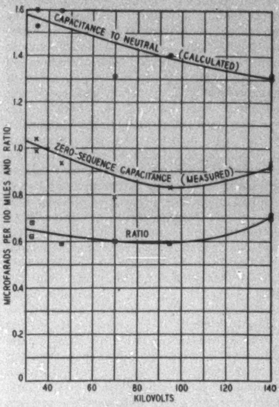
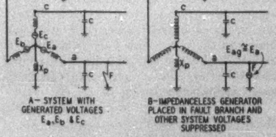


Figure 2

P. A. Jeanne (Bell Telephone Laboratories, Inc., New York, N. Y.): I have been quite interested in the recent activity in the application of Petersen coils in this country, particularly as it has been my privilege, as a member of project committee 2*J* of the joint subcommittee on development and research of the Edison Electric Institute and Bell System, to participate in studies of current-limiting devices, including the tests on the first of the more recent Petersen-coil installations in this country, made in 1931. I think that Mr. Champe and Mr. Von Voigtlander have brought out quite well most of the features requiring consideration in the application of Petersen coils. The comments which I have on the paper are made chiefly from the standpoint of clarifying certain ideas. In the introduction to Mr. Champe's paper (advance copy), the statement is made that with a Petersen-coil system the inductive reactance, most of which will be in the Petersen coil or coils, will be approximately equal to the capacitive reactance of the two unfaulted conductors to ground. I think this statement requires modification because one might assume from it that the proper reactance for the Petersen coil would be one determined from the sum of the direct capacitance to ground of the sound phases, while in fact it is determined from the sum of the direct capacitance to ground of all three phases. I think perhaps the author had in mind the fact that in a Petersen-coil system the charging current of the two sound phases equals the Petersen-coil current, a fact which might readily lead to the statement cited. Perhaps I can make this clear by reference to figure 3. Figure 3(*a*) represents a balanced three-phase power system grounded through a reactor and with a fault on phase *A*. Now if the only voltage in the system were that in phase *A* we could immediately see that for minimum current in the fault the neutral reactance should just equal in magnitude the capacitive reactance of the two sound phases to ground, as that would provide the parallel resonant condition sought. However the presence of the voltages E_b and E_c in the sound phase branches subjects them to higher voltages to ground than that across the reactor and the requirement for resonance is not immediately apparent. Now by a well-known theorem, the current in a particular branch of a network can be determined if an electromotive force equal to that which would exist across an open circuit in that branch is introduced into the branch and the other voltages in the system suppressed. If we apply this idea to the circuit under consideration, as in figure 3(*b*), the current in the fault branch will be the same as it would be with the regular voltages impressed as in figure 3(*a*). It can be seen immediately that, in the circuit of figure 3(*b*), the direct capacitances-to-ground of all three conductors are in parallel with the reactor, so that to secure the parallel resonant condition desired, the reactance of the Petersen coil must be made equal in magnitude to the capacitive reactance of the sum of the direct capacitance to ground of all *three* conductors. The same thing could of course be shown by



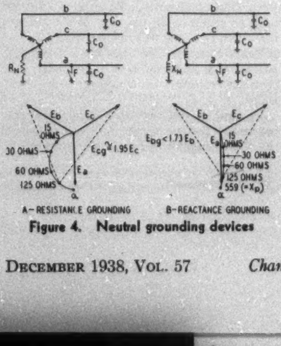
A— SYSTEM WITH GENERATED VOLTAGES E_a, E_b & E_c

B—IMPEDANCELESS GENERATOR PLACED IN FAULT BRANCH AND OTHER SYSTEM VOLTAGES SUPPRESSED

Figure 3. Petersen coil

solving the network of figure 3(*a*) for a value of reactance which would make the current through the reactor equal to the vector sum of the charging currents of the two sound phases. The authors have used the proper capacitance in their calculations and it is only the statement in the early part of the paper that I feel needs modification.

The second comment I have applies to the discussion of overvoltages under fundamental frequency conditions. While I agree that on an extensive Petersen-coil system the voltage at points remote from the fault may exceed line-to-line voltage, I think it should be made clear that the type of voltage rise the authors discuss is not peculiar to a Petersen-coil system. The effect is the well-known tendency of a long line, energized to ground at the sending end and open at the receiving end, to show a rise in voltage at the open end. If the effect is sufficiently large in the case of a Petersen coil installation to require attention, it would also probably require attention if the same system were grounded at the same point in any other manner. For example, if the system were solidly grounded at one point only, the voltage rise at points remote from the ground might be sufficient to prevent its being treated as a solidly grounded system. In this conection it is interesting to note the tendency of a Petersen coil to establish, at the point of fault, line-to-ground voltages on the two sound phases which are equal to each other and no greater than line-to-line voltage, a condition which other types of neutral impedance may fail to accomplish. Figure 4, prepared for the system assumed in part II of the paper, illustrates the point I wish to make. At the upper left is indicated a neutral resistance grounded system. If, for the sake of convenience, we assume that the neutral remains fixed and allow the point representing the earth to shift with varying voltage drop through the resistor, we can conveniently represent the variation in the line-to-ground voltages of the sound phases. As the neutral resistance is varied, the point representing the earth moves along the circular locus in the diagram in the lower left, and the potentials to the sound phases are measured from the point on the arc corresponding to the value of resistance for which the voltages are desired. This diagram shows quite clearly that with neutral resistance grounding, the line-to-ground voltages of the sound phases are unequal and one of them may considerably exceed the line-to-line voltages, as in the case of phase-*C* voltage when the neutral resistance is 125 ohms. In the right half of this figure are shown the corresponding diagrams for reactance grounding. It is seen here that as the reactance increases, the locus for the "earth" remains very close to the vector E_A and that the line voltages do not depart very much from balance. In the particular example, the actual departure is even less than shown by the diagram which was distorted somewhat for the sake of clarity. In some types of systems, however, the unbalance may be considerably greater. It is seen that as the value of reactance approaches that required for a Petersen coil, the unbalance tends to decrease, and when it is at the value corresponding to the Petersen coil the line-to-ground voltages are essentially balanced and equal to line-to-line voltage. This is an effect typical of the Petersen coil, and except as propagation effects on long lines may come into play, Petersen coils tend, under fault conditions, to maintain line-to-ground voltage on sound phases substantially equal to line-to-line voltage.



A—RESISTANCE GROUNDING B—REACTANCE GROUNDING

Figure 4. Neutral grounding devices

S. B. Griscom (nonmember; Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The authors have presented very interesting papers on operating features and computation of Petersen coils. In part I, under "Effect of System Operation," some of the pros and cons concerning the application of coils are given. It seems to me that under this heading a few other factors should be pointed out. One of these for lack of a better term will be called "adaptability to interconnection." Power systems having various types of grounding (solid, resistance, or reactance) may be metallically connected together with but slight changes in the relaying in the vicinity of the tie-in point. With a Petersen-coil-equipped system, however, it is necessary that both systems be equipped with the coils, or else use isolating transformers or their equivalent in making the connection. Thus, all new additions to the system or future interconnections may require the installation of additional equipment. The desirability for Petersen coils should also be considered in relation to the fact that American practice usually provides multiple feeds to important loads to insure continuity not only for transitory but also prolonged faults in case of physical line trouble. The trend toward higher relay and breaker speeds has greatly minimized the shock to the system for faults. A ground fault, unless very close to a full capacity solidly grounded transformer is inherently less severe than a phase-to-phase fault, and very much less severe than a three-phase fault, and the latter must be provided for in any case.

The information given under "Accuracy of Computations," part II, is very interesting. In view of the observed fact that calculated admittances are always lower than test data, I am wondering whether the authors have been able to trace this discrepancy to any commonly neglected factors, such as capacitance of insulator strings, height of vegetation, etc. I understand that in some cases, systems operating Petersen coils have found it necessary to change the compensation due to growth of corn, or the occurrence of a heavy snowfall. In the latter case it would appear more likely that snow or sleet on the conductors would be more effective in changing the zero-sequence capacitance, than a nominal depth of snow on the ground. The writer would like to ask whether any such effect has been noticed on the portion of the Consumers Power 140-kv system that is Petersen-coil equipped.

W. C. Champe and F. Von Voigtlander: Mr. A. W. Gothberg has suggested the use of a suitable grounded gap to bridge the insulator of the top conductor of a wood-pole line so that this conductor would act to shield the other two from high induced potentials. The resulting single-phase fault to ground of the combination conductor and shield wire, when discharging a high potential, would be cleared by the Petersen coil. A similar plan has been advanced by the authors and others, and also the possibility of reducing the insulation of the top conductor on steel tower lines without ground wires has been considered on systems equipped with Petersen coils. Though the authors have not had the opportunity of observing such schemes in actual use, it would appear that they would be rather successful in reducing line outages due to lightning and that they would greatly extend the field of usefulness of Petersen coils.

While on highly insulated wood-pole lines, lightning outages may be predominantly phase-to-phase faults, the fact still remains that there are thousands of miles of moderate voltage wood-pole lines in this country on which transitory ground faults are an appreciable proportion of the total faults experienced. This ratio of transitory ground faults to total faults becomes an important criterion for successful Petersen-coil installation. A ratio of transitory ground faults to total faults of as high as 50 per cent is not at all uncommon, particularly on lines of older construction. On such lines Petersen coils have proved eminently effective.

Mr. Griscom has suggested a factor aptly called by him "Adaptability to Interconnection," which is an important consideration in the application of Petersen coils to systems which may be switched onto other systems not so equipped. Where it is necessary to switch a Petersen-coil-equipped system onto such other systems, the switching must be done through transformers having a high zero-sequence impedance, if the effectiveness of the Petersen coils are to be retained, unless the interconnected system is non-grounded and is within the tuning range of the coils.

Mr. Griscom asked regarding the effect of the height of vegetation, increased conductor diameter due to snow or sleet, occurrence of heavy snowfalls, etc., on the tuning of the Consumers Power Company's 140-kv Petersen-coil system. While these factors have an effect on the system admittance to ground, such effects have not been observed to an extent sufficient to warrant retuning of the coils.

Messrs. Griscom, Hunter, and Lewis have commented on the accuracy of computations and on the fact that the observed values of system admittance to ground have always been found to be greater than calculated values. From the tests and calculations which have been made, the effects of height of vegetation, capacitance of insulator strings, etc., have not been found to be sufficient to account for these differences between observed and calculated admittances.

In the authors' experience, the quadrature components of fault current have been consistently predicted within ten per cent of measured values. In such calculations, the system admittances have been increased ten per cent above the values computed by standard methods, since, as pointed out in the paper, the authors have found the measured values to be consistently ten per cent greater than those calculated on the systems with which they are familiar. In the example in the paper, this ten per cent increase in admittance was not made, as experience on other systems may show a somewhat different relation between measured and calculated admittance. The quadrature component of fault current was calculated in the example as 34 amperes. With the admittance increased ten per cent this current component becomes 38 amperes, which could be expected to be within ten per cent of the measured value on the basis of the authors' experience and about 13 per cent on the basis of Mr. Hunter's experience. The discrepancy between the measured and calculated total ground fault current in the Colorado example cited by Mr. Lewis can probably be ascribed to the magnitude of the in-phase components, since the measured and calculated quadrature components checked quite well. It is hoped that further investigation into the differences between calculated and measured values of line admittances will be made and that the results of such investigations will be made available to members of the Institute. As observed in the paper, the computation of the in-phase components is subject to large error, especially for the smaller values, as the effect of the intangibles on the magnitude of these components then becomes larger than the effect of the system resistances.

Messrs. Hardaway and Hunter have discussed the magnitudes of the in-phase components of fault current on operating Petersen-coil systems. Mr. Hardaway reports in-phase fault current arcs of from 20 to 25 amperes are being cleared without difficulty on the Colorado 100-kv system, and points out that fault currents of from 40 to 50 amperes are being interrupted on the Michigan 140-kv system. The maximum values of in-phase component of ground fault current which can be tolerated for a given system equipped with Petersen coils has not been even approximately established, to the authors' knowledge. Experience has shown that in-phase transitory arcs up to 50 amperes are generally successfully extinguished, though during quiet conditions, such arcs have been known to continue for over 800 cycles. Wind and weather conditions have a very important effect on the behavior of arcs of this sort, and in any test to determine the range of maximum current which can be extinguished wind and weather conditions would have to be considered.

Tests have been made on long and extensive medium-voltage systems where the resistance to reactance ratio was high but corona was no factor, in which sufficiently large in-phase currents were measured to make questionable the successful operation of Petersen coils. On extensive high-voltage systems where corona was present, in-phase components as high as 160 amperes have been calculated. However, in the majority of systems encountered in practice, particularly at voltages under 66 kv, the in-phase components are probably of only secondary importance.

Messrs. Hunter and Jeanne made some very interesting comparisons of system voltages to ground for isolated-neutral, grounded-neutral, and Petersen-coil-equipped systems. In Mr. Hunter's tabulation it is interesting to note for the case of the isolated neutral that as the number of generators on the system is reduced from three to one, the positive- and negative-phase-sequence impedances of the system increase to a point where they are of appreciable magnitude compared to the zero-sequence impedance, resulting in considerably higher system voltages to ground. Incidentally phase-to-phase voltages would also be found to be considerably higher than normal for the assumed ground-fault conditions.

Mr. Hunter asked for the authors' further comment on the high transient overvoltages coincident with line-to-ground faults. The authors believe these transients to be of the nature of switching surges, due largely to the rearrangement of line capacitances to ground incident to the occurrence of the fault. As Mr. Hunter observed, these transients are of too short duration to be accurately recorded on conventional magnetic oscillographs. The authors have not carried out any extensive investigation of these transients, either mathematically or in tests, but have observed what magnitudes may be experienced by the use of surge recorders or klydonographs on systems during ground-fault tests. Such values as have been observed, though of surprising magnitude, have been found to be well within the limits of insulation strength of connected apparatus and equipment. No doubt these transients also are mitigated by the Petersen coils.

Mr. Jeanne observed that the reactance of Petersen coils is determined from the direct capacitance to ground of all three phases of a system, and not from that of only the two unfaulted conductors as might be inferred from the paper. The authors of course agree with this, having the point in mind, as Mr. Jeanne mentioned, that the coil current is equal to the quadrature component of the charging current of the two unfaulted conductors. Mr. Jeanne's discussion and diagrams of these relations and the effect of neutral resistance and reactance on the system voltages to ground are very interesting. They demonstrate the points in question very well, and are worthy of detailed study by everyone interested in this subject.

The empirical relations which Mr. Lewis has shown between the calculated charging current and the measured Petersen-coil current, and the relations between the calculated capacitance to neutral and the zero-sequence capacitance are based on a very wide experience in this field and on a considerable number of tests. They should, therefore, be a useful guide in working out these values for any given power system.

The authors agree with Messrs. Rankin and Neidig regarding the use of lightning arresters rated at less than line-to-line voltage on Petersen-coil systems. However, there appears to be some misinterpretation on the matter of permissible lightning-arrester ratings, as the point intended in the paper was that multiple Petersen coils may permit the use of lightning arresters with a rating lower than those which might be used for a totally isolated system. That is, on a totally isolated neutral system, certain of the lightning arresters might well be required to have ratings considerably in excess of that based on normal line-to-line voltage, whereas on a system equipped with multiple Petersen coils, the rating based on normal line-to-line voltage would probably be adequate. For example, normally on a 33-kv isolated-neutral system, 37-kv ligntning arresters would be used. However, if it were a large and extensive system, 43-kv arresters might be required at some stations to interrupt high dynamic overvoltages. If this system were equipped with only one Petersen coil, a number of the 43-kv arresters might still be required, but if it were equipped with multiple Petersen coils, the use of 37-kv arresters might well be feasible throughout.

It is presumed that the maximum recorded voltages mentioned as never having been found to exceed 1.7 times normal line neutral voltage, refer to a system equipped with multiple Petersen coils.

Transpositions are of considerable importance in a Petersen coil application, as their proper use reduces the normal residual voltage of the system and thereby limits the continual current through the coils and restricts neutral displacement during normal system conditions.

The number of coils required on a system would be a function of the possible sectionalized operation of the system. Whether or not all grounding transformers should be equipped with Petersen coils would be a matter for individual engineering judgment on the system involved.

Since there is but little maintenance work to be done in connection with Petersen coils, it would appear difficult to justify any but the simplest means of by-passing the coil and grounding the neutral. Justification for additional equipment would depend largely on local conditions.

The comments of Messrs. Rankin and Neidig on the results of operation with a 66-kv system equipped with three Petersen coils show the effectiveness of this installation. The results which have been obtained with as much as 25 per cent of the system for which the coils were tuned, disconnected without retuning the coils are interesting, particularly as all cases of faults were successfully cleared in comparatively short times.

In closing, the authors extend their appreciation for the many interesting discussions of this paper that were submitted.

The Fundamental Principles of Fluorescence

By GORTON R. FONDA
NONMEMBER AIEE

FLUORESCENCE is a property of many substances. It consists of an emission of light under excitation by radiation whose wave length falls within a critical absorption band. When the emission persists for an appreciable period after the existing radiation has been extinguished, it is known as phosphorescence.

At a low order of brightness fluorescence is encountered in a great number of the organic products of our daily life, such as seeds, plants, woods, eggs, milk products, lubricating oils, print and paper. Their emission exhibits characteristic features that make it possible to determine variations in quality and type.[1] Indeed, fluorescence has even been developed as a tool in microchemical analysis for the detection and estimation of several of the metals as well as of many organic compounds.[2] It appears also among many minerals. Especial interest, however, attaches to artificially prepared products. They include solutions, both liquid and solid, and also solid inorganic substances, such as sulfides, oxides, tungstates, and silicates. Fluorescent solids are generally known as "phosphors."

There are two striking characteristics that one notes in surveying the artificial products. One notices first that fluorescence is frequently associated with the presence of a foreign ingredient at a very low concentration. In inorganic phosphors this foreign ingredient is an appropriate metal at a concentration of one per cent or less. In organic dye solutions, which may be either liquid or solid, it is a dye at a concentration much less than one per cent, with the further limitation that that dye shall be one of a few whose selection depends upon the presence in its molecule of certain derivatives of benzene.

This peculiarity of fluorescence associated with the presence of a specific ingredient at low concentration suggests an analogy with the behavior of a vapor or gas at low pressure within an enclosing tube. In both cases luminescence results by illumination with the light of a critical wave length, and in both cases the luminescence formed decreases in intensity as the concentration of the effective element is raised, whether it be a foreign ingredient in a solid or the pressure of gas in a tube. In the case of a gas the phenomenon is known as "resonance radiation." The illumination exciting it must have a wave length corresponding to an absorption band of the gas. In this way it induces resonance in the atoms or molecules of the gas, exciting them to such an extent through the absorption of energy that an electron is driven out to an outer orbit. The condition for this is that

$$E = h\nu$$

where E is the energy of a quantum of the exciting radiation, and ν is the frequency corresponding to the wave length at which energy can be absorbed to excite the atom to resonance. The luminescence arises from the return of the excited electron to its normal orbit, and this return is accompanied by an emission of energy of the same frequency. Consequently, the luminescence is of the same wave length as that of the exciting radiation.

The condition in a phosphor is more complex because the basic substance, whether solid or liquid, is itself made up of atomic groups at a relatively high concentration, and completely enveloping the scattered, occasional particles of the active ingredient. This characteristic, however, is so far from breaking the analogy with resonance radiation in a gas at very low pressure that it actually serves to strengthen it by acting as basis for an explanation of the one outstanding difference between the two. The explanation is derived from the second observation that becomes apparent to the investigator in his endeavor to pick out the salient features of fluorescence. It is the fact that the fluorescence is not monochromatic, as that of the exciting radiation may be, but extends over a broad band of the spectrum and, in general, is displaced to a position of longer wave length than that of the exciting radiation. This latter characteristic is known as Stokes' Law.

A survey of some characteristic features of resonance radiation will serve to bridge the gap between it and fluorescence, and to demonstrate the basis for the transition to the condition described by Stokes' Law.

The simplest case is that of pure sodium vapor at very low concentration—so low in fact that the time during which an electron stays in an excited state shall be less than the time between collision of the excited atom with some neighboring atom. The luminescence of sodium vapor, such as is exhibited in the sodium lamp, is made up of the *D*-line doublet, 5,890 angstrom units and 5,896 angstrom units. The emission of this doublet, as illustrated in figure 1. is the result of the return of excited electrons from two energy levels lying close together, the $2^2P_{1/2}$ and the $2^2P_{3/2}$, to the lower normal level $1^2S_{1/2}$. When the vapor is sufficiently low in pressure, only one of these lines can be excited as resonance radiation by the absorption of light of corresponding wave length. This is as theory would predict. When, however, the pressure is raised to 0.015 millimeters, corresponding to a temperature of 300 degrees centigrade, or when a fraction of a millimeter of hydrogen is introduced, both lines appear, even though the exciting radiation is limited to one line. This is due to a collision of some of the excited atoms with neighboring atoms. Such a shock leads to an energy transfer sufficient to throw the excited electron into the closely adjacent energy level, for the energy difference between the two states yielding the doublet corresponds to only 0.01 volt. The result illustrates for a system as simple as sodium vapor how sensitive the character of its radiation is to the influence exerted by neighboring atoms upon one another, even while at a relatively low concentration.

The condition becomes more complex for a vapor such as iodine which is in the molecular rather than the atomic state. In iodine vapor the two atoms constituting the diatomic molecule are free to vibrate back and forth with respect to one another, making various vibrational energy states possible. Furthermore, the

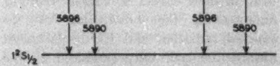


Figure 1. Energy-level diagram for resonance radiation of sodium vapor

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1. For all numbered references, see list at end of paper.

Figure 2. Resonance spectrum of iodine vapor, as affected by presence of helium (From R. W. Wood and J. Franck, Verh. d. D. Phys. Ges., 13, 85, 1911)

A—No helium
B—Two millimeters of helium
C—Ten millimeters of helium

molecule can rotate about the axis uniting its two atoms, thus giving rise also to various states of rotational energy. Consequently, there are not only the usual number of electronic energy levels, but each one of these levels is subdivided into additional ones corresponding to the possible vibrational energy states, and each one of these is subdivided again into a series of rotational energy levels. For iodine vapor the case is rendered so complex by these features that its pure resonance spectrum no longer consists of a single line but of a succession of doublets.

A more significant feature, however, is shown by an increase in pressure sufficient to lead to collision of excited iodine molecules with neighboring molecules. The resulting energy interchange is such as to shift the excited electrons to adjacent vibrational and rotational states. Consequently, when the electrons return to their normal orbit, it will no longer be by a succession of definite, specific jumps, leading to an emission of clear-cut doublets, but rather by jumps of variable magnitude, the energy of each of which is determined by that of the abnormal excited state attained in the collision. These states are so close together that the resulting emission can no longer be resolved into lines, but appears as continuous radiation arranged as bands at the location of the expected doublet. There is still another effect associated with high pressure. As the result of collision, the excited electron itself loses energy on its return to the normal orbit, leaving a smaller amount available for emission as light. Consequently, the emission is displaced toward the red. Both of these effects, so characteristic of fluorescence, the broadening of the emitted radiation and its displacement toward the red, were produced in the resonance spectrum of iodine vapor by the introduction of helium, as well as by increases in the vapor pressure of the iodine,[3] as shown in the photographs of figure 2.

These observations on resonance radiation illustrate how complications arise even in the gaseous state, as one passes from a monatomic to a diatomic vapor and from very low pressures to pressures of a few millimeters. It can, therefore, be readily comprehended that, as the concentration of atoms and molecules is still further increased to such a point that their condition becomes that of a liquid or a solid, the mutual interplay of forces between them will become much more intricate and involved, and their effect still more pronounced on the character of the emitted light. The theoretical development in recent years has, in fact, shown that the energy states in a solid can no longer be represented by a few lines of discrete energy values, as in a monatomic gas at low pressure, or even by large groups of lines corresponding to the vibrational and rotational states of a diatomic gas. The interplay of forces between atoms as contiguous as in a solid is so manifold that each group of electronic levels becomes packed with many lines and develops into what is virtually a continuous band of energy. This condition is illustrated in figure 3 by contrast with that for a vapor already brought out in figure 1. A detailed discussion of the situation is given in a recent review by Seitz and Johnson.[4]

In figure 3 two such bands are shown. The lower one represents the outermost energy state under normal, unexcited conditions. The upper is referred to as a "normally empty band." It represents the energy state into which an electron would be driven if excitation could occur. In solids which are insulators at room temperature, the bands are so far apart in energy content that the available processes for excitation are insufficient to raise an electron from the filled band to the upper unfilled one. The case becomes different, however, if in some way a localized state may be inserted between the two bands. Such a condition is produced for instance when a small amount of an impurity is dissolved in the crystal. Some of the energy states of this impurity will of course fall within the normal banded levels of the solid, but under appropriate choice the uppermost filled level of the impurity will appear as a discrete, localized state, lying above the filled band of the crystal, as shown in figure 3. Its position with reference to the upper unfilled band of the solid, can be assumed to be such as to allow of excitation under radiation of appropriate wave length, causing an electron to make the jump from this localized level into the unfilled band. The consequence will be that an emission of fluorescence will occur, associated purely with this impurity atom. The excited electron will descend by a series of thermal interchanges to the bottom of the unfilled band. Consequently, the energy emitted by its return from this point to its normal state will be lower than the energy of excitation, and the resulting fluorescence will be displaced toward longer wave lengths.

Complicating features due to the proximity of neighboring atoms of the solid have the effect of broadening the emission into a band. The cause of this effect, as well as the extention to longer wave length, can be more clearly discerned by consideration of the Franck-Condon Principle.

In the diagram of figure 4 the abscissae denote the distance between an atom of the foreign metal and, for simplicity's sake, one of its neighbors. Due to thermal vibrations and under the alternate forces of attraction and repulsion, the distance between the atoms will vary periodically so that the potential energy may be represented by the ordinates of the two curves, *N* for the normal condition of the metal atom, and *E* for its excited state. Consider that the vibra-

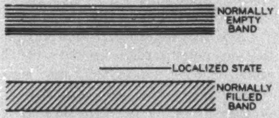

Figure 3. Energy bands for a solid

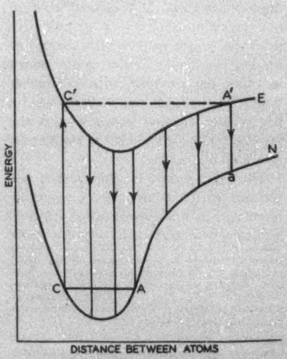

Figure 4. Excitation and emission of a phosphor according to Franck-Condon Principle

Table I. Fluorescence of Rhodamine B at Various Concentrations in Cellulose Acetate Films

Molar Concentration	Peak Wave Length Fluorescence (Angstrom Units)	Yield of Fluorescence (Per Cent of Radiant Energy)
$4(10)^{-6}$	5,900	5.4
$4(10)^{-5}$	5,930	28.3
$4(10)^{-4}$	6,050	44.0
$4(10)^{-3}$	6,210	4.6
$3(10)^{-2}$	6,250	1.0

tion is such as to lead to maximum oscillations between the points C and A. The Franck-Condon Principle prescribes that the time of excitation is so short as compared with the time of vibration that a transition from the lower to the upper level will take place along a vertical line, such as CC'. While in the excited state vibrations will continue between points of equal potential energy, such as C' and A'. As the duration of the excited state persists over a finite time, the return of the electron to its normal position may be represented as occuring from any position along the curve E between the points C' and A'. It is an inherent feature of the Franck-Condon Principle that the curves E and N shall have the different shapes shown in figure 4 as representative of the difference in the interplay of forces between an atom and its neighbors, depending upon whether the atom is in the normal or excited state. Consequently, the energy emitted by the return of the electron will not be equal to the energy of excitation, represented by the line CC', but will have a succession of lower energy values depending upon the point on curve E from which its return starts and extending as low as the transition $A'a$. Such a process would therefore yield a continuous band of emitted fluorescence extending to longer wave lengths than that of the exciting monochromatic radiation. This viewpoint has been given quantitative treatment by Seitz[5] for the case of the halide phosphors, developed experimentally by the school of Pohl and Gudden.[6]

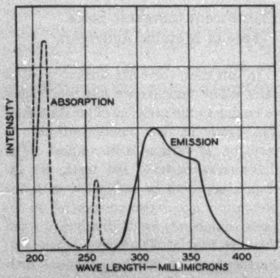

Figure 5. Absorption and emission spectra of potassium bromide activated with 0.003 per cent thallium (From W. von Meyeren, Z. Phys., 61, 321, 1930)

It has been convenient to visualize the localized state as one induced by the presence of an impurity atom. It is conceivable, however, that it might be realized in other ways leading to irregularities in the lattice, such as by departures from stoichiometric proportions in the composition of the solid, by a low concentration of an allotropic form of the solid, by the dispersion of the active molecules in a solution, or by their adsorption in dispersed form on a solid. In the discussion that follows of various types of fluorescence, the question is to be kept always in mind—what is the feature that serves as source of fluorescence by introducing a foreign condition into a solid or liquid?

Fluorescent Organic Bodies

There are many organic compounds which exhibit a low fluorescence. Benzene can be taken as an example. As a vapor it can be excited by illumination to yield a purely resonance radiation, also termed fluorescence, consisting of a succession of narrow bands arranged in groups. At closer proximity of its molecules, such as in the solid or liquid condition at zero degrees centigrade or in solution in alcohol, the subsidiary bands of a group agglomerate into a broad band, and these bands become displaced toward longer wave lengths, a trend that becomes still more pronounced as the concentration of a solution is increased. The same observation has been made for all aromatic substances studied, as well as for benzene, namely, that the fluorescence bands are displaced to longer wave lengths as the physical state is changed from a vapor to a liquid or solid solution.[7] The results serve again to illustrate the transition from pure resonance radiation to fluorescence, involving the broadening of bands and the shift in wave length that are consequences of slight increases in concentration. In this type of substance the active group seems to be the molecule itself of the compound, for the fluorescence is virtually extinguished at such high concentrations as are present in a pure liquid. It appears at its best in dilute solutions or in an adsorbed film at high dispersion on cellulose or silk fibers. Variations in the character of this type of fluorescence have been particularly applied by Haittinger to a system of organic analysis.[2]

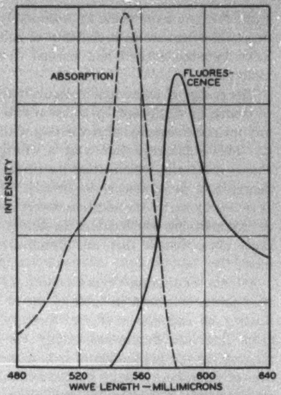

Figure 6. Absorption and fluorescence spectra of rhodamine in alcohol solution (From A. Poritsky, Journ. Frankl. Inst., 197, 527, 1924)

A much higher intensity of fluorescence is exhibited by certain of the organic dyes. It appears only in liquid and solid solutions, and is completely absent in the pure, solid dye. Rhodamine can be taken as a typical example, especially because it was one of the first to be considered for commercial application in fluorescence from a proposal made by Cooper-Hewitt.[8] Its fluorescence can be excited by the ultraviolet, but is more generally excited through radiation with green light lying within the range of its absorption band. The relation between this band and the fluorescence is brought out in figure 6. Each is seen to be roughly the mirror image of the other. The full fluorescent spectrum is excited by any wave length lying within the absorption spectrum.[9] As the concentration of the solution is increased, the band of fluorescence passes through an optimum in intensity and at the same time is displaced toward the red, as brought out in table I for films of rhodamine B in cellulose acetate excited by the mercury green line.[10] The concentrations given apply to the alcoholic solution before evaporation to yield the solid film.

The existence of an optimum intensity of fluorescence at a critical concentration has been found to be a characteristic common to the solutions of all such dyes and to fit the Brunighaus equation[11] $I = kce^{-Bc}$, where I denotes the intensity of fluorescence and c the dye concentration. The decrease in fluorescence beyond the optimum value is presumably due to the increase in probability of collision between excited and normal dye molecules.

The energy of an excited molecule may of course be dissipated by collision with any molecules capable of resonating with it. This influence is shown by a wrong choice of solvent, which either lowers the fluorescence or destroys it utterly.[10,12] It is also shown by the addition of certain foreign materials which reduce the fluorescence even though they are chemically inert.[7,13]

At the optimum concentration of rhodamine, as shown in table I, the efficiency of conversion of the mercury green line into fluorescent energy corresponds to the production of one quantum of fluorescent energy by the action of 1.6 quanta of absorbed energy, a conversion that is approaching closely that which would be theoretically attainable.

Fluorescent Inorganic Substances Containing Metallic Activators

Mention has already been made of the alkaline halide phosphors developed by the school of Pohl and Gudden.[6] Fluorescence is absent in the pure halides but appears when a trace of thallium is introduced. The characteristics are presented in the curves of figure 5 for the typical case of potassium bromide. The presence of the thallium gives rise to two new absorption bands. Fluorescence is excited by radiation of any wave length lying within these bands. It seems clear therefore that the fluorescence arises from excitation of the thallium, and the theoretical treatment by Seitz[5] furnishes a quantitative basis for this conclusion.

In the experiments described the amount of thallium in the halides was limited to very small concentrations that would not interfere with the formation of single crystals. The writer has investigated the effect of larger amounts in microcrystals. In general an optimum concentration was found, as is brought out in table II for the case of caesium iodide excited by the mercury line 2,536 angstrom units.

The dependence of fluorescence upon the presence of a foreign metal at low concentration is exhibited in other inorganic materials. For the alkaline earth sulfides and zinc sulfide, extensively investigated by the school of Lenard,[14] these activators include manganese, copper, silver, antimony, lead, and bismuth. The pure sulfides, with the possible exception of zinc sulfide, are devoid of fluorescence in the absence of an activator. The color of their fluorescence varies with the activator used. The fluorescence increases to an optimum value at a critical concentration of the foreign metal and decreases to extinction at higher concentrations, just as for the halides.

Many of the oxides also may be made fluorescent by addition of a foreign metal. Calcium oxide is a typical case. It is activated by manganese, copper, lead, bismuth, and antimony, each yielding a different color. In the extensive investigation by Ewles[15] an optimum concentration was found for each of the last three metals.

The existence of an optimum content of a foreign metal in these diverse types of inorganic compounds is obviously related to the effect of an optimum concentration in solutions of dyes. The fact that the fluorescence reaches a saturated value and then decreases for larger amounts of the metal signifies again that an activated atom can lose its energy when a neighboring atom of the metal approaches it too closely. Unpublished observations by the writer on related effects indicate that this loss is due to the thermal vibrations of the atoms, and that its extent depends upon the amplitude of these vibrations.

Manganese finds very extensive use as an activator. Besides the sulfides and oxides, it has been found also to activate certain sulfates, phosphates, and silicates.[16] In the case of zinc silicate, the writer has found that the fluorescence is displaced toward the red as the concentration of manganese is increased, as brought out in table III.

This result is important in that it demonstrates that the same effect is observable in the fluorescence of inorganic solids as in that of organic solutions and in the resonance radiation of vapors—namely, a shift in the emitted radiation to longer wave lengths that accompanies an increase in concentration of the active ingredient.

A natural query arises as to the condition in which the foreign metal activator is incorporated in the basic substance. Experiments by the writer on X-ray analysis of zinc silicate phosphors have demonstrated that manganese atoms enter into the crystal as integral parts by substitution for occasional zinc atoms. The result is a slight extension in lattice spacing, due to the larger size of the manganese ion, as compared with the zinc.

Table II. Fluorescence of Caesium Iodide Activated With Various Percentages of Thallium

Percentages of Thallium by Weight	Relative Fluorescence
0.07	88
0.22	100
0.37	84
1.11	39
8.70	5

Table III. Effect of Manganese Concentration Upon the Color of Fluorescence in Zinc Silicate Phosphors

Percentage of Manganese	Peak in Fluorescence (Angstrom Units)
0.1	5,270
0.4	5,280
0.9	5,290
1.4	5,310
2.3	5,330
4.5	5,350

In a different way Riehl has arrived at the same conclusion for zinc sulfide activated with copper.[17] The crystals of the phosphor were treated with hydrochloric acid so as to dissolve them slowly. As they became smaller, however, the fluorescence characteristic of copper activation continued and persisted almost up to complete solution. This is direct evidence that the copper had penetrated the lattice rather than formed as an adsorbed layer on the surface.

There are many natural minerals that fluoresce, one of the most notable being a variety of willemite from New Jersey known as Franklinite. So far as these minerals have been investigated, they have been found to contain a foreign metal impurity as source of the fluorescence.[7]

The fluorescent glasses fall in this same classification, for their luminescence under radiation is due to activation by a foreign metal at very low concentration. It may be uranium or even one of the common metals such as manganese.[18]

Fluorescent Inorganic Salts Free of Metallic Activators

In this class there are several types of salts whose fluorescence has been found to be due to the presence of an abnormality, again at low concentrations. It may be a foreign acid radical. The fluorescence in bone and teeth, for instance, seems to be associated with the presence of about two per cent of an organic compound, for it disappears when they are fired at 600 degrees centigrade. Furthermore, it can be produced artificially by the incorporation in calcium phosphate of a small amount of an organic product.[19] Doctor Byler of this laboratory [General Electric Company, Schenectady, N. Y.] has just shown that the fluorescence of calcium phosphate may also be induced by incorporating a critical amount of the acid phosphate in the hydroxy phosphate, and that similar complex salts of other metals can be formed at such critical compositions that fluorescence results.[20]

A related case is that of zinc carbonate which developed fluorescence when it had been fired sufficiently to bring about partial decomposition into the oxide. An optimum fluorescence resulted at a composition of 11 molar per cent zinc oxide combined with 89 molar per cent zinc carbonate.[21]

Still another type of abnormality is represented by the presence in a salt of a small amount of an allotropic modification of that same salt. Doctor Byler developed fluorescence in barium chloride, for instance, by firing it for a short time just above its transition point. Another instance is furnished by the long controversy over the fluorescence of zinc sulfide. The variations in the intensity and color of its fluorescence that have been produced by altering the amount and kind of a foreign metal impurity demonstrate that activation of zinc sulfide by a foreign metal actually exists. However, a low degree of blue fluorescence arises on firing zinc sulfide to which no metallic impurity has consciously been added. There is a suspicion that this arises from the presence of a residual impurity which the chemical purification has not been capable of removing. Schleede, however, has demonstrated that the fluorescence of the purported pure zinc sulfide is at a maximum after a brief firing at 1,000 degrees centigrade, which served to convert a part of it from the cubic to the hexagonal form. He concluded that the fluorescence arose from this intermixture of different crystalline types, for it disappeared under slightly higher firing at temperatures most suitable for the development of fluorescence in the presence of foreign metals.[22] He ascribed the loss in fluorescence of the pure sulfide at such temperatures to its complete conversion to the hexagonal form.

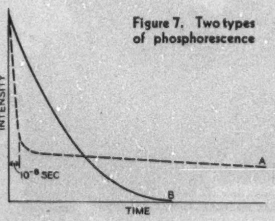

Figure 7. Two types of phosphorescence

There are many metals which can be identified by the fluorescence of their salts, most especially those with certain organic acids. In general, however, this fluorescence is observed as adsorbed films on a suitable base, such as filter paper, or in solution. It would seem to be due, therefore, to a fluorescence of the molecule under the condition of sufficient dispersion to prevent disturbances by collision.[2]

There are, however, other cases in which fluorescence is associated with pure inorganic solids which are not yet known to contain any abnormalities. Such are the salts of the rare earths and of the uranyl radical and also the tungstates and molybdates.[14] It is reasonable to expect that the presence of some abnormality will eventually be found in them also.

Phosphorescence

The light emitted during radiation by the exciting source is known as fluorescence. In many cases the fluorescent substance continues to emit light after the exciting source is extinguished. Such a delayed emission of light is known as phosphorescence. The two are intimately related as manifestation of the same fundamental phenomenon, namely, excitation of an atom. When the excited electron returns directly to its normal orbit, simply fluorescence is observed as the result of what Lenard calls an instantaneous process. When, however, the return of the electron is for any reason delayed, thus giving rise to phosphorescence, then, by way of distinction, Lenard refers to it as the result of a delay process.

Actually fluorescence may persist for an instant, of the order of 10^{-8} second, after the exciting source is extinguished. When no luminescence beyond this period is observed, then one can be assured that the instantaneous process is the only one involved. When phosphorescence is manifested, however, it is clear that both processes are going on simultaneously during direct illumination. Their relative proportions in two different phosphors can be estimated by measuring the rates at which their phosphorescence decays after extinction of the exciting light. If one substance shows a sharp initial decay, less than about 10^{-8} second, such as that represented by the first portion of curve *A* in figure 7, to be followed by a slow decay which begins at low intensity, then it is clear that the light emitted from it during illumination is produced to a relatively greater extent by the instantaneous process.

The delay process itself is characterized by differences in the speed with which it is carried out. This is manifested by differences in the character of the decay curve. In figure 7, for instance, even though the delay process is predominant in *B*, yet the speed of its decay is relatively so high that its phosphorescence has fallen to zero at a time when the longer life phosphorescence from *A* is still observable.

In the case of a sulfide phosphor, there are generally three different types of phosphorescence, each excited by a narrow band of differing spectral distribution. The instantaneous process giving rise to fluorescence is excited by a broad band whose spectral distribution includes all of the narrower bands which lead to phosphorescence. The halides offer a contrasting case. In them there are two distinct absorption bands, as brought out in figure 5. Excitation by the one of longer wave length yields only fluorescence, whereas excitation in the shorter band leads to both fluorescence and phosphorescence as well.

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A Memory Attachment for Oscilloscopes

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THE DEVICE described in this paper, called a "memnoscope," is a system for obtaining an oscillogram of random phenomena and events immediately preceding and following its occurrence.

This system was developed and has been used to study arc-back phenomena in mercury-arc rectifiers. Other methods, such as the use of long transmission lines and high-persistence cathode-ray tube screens[2] have been used to obtain oscillograms of random phenomena.

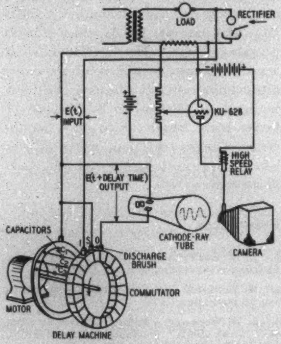


Figure 1. Memnoscope: A system for oscillograms of random events

It is possible to obtain volt-current-time data on one film because the discharge of each condenser element of the memnoscope gives a dot on the oscilloscope record and a knowledge of the commutator speed and number of condensers gives time (see figure 4*a*). Information on random transmission-line faults can be obtained from normal conditions, through fault time and back to normal.

Description of Complete System

In figure 1 is shown a rectifier which is a source of a random[1] occurrence commonly known as arc-back. If it is desired to obtain an oscillogram of conditions in a rectifier immediately preceding and following the arc-back, a system as shown in figure 1 can be used.

The grid of the thyratron is excited by the random phenomenon and the plate current opens the camera shutter. A picture is obtained of the motion of the spot on the cathode-ray-tube screen. Since the motion of the spot is determined by potentials of the rotating capacitors and if rotation is as shown in the figure, the picture taken will be of events occurring prior to the time the random phenomenon occurred. The heart of the system is the rotating capacitor network designated in figure 1 as "delay machine."

Construction of Delay Machine

To obtain satisfactory oscillograms at 60 cycles, for the conditions investigated, a machine with 147 capacitors was constructed. This machine was built using a standard commutator with 147 bars and 147 0.01-microfarad capacitors mounted around the shaft. One terminal of each capacitor was connected to a commutator segment; the other terminal was connected to a collector ring. The whole assembly was balanced and then mounted in a standard motor frame of suitable size. The size of the completed capacitor unit, which is the machine in the foreground, and the driving motor can be seen from figure 2. The capacitors are of the commercial paper-wax tubular type, $^1/_3$ inch in diameter and one inch long.

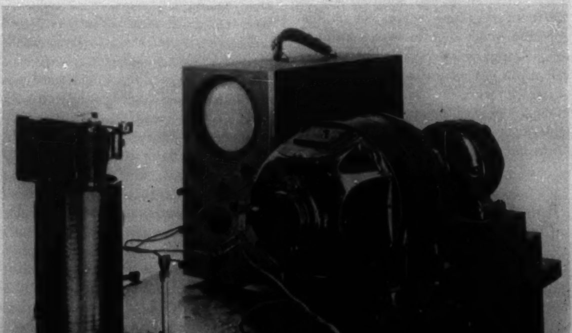
The brushes are mounted close together in order to utilize as many of the total number of capacitors as possible. With rotation as shown in figure 1, I is the input brush, S is the short-circuiting brush, and O is the output brush.

Operation of Machine

Assume the direction of rotation as shown in figure 1 and a voltage $E\ (t)$ across the input terminals. The resistance and inductance of the input circuit are negligible; then the capacitor, C_1, at the same time it leaves the brush I will be charged to a voltage $E\ (t_1)$. After a time very nearly $1/rps$ seconds (the delay time) this capacitor, C_1, will be under brush O and the voltage $E\ (t_1)$ can be recorded. Before C_1 can reach the input brush again, it is discharged by the short circuiting brush S bringing $E\ (t_1)$ to zero and C_1 is ready for another transfer of charge.

All the capacitors C_1, C_2, C_3, C_4 . . . . . . transfer their specific charges once per revolution and a stepped wave $E'(t)$ is obtained at the output end. This wave is $1/rps$ second behind the wave $E(t)$. The number of electrical degrees per step is $\theta = f/rps \times 360/b$. Where f = frequency and b = total number of capacitors in the machine. For 1,800 rpm and a 60-cycle wave $\theta = 4.90$ degrees. This means that there will be 73.5 steps per cycle of a 60-cycle wave.

Figure 2. Apparatus as used to obtain pictures. The larger machine contains rotating capacitor network



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[2] For all numbered references, see list at end of paper.

The delay time is that of one revolution of the commutator corrected for the total span for the three brushes. In this case it is $t_m = 1/rps \times 142/147 = 0.0322$ second for 1,800 rpm. This time is nearly two cycles of 60-cycle frequency.

Oscillograms, figure 4, have been obtained using a cathode-ray tube, with the output voltage connected directly to the plates. The input voltage required under these conditions is 50 volts for one-inch deflection on the cathode-ray-tube screen.

The machine can be used with the magnetic-type oscillograph if a suitable coupling means is used between the output of the machine and the oscillograph. By using more than one group of three brushes on the commutator, more than one picture can be taken at the same time. Also, more than one commutator can be mounted on the same shaft if more records are required.

Response Characteristics—Input End of Network

Equation 5 (see appendix) indicates that if a wave definition p (number of steps per cycle of highest frequency to be reproduced) is required, the RC product should be decreased as the frequency desired is increased. It is desirable to use as small a value of resistance in the input circuit as possible. If a resistance potentiometer is used to supply voltage to the network, the resistance of this input device is to be included in determining the frequency response.

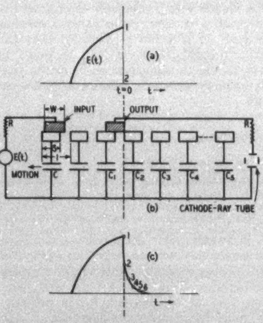

Figure 3. Distortion due to output brush making contact to more than one capacitor at a time

It is also advantageous to use a subtended brush angle of such a value that only one or two capacitors are in the brush circuit. The value of C, in equation 5, is the total capacity under the input brush.

It is not difficult to obtain good response as far as the RC value is concerned. The value of p, which determines the definition of the wave, is more important since the maximum frequency which can be transmitted by the network is determined by the number of capacitors required per cycle to make the frequency recognizable in the output circuit. These considerations determine the product pf. After the value of pf is chosen, the maximum value RC, which may be used as far as the input circuit is concerned, can be obtained from equation 5.

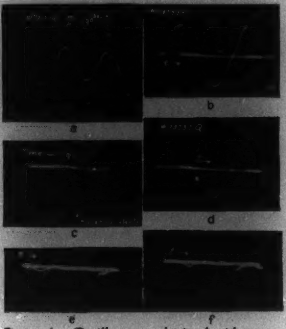

Figure 4. Oscillograms obtained with memnoscope

(*a*)—60-cycle transient

(*b*)—Wave as in figure 3*a* obtained without machine

(*d*)—Same wave as (*b*) with machine

(*c*), (*e*), (*f*)—Arc-back oscillograms of anode-to-cathode voltage of a mercury-arc rectifier

The Output End of Network

If contact is made at all times to not more than one capacitor, the output voltage wave into the receiving device will not be distorted provided p, the number of capacitors per cycle, is of sufficient number for the frequency transmitted.

If, however, the output brush makes contact with more than one capacitor at a time, then there will be a division of the charge and distortion will result. The effect of double contact can be seen from a consideration of a wave distribution as shown in figure 3*a*.

The action is as follows:

At $t = 0$ the voltage on capacitor C_1 (see figure 3*b*) is E_1. At the time contact is made to C_2, the voltage at the receiving device should be equal to zero according to point 2 on figure 3*a*. However, if the brush makes contact to C_1 at this time, as well as to C_2, C_2 will be charged to a voltage $E_2 = E^1/_2$. This transfer of charge by the brush continues and a distorted wave as shown in figure 3*c* will be obtained rather than the correct wave, figure 3*a*.

If a high-resistance brush material is used, the wave distortion will be reduced. Calculation shows that the resistance through the brush from bar to bar would have to be 1.25×10^6 ohms in order to make E_2 only one per cent of E_1 for the time t_1 used in this machine. A portion of this resistance would occur in the output circuit and distortion there would be enhanced.

Equation 7 in the appendix is for the wave from $t = 0$. This equation indicates that the maximum distortion occurs when contact is made to two bars at a time. For contact to more than two bars at a time, the distortion is reduced slowly; that is, as the log of a function of the number of bars spanned by the brush. Contact to only one bar at a time is best for minimum distortion.

The Short-Circuiting Brush

This brush circuit should have low resistance and inductance. Increasing the number of bars spanned by this brush does not affect the time constant unless the resistance is decreased in proportion. Brushes mounted axially with the commutator and separate and short leads from each brush to the common side of the circuit is possibly the best way to discharge capacitors.

Results With Memnoscope

Figure 4 shows the type of films obtained with the camera using a cathode-ray tube with a blue-trace screen.

Figure 4*a* shows a 60-cycle transient crossing the cathode-ray-tube screen two times. Each dot on the film represents the voltage of a capacitor.

Figures 4*b* and 4*d* are for waves of the type in figure 3*a*. These pictures were taken, respectively, without and with the memnoscope. They represent the anode-to-cathode voltage of an ignitron operating from a 60-cycle source of power.

Figures 4*c*, 4*e*, and 4*f* are oscillograms of random arc-backs of a rectifier. The films show anode-to-cathode voltage changing from normal inverse voltage to an arc-back. Films *c* and *f* show that the arc-back occurs near the crest of the wave, and *e* shows one starting at one-half crest voltage.

Value of Memnoscope in Research

The memnoscope has proved its value in rectifier research. The opinion has been that arc-banks in rectifiers occurred only during transition time. Oscillograms obtained with the memnoscope show that arc-backs do occur at other times during the inverse cycle. Oscillograms have given information which has led to improved designs and to very substantial increases in the ratings of ignitrons.

Appendix

To determine the frequency response at the input end, we may use the equation for voltage across a capacitor charged through a resistance r. If the input voltage is $E \cos(2\pi ft + \phi)$ the voltage at any time t is, if charge on $C = 0$ at $t = 0$

$$e_c = \frac{E}{2\pi fCZ}\left[\sin(2\pi ft + \phi) - \sin\phi \cdot \epsilon^{\frac{-t}{rc}}\right]$$

where

$$Z = \sqrt{r^2 + \left(\frac{1}{2\pi fc}\right)^2}$$

$$\phi = \psi - \tan^{-1}\frac{1}{2\pi frc} \tag{1}$$

ψ = phase of E at $t = 0$

In this case the voltage impressed on the capacitor depends on the input voltage at the time contact is made. If the time duration of contact is t_1, when the brush leaves the capacitor being charged, the voltage on the capacitor will be

$$e_c = \frac{E}{2\pi fCZ}\left[\sin(2\pi ft_1 + \phi) - \sin\phi \cdot \epsilon^{\frac{-t_1}{rc}}\right] \tag{2}$$

t_1, the time duration of contact in terms of the frequency f, and p, the number of capacitors per cycle is

$$t_1 = \frac{1}{pf} \tag{3}$$

and

$$\frac{t_1}{rc} = \frac{1}{pfrc} \tag{4}$$

Substituting in (2), e_c becomes

$$e_c = \frac{E}{\sqrt{1 + (2\pi frc)^2}} \times \left[\sin\left(\frac{2\pi}{p} + \phi\right) - \sin\ \phi \cdot \epsilon^{-\frac{1}{pfrc}}\right] \tag{5}$$

This equation shows that a phase distortion and an amplitude distortion can occur. Since p must be made sufficiently large to give a recognizable wave, when the frequency desired is increased, the rc value must be decreased in order to make both the second term under the radical and the second term in the brackets negligible compared to other terms in the above equation.

The time of contact t_1, see figure 3*b*, in terms of commutator bar, brush, and insulation dimensions is

$$t_1 = \frac{w + s}{(rps)bl} \tag{6}$$

Using this value for t_1 we can write (5) in terms of the dimensions, speed, and number of bars.

If contact is made at the output end to more than one capacitor at a time distortion will occur. If we consider an input wave as shown in figure 3*a*, and contact to two capacitors at a time, the output wave will be as shown in figure 3*c*.

This wave can be represented by equation (7), assuming $C_1 = C_2 = C_3$, etc., and a brush of negligible resistance

$$E_n = \frac{E_1}{2^{n-1}} = E_1\epsilon^{-(n-1)\log 2} \tag{7}$$

where E_n is the voltage of the nth capacitor, and $n = 1$ for C_1, $n = 2$ for C_2, etc., and E_1 = constant. This gives an exponential curve for the output end, instead of a vertical line as desired.

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Discussion

H. P. Kuehni (General Electric Company, Schenectady, N. Y.): In the device described in this paper an electrical phenomenon to be recorded is first impressed on a set of rotating capacitors which are connected to commutator segments. Each capacitor is charged through the input commutator brush to a potential proportional to that of the electrical phenomenon at the time of brush contact. By means of a second commutator brush, which is advanced in the direction of rotation, the charged capacitors consecutively are connected to the oscillograph, thus recording in steps the event a definite time after its occurrence.

The accuracy of reproduction and frequency response of this device are largely functions of the number and size of the capacitors used, the speed of rotation, and the input and output circuit time constants, as well as the energy available from the circuit to be studied. For practical and economic reasons the number of capacitors, commutator segments, and speed of rotation obviously are quite limited. In the author's machine 147 0.01-microfarad capacitors were driven at a speed of 1,800 rpm. Under these conditions a 60-cycle wave is reproduced with 73.5 steps of 4.9 electrical degrees per step. However, a 600-cycle wave, for example, would be reproduced considerably distorted by only 7.35 steps with 49 electrical degrees per step. It is to be expected, therefore, that even though the circuit time constants can be made small to satisfy the response requirements, the limited practical number of capacitors and commutator segments render this type of recording device suitable for moderate frequencies only. It is questionable whether a device of this type can be built economically for high-frequency response as required, for example, in the recording of surges of short duration, switching transients, and the like.

For the recording of random phenomena Doctor A. W. Hull[1] made use of the afterglow of the fluorescent cathode-ray-tube screen. The recording photographic camera, being triggered through a thyratron by the disturbance itself, photographs the afterglow of the cathode-ray trace or the so-called tube screen "memory" of the event. The screen substance willemite, which is widely used in commercial cathode-ray tubes, has an afterglow of approximately $^1/_{20}$ second which is ample time for the camera to catch. An automatic three-tube cathode-ray oscillograph built on the screen "memory" principle has been in continuous and highly successful use for nearly two years in the General Electric Company for the study of random phenomena pertaining to power rectifiers and tube circuits. The cathode-tube screen afterglow can, of course, be used with any cathode-ray oscillograph providing the tube voltage is sufficiently high to produce a recordable trace of the transient. With the General Electric high-speed hot-cathode type-*HC*-15 cathode-ray oscillograph,[2] using a 15,000-volt permanently sealed glass cathode-ray tube, successful photographic memory records were obtained in experimental work of a transient as fast as a 1 × 5-microsecond impulse wave by triggering the camera with a relay energized from a charged capacitor through a thyratron fired by the impulse itself. The cathode-ray tube "memory" principle has practically unlimited frequency response and outside of a simple relay, capacitor thyratron circuit, and a camera, no extra equipment is needed. It is believed doubtful that the author's capacitor device can be made to have as wide a frequency range as the simple memory principle described by Doctor Hull.

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W. E. Pakala: Mr. Kuehni has questioned the economical use, at high frequencies, of the rotating capacitor network for memory oscillograms of random events, and mentions the use of high-persistence cathode-ray tubes for memory oscillograms.

The machine described was designed to give information as to conditions existing for two to four cycles prior to random arc-back in a mercury-arc or ignitron rectifier operating from a 60-cycle power source. For these conditions the capacitor machine has been very useful and entirely satisfactory.

In the cathode-ray-tube method of obtaining "memory" oscillograms of random events, it is necessary to keep the tube fully energized and the recurrent phenomena on the screen. This means that the recurrent or normal trace, while waiting for random

Remote Control of Network Protectors

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Synopsis: Two low-voltage a-c networks have been installed in Tennessee, using a radically different protective scheme. The customary master relays, phasing relays, and associated current transformers have been omitted from the network protectors using in their place a remote-control system of pilot wires operated from relays at the supply substation which protect the high-voltage cable feeders.

In contrast with previous pilot-wire-protected network systems utilizing looped primary circuits with secondary transformer fuses[1,2], these networks retain the radial supply and electrically operated secondary air circuit breakers of the more conventional systems.

Previous experience has pointed to operating advantages of a remote control system provided a thoroughly reliable installation could be secured at a reasonable cost. The system described does not exceed the cost of conventional relay installations and it is felt that inherent hazards to reliability have been overcome.

LOAD GROWTH in the business district of Nashville, Tenn., had made necessary the rehabilitation of the existing underground supply system consisting of overloaded 2.3-kv radial cables. Comparative cost estimates showed decided advantage of replacing the entire 2.3-kv system with its crowded ducts and aged cables with a low-voltage a-c network system supplied from four 13.2-kv cable feeders.

Successful operating experience with d-c pilot-wire systems for tie-line protection over a period of five years[3] led to the consideration of remote-control pilot wires for the proposed network system instead of master relays and phasing relays. Conditions particularly favorable to remote control were the fact that an entirely new network system was contemplated making the conversion of existing equipment unnecessary, the supply substation is located immediately adjacent to the network area, the fact that replacement of 2.3-kv cables with 13.2-kv cables would release ducts which would be idle except for remote-control cables, and that the arrangement of duct banks resulted in several "inside" ducts which would be of little use in power-carrying capacity.

Three of the four 13.2-kv cable feeders were connected in service on February 6, 1937, with the initial network step consisting of five units. At the close of 1937 a total of 34 units had been installed on the four cable feeders. Construction is proceeding in a steady program.

In Chattanooga a low-voltage network system is being installed to supplement a 2.3-kv radial underground system. By the end of 1937 a total of eight units had been installed on three 11-kv cable feeders. It is planned that the network will eventually replace all of the 2.3-kv radial underground system.

Operating Requirements

Network experience over a period of years has resulted in a rather imposing list of requirements for the network control relays, such as:

1. The network protector must trip for any short circuit or ground on the high-voltage cable feeder.
2. The network protector should trip on transformer magnetizing current or cable charging current with the supply breaker open.
3. The network protector should not trip due to small phase-angle differences in supply of various feeders.
4. The network protector must not trip during a fault on the secondary network.
5. The network protector must not trip during a fault on any other high-voltage feeder.
6. The network protector should not trip on power reversals at light loads or in case of regenerative loads.
7. The network protector should close when the transformer is energized with voltage of such magnitude and phase relation that power will flow from the transformer to the network.
8. The network protector must not close if resulting power flow would be such as to operate the sensitive tripping relay.
9. The network protector must not pump when the high-voltage cable feeder is energized through one network unit which has failed to open.

It is evident that limits usually imposed on the closing of a network protector are based on imperfect selectivity in its opening characteristics.

A tremendous amount of development work has been done by operating engineers and manufacturers in evolving relays or systems which could meet these requirements, and the remarkably good record of network relay performance[4] is a tribute to their work. One solution[5] has been to apply relays which are sensitive primarily to negative-phase-sequence quantities. Naturally such relays will not meet the requirements of opening on transformer magnetizing current, but operation approaching this can be achieved by providing a source of negative-sequence power at the substation. Another approach[6] has been to install power devices in the primary system for controlling phase-angle displacement of the various feeders supplying a common network. Doubt has been expressed regarding the universal application of either of these methods.

One engineer[7] has compared the operation of a network system to the conventional transmission network in which all switches are operated closed except for inspection or repairs or in event of system trouble, in which case only the circuit breakers which are necessary to isolate the fault are opened. Accordingly it has been suggested that requirements of network relays be reduced to (1) the network protector should close when its transformer is supplied with normal three-phase voltage in excess of the network voltage by a predetermined value (2) the

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The authors wish to acknowledge the assistance and co-operation of the underground distribution, design engineering, and field test personnel of The Tennessee Electric Power Company in the development and testing of this protective scheme, and the co-operation and encouragement of engineers of several utility companies. They also wish to acknowledge co-operation of the engineers of the General Electric Company in furnishing information on various other proposed methods of network protection.

[1] For all numbered references, see list at end of paper.

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phenomena, will have some effect on the interpretation of films, since the repeated trace will photograph much better than the transients, which are single sweeps on the cathode-ray-tube screen. In other words, there is some "burning in" effect because of recurrent traces, whereas the intensity of the transient trace decays exponentially.

If a rotating capacitor delay machine is used with a cathode-ray tube, the intensity can be kept at a low level until the camera shutter is opened and high-speed cathode-ray screens can be used, giving pictures of only the trace desired. Another advantage is that a magnetic oscillograph can be used and oscillograms of several currents and voltages obtained simultaneously as economically as with, say, a battery of six cathode-ray tubes.

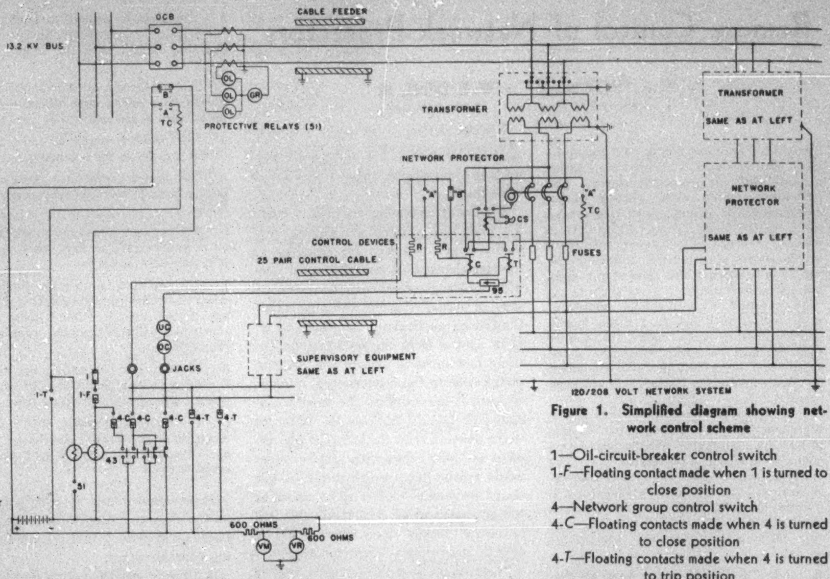

Figure 1. Simplified diagram showing network control scheme

1—Oil-circuit-breaker control switch
1-*F*—Floating contact made when 1 is turned to close position
4—Network group control switch
4-*C*—Floating contacts made when 4 is turned to close position
4-*T*—Floating contacts made when 4 is turned to trip position
43—Series-shunt reversing relay
51—Feeder protective relays
95—Copper-oxide rectifier with mid tap
C—Remote-control closing relay
T—Remote-control tripping relay
CS—Network protector motor cut-off switch
R--4,100-ohm resistor
OC—Overcurrent supervisory relay
UC—Undercurrent supervisory relay
TC—Trip coil
VM—High-resistance voltmeter
VR—Sensitive voltage relay

network protector should trip in case of a high-voltage fault or when opening means are applied at the supply substation.

A network primary feeder with its associated network transformers may be compared to a transformer in a power station which is operated in parallel with other units. Individual control switches provide separate manual control of the high-voltage and low-voltage breakers but internal faults trip both breakers.

This is the general type of control which has been provided for the remote-control network system, except that only two wires are used to connect each low-voltage air circuit breaker with the supply substation equipment resulting in depending on protective relays on a high-voltage feeder alone to determine short circuits in the protected zone.

Description of Control Scheme

Each feeder control panel at the supply substation is provided with two standard-size control switches, one to control the high-voltage oil circuit breaker for the cable feeder and the other to control the entire group of low-voltage network breakers associated with that feeder. Operation of overload or sensitive ground relays applied to the high-voltage cable feeder results in tripping the feeder oil circuit breaker and also tripping the entire low-voltage group, causing green lamps on both control switches to burn and sounding a station alarm which may be silenced by turning both control switches to the trip position. After trouble has been located and repaired the operator can energize the feeder and then close the network switches immediately or if desired he can energize the feeder and wait for a given test period before connecting the feeder to the network.

Among requirements sometimes listed for network master relays is that they shall provide a check on any possible incorrect phasing on the high-voltage or low-voltage side of a network protector during construction. This function is really a by-product of the opening and closing characteristics of the network relays and those who have had experience with incorrect factory wiring of distance relays and similar equipment would not like to trust such devices for a definite check on the adequacy of construction phasing. At the present time our construction forces prefer to check phasing electrically across a network protector with fuses removed but provision has been made for checking phasing of any new network unit or of any splice in a primary cable using equipment at the substation. Instead of using one potential transformer on each underground cable feeder, two such transformers are connected in open delta and a swinging panel with two 360-degree synchroscopes arranged with transfer switches so that a feeder cable may be energized from a new network unit or from a network unit beyond a new cable splice and exact phase relations checked at the substation.

Each primary radial feeder is protected by three inverse-type overload relays with instantaneous attachments and one inverse-type sensitive ground relay with instantaneous attachment (see figure 1). The contacts of all four protective relays are connected to the trip coil of the oil circuit breaker through a series coil of a battery reversing relay. This relay is also provided with a shunt coil which seals in from momentary operation of the series coil until it is released by turning the network group control switch to the trip position. The shunt coil is also operated from a "*b*" finger of the oil circuit breaker through a floating contact of the control switch, guarding against leaving the primary cable energized from the network (without normal protection) in case the feeder oil circuit breaker should open accidentally.

The reversing relay controls the group of network air circuit breakers on a feeder by reversing 125 volts d-c battery supply to the remote control pairs. Control battery is carried to each network protector on an individual pair of wires by means of number 22 gauge double paper-insulated lead-covered cable located in the same underground duct system with the primary supply cables. For the main runs one 25-pair cable is used with each high-voltage feeder and 10-pair cables are used for lateral runs to each network unit. One common pair for each feeder is terminated in a "condulet" type receptacle at every network protector for plugging in portable telephone equipment. Each control cable has four spare pairs terminated in each network protector.

Inside each network protector housing all relays, current transformers and associated wiring have been removed, leaving essentially an electrically operated air circuit breaker. Two small contactors are added operating in conjunction with one copper-oxide rectifier with mid tap and two fixed resistors to give closing or tripping impulses to the network air circuit breaker depending on polarity of direct current supplied at the substation. As a tripping or closing operation of the air circuit breaker is completed the auxiliary switch inserts a fixed resistance to reduce current in the pilot wire circuit from the operating value to the supervisory value. Figure 2 shows the contactors (*C* and *T*) mounted in the network protector housing. The rectifier and resistors are shown at the lower left. Interruption of d-c supply or the failure of remote-control pairs does not result in operation of the air circuit breakers but merely removes control from them.

Figure 2. Mounting of remote control equipment in standard protector unit

Continuous audible and visible supervision of each pilot-wire circuit[3] is provided by means of a small overcurrent and undercurrent contactor for each pair located at the substation. Remote-control operating current is 130 milliamperes which is reduced immediately through air circuit breaker auxiliary switches to 25 milliamperes. The closing and tripping contactors at the network protectors operate on 75 milliamperes and drop out on 45 milliamperes, while at the substation the overcurrent contactors pick up at 75 milliamperes and drop out at 45 milliamperes and the undercurrent contactor picks up at 18 milliamperes and drops out at 11 milliamperes. A control alarm buzzer at the substation operates if current is above 75 milliamperes or below 11 milliamperes, or in case of a ground on any portion of the battery circuit. A row of telephone-type white supervisory lamps (see figure 3) is provided on the substation feeder switchboard panel with one lamp for each network protector. These lamps burn normally regardless of the position of the network protector but are extinguished if the overcurrent or undercurrent contactors indicate an abnormal condition. Although the alarm system is primarily for the purpose of supervising the remote-control circuits it also calls attention to the failure of an air circuit breaker to open or close (as directed by substation equipment), since the operating current is chosen to pick up the overcurrent contactor.

Under each supervisory lamp on the switchboard panel are located a set of telephone-type jacks for isolation and tests of the individual control pairs. One zero center d-c milliammeter is provided which can be used by means of a cord and plug to read current in any control pair. Thus the operator can determine readily whether an alarm indicates an open circuit, short circuit, or failure of the network air circuit breaker to operate.

An interesting by-product of individual control-circuit supervision is individual remote control of the network protector units. The same telephone jacks used for test and isolation of the control pairs can be used by means of a special plug to operate one or more network units to a position other than that indicated by the principal control switch. When the special plug is removed from the test jack the network protector returns to the position called for by the main control switch. Naturally the automatic battery reversing relay takes precedence over the individual jacks as well as over the main control switch, so that network protectors always trip in case of power trouble regardless of manual operation from the substation.

Figure 3. Switchboard arrangement of control equipment for Nashville network. Each panel controls one primary feeder and its network protectors

Figure 4 shows a front view of switchboard at the supply substation of the Chattanooga network. The control scheme is quite similar to the Nashville installation except that a row of keys has been provided for individual control of the network units using jacks merely for tests and isolation. A further difference is that the single white supervisory lamp for each network protector has been replaced by a supervisory red lamp and a green lamp, these lamps being operated from the same contacts on the overcurrent and undercurrent relays which are used in the Nashville scheme to operate the white supervisory lamps. The red and green lamps are connected at the substation end of the remote control pairs (between substation battery and the supervisory relays) so that a somewhat better indication of the position of the network protectors is given as long as the control circuit is intact and the protector operates correctly. Naturally complete indication

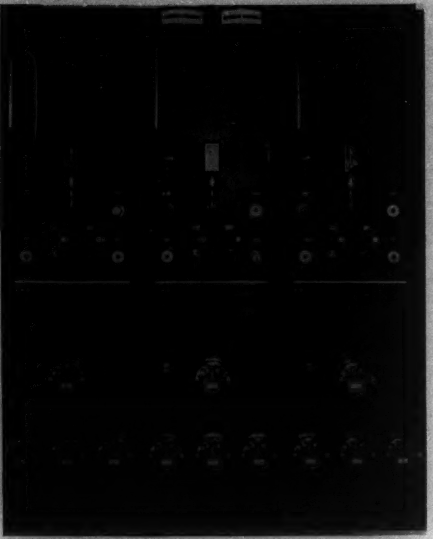

Figure 4. Switchboard arrangement of control equipment for Chattanooga network. Each panel controls one primary feeder and its network protectors

of the network switch position with the control circuit in trouble would require additional control wires. If the control pair is in trouble or if the network protector fails to move to the required position then the red and green lights at the substation burn dimly in series.

It may be noted that telephone type equipment has been mounted on the standard ebony asbestos switchboard by using a center section of thin bakelite arranged flush with the front of the $1^1/_2$-inches-thick composition panel.

Relative Cost of Remote Control Equipment

At the present time standard network protectors without relays or current transformers are being purchased from two leading manufacturers with a price reduction of about $400 per protector. Estimates based on the Nashville completed network with a total of 60 network protectors show that the pilot-wire cables and associated equipment can be installed for about 60 per cent of the price reduction on the 60 network units. These estimates include cost of the control cable and its complete installation with fireproofing but do not include any allocated cost for duct space. No credit is taken for the cost of telephone pairs. Cost of terminal equipment in the network protectors and at the substation is included. Many of the parts are quantity-production items used in communication and automatic control.

In cases where duct space for control-cable circuits is not readily available at moderate cost, consideration should be given to leasing a remote control pair for each network protector from the local communication company. Aside from the cost comparison, the advantage of relative freedom of exposure to power system troubles can be weighed against possible control-circuit outages arising from telephone protective equipment or maintenance procedure on the part of the communication company.

While variations in cable installation cost may be expected the resulting figures would still show positively that the remote-control system would not cost more than the conventional system. The fact that the selling price of the equipment omitted from the network protectors is nearly $600, whereas the present omission price is two-thirds as much points to probable additional economies in the future.

Reliability

It is generally conceded that the frequency of inspections of equipment which is likely to cause trouble has a pronounced bearing on the probability of failure of protective equipment to perform properly during system trouble. Attention has been called to this recently in connection with low-voltage a-c network operation.[4] Experience with transmission relays has emphasized the need of frequent inspections of protective devices and in the case of protective equipment which is difficult to inspect and which is exposed to hazards extending over several miles such as a pilot-wire circuit the use of continuous audible and visible supervision has been found highly effective. Although these conclusions were reached through operating experience and study, the curve of figure 5 has been prepared to illustrate the theoretical relation between the time required to locate and clear protective trouble and the probability of failure of the protective equipment to clear primary short circuits. The time of one hour at one end of the curve is taken as a typical value for remote-control equipment since within this length of time the operator can determine that an individual control pair is defective (grounded, short-circuited, or open-circuited) and can arrange to have this individual network unit removed from service even under peak load conditions such that it would be undesirable to remove an entire network feeder from service.

Experience of communication companies indicates that many troubles with underground lead-covered cables come about through the entrance of moisture into the cable through small defects in the lead sheaths. Several hours are usually required after the failure of one pair before the entire cable is involved. This was borne out in striking manner by the one case of control-cable trouble which has been experienced since the Nashville network system was started. A splice was known to be defective (due to special local causes) but construction was at such a stage that it was decided to leave the splice in service over a period of time and observe results. After the first pair caused operation of an overcurrent relay (indicating a high-resistance short circuit) a period of six days elapsed before another pair was involved. After an additional period of 12 days two other pairs were involved and at that time the defective splice was repaired. This trouble was due to the entrance of moisture through a defective splice into one of the 25-pair control cables.

Another case of trouble was discovered by means of the ground alarm. Isolation of various pairs at the substation by means of test jacks immediately located the faulty circuit which was then operated to trip the network protector and isolate it from battery supply. An investigation showed a vault half full of water and a defective network-protector housing had permitted moisture to collect on the control wiring resulting in the indication of a high-resistance ground.

The adequacy of supervision for preventing protective failures rests on troubles in the primary equipment and control equipment being reasonably independent of one another, and precautions have been taken to insure this condition. Special short-circuit tests were made to determine the possible difference of ground potential between the supply substation and network units from which calculations indicate a maximum difference of 200 volts under the most unfavorable short-circuit conditions.

One chief cause of concern has been the likelihood of primary cable burning control pairs in two or destroying a network control cable. The choice of control scheme has reduced the possible damage from such a contingency. In case of a primary cable burning its associated control cable, correct operation is secured if the tripping impulse is given to the network protectors before any control cable pairs are damaged. The use of instantaneous devices on the protective relays and high-speed control equipment insures that the trip plungers at the air circuit breakers will be in motion less than four cycles after the beginning of the fault. Since interruption of d-c supply does not cause the network protectors to open, the damaging by a power cable of a control cable associated with another feeder does not impair operation as long as the control cable can be repaired and restored to service before the occurrence of a second high-voltage fault.

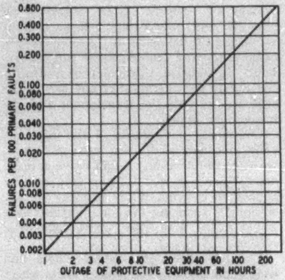

Figure 5. Curve showing probable failures per 100 primary feeder faults of network protector units to trip due to control pair and control equipment failures extending over various periods of time

In order to minimize the possibility of any damage to control cables from power equipment the control cables have been fireproofed at every vault in the same manner as the power cables and wherever feasible they are located not too close to the power cables or splices. As a check on the adequacy of such fireproofing, short-circuit tests were arranged at system voltage, using short sections of three conductor power cable and splices of such cable placed within five inches (surface to surface) on racks from a section of fireproofed control cable. An internal fault from one conductor to sheath was made before the test in each sample of cable and oscillograph elements arranged to record any change in normal direct current in any pairs of the control cable and also to record through a current transformer the passage of any current to the sheath of the control cable.

A series of 15 short-circuit tests were made energizing the faulted cable specimen with a three-phase grounded-neutral 11-kv power source of sufficient capacity to provide a ground current of 7,000 amperes. The duration of these short circuits was varied from 5 cycles to 30 cycles and in no case was there any disturbance to the control current and in none of the tests did the arc cause any current to flow in the control-cable sheath. Figure 6 illustrates the arrangement of test cables and shows burning of adjacent surfaces after a short-circuit test.

The extremely remote possibility of an entire control cable being out of service during a primary fault was examined by calculating the division of fault current through the entire network with a short on a primary feeder. Studies showed that all protector fuses on the faulted feeder would blow without damage to equipment. This situation should apply to any network designed for adequate burn-off characteristics.

Many failures of network protectors to clear primary faults are due to troubles in the network protector mechanisms. Since more than one-half of all network operations are classed as "miscellaneous" or unintentional[4] a control system which eliminates practically all of such operations should result in improving the performance records of network protector mechanisms for the same frequency of maintenance.

Maintenance

All devices have been applied with ample margin, especially those located inside the network protector housings. For example, the continuous current through the copper-oxide rectifier is less than 25 per cent of the rated current of this unit. The normal rectifier resistance is under 10 per cent of the total series circuit permitting wide variations with temperature without seriously affecting the value of operating current. Although it is not required for reliable action, the pilot wire operating current may be held within 10 per cent of its rated value by means of tapped resistors located in the substation even if aging of a rectifier unit should raise its forward resistance to as much as five times the normal value. It is evident that there are no fine adjustments to be made to the protective equipment even on the more accessible apparatus located at the substation. Protective maintenance then becomes primarily a matter of repairing any failures in the control cable.

A control system which eliminates practically all unintentional protector operations should reduce trouble and expense of maintenance on the network protector mechanisms. It has been noted on power systems that an oil circuit breaker which is approaching a stage of uncertain operation due to sticking, weak springs, wearing of latches and rollers, or similar causes will frequently give forewarning of such a condition by operating sluggishly on the tripping operation if it has been standing closed for sometime (although subsequent operations for tests may be at normal speed), or by failing to

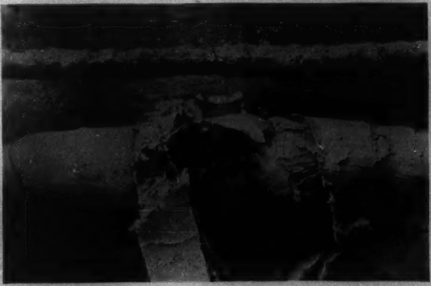

Figure 6. Tests illustrating integrity of remote control when exposed to a 7,000-ampere short circuit on adjacent power cable

latch on the first attempt to close it. It seems entirely probable that a system which calls audible and visible attention of the operator to any initial failure to latch or to any sluggishness in opening of a particular network protector will aid in a program of maintenance which should forestall many mechanical failures.

In contrast with a remote-control system the conventional network-protector relays must be adjusted very closely and slight variations in their characteristic due to temperature, corrosion, and loose connections, are quite serious. Many have found the maintenance and tests of network relays to be extremely difficult when performed at the network protector location[8] and two leading manufacturers now supply readily detachable relays so that these may be checked in the laboratory and then installed in the network protectors.

Additional Operating Features

Design of protective equipment has been concentrated on securing a thoroughly reliable installation at reasonable cost and it is planned that no other features will be added until after considerable experience and study of the present installations. However consideration may be given in the future to the installation of temperature and pressure relays in each network transformer using existing remote-control pairs. One possibility would be to permit the temperature and pressure contacts to send an alarm or tripping impulse over a common pair for all the units on one feeder cable with auxiliary contacts to trip the protector at the transformer involved and open its d-c control circuit thus providing instantaneous indications at the substation of the number and location of transformers involved without the use of coding. Consideration has sometimes been given to differential protection of network transformers but there has been no means of tripping the high-voltage feeder in case of low-voltage transformer trouble. One common pair for each feeder could be used for such a purpose, using the individual control pair for indicating the location of the transformer which had caused the feeder to trip.

Individual control of the network units can be used to considerable advantage without installing additional equipment. Consideration is being given to the installation at the substation of three sensitive single-phase wattmeters of low current range which may be connected to the current transformers on any feeder by means of transfer switches. Auxiliary current transformers would be connected wye-delta and used to bring wattmeter current in phase with current at the network-protector terminals. After a secondary fault or on any routine basis the operator can open a group of network protectors associated with the feeder and then close one of them with individual control jacks. The wattmeters may then be switched to the feeder supplying the one network unit and wattmeter readings would instantly show if there were any fuses which had blown during secondary trouble or due to mechanical defects.[9] Readings could be taken one at a time on all the network units on one feeder in a few minutes. Such a procedure would also indicate possible overloads on any transformer unit. This scheme has been tried using a special portable wattmeter and indications of a blown fuse are quite positive. Readings obtained from this method from the substation on individual units check very closely with current readings taken at the vaults.

Conclusions

1. The remote-control system described can be applied to the protection of many network systems at reasonable cost and in some cases at a substantial saving in cost over systems with conventional relays.

2. The remote-control system offers distinctive advantages in simplicity, flexibility, freedom from pumping, decreased maintenance trouble and expense on relays and protector mechanisms and suitability for a wide range of supply systems.

3. Operating experience must provide the final proof of the adequacy of any protective system, and the relatively low rate of primary failure on an underground network may require that such operating experience continue over a period of several years for conclusive results. Nevertheless it is felt that careful consideration has been given to all features and that operating and short-circuit tests have been carried far enough to predict excellent operation. Previous experience with pilot-wire circuits and their supervision points toward a protective design of high reliability.

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Discussion

L. H. Hill (Allis-Chalmers Manufacturing Company, Milwaukee, Wis.): Messrs. Brownlee and Dent have developed a low-voltage a-c network system devoid of all sensitive and unnecessary relays, without sacrificing the flexibility of the more complicated automatic non-pilot-wire control systems. In addition it provides the means for controlling the operation of any one of the switches between the transformer and the low-voltage mains.

Continuous supervision of the pilot circuits, and their subdivision into small parts, without complication, should prove entirely satisfactory. It is a scheme that may be applied to systems of any primary voltage without introducing the complications that would follow with other known pilot-wire systems.

The Brownlee-Dent system should find wide application in the smaller areas where the cost of duct space is nominal or of course also in larger areas if pilot wires can be economically installed.

J. B. Hodtum (Allis-Chalmers Manufacturing Company, Pittsburgh, Pa.): From a description of the authors' pilot-wire-controlled system, it appears that they have greatly simplified the operation of low-voltage a-c network systems. The third conclusion of their paper will really prove the merits of this system.

There is some background of course for the use of pilot-wire-controlled circuits as they have had wide use in Europe, especially in England. It seems that the authors have taken every possible precaution to supervise continuously and to guard against failure of any part of the control system involving the entire feeder. As pointed out, the control system may be extended to include, for example, differential protection of individual transformers which may even be made automatically to disconnect a feeder. It is questionable, however, whether automatic tripping of a feeder due to trouble, in a single unit, is advisable.

It appears that this system may be an advance in the art in that it permits individual transformers to carry loads in the reverse direction, since such operation would really mean that energy is being transferred from one part of the low-voltage system to another part over the high-voltage feeders instead of through the low-voltage network itself.

E. E. George (Tennessee Electric Power Company, Chattanooga): The authors have touched lightly on several important features of this new remote-control system. One feature is the possible use of control cables leased from a communication company. In Chattanooga and Nashville it was found that the telephone company could reach each transformer vault with a control pair over an underground route separate from the power route. This installation would usually be through the building adjacent to the transformer vault. However, the availability of a large amount of excess duct space made power-company-owned control cables somewhat cheaper.

If the communication company could establish a group rate for several pairs in the same area or in the same cable, avoiding long central-office loops there ought to be a considerable field of future business in certain cities that might use this new network control.

A feature that has appealed strongly to the operating, maintenance, and construction departments has been the availability of telephone service in each transformer vault. The portable telephones used by the underground crews and test engineers are furnished by the telephone company and permit ready communication between vaults and with the distribution dispatcher, assistant dispatchers, maintenance supervisors, and all other points.

This new system with its manual group or individual control of protector opening and closing by the operator at the supply station may be attractive to the metropolitan companies requiring sectionalizing in case of shutdown. We would like to have comments from those more familiar with this problem. Although about everything possible has been done to test out the new system, the real test will be the emergencies that may occur in the future without engineering, test, dispatching, and maintenance departments right on the scene. Such emergencies occur rarely, and it is hoped that other companies will make installations of this type of remote control so as to expedite the development work and to provide service for the benefit of the industry in general.

L. F. Kennedy (General Electric Company, Philadelphia, Pa.): The authors have made the statement that the limits placed on the closing characteristic of the network relay are due to the imperfect selectivity of its tripping characteristic. I should rather say that all the limits under which the network relay must operate, either to trip or reclose, are brought about by combining in one device both protective and control functions. It naturally follows that the device which meets these requirements needs more maintenance to insure reliable operation. The authors' familiarity with other protective relays will bear out the statement that as the operating requirements become more severe, the maintenance is increased.

The authors have installed the pilot-wire system at a saving in cost over the conventional network protector. At the same time they indicate savings in maintenance through the use of their system. But this has been done by setting up a new set of requirements, or rather the waiving of some of those normally met by low-voltage network protectors, and it would seem proper that any economic comparison then should be made between a network protector and a pilot system fulfilling the same requirements.

Relay equipment to trip on feeder faults only would be relatively simple, and if reclosing were permitted whenever the feeder voltage is approximately normal (corresponding to primary network practice), the over-all relay picture would be considerably changed. With such equipment relay and protector maintenance could be reduced and, since the protectors would operate only under short-circuit conditions, longer life would be obtained.

The pilot system operates basically in the same manner as the protector but in addition provides continual indication of each protector position and permits manual operation of the protectors either singly or by groups. The value of indication and control, even if considered to be small, is an item favorable to the pilot system.

It would seem necessary, therefore, to draw up a new set of operating requirements wherein the desired relay characteristics are definitely shown and then compare installed cost and maintenance of the two systems. If on such a comparison the pilot system is shown to be less expensive, it can unquestionably be justified. If the pilot system is more costly, it may still be justified by evaluating the indicating and control features.

I have purposely omitted any consideration of relative reliability in the foregoing comments. With simpler relay equipment I should be inclined to consider it more reliable than the pilot, but this is admittedly largely my own personal opinion. Certainly experience of the past few years has demonstrated that a pilot system of this type will show a high degree of successful performance. Given an equal amount of experience with both systems, perhaps the reliability should be considered equal.

To me the successful operation of the authors' system opens up the whole question of operating requirements of low-voltage networks. It is perhaps possible that these requirements have been made too severe and that we may see in the near future a general lessening of them accompanied by a reduction in maintenance expense on the whole system.

F. Von Voigtlander (The Commonwealth and Southern Corporation, Jackson, Mich.): The system of remote control of network protectors described in this paper appears to be a considerable advance in control systems of this type, and at the same time it achieves that goal sought by all of us, that of obtaining something better at a lesser cost compared to conventional methods.

The installation cited for a basis of comparative costs, involves a total of 60 network-protector units. There is, no doubt, a minimum system to which it would be economical to apply a network control scheme of this type. I would like to ask the authors' opinion of the approximate size of such a minimum system.

T. G. LeClair (Commonwealth Edison Company, Chicago, Ill.): A pilot-wire scheme for the control of network protectors is naturally of more interest to a Chicago engineer than any other because the Commonwealth Edison Company has made wider use of pilot-wire relay protection than perhaps any other metropolitan utility. Also, the operating record of the pilot-wire relaying system has been much better than that of other relaying systems. For many companies the cost of the pilot-wire control scheme for networks would be materially increased because of the duct cost which must be added to the cable cost. However, for larger cities a majority of the pilot cable would be installed in large duct sections where the central ducts cannot be used for power cables due to temperature limitations, but are available for pilot or control cable. Therefore, a large part of the duct cost charged to the pilot cable would be more theoretical than real.

The pilot-wire system suggested by Messrs. Brownlee and Dent might even be extended to include greater protection than at present by the addition of differential relays on the network transformers and a tripping circuit which would trip the primary and all the secondary breakers automatically for a fault on the transformer secondary in the zone now unprotected.

One missing feature of the scheme suggested in the paper is the inability to trip on back-feed through the network units without the use of the control equipment at the supply end. This is probably only of importance for a system fed from two independent sources where it is desirable automatically to transfer the entire network load to one source in the event of a major catastrophe which would make the feeder circuit breakers inoperative due to fire or explosion in the other source.

An item of maintenance expense which might be important in metropolitan systems is the relatively large number of cable cutovers which would be necessary on the pilot cable as the network system grows and transformers are frequently transferred from one feeder to another, either singly or in groups.

Chase Hutchinson (Tennessee Public Service Company, Knoxville): This type of system should appeal to many operating men since it offers a means of securing almost any desired information on vault conditions and at a low cost. Load, temperature, pressure, and other indications of condition can be transmitted, either to inform the operator, trip the vault protector, or trip the entire circuit. It eliminates much of the loss of time in checking protector positions and almost immediately locates a protector which sticks in the closed position.

Balancing some of these attractive advantages are some disadvantages the writer believes deserve careful consideration:

1. What is going to happen if a street cave-in or duct-line failure occurs? Can the network system

be depended upon successfully to blow simultaneously the required number of protector fuses?

2. In most cases the question of duct cost for the pilot cable cannot be neglected.

3. Since the pilot wire cost varies with the length of the feeder, and the network relay cost varies with the number of transformers, it appears the "beginning" cost of the pilot wire system might be higher than the conventional protector.

4. In attempting to peer into the future capital cost of these systems, one is inclined to argue thus: The pilot wire system is largely made up of "production items," and to a considerable extent is assembled in the various transformer vaults and manholes. The cost of this type of work is apt to increase in future years. On the other hand, the conventional protector has had to stand an enormous development and advertising cost. The present version is reliable, and with general acceptance of the system at hand it seems reasonable to believe the cost of protectors will decrease in the future years.

5. The spot network using pilot-wire protectors may be prohibitively expensive.

It appears to the writer that the system devised by Brownlee and Dent is comparable to the conventional network protector system equipped with supervisory control, consequently, where supervision of the protector and vaults is considered necessary, the scheme appears to offer a lower cost method.

J. H. Neher (Philadelphia Electric Company, Philadelphia, Pa.): The use of low-voltage a-c networks is in general limited to relatively large urban load centers, and as a result not many relay engineers have need to study this phase of the relay art. Despite this fact, however, all relay engineers can well profit by a careful study of this paper. The straightforward manner in which the authors, unencumbered by previous notions as to how network feeders could be relayed, have produced a new system, designed from the ground up to do the specific work at hand, should be a good guide in the design of other types of relay systems.

The authors' tests demonstrating the independence of the control circuits with respect to short circuits in the power circuits, are most interesting. The results of these tests, together with the relatively low cost of installation of the control cables, open up a large field for the use of power company owned-and-installed communication circuits for relay protection and remote control.

W. R. Brownlee and **W. E. Dent, Jr.**: The beginning cost of a remote-control network system will naturally be greater per network transformer than the complete installation as pointed out by Mr. Hutchinson. The answer to this question and also to Mr. Von Voigtlander's question concerning the minimum size network system to which remote control could be applied economically is best given by considering specifically the present status of the Chattanooga network. The control equipment cost per network transformer is determined partly by the fixed terminal cost at the substation, but more by the length of control cables as determined by the distance from the supply station to the beginning load center. In the case of the Chattanooga network the supply substation for the present three-cable system is 0.8 mile from the load center of a small area, resulting in a control equipment cost of almost exactly the value of the price reduction of the network protectors. The demand of the present Chattanooga network is 1,500 kw which could be considered as the minimum system (based on cost alone) with the supply point approximately one mile from the load center.

As pointed out by Mr. LeClair and Mr. Hodtum the remote-control system lends itself to the ready application of differential protection of network transformers. In conventional systems the protectors or backup fuses will usually separate the network from a short circuit on the secondary of a transformer but proper co-ordination of the high-voltage feeder relays prevents automatic disconnection of the primary feeder in case of transformer secondary trouble. Probably protection of this zone has been omitted due both to the inherent difficulty of such protection and to the infrequency of transformer faults involving the secondary only.

The authors do not agree altogether with Mr. Hodtum that differential protection of network transformers should merely sound an alarm at the substation. Present remote indicating equipment transmitting indications of over-temperature or over-pressure of a network transformer do not usually trip feeder breakers, since their indication is not altogether positive and since it may indicate merely a secondary fault between turns which would gradually build up pressure over a period of minutes or even hours. Since most of these present systems are looped through all transformer vaults and depend on coded signals for determination of the transformer in trouble they are not suitable for instantaneous disconnection of a feeder if such were desired. It is the authors' opinion that any fault in a transformer sufficient to operate a differential relay requires immediate action and should be disconnected within a few cycles. Probably the general application of differential protection will be governed by the frequency and seriousness of faults beginning on transformer secondaries and also by the effectiveness of temperature and pressure devices, many of which have been installed in the last three or four years.

Mr. LeClair raises a pertinent question in cases where a network is supplied from two or more busses in the same or different substations. Naturally a major failure of a feeder oil circuit breaker could prevent this breaker from clearing a short circuit or even from opening mechanically. Usually differential or backup-type bus protection will be employed to disconnect all sources of power from the bus and extra contacts of the bus protection auxiliary tripping relay should be connected to the shunt coil of the reversing relay (device 43 of figure 1) of each feeder, thereby preventing the network from feeding the short circuit regardless of the condition or position of the feeder breaker.

Mr. Hutchinson's question regarding duct-line failures apparently considers a serious and infrequent disturbance of such a nature as to destroy first one or more control cables and second the power cables associated with those particular control cables. The calculations mentioned in the paper in connection with this extremely remote possibility considered a fault on one of the three or four primary cables with its protection completely removed and it was found that the backup fuses would operate without damage to equipment. These network systems have not been designed to operate with two cables out of service on a three-cable or four-cable system and consequently no studies have been made of simultaneous faults on two or more primary cables with protection previously removed.

Mr. LeClair's question regarding maintenance expense of transferring control cables when network transformers are transferred from one feeder to another is rather difficult to answer. Obviously a reconnection of cables at the network units would involve appreciable maintenance expense. One possibility would be to leave the secondary cables connected at the network transformer end and merely exchange pairs between adjacent control panels at the substation end. Proper coding of the control cables would permit ready identification of pairs from a key drawing showing the routing of each separate control pair. The result of having three or four pairs of one feeder located in the 25-pair control cable for another feeder is not as serious as it might seem. The resistance of terminal equipment at the substation in series with each control pair is enough to limit short-circuit current in a control cable to a few tenths of an ampere. The voltage on the pairs is maintained at approximately 65 volts to ground through a resistance of some 40,000 ohms (see figure 1). Furthermore the accidental short-circuiting, open-circuiting, or grounding of a control pair does not result in improper operation of any network protector and accordingly all construction splicing and maintenance of control cables can be made if desired without removing battery supply from the cables.

The authors cannot agree with Mr. Kennedy that any network operating requirements have been waived, but are convinced that the long list of requirements of conventional relays are solely for the purpose of fulfilling properly the simple operating requirements outlined in the paper. Even if operators were willing to waive the disconnecting function of network protection, and send a man to each vault whenever a clearance is required, it is doubtful if much simpler equipment can be readily designed to trip on feeder faults only. That is, conventional relays set insensitive enough to prevent improper operation on various power reversals might not be sensitive enough to operate in case of a ground on a primary cable using customary delta-wye network transformers and shielded primary cable. The authors are pleased to have a manufacturer who sells large numbers of network protectors confirm their opinion that protector maintenance will be reduced and operation improved if all "miscellaneous" or unnecessary operations are eliminated.

Positive-Grid Characteristics of Triodes

By RICHARD W. PORTER
ASSOCIATE AIEE

Synopsis: The total emission current in a vacuum tube can be expressed by the familiar space-charge equation. This paper shows theoretically that the ratio of plate current to total current should be a function only of the ratio of grid to plate voltage, and introduces experimental data to show that the function can be represented by a single exponential term. The exponential function is combined with the space-charge equation to obtain an empirical equation for the positive-grid plate-current characteristic. A simple, experimental method of obtaining the constants in this equation is described.

THE NEED for higher efficiency in radio-frequency power amplifiers makes it desirable to operate vacuum tubes with grids strongly positive for a short interval in each cycle. When the grid of a tube becomes positive, not all the electrons which leave the cathode will go to the plate; some of them will be collected by the grid. Therefore, the plate-current equation[1] which has been used for negative and low positive grid potentials assuming negligible grid current, will not apply. This paper presents a modification of the usual equation which takes account of the division of current between grid and plate. The object is to obtain a simple expression which will give approximately correct values of plate current when the grid is strongly positive, rather than to attempt to analyze in detail the various phenomena which occur within the tube.

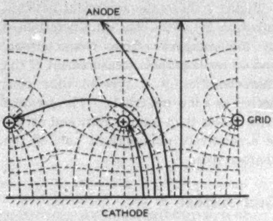

Figure 1. Possible electron paths in tube with positive grid

Total Space Current

It has generally been assumed that the basic space-charge equation

$$i = k(e_p + \mu e_g)^n$$

holds equally well for both positive and negative grid potentials, provided the current i includes all the electrons which leave the cathode, both those that go to make up the grid current and those that go to the plate. This assumption follows from the usual derivations based on the work of Langmuir[2] and Van der Bijl,[3] provided there be sufficient emission from the cathode to maintain the space charge, and that there be no unusual interaction between electrons after they leave the cathode space-charge region. Its validity is strikingly illustrated by the constancy of the total current in the following data for the Radiotron *UV* 207 in which grid and plate voltages were varied holding constant the sum of the plate voltage plus the grid voltage times the amplification factor.

The equation for the total space current can therefore be written

$$i_p + i_g = k(e_p + \mu e_g)^n \qquad (1)$$

where k, μ, and n are constant and have the same values when the grid is positive as when it is negative.

Division of Current Between Grid and Plate

In considering the problem of current division between the grid and the plate, first examine the path of a single electron which is emitted from some point on the cathode with negligible velocity. It will move as shown in figure 1 through a complicated electric field pattern, perhaps reaching the grid, perhaps the plate. If the grid and plate voltages are increased but kept in the same ratio, the magnitude of the field at any point will increase, but the field pattern will remain the same. Under these conditions, it seems reasonable that the electron paths also should remain the same, although the velocities may increase. This fact is proved mathematically in appendix *A*. The destination of any particular electron, therefore, will depend only on the ratio of the grid voltage to the plate voltage.

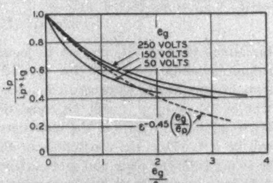

Figure 2. Current division in type-46 tube with grids connected together

Generalizing from the single electron to many electrons, there follows the important conclusion that the division of current between grid and plate should be a function only of the ratio of grid to plate voltage, and for any given ratio should be independent of the magnitudes of these voltages. The assumptions implied in this generalization are, (1) no emission of electrons by grid or plate, (2) no interaction between electrons after they leave the cathode space-charge region, (3) negligible initial velocity, (4) no interaction between the electrons and gaseous molecules. The nature of the functional relationship must be such that the ratio of plate current to total current will be unity when the grid voltage is zero, and will approach but never become zero as the ratio of grid to plate voltage becomes large.

Experimental Data

Data taken by the writer for the type-46 tube at high positive grid voltages tend

Table I

e_p	e_g	$e_p + \mu e_g$	i_p	$i_p + i_g$
500 v	500 v	10,500 v	2.3 a	3.35 a
2,500	400	10,500	3.1	3.20
4,500	300	10,500	3.25	3.25
6,500	200	10,500	3.30	3.30

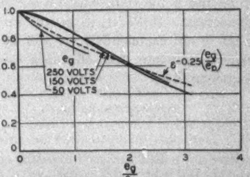

Figure 3. Current division in type-46 tube with grid 2 connected to plate

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RICHARD W. PORTER is employed in the engineering general department of the General Electric Company, Schenectady, N. Y. This paper is part of a dissertation presented for the degree of doctor of philosophy at Yale University.

1. For all numbered references, see list at end of paper.

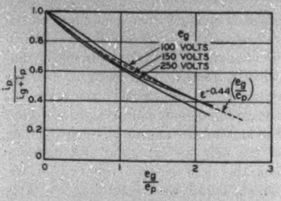

Figure 4. Current division in Westinghouse EX 50 50-watt triode

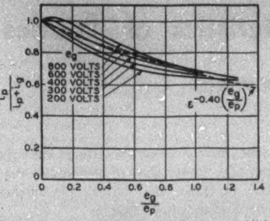

Figure 5. Current division in Radiotron UV 207, SKW triode

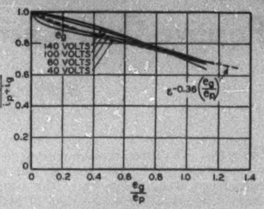

Figure 6. Current division in General Electric FP 169, 150-watt triode

to confirm these predictions and at the same time to justify the assumptions on which they were based. See figures 2 and 3, in which the ratio of currents is shown as a function of the ratio of voltages for various values of grid voltage. Except for the secondary effects which were assumed negligible, all the curves for each tube should be exactly the same. Similar curves have been plotted from data taken by Kozanowski and Mouromtseff[5] for the Westinghouse *EX* 50 tube and from published data on various RCA and General Electric tubes. Of these latter the *UV* 207 shows the highest discrepancy and the *FP* 169 the least. It is significant that the *UV* 207 characteristics show large secondary emission currents from the grid, particularly in the region of largest discrepancy, whereas secondary emission is not prominent in the *FP* 169. Data for the *EX* 50, the *UV* 207, and the *FP* 169 are shown in figures 4, 5, and 6.

These comparisons lead to the conclusion, perhaps already foreseen, that secondary emission is the principal reason for the failure of the current ratio to be a simple function of the voltage ratio. However, the greatest divergence observed was only about 17 per cent, or nine per cent variation from the mean. For the region near $E_g = E_p$, the greatest variation to be found in the published data is less than four per cent from the mean. Errors in measurement and drafting are sometimes as great as this.

The exact nature of the relationship between the current ratio and the voltage ratio depends on the construction of the tube and is too complicated a matter to treat here. However, its general form is that of an exponential. It will be assumed therefore that all tubes can be represented by

$$\frac{i_p}{i_p + i_g} = \epsilon^{-\alpha e_g/e_p} \qquad (2)$$

where α is a constant used to fit the equation to the data. For practical purposes it is usually well to choose α so the curve will match the data at $e_g = e_p$, for it is in this region that a tube delivers most of its useful power as a high-efficiency radio-frequency power amplifier. Representative exponentials are shown in dashed lines in figures 2 to 6. The error in this assumption is seen to be approximately the same as that introduced by assuming the existence of a simple functional relationship between ratios.

Plate Current

Combining the expression for total space current (1) with that for the fraction which goes to the plate (2), we obtain an equation for the plate current alone

$$i_p = k(e_p + \mu e_g)^n \epsilon^{-\alpha e_g/e_p} \qquad (3)$$

This equation as it stands is correct only when e_g and e_p are both positive. When the grid is negative the exponential term remains equal to one and when the plate is negative, it is zero.

It is not necessary to have complete oscillographic data on a tube in order to evaluate the constants of equation (3). They can be determined by a few simple measurements of plate and grid current at low voltage. For the composite diode characteristic $e_g = e_p$ equations (1) and (3) become

$$i_p = k(1 + \mu)^n e_g^n \epsilon^{-\alpha} \qquad (4)$$

$$i_p + i_g = k(1 + \mu)^n e_g^n \qquad (5)$$

Taking the logarithm of both sides,

$$\log i_p = \log \left\{k(1 + \mu)^n\right\} + n \log e_g - \alpha \qquad (6)$$

$$\log (i_p + i_g) = \log \left\{k(1 + \mu)^n\right\} + n \log e_g \qquad (7)$$

Equations (4) and (5), plotted to logarithmic co-ordinates, will be parallel straight lines having slope n. The intercept of (5) at $e_g = 1$ will be equal to $k(1 + \mu)^n$ and the ratio of the intercepts will be $\epsilon^{-\alpha}$. Thus the constants can all be found from the plate and grid current data for a composite diode characteristic and the value of μ, which may be determined in any of the usual ways.

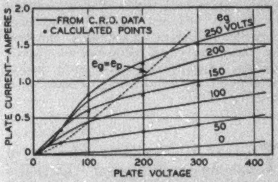

Figure 7. Positive-grid plate characteristic of type-46 tube, grid 2 connected to plate, showing comparison with calculated points

Figure 7 shows the agreement between equation (3), with constants determined by the composite diode characteristic, and an oscillographic measurement of the characteristics of a type-46 tube connected as a low-mu triode. The greatest error is less than ten per cent and occurs in a relatively unimportant part of the characteristic.

Grid Current

Subtracting equations (1) and (3) the grid current is found to be

$$i_g = k(e_p + \mu e_g)^n [1 - \epsilon^{-\alpha e_g/e_p}] \qquad (8)$$

When the constants for this equation are determined from the diode characteristic, it will hold very well for points near $e_g = e_p$ where the grid current is large. Although not nearly so accurate as the plate-current equation, the grid-current expression is a welcome aid in approximating the grid current and driving power in radio-frequency amplifiers.

Summary

It has been shown that the equation

$$i_p = k(e_p + \mu e_g)\epsilon^{-\alpha e_g/e_p}$$

will give values for the plate current of a triode in which the plate and grid are both positive, correct to within about ten per cent for a wide variety of tubes and voltages. The two most important approximations to which it is subject are (1) that secondary emission from the grid is negligible, (2) that the relation between $i_p/(i_p + i_g)$ and e_g/e_p can be represented by an exponential for any tube. Values for the constants in this equation were determined from a composite diode characteristic at low voltage and found to be accurate at much higher voltage and current. Finally it was seen that the grid current could be obtained approximately by taking the difference between this expression and that for the total space current.

This mathematical expression for positive-grid tube characteristics is extremely simple in form and sufficiently accurate for engineering purposes. It is valuable, because it avoids complicated and expensive oscillographic measurements of tube characteristics, and because it provides a new basis for attacking practical vacuum-tube problems.

Appendix A

Consider the path of a single electron which starts from some point on the cathode with negligible velocity. Its dynamical equations in rectangular co-ordinates will be

$$\frac{d^2y}{dt^2} = \frac{e}{m} E_y(x, y) \qquad (1a)$$

$$\frac{d^2x}{dt^2} = \frac{e}{m} E_x(x, y) \qquad (2a)$$

where $E_y(x, y)$ and $E_x(x, y)$ are scalar functions of the co-ordinates x and y, equal to the field intensity along the x and y axes, respectively. It is assumed that E_z is zero.

Equation (1*a*) may be written in the form,

$$d\left(\frac{dy}{dt}\right) = \left(\frac{e}{m}\right) E_y\,(x, y)\, dt \qquad (3a)$$

or

$$\left(\frac{dy}{dt}\right) d\left(\frac{dy}{dt}\right) = \left(\frac{e}{m}\right) E_y(x, y)\, dy \qquad (4a)$$

Integrating (4*a*)

$$\frac{1}{2}\left(\frac{dy}{dt}\right)^2 = \left(\frac{e}{m}\right)\int_{y_0}^{y} E_y\,(x, y)\, dy \qquad (5a)$$

Similarly from equation (2*a*)

$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 = \left(\frac{e}{m}\right)\int_{x_0}^{x} E_x\,(x, y)\, dx \qquad (6a)$$

Dividing (5*a*) by (6*a*)

$$\left(\frac{dy}{dx}\right)^2 = \frac{\int_{y_0}^{y} E_y(x, y)\, dy}{\int_{x_0}^{x} E_x(x, y)\, dx} \qquad (7a)$$

Equation (7*a*) is the differential equation of the path of the electron, the time element having been eliminated. Now if the grid and plate potentials are increased or decreased in the same ratio, E_y and E_x will both be increased or decreased by the same constant multiplier, which may be removed from the integral signs and divided out. Consequently, for any tube the path of a single charged particle emitted by the cathode is a function only of the ratio of electrode voltages, and not of the magnitudes.

Nomenclature

i = current in amperes
i_g = grid current in amperes
i_p = plate current in amperes
μ = constant, amplification factor
k = constant, conductance factor
n = constant, space-charge exponent theoretically equal to 3/2
α = constant, distribution factor
e_p = plate voltage
e_g = grid voltage
e/m = ratio of charge to mass for an electron
E_y = y-component of electric field intensity
E_x = x-component of electric field intensity

References

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3. **Van der Bijl**, *Verh. D. Phys. Ges.*, volume 15 1913, page 338; *Phys. Rev.*, volume 12, 1918, page 180; Thermionic Vacuum Tubes, chapter IV.

4. **Maxwell**, Electricity and Magnetism, volume I, page 310.

5. **Kozanowski** and **Mouromtseff**, P.I.R.E., August 1933, page 1082.

6. **Chaffee**, Theory of Thermionic Vacuum Tubes, chapter VII.

7. **Morecroft**, Principles of Radio Communication, chapter VI.

Discussion

Karl Spangenberg (Stanford University, Calif.): Mr. Porter is to be commended for a clear-cut presentation of the factors involved in the positive-grid characteristics of triodes.

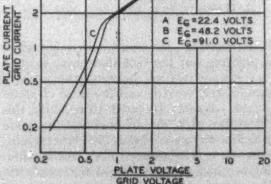

Figure 1

A limitation of the particular fundamental relationship assumed may however be indicated. The assumed relationship is of the form

$$y = e^{-\alpha x}$$

This form permits of the selection of but one parameter in fitting the empirical formula to the observed experimental data. That is, it is only possible to fit the empirical formula to the observed data at one point. Since a choice of ordinate and curvature are not independent in the assumed form the rest of the curve may or may not fit the experimental form.

The fundamental relationship assumed by Porter for the ratio of plate to space-current leads to an admitted error of considerable magnitude in the derived expression for grid current. This may be expected from the fact that while the diversion of a fraction of the space current to the grid is a second-order effect as far as plate current is concerned, it is a first-order effect for grid current.

A possible alternative basic relation is one of the form

$$i_p/i_g = f(e_p/e_g)$$

This form was proposed by early German investigators Tank[1] and Lange.[2] They showed that the functional form was a simple power of the ratio e_p/e_g for which the exponent has very nearly the value ½ for all tubes. The coefficient, however, varies from tube to tube. This relation has been verified by the writer for small modern transmitting tubes.[3] Essentially the same result was arrived at independently by Myers.[4] This permits the use of the basic equation

$$i_p/i_g = \delta(e_p/e_g)^n$$

which permits the selection of two parameters, the exponent and the coefficient.

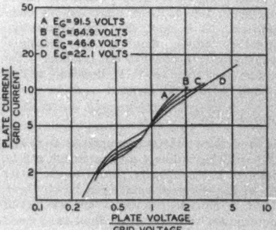

Figure 2

This form further expresses the relation in terms of the current ratio which experiences the greatest variation with voltage.

The coefficient and exponent in the above form are readily obtained by plotting i_p/i_g against e_p/e_g on log-log paper. The exponent is the slope of the straight-line characteristic obtained and the coefficient is the intercept upon the ordinate at $e_p/e_g = 1$. Such a plot is shown in figure 1. It will be observed that for values of e_p/e_g greater than one the curves are almost coincident straight lines with about the same slope which measures to be nearly $^1/_2$. It should be mentioned that no satisfactory theoretical explanation has as yet been given for the apparent universality of this value of the exponent in this region.

A further advantage of a plot of the type shown in figure 1 which may be mentioned is that it reveals readily any changes in the functional form which may occur in the ratio i_p/i_g. Such a change occurs in the curves of figure 1 for values of e_p/e_g of about 0.8. For values of e_p/e_g less than 0.8 the exponent as determined by the slope of the curves has a new value. The change to the new exponent occurs when the ratio e_p/e_g becomes sufficiently small so that some of the electrons which initially miss the grid are deflected strongly enough so that they are unable to reach the plate and fall back into the grid wires. For values of e_p/e_g greater than 0.8 all the electrons which initially miss the grid wires are received by the plate.

It should be pointed out that the coincidence of the curves in a plot like that of figure 1 depends upon the complete absence of secondary emission. Even the slightest traces of secondary emission will destroy the coincidence. Figure 2 shows the form of the curves for a tube with a slight amount of secondary emission. It will be observed that the curves are no longer coincident except in the vicinity of e_p/e_g where the exchange of secondary electrons between plate and grid tends to compensate. All of the curves given by Mr. Porter show considerable secondary emission by their similarity to the curves of figure 2 when plotted in that form. No satisfactory expression for current division in the presence of secondary emission has as yet been found.

It was not found possible to make the data of figure 1 fit Mr. Porter's equation because the value of α which made the curve have the proper value for $e_p/e_g = 1$ gave a curve whose curvature was much too small.

This is shown in figure 3 where the curves of figure 1 have been replotted for comparison. It is seen that the exponential function which matches the measured values for e_p/e_g equal to one is a poor fit for values less than this. If, however, an empirical curve of the form

$$y = 1 - kx^n$$

is used there are two parameters to select. A particular ordinate and curvature can be selected independently, and a much better fit to the observed values can be obtained. In this empirical formula the coefficient determines the intercept upon $e_p/e_g = 1$ and the exponent the curvature. This form permits the choice of the proper curvature from a whole family of curves with the required exponent.

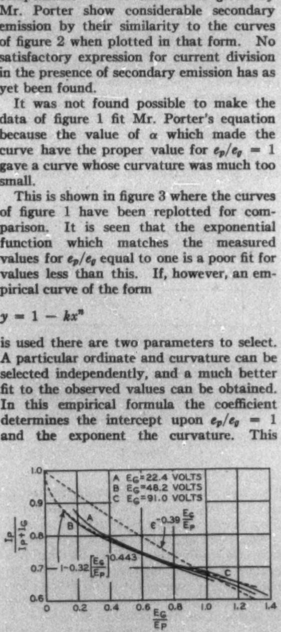

Figure 3

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1. **F. Tank,** *Jahr. der Draht. Tel. und Tel.*, volume 20, 1922, page 80.

2. **H. Lange,** *Seit. für Hochfrequenz.*, April 1928.

3. **W. L. Everitt** and **K. Spangenberg,** *Proc. I.R.E.*, May 1938.

4. **D. M. Myers,** *Proc.* Phys. Soc., May 1937.

R. W. Porter: Mr. Spangenberg is correct in pointing out the fact that there exists a very simple relation between the ratio of plate-to-grid current and the ratio of plate-to-grid voltage. In order to combine this expression with the space-charge equation in such a way as to obtain the plate current, however, it is necessary to perform manipulations which destroy some of the simplicity. The empirical form shown in figure 3 will not hold for very large ratios of grid-to-plate voltage since it indicates the possibility of negative plate currents.

In general all these expressions lead to difficult integrations when applied to radio-frequency design problems, so frequently it is of great advantage to have only one parameter involving division of current between grid and plate even if the empirical curve does not match the data perfectly at all points. The chief justification for permitting a certain amount of mismatch is that under normal operating conditions most of the plate and grid current will flow when the ratio of grid-to-plate voltage is nearly one. Hence the error in design calculations can be made small even when a single parameter is used. A second consideration is that secondary emission has a tendency to increase the ratio $i_p/(i_p + i_g)$ for small values of e_g/e_p and to decrease it for high values. Consequently any spreading of the curves shown in figure 3 as a result of secondary emission will be in the direction of the exponential curve.

Tests on and Performance of a High-Speed Multibreak 138-Kv Oil Circuit Breaker

By PHILIP SPORN
FELLOW AIEE

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MEMBER AIEE

The History and Results of High-Speed Breaker Development

HIGH-speed-oil-circuit-breaker development was really started in the period 1929–1930 and the first tests with high-speed 138-kv breakers were made at Philo in July 1930. The results of these tests, both from a breaker-design standpoint and from a system viewpoint, were described some seven years ago.[1] At that time it was definitely expected that operating benefits would be obtained out of the additional high speed but the problem of high speed was not yet then an acute or pressing one.

Parallel with that, although really anteceding that, was the development of high-speed relaying. This was first developed in what proved to be an impractical form in 1926. The original aim at that time was for a universal relay system that would operate, that is, would give an impulse to the breaker trip circuit within 20 cycles. The two developments went along in parallel although actual practical achievement in breaker speeds was attained much earlier than the relaying. However, by 1935 both achievements had been brought to such a state of perfection that it was entirely a routine matter to be able to get a six-to-eight-cycle 132-kv breaker in fairly standard construction and to get a relay system that would give universal protection in a period of one cycle or less. The history of the relay development has already been fully described.[2] By that time also the development in breakers had been carried through not only in the 138-kv classes but in the lower voltage classes as well. For special services, such as the Boulder Dam job, breakers of much faster speed were needed and three-cycle breakers of the impulse type were actually developed and built.[3]

Both of these developments have had the widest influence on the development

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1. For all numbered references, see list at end of paper.

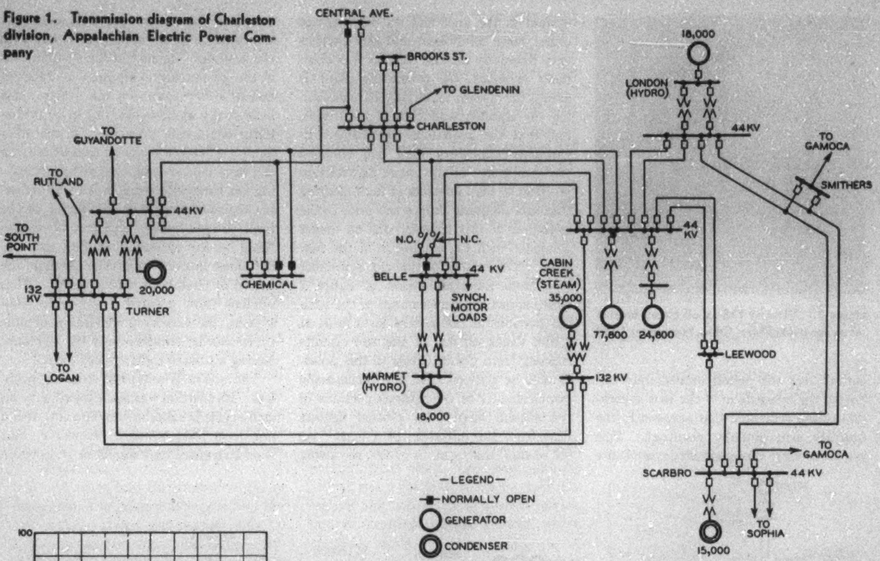
CENTRAL AVE.
BROOKS ST.
TO GLENDENIN
CHARLESTON
18,000
LONDON (HYDRO)
44 KV
TO GAMOCA
SMITHERS
TO GUYANDOTTE
TO RUTLAND
44 KV
TO SOUTH POINT
132 KV
20,000
TURNER
TO LOGAN
CHEMICAL
N.O.
N.C.
BELLE
44 KV
SYNCH. MOTOR LOADS
MARMET (HYDRO)
18,000
CABIN CREEK (STEAM)
35,000
77,800
24,800
44 KV
132 KV
LEEWOOD
SCARBRO
TO GAMOCA
44 KV
TO SOPHIA
15,000
—LEGEND—
NORMALLY OPEN
GENERATOR
CONDENSER

Figure 1. Transmission diagram of Charleston division, Appalachian Electric Power Company

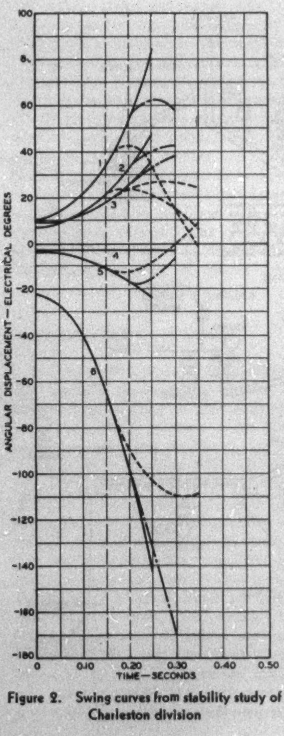

Figure 2. Swing curves from stability study of Charleston division

1—London	4—Turner 132-kv system
2—Marmet	5—Scarbro condenser
3—Cabin Creek	6—Belle motor load

——— Fault on

— — — Fault cleared by simultaneous opening of breakers in 0.20 second = 12 cycles

- - - - Fault cleared by simultaneous opening of breakers in 0.15 second = 9 cycles

and successful performance of modern transmission lines and networks. The effect on electric-power-system development and operation of these developments in breakers and relays can best be illustrated by a few typical examples of problems arising in transmission system operation and performance where the solution from a practical standpoint can be obtained only by a faster clearing of faults, which in turn can be obtained only by a high-speed breaker and in many cases only by a high-speed breaker coupled with a fast relay system.

Figure 1 shows a portion of the transmission system of the Appalachian Electric Power Company which operates in Virginia and West Virginia. This transmission system also extends to the affiliated companies, the Kentucky and West Virginia Power Company, Incorporated, and Kingsport Utilities, Incorporated, operating in Kentucky and Tennessee, respectively. The portion that is of particular interest is that part of the Charleston division lying between the Turner substation, which is west of Charleston, W. Va., and the Cabin Creek plant located at Cabin Creek Junction, W. Va., which is the principal steam plant serving that region and the immediately surrounding area. The loads in this territory are comparatively heavy, being predominantly coal and chemical operations. At the Belle substation, some six miles from Cabin Creek, a single load which some eight or nine years ago started at about 3,000 kw has now increased to approximately 42,000 kw, the principal use of power being in the operation of synchronous motors as large as 2,700 horsepower which drive high-pressure (1,200-atmosphere) compressors. Although some difficulty was experienced with surges in the early days when this load was building up, the problem seemed to have been entirely cleared up after an extensive investigation was carried out some five years ago, which showed the desirability of making some control changes on the motors and speeding up of the breakers on all the lines in the immediate neighborhood of the load.

About $2^{1}/_{2}$ years ago an additional source of generation was brought in almost directly on the bus bar of the main substation supplying this load. It was expected that this would have a beneficial effect on the stability of the supply system, but a series of disturbances that oc-

Figure 3. View of 138-kv oil circuit breaker set up for test at Philo, Ohio, November 1937

curred over the period immediately following the bringing in of the new generation source indicated that apparently the contrary had actually occurred. This led to a very thorough analysis of the system in the area and its performance under surge conditions, and this analysis very definitely showed that as was originally expected, the generating plant is aiding the system materially in maintaining the stability and voltage conditions, but that the growth of the large synchronous-compressor load had changed the complexity of the entire distribution problem so that a system of fault clearing that was adequate three years prior to the initiation of this trouble, was no longer adequate with the coming in of the new plant. The reason for this can be further seen from an examination of figure 2 which shows the performance of the various portions of the system for a fault at Cabin Creek on one of the two circuits running from Cabin Creek to this point. It will be noticed that the phase-angle displacements of the various portions of the system have been plotted against time from the initiation of a fault. At 0.2 second, that is at 12 cycles, the swing curve of the motor (curve 6) definitely shows the motors will fall out of step with the system. The rest of the system, that is the generating equipment at Marmet and at other points on the system, is a little shaky at this point but in all probability could ride through. On the other hand it will be clearly seen that by cutting the time 0.05 second, that is by decreasing the time only three cycles and reducing the maximum time of clearing of the fault to nine cycles (0.15 second) everything on the system, including the synchronous motors, will ride through the period of trouble in perfect safety. It is obvious that, granted a one-cycle relay system, the necessary maximum of nine cycles can be obtained only by a breaker having a time of eight cycles or less.

The above is a typical stability problem. Its solution was made possible by an eight-cycle breaker in combination with a one-cycle relay system. However, that is no guarantee that similar or other more

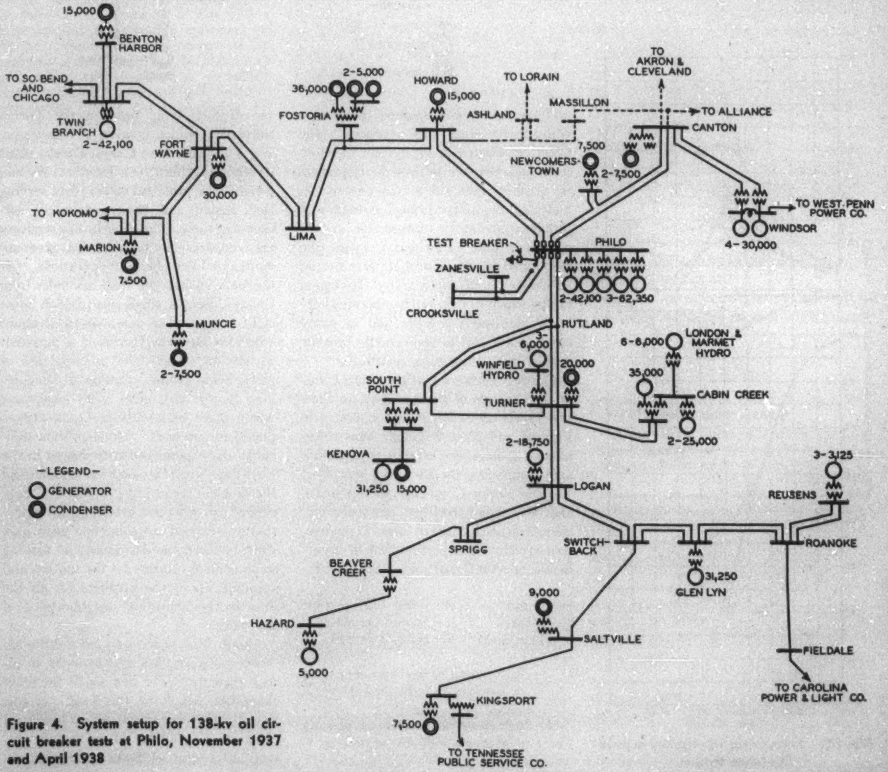
15,000
BENTON HARBOR
TO SO. BEND AND CHICAGO
TWIN BRANCH
2-42,100
FORT WAYNE
30,000
TO KOKOMO
MARION
7,500
MUNCIE
2-7,500
LIMA
2-5,000
36,000
FOSTORIA
HOWARD
15,000
TO LORAIN
ASHLAND
MASSILLON
TO AKRON & CLEVELAND
TO ALLIANCE
CANTON
7,500
NEWCOMERS-TOWN
2-7,500
TO WEST. PENN POWER CO.
WINDSOR
4-30,000
TEST BREAKER
PHILO
ZANESVILLE
2-42,100 3-62,350
CROOKSVILLE
RUTLAND
3-6,000
WINFIELD HYDRO
20,000
6-6,000
LONDON & MARMET HYDRO
35,000
CABIN CREEK
SOUTH POINT
TURNER
2-25,000
KENOVA
31,250 15,000
2-18,750
LOGAN
3-3,125
REUSENS
—LEGEND—
GENERATOR
CONDENSER
SPRIGG
SWITCH-BACK
ROANOKE
31,250
GLEN LYN
BEAVER CREEK
9,000
SALTVILLE
HAZARD
5,000
FIELDALE
TO CAROLINA POWER & LIGHT CO.
KINGSPORT
7,500
TO TENNESSEE PUBLIC SERVICE CO.

Figure 4. System setup for 138-kv oil circuit breaker tests at Philo, November 1937 and April 1938

involved problems in the future will not call for even faster time. If it is granted that one-cycle relay time is about as fast as can practically be sought, it is obvious that more breaker speed and speed beyond eight cycles is needed.

As a matter of fact such a need arose in connection with the development of ultrahigh-speed reclosing of transmission lines.[4] Here, although ultrahigh-speed reclosing was successfully accomplished on a breaker having very little better time than eight cycles, it was evident that the amount of waiting time with the line de-energized, necessary to bring about deionization, was not only a function of the arc current and the atmospheric conditions, but was also dependent upon the duration of arcing. Obviously if the arcing time was reduced, the total time necessary for reclosure would be reduced not only by that time but also by the additional gain as a result of the reduced waiting time needed with line de-energization.

A realization that the solution of many transmission problems, as typified by the two outlined above, was possible only by virtue of the fact that high-speed interruption was available when needed led to a decision some two years ago to apply the knowledge gained in the development of the impulse breaker to the development of a breaker having substantially higher speeds than appeared possible with the conventional oil blast breaker, and to do so along lines that would give the breaker no insurmountable economic handicaps in finding application on normal and commercial power systems. The results of that development are being described in the companion paper of Spurck and Strang.

Aims and Objects of Tests

The object of the tests herein described was basically to determine whether a breaker designed along the lines outlined[5] would successfully perform under the severe duty and test conditions that it was possible to set up at Philo. It was felt that in the light of the broad background of experience on breaker developments and tests obtained over a period of some 12 years, it was not safe to assume that the design difficulties encountered in making a change such as is involved in the development described could be overcome without thorough field testing. It was felt that these difficulties were so great that even with the fullest theoretical background and with the maximum amount possible in factory testing, it was not reasonable to expect even the most skilled designer to take care with finality of all the problems that were bound to arise. Hence the only safe basis of procedure, particularly since the equipment was protective equipment, was to subject the design to severe field tests and thus bring to light initial design errors and weaknesses. Past experience had indicated that those were almost inevitable and that designs put through the finishing developmental stages made possible by such tests were far more likely to operate trouble-free and completely successfully than those that were not subjected to the rigid experience of full field tests.

In short, it was hoped through this development and these tests to get a breaker having a time possibly close to four cycles in a design sufficiently simple, however, as to involve no appreciable, if any, increase in cost. Past experience had indicated that with a growing power system, having available such designs on an economical basis when they were actually needed was a material factor in maintaining not only reliability of system service, but in keeping system growth within economic limits.

System Setup for Tests

The Philo plant on the system of The Ohio Power Company provides the greatest concentration of power on the American Gas and Electric Company 132-kv system and was, therefore, chosen again, as in 1930, as the best place to test the new oil circuit breaker. As a matter of fact, the test breaker was set up in almost the same location with respect to the 132-kv switchyard as in 1930,[1] and it was connected in the same way to the transfer bus so as to make use of one of the Crooksville oil circuit breakers as a back-up breaker. A view of the breaker setup for test is shown in figure 3.

Likewise, the system setup as shown in figure 4 did not differ materially from that available for the 1930 tests, although a number of important system changes have been made since then. These include the double circuiting of the line from Philo all the way to Twin Branch, the installation of a 36,000-kva synchronous condenser at Fostoria, the placing in service of the hydro plants at London, Marmet, and Winfield, and the erection of a new 132-kv line from Turner to South Point. Except for the Howard line, which contributed substantially more current to the short circuit because of double circuiting, the effect of these changes in system setup and generation made very little difference in the short-circuit current contributions from these lines for a short circuit at Philo. At the Philo plant although the electrical end of the plant itself is exactly the same as in 1930, the actual short-circuit current supplied by the generators, as well as the total short-circuit current, was substantially greater due to the increased mechanical speed of the breaker and the consequent smaller decrement occurring prior to parting of the breaker contacts. This factor, together with the increase in system capacity mentioned above, made it possible to record tests as high as 2,000,000 kva based upon the initial current in the arc.

Description of the New Breaker

A detailed description of the design and theory of the new multibreak inter-

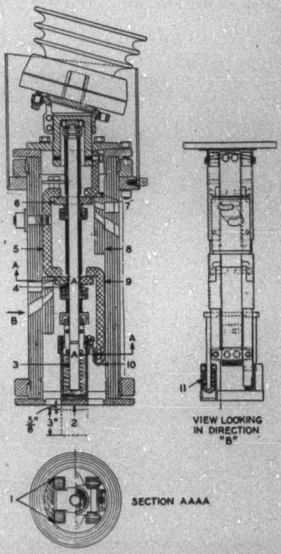

Figure 5. Cross section of multibreak, high-speed interrupter used in oil circuit breaker tested at Philo, April 24, 1938

1—Contact support rods
2—Contact cap
3—Springs
4—Contact pressure springs
5—Stationary contact bar
6—Movable contact
7—Stationary contact bar
8—Port segment
9—Stationary contact bar
10—Flexible connection
11—Opening springs

rupter, with particular reference to the six-break, 230-kv type, is given in the Spurck and Strang paper. The interrupters used in the test breakers at Philo were, of course, of the four-break design and were installed in a 138-kv conventional type of oil circuit breaker using standard 60-inch round tanks as shown in figure 3. A cross section of the interrupter itself is shown in figure 5, and a photograph of the interior parts alongside of the enclosing cylinder is shown in figure 6. The latter shows the general appearance of the interrupter, including the double-port opening in the side of the enclosing shell, whereas the cross section (figure 5) shows the arrangement of the four breaks, current carrying parts, location of port openings with respect to the interrupting breaks, and the compression springs along the central insulating rod.

The high-speed operating mechanism used on this breaker was similar to that shown schematically in figure 4 of the Spurck and Strang paper, except that it was designed for ultrahigh-speed reclosing, although most of the parts necessary for reclosing service were omitted. As shown in figure 3, this mechanism was mounted on the end of number 1 tank of the breaker.

Test Program and Results

Three series of tests, the first on November 20 and 21, 1937, the second on November 28, 1937, and the third and final series on April 24, 1938, were carried out on this breaker. This program may be summarized briefly as follows:

(*a*). Interrupters of the single-port design as described by Spurck and Strang were used in the first series of tests but failed to withstand the internal pressures developed in connection with currents somewhat above their rating.

(*b*). In the second series of tests, using two poles only, one pole was equipped with single-port interrupters, while the second pole was equipped with double-port interrupters and larger total port area. As in the previous tests, the single-port interrupters failed to withstand the pressures developed at currents somewhat above their ratings, whereas the double-port interrupters performed successfully and the series of tests was completed on a single-phase basis.

(*c*). The third and final series of tests was carried out on a complete three-phase basis, using interrupters with double-port openings and substantially strengthened enclosing cylinders, together with other minor modifications. The breaker came through these tests in a completely successful manner.

Detailed results of these tests are given in tables I, II, and III covering the first, second, and third series of tests, respectively. These results will be described in more detail below:

TESTS OF NOVEMBER 20 AND 21, 1937

As shown in table I, six preliminary tests were made, using one generator only, the first test primarily to check the test setup, and the other five with varying excitations on the generator to furnish information in connection with generator reactance characteristics. The remainder of the tests on November 20, included, first, a series of eight shots with two

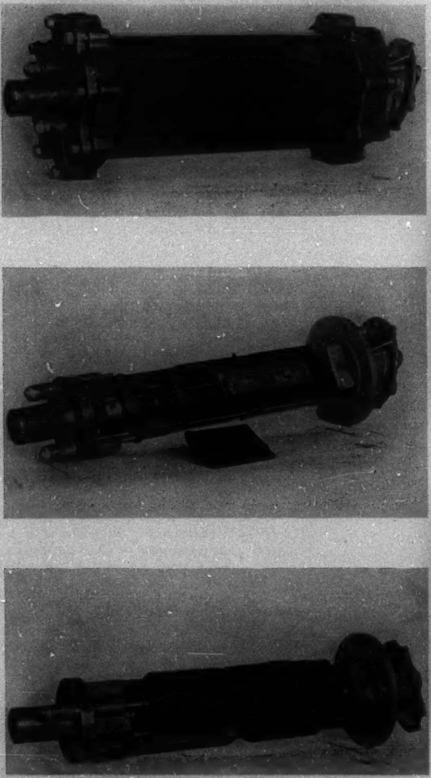

Figure 6. View of interior parts and enclosing cylinder of interrupter after tests at Philo, April 24, 1938

Figure 7. Oscillogram of test number 7, November 20, 1937, showing interruption of 1,500 amperes in 3.5 cycles

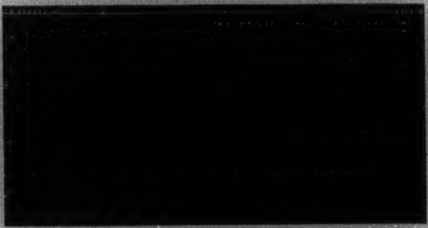

Figure 8. Oscillogram of test number 38, November 28, 1937, showing interruption of 8,500 amperes, single phase, in 3.7 cycles

Table I. First Series of Tests—November 20 and 21, 1937

Test Number	Type	Machines at Philo (Kva)	Lines Connected to 132-Kv Bus	Initial Current in Arc 1	Initial Current in Arc 2	Initial Current in Arc 3	Breaker Time (Cycles) 1	Breaker Time (Cycles) 2	Breaker Time (Cycles) 3	Equivalent Kva Interrupted	Remarks
1	O	1 42,100	None	1,000	940	990	Delayed tripping			230,000	Tests for generator reactance data
1A	O	1 42,100	None				Delayed tripping				
2	O	1 42,100	None	92	92	92	Delayed tripping			6,400	0.25 normal voltage
3	O	1 42,100	None	170	170	170	Delayed tripping			21,000	0.50 normal voltage
4	O	1 42,100	None	240	240	240	Delayed tripping			42,000	0.75 normal voltage
5	CO	1 42,100	None				Delayed tripping				1.00 normal voltage
6	O	1 42,100	None				Delayed tripping				1.10 normal voltage
7	O	2 42,100	None	1,500	1,400	1,200	3.7	3.5	3.4	360,000	Cleared OK
8	O	2 42,100	None	1,500	1,300	1,400	3.8	4.1	3.4	360,000	Cleared OK
9	CO	2 42,100	None	1,400	1,500	1,500	4.1	4.0	3.9	360,000	Cleared OK
10	CO	2 42,100	None	1,450	1,200	1,600	4.6	4.4	4.2	380,000	Cleared OK
11	O	2 42,100	1—Canton line open at far end	1,600	1,450	1,200	3.0	3.5	3.8	380,000	Cleared OK
12	O	2 42,100		1,200	1,400	1,600	4.2	3.6	3.4	380,000	Cleared OK
13	CO	2 42,100		1,300	1,350	1,350	4.3	3.7	3.9	320,000	Cleared OK
14	CO	2 42,100		1,200	1,400	1,300	4.3	3.7	4.0	335,000	Cleared OK
15	O	1 187,000	2—Howard, 2—Rutland closed to system	4,200	3,900	4,700	3.9	3.3	4.2	1,100,000	Cleared OK
16	O	1 187,000		4,300	4,300	4,200	4.2	3.6	3.9	1,000,000	Cleared OK
17	CO	1 187,000		4,500	4,000	5,300	3.1	2.5	1.5	1,270,000	Cleared OK Poor contact number 1 pole
18	GO	1 187,000		4,000	4,800	3,800	3.0	1.4	2.7	1,150,000	Cleared OK
19	O	1 187,000 1 42,100	All except Crooksville	6,500	6,700	6,350	3.5	3.4	3.8	1,600,000	Cleared OK Poor contact number 1 pole
20	O			6,800	6,700	6,300	3.6	3.5	3.3	1,620,000	Cleared OK
21	CO			5,900	7,800	6,600	6.0	0.5	0.3	1,860,000	Cleared OK. Heavy jar, poor contact number 1 and number 3 poles
22	CO	1 187,000 1 42,100	All except Crooksville	0	5,800	6,400		3.5	3.4	1,530,000	Cleared OK. No current number 1 pole. Poor contact number 3 pole Interrupter number 1 pole shattered on test number 21
23	O	1 187,000 2 42,100	All	0	0	8,000			3.6	1,900,000	Cleared OK. Tests number 3 pole only
24			All	0	0	7,600			3.2	1,820,000	Cleared OK
25			All	0	0	7,300			3.3	1,750,000	Cleared OK. Heavy jar. Interrupter number 3 pole shattered

generators only, including four tests with and four without a transmission line connected to the 132-kv bus to furnish a comparison between high- and low-voltage recovery rates. While the longest breaker time occurred with the high recovery rate setup where no lines were connected, the average results showed no consistent difference. This comparison, of course, was not particularly satisfactory because of the relatively low short-circuit current involved. It was not feasible to make such a comparison at higher levels of short-circuit current. All of these tests, 1 to 14, inclusive, in table I, were made on a setup completely isolated from the rest of the system.

Tests 15 to 18, inclusive, at a duty slightly above 1,000,000 kva, were carried out on November 21 without incident, with breaker time from less than two cycles to slightly over four cycles. On the next four shots, tests 19 to 22, inclusive, with slightly less than the full system capacity, the first two were entirely successful at approximately 1,600,000 kva, whereas test number 21 showed evidence of distress, and on test number 22 no current was obtained in phase 1 as a result of the complete shattering of one of the interrupters in that phase on the previous test. Three more shots were then carried out on a single-pole basis, the last of which at 1,750,000 kva destroyed one of the remaining interrupters, thus concluding the first series of tests.

Figure 7 shows an oscillogram of test number 7, a typical light shot of this series. The points at which tripping, contact parting, and interruption occurred and the actual breaker time are indicated on the oscillogram.

Tests of November 28, 1937

Between November 21 and November 28, two sets of interrupters sufficient to equip two breaker poles were reconstructed one with single-port openings approximately the same as on the initial tests, and the other with double-port openings and increased port area. As shown in table II, the interrupters with a single port, after withstanding four shots at light duty and two shots slightly above the full rating, failed on test number 32, causing mechanical damage to the interrupter and allowing arcing in that pole to continue for something more than thirty cycles until the back-up breaker opened the circuit. Subsequent examination of the interrupters in pole 2 indicated that electrical arc-over as well as mechanical damage had occurred.

Single-phase tests were then continued on the single pole with the double-port interrupters, concluding successfully a series of six with the full system capacity and a maximum recorded value of 2,000,000 kva on the last two shots. An oscillogram showing one of these last two heavy shots, test number 38, is presented in figure 8. As noted on the oscillogram, the opening time of the breaker in this case was 3.7 cycles.

Tests of April 24, 1938

While the series of single-phase tests on the double-port interrupters as completed on November 28, 1937, was entirely successful, nevertheless it was felt that, as a definite proof of the final design, it would be advisable to carry out a series of three-phase tests on the breaker equipped throughout with the double-port interrupters. This was particularly so since in the final interrupters, the designers, taking advantage of the information gained from previous tests, had made a number of modifications, such as increased strength of enclosing cylinders, improved electrical insulation, and other changes of a minor character.

The results of this final series of tests are shown in table III. These show that the breaker in its final form gave a most creditable performance. As shown in this table, after two preliminary shots with two generators only, the breaker was subjected immediately to four tests at duties between 1,400,000 and 1,500,000 kva, followed by six shots at the full system capacity, with a maximum of about 2,000,000 kva. As shown also in table III, on the last ten shots which were taken at duties from 1,400,000 to 2,000,-

Table II. Second Series of Tests—November 28, 1937

Test Number	Type	Machines at Philo (Kva)	Lines Connected to 132-Kv Bus	Initial Current in Arc 1	Initial Current in Arc 2	Initial Current in Arc 3	Breaker Time (Cycles) 1	Breaker Time (Cycles) 2	Breaker Time (Cycles) 3	Equivalent Kva Interrupted	Remarks
26	O	2 42,100	None	This pole not connected	1,800	2,000		3.7	4.0	480,000	Cleared OK
27	O	2 42,100	None		1,850	1,600		3.6	3.3	440,000	Cleared OK
28	CO	2 42,100	None		1,900	1,850		3.8	5.0	450,000	Cleared OK
29	CO	2 42,100	None		1,600	1,900		4.0	4.7	450,000	Cleared OK
30	O	1 187,000 1 42,100	All except Crooksville	This pole not connected	6,700	6,300		3.5	3.3	1,600,000	Cleared OK [pole 3
31	O				6,500			3.3		1,550.000	Cleared OK No oscillogram
32	CO				6,200	6,900			3.3	1,650,000	Arced to end of stroke on pole 2. Sustained oil spray from tank 2. Cleared by back-up breaker. Interrupters in pole 2 damaged
33	O	1 187,000 1 42,100	All except Crooksville	Poles 1 and 2 disconnected		5,900			3.3	1,400,000	Cleared OK Pole 3 only with doubleport interrupters on remainder of tests
34	CO					6,000			3.5	1,430,000	Cleared OK
35	O	1 187,000 2 42,100	All	Poles 1 and 2 disconnected		8,200			3.5	1,960,000	Cleared OK Added solid neutral ground to 2–63,000 kva banks not previously grounded. All other banks grounded throughout.
36	CO					7,700			3.9	1,840,000	Cleared OK
37	O					8,500			3.3	2,000,000	Cleared OK
38	CO					8,500			3.7	2,000,000	Cleared OK

000 kva, the breaker time varied from the maximum of 3.5 cycles to a minimum of less than two cycles. On the first two shots, however, which were made at light duty and, incidentally, at a high recovery rate, the breaker time, as would be expected, is somewhat longer. On shot number 2 this time reached a maximum of 5.7 cycles on pole number 2, which interrupted 1,400 amperes or the equivalent of 335,000 kva. Inasmuch as the duty in this case was actually less than 25 per cent of the breaker rating, the five-cycle speed rating of the breaker was, therefore, not actually exceeded under the prevailing standard rules for breaker rating. A typical oscillogram taken during this series of tests, showing the interruption on test number 8, is reproduced in figure 9.

The entire series of tests was carried out without any indication of distress, the only outward evidence of circuit interruption being the noticeable jar transmitted through the cinder-filled ground occupied by the test setup and a small amount of smoke or vapor issuing from the vent pipes on the tanks.

Following the tests, all of the interrupters were removed from the tanks for inspection and were found to be in excellent condition, showing no signs of distress whatever. A photograph of one of the interrupters taken after the completion of the tests is shown in figure 6. This is typical of all of the interrupters removed from this breaker. With the small amount of burning indicated on the contacts, it appeared that the interrupters could have withstood many more tests without maintenance or adjustment.

As would be expected, oil samples taken immediately after the tests showed no noticeable discoloration and tested an average of 30 kv, the same as at the beginning of the tests.

Effects of Tests on System Operation

In order to minimize the possibility of disturbances to customers during these tests, special arrangements were made to take care of certain areas electrically close to the Philo bus and thereby subject to the greatest voltage dips. The Newark area was taken care of by operating the Newark steam plant, tied in with the system only through a 66-kv line extending north to Howard substation and entirely disconnected from the normal 132-kv source at Crooksville. On the other hand, the town of Zanesville, which included among its factories a number of glass plants, which are generally particularly susceptible to voltage dips, had to be subjected to the full disturbance, as there was no other way to supply the town. Therefore, as a precautionary measure, the large industrial customers were notified as to the exact time of carrying out the shots with full system capacity in order that they could have operating men on hand to restart any motors which might possibly drop out because of some unforeseen contingency or because of, perhaps, some instantaneous under-voltage releases.

Except for these precautions, the effect of the tests on the system may be said to have been practically negligible as far as any harmful effects are concerned.

Conclusions

This entire program of field tests in connection with the development of this new high-speed interrupter has again demonstrated what was brought out in the beginning of this paper, namely, that no matter how skilled the designers may be, and with the most exhaustive research testing that is possible in factory testing laboratories, it is still necessary to look

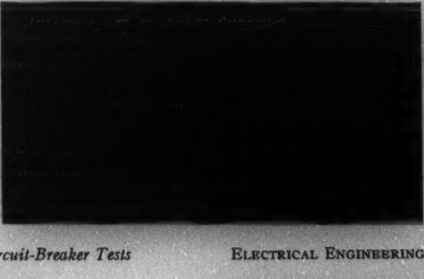

Figure 9. Oscillogram of test number 8, April 24, 1938, showing interruption of 7,700 amperes in 3.25 cycles

Table III. Third Series of Tests—April 24, 1938

Test Number	Type	Machines at Philo (Kva)	Lines Connected to 132-Kv Bus	Initial Current in Arc 1	2	3	Breaker Time (Cycles) 1	2	3	Equivalent Kva Interrupted	Remarks
1	O	2 42,100	None	1,600	1,400	1,500	3.6	3.6	3.7	380,000	Cleared OK
2	CO	2 42,100	None	1,600	1,400	1,300	4.2	5.8	5.0	380,000	Cleared OK
3	O	1 187,000	All except Crooksville	5,700	5,700	6,500	3.2	3.0	3.3	1,550,000	Cleared OK
4	O	1 187,000		6,000			3.5			1,430,000	Cleared OK. No record poles 2 and 3
5	CO	1 187,000		5,700	5,700	5,800	3.4	3.2	2.6	1,380,000	Cleared OK
6	CO	1 187,000		5,900	5,600	5,800	3.5	3.3	3.2	1,400,000	Cleared OK
7	O	1 187,000 2 42,100	All	6,700	7,100	7,400	2.9	3.2	3.0	1,760,000	Cleared OK
8	O			6,800	7,100	7,700	2.9	3.3	3.1	1,840,000	Cleared OK
9	CO			8,200	7,500	7,300	3.2	1.5	1.3	1,960,000	Cleared OK
10	CO	1 187,000 2 42,100	All	8,200	7,300	7,200	2.7	1.7	1.4	1,960,000	Cleared OK
11	CO			7,000	7,300	7,700	3.5	1.3	1.6	1,840,000	Cleared OK
12	CO			7,500	8,500	8,500	3.4	2.7	1.6	2,000,000	Cleared OK

to actual high-capacity field tests for ultimate confirmation of circuit breaker designs and to bring out possible weaknesses which otherwise may escape disclosure. In this particular development there is no question but that the fundamental design of the interrupter and, in general, its details had been worked out on a sound basis. It is no reflection whatever on the designers that unforeseen weaknesses were disclosed in the tests; on the contrary, it is proof of the soundness of the development that comparatively minor changes based upon the information obtained from the tests were all that were necessary to enable the final design to go through what is believed to be the most severe series of tests yet made on a high-voltage circuit breaker and do so with perfect performance.

It is also interesting to look at this development in the light of what has recently been said and written in considerable volume on the subject of oilless and oil-poor circuit breakers. A great deal of development work, largely abroad, but in an appreciable amount also in this country, has been going on in this field and many engineers have felt that such oilless or oil-poor breakers might eventually become the final answer to the circuit-breaker problem. Here we have, however, in an oil circuit breaker of the conventional type, the culmination of developments which have been carried along for some years in an orderly manner, resulting in a breaker which, with a rating of 1,500,000 kva, has demonstrated its ability to interrupt repeatedly and with no apparent effort, short circuits at and well above its rating, up to maximum values of 2,000,000 kva, with speeds consistently under four cycles. Viewed from this angle, we cannot help but feel that this development is truly a tribute to the level-headed skill and ingenuity of our American designers.

It goes without saying that the carrying out of tests of this nature represents not only an actual expense in time, construction, and even fuel cost, but also a considerable amount of stress and disturbance to the personnel operating a large power system. For all of this, however, we feel that we have been well repaid, first, because of the added confidence in the ability of the equipment and the system as a whole to handle disturbances smoothly gained by the operating forces and, second, for having been of assistance in putting the final necessary touches on a development which is without doubt a notable one, and one which will become of considerable value both now and in the future as a much needed tool in the development, expansion, and safe operation of large inter-connected power systems.

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5. **R. M. Spurck and H. E. Strang,** A NEW MULTIBREAK INTERRUPTER FOR FAST CLEARING OIL CIRCUIT BREAKERS, presented at the 1938 summer convention of the AIEE.

Discussion

J. B. MacNeill: See discussion, page 710.

W. F. Sims: See discussion, page 711.

S. B. Crary (General Electric Company, Schenectady, N. Y.): Power-system stability has been an important factor in determining the maximum allowable fault-clearing times. If we consider generator stability only and not load stability, total fault clearing times of the order of nine cycles for complete clearing of the fault appear to be adequate for even the most severe faults on the majority of power systems. Exceptions exist, of course, when the transmission distances are great and the steady-state power margins correspondingly low. However, fault-clearing times less than that calculated as the maximum allowable for generator stability are desirable in that a margin of safety is obtained which allows for unforseen extreme loadings and operation. Also the lower the switch time the greater the time or flexibility which can be allowed for relaying. In some cases this will allow for sequential switching out of the faulty section rather than requiring simultaneous switching.

The paper by Messrs. Sporn and St. Clair discusses a problem which it is believed will become of increasing importance. This is the ability to remove severe faults quickly enough so that important synchronous motor loads will not be lost. A synchronous motor is particularly liable to loss of synchronism for the following reasons.

1. When the fault is on, the synchronous motor tends to decelerate due to its shaft load and the losses due to the flow of fault current. This is not the case for a generator as the losses due to the flow of fault current decrease the acceleration and therefore are beneficial.

2. The size of the individual synchronous motors is usually small, so that the field flux decays more rapidly than in a generator due to the armature reaction created by the flow of fault current and the swing or oscillation after the fault is cleared.

3. The per-unit effective inertia of the rotating element is usually much less than that of generators. Exceptions exist, of course, for the case of high-inertia loads. This means that the angular displacement, in general, increases more rapidly.

4. Synchronous motors are not usually provided with voltage regulators. This combined with the low field time constant, reduces their synchronous strength appreciably during the period the fault is on and immediately after it is cleared.

5. Operators of industrial plants are not always cognizant of the stability problem or the importance of it, so they are apt to purchase and install motors which do not necessarily have high pull-out torques. A large group of motors of only ordinary characteristics or a motor connected through a long line to a system may have very little stability margin even under steady-state conditions.

All these factors combine to make the synchronous-motor load particularly susceptible to pulling out of step. A most effective manner to prevent their losing synchronism is by quick switching. Because of the nature of synchronous motor loads, the fault-clearing times for severe faults and locations relative to the synchronous motor load must be quicker than that required in general for corresponding severe faults relative to the generating station. Fault clearing times of less than nine cycles then become highly desirable.

If a power system has connected to it important synchronous-motor loads which require a high degree of service, then the need for very quick switching will undoubtedly exist for severe faults relative to the synchronous motor load.

L. F. Hunt (Southern California Edison Company, Ltd., Los Angeles, Calif.): The authors should be commended for their paper, and they should receive congratulations for their courage in pioneering actual system oil-circuit-breaker-short-circuit tests. The work involved in lining up a system for short-circuit tests is enormous, and the reading of a paper such as this really does not give the reader a full understanding of the intricacy of such tests.

The engineers of the Southern California Edison Company, Limited, agree with the authors that the final proving ground for oil switches is the actual system short-circuit tests. In fact, the latest policy has been to make system tests on breakers to ascertain complete information of their operation.

If I may digress from the paper a small amount, I would like to mention that we have conducted many tests on switches, and more recently have conducted about five series of tests on 230-kv breakers on our 220-kv system. Two series of tests were made on 230-kv breakers in accordance with the ones discussed in the paper, except having six breaks per pole instead of four. The first series was on April 25, 1937. The results of these tests read like a carbon copy of the authors' first series of tests November 20 and 21, 1937, except we were more fortunate in that we did not require the operation of a back-up breaker. Then on October 17, 1937, we ran the second series of tests with a much improved mechanical designed interrupter described in the Spurk and Strang paper. These tests also were successful, as in the case of the authors' third series of tests. You will note that our final tests were made prior to the authors' first test.

Just one question occurs to me. In the case of a failure on the test switch, how sure are you that the back-up breaker will take the rap and clear successfully?

Again, I wish to congratulate the authors and their men for the courage of these and other tests. The valuable information gained will give all switch users better switches.

D. C. Prince (General Electric Company, Philadelphia, Pa.): It seems appropriate to comment at this time on the debt of gratitude which switchgear manufacturers owe to operating engineers like Mr. Sporn, who have the courage and willingness to subject their systems to high-power field testing. A high-power test laboratory is a very expensive thing. The largest laboratories in this country have a symmetrical capacity a little over 500,000 kva and can deliver about 1,000,000 kva with a full offset wave. This is small compared with a 2,500,000-kva-rated breaker.

Various methods are employed to substantiate ratings beyond test plant capacity. These include pretripping, two-part tests, and other special arrangements. While there is good reason for having confidence in the data obtained from such tests, yet we all like to see how a circuit breaker behaves when it clears a fault on an actual system of large capacity.

Without field tests a new design may operate for months or years before its quality is tried by a real fault. The 287-kv transmission lines from Boulder to Los Angeles were so carefully protected by ground wires and counterpoises, and so well built that the circuit breakers were not called into automatic operation for nearly a year and a half. Then the flood came and there was a washout on both lines and the circuit breakers isolated the faulted sections with hardly a blink of the lights. When the actual emergency arose of course there was no oscillographic equipment hooked up so that no one knows what actually did happen.

The great power concentration around Philo actually delivered 2,000,000 kva to a fault which was cleared by the test breaker with all important events measured. Due to Mr. Sporn's courage and thirst for facts, we have an assurance of protection under actual operating conditions even though these conditions may include a catastrophe of nature such as the flood which damaged Boulder Dam–Los Angeles lines.

V. M. Marquis (American Gas and Electric Service Corporation, New York, N. Y.): A great many of us believe that the oil circuit breaker as an interrupting device represents a notable engineering achievement. To forcefully bring this out, it would be interesting to compare its performance with that of other methods of stopping the flow of energy, such as, for example, with a high-pressure valve, a brake, or a fire extinguisher. However, to make a comparison between the oil circuit breaker and other devices or mechanisms for interrupting the flow of energy is rather difficult. In the case of the oil circuit breaker, the actual energy in the arc is low, perhaps not more than 1,000 kilowatt-seconds. The power factor of the faulted circuit is probably ten per cent or less. Furthermore, the circuit breaker resorts to the trick of interrupting the arc when the current passes through zero. Granting all of this, it must be remembered that when the circuit breaker is attempting to extinguish the arc, the system voltage (in this case 132,000 volts) is at all times attempting to recover and although the current is interrupted when it passes through zero, it must be further borne in mind that the three phases do not open simultaneously, and therefore instantaneously there is an interchange of single-phase power (the mean of which is zero) between the phases which in kilowatts may possibly be equal to the kilovolt-amperes interrupted. All of this results in heavy forces which the oil circuit breaker must withstand because of the magnetic field and the making of room for the gas bubbles as fast as they are formed.

A comparison, although perhaps rather crude, might be made between the oil circuit breaker and the latest devices for extinguishing oil fires. We have made a rough comparison on this basis with some recent tests in which large oil fires were extinguished by means of water from high-pressure nozzles. The calculated equivalent of mechanical power represented by the fires in these tests, based on burning ten gallons of oil per minute, is approximately 25,000 kw, and the best performance obtained in putting out these fires was approximately 20 seconds. On this basis it is obvious that the circuit breaker compares most favorably in doing its job in three cycles or $^{1}/_{400}$ of the time taken to extinguish the fire.

This breaker is reported to have interrupted a 2,000,000-kva short circuit in approximately three cycles, and as shown by the film, a puff of smoke from the vents was the only visible sign that anything had happened. It may be of interest to compare this with the oil fire by showing a short section of film of the oil fire tests already referred to. Regardless of the accuracy of the comparison, I think we can all agree that the modern oil circuit breaker performs efficiently a most difficult job in an extremely short period of time.

Philip Sporn and **H. P. St. Clair:** In closing the discussion on this paper, the authors would like, first of all, to thank those who have contributed some very interesting and helpful discussions. It is particularly gratifying to find that these contributors have shown by their discussions, as well as by direct expression, that they fully appreciate the scope and importance of an undertaking such as that described.

Mr. Crary, we believe, has brought out the importance of high-speed switching in connection with the problem of stability involving synchronous-motor loads. We have already included in the paper some discussion of one such problem which was actually solved by speeding up circuit-breaker operation and we might say that there are many synchronous motor applications on almost any system which will, in general, be affected by the speed of switching. This problem, in fact, we consider to be one of the major arguments in favor of high-speed switching.

Mr. Sims shows a keen appreciation of the development work which has been done both by the manufacturers on this breaker and by our company in bringing this to a successful conclusion by means of test. In pointing out that the design of the new interrupter is such that it may be applied to replace corresponding parts of older breakers at a very great saving over the complete replacement of breakers, Mr. Sims has emphasized again the credit which we have given in the paper to our American designers for the progress they have made in improving the conventional oil circuit breaker along conventional lines.

We are very glad indeed to note from Mr. Hunt's discussion that the engineers of the Southern California Company, Limited, have arrived at the conclusion that the only reliable final proving ground for oil circuit breakers is actual short-circuit tests and that it is the policy of the company at the present time to make such tests on the system. As we have stated many times before, we are firmly convinced of the soundness of this position and we fully expect to continue making system short circuit tests whenever it is necessary to prove out a new

A New Multibreak Interrupter for Fast-Clearing Oil Circuit Breakers

By R. M. SPURCK
FELLOW AIEE

H. E. STRANG
ASSOCIATE AIEE

Synopsis: A description of a new multibreak interrupter for fast clearing of short circuits applicable to high-voltage conventional tank-type oil circuit breakers. Breakers equipped with such interrupters were subjected to interrupting tests on large 138-kv and 230-kv systems during which short circuits as high as 2,000,000 kva were interrupted in less than five cycles. The results of those interrupting tests are shown and discussed.

IT IS generally recognized that fast clearing of short circuits on power systems is desirable because among other benefits, shortening the duration of short circuits reduces the:

(*a*) Tendency of the system to become unstable.

(*b*) Magnitude and duration of voltage dips.

(*c*) Burning of the conductors at the point of short circuit.

(*d*) Potential damage to apparatus.

As the breaker interrupting time, that is, the time measured from the instant the trip coil is energized until the short circuit is interrupted, is an important part of the time required to clear a short circuit, much attention has recently been given to the development of breakers having short interrupting times. Most breakers sold today for all voltages and interrupting ratings, have a rated interrupting time of eight cycles, 0.133 second. Some few breakers for special applications have had interrupting time ratings of three cycles, 0.05 second. Breakers of that time rating, utilizing the impulse oil-blast principle in which oil is forced between the contacts by a mechanically driven piston, are used on the 287.5-kv transmission line between Boulder Dam and Los Angeles[1] and on the 138-kv side of the autotransformers of the Los Angeles end of that line. Also, many single-phase impulse oil-blast-type breakers having an interrupting time of one cycle, 0.04 second, are in operation on 11,000-volt single-phase 25-cycle railway service.[2] During the last two years, a need appeared for breakers of a time rating intermediate between that of standard breakers rated eight cycles and that of the special very expensive breakers rated three cycles. When the possibility of meeting this need was reviewed, it was natural to try to adopt, as far as applicable, the principles already established in the superspeed high-voltage impulse breakers. Pressure-generating breaks[3] were selected as the driving force for the oil blast rather than the mechanical driven piston of the impulse breaker. This method has the proved reliability of thousands of applications. In the absence of extreme speed requirements, this self-generated oil-blast action seemed to be fast enough.

High-voltage multibreak impulse breakers use a cylindrical insulating interrupting chamber in which a number of interrupting contacts connected in series are placed adjacent to port openings in the cylinder wall. The logical modification in such an interrupter to utilize the arc-pressure oil-motivating means was to associate a pressure-generating gap with each interrupting gap.

It seemed expedient to make such interrupters adaptable to conventional tank-type breakers because; first, it permitted the use of conventional bushing current transformers in place of the more expensive self-contained tripping transformers required by the high-voltage impulse breaker, and second, it made possible the modernization of large numbers of circuit breakers already installed so that operators could secure faster operating time without the expense of completely new apparatus.

Several different designs of multibreak interrupters of cylindrical shape adapted to tank-type breakers and utilizing the cross-blast principle with self-contained pressure generation, were subjected to interrupting tests at the high-capacity testing station at Schenectady. From those tests, the number of breaks required for various voltages was determined and the best features of all the types, such as shape, number, and location of port openings, were combined in arriving at a design which would interrupt effectively in minimum time.

Interrupters of the above described general design were subjected to many

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R. M. SPURCK is manager of sales, circuit breaker section, General Electric Company, Philadelphia, Pa.; H. E. STRANG is managing engineer, large oil circuit breaker division of the same company.

[1] For all numbered references, see list at end of paper.

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development in which we are interested.

In regard to Mr. Hunt's question as to how sure we are that the back-up breaker will take the rap and clear successfully in case of a failure on a test switch, the final answer to this question is that we are not positively certain. However, the back-up breaker used is one that has been in service and has performed successfully on many previous occasions during normal system operation and we have a fairly high degree of confidence that it will interrupt the short circuit. For one thing, the duty on the back-up breaker in such an event would be appreciably less than that involved in a quick clearing of the test breaker because of the longer time and resulting decrement. But, and this is more important, since the back-up breaker is one of the regular breakers on which is placed dependence for normal protection of the system, we feel there should be no question regarding its ability to handle the duty imposed upon it and if it is not capable of doing so, there really is no better time to find it out than under staged short-circuit conditions when everyone is on his toes and prepared for whatever may happen.

Mr. MacNeill has brought out a number of interesting points, particularly in comparing the present interrupter with the multibreaker design used in the Boulder Dam 287-kv breakers. Since most of these points have to do mainly with design details, I shall leave them for the consideration of the authors of our companion paper, Messrs. Spurck and Strang. Such discussions are always helpful in clarifying ideas and theories in connection with new developments.

Mr. Prince has pointed out that the circuit breakers installed on the Boulder Dam–Los Angeles 287-kv transmission line were not called upon to operate on a short circuit for nearly $1^1/_2$ years after installation. While we are not nearly so fortunate on our own system in having such infrequent faults, at the same time circuit breakers may wait a long time for anything like a severe test, first, because interrupting capacities are usually figured on a more or less future setup and, second, because even though faults may occur frequently enough, the location of these faults is rarely such as to give rise to the maximum possible fault current. This only confirms the wisdom of making system short-circuit tests to find out before one is lulled into a false sense of security and before there is a chance of an expensive failure, whether a given circuit breaker or circuit-breaker design is capable of performing as it is expected to perform.

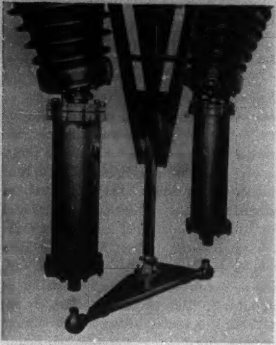


Figure 1. Six-break single-port interrupters installed on a 230-kv oil circuit breaker

interrupting tests at low currents and high currents in the Schenectady testing station. They were also given preliminary field tests before the final design was submitted for the field tests at Saugus and at Philo herein described.

In the tests at Schenectady, attention was not centered alone on the ability of the interrupters to handle heavy short-circuit currents but also on the ability to interrupt light currents as well. For example, the six-break interrupter was tested successfully on currents ranging from 20 amperes to 12,000 amperes and the four-break design from 25 amperes up to 27,000 amperes.

From those tests, it was determined that six breaks per terminal, that is, three pressure generating and three interrupting breaks, were adequate for 230 kv applications and four breaks for 138-kv service without resorting to voltage grading other than that resulting from the normal arrangement of parts.

Multibreak interrupters with cross-blast oil flow not requiring a mechanically driven piston for oil motivation and applicable to conventional tank-type high-voltage breakers are now available for breakers, or the modernization of breakers, rated from 115 kv to 230 kv in interrupting ratings up to 2,500,000 kva.

Description of Interrupter

A pair of these interrupters attached to one pole of a 230-kv oil circuit breaker is shown in figure 1. In figure 2 the interrupter is shown in cross section.

The main housing of the interrupter is a thick walled cylinder of insulating material having an unusually high dielectric and mechanical strength. This insulating housing is readily removable without removing the interrupter from the bushing. Figure 3 shows a six-break interrupter with housing removed and partially disassembled. When the housing is removed, all contact parts are still in place but exposed for inspection. Stationary members of the conducting parts are supported on four vertical insulating posts, each fastened at the top to the upper cover of the interrupter, and at the bottom to a retainer. Extending through the center of the interrupter is an insulating rod which carries the moving contacts. This insulating rod, which projects out of the lower end of the interrupter, terminates in a metal cap which is the contact engaged by the crosshead. A flexible lead from this cap carries the current from the cap to the main current path of the interrupter. Heavy pressure for each contact is obtained by inserting a spring between each contact and its fastening to the center operating rod. Interrupting contact faces are of Elkonite which is outstanding in its ability to withstand severe and repeated arcing with minimum loss of material.

When the breaker opens, the crosshead moves downward thus permitting a spring inside the interrupter to force the center rod supporting the interrupting contacts downward and part the contacts inside the interrupter. The arcs formed in the interrupter decompose oil into gas which creates pressure and forces oil out through the ports. The ports are so located in relation to the contacts that the flow of oil is directed between the contacts. Thus, during the time current is flowing, both oil and gas are flowing out of the port. As current zero is approached, less gas and more oil are carried over the contacts and out the port. It is important to bring out that the bleeding of gas from the interrupter takes place at the contacts adjacent to the port. Gas created at the contacts which have no port adjacent to them is stored in the chamber so that at current zero when no gas is generated, the stored pressure inside of the chamber is sufficient to drive oil into the space between the contacts adjacent to the port. Only a small contact separation is required, therefore, to allow sufficient oil insulation between the contacts to prevent the recovery voltage from breaking down the di-

Table I. Approximate Short-Circuit Values Obtainable by Variation of System Currents

Setup	Line to Ground Short-Circuit Current (Amperes)	Equivalent Three-Phase Kilovolt-Amperes
1	1,000	375,000
2	1,400	525,000
3	2,700	1,000,000
4	4,500	1,700,000
5	Line charging current approximately 50 amperes	

NOTE: Setup 1 utilized a feed from the 66-kv system through step-up transformers at Saugus to 220 kv. All other test arrangements, including that for line charging current, used lines of the 230-kv system only.

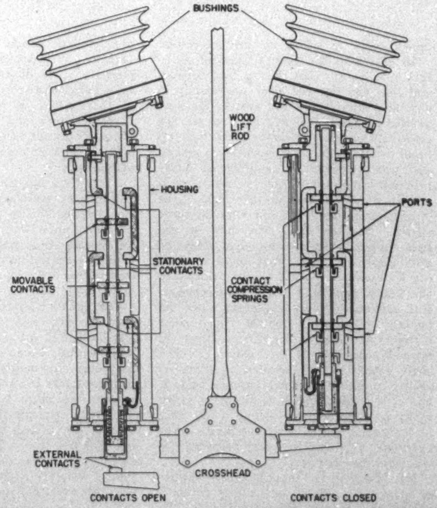


Figure 2. Cross section of six-break single-port interrupters for 230-kv oil circuit breakers. Contacts shown in open and closed positions

Table II. Interrupting Tests on a 220-Kv System

Test Number	Initial Voltage (Kilovolts)	Number of Ports Per Break	Tank	Duty**	Current Interrupted (Root-Mean-Square Amperes)	Equivalent Three-Phase Kilovolt-amperes	Breaker Interrupting Time (Cycles*)
1	216	1	*A*	Opening	1,032	386,000	4.22
1*A*	216	1	*A*	CO	1,010	378,000	3.74
2	216	1	*A*	Opening	1,414	529,000	3.57
2*A*	216	1	*A*	CO	1,414	529,000	3.44
3	216	1	*A*	Opening	2,680	1,000,000	3.5
3*A*	216	1	*A*	CO	2,680	1,000,000	3.27
4	216	1	*A*	Opening	4,400	1,640,000	2.96
4*A*	216	1	*A*	CO		1,760,000†	
15-second interval between tests 4A and 4A-1							
4*A*-1	216	1	*A*	CO	4,700	1,760,000	1.37
5	216	1	*C*	Opening	4,800	1,800,000	3.05
5*A*-1	216	1	*C*	CO	4,550	1,700,000	2.15
15-second interval between tests 5A-1 and 5A-2							
5*A*-2	216	1	*C*	CO	4,550	1,700,000	1.96
6	216	2	*B*	Opening	4,600	1,720,000	3.1
6*A*-1	216	2	*B*	CO	4,700	1,760,000	2.89
15 second interval between tests 6A-1 and 6A-2							
6*A*-2	216	2	*B*	CO	4,700	1,760,000	2.45

* 50-cycle system.

** Opening means breaker initially in closed position and tripped on occurrence of fault. CO means breaker is closed on a fault and then tripped open.

† No record. Estimated value.

electric between the contacts. This interrupting action is entirely in accordance with the oil-blast principle of interruption.

Proper arrangement of the port or ports in relation to the contacts results in an effective device which interrupts the current at an early current zero after the contacts part, as shown in the results of the field tests. Interruption of the circuit generally occurs before the crosshead contact parts from the exterior contact at the bottom of the interrupter. After interruption, the crosshead moves downward to the full stroke of the breaker, a distance sufficient to withstand the required insulation tests with the breaker in the open position.

Field Interrupting Tests at Saugus

On October 17, 1937, at the Saugus substation of the Southern California Edison Company, Ltd., a series of interrupting tests at 220 kv, frequency 50 cycles, was made on multibreak interrupters of the self-pressure-generating cross-blast type installed in a rebuilt triple-pole 230-kv type *FHKO*-139 oil circuit breaker having 108-inch-diameter round tanks. This oil circuit breaker was supplied in 1929. When originally put in service, it was equipped with plain explosion chambers for arc control and had an interrupting rating of 2,500,000 kva on the then standard CO–2 minute–CO duty cycle.

When tested, each pole of the breaker was equipped with six-break multibreak interrupters. Tanks *A* and *C* were provided with interrupters having single ports adjacent to each break and tank *B* with double ports. The original centrifugal-type closing mechanism was replaced by a cam-type mechanism of new design. This cam mechanism closes the breaker in about one-half second as compared with a closing time of about one second required with the original mechanism. Clear break distance from the external contact of each interrupter to the crosshead contact, with the breaker in the open position, was 32 inches. The six gaps of each interrupter add approximately 20 inches to that clear-break distance thus making the total clearance 52 inches from the live part of the lower end of the bushing to the crosshead in the open position.

Cam mechanisms for the operation of high-voltage breakers, are a recent development designed to improve on solenoid and centrifugal motor-type mechanisms in the following respects:

(*a*) Faster closing, or shorter closing time.

(*b*) Better proportioning of the driving output of the mechanism to the load requirements of the breaker.

(*c*) Lower operating currents for the same duty.

(*d*) Reducing slam and overtravel at the end of the closing stroke.

Figure 3. Six-break interrupter for 230-kv oil circuit breakers with housing removed and partially disassembled

Figure 4 shows a diagram of a cam motor mechanism of the type used on the test breaker. The driving element of this mechanism consists of a specially designed high-starting-torque motor arranged to rotate a cam approximately 270 degrees through a spur-gear reduction. Such a driving unit is substituted for the solenoid coil and plunger of solenoid mechanisms, thereby utilizing the same simple and reliable linkage used on modern solenoid mechanisms and retaining the feature of being mechanically trip-free in all positions. Tests show that these mechanisms which combine the features of the present improved solenoid linkage with the motor-driven cam, eliminate the slow starting of solenoids caused by the slow current build up and permit proportioning of the cam in accordance with actual output requirements. The result is the attainment of the objectives noted above.

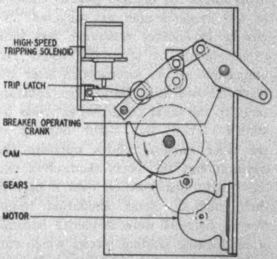


Figure 4. Diagram of cam-type operating mechanism for oil circuit breakers

The Saugus substation is a switching station for 66-kv and 230-kv lines. Oil circuit breakers and disconnecting switches in the station are so arranged that various lines entering the station can be connected individually or in combination to the test breaker in such a way that single-phase line-to-ground short circuits on the 220-kv system would produce approximately the equivalent three-phase values shown in table I.

All of the tests were made in one day.

No inspection of contacts or adjustments were made during the tests. Table II shows the results of the interrupting test, table III the charging current tests. The oil in all tanks tested at 30 kv between one-inch-diameter flat disks spaced 0.1 inch apart at the beginning of the tests and also at the conclusion of the tests. Oil color remained substantially unchanged during the tests. In order to check the condition of the contacts as the tests progressed, contact resistance measurements were made at frequent intervals during the tests. These tests were made by measuring the voltage drop across the breaker pole at currents of 400 amperes and 600 amperes. No change in contact resistance was indicated during or after the tests. Examination of the interrupters after the tests showed that they were substantially in the same condition as when installed. There was slight burning of the fiber burning plates opposite the contacts but the ports themselves had only slight discoloration over a part of their surface. Some insulation re-enforcing barriers were displaced because of inadequate holding screws which can easily be strengthened in future designs. Contacts were only slightly burned, as shown on the photograph of representative contact samples, figure 7. This contact burning is considered so minor that it is estimated that the contacts could have been subjected to ten times as many operations without impairing their current-carrying ability.

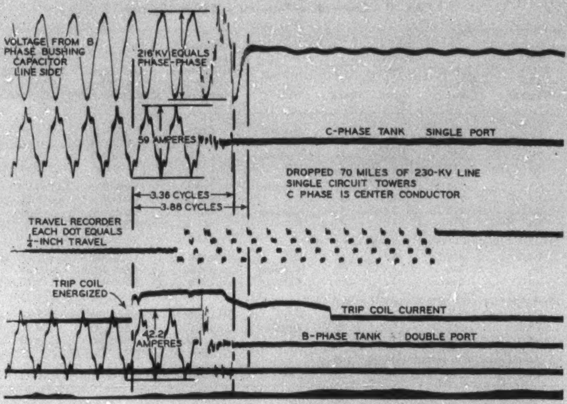
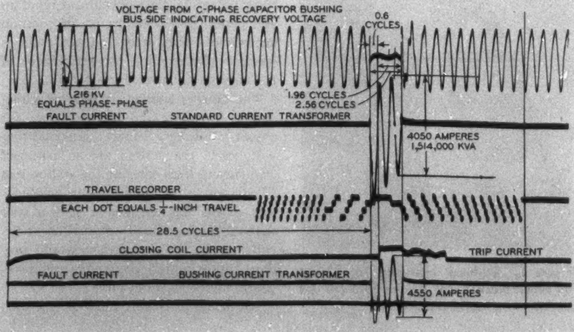


Figure 6. Oscillogram of test number 8 recording interruption of line charging current of 70 miles of line

Line voltage—216 kv
Current phase *B*—42.2 amperes
Interrupting time phase *B*—3.88 cycles
Current phase *C*—59.0 amperes
Interrupting time phase *C*—3.36 cycles
50-cycle system

Oscillograms recording current, voltage, crosshead travel, trip coil current, etc., were taken for all short-circuit tests. Short-circuit currents transformed by a bushing-type current transformer and by a conventional wound transformer, were recorded on the oscillograph. The currents indicated by the bushing current transformer were, in general, slightly higher than those indicated by the wound transformer. Values shown in table I are calculated from the currents registered from the bushing-type transformer. The oscillogram of test 5*A*-2 shown on figure 5, is typical of those obtained on the short-circuit tests. This test is of particular interest in that it represents a CO test during which 1,700,000 kva was interrupted and which was made 15 seconds after test 5*A*-1 a CO operation interrupting 1,700,000 kva. In this latter test, the total time of short circuit was 2.56 cycles from the time the short circuit was established to interruption, including relay time.

Figure 5. Oscillogram of test number 5A-2. Test breaker closed on, and interrupted a short circuit of 1,700,000 kva at 216 kv. Breaker interrupting time 1.96 cycles, total short-circuit duration 2.56 cycles. 50 cycle system



Line-charging-current test oscillograms included records of voltage, crosshead travel and the charging current in each of two lines. Oscillogram of test Number 8, an opening operation designated as figure 6, is representative of the line charging current tests. Although the oscillogram tells its own story, it is not considered superfluous to call attention to the disappearance of the normal-frequency current and the appearance of high-frequency high-current oscillations in the oscillograph line-current record after the breaker contacts parted. Furthermore, although the breaker interrupting time when interrupting line charging current (maximum recorded value 3.88 cycles) was longer than the breaker interrupting time on large short circuits, it was, nevertheless, slightly shorter than the interrupting time of the CO tests on short circuit of approximately 386,000 kva on which the breaker operating time was 4.22 cycles and which was the maximum value recorded for any test.

Two series of 138-kv field tests were made on the four-break interrupters at the Philo station at The Ohio Power Company. The details of these tests including a description of the system setup and a tabulation of results are given in a companion paper by Sporn and St. Clair.[4]

Table III. Line-Charging-Current Interrupting Tests on a 230-Kv System

Test Number	Kilovolts	Number of Ports Per Break	Tank	Kind of Test	Line Charging Current (Amperes)	Breaker Interrupting Time (Cycles*)
7	216	1	*C*	Opening	43.2	3.08
7	216	2	*B*	Opening	57.0	3.08
8	216	1	*C*	Opening	59.0	3.36
8	216	2	*B*	Opening	42.2	3.88
9	216	1	*C*	Closing	57.6	
9	216	2	*B*	Closing	43.2	
10	216	1	*C*	Opening	No record**	
10	216	2	*B*	Opening	No record**	

* 50-cycle system.
** Same connections as for previous tests.

In brief, the tests were made on a new 1,500,000-kva 60-inch tank breaker equipped with a new high-speed trip cam mechanism and four-break interrupters which, in turn, were built with single port openings adjacent to each interrupting break and of somewhat lighter construction than those used on the circuit breaker tested at the Saugus substation. Twenty-one three-phase tests were made both opening and closing-opening (CO) at capacities ranging from 239,000 to 1,860,000 kva with no delay or inspection. Although the twenty-first test cleared the circuit without distress, it was found on the next test that one pole was open-circuited due, as found by later inspection, to a ruptured cylinder occurring on the twenty-first test.

After disconnecting two poles, including the damaged one, the tests were continued single phase to ground on one pole until on the twenty-fifth test at 1,910,000 kva, or 25 per cent above the rating of the breaker, the interrupters in this pole also suffered mechanical damage. However, the breaker interrupted the circuit in the normal time of 3.3 cycles with no outward evidence of distress.

Although these tests indicated gratifying performance at capacities considerably above the breaker rating and with maximum operating time of 4.6 cycles, it was considered quite obvious that certain modifications and additional tests would have to be made. Accordingly, after making two modifications in the interrupters—(1) increased size of vents to reduce pressures at high currents; and (2) slight rearrangement of insulating parts to obtain better arc control, a second series of tests was carried out on two poles only of the three-pole breaker, one pole equipped with interrupters modified by having two port openings, instead of one, adjacent to each interrupting break. These extra openings provide additional venting to relieve the pressure inside the cylinder, and at the same time the wall between them acts as an arc splitter, the efficacy of which has been demonstrated previously. The other pole was equipped with single port interrupters substantially the same as in the previous series of tests. After six two-phase-to-ground tests, going to values above the rating of the breaker, the single port interrupters again suffered mechanical damage and the tests were continued single phase to ground on the double port interrupters, completing the schedule of 13 tests without further trouble. In all of these 13 tests at duties from 382,000 to 2,030,000 kva, the maximum interrupting time was five cycles and out of the nine tests which were above 1,400,000 kva, the maximum time was 3.9 cycles. After these tests, the oil showed only slight deterioration, and, as in the case of the Saugus tests, the Elkonite-surfaced contacts showed practically no burning nor did any of the other parts of the interrupter show any deterioration or signs of distress.

Since 1926, three major series of oil-circuit-breaker interrupting tests have been made on the 138-kv system of the Ohio Power Company. Figure 8 compares the results of these three series of tests and shows a reduction in oil-circuit-breaker interrupting time that has been realized since the first series in 1926. Figure 8 also gives an indication of the increase in power concentration on the system. During the tests made in 1926 at Canton,[5] a circuit breaker equipped with plain explosion chambers interrupted up to 750,000 kva at 138 kv in 15 to 20 cycles. This represented a marked improvement in performance over the plain break contacts of an earlier day. By 1930,[6] about 1,750,000 kva was available for field testing at Philo, and oil blast chambers gave over-all operating times of eight cycles or less. In 1937, the system short circuit had grown to over 2,000,000 kva, providing the largest amount of power ever made available for oil circuit breaker field tests. During these tests, the double-port multibreak interrupter successfully cleared all currents over this range with a maximum breaker interrupting time of five cycles. Contrary to what might be expected, there is scarcely any measurable difference in breaker interrupting time of the single and double port interrupters.

Figure 7. Representative contacts of interrupter after tests at Saugus substation

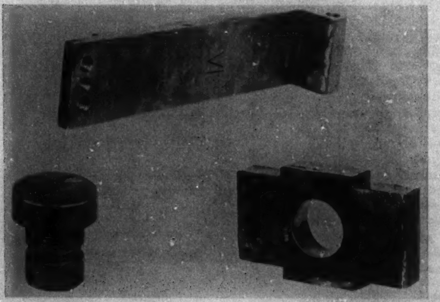
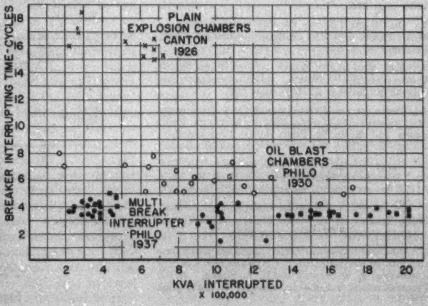


Figure 8. Comparison of oil circuit breaker interrupting time as shown by tests on a 138-kv system

Dark circles—single ports
Dark squares—double ports



The consistency of performance is best indicated by the fact that above 1,300,000 kva every operation fell in a narrow band between three and four cycles.

Conclusion

The multibreak interrupter follows logically the development of earlier oil-blast chambers and the superspeed impulse breaker. Proved by both laboratory and field tests, it makes available performance closely approaching the impulse designs for voltages 115 kv and above, but in circuit breakers of relatively conventional and inherently less costly design. It also affords a means of obtaining the advantages of this still higher speed operation on many breakers already in service when changing system conditions indicate such a need. Many existing circuit breakers may be modernized with these chambers to secure the superior performance demanded by a progressive and growing industry.

Acknowledgment

The authors thank and express their appreciation to the organizations of the American Gas and Electric Company, the Ohio Power Company, and the Southern California Edison Company, Ltd., whose courage, planning, and co-operation made these tests possible. It is a distinct tribute to the sound engineering and planning of the designers and operators of those systems, that system short-circuit tests involving short-circuit values of approximately 2,000,000 kva were made without delay, slight system disturbance, and no interference with customer's service. Tests of this kind are potent factors in the development of apparatus adequate for the present day high standards of system operation.

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6. Oil Circuit Breaker Tests, Philo, 1930, Philip Sporn and H. P. St. Clair. AIEE Transactions, volume 50, 1931, page 498.

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Discussion

J. B. MacNeill (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The paper by Spurck and Strang and that by Sporn and St. Clair, both on the subject of high-voltage circuit interruption, are valuable additions to this art and demonstrate a high grade of ability in modern interrupters built along conventional lines.

It has been realized for some years that to obtain maximum interrupting speed on high-voltage circuits it is necessary to introduce multiple breaks. The idea of using groups of series interrupters isolated in the open position by a moving contact arm was developed several years ago.

Figure 1 of this discussion shows an early construction of this method. By it is obtained the necessary increase of total contact speed while maintaining reasonable velocities of the heavier moving parts. The follow-up action of the moving contact establishes a large open break after interruption has taken place. A construction having adequate 60-cycle and impulse voltage tests is thus secured with the required high speed of circuit interruption. Since the weight of moving contact material approaches that of an ordinary two-break device, there is no sacrifice in the closing speed necessary for synchronizing and high-speed reclosure.

An outstanding application of this principle was employed in the 287,000-volt breakers at Boulder Dam, which were covered in the paper by Wilcox and Leeds, presented at the Pasadena convention in 1936. Figure 2 shows the arrangement of contacts there used. With this construction circuit breaker times of well under three cycles have been obtained for the highest commercial voltages. Figure 3 shows an oscillogram of one-half a pole unit opening 2,900 amperes at 132 kv at a total time of 2.45 cycles on a 60-cycle circuit.

There has been some speculation as to the possibility of securing this type of performance in dead-tank structures at high voltages. Previous AIEE papers have referred to the unequal distribution of voltages between breaks, particularly with grounded short circuits. No particular means for taking care of this problem is evident in the design described by the authors. We would like to have their ideas as to the voltage distribution which takes place between the breaks with their designs under different conditions of short circuit. Is 90 per cent of the voltage across the ungrounded side in case of a grounded short circuit?

Means for securing suitable voltage distribution in such structures have been

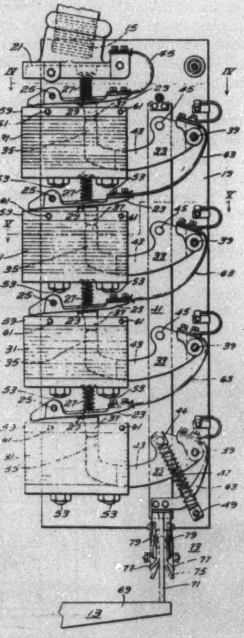

Figure 1

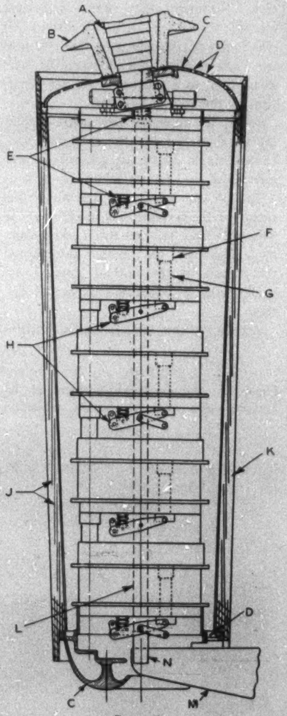

Figure 2

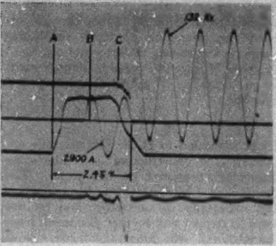

Figure 3

known for years and are useful in securing 60-cycle voltage tests and impulse tests as well as during arc interruption. Figure 4 shows a disclosure of the fundamental idea of electrostatic distribution between arcs as applied to oil circuit breakers made approximately ten years ago. The complete theory was covered in an AIEE paper by R. C. Van Sickle published in Electrical Engineering for August 1937.

The papers by Sporn and St. Clair and by Spurck and Strang are noteworthy as showing variations of the construction referred to above and also applied to dead-tank circuit breakers. We hold that it is fortunate that the improved operation cited in the test results given in these papers has been obtained in conventional constructions which have a direct bearing on many existing American high-voltage stations, and which are usable in much old apparatus, as well as that offered for future sale. It would be unfortunate if ideas previously expressed to the effect that high-speed operation is obtainable only in porcelain structures should gain ground. The work of the past four years points to the conclusion that the highest grade of switchgear performance is obtainable, and with large factors of safety, in a standard dead-tank unit.

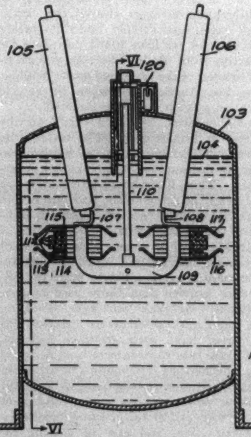

Figure 4

The writers speak of the 138-kv breaker as a four-break design and of the 230-kv type as a six-break design. They refer, in using these designations, to the number of points at which circuit rupture is accomplished. However, it seems to me that the 138-kv design has 10 breaks, consisting of 4 circuit-interruption breaks, 4 gas-generating breaks and, in addition, the 2 long isolating breaks. In the same way the 230-kv design has 14 breaks consisting of 6 circuit-interrupting breaks, 6 gas-generating breaks, and the 2 isolating breaks. Since the circuit presumably is interrupted in practically all cases before the isolating breaks get under way, there will be no contact depreciation at these points, but the other breaks share equally in contact burning and oil depreciation and we feel therefore that both the circuit-interrupting breaks and the gas-generating breaks should be recognized.

Both designs incorporate a number of breaks in a single container and reference is made in each paper to difficulties with this container during the process of development. Failure of the container places one-half of the pole unit out of service. Would not a rearrangement of the interrupters in individual containers, such as shown in the design of figure 2 give greater mechanical strength and greater assurance of operation in case of breakage of an individual unit?

W. F. Sims (Commonwealth Edison Company, Chicago, Ill.): The papers presented by Mr. Spurck and Mr. Sporn cover the development and test of an important advance in the art of arc interruption and bring to mind a few thoughts that should be of interest in this connection.

During the long period of financial stress to which we have all been subjected the manufacturers have gone steadily ahead with constructive research and development in bringing out improved devices of greater interrupting duty and shorter arcing time. They should be commended for their foresight in carrying on this work under adverse conditions, which insures to the utilities the availability of new equipment to meet the more severe and ever changing requirements. Because of this attitude on the part of the manufacturers we can have full confidence that new equipment will continue to be produced as required.

In connection with the tests on this equipment, it is a noteworthy fact that these two companies had such confidence in the soundness of the design and in the integrity of construction of their systems as to be willing to permit short circuits to be placed on their systems under maximum conditions. Much credit is due these companies for their courage and willingness to carry out these tests in the field under the conditions to be met in actual operation.

The high-power laboratories at the factories, which represent large investments on the part of the manufacturers, are of great value in the working out of new designs and in testing their component parts. However, it is not economically feasible to provide laboratories that would subject the equipment to the same conditions that would have to be met in service. The interests of both the manufacturers and the utilities are very greatly advanced because of the willingness of these companies to permit these field tests to be made upon their systems.

It is important to note that the manufacturers in their developments have designed these improved arc interrupting devices in such manner that they can be used to replace corresponding parts of older design and can be installed within the structures of existing equipment. This is an economic achievement which will save the utilities large investment expenditures when the breakers now on their systems become inadequate to meet the more severe conditions brought about by system growth.

E. E. George (Tennessee Electric Power Company, Chattanooga): One of the most interesting features of high-speed breakers associated with high-speed relays is the ease with which staged tests can be made on week days during peak load conditions at any point on the system. When making staged tests ten years ago it was frequently necessary to select an isolated corner of the transmission system, provide a special generating setup, change many relay settings, and schedule the tests between midnight and 5:00 a.m. on Sunday or Monday when the industrial load would be at a minimum. Even then power sales departments and customers' electricians needed considerable reassurance.

With eight-cycle breakers and high-speed relay, tests are now made satisfactorily during peak load periods at the heart of the system without disturbance to customers and without re-arrangement of system connections or generating setup.

H. A. Lott (Southern California Edison Company, Ltd., Los Angeles): Engineers of the Southern California Edison Company, Ltd., have been interested in the modernization of existing 220-kv breakers as an essential part of the program for continued improvement in continuity of service. The first step in the program of modernization was made in 1932, when six 220-kv breakers were rebuilt on the basis of 12 cycles, on a 50-cycle base. Subsequent development and experience resulted in revised specifications calling for eight-cycle interruption, whereas more recent development and experience indicated a still further reduction in over-all time of clearing not to exceed five cycles as being the probable practical limit for the clearance of short circuits by the addition of modern interrupters to existing breakers.

The modernization program has continued since 1932. At present, 62 out of a total of 103 220-kv breakers have been rebuilt to 12 cycles or better. The program has progressed at a conservative rate in order to take advantage of new developments, and also to receive some operating experience with higher speed clearances, which would tend to establish a practical balance between the added cost of still higher interrupting speeds and the necessity for such higher speeds.

The Southern California Edison 220-kv system is characterized by extensive high-voltage transmission lines. The generation is predominantly hydroelectric, with 399,000 kilowatts concentrated in the five Big Creek plants located 250 miles from the load center. The hydro generation is supplemented by 355,000 kilowatts of steam generation located at Long Beach, 25 miles from the load center. These two major plants, which are 275 miles apart, are interconnected with 825 circuit miles of 220,000-volt transmission line. This portion of the system, together with the seven step-down substations and 400,000 kvs of synchronous condenser capacity, must transmit and transform 85 per cent of all of the energy generated by the Edison system.

The importance of fast clearances on this system is evident when under maximum short-circuit conditions of $2^1/_4$ million kva an over-all clearance of five cycles is required in order to prevent instability following multiphase short circuits during heavy load conditions. During the past six years, 116 short circuits have been cleared from the 220-kv system. Four of the short circuits started multiphase, while 11 of the short circuits developed into multiphase after an average duration of about 15 cycles; the remainder were single conductor to ground short circuits. If it is possible to clear all faults in 12 cycles, we can expect to have two cases of instability each three years, based on the past six years experience, whereas clearances in the order of 5 to 8 cycles would be expected to eliminate most of the cases of instability caused by short circuits which start multiphase.

Since the program of rebuilding breakers was started, it has been found that the damage at the point of fault is now practically negligible, when the clearance is effected in 15 cycles or less. The damage at points of flashover is so slight that it is now necessary to climb towers and make a close inspection in order to locate the trouble. An effort is now being made to determine the approximate location of such faults by utilizing recorded values of ground current.

In order to assist in the development of high-speed interrupters, it was realized that it would be necessary to supplement factory tests, where certain limitations in both voltage and capacity exist, by field tests under actual operating conditions, where maximum short-circuit duties of $2^1/_4$ million kva were available. Accordingly, the first series of field tests at 220 kv were made at our Saugus substation in October of 1936, when a total of 15 short circuits, varying from 300,000 kva to 1,700,000 kva were made without indication of instability or without inconvenience to consumer's service. From these tests, it was evident that if the over-all time of clearance was in the order of five to eight cycles, similar tests could be carried to the maximum values of $2^1/_4$ million kva without instability or inconvenience to consumer's service.

Subsequent to the 1936 tests, the first series of tests on the "New Multibreak Interrupter" described in the paper presented by Messrs. R. M. Spurk and H. E. Strang, were made in April of 1937, with short circuits varying from 300,000 kva to 1,700,000 kva. These tests proved the electrical performance of the new interrupter, but indicated a mechanical weakness that could be readily overcome. The second series of tests on this type of interrupter was made in October of 1937, with the results as described by the authors.

During the tests at Saugus, the system generating capacity, interconnected during each test, was 560,000 kva; of which 360,000 kva was at Big Creek, 210 miles from the point of fault, and 200,000 kva was at Long Beach, 65 miles from the point of fault. Including the synchronous condenser capacity of 400,000 kva, which was in operation at the time of test, the total synchronous capacity was 960,000 kva.

The Saugus 220-kv construction is our standard 220 kv double-bus construction with two breakers, one to each bus for each position, which greatly facilitated the switching necessary to obtain the various values of short-circuit current. A complete duplicate opening and closing circuit for both the test and back-up breakers, together with the test relays, oscillograph, etc., were set up in a temporary relay house just outside the bus yard. A portable dark room was installed near the relay house to provide facilities for developing films between each test. The several values of short-circuit current were obtained by opening lines of different lengths from the load bus and closing them to the test bus at Saugus. The maximum values were obtained by paralleling the load bus and the test bus.

The test was completely planned and a printed program giving each step in the operating procedure was distributed to the system dispatchers, and all operators concerned in advance of the test. Telechron clocks were synchronized before the test started, and the 220-kv station operators were notified about five minutes prior to each test, of the exact time of each test. One man was selected to maintain all contact between the test location and the system load dispatchers, so that after each system setup had been completed, a time was selected and the short circuits applied without further instructions. As a result, the tests progressed rapidly. Four different line and bus arrangements were made to obtain the short-circuit-current values, and 15 short circuits were imposed and cleared between 9:15 a.m. and 2:00 p.m.

The results, from an operating standpoint, were ideal in every respect. There was no dip in system speed, and a maximum voltage dip of only 7 per cent was recorded under the most severe condition. There were no consumer complaints, since in every one of the 15 cases, the short circuit was cleared with but a barely perceptible flicker of consumer's lights. It was again established that short circuits would not impose an inconvenience to consumer's service if cleared by high-speed relay and breaker operation. The performance of the switch was such that following an inspection of the contacts, the breaker was placed in routine service without requiring any further adjustments.

The authors are to be congratulated for their presentation of such an interesting paper, marking another step toward better continuity of service through the medium of improvement in circuit breaker design.

A. C. Schwager (Pacific Electric Manufacturing Corporation, San Francisco, Calif.): It is gratifying to notice the improvement obtained with the fast-clearing circuit breaker described by the authors over the two-break oil-blast breaker. The reduction in opening time is accomplished mainly by the use of the multibreak principle which seems to become the conventional means among all manufacturers for obtaining fast interruption. For an analysis of the effectiveness of the device described it would be interesting to know the arcing times present during the interruptions listed in table II and the possible improvement in performance if potential grading had been used.

R. M. Spurck and H. E. Strang: We appreciate the constructive and interesting discussions presented on this paper. The discussions of Messrs. George Lott and Sims add materially to the paper and do not seem to require any further discussion from the authors. Mr. Lott's comments are of particular importance as they apply to the Southern California Edison System on which the 220-kv tests were made.

Messrs. MacNeill and Schwager ask about the potential distribution across the contacts. As stated in the paper, no means for grading the voltage across the various breaks was provided except that resulting from the normal arrangement of parts. The natural grading so obtained was so effective, as evidenced by the excellent performance of the breaker, that the additional application of artificial grading was not justified. In general, the arcing times obtained during the tests listed in table I were between one-half cycle and one cycle.

We emphasize that breakers with this type of interrupter are intended for five-cycle operation. Breakers for three-cycle operation at 287 kv, as used on the Boulder Dam, Los Angeles line, are of the impulse type described by Mr. D. C. Prince before the Institute in 1935. (D. C. Prince, "Circuit Breakers for Boulder Dam Line," AIEE TRANSACTIONS, volume 54, 1935, page 366). In that paper, the results of various tests are listed. One test shows one-half of one pole of the breaker clearing 730 amperes at 264 kv in 2.45 cycles. That performance is much better than the performance shown on the oscillogram presented by Mr. MacNeill in his discussion, where one-half of a pole unit of a 287-kv breaker interrupted 2,990 amperes at 132 kv in 2.45 cycles. Even though the multibreak interrupter cleared the circuit in all cases substantially under its rating of five cycles, we have found no reason for departing from the impulse-type porcelain-clad breaker of the type used on the Boulder Dam, Los Angeles line when three-cycle operation at high voltage is required.

Some question has been raised on the number of breaks used. The text of the paper and the captions of the illustrations refer to the number of breaks per interrupter. Thus, if six-break interrupters are used as for the 220-kv tests, there are twelve breaks total in the two interrupters in addition to the isolating breaks.

Mr. MacNeill discusses the advantages of multibreak interrupters which do not require main moving contacts more complicated or heavier than those used in conventional two-break tank-type breakers. The multibreak interrupters described in our paper have such desirable features. Also, these interrupters have been designed with one housing surrounding a group of breaks rather with one housing for each break or pair of breaks in the interest of greater strength of the housing, simplicity of construction, and ease of housing removal for inspection.

A Symbolic Analysis of Relay and Switching Circuits

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I. Introduction

IN THE CONTROL and protective circuits of complex electrical systems it is frequently necessary to make intricate interconnections of relay contacts and switches. Examples of these circuits occur in automatic telephone exchanges, industrial motor-control equipment, and in almost any circuits designed to perform complex operations automatically. In this paper a mathematical analysis of certain of the properties of such networks will be made. Particular attention will be given to the problem of network synthesis. Given certain characteristics, it is required to find a circuit incorporating these characteristics. The solution of this type of problem is not unique and methods of finding those particular circuits requiring the least number of relay contacts and switch blades will be studied. Methods will also be described for finding any number of circuits equivalent to a given circuit in all operating characteristics. It will be shown that several of the well-known theorems on impedance networks have roughly analogous theorems in relay circuits. Notable among these are the delta-wye and star-mesh transformations, and the duality theorem.

The method of attack on these problems may be described briefly as follows: any circuit is represented by a set of equations, the terms of the equations corresponding to the various relays and switches in the circuit. A calculus is developed for manipulating these equations by simple mathematical processes, most of which are similar to ordinary algebraic algorisms. This calculus is shown to be exactly analogous to the calculus of propositions used in the symbolic study of logic. For the synthesis problem the desired characteristics are first written as a system of equations, and the equations are then manipulated into the form representing the simplest circuit. The circuit may then be immediately drawn from the equations. By this method it is always possible to find the simplest circuit containing only series and parallel connections, and in some cases the simplest circuit containing any type of connection.

Our notation is taken chiefly from symbolic logic. Of the many systems in common use we have chosen the one which seems simplest and most suggestive for our interpretation. Some of our phraseology, as node, mesh, delta, wye, etc., is borrowed from ordinary network theory for similar concepts in switching circuits.

II. Series-Parallel Two-Terminal Circuits

Fundamental Definitions and Postulates

We shall limit our treatment to circuits containing only relay contacts and switches, and therefore at any given time the circuit between any two terminals must be either open (infinite impedance) or closed (zero impedance). Let us associate a symbol X_{ab} or more simply X, with the terminals a and b. This variable, a function of time, will be called the hindrance of the two-terminal circuit a-b. The symbol 0 (zero) will be used to represent the hindrance of a closed circuit, and the symbol 1 (unity) to represent the hindrance of an open circuit. Thus when the circuit a-b is open $X_{ab} = 1$ and when closed $X_{ab} = 0$. Two hindrances X_{ab} and X_{cd} will be said to be equal if whenever the circuit a-b is open, the circuit c-d is open, and whenever a-b is closed, c-d is closed. Now let the symbol + (plus) be defined to mean the series connection of the two-terminal circuits whose hindrances are added together. Thus $X_{ab} + X_{cd}$ is the hindrance of the circuit a-d when b and c are connected together. Similarly the product of two hindrances $X_{ab} \cdot X_{cd}$ or more briefly $X_{ab}X_{cd}$ will be defined to mean the hindrance of the circuit formed by connecting the circuits a-b and c-d in parallel. A relay contact or switch will be represented in a circuit by the symbol in figure 1, the letter being the corresponding hindrance function. Figure 2 shows the interpretation of the plus sign and figure 3 the multiplication sign. This choice of symbols makes the manipulation of hindrances very similar to ordinary numerical algebra.

It is evident that with the above definitions the following postulates will hold:

Postulates

1.	a.	$0 \cdot 0 = 0$	A closed circuit in parallel with a closed circuit is a closed circuit.
	b.	$1 + 1 = 1$	An open circuit in series with an open circuit is an open circuit.
2.	a.	$1 + 0 = 0 + 1 = 1$	An open circuit in series with a closed circuit in either order (i.e., whether the open circuit is to the right or left of the closed circuit) is an open circuit.
	b.	$0 \cdot 1 = 1 \cdot 0 = 0$	A closed circuit in parallel with an open circuit in either order is a closed circuit.
3.	a.	$0 + 0 = 0$	A closed circuit in series with a closed circuit is a closed circuit.
	b.	$1 \cdot 1 = 1$	An open circuit in parallel with an open circuit is an open circuit.

4. At any given time either $X = 0$ or $X = 1$.

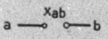
These are sufficient to develop all the theorems which will be used in connection with circuits containing only series and parallel connections. The postulates are arranged in pairs to emphasize a duality relationship between the operations of addition and multiplication and the quantities zero and one. Thus, if in any of the a postulates the zero's are replaced by one's and the multiplications by additions and vice versa, the corresponding b postulate will result. This fact is of great importance. It gives each theorem a dual theorem, it being necessary to prove only one to establish both. The only one of these postulates which differs from ordinary algebra is 1b. However, this enables great simplifications in the manipulation of these symbols.

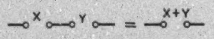
Paper number 38-80, recommended by the AIEE committees on communication and basic sciences and presented at the AIEE summer convention, Washington, D. C., June 20–24, 1938. Manuscript submitted March 1, 1938; made available for preprinting May 27, 1938.

Claude E. Shannon is a research assistant in the department of electrical engineering at Massachusetts Institute of Technology, Cambridge. This paper is an abstract of a thesis presented at MIT for the degree of master of science. The author is indebted to Doctor F. L. Hitchcock, Doctor Vannevar Bush, and Doctor S. H. Caldwell, all of MIT, for helpful encouragement and criticism.

THEOREMS

In this section a number of theorems governing the combination of hindrances will be given. Inasmuch as any of the theorems may be proved by a very simple process, the proofs will not be given except for an illustrative example. The method of proof is that of "perfect induction," i.e., the verification of the theorem for all possible cases. Since by postulate 4 each variable is limited to the values 0 and 1, this is a simple matter. Some of the theorems may be proved more elegantly by recourse to previous theorems, but the method of perfect induction is so universal that it is probably to be preferred.





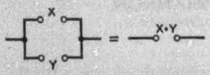


Figure 1 (left). Symbol for hindrance function

Figure 2 (middle). Interpretation of addition

Figure 3 (right). Interpretation of multiplication

$$X + Y = Y + X \tag{1a}$$

$$XY = YX \tag{1b}$$

$$X + (Y + Z) = (X + Y) + Z \tag{2a}$$

$$X(YZ) = (XY)Z \tag{2b}$$

$$X(Y + Z) = XY + XZ \tag{3a}$$

$$X + YZ = (X + Y)(X + Z) \tag{3b}$$

$$1 \cdot X = X \tag{4a}$$

$$0 + X = X \tag{4b}$$

$$1 + X = 1 \tag{5a}$$

$$0 \cdot X = 0 \tag{5b}$$

For example, to prove theorem 4*a*, note that X is either 0 or 1. If it is 0, the theorem follows from postulate 2*b*; if 1, it follows from postulate 3*b*. Theorem 4*b* now follows by the duality principle, replacing the 1 by 0 and the · by +.

Due to the associative laws (2*a* and 2*b*) parentheses may be omitted in a sum or product of several terms without ambiguity. The Σ and Π symbols will be used as in ordinary algebra.

The distributive law (3*a*) makes it possible to "multiply out" products and to factor sums. The dual of this theorem (3*b*), however, is not true in numerical algebra.

We shall now define a new operation to be called negation. The negative of a hindrance X will be written X' and is defined as a variable which is equal to 1 when X equals 0 and equal to 0 when X equals 1. If X is the hindrance of the make contacts of a relay, then X' is the hindrance of the break contacts of the same relay. The definition of the negative of a hindrance gives the following theorems:

$$X + X' = 1 \tag{6a}$$

$$XX' = 0 \tag{6b}$$

$$0' = 1 \tag{7a}$$

$$1' = 0 \tag{7b}$$

$$(X')' = X \tag{8}$$

ANALOGUE WITH THE CALCULUS OF PROPOSITIONS

We are now in a position to demonstrate the equivalence of this calculus with certain elementary parts of the calculus of propositions. The algebra of logic[1–3] originated by George Boole, is a symbolic method of investigating logical relationships. The symbols of Boolean algebra admit of two logical interpretations. If interpreted in terms of classes, the variables are not limited to the two possible values 0 and 1. This interpretation is known as the algebra of classes. If, however, the terms are taken to represent propositions, we have the calculus of propositions in which variables are limited to the values 0 and 1,* as are the hindrance functions above. Usually the two subjects are developed simultaneously from the same set of postulates, except for the addition in the case of the calculus of propositions of a postulate equivalent to postulate 4 above. E. V. Huntington[4] gives the following set of postulates for symbolic logic:

1. The class K contains at least two distinct elements.
2. If a and b are in the class K then $a + b$ is in the class K.
3. $a + b = b + a$
4. $(a + b) + c = a + (b + c)$
5. $a + a = a$
6. $ab + ab' = a$ where ab is defined as $(a' + b')'$

If we let the class K be the class consisting of the two elements 0 and 1, then these postulates follow from those given in the first section. Also postulates 1, 2, and 3 given there can be deduced from Huntington's postulates. Adding 4 and restricting our discussion to the calculus of propositions, it is evident that a perfect analogy exists between the calculus for switching circuits and this branch of symbolic logic.** The two interpretations of the symbols are shown in table I.

Due to this analogy any theorem of the calculus of propositions is also a true theorem if interpreted in terms of relay circuits. The remaining theorems in this section are taken directly from this field.

De Morgan's theorem:

$$(X + Y + Z \ldots)' = X' \cdot Y' \cdot Z' \ldots \tag{9a}$$

$$(X \cdot Y \cdot Z \ldots)' = X' + Y' + Z' + \ldots \tag{9b}$$

This theorem gives the negative of a sum or product in terms of the negatives of the summands or factors. It may be easily verified for two terms by substituting all possible values and then extended to any number n of variables by mathematical induction.

A function of certain variables $X_1, X_2, \ldots\ldots X_n$ is any expression formed from the variables with the operations of addition, multiplication, and negation.

Table I. Analogue Between the Calculus of Propositions and the Symbolic Relay Analysis

Symbol	Interpretation in Relay Circuits	Interpretation in the Calculus of Propositions
X	The circuit X	The proposition X
0	The circuit is closed	The proposition is false
1	The circuit is open	The proposition is true
$X + Y$	The series connection of circuits X Y	The proposition which is true if either X or Y is true
XY	The parallel connection of circuits X and Y	The proposition which is true if both X and Y are true
X'	The circuit which is open when X is closed, and closed when X is open	The contradictory of proposition X
=	The circuits open and close simultaneously	Each proposition implies the other

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[1] For all numbered references, see list at end of paper.

* This refers only to the classical theory of the calculus of propositions. Recently some work has been done with logical systems in which propositions may have more than two "truth values."

** This analogy may also be seen from a slightly different viewpoint. Instead of associating X_{ab} directly with the circuit *a-b* let X_{ab} represent the *proposition* that the circuit *a-b* is open. Then all the symbols are directly interpreted as propositions and the operations of addition and multiplication will be seen to represent series and parallel connections.

The notation $f(X_1, X_2, \ldots\ldots X_n)$ will be used to represent a function. Thus we might have $f(X, Y, Z) = XY + X'(Y' + Z')$. In infinitesimal calculus it is shown that any function (providing it is continuous and all derivatives are continuous) may be expanded in a Taylor series. A somewhat similar expansion is possible in the calculus of propositions. To develop the series expansion of functions first note the following equations.

$$f(X_1, X_2, \ldots X_n) = X_1 \cdot f(1, X_2 \ldots X_n) + X_1' \cdot f(0, X_2 \ldots X_n) \qquad \mathbf{(10a)}$$

$$(X_1 \ldots X_n) = [f(0, X_2 \ldots X_n) + X_1] \cdot [f(1, X_2 \ldots X_n) + X_1'] \qquad \mathbf{(10b)}$$

These reduce to identities if we let X_1 equal either 0 or 1. In these equations the function f is said to be expanded about X_1. The coefficients of X_1 and X_1' in 10*a* are functions of the $(n-1)$ variables $X_2 \ldots X_n$ and may thus be expanded about any of these variables in the same manner. The additive terms in 10*b* also may be expanded in this manner. Expanding about X_2 we have:

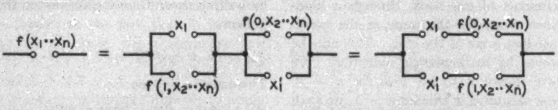


Figure 4. Expansion about one variable

$$\begin{aligned} f(X_1 \ldots X_n) = {} & X_1X_2f(1, 1, X_3 \ldots X_n) + \\ & X_1X_2'f(1, 0, X_3 \ldots X_n) + \\ & X_1'X_2f(0, 1, X_3 \ldots X_n) + \\ & X_1'X_2'f(0, 0, X_3 \ldots X_n) \end{aligned} \qquad \mathbf{(11a)}$$

$$\begin{aligned} f(X_1 \ldots X_n) = {} & [X_1 + X_2 + f(0, 0, X_3 \ldots X_n)] \cdot \\ & [X_1 + X_2' + f(0, 1, X_3 \ldots X_n)] \cdot \\ & [X_1' + X_2 + f(1, 0, X_3 \ldots X_n)] \cdot \\ & [X_1' + X_2' + f(1, 1, X_3 \ldots X_n)] \end{aligned} \qquad \mathbf{(11b)}$$

Continuing this process n times we will arrive at the complete series expansion having the form:

$$\begin{aligned} f(X_1 \ldots X_n) = {} & f(1, 1, 1 \ldots 1)\, X_1X_2 \ldots X_n + \\ & f(0, 1, 1 \ldots 1)\, X_1'X_2 \ldots X_n + \ldots + \\ & f(0, 0, 0 \ldots 0) X_1'X_2' \ldots X_n' \end{aligned} \qquad \mathbf{(12a)}$$

$$\begin{aligned} f(X_1 \ldots X_n) = {} & [X_1 + X_2 + \ldots X_n + \\ & f(0, 0, 0 \ldots 0)] \cdot \ldots \cdot [X_1' + X_2' \ldots + \\ & X_n' + f(1, 1 \ldots 1)] \end{aligned} \qquad \mathbf{(12b)}$$

By 12*a*, f is equal to the sum of the products formed by permuting primes on the terms of $X_1X_2 \ldots X_n$ in all possible ways and giving each product a coefficient equal to the value of the function when that product is 1. Similarly for 12*b*.

As an application of the series expansion it should be noted that if we wish to find a circuit representing any given function we can always expand the function by either 10*a* or 10*b* in such a way that any given variable appears at most twice, once as a make contact and once as a break contact. This is shown in figure 4. Similarly by 11 any other variable need appear no more than four times (two make and two break contacts), etc.

A generalization of De Morgan's theorem is represented symbolically in the following equation:

$$f(X_1, X_2 \ldots X_n, +, \cdot)' = f(X_1', X_2' \ldots X_n', \cdot, +) \qquad \mathbf{(13)}$$

By this we mean that the negative of any function may be obtained by replacing each variable by its negative and interchanging the + and · symbols. Explicit and implicit parentheses will, of course, remain in the same places. For example, the negative of $X + Y \cdot (Z + WX')$ will be $X'[Y' + Z'(W' + X)]$.

Some other theorems useful in simplifying expressions are given below:

$$X = X + X = X + X + X = \text{etc.} \qquad \mathbf{(14a)}$$

$$X = X \cdot X = X \cdot X \cdot X = \text{etc.} \qquad \mathbf{(14b)}$$

$$X + XY = X \qquad \mathbf{(15a)}$$

$$X(X + Y) = X \qquad \mathbf{(15b)}$$

$$XY + X'Z = XY + X'Z + YZ \qquad \mathbf{(16a)}$$

$$(X + Y)(X' + Z) = (X + Y)(X' + Z)(Y + Z) \qquad \mathbf{(16b)}$$

$$Xf(X, Y, Z, \ldots) = Xf(1, Y, Z, \ldots) \qquad \mathbf{(17a)}$$

$$X + f(X, Y, Z, \ldots) = X + f(0, Y, Z, \ldots) \qquad \mathbf{(17b)}$$

$$X'f(X, Y, Z, \ldots) = X'f(0, Y, Z, \ldots) \qquad \mathbf{(18a)}$$

$$X' + f(X, Y, Z, \ldots) = X' + f(1, Y, Z, \ldots) \qquad \mathbf{(18b)}$$

All of these theorems may be proved by the method of perfect induction.

Any expression formed with the operations of addition, multiplication, and negation represents explicitly a circuit containing only series and parallel connections. Such a circuit will be called a series-parallel circuit. Each letter in an expression of this sort represents a make or break relay contact, or a switch blade and contact. To find the circuit requiring the least number of contacts, it is therefore necessary to manipulate the expression into the form in which the least number of letters appear. The theorems given above are always sufficient to do this. A little practice in the manipulation of these symbols is all that is required. Fortunately most of the theorems are exactly the same as those of numerical algebra—the associative, commutative, and distributive laws of algebra hold here. The writer has found theorems 3, 6, 9, 14, 15, 16*a*, 17, and 18 to be especially useful in the simplification of complex expressions.

Frequently a function may be written in several ways, each requiring the same minimum number of elements. In such a case the choice of circuit may be made arbitrarily from among these, or from other considerations.

As an example of the simplification of expressions consider the circuit shown in figure 5. The hindrance function X_{ab} for this circuit will be:

$$\begin{aligned} X_{ab} &= W + W'(X + Y) + (X + Z) \cdot \\ &\qquad (S + W' + Z)(Z' + Y + S'V) \\ &= W + X + Y + (X + Z) \cdot \\ &\qquad (S + 1 + Z)(Z' + Y + S'V) \\ &= W + X + Y + Z \cdot (Z' + S'V) \end{aligned}$$

These reductions were made with 17*b* using first W, then X and Y as the "X" of 17*b*. Now multiplying out:

$$\begin{aligned} X_{ab} &= W + X + Y + ZZ' + ZS'V \\ &= W + X + Y + ZS'V \end{aligned}$$

The circuit corresponding to this expression is shown in figure 6. Note the large reduction in the number of elements.

It is convenient in drawing circuits to label a relay with the same letter as the

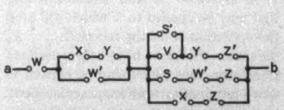


Figure 5. Circuit to be simplified

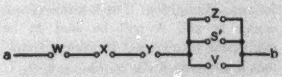


Figure 6. Simplification of figure 5

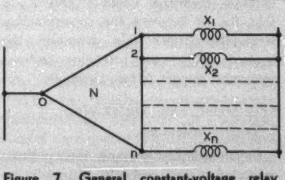


Figure 7. General constant-voltage relay circuit

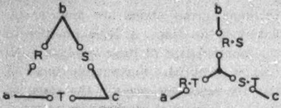


Figure 8. Delta-wye transformation

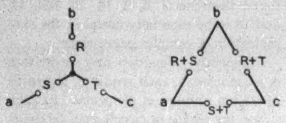


Figure 9. Wye-delta transformation

hindrance of make contacts of the relay. Thus if a relay is connected to a source of voltage through a network whose hindrance function is X, the relay and any make contacts on it would be labeled X. Break contacts would be labeled X'. This assumes that the relay operates instantly and that the make contacts close and the break contacts open simultaneously. Cases in which there is a time delay will be treated later.

III. Multiterminal and Non-Series-Parallel Circuits

Equivalence of *n*-Terminal Networks

The usual relay control circuit will take the form of figure 7, where $X_1, X_2, \ldots X_n$ are relays or other devices controlled by the circuit and N is a network of relay contacts and switches. It is desirable to find transformations that may be applied to N which will keep the operation of *all* the relays $X_1 \ldots . X_n$ the same. So far we have only considered transformations which may be applied to a two-terminal network keeping the operation of one relay in series with this network the same. To this end we define equivalence of n-terminal networks as follows. Definition: Two n-terminal networks M and N will be said to be equivalent with respect to these n terminals if and only if $X_{jk} = Y_{jk}$; $j, k = 1, 2, 3 \ldots n$, where X_{jk} is the hindrance of N (considered a two-terminal network) between terminals j and k, and Y_{jk} is that for M between the corresponding terminals. Under this definition the equivalences of the preceding sections were with respect to two terminals.

Star-Mesh and Delta-Wye Transformations

As in ordinary network theory there exist star-to-mesh and delta-to-wye transformations. In impedance circuits these transformations, if they exist, are unique. In hindrance networks the transformations always exist and are not unique. Those given here are the simplest in that they require the least number of elements. The delta-to-wye transformation is shown in figure 8. These two networks are equivalent with respect to the three terminals a, b, and c, since by the distributive law $X_{ab} = R(S + T) = RS + RT$ and similarly for the other pairs of terminals a-c and b-c.

The wye-to-delta transformation is shown in figure 9. This follows from the fact that $X_{ab} = R + S = (R + S) \cdot (R + T + T + S)$, etc. An n-point star also has a mesh equivalent with the central junction point eliminated. This is formed exactly as in the simple three-point star, by connecting each pair of terminals of the mesh through a hindrance which is the sum of the corresponding arms of the star. This may be proved by mathematical induction. We have shown it to be true for $n = 3$. Now assuming it true for $n - 1$, we shall prove it for n. Suppose we construct a mesh circuit from the given n – point star according to this method. Each corner of the mesh will be an $n - 1$-point star and since we have assumed the theorem true for $n - 1$ we may replace the nth corner by its mesh equivalent. If Y_{0j} was the hindrance of the original star from the central node 0 to the point j, then the reduced mesh will have the hindrance $(Y_{0s} + Y_{0r}) \cdot (Y_{0s} + Y_{0n} + Y_{0r} + Y_{0n})$ connecting nodes r and s. But this reduces to $Y_{0s}Y_{0r}$ which is the correct value, since the original n-point star with the nth arm deleted becomes an $n - 1$-point star and by our assumption may be replaced by a mesh having this hindrance connecting nodes r and s. Therefore the two networks are equivalent with respect to the first $n - 1$ terminals. By eliminating other nodes than the nth, or by symmetry, the equivalence with respect to all n terminals is demonstrated.

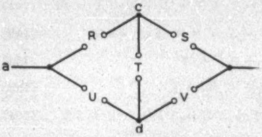


Figure 10. Non-series-parallel circuit

Hindrance Function of a Non-Series-Parallel Network

The methods of part II were not sufficient to handle circuits which contained connections other than those of a series-parallel type. The "bridge" of figure 10, for example, is a non-series-parallel network. These networks will be treated by first reducing to an equivalent series-parallel circuit. Three methods have been developed for finding the equivalent of a network such as the bridge.

The first is the obvious method of applying the transformations until the network is of the series-parallel type and then writing the hindrance function by inspection. This process is exactly the same as is used in simplifying the complex impedance networks. To apply this to the circuit of figure 10, first we may eliminate the node c, by applying the star-to-mesh transformation to the star a-c, b-c, d-c. This gives the network of figure 11. The hindrance function may be written down from inspection for this network.

$$X_{ab} = (R + S)[U(R + T) + V(T + S)]$$

This may be written:

$$\begin{aligned} X_{ab} &= RU + SV + RTV + STU \\ &= R(U + TV) + S(V + TU) \end{aligned}$$

The second method of analysis is to draw all possible paths between the points under consideration through the network. These paths are drawn along the lines representing the component hindrance elements of the circuit. If any one of these paths has zero hindrance, the required function must be zero. Hence if the result is written as a product, the hindrance of each path will be a factor of this product. The required result may therefore be written as the product of the hindrances of all possible paths between the two points. Paths which touch the same point more than once need not be considered. In figure 12 this method is applied to the bridge. The paths are shown dotted. The function is therefore given by:

$$\begin{aligned} X_{ab} &= (R + S)(U + V)(R + T + V) \cdot \\ &\qquad (U + T + S \\ &= RU + SV + RTV + UTS \\ &= R(U + TV) + S(V + TU) \end{aligned}$$

The same result is thus obtained as with the first method.

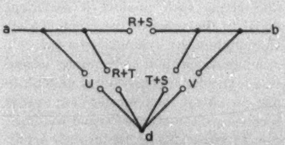


Figure 11. Hindrance function by means of transformations

The third method is to draw all possible lines which would break the circuit between the points under consideration, making the lines go through the hindrances of the circuit. The result is written as a sum, each term corresponding to a certain line. These terms are the products of all the hindrances on the line. The justification of the method is similar to that for the second method. This method is applied to the bridge in figure 13.

This again gives for the hindrance of the network:

$$\begin{aligned} X_{ab} &= RU + SV + RTV + STU \\ &= R(U + TV) + S(V + TU) \end{aligned}$$

The third method is usually the most convenient and rapid, for it gives the result directly as a sum. It seems much easier to handle sums than products due, no doubt, to the fact that in ordinary algebra we have the distributive law $X(Y + Z) = XY + XZ$, but not its dual $X + YZ = (X + Y)(X + Z)$. It is, however, sometimes difficult to apply the third method to nonplanar networks (networks which cannot be drawn on a plane without crossing lines) and in this case one of the other two methods may be used.

SIMULTANEOUS EQUATIONS

In analyzing a given circuit it is convenient to divide the various variables into two classes. Hindrance elements which are directly controlled by a source external to the circuit under consideration will be called independent variables. These will include hand-operated switches, contacts on external relays, etc. Relays and other devices controlled by the network will be called dependent variables. We shall, in general, use the earlier letters of the alphabet to represent independent variables and the later letters for dependent variables. In figure 7 the dependent variables are $X_1, X_2 \ldots X_n$. X_k will evidently be operated if and only if $X_{0k} = 0$, where X_{0k} is the hindrance function of N between terminals 0 and k. That is:

$$X_k = X_{0k} \qquad k = 1, 2, \ldots n$$

This is a system of equations which completely define the operation of the system. The right-hand members will be known functions involving the various dependent and independent variables and given the starting conditions and the values of the independent variables the dependent variables may be computed.

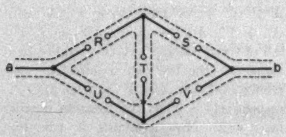


Figure 12. Hindrance function as a product of sums

A transformation will now be described for reducing the number of elements required to realize a set of simultaneous equations. This transformation keeps X_{0k} ($k = 1, 2 \ldots n$) invariant, but X_{jk} ($j, k = 1, 2 \ldots n$) may be changed, so that the new network may not be equivalent in the strict sense defined to the old one. The operation of all the relays will be the same, however. This simplification is only applicable if the X_{0k} functions are written as sums and certain terms are common to two or more equations. For example suppose the set of equations is as follows:

$$\begin{aligned} W &= A + B + CW \\ X &= A + B + WX \\ Y &= A + CY \\ Z &= EZ + F \end{aligned}$$

This may be realized with the circuit of figure 14, using only one A element for the three places where A occurs and only one B element for its two appearances. The justification is quite obvious. This may be indicated symbolically by drawing a vertical line after the terms common to the various equations, as shown below.

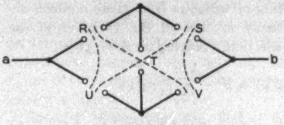


Figure 13. Hindrance function as a sum of products

$$\begin{array}{ll|l|l} W = & & B + & CW \\ X = & A + & & WX \\ Y = & & CY & \\ Z = & F + & EZ & \end{array}$$

It follows from the principle of duality that if we had defined multiplication to represent series connection, and addition for parallel connection, exactly the same theorems of manipulation would be obtained. There were two reasons for choosing the definitions given. First, as has been mentioned, it is easier to manipulate sums than products and the transformation just described can only be applied to sums (for constant-current relay circuits this condition is exactly reversed), and second, this choice makes the hindrance functions closely analogous to impedances. Under the alternative definitions they would be more similar to admittances, which are less commonly used.

Sometimes the relation $XY' = 0$ obtains between two relays X and Y. This is true if Y can operate only if X is operated. This frequently occurs in what is known as a sequential system. In a circuit of this type the relays can only operate in a certain order or sequence, the operation of one relay in general "preparing" the circuit so that the next in order can operate. If X precedes Y in the sequence and both are constrained to remain operated until the sequence is finished then this condition will be fulfilled. In such a case the following equations hold and may sometimes be used for simplification of expressions. If $XY' = 0$, then

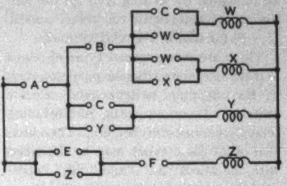


Figure 14. Example of reduction of simultaneous equations

$$\begin{aligned} X'Y' &= Y' \\ XY &= X \\ X' + Y &= 1 \\ X' + Y' &= X' \\ X + Y &= Y \end{aligned}$$

These may be proved by adding $XY' = 0$ to the left-hand member or multiplying it by $X' + Y = 1$, thus not changing the value. For example to prove the first one, add XY' to $X'Y'$ and factor.

SPECIAL TYPES OF RELAYS AND SWITCHES

In certain types of circuits it is necessary to preserve a definite sequential relation in the operation of the contacts of a relay. This is done with make-before-break (or continuity) and break-make (or transfer) contacts. In handling this type of circuit the simplest method seems to be to assume in setting up the equations that the make and break contacts operate simultaneously, and after all simplifications of the equations have been made and the resulting circuit drawn, the required type of contact sequence is found from inspection.

Relays having a time delay in operating or deoperating may be treated similarly or by shifting the time axis. Thus if a relay coil is connected to a battery through a hindrance X, and the relay has a delay of p seconds in operating and releasing, then the hindrance function of the contacts of the relay will also be X, but at a time p seconds later. This may be indicated by writing $X(t)$ for the hindrance in series with the relay, and $X(t-p)$ for that of the relay contacts.

There are many special types of relays and switches for particular purposes, such as the stepping switches and selector switches of various sorts, multiwinding relays, cross-bar switches, etc. The operation of all these types may be described with the words "or," "and," "if," "operated," and "not operated." This is a sufficient condition that they may be described in terms of hindrance functions with the operations of addition, multiplication, negation, and equality. Thus a two-winding relay might be so constructed that it is operated if the first *or* the second winding is operated (activated) and the first *and* the second windings are not operated. If the first winding is X and the second Y, the hindrance function of make contacts on the relay will then be $XY + X'Y'$. Usually, however, these special relays occur only at the end of a complex circuit and may be omitted entirely from the calculations to be added after the rest of the circuit is designed.

Sometimes a relay X is to operate when a circuit R closes and to remain closed independent of R until a circuit S opens. Such a circuit is known as a lock-in circuit. Its equation is:

$$X = RX + S$$

Replacing X by X' gives:

$$X' = RX' + S$$

or

$$X = (R' + X)S'$$

In this case X is *opened* when R closes and remains open until S opens.

IV. Synthesis of Networks

Some General Theorems on Networks and Functions

It has been shown that any function may be expanded in a series consisting of a sum of products, each product being of the form $X_1X_2 \ldots X_n$ with some permutation of primes on the letters, and each product having the coefficient 0 or 1. Now since each of the n variables may or may not have a prime, there is a total of 2^n different products of this form. Similarly each product may have the coefficient 0 or the coefficient 1 so there are 2^{2^n} possible sums of this sort. Hence we have the theorem: The number of functions obtainable from n variables is 2^{2^n}.

Each of these sums will represent a different function, but some of the functions may actually involve less than n variables (that is, they are of such a form that for one or more of the n variables, say X_k, we have identically $f|_{X_k=0} = f|_{X_k=1}$ so that under no conditions does the value of the function depend on the value X_k). Thus for two variables, X and Y, among the 16 functions obtained will be X, Y, X', Y', 0, and 1 which do not involve both X and Y. To find the number of functions which actually involve all of the n variables we proceed as follows. Let $\phi(n)$ be the number. Then by the theorem just given:

$$2^{2^n} = \sum_{k=0}^{n} \binom{n}{k} \phi(k)$$

where $\binom{n}{k} = n!/k!(n-k)!$ is the number of combinations of n things taken k at a time. That is, the total number of functions obtainable from n variables is equal to the sum of the numbers of those functions obtainable from each possible selection of variables from these n which actually involve all the variables in the selection. Solving for $\phi(n)$ gives:

$$\phi(n) = 2^{2^n} - \sum_{k=0}^{n-1} \binom{n}{k} \phi(k)$$

By substituting for $\phi(n-1)$ on the right the similar expression found by replacing n by $n-1$ in this equation, then similarly substituting for $\phi(n-2)$ in the expression thus obtained, etc., an equation may be obtained involving only $\phi(n)$. This equation may then be simplified to the form:

$$\phi(n) = \sum_{k=0}^{n} \binom{n}{k} 2^{2^k}(-1)^{n-k}$$

As n increases this expression approaches its leading term 2^{2^n} asymptotically. The error in using only this term for $n = 5$ is less than 0.01 per cent.

We shall now determine those functions of n variables which require the most relay contacts to realize, and find the number of contacts required. In order to do this, it is necessary to define a function of two variables known as the sum modulo two or disjunct of the variables. This function is written $X_1 \oplus X_2$ and is defined by the equation:

$$X_1 \oplus X_2 = X_1X_2' + X_1'X_2$$

It is easy to show that the sum modulo two obeys the commutative, associative, and the distributive law with respect to multiplication, that is,

$$X_1 \oplus X_2 = X_2 \oplus X_1$$
$$(X_1 \oplus X_2) \oplus X_3 = X_1 \oplus (X_2 \oplus X_3)$$
$$X_1(X_2 \oplus X_3) = X_1X_2 \oplus X_1X_3$$

Also:

$$(X_1 \oplus X_2)' = X_1 \oplus X_2' = X_1' \oplus X_2$$
$$X_1 \oplus 0 = X_1$$
$$X_1 \oplus 1 = X_1'$$

Since the sum modulo two obeys the associative law, we may omit parentheses in a sum of several terms without ambiguity. The sum modulo two of the n variables $X_1, X_2 \ldots X_n$ will for convenience be written:

$$X_1 \oplus X_2 \oplus X_3 \ldots \oplus X_n = \overset{n}{\underset{k=1}{\Xi}} X_k$$

Theorem: The two functions of n variables which require the most elements (relay contacts) in a series-parallel realization are $\overset{n}{\underset{1}{\Xi}} X_k$ and $\left(\overset{n}{\underset{1}{\Xi}} X_k\right)'$, each of which requires $(3 \cdot 2^{n-1} - 2)$ elements.

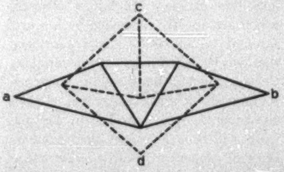


Figure 17. Superposition of a network and its dual

This will be proved by mathematical induction. First note that it is true for $n = 2$. There are ten functions involving two variables, namely, XY, $X + Y$, $X'Y$, $X' + Y$, XY', $X + Y'$, $X'Y'$, $X' + Y'$, $XY' + X'Y$, $XY + X'Y'$. All of these but the last two require two elements; the last two require four elements and are $X \oplus Y$ and $(X \oplus Y)'$, respectively. Thus the theorem is true for $n = 2$. Now assuming it true for $n - 1$, we shall prove it true for n and thus complete the induction. Any func-

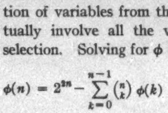


Figure 15 (left). Planar network for illustration of duality theorem

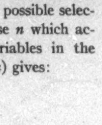


Figure 16 (right). Dual of figure 15

tion of n variables may be expanded about the nth variable as follows:

$$f(X_1, X_2 \ldots X_n) = f = X_n f(X_1 \ldots X_{n-1}, 1) + X_n' f(X_1 \ldots X_{n-1}, 0) \quad (19)$$

Now the terms $f(X_1 \ldots X_{n-1}, 1)$ and $f(X_1 \ldots X_{n-1}, 0)$ are functions of $n-1$ variables and if they individually require the most elements for $n-1$ variables, then f will require the most elements for n variables, providing there is no other method of writing f so that less elements are required. We have assumed that the most elements for $n-1$ variables are required by $\overset{n-1}{\underset{1}{\Xi}} X_k$ and its negative. If we, therefore, substitute for $f(X_1 \ldots X_{n-1}, 1)$ the function $\overset{n-1}{\underset{1}{\Xi}} X_k$ and for $f(X_1 \ldots X_{n-1}, 0)$ the function $\left(\overset{n-1}{\underset{1}{\Xi}} X_k\right)'$ we find:

$$= X_n \overset{n-1}{\underset{1}{\Xi}} X_k + X_n' \left(\overset{n-1}{\underset{1}{\Xi}} X_k\right)' = \left(\overset{n}{\underset{1}{\Xi}} X_k\right)'$$

From the symmetry of this function there is no other way of expanding which will reduce the number of elements. If the functions are substituted in the other order we get:

$$f = X_n \left(\overset{n-1}{\underset{1}{\Xi}} X_k\right)' + X_n' \overset{n-1}{\underset{1}{\Xi}} X_k = \overset{n}{\underset{1}{\Xi}} X_k$$

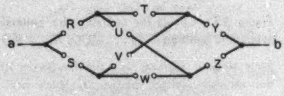


Figure 18. Nonplanar network

This completes the proof that these functions require the most elements.

To show that each requires $(3 \cdot 2^{n-1} - 2)$ elements, let the number of elements required be denoted by $s(n)$. Then from (19) we get the difference equation:

$$s(n) = 2s(n-1) + 2$$

with $s(2) = 4$. This is linear, with constant coefficients, and may be solved by the usual methods. The solution is:

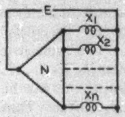
$$s(n) = 3 \cdot 2^{n-1} - 2$$

as may easily be verified by substituting in the difference equation and boundary condition.

Note that the above only applies to a series-parallel realization. In a later section it will be shown that the function $\overset{n}{\underset{1}{\Xi}} X_k$ and its negative may be realized with $4(n-1)$ elements using a more general type of circuit. The function requiring the most elements using any type of circuit has not as yet been determined.

Dual Networks

The negative of any network may be found by De Morgan's theorem, but the network must first be transformed into an equivalent series-parallel circuit (unless it is already of this type). A theorem will be developed with which the negative of any planar two-terminal circuit may be found directly. As a corollary a method of finding a constant-current circuit equivalent to a given constant-voltage circuit and vice versa will be given.



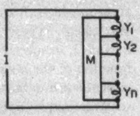


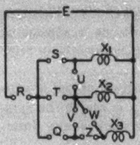
Figure 19 (left). General constant-voltage relay circuit

Figure 20 (right). General constant-current relay circuit

Let N represent a planar network of hindrances, with the function X_{ab} between the terminals a and b which are on the outer edge of the network. For definiteness consider the network of figure 15 (here the hindrances are shown merely as lines).

Now let M represent the dual of N as found by the following process; for each contour or mesh of N assign a node or junction point of M. For each element of N, say X_k, separating the contours r and s there corresponds an element X_k' connecting the nodes r and s of M. The area exterior to N is to be considered as two meshes, c and d, corresponding to nodes c and d of M. Thus the dual of figure 15 is the network of figure 16.

Theorem: If M and N bear this duality relationship, then $X_{ab} = X_{cd}'$. To prove this, let the network M be superimposed upon N, the nodes of M within the corresponding meshes of N and corresponding elements crossing. For the network of figure 15, this is shown in figure 17 with N solid and M dotted. Incidentally, the easiest method of finding the dual of a network (whether of this type or an impedance network) is to draw the required network superimposed on the given network. Now, if $X_{ab} = 0$, then there must be some path from a to b along the lines of N such that every element on this path equals zero. But this path represents a path *across* M dividing the circuit from c to d along which every element of M is one. Hence $X_{cd} = 1$. Similarly, if $X_{cd} = 0$, then $X_{ab} = 1$, and it follows that $X_{ab} = X_{cd}'$.



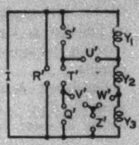


Figure 21 (left). Simple constant-voltage system

Figure 22 (right). Constant-current system equivalent to figure 21

It is evident from this theorem that a negative for any planar network may be realized with the same number of elements as the given network.[†]

In a constant-voltage relay system all the relays are in parallel across the line. To open a relay a series connection is opened. The general constant-voltage system is shown in figure 19. In a constant-current system the relays are all in series in the line. To de-operate a relay it is short-circuited. The general constant-current circuit corresponding to figure 19 is shown in figure 20. If the relay Y_k of figure 20 is to be operated whenever the relay X_k of figure 19 is operated and not otherwise, then evidently the hindrance in parallel with Y_k which short-circuits it must be the negative of the hindrance in series with X_k which connects it across the voltage source. If this is true for all the relays, we shall say that the constant-current and constant-voltage systems are equivalent. The above theorem may be used to find equivalent circuits of this sort, for if we make the networks N and M of figures 19 and 20 duals in the sense described, with X_k and Y_k as corresponding elements, then the condition will be satisfied. A simple example of this is shown in figures 21 and 22.

Synthesis of the General Symmetric Function

It has been shown that any function represents explicitly a series-parallel circuit. The series-parallel realization may require more elements, however, than some other network representing the same function. In this section a method will be given for finding a circuit representing a certain type of function which in general is much more economical of elements than the best series-parallel circuit. This type of function is known as a symmetric function and appears frequently in relay circuits.

[†] This is not in general true if the word "planar" is omitted. The nonplanar network X_{ab}, of figure 18, for example, has no negative containing only eight elements.

Definition: A function of the n variables $X_1, X_2 \ldots X_n$ is said to be symmetric in these variables if any interchange of the variables leaves the function identically the same. Thus $XY + XZ + YZ$ is symmetric in the variables X, Y, and Z. Since any permutation of variables may be obtained by successive interchanges of two variables, a necessary and sufficient condition that a function be symmetric is that any interchange of two variables leaves the function unaltered.

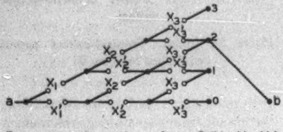


Figure 23. Circuit for realizing $S_2(X_1, X_2, X_3)$

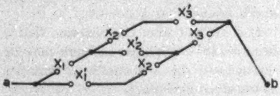


Figure 24. Simplification of figure 23

By proper selection of the variables many apparently unsymmetric functions may be made symmetric. For example, $XY'Z + X'YZ + X'Y'Z'$ although not symmetric in X, Y, and Z is symmetric in X, Y, and Z'. It is also sometimes possible to write an unsymmetric function as a symmetric function multiplied by a simple term or added to a simple term. In such a case the symmetric part may be realized with the methods to be described, and the additional term supplied as a series or parallel connection.

The following theorem forms the basis of the method of design which has been developed.

Theorem: A necessary and sufficient condition that a function be symmetric is that it may be specified by stating a set of numbers $a_1, a_2 \ldots a_k$ such that if exactly a_j $(j = 1, 2, 3 \ldots k)$ of the variables are zero, then the function is zero and not otherwise. This follows easily from the definition. The set of numbers $a_1, a_2 \ldots a_k$ may be any set of numbers selected from the numbers 0 to n, inclusive, where n is the number of variables in the symmetric function. For convenience, they will be called the a-numbers of the function. The symmetric function $XY + XZ + YZ$ has the a-numbers 2 and 3, since the function is zero if just two of the variables are zero or if three are zero, but not if none or if one is zero. To find the a-numbers of a given symmetric function it is merely necessary to evaluate the function with $0, 1 \ldots n$ of the variables zero. Those numbers for which the result is zero are the a-numbers of the function.

Theorem: There are 2^{n+1} symmetric functions of n variables. This follows from the fact that there are $n + 1$ numbers, each of which may be taken or not in our selection of a-numbers. Two of the functions are trivial, however, namely, those in which all and none of the numbers are taken. These give the "functions" 0 and 1, respectively. The symmetric function of the n variables $X_1, X_2 \ldots X_n$ with the a-numbers $a_1, a_2 \ldots a_k$ will be written $S_{a_1a_2 \ldots a_k}(X_1, X_2 \ldots X_n)$. Thus the example given would be $S_{2,3}(X, Y, Z)$. The circuit which has been developed for realizing the general symmetric function is based on the a-numbers of the function and we shall now assume that they are known.

Theorem: The sum of two given symmetric functions of the same set of variables is a symmetric function of these variables having for a-numbers those numbers common to the two given functions. Thus $S_{1,2,3}(X_1 \ldots X_6) + S_{2,3,5}(X_1 \ldots X_6) = S_{2,3}(X_1 \ldots X_6)$.

Theorem: The product of two given symmetric functions of the same set of variables is a symmetric function of these variables with all the numbers appearing in either or both of the given functions for a-numbers. Thus $S_{1,2,3}(X_1 \ldots X_6) \cdot S_{2,3,5}(X_1 \ldots X_6) = S_{1,2,3,5}(X_1 \ldots X_6)$. To prove these theorems, note that a product is zero if either factor is zero, while a sum is zero only if both terms are zero.

Theorem: The negative of a symmetric function of n variables is a symmetric function of these variables having for a-numbers all the numbers from 0 to n, inclusive, which are not in the a-numbers of the given function. Thus $S'_{2,3,5}(X_1 \ldots X_6) = S_{0,1,4,6}(X_1 \ldots X_6)$.

Before considering the synthesis of the general symmetric function $S_{a_1,a_2 \ldots a_k}(X_1, X_2 \ldots X_n)$ a simple example will be given. Suppose the function $S_2(X_1, X_2, X_3)$ is to be realized. This means that we must construct a circuit which will be closed when any two of the variables X_1, X_2, X_3 are zero, but open if none, or one or three are zero. A circuit for this purpose is shown in figure 23. This circuit may be divided into three bays, one for each variable, and four levels marked 0, 1, 2, and 3 at the right. The terminal b is connected to the levels corresponding to the a-numbers of the required function, in this case to the level marked 2. The line coming in at first encounters a pair of hindrances X_1 and X_1'. If $X_1 = 0$, the line is switched up to the level marked 1, meaning that one of the variables is zero; if not it stays at the same level. Next we come to hindrances X_2 and X_2'. If $X_2 = 0$, the line is switched up a level; if not, it stays at the same level. X_3 has a similar effect. Finally reaching the right-hand set of terminals, the line has been switched up to a level equal to the total number variables which are zero. Since terminal b is connected to the level marked 2, the circuit a-b will be completed if and only if 2 of the variables are zero. If $S_{0,3}(X_1, X_2, X_3)$ had been desired, terminal b would be connected to both levels 0 and 3. In figure 23 certain of the elements are evidently superfluous. The circuit may be simplified to the form of figure 24.

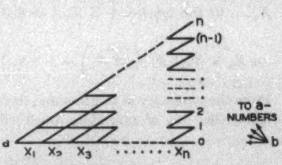


Figure 25. Circuit for realizing the general symmetric function $S_{a_1a_2 \ldots a_k}(X_1, X_2, \ldots X_n)$

Each sloping element has the hindrance of the variable written below it; each horizontal element has the negative of this hindrance. This convention will be used on most symmetric-function drawings

For the general function exactly the same method is followed. Using the general circuit for n variables of figure 25, the terminal b is connected to the levels corresponding to the a-numbers of the desired symmetric function. In figure 25 the hindrances are represented merely by lines, and the letters are omitted from the circuit, but the hindrance of each line may easily be seen by generalizing figure 23. After terminal b is connected, all superfluous elements may be deleted.

In certain cases it is possible to greatly simplify the circuit by shifting the levels down. Suppose the function $S_{0,3,6}(X_1 \ldots X_6)$ is desired. Instead of continuing the circuit up to the sixth level, we connect the second level back down to the zero level as shown in figure 26. The zero level then also becomes the third level and the sixth level. With terminal b connected to this level, we have realized

Table II. Relation of Operating Characteristics and Equations

Symbol	In Terms of Operation	In Terms of Nonoperation
X	The switch or relay X is operated	The switch or relay X is not operated
$=$	If	If
X'	The switch or relay X is not operated	The switch or relay X is operated
$\cdot$	Or	And
$+$	And	Or
$(-\ -)'$	The circuit (— —) is not closed, or apply De Morgan's theorem	The circuit (— —) is closed, or apply De Morgan's theorem
$X(t-p)$	X has been operated for at least p seconds	X has been open for at least p seconds

If the dependent variable appears in its own defining function (as in a lock-in circuit) strict adherence to the above leads to confusing sentences. In such cases the following equivalents should be used.

$X = RX + S$	X is operated when R is closed (providing S is closed) and remains so independent of R until S opens	
$X = (R' + X)S'$		X is opened when R is closed (providing S is closed) and remains so independent of R until S opens

In using this table it is usually best to write the function under consideration either as a sum of pure products or as a product of pure sums. In the case of a sum of products the characteristics should be defined in terms of nonoperation; for a product of sums in terms of operation. If this is not done it is difficult to give implicit and explicit parentheses the proper significance.

the function with a great saving of elements. Eliminating unnecessary elements the circuit of figure 27 is obtained. This device is especially useful if the a-numbers form an arithmetic progression, although it can sometimes be applied in other cases.

The functions $\overset{n}{\underset{1}{\Xi}}X_k$ and $\left(\overset{n}{\underset{1}{\Xi}}X_k\right)'$ which were shown to require the most elements for a series parallel realization have very simple circuits when developed in this manner. It can be easily shown that if n is even, then $\overset{n}{\underset{1}{\Xi}}X_k$ is the symmetric function with all the even numbers for a-numbers, if n is odd it has all the odd numbers for a-numbers. The function $\left(\overset{n}{\underset{1}{\Xi}}X_k\right)'$ is, of course, just the opposite. Using the shifting-down process the circuits are as shown in figures 28 and 29. These circuits each require $4(n-1)$ elements. They will be recognized as the familiar circuit for controlling a light from n points, using $(n-2)$ double-pole double-throw switches and two single-pole-double-throw switches. If at any one of the points the position of the switch is changed, the total number of variables which equal zero is changed by one, so that if the light is on, it will be turned off and if already off, it will be turned on.

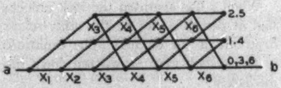


Figure 26. Circuit for $S_{0,3,6}(X_1 \ldots X_6)$ using the "shifting down" process

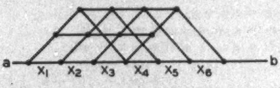


Figure 27. Simplification of figure 26

More than one symmetric function of a certain set of variables may be realized with just one circuit of the form of figure 25, providing the different functions have no a-numbers in common. If there are common a-numbers the levels may be shifted down, or an extra relay may be added so that one circuit is still sufficient.

The general network of figure 25 contains $n(n+1)$ elements. We will show that for any given selection of a-numbers, at least n of the elements will be superfluous. Each number from 1 to $n-1$, inclusive, which is not in the set of a-numbers produces two unnecessary elements; 0 or n missing will produce one unnecessary element. However, if two of the a-numbers differ by only one, then two elements will be superfluous. If more than two of the a-numbers are adjacent, or if two or more adjacent numbers are missing, then more than one element apiece will be superfluous. It is evident then that the worst case will be that in which the a-numbers are all the odd numbers or all the even numbers from 0 to n. In each of these cases it is easily seen that n of the elements will be superfluous. In these cases the shifting down process may be used if $n > 2$ so that the maximum of n^2 elements will be needed only for the four particular functions X, X', $X \oplus Y$, and $(X \oplus Y)'$.

EQUATIONS FROM GIVEN OPERATING CHARACTERISTICS

In general, there is a certain set of independent variables A, B, C . . . which may be switches, externally operated or protective relays. There is also a set of dependent variables x, y, z . . . which represent relays, motors or other devices to be controlled by the circuit. It is required to find a network which gives, for each possible combination of values of the independent variables, the correct values for all the dependent variables. The following principles give the general method of solution.

1. Additional dependent variables must be introduced for each added phase of operation of sequential system. Thus if it is desired to construct a system which operates in three steps, two additional variables must be introduced to represent the beginning of the last two steps. These additional variables may represent contacts on a stepping switch or relays which lock in sequentially. Similarly each required time delay will require a new variable, representing a time delay relay of some sort. Other forms of relays which may be necessary will usually be obvious from the nature of the problem.

2. The hindrance equations for each of the dependent variables should now be written down. These functions may involve any of the variables, dependent or independent, including the variable whose function is being determined (as, for example, in a lock-in circuit). The conditions may be either conditions for operation or for nonoperation. Equations are written from operating characteristics according to table II. To illustrate the use of this table suppose a relay U is to operate if x is operated and y or z is operated and v or w or z is not operated. The expression for A will be:

$$U = x + yz + v'w'z'$$

Lock-in relay equations have already been discussed. It does not, of course, matter if the same conditions are put in the expression more than once—all superfluous material will disappear in the final simplification.

3. The expressions for the various dependent variables should next be simplified as much as possible by means of the theorems on manipulation of these quantities. Just how much this can be done depends somewhat on the ingenuity of the designer.

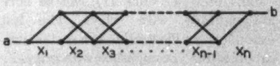


Figure 28. $\overset{n}{\underset{1}{\Xi}}X_k$ for n odd; $\left(\overset{n}{\underset{1}{\Xi}}X_k\right)'$ for n even

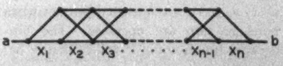


Figure 29. $\left(\overset{n}{\underset{1}{\Xi}}X_k\right)$ for n even; $\left(\overset{n}{\underset{1}{\Xi}}X_k\right)'$ for n odd

4. The resulting circuit should now be drawn. Any necessary additions dictated

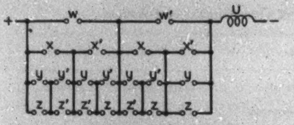


Figure 30. Series-parallel realization of selective circuit

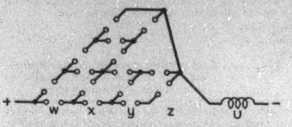


Figure 31. Selective circuit from symmetric-function method

by practical considerations such as current-carrying ability, sequence of contact operation, etc., should be made.

V. Illustrative Examples

In this section several problems will be solved with the methods which have been developed. The examples are intended more to illustrate the use of the calculus in actual problems and to show the versatility of relay and switching circuits than to describe practical devices.

It is possible to perform complex mathematical operations by means of relay circuits. Numbers may be represented by the positions of relays or stepping switches, and interconnections between sets of relays can be made to represent various mathematical operations. In fact, any operation that can be completely described in a finite number of steps using the words "if," "or," "and," etc. (see table II), can be done automatically with relays. The last example is an illustration of a mathematical operation accomplished with relays.

A SELECTIVE CIRCUIT

A relay U is to operate when any one, any three or when all four of the relays w, x, y, and z are operated but not when none or two are operated. The hinderance function for U will evidently be:

$$U = wxyz + w'x'yz + w'xy'z + w'xyz' + wx'y'z + wx'yz' + wxy'z'$$

Reducing to the simplest series-parallel form:

$$U = w[x(yz + y'z') + x'(y'z + yz')] + w'[x(y'z + yz') + x'yz]$$

This circuit is shown in figure 30. It requires 20 elements. However, using the symmetric-function method, we may write for U:

$$U = S_{1,3,4}(w, x, y, z)$$

This circuit (figure 31) contains only 15 elements. A still further reduction may be made with the following device. First write:

$$U' = S_{0,2}(w, x, y, z)$$

This has the circuit of figure 32. What is required is the negative of this function. This is a planar network and we may apply the theorem on the dual of a network, thus obtaining the circuit shown in figure 33. This contains 14 elements and is probably the most economical circuit of any sort.

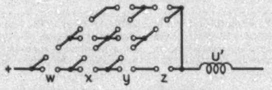


Figure 32. Negative of selective circuit from symmetric-function method

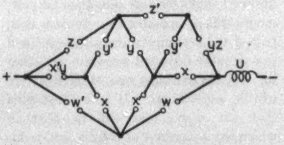


Figure 33. Dual of figure 32

DESIGN OF AN ELECTRIC COMBINATION LOCK

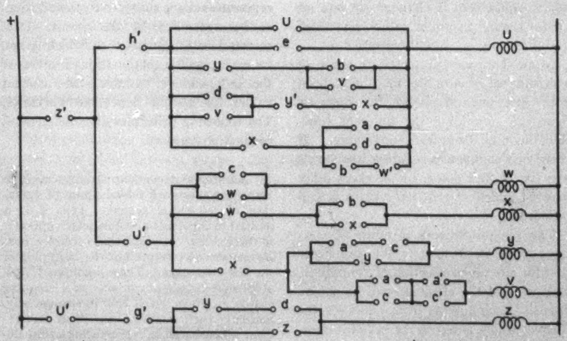
An electric lock is to be constructed with the following characteristics. There are to be five pushbutton switches available on the front of the lock. These will be labeled a, b, c, d, e. To operate the lock the buttons must be pressed in the following order: c, b, a, and c simultaneously, d. When operated in this sequence the lock is to unlock, but if any button is pressed incorrectly an alarm U is to operate. To relock the system a switch g must be operated. To release the alarm once it has started a switch h must be operated. This being a sequential system either a stepping switch or additional sequential relays are required. Using sequential relays let them be denoted by w, x, y and z corresponding, respectively, to the correct sequence of operating the push buttons. An additional time-delay relay is also required due to the third step in the operation. Obviously, even in correct operation a and c cannot be pressed at exactly the same time, but if only one is pressed and held down the alarm should operate. Therefore assume an auxiliary time delay relay v which will operate if either a or c alone is pressed at the end of step 2 and held down longer than time s the delay of the relay.

When z has operated the lock unlocks and at this point let all the other relays drop out of the circuit. The equations of the system may be written down immediately:

$$w = cw + z' + U'$$
$$x = bx + w + z' + U'$$
$$y = (a + c)y + x + z' + U'$$
$$z = z(d + y) + g' + U'$$
$$v = x + ac + a'c' + z' + U'$$
$$U = e(w' + abd)(w + x' + ad) \cdot [x + y' + dv(t - s)] \cdot [y + bv(t - s)]U + h' + z'$$

These expressions can be simplified considerably, first by combining the second and third factors in the first term of U,

Figure 34. Combination-lock circuit



Z AND Z' MAKE BEFORE BREAK

U AND U' MAKE BEFORE BREAK

and then by factoring out the common terms of the several functions. The final simplified form is as below:

$$\begin{array}{ll}
U = & \\
w = & \\
x = Z' + & \left| \begin{array}{l} h' + e[ad(b+w')+x']\cdot(x+y'+dv)(y+vb)U \\ U' + \left| \begin{array}{l} cw \\ bx + w \\ x + \left| \begin{array}{l} (a+c)y \\ ac + a'c' \end{array}\right. \end{array}\right. \end{array}\right. \\
y = & \\
v = & \\
z = g' + (y+d)z + U' &
\end{array}$$

This corresponds to the circuit of figure 34.

ELECTRIC ADDER TO THE BASE TWO

A circuit is to be designed that will automatically add two numbers, using only relays and switches. Although any numbering base could be used the circuit is greatly simplified by using the scale of two. Each digit is thus either 0 or 1; the number whose digits in order are $a_k, a_{k-1}, a_{k-2}, \ldots a_2, a_1, a_0$ has the value

$$\sum_{j=0}^{k} a_j 2^j$$

Let the two numbers which are to be added be represented by a series of switches; $a_k, a_{k-1}, \ldots a_1, a_0$ representing the various digits of one of the numbers and $b_k, b_{k-1}, \ldots b_1, b_0$ the digits of the other number. The sum will be represented by the positions of a set of relays $s_{k+1}, s_k, s_{k-1} \ldots s_1, s_0$. A number which is carried to the jth column from the $(j-1)$th column will be represented by a relay c_j. If the value of any digit is zero, the corresponding relay or switch will be taken to be in the position of zero hindrance; if one, in the position where the hindrance is one. The actual addition is shown below:

$$\begin{array}{llll}
c_{k+1} & c_k \quad c_{j+1}c_j \quad c_2c_1 & & \text{Carried numbers} \\
 & a_k \text{---} a_{j+1}a_j \text{---} a_2a_1a_0 & & \text{First number} \\
 & b_k \quad b_{j+1}b_j \quad b_2b_1b_0 & & \text{Second number} \\
\hline
c_{k+1} & S_k \text{---} S_{j+1}S_j \text{---} S_2S_1S_0 & & \text{Sum} \\
\text{or} & & & \\
S_{k+1} & & &
\end{array}$$

Starting from the right, s_0 is one if a_0 is one and b_0 is zero or if a_0 is zero and b_0 one but not otherwise.
Hence:

$$s_0 = a_0b_0' + a_0'b_0 = a_0 \oplus b_0$$

c_1 is one if both a_0 and b_0 are one but not otherwise.

$$c_1 = a_0 \cdot b_0$$

s_j is one if just one of a_j, b_j, c_j is one, or if all three are one.

$$S_j = S_{1,3}(a_j, b_j, c_j) \qquad j = 1, 2, \ldots k$$

c_{j+1} is one if two or if three of these variables are one.

$$c_{j+1} = S_{2,3}(a_j, b_j, c_j) \qquad j = 1, 2, \ldots k$$

Using the method of symmetric functions, and shifting down for s_j gives the circuits of figure 35. Eliminating superfluous elements we arrive at figure 36.

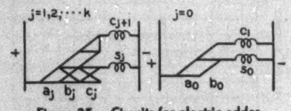


Figure 35. Circuits for electric adder

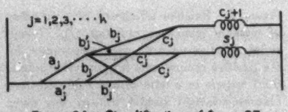


Figure 36. Simplification of figure 35

References

1. A complete bibliography of the literature of symbolic logic is given in the *Journal of Symbolic Logic*, volume 1, number 4, December 1936. Those elementary parts of the theory that are useful in connection with relay circuits are well treated in the two following references.

2. THE ALGEBRA OF LOGIC, Louis Cauturat. The Open Court Publishing Company.

3. UNIVERSAL ALGEBRA, A. N. Whitehead. Cambridge, at the University Press, volume 1, book III, chapters I and II, pages 35–82.

4. E. V. Huntington, *Transactions* of the American Mathematical Society, volume 35, 1933, pages 274–304. The postulates referred to are the fourth set, given on page 280.

Low-Voltage Fluorescent Lamps

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Introduction

FLUORESCENT lamps have been developed, simple in design, and operating through suitable ballast from 115- and 230-volt a-c circuits. They are tubular in form, and the fluorescent material is energized by a hot-cathode positive-column electric discharge. Just as a transformer converts one voltage to another, the fluorescent powder adhering to the inner wall of the bulb converts the invisible ultraviolet radiation present in a low-pressure mercury discharge into visible radiation, or light. Recent progress in the development of these lamps has resulted from the gradual increase in knowledge pertaining to hot cathodes, to the production of short-wave ultraviolet, and to the manufacture of efficient fluorescent powders. These advances have been applied in a lamp to produce colored light many times more efficiently than do present light sources. In addition, fluorescence makes possible for the first time an efficient, practicable, low-wattage white light matching daylight in appearance. Much of the development has been concentrated on the electrical characteristics of the lamps, and on the design of efficient ballast equipment. The lamps described are in one of several classes which are under development. In some respects they are experimental and may be changed before being commercialized.

Lamp Description

To obtain the positive-column type of discharge, found most efficient for this lamp, the tube length should be at least several times its diameter. One electrode is sealed in at each end of the tube, and serves alternately as cathode and anode. The electrodes are small coils of tungsten wire (see figure 1) coated with emission material, such as barium and strontium carbonates, which in manufacture are broken down into oxides. The tungsten wire is small enough not only to be heated sufficiently by the energy of the discharge to provide good emission, but also to heat up quickly when the lamp is started. In general, only one or two turns of the tungsten coils are heated to incandescence by the discharge, but the hot spots shift from one section to another during the life of the lamp.

The inner wall of the tube is coated with a thin layer of fluorescent material (also called the phosphor). Inside coating, in addition to its usual advantages, is essential in this case because the ultraviolet radiation which energizes the phosphor

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will not pass through ordinary glass. The coating must be thick enough to absorb the impinging ultraviolet, yet thin enough in every section not to absorb too much of its own light, or of the light internally directed from the opposite side of the tube. Considerable effort has been spent on the technique of obtaining coatings smooth and uniform in appearance, and lamps of uniform color and efficiency.

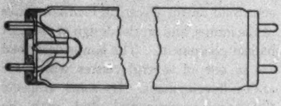

Figure 1. Construction of a fluorescent lamp. The left of the diagram represents the cross section of an end. The right of the diagram shows the completed lamp appearance

The assembled lamp, consisting only of this coated tube and the sealed-in electrode at each end, is evacuated, and the carbonates on the electrodes are decomposed as in radio-tube manufacture. A little argon gas (to facilitate starting) and a small drop of mercury are admitted to the tube. These remain without cleanup for the life of the lamp. A sturdy yet simple construction has resulted: a white glass tube with a cap or base on each end (see figure 2). Since there is no filament or supporting structure of any kind between the electrodes, the lamp is far more rugged than filament lamps of the same size and shape, and will withstand far more shock and vibration.

Figure 2. Photograph of a fluorescent lamp

Lamp Design

The choice of lamp dimensions and electrical values is determined not only by the maximum luminous efficiency, but by numerous other factors, such as a useful total light output, adaptability to starting and running on standard line voltages, suitable voltage drop and minimum wattage loss in ballast equipment, esthetically pleasing proportions, low surface brightnesses, and commercial adaptability to manufacture, shipment, and use. Any choice requires compromises, and only a few variables can be considered here. Most development has centered around tubes of one-inch diameter with currents of about 0.25 ampere.

Mercury is used as the vapor for the discharge in fluorescent lamps because of its high efficiency in producing short ultraviolet radiation of one wave length, its resonance line, 2,537 angstrom units (see figure 3). Fluorescent materials have been selected, such as that shown in figure 4, which produce light most efficiently for ultraviolet radiation of about this wave length, thus forming an ideal combination. No other measured spectral line of the low-pressure discharge in mercury vapor amounts to more than two per cent of the input watts. Consequently, the 2,537 angstrom-unit line can reasonably be considered as the chief exciter of fluorescence in these lamps. The presence of fluorescent material on the inside of the tube has no appreciable effect on the operation or characteristics of the discharge.

Temperature is a critical factor in the efficiency of production of 2,537 angstrom units in a low-pressure mercury discharge. Figure 5 shows that the efficiency is highest at 40 degrees centigrade. Consequently, a lamp at the most efficient temperature will run only 15–20 degrees centigrade hotter than normal room temperature, and to that extent is a "cold light." Lamps must run hotter than this optimum point to obtain higher wattage and higher light output in tubes of the same dimensions. By using a wattage lower than that giving the optimum temperature, efficiency losses from low temperature may be compensated by the advantage of lower current density (figure 3). The one-inch-diameter lamps listed in table I operate at the optimum point; the $1^1/_2$-inch-diameter lamps operate below it. The larger diameter was chosen to secure lower surface brightness and less glare. At lower temperatures, such as zero degrees centigrade, very little ultraviolet is produced because insufficient mercury is vaporized. To maintain normal light output at such an ambient temperature, the lamp must be jacketed or shielded to conserve heat.

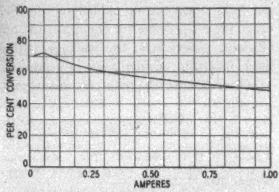

Figure 3. Percentage of total energy input converted to mercury resonance radiation (2,537 angstrom units) versus current, for the positive column of a mercury discharge in a one-inch-diameter tube held at 40 degrees centigrade

Table I. Characteristics of Some Fluorescent Lamps

Length (Inches)	Diameter (Inches)	Watts	Volts	Amperes	Approximate Lumens Per Watt of a Green Lamp
18	1	15	63	0.27	60
18	$1^1/_2$	15	50	0.33	60
24	$1^1/_2$	20	65	0.35	65
36	1	30	115	0.30	70

Another design factor is lamp voltage. There is an almost constant voltage drop at the electrodes which is practically lost as far as ultraviolet and light production is concerned. Consequently, the higher the lamp voltage is raised by increasing length or decreasing diameter, the smaller in percentage this loss becomes (figure 6). It is quite possible to make a lamp of the

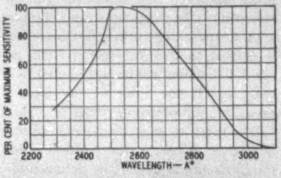

Figure 4. Relative sensitivity of the zinc beryllium silicate phosphor to ultraviolet radiation of various wave lengths. The peak at or near 2,537 angstrom units is characteristic of several phosphors

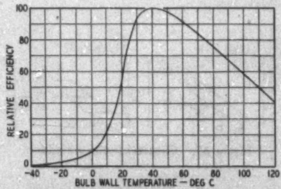

Figure 5. Relative efficiency of production of mercury resonance radiation (2,537 angstrom units) versus bulb wall temperature, for the positive column of a mercury discharge at 0.25 ampere in a tube of one inch diameter. Fluorescent light output follows this curve closely

bulbular type similar in shape to those now used on incandescent-filament lamps, but the cathode drop becomes a large percentage of the total drop and luminous efficiency is low. Another reason favoring high lamp voltage is that auxiliary loss becomes less as lamp voltage becomes a greater proportion of line voltage. It is necessary for good stability of this type of discharge, when a simple circuit is used, that the lamp voltage shall not greatly exceed two-thirds of the available supply voltage, the remainder being taken up by suitable reactance. The bulbular, low-voltage lamp mentioned above would suffer another disadvantage from the greater voltage drop and wattage loss in the ballast from 115-volt lines.

Electrical Characteristics of Lamps and Circuits

The operating voltage of a fluorescent lamp may be divided into two components, the electrode drop and the potential gradient of the positive column. The former is fixed somewhere between 12 and 18 volts, dependent on electrode size, cathode emission, and filling gas. The gradient is the voltage required per inch of tube to make up for energy losses, such as the neutralization of ions at the walls, and the conversion of energy into radiation. The following empirical equations were found to govern for the current range 0.10–1.00 ampere in experimental lamps:

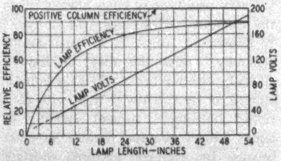

Figure 6. Lamp efficiency relative to positive-column efficiency, and volts for various lamp lengths. The arc gap is about $1\frac{3}{4}$ inch less than over-all lamp length

For one-inch tube diameter,

$$E = \frac{2.73l}{0.56 + I} + 12.5 \qquad (1)$$

For $1^1/_2$-inch tube diameter,

$$E = \frac{2.83l}{0.75 + I} + 12.5 \qquad (2)$$

Where E is the lamp voltage, l the arc gap in inches, and I the current in amperes. See figures 6 and 7.

The wave shapes are characteristic of the low-pressure discharge in mercury vapor (figure 8). Lamp current and lamp voltage are in phase for resistance ballast, but wave-shape distortion results in a lamp power-factor of 0.88–0.90, defined as lamp watts divided by lamp root-mean-square volt-amperes. Lamp power factor varies somewhat with the source of supply and the type of ballasting equipment used. High-resistance voltage coils must be used in measurement to prevent serious diversion of lamp current and alteration in wave shape; 100 ohms per volt has been the minimum resistance used. The voltage equations 1 and 2 multiplied by current and by 0.90, lamp power factor, give wattage values. See also figure 7.

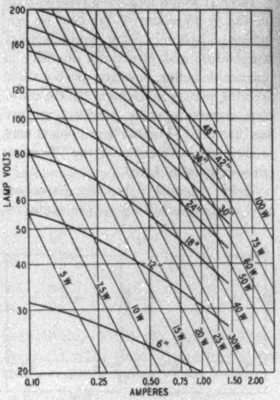

Figure 7. Volts, amperes, and watts for one-inch-diameter lamps of various lengths. Lamp power factor is 0.90

The starting voltage of a fluorescent lamp without preheating of the electrodes (cold starting) is about four times the operating voltage. For example, the 18-inch-long one-inch-diameter lamp requires about 250 volts to initiate the discharge. Even at this voltage, a nearby plate or other metal surface connected to one end of the lamp or line, or to ground, is usually required. Several types of separate-winding or autotype high-leakage-reactance transformers were tried to provide automatically the required starting and operating voltages. But this method required at least 21 watts, and usually 22–27 watts over-all to operate one 15-watt lamp. A semiresonant circuit as shown in figure 9*B* and described below, but with only one connection to each electrode, reduced over-all wattage to 17–18.

Recent work has concentrated on preheated-electrode (hot) starting because lamp life is greatly prolonged, and end blackening reduced, by eliminating the sputtering of electrode material that occurs with cold starting. Also the required starting voltage is reduced, and may drop to 115 volts with sufficient electrode heating. The circuit most easily explained is shown in figure 9*A*. Electrode and reactor design are so correlated that the electrodes are heated to incandescence when the switch is closed. Opening the switch by hand, or automatically by a thermal switch after one or two seconds of heating, produces a surge from the reactor which easily starts the discharge. A modification of this circuit is a vibrator element built on the reactor or separately. In this case repeated makes and breaks occur, with the electrodes gaining heat at each contact, until the discharge starts. The operating current then produces a field strong enough to hold the vibrator contacts open. The simple reactor loses from one to two watts, dependent on size. The semiresonant circuit developed by Mr. M. A. Edwards of Schenectady and shown in figure 9*B*, provides simultaneously some voltage boost and electrode-heating current, low wattage loss, and automatic action with no moving parts. By suitable choice of inductance and capacitance values, and by correct design for the saturation characteristics of the inductance, a combined value of voltage across the lamp and electrode-heating current assures lamp starting. As soon as the lamp starts, it effectively shunts the parallel circuit, and runs at an operating current determined by the series unit. Energy lost in the parallel circuit is less than a watt. The whole circuit consumes 17–18 watts over-all for a 15-watt lamp.

Figure 8. Oscillogram of voltage and current of fluorescent lamp operating from simple inductance (circuit of figure 9*A*); 64 root-mean-square volts, 0.26 root-mean-square amperes

The power factor of most fluorescent lamp circuits is closely given by the ratio of lamp to line volts. A 65-volt lamp on the circuit of figure 9*A*, with 115 volt line, gives about 0.57 lagging power factor. If capacitance predominates, the power factor is the same value, but leading. Although this value of power factor may be considered low, it is much better than the values of 0.20–0.26 obtained with the high-leakage-reactance transformers first tried for cold starting. Many variants and combinations of circuits are possible.

Characteristics of the Light Output

The outstanding advantage of the fluorescent lamp is its ability to produce colored light at efficiencies far exceeding those of filament lamps. Experimental 15-watt green fluorescent lamps have operated at 70 lumens per watt compared with about 0.3 lumen per watt for 60-watt filament lamps, an efficiency gain of about two hundredfold. In blue, corresponding efficiencies are 18 against about 0.3, a gain of about fiftyfold. In other colors, the gain is less spectacular, but still several fold; 15-watt fluorescent lamps of the same color as daylight have been made with efficiencies of about 30 lumens per watt, to give 450 lumens. Obtaining the same amount of light from a filament lamp, with a suitable absorbing filter to duplicate daylight, requires 150 watts; that is, ten times as much wattage and heat.

One basis for such great gains in special colors is the fact that phosphors inherently produce a certain color, determined by their atomic and crystal structure. As in figure 10, spectral energy distribution measurements typically show a peak region (at 5,950 angstrom units, yellow, in this case). In filament lamps, color must be produced from a source giving a continuous spectrum by the subtractive method of using a filter which absorbs all light except that of the desired color. The second basis for efficiency gains is the high percentage of input energy converted into usable radiant energy. A 40-watt filament lamp, for example, converts about seven per cent of its input energy into visible radiant energy. Most of the remainder is converted into infrared radiation, which is not convertible into light. The 15-watt fluorescent lamp converts about 50 per cent of its input energy into one ultraviolet line. Despite the efficiencies of utilization shown in table II, the net percentage of energy available for light is about twice that of filament lamps.

Table II. Approximate Energy Utilization in a 15-Watt Fluorescent Lamp

	Efficiency of Process (Per Cent)
Conversion to 2,537 angstrom units	50
Quantum conversion of 2,537 angstrom units to visible radiation (5,100 angstrom units)	50
Perfection of phosphor	60
Transmission of envelope (covers bulb and base absorption)	90
Over-all efficiency	13

Table II would indicate a possibility of increasing efficiencies 65 per cent by perfecting phosphors. Since requirements of temperature and ultraviolet intensity in practicable lamps, however, are not those favoring perfect conversion by the phosphor, expectations for practical use are less than this. Further development may produce improvements in the efficiency of the other items listed.

Under external light, the phosphors used in these lamps all appear white, regardless of their color when fluorescing. As a result of this whiteness, the familiar yellow, green, and blue lines of the mercury discharge pass through the fluorescent coating on the tube with little absorption, and contribute about 3.5 lumens per watt to lamp efficiency. Figure 10 shows these lines superposed on the fluorescent spectrum. Because of their intensity, the production of fluorescent lamps with a narrow band of colored light, such as deep red, may require a colored enamel or filter. For the opposite case, where it is desired to avoid any sharp peak of energy, as for daylight lamps, several phosphors may be mixed together to give a more nearly uniform spectral energy distribution.

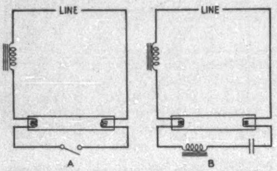

Figure 9. Operating circuits for fluorescent lamps

Miscellaneous

Some phosphors, such as the green and pink, inherently possess enough lag, continuing to give off light after the exciting source is removed (phosphorescence), that 15-watt fluorescent lamps using them show no more flicker on 60-cycle supply than do regular 25-watt 115-volt filament lamps. Other phosphors, such as blue, have no measurable lag, and follow closely the starting and extinguishing of the discharge on each half cycle.

The condensation of mercury at low temperatures not only lowers the production of ultraviolet and of light (figure 5), but raises the starting voltage enough that special ballast equipment may be required. Hence, indoors, and outdoors in mild weather are the best fields of use.

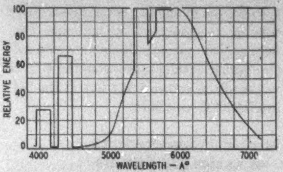

Figure 10. Spectral energy distribution of the radiation from a fluorescent lamp coated with zinc beryllium silicate phosphor. The curve for the phosphor alone is smooth. The block peaks are representations of the visible lines from the mercury discharge, plotted as though 200 angstrom units wide. The combined radiation matches the color, but not the spectral energy distribution, of a black body at about 2,800 degrees Kelvin

Every new type of lamp always raises the question, "How long will it live?" In fluorescent lamps, as in vapor lamps generally, useful life may be limited either by the number of times the lamp is started, or by the loss of light with continued burning. Frequent starting of lamps causes eventual failure to start because the emission material sputters off. Where lamps are burned continuously, or for several hours on each start, they may live several thousand hours, but at steadily decreasing light outputs until replacement is economically justifiable. The amount of decrease varies with different phosphors. Fluorescent lamps under average conditions are expected to burn longer than filament lamps and to have a lumen maintenance comparable to that of vacuum type filament lamps.

Another quite different type of renewable low-voltage fluorescent lamp has been developed concurrently by T. E. Foulke of Hoboken, New Jersey. The lamp development here reported has been the joint work of many participants. Special acknowledgment for all measurements on spectral energies is due B. T. Barnes of this laboratory.

Graphical Field-Plotting Methods in Engineering

By HILLEL PORITSKY
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1. Introduction

IT IS THE OBJECT of this paper to exhibit the great range of problems, often very difficult analytically, which can be solved with comparative ease by graphical means. The paper is based on several lectures presented to the electrical and mechanical sections of the advanced course in engineering of the General Electric Company. Due to lack of space the theory will be presented rather briefly, though enough of it will be included to make the treatment complete. Some of the solutions presented are believed to be new.

2. The Laplace Equation and the Small-Square Field Plots

The most familiar examples of field plotting are in connection with solutions of the Laplace equation:

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \qquad (1)$$

whose solutions are known as "harmonic" functions. This equation as well as its three-dimensional analogue has a wide range of application, applying equally well to gravitational, electrostatic, and magnetic fields in homogeneous media, outside of the region of attracting masses, charges, poles, and electric currents, respectively. These fields are characterized by the possession of a scalar potential and by conservation of flux. In the two-dimensional case, if X, Y are the field components, these properties are expressed by the fact that the two curve integrals

$$v = -\int_C F_s ds = -\int_C X dx + Y dy \qquad (2)$$

$$u = \int_C F_n ds = \int_C X dy - Y dx \qquad (3)$$

are independent of the path of integration C, depending only on the end points. The first expression is the integral of the tangential component of the field and represents the work done in moving a unit active material (unit mass, charge, pole) along C; the second integral represents the "flux" crossing C and is the integral of the normal component of the field over C. With the lower end point of C held fast and the upper one varying arbitrarily the integrals (2), (3) define two true point functions v, u; v is the potential, u the flux function. In terms of u, v the field components X, Y are given by

$$X = -\frac{\partial v}{\partial x}, \quad Y = -\frac{\partial v}{\partial y} \qquad (4)$$

$$X = \frac{\partial y}{\partial u}, \quad Y = -\frac{\partial u}{\partial x} \qquad (5)$$

The conditions on the field leading to the independence of the integrals (2), (3) of the path of integration are

$$\frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x} = 0 \qquad (6)$$

$$\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} = 0 \qquad (7)$$

respectively. From (4) and (7) is deduced (1), while from (5) and (6) follows that u is also harmonic:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad (8)$$

From (4), (5) follows that u, v satisfy the "Cauchy-Riemann" equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \qquad (9)$$

Solutions of these are known as "conjugate harmonic" functions.

From their definitions is seen that the equipotentials v = constant are everywhere perpendicular to the field, while the curves u = constant are everywhere tangent to it and thus reduce to lines of force.

By picking C in (2) along a line of force, then along an equipotential in (3) one obtains from (2), (3)

$$F = \frac{\partial v}{\partial n} = \frac{\partial u}{\partial s} \qquad (10)$$

where F is the field strength $\sqrt{X^2 + Y^2}$, and dn, ds are the elements of length normal to and parallel to the equipotentials, respectively. Hence, approximately, for small Δu, Δv

$$F = \frac{\Delta v}{\Delta n} = \frac{\Delta u}{\Delta s} \qquad (11)$$

If, therefore, we consider the equipotentials and flux lines

$$u = u_0 + m\delta, \quad v = v_0 + n\delta \qquad (12)$$

where m, n are integers and δ a small constant, then between adjacent curves $\Delta u = \Delta v = \delta$, and from (9) follows

$$\Delta n = \Delta s \qquad (13)$$

that is the curves (12) break up the plane into small squares. This is the basis of the familiar field-plotting methods used in this case.

While powerful analytical methods are available for the solution of Laplace's equation, many problems arise for which the graphical methods are the only convenient ones. Often the analytical solutions are suited only to special simple boundaries; the graphical ones, on the other hand, apply with equal facility to all kinds of boundaries.

In the applications to gravitation, electrostatics, and magnetism, the existence of a scalar potential and the conservation of flux can be inferred directly from the inverse square law; in case of magnetic fields due to electric currents, indirectly, by the use of dipoles. There are cases where other reasons must be advanced for the applicability of (1). Thus in the flow of an ideal, incompressible fluid, with X, Y denoting the velocity components, the conservation of flux obviously expresses the conditions for conservation of fluid mass. (The name "flux," in fact, owes its origin to the analogy to flow of a fluid.) The existence of the scalar potential, however—the velocity potential—was at first purely postulated but later it was proved by the circulation theorem of Kelvin and Helmholtz, though its violation by actual fluids was not explained fully till comparatively recently.

In conduction of currents in thin plates, if X, Y denote the components of current-flow density across section perpendicular to the axes, the flux condition is again obvious, due to conservation of electricity. If V is the electric potential at any point, then by Ohm's law,

$$X = -kt\frac{\partial V}{\partial x}, \quad Y = -kt\frac{\partial V}{\partial y} \qquad (14)$$

where k is the conductivity and t the plate

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thickness. For uniform k and t, v may be identified with $kt\ V$ and equation 1 is obtained. For nonuniform k or t or both, (14) combined with (7) leads to

$$\frac{\partial}{\partial x}\left(kt\frac{\partial V}{\partial x}\right)+\frac{\partial}{\partial y}\left(kt\frac{\partial V}{\partial y}\right)=0 \qquad (15)$$

A somewhat unrelated application of (1) is in connection with small displacements of flexible membranes initially stretched to a constant tension in a plane, then deflected slightly by normal displacements of edges. If v is the normal deflection, its derivatives $\partial v/\partial x, \partial v/\partial y$ are shown to be proportional to the normal components of tension across sections parallel to the axes, while (1) follows from the equilibrium of the forces acting on an element.

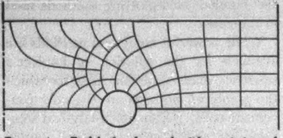

Figure 1. Field of a charged grid consisting of circular rings

3. Axially Symmetric Fields

We consider next axially symmetric fields possessing a scalar potential v and conservation of flux. If r, θ, z are cylindrical co-ordinates, v depends on r and z but is independent of θ. The aspect of the field in one half plane through the axis of symmetry (say, the plane $\theta = 0$ in which the x-axis lies) is sufficient for specifying the whole field.

In such a half plane consider the equipotentials for which the potential v differs again by the same constant amount of Δv; also pick lines of force such that when revolved about the axis of symmetry they yield tubes of flux, each enclosing the *same* amount of flux Δu. The equipotentials and flux lines now break up the half plane into rectangles whose sides Δn, Δs are no longer equal to each other, but have a ratio varying as r, the distance from the axis, thus

$$\frac{\Delta n}{\Delta s}=kr, \quad k=\frac{2\pi\,\Delta v}{\Delta u} \qquad (16)$$

Indeed, the field strength H is given by

$$H=\frac{\partial v}{\partial n} \qquad (17)$$

while the flux crossing the element of area obtained by revolving ds about the axis is

$$du = 2\pi\, r\, H\, ds$$

so that

$$H=\frac{1}{2\pi r}\,\frac{\partial u}{\partial s} \qquad (18)$$

From (17), (18) one readily derives (16).

Fields of axial symmetry are obviously equivalent to conduction problems for thin plates whose thickness varies as the distance from a fixed line or edge. By reference to (15) it will be seen that the potential satisfies not (1) but

$$\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)+\frac{\partial}{\partial z}\left(r\frac{\partial v}{\partial z}\right)=0 \qquad (19)$$

The flux function of such fields is sometimes referred to as the Stokes function.

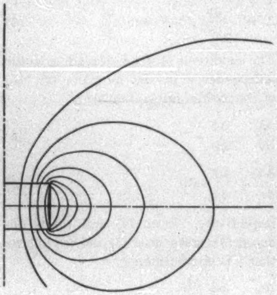

Figure 2. Solenoid in air

Plots of fields of axial symmetry are more difficult to make than two-dimensional small-square plots. In making such a plot it is convenient to mark a distance from the axis at which kr is taken as one so that the rectangles reduce to squares. At double the distance the rectangles will then have their sides in the ratio two to one, and so forth. Right near the axis, due to the vanishing of r, it is convenient to drop the rule (16) and replace it by the requirement that successive flux lines are at distances from the axis in ratio

$$1^{1/2}:2^{1/2}:3^{1/2}:\ldots \qquad (20)$$

This result is obtained by considering a uniform field parallel to the axis and successive cylindrical shells enclosing equal amounts of flux Δu.

As usual, if there are no finite boundaries, there are conditions "at infinity" to consider at the boundary of the plot. These vary with the nature of the problem. If the field becomes perpendicular to the axis and uniform (see figure 1), the equipotentials become parallel to the axis and at distances varying in geometric series. If the field approaches that of a point charge on the axis of symmetry, the equipotentials become circular while the lines of flux approach those radial lines that correspond to colatitude angles Ω for which $\cos\Omega$ increases by constant amounts. In case of solenoids with air core or iron core the field at infinity is like that of a dipole for which

$$v = \text{constant} \times \cos\Omega/R^2, \quad R=\sqrt{r^2+z^2} \qquad (21)$$

This is also the aspect of the field at infinity for a condenser of axial symmetry with equal positive and negative charges.

Figure 1 represents the field near a grid consisting of equidistant concentric rings; figure 2 the field of a solenoid with air core; figure 3 that of a solenoid on an iron core ($\mu = \infty$). The inductances can be readily found from the plot and check reasonably with measured values and tables.

4. Plane Fields in Current-Carrying Regions—the Poisson Equation

Inside current-carrying regions the magnetic field fails to possess a scalar potential, retaining, however, the constancy of flux property. This is tied up with the fact that the "work-integral" $\int F_s ds$ *is* dependent upon the path of integration. For the two-dimensional case equations 2, 4, 6 are now discarded, while (3), (5), (7) still apply. Equation 6, in fact, is replaced by the relation

$$\frac{\partial Y}{\partial x}-\frac{\partial X}{\partial y}=4\pi i \qquad (22)$$

where i is the density of the current (extending indefinitely in the direction of the z-axis) and X, Y the components of the nagnetic field. This relation is obtained by carrying a unit pole around an element of area $dx\ dy$ and equating the work done

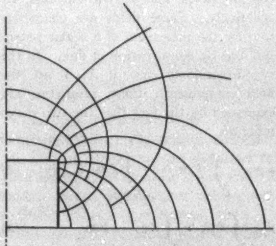

Figure 3. Solenoid on iron core

to 4π times the current enclosed. Substituting from (5) one obtains

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -4\pi i \qquad (23)$$

This now replaces Laplace's equation 8.

In electrostatic or gravitational problems, within regions occupied by charges or attracting masses, the scalar potential still exists, but the flux constancy is modified so that an equation similar to (23), known as Poisson's equation results for the potential.

Technically of interest is the case where i is constant over rectangular regions and zero elsewhere. Rules have been given for plotting solutions of (23) but they are by no means simple, and the procedure is quite complex. A method based on superposition will now be outlined.

This method is based upon the fact that equation 23 is linear in u but not homogeneous. Any particular solution u of (23) can be written as

$$u = u_1 + u_2 \qquad (24)$$

where u_1 is any other solution of (23) and u_2 is a solution of the homogeneous (Laplace) equation (8). In many cases convenient particular solutions of (23) are available in analytical form. The correcting u_2 plot, since it satisfies Laplace's equation is of the small curvilinear square type. Now the desired solution u must satisfy certain boundary conditions; the special solution u_1 will in general fail to satisfy these conditions. Thus u_2 must be such as to correct for this fact in order that $u = u_1 + u_2$ does fulfill these boundary conditions.

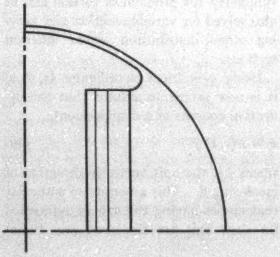

Figure 4. Two-pole machine—preliminary plot

Quite often the boundary conditions are such that

$$\frac{\partial u}{\partial n} = 0 \qquad (25)$$

corresponding to infinitely permeable iron. In such cases then u_2 must be chosen from the boundary condition

$$\frac{\partial u_2}{\partial n} = -\frac{\partial u_1}{\partial n} \qquad (26)$$

This condition may be replaced by a condition for v_2, the conjugate harmonic of u_2, whose tangential derivative may thus be found from (26). From this tangential derivative v_2 itself may be found by integration. This is more convenient than (26).

The addition of the u_1, u_2, values is carried out graphically by drawing diagonals of the small parallelograms formed by the curves u_1 = constant, u_2 = constant, drawn for equidistant right-hand constants. This is indicated in figure 6 in which the plot of the field in a two-pole machine with field coil is shown. Figure 4 gives the u_1-plot which in this case is picked so as to depend on one co-ordinate only (x) and so as to vanish outside the current-carrying region. Figure 5 indicates the correcting u_2-plot along with its conjugate harmonic v_2.

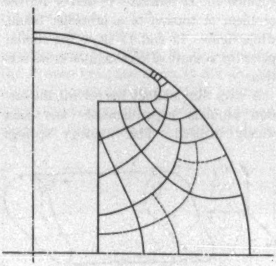

Figure 5. Two-pole machine—complementary plot

The example just described is a very simple one because the coil is backed on three sides by iron. In general cases the current carrying regions are removed from the iron boundaries. In such cases u_1 may be taken as the flux function corresponding to a rectangular conductor in free space. An analytic solution of this is available and plots for certain cases may be found in the literature. (See "Graphical Determination of Magnetic Fields," by A. R. Stevenson and R. H. Park, *G. E. Review*, February-March, 1928.) Figures 7 and 8 show the u_1 and the complete $u = u_1 + u_2$ plots for the case of a coil in a rectangular slot. In applying (26) both u_1 and u_2 are replaced by their conjugate harmonic potentials v_1 and v_2, and the conditions (25), (26) by the constancy of $v_1 + v_2$ along each iron boundaries.

5. Torsion of Shafts of Constant Cross Section—Viscous Flow in Pipes

Poisson's equation with a constant right-hand member also occurs in elasticity in the problem of torsion of shafts (or prisms) of constant cross section. Let t be the twist or rotation per unit length of the axis. Choosing the latter as the z-axis the displacement components are given by

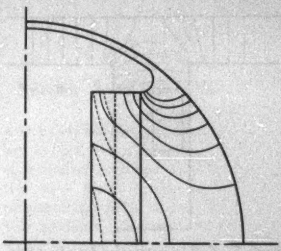

Figure 6. Two-pole machine—final field plot

$$\left.\begin{aligned} u &= -tyz \\ v &= txz \\ w &= t\,\phi(x, y) \end{aligned}\right\} \qquad (27)$$

Here the first two equations represent a rotation of magnitude tz about the z-axis, while the last one assumes a warping of cross section proportional to t but independent of z. Substituting in the equations of elastic equilibrium (G = shear modulus, σ = Poisson's ratio)

$$G\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(u, v, w) + \frac{1}{1-2\sigma}\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right] = 0 \qquad (28)$$

there follows that ϕ is harmonic. The various strain components are seen to vanish except for the shears in the xz- and yz-planes. Hence all the stress components vanish except for X_z and Y_z. Solving for these one obtains

$$\left.\begin{aligned} X_z &= G\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = Gt\left(\frac{\partial \phi}{\partial x} - y\right) \\ Y_z &= G\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = Gt\left(\frac{\partial \phi}{\partial y} + x\right) \end{aligned}\right\} \qquad (29)$$

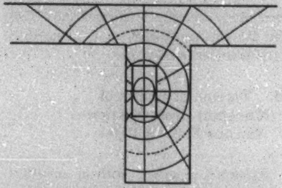

Figure 7. Coil in rectangular slot—preliminary plot (field of coil in air)

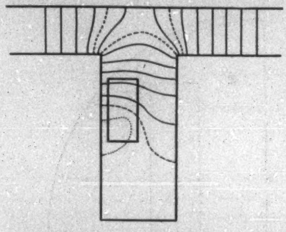

Figure 8. Coil in rectangular slot—final field plot

If ψ is the conjugate harmonic function of ϕ, the above become

$$\left.\begin{aligned} X_z &= Gt\left(\frac{\partial\psi}{\partial y} - y\right) \\ Y_z &= Gt\left(-\frac{\partial\psi}{\partial x} + x\right) \end{aligned}\right\} \tag{30}$$

and introducing

$$\Psi = \psi - \frac{1}{2}(x^2 + y^2) \tag{31}$$

$$X_z = Gt\frac{\partial\Psi}{\partial y}, \qquad Y_z = -Gt\frac{\partial\Psi}{\partial x} \tag{32}$$

Comparing with (5) one concludes that $Gt\,\Psi$ is a "flux function" for the vector X_z, Y_z. The latter represents the stress vector across a shaft section, so that this vector is tangent to the curves Ψ = constant and in magnitude is equal to the gradient of Ψ. On this account Ψ is called the "stress function." Since ψ is harmonic it follows from (31) that Ψ satisfies the Poisson equation

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} = -2 \tag{33}$$

Along the boundary of the shaft section the shear vector must be tangent to the boundary curve, since any normal shear component would also show up as a shear across the shaft boundary, whereas that boundary is, of course, free from stress (see figure 9). Hence Ψ is constant along the boundary, and (for simply connected regions) could be taken as equal to zero there. It follows then that ψ could be taken as equal to $^1/_2\,(x^2 + y^2)$ over the boundary. Once Ψ is determined, the stress distribution is known, and its resultant and moment or torque may be determined. The former vanishes while the latter is given by

$$Q = 2Gt\iint \Psi\,dx\,dy \tag{34}$$

so that the stiffness constant of the shaft per unit length of axis is given by

$$Q/t = 2G\iint \Psi\,dx\,dy \tag{35}$$

The graphical method outlined in the preceding section can be applied very nicely toward the solution of (33). A convenient particular solution is $\Psi_1 = -x^2$, or, what is suggested by the analysis, $\Psi_1 = -(x^2 + y^2)/2$. The complementary harmonic function Ψ_2 then takes on values equal to $-\Psi_1$ along the boundary. Figures 10, 11 indicate Ψ_2 and Ψ for the problem of torsion of a propeller blade, while figures 12 and 13 show the similar plots for a shaft of rectangular cross section.

A soap film analogy for solving the torsion problem experimentally has been widely utilized. This analogy springs from the fact that the normal deflection w of a loaded stretched membrane satisfies the equation

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{p}{T} \tag{36}$$

where p is the pressure and T the tension. It is believed that the graphical method above outlined is far more convenient and less time-consuming than the soap film method.

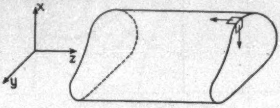

Figure 9

The laminar (non-turbulent) flow of an incompressible viscous fluid in a pipe of arbitrary but uniform cross section can also be reduced to a similar Poisson equation. The hydrodynamic equations, in absence of acceleration and external forces, are

$$\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\right) = \mu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(u, v, w) \tag{37}$$

where μ is the coefficient of viscosity, p the pressure, and u, v, w are the components of velocity. Placing the tube axis along the z-axis, so that u, v vanish while w reduces to a function of x, y only, one obtains from (37)

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial z} \tag{38}$$

Since the pressure gradient is constant both across the pipe section and along the axis, the right-hand side of (38) is truly constant. The analogy to the above problems is obvious, and the same graphical methods are applicable. In fact, figure 11, 13 may be considered to yield velocity profiles for the flow in channels of the shape given by the boundary, while the torsional shear and torque turn out to be analogous to the viscous shear and total flow.

6. Torsion of Shafts of Circular but Variable Cross Section

In the preceding section the shaft cross section was arbitrary, but had to be constant along the shaft; the state of stress, likewise, was assumed to be the same at all cross sections. For axial symmetry the problem of torsion can be also solved for variable section and varying stress distribution across different sections.

Using cylindrical co-ordinates (r, θ, z) it is now proper to assume that the deflection consists of a displacement

$$\mathbf{v} = v\,(r, z)\,\mathbf{j} \tag{39}$$

where $\mathbf{j}$ is the unit vector in direction of increasing θ. This amounts to assuming that circles having the axis of symmetry for their axis are displaced into them-

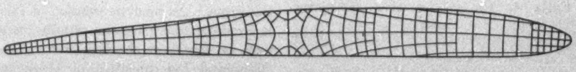

Figure 10. Torsion of propeller blade—complementary plot

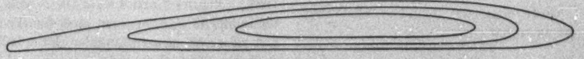

Figure 11. Torsion of propeller blade—plot of stress function

selves by undergoing an angular deflection

$$\omega = v/r \tag{40}$$

Substituting (39) in the elasticity equations (28) written perferably in vector form

$$G\left[\nabla^2 v + \frac{1}{1-2}\nabla(\nabla \cdot v)\right] \tag{41}$$

one finds that the divergence $\nabla \cdot v$ vanishes, (this is also obvious from the nature of the displacement) and hence

$$0 = \nabla^2 v =$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)(v\,j)$$

$$= j\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{\partial r}\frac{\partial v}{r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2}\right] \tag{42}$$

The resulting equation in v may be written thus:

$$\frac{\partial}{\partial r}\left[r^3\frac{\partial(v/r)}{\partial r}\right] + \frac{\partial}{\partial z}\left[r^3\frac{\partial(v/r)}{\partial z}\right] = 0 \tag{43}$$

Comparing (43) with (15) we conclude that the angular deflection $\omega = v/r$ satisfies the same equation as the potential in a conduction problem in a thin plate whose thickness varies as r^3, the cube of the distance from the shaft axis.

Computation of the (strains and) stresses yields

$$p_{r\theta} = G\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right), \quad p_{\theta z} = G\frac{\partial v}{\partial z} \tag{44}$$

and shows that all the other stress components vanish. Putting (44) in the form

$$\frac{p_{r\theta}}{rG} = \frac{\partial \omega}{\partial r}, \quad \frac{p_{\theta z}}{rG} = \frac{\partial \omega}{\partial z} \tag{45}$$

one concludes that in the analogy between the torsion problem and conduction problem the vector $(p_{r\theta}/r,\ p_{\theta z}/r)$ corresponds to the voltage gradient and hence to the vector of current density per unit area of the plate section, while the direction of the resultant shear across a plane θ = constant is the same as the direction of the current flow. The current components I_r, I_z across *complete* plate sections (per unit perimeter) are proportional to

$$r^3(\partial\omega/\partial r), \quad r^3(\partial\omega/\partial z) \tag{46}$$

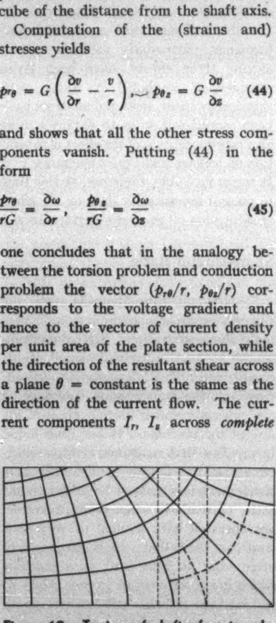

Figure 12. Torsion of shaft of rectangular section—complementary plot

and hence to

$$r^3 p_{r\theta}, \quad r^3 p_{\theta z} \tag{47}$$

It is possible to introduce a flux function u such that u = constant results in the flow lines. Moreover, the surface of rotation obtained by rotating a flow line around the axis of symmetry in the original shaft has no stresses transmitted across it. Hence the boundary of the shaft itself corresponds to a flow line except where the traction is applied, while if a shell bounded by two such surfaces is cut out, it will presumably hold itself in equilibrium by means of the tractions over its ends. The increase in the flux function between two successive flow lines, it is readily shown from (47), is equal to the torque or moment of the shearing stresses acting across a section of the corresponding shell.

While the analogy to a conduction problem can be utilized for determining the stresses in actual cases, it can also be used for deriving a rule of field plotting. A proof similar to that of section 3 shows that for the conduction problem in a plate of varying thickness, t, the curves $u = u_0 + m(\Delta u)$, $v = v_0 + n(\Delta u)$ cut the figure into rectangles such that $\Delta n/\Delta s$ vary as t. Thus in the present case the lines of equal angular deflection ω and lines of shear divide the shaft section into rectangles such that

$$\frac{\Delta n}{\Delta s} = kr^3 \tag{48}$$

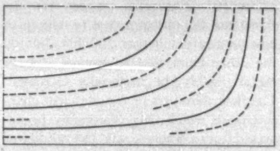

Figure 13. Torsion of shaft of rectangular section—plot of stress function

A plot of the stress problem of a short circular shaft with the torque applied by means of infinitely stiff wheels near the ends is shown in figure 14.

7. Other Problems Reducible to Field-Plotting Methods

Without going into detail we discuss briefly in this concluding section other problems capable of graphical treatment.

Fields possessing flux conservation and a potential v which varies as cos $n\theta$ yield a Laplace equation $V^2 v = 0$ the form

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{n^2}{r^2}v = 0 \tag{49}$$

For $n = 0$ these fields reduce, of course, to the case of axial symmetry. For $n = 1$ (49) will be recognized as equivalent to (42). Thus the graphical method of torsion of a circular shaft applies in this case. Fields varying as cos θ approximate the fields at the end of two-pole machines. Similarly any value of n can be shown to be reducible to a conduction problem in a strip of proper thickness.

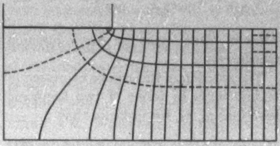

Figure 14. Torsion of short circular shaft

Two dimensional fields in regions with continuously varying dielectric constant ϵ or permeability μ can be reduced to graphical rules analogous to those for conduction problems in plates of variable thickness. Abrupt variations of ϵ or μ across boundaries reduce to certain "refraction" rules, whereby small squares on one side become rectangles of a definite ratio of sides on the other side.

Eddy-current problems can sometimes be reduced to graphical methods, though, in general, the rules are far too complicated to be useful. Similar remarks apply to wave propagation and radiation fields.

In sections 5 and 6 several elasticity problems were reduced to the Poisson and (vector) Laplace equations, and graphical methods were obtained for them based on these equations. Another problem in elasticity which admits of a similar treatment is the exact problem of bending of a cantilever of any cross section by means of a terminal load. This problem is generally treated by the approximate methods of beam theory which do not allow the precise determination of the shear distribution. However, an exact treatment is possible in terms of a harmonic "bending function" χ satisfying a proper boundary condition (Love, "Elasticity," chapter XV, section 229). The determination of χ can be carried out graphically in a manner somewhat similar to that of section 5.

The reduction of several elasticity

problems to the Laplace equation is, however, somewhat exceptional. In general, elasticity problems reduce to the *repeated* Laplace equation whose two-dimensional form is

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 F = \left(\frac{\partial^4}{\partial x^4}+2\,\frac{\partial^4}{\partial x^2\partial y^2}+\frac{\partial^4}{\partial y^4}\right)F = 0 \qquad (50)$$

Thus in plane stress the Airy's function satisfies (50).

The solution of this can be expressed in several ways in terms of two harmonic functions u_1, u_2, for instance thus

$$F = xu_1 + u_2 \qquad (51)$$

so that the stress determination can be reduced to the construction of two plots of the small-square type, one for u_1 and the other for u_2. However, when the boundary conditions utilizing the applied tractions are expressed in terms of u_1 and u_2 they turn out to be of a complicated nature which renders the above graphical method rather difficult to apply in its present form. However, for a partial solution of the plane stress problem the small-square plot is useful in determining the sum of the principal stresses in terms of its boundary values (this is harmonic), and for simplifying a complicated boundary into a simpler one without at the same time unduly complicating the transformation of (50).

In conclusion the author gratefully acknowledges the help of Walter C. Johnson, of the advanced course, for assistance in the preparation of the diagrams. He further expresses his appreciation to the Institute for the privilege of presenting this discussion.

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Stability Characteristics of Turbine Generators

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IT IS believed that many worth-while savings and improvements can be obtained by considering more carefully than has been done in the past, what constitutes rational turbine-generator stability requirements. Keeping this objective in view, this paper reviews and analyzes the steady-state stability characteristics of turbine generators taking the effects of saturation into account.

The need for such an analysis has become particularly important with the recent trend toward 3,600-rpm units of large capacity. With the development of such units, the designer is forced in the direction of reducing the weight of the rotor per kilovolt-ampere in order to keep the length and diameter within reasonable limits. However, the reduction in weight of the rotor tends to reduce the short-circuit ratio, which has long been used as a measure of the stability characteristics. Naturally, neither the designer nor the operator care to relinquish any necessary inherent stability, but on the other hand the mechanical stresses cannot, safely, be materially increased. The determination of the stability characteristics which may reasonably be expected from a machine, and the validity of short-circuit ratio as a criterion for steady-state stability, were considered sufficiently important questions to justify making an analysis.

The steady-state stability characteristics are by far the most important from a design standpoint. The transient characteristics are always comparatively good because of the inherently low transient reactance of the generator and the comparatively high inertia of the rotating element. The noncondensing turbine and its generator is an exception in the latter respect in that its effective inertia is considerably less than that of a condensing turbine and its generator, being comparable to that of a water wheel with its generator. However, the transient characteristics are in most cases determined largely by the fault clearing times and station and system design, and are only slightly affected by the machine characteristics.

Although this paper deals primarily with the turbine generator or cylindrical-rotor type of generator, the results also apply in general to the water-wheel or salient-pole generator.

Previous analyses[1–4] have shown the effects of voltage regulators, machine reactances, types of load, and in an approximate manner, saturation, on the stability characteristics. More recently the effects of saturation on machine performance, particularly methods of analyzing these effects, have been given considerable study.[5,6,9–12] Along with this development, tests and analyses have been carried out so as to obtain a more accurate conception of the effect of saturation on leakage and Potier reactance.[13–15] It seems desirable, therefore, at this time to present quantitative data on the effect of saturation on generator performance.

Short-Circuit Ratio

Short-circuit ratio depends upon the synchronous reactance and no-load normal-voltage saturation. Since the saturation at no load is normally small, short-circuit ratio thus depends almost entirely on the synchronous reactance of the generator and is an indication of the size of the machine. It has been found in practice that machines of high short-circuit ratio are in general inherently more stable than machines of low short-circuit ratio, particularly when these generators are operated with manual or very slow control of excitation. Slow response may often exist at light load on a machine which has a self-excited exciter while at full load the response will be more rapid.

It is necessary that a manually controlled generator be able to carry, with

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The authors wish to acknowledge the co-operation and many helpful suggestions of M. A. Savage of the General Electric Company and S. B. Farnham of the same company for his assistance in making many of the calculations.

[1] For all numbered references, see list at end of paper.

no change of field excitation, the load changes to which it may be subjected following a system disturbance. To do this, the machine should have a high short-circuit ratio, since short-circuit ratio and field excitation are practically the only machine characteristics which determine the pull-out power. Under these conditions pull-out occurs with the generator attempting to carry a high kilowatt load with low excitation, resulting in leading terminal power factor or low terminal voltage or both, with consequent low saturation in the machine. Since short-circuit ratio is, in effect, determined by synchronous reactance modified by no-load normal-voltage saturation, it provides a fair measure of the relative stability characteristics of machines operating under manual control of field excitation.

However, even in this case, short-circuit ratio has its limitations because the field excitation for a given load condition is a function of the load saturation for that load. Therefore, two machines which have identical short-circuit ratios but different load saturations can pick up different increments of load from the same initial loading without loss of synchronism, the machine having the largest load saturation being able to pick up the greatest amount of load.

There are two advantages in having a machine of high short-circuit ratio which are worthy of mention. First, under some conditions of operation, particularly light kilowatt load, a turbine generator may be operated underexcited in order to hold the voltage at its terminals down to normal. Under these conditions the amount of corrective kilovolt-amperes which the generator is able to supply to the system for a given kilowatt load is a function of the short-circuit ratio as there is little or no saturation existing in the machine. Also, the line charging capacity of a machine is determined principally by its synchronous reactance, which is approximately specified by the short-circuit ratio. On the other hand, a generator of high short-circuit ratio is an expensive machine, tending to have inherently greater losses and higher short-circuit current. These disadvantages in many cases outweigh the advantage of having greater reactive corrective capacity at light loads.

Voltage Regulators

For machines that are operated with voltage regulators and exciters of moderate response and that are not required to operate appreciably underexcited, the need for machines of high short-circuit ratio has been in many cases greatly overemphasized. Furthermore, short-circuit ratio is not a true criterion for the relative stability characteristics of machines operating with voltage regulators. The use of generator voltage regulators with pilot-excited exciters has become increasingly more general, particularly for new machines. This makes possible a more economical design, in that the regulator is allowed to take its proper share of responsibility in maintaining the stability of the system, a responsibility which can now be confidently assigned to the modern regulator and exciter. By shifting part of the responsibility for maintaining synchronism onto the excitation system, it is possible to build a generator of reduced weight and size, having inherently lower stresses, and higher efficiency.

The function of the regulator, as used in this discussion, is to make changes in field excitation so as to maintain terminal or system voltage following load changes, as distinguished from the operation of a regulator which is capable of maintaining stability under the condition of "dynamic stability." A regulator capable of making continual and sufficiently rapid changes in field excitation is able to maintain the stability of a generator above the ordinary static stability limits which are discussed in this paper. However, such operation is not considered here.

There are a few systems, those which serve large metropolitan areas, which depend upon manual control of the excitation system. For these systems there has been a feeling in the past that machines of around unity short-circuit ratio should be used. This was based on experiences involving the loss of an appreciable amount of generation due to a fault or interruption of an interconnection, with subsequent increase of load on the remaining machines. The machines which before the disturbance were operating at light load would attempt to pick up the largest increment of load. It was found that if these initially lightly loaded machines had low short-circuit ratios, they were more apt to lose synchronism. This, of course, aggravated the disturbance and naturally resulted in a strong preference for machines of high short-circuit ratio.

Disturbances of this type can in general be successfully handled by automatic voltage regulators. Also, it is believed that by proper system design and connection of generating capacity relative to the load areas, the amount of load that must be picked up in any given area may be made comparable to the generating capacity in that area. This not only aids in riding through disturbances resulting in the breaking up of the system into sections, but also allows a more economical utilization of generating facilities in that the required amount of spinning reserve is reduced, and the machines may therefore be operated more nearly at their most economical loading. It is believed that if these factors are given proper consideration, along with the fact that a high-short-circuit-ratio machine is a more expensive generator, there will be little reason for going to short-circuit ratios higher than 0.8 for a generator feeding directly into a metropolitan area.

For those machines provided with voltage regulators which are located relatively close to their load and have loads of inherently stable characteristics, such as lighting, rectifier, and converter load, little attention need be paid to the stability characteristics. In fact, it is reported that loads of this type are being successfully carried in Europe by machines with short-circuit ratios as low as 0.4, while in the United States machines having short-circuit ratios of 0.6 are operating satisfactorily. For generators located relatively distant from their loads or which have a large proportion of synchronous motor load, the stability characteristics are of importance. For these cases in which stability is an important factor, short-circuit ratio should not be used as an indication of the relative stability since pull-out occurs under a condition of higher saturation than that corresponding to no-load normal-voltage saturation.

Stability Criterion

It has been realized for some time that short-circuit ratio does not properly indicate the stability characteristics of a generator which has means of increasing the field excitation corresponding to changes in kilowatt load, and that other factors enter into the determination of the relative stability characteristics. To specify the stability characteristics more completely, another quantity, saturation factor, is used in addition to short-circuit ratio; short-circuit ratio depending upon the synchronous reactance and no-load saturation, and saturation factor on the slope of the open-circuit saturation curve at no-load normal voltage.

Short-circuit ratio and saturation factor depend upon the no-load saturation characteristics. In reference 2, saturation factor was used in an approximate criterion for determining the minimum

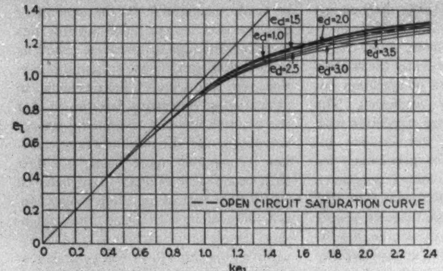

Figure 1. Turbine-generator saturation characteristics

Short-circuit ratio = 0.66 x_d = 1.64 x_l = 0.15

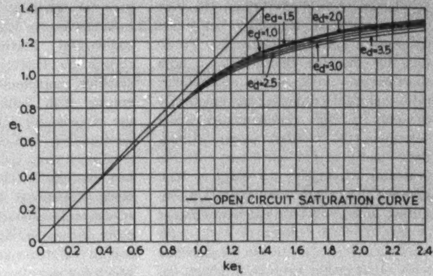

Figure 2. Turbine-generator saturation characteristics

Short-circuit ratio = 0.85 x_d = 1.29 x_l = 0.13

short-circuit ratio. However, with the more general use of voltage regulators, it has become evident that, instead of the no-load saturation characteristics, the load saturation characteristics should be used, as it is the latter which determine the pull-out under load. The factors which determine the stability characteristics under load are the synchronous and leakage reactances, the amount of load saturation, and the rate of change of saturation under load. All these factors should be weighted if a single quantity for determining the stability characteristics of alternators under load is to be obtained. An investigation[5] revealed that the weighting to be given these factors depended not only on the load the machine was carrying but also upon the system and type of load connected to its terminals.

However, this previous investigation did suggest an approximate value of average equivalent reactance for turbine generators operating under load at normal power factors connected to a system through impedance which included all of the above machine factors. This criterion is given by the formula*

$$x_{eq} = x_l + \frac{x_d - x_l}{k\sqrt{1 + a/b}} \tag{1}$$

which can be determined approximately from the no-load saturation curve as shown by figure 18. In this paper are presented the results of calculations which show that this criterion does in general give, with fair accuracy, the approximate stability limits, and is conservative. A method of testing for this reactance using a zero-power-factor volt-ampere characteristic, as well as a method for its approximate calculation from the open-circuit saturation curve, are presented in appendices *C* and *D*, respectively.

Results of Analysis

An analysis of the steady-state stability characteristics of turbine generators is presented under the following five divisions.

A. Synchronous load without generator voltage regulators

B. Synchronous load with generator voltage regulators

C. Induction motor load with generator voltage regulators

D. Equivalent reactance for synchronous load

E. Equivalent reactance for induction motor load

A. SYNCHRONOUS LOAD WITHOUT GENERATOR VOLTAGE REGULATOR

In studying the stability characteristics of turbine generators, it was considered advisable to consider the case in which the generator is provided with a voltage regulator of modern design such that it can successfully follow the load changes to which the generator is subjected, and also the case in which the generator excitation is under manual control. Results obtained from these two divisions of the study, i. e., synchronous load without regulators and synchronous load with regulators, should indicate therefore the maximum benefit to be obtained by the use of a voltage regulator. Slow response or sluggish excitation systems would be intermediate between these two limits.

The first division of the analysis to be discussed is synchronous load without generator voltage regulators. This part of the analysis was made by determining the maximum amount of power that a generator could deliver to an infinite bus through a given amount of external reactance, starting from different initial load conditions. The external reactance may be taken as representing a power system or a synchronous motor load.

Figures 4 and 5 show the maximum load that can be carried for three machines having short-circuit ratios of 0.66, 0.85, and 1.03 for varying amounts of initial load, with zero external reactance $x_e = 0$. The excitation for these machines was held constant corresponding to that required for the initial load at the power factor indicated on the figure (0.8 for figure 4 and 1.0 for figure 5). The characteristics, including the load saturation curves, necessary for determining these limits are given on figures 1, 2, and 3.

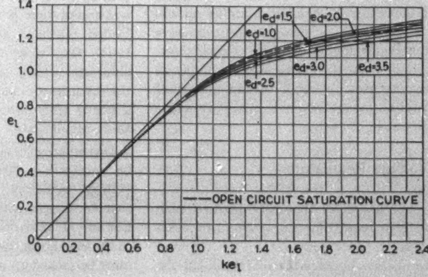

Figure 3. Turbine-generator saturation characteristics

Short-circuit ratio = 1.03
x_d = 1.09
x_l = 0.11

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* See nomenclature for definition of symbols.

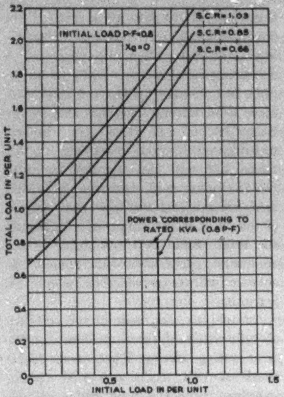

Figure 4. Steady-state power limits of turbine generators with excitation fixed at values corresponding to indicated initial load at 0.8 power factor and 1.0 initial terminal voltage

Saturation characteristics shown on figures 1, 2, and 3. Generator connected directly to infinite bus

In this analysis, the same machine reactances and load saturation curves for a machine of given short-circuit ratio were used for two cases:

(*a*) When the initial load was 0.8 power factor, and

(*b*) When it was 1.0 power factor.

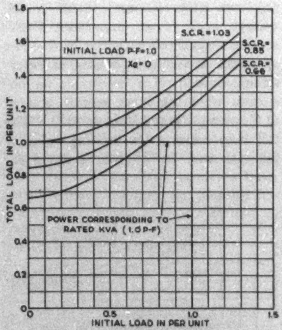

Figure 5. Steady-state power limits of turbine generators with excitation fixed at values corresponding to indicated initial load at 1.0 power factor and 1.0 initial terminal voltage

Saturation characteristics shown on figures 1, 2, and 3. Generator connected directly to infinite bus

Since the saturation characteristics were obtained from machines having a normal rating of 0.8 power factor, the stability characteristics for 1.0 power factor correspond to the condition of an 0.8-power-factor machine operating at 1.0 power factor. However, results obtained for these machines operating at 1.0 power factor can be interpreted as applying to machines designed for 1.0-power-factor operation, except that for the same kilovolt-ampere load the 1.0-power-factor machines studied in this paper have less load saturation than the corresponding 0.8-power-factor machines. The method of determining the limits is given in appendix *A*.

It will be noted that for the case of 0.8 power factor and zero external reactance, as given by figure 4, with zero initial load and an excitation corresponding to normal voltage, the 66 per cent short-circuit-ratio machine is able to pick up approximately 66 per cent load, the 85 per cent short-circuit-ratio machine 85 per cent load, and the 103 per cent short-circuit-ratio machine 103 per cent load. These are loads in per cent of the kilovolt-ampere base.

With an excitation corresponding to normal-voltage no load the ratio of the per unit field excitation to the synchronous reactance is the short-circuit ratio. Therefore if the terminal voltage is unity. pull-out occurs at a load equal to the short-circuit ratio on the basis that no saturation exists at pull-out, which is practically the case. As the initial load is increased the increment of load which can be picked up is decreased, while the total load which the generator can deliver without change of excitation is increased, because of the greater amount of excitation present. Since these calculations were made under the condition of zero external reactance, normal voltage being maintained at the terminals of the machine, figures 4 and 5 indicate the upper limit of power that can be carried on the basis of fixed field excitation. That is, if the voltage at the terminals were allowed to drop, the maximum amount of power that the machine could deliver would be decreased. Accordingly, if it is desired to pick up normal load on a unit without a voltage regulator, which is initially only lightly loaded, short-circuit ratios of the order of unity or greater would be required.

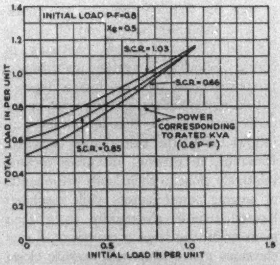

Figure 6. Steady-state power limits of turbine generators with excitation fixed at values corresponding to indicated initial load at 0.8 power factor and 1.0 initial terminal voltage

Saturation characteristics shown on figures 1, 2, and 3. Generator connected to infinite bus through reactance $x_e = 0.5$

Figure 5 is for the same conditions as figure 4 except the generator load is initially at 1.0 power factor rather than 0.8. Under these conditions the load that can be picked up from a zero initial load is the same as that for the 0.8-power-factor case, while for higher loads the increment of load and the total load which can be delivered at the initial constant excitation are less than for 0.8 power factor.

Subsequent to a system disturbance, a generator may be required to pick up a load equal or corresponding to that with all steam valves wide open and therefore may be expected to carry loads equal to or greater than its kilovolt-ampere rating. It is apparent from figures 4 and 5 that the ability of a machine to do this is increased with increase in short-circuit ratio, and also this ability is increased if the initial load is a lagging-power-factor load rather than a unity or leading-power-factor load. These results are borne out in practice by operating instructions

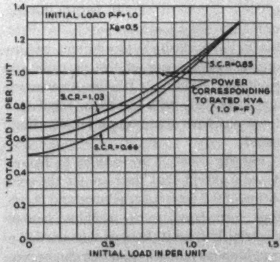

Figure 7. Steady-state power limits of turbine generators with excitation fixed at values corresponding to indicated initial load at 1.0 power factor and 1.0 initial terminal voltage

Saturation characteristics shown on figures 1, 2, and 3. Generator connected to infinite bus through reactance $x_e = 0.5$

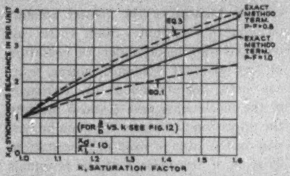

Figure 8. Maximum permissible synchronous reactance x_d for stability as a function of saturation factor k when generator is connected through unit external reactance to an infinite bus and is delivering rated current at unit terminal voltage

that require for machines under manual control of field excitation that the field current on a generator shall at no time be reduced to such an extent as to operate a machine leading, in the underexcited direction. These results also indicate the desirability from a stability standpoint, when machines of different short-circuit ratios are used under manual control, of having operating instructions such that the relative field currents of the machines are determined on the basis of their short-circuit ratios and their relative kilowatt loadings as well as upon other considerations.

The curves indicate the desirability of operating the lightly loaded units of a station at relatively lower lagging power factors, as these are the units which may be required to pick up the largest increments of load subsequent to a system disturbance and may therefore be the most likely ones to pull out of step. If these can be operated at lower power factors, the limits as shown on figures 4 and 5 do not necessarily apply for the lightly loaded condition. However, such overexcited operation of the lightly loaded unit may not always be possible at light loads, particularly if the system has an appreciable amount of line or cable capacitance loading. If the generators are operated with voltage regulators these problems are of course largely eliminated.

Figures 6 and 7 are similar to figures 4 and 5 except that they are for the condition of 0.5 external reactance and therefore correspond to a machine remote electrically from its load or connected to a synchronous motor load which can be represented by this amount of external reactance. It should be noted that in this case the amount of load which can be picked up for a given initial load is considerably reduced and that the difference between the three short-circuit ratios studied is greatly reduced. This indicates that for generators delivering power to a motor load or connected remote from a load, short-circuit ratios greater than unity would be required in order that the machines be able to pick up full load from an initially lightly loaded condition unless, of course, the machines are provided with voltage regulators or operated overexcited at the light loads. It is interesting to note that figures 6 and 7 for the higher initial loadings show only small differences in the total power that can be carried for the different short-circuit ratios.

B. Synchronous Load With Generator Voltage Regulators

Under this section, the case in which the generators are provided with voltage regulators and delivering power through an external reactance to a point of maintained voltage is studied. The generator excitation system is assumed to be such that pull-out occurs with normal voltage at the terminals of the machine. In order to show the effect of power factor on pull-out it was considered advisable to plot the stability limit for the conditions of machines pulling out of step at 0.8 and 1.0 power factor. Actually, of course, following a system disturbance, the power factor on a machine is dependent on many factors and may not necessarily correspond to that for normal conditions before the disturbance occurred. However, in general, machines located relatively close to their loads will operate at lagging power factors while machines located remote electrically from their loads or having large proportions of synchronous motor loads will operate closer to unity power factor.

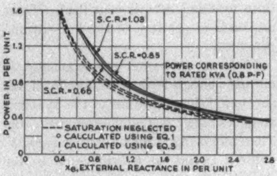

Figure 9. Steady-state power limits of turbine generators with various short-circuit ratios operating at normal terminal voltage and 0.8 power factor

Saturation characteristics shown on figures 1, 2, and 3

The effect of generator saturation on the stability limit when the generator is connected to synchronous load or to a large power system through a reactance tie is shown in figure 8. The full curves show the maximum permissible generator synchronous reactance for stability as a function of the saturation factor k, for 0.8 lag and 1.0 power factor at the generator terminals, when the generator is delivering full load ($i = 1.0$) to an infinite bus through 1.0 per unit reactance. Since k for normal machines may lie between 1.1 and 1.25 at full load, the permissible synchronous reactance lies between 1.45 and 2.05 per unit for unity power factor and between 1.65 and 2.45 per unit for 0.8 lag power factor. If the effect of saturation were neglected the error would be considerable as the maximum permissible synchronous reactance indicated would then be only 1.0 per unit.

An analysis was made of the stability characteristics of three machines of the same rating having synchronous reactances of 1.64, 1.29, and 1.09 (corresponding to short circuit ratios of 0.66, 0.85, and 1.03, respectively) and having, when operating at 0.8 power factor, approximately the same degree of saturation under load. When these machines are operated at unity power factor the amount of load saturation is greatly reduced. The saturation characteristics for these three machines are given on figures 1, 2, and 3, respectively. These characteristics are drawn in a manner similar to that used in reference 5. The dotted curve in each of these figures is the open-circuit saturation curve.

Figure 9 shows the steady-state power limit of these three machines as a function of the external reactance. This external reactance is the reactance between the generator terminals and a point of maintained voltage, which may, therefore, represent either a large system or the voltage back of an impedance representing a synchronous motor load. (See appendix A for method used in determining limits.) The conditions for operation were taken as normal voltage (1.0), normal power factor (0.8). The full lines indicate the limiting value of kilowatt load for a given external re-

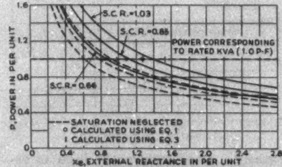

Figure 10. Steady-state power limits of turbine generators with various short-circuit ratios operating at normal terminal voltage and 1.0 power factor

Saturation characteristics shown on figures 1, 2, and 3 actance for each of the three machines. It will be noted that for this case of machines operating under the condition of 0.8 terminal power factor at pull out, there is very little difference in the power limit with change in synchronous reactance. The dotted lines correspond to the condition when saturation is neglected, the power limit being then determined by synchronous reactance. This would be comparable to the limit determined by short-circuit ratio.

Following a system disturbance the output of the generator may be increased to a load corresponding to the condition with all steam valves wide open, which may be five to ten per cent above the kilowatt rating, or as high as the kilovolt-ampere rating of the generator, depending upon the turbine design. (A possible exception is the case of a topping unit which may be subjected, subsequent to a disturbance, to low back pressure resulting in high turbine torque and high electrical output. This, however, is an unusual type of disturbance which can usually be adequately taken care of in the design of the steam system and control.) Accordingly, the limiting values of external reactance, x_e, for purposes of discussion may be considered at the condition of $P = 1.0$ in figure 9, corresponding to a kilowatt load equal to the kilovolt-ampere rating. For this load the permissible value of external reactance varies less than ten per cent for the three machines studied. Therefore, for all practical purposes there is no appreciable difference between the stability of a 1.03 or a 0.66 short-circuit-ratio machine if pull-out occurs at normal voltage and 0.8 power factor.

Figure 10 presents the results of a set of calculations similar to that given in figure 9, except that the machine pulls out under the condition of 1.0 power factor instead of 0.8 power factor. It will be noted that at unity power factor there is a greater difference in the stability limits of the three machines for a given external reactance, x_e, than for the case of pull-out at 0.8 power factor. Also the effect of saturation is greater even though the degree of saturation is less. This indicates that a greater increase in power limit can be obtained for a given decrease in generator reactance, if the machine operates at unity power factor near the pull-out point. Since machines which are located relatively distant from their loads operate at higher power factors, these results present additional evidence that these machines are, in general, the ones which should be given the most consideration from a stability standpoint.

During or subsequent to a disturbance which may result in instability the terminal voltage usually tends to be lower than normal. This means, therefore, that with a good excitation system of moderate response and ample ceiling, the power factor at the generator terminals after the major electrical transients have disappeared will usually be lagging even though the machine may operate at unity power factor under normal load conditions. Accordingly, it can be expected that under these conditions of lagging power factor and high excitation the effect of change in synchronous reactance or short-circuit ratio will not be as great as indicated by figure 10 but will tend to be more nearly like figure 9.

The results shown in figure 9 are for machines which were designed with comparable efficiencies and about the same degree of load saturation at 0.8 power factor load. Figure 10 is for the same machines studied in figure 9 except they are operated at 1.0 power factor and therefore have comparatively small saturation.

C. Induction Motor Load With Generator Voltage Regulators

The effect of generator saturation and reactance on the stability characteristics when the generator is connected to an induction motor load was studied for two conditions; one, when the induction motor load was connected directly to the terminals of the generator, and another when it was connected through line and transformer impedance. The full curve of figure 13 shows the maximum permissible synchronous reactance for stability as a function of the saturation factor k, when a generator is directly connected to an induction motor delivering full load at constant shaft torque. (Appendix B describes the method of calculation.) In calculating the results shown in this figure an approximate relation between k and a/b, figure 12, was used. The relation between k and a/b given in figure 12 is an average arrived at from several different machine designs. The induction motor characteristics used in the calculations are shown in figure 13. As normal values of k vary from 1.10 to 1.25 at full load the permissible synchronous reactance may vary from 2.00 to 3.00. As seen from this curve, if conclusions were based on neglecting the effect of saturation considerable error would be obtained, as the maximum permissible synchronous reactance would appear to be only 1.30.

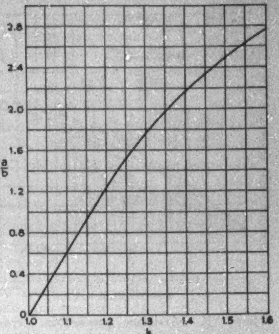

Figure 12. Average relation between saturation factors used in calculation of induction motor stability limits shown on figure 13

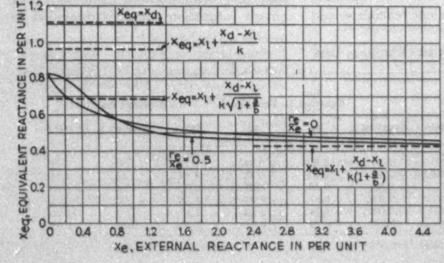

Figure 11. Equivalent reactance for a turbine generator operating at normal terminal voltage and 0.8 power factor

Saturation characteristics shown on figure 5, reference 5

Another set of calculations was made to determine the per cent direct-connected induction-motor overload that could be carried by each of the three different generators having synchronous reactances of 1.09, 1.29, and 1.64. The results of these calculations indicated that kilowatt loads in excess of 200 per cent of generator rating could be carried by any of the three machines without loss of synchronism. This indicated the inherently stable characteristics of induction motor loads when connected directly to the terminals of a turbine generator having a voltage regulator or close manual control.

Next, on the basis of induction motor loads equal to the rating of the generators,

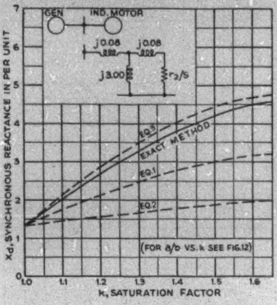

Figure 13. Maximum permissible synchronous reactance of generator for stability as a function of saturation factor k

Generator directly connected to an induction motor which is delivering full load with constant shaft torque ($P = 0.89$ in per unit)

the maximum permissible amounts of impedance (viewed from the generator end of the line) for stability between the generator and the motor terminals were determined. The equivalent induction motor was taken as fully loaded. Voltage was taken as normal at the generator and motor terminals, a transformer having the required turn ratio to accomplish this being assumed in the line. The results are shown in figure 14 for the condition of $r_e/x_e = 0$ and 0.5, which covers the usual range of these quantities met in practice. As seen from these results there is not a great difference in permissible impedance with change in generator reactance. There would be still less difference if the induction motor representing the load was taken to be operating at partial load, which conforms more closely to the case met in practice for a composite group of motors.

D. Equivalent Reactance for Synchronous Load

From the results of reference 5 the following information was obtained.

(*a*) No single reactance can be used to represent exactly a synchronous generator under steady-state conditions even for a given set of load conditions. The equivalent reactance to be used depends on the connected or external system as well as on the machine reactances and saturation characteristics.

(*b*) In order to obtain a true equivalent reactance which gives the correct angular displacement as well as the correct current and voltage response, it is necessary to use two reactances.[6] However, for all practical purposes, for determining the effect of the machine on the system and for determining the stability limit, the machine can be represented by one equivalent reactance which gives the same voltage and current response at its terminals as the actual machine for small changes.

(*c*) It was found that the range of this reactance which gives the correct current and voltage response is from

$$x_{eq} = x_l + \frac{x_d - x_l}{k} \tag{2}$$

to

$$x_{eq} = x_l + \frac{x_d - x_l}{k\left(1 + \dfrac{a}{b}\right)} \tag{3}$$

depending upon whether it is a synchronous generator or condenser and what external system and load is connected to it.

(*d*) It was recommended that for purposes of approximately determining the stability characteristics of turbine generators a value

$$x_{eq} = x_l + \frac{x_d - x_l}{k\sqrt{1 + a/b}} \tag{1}$$

be used, which is an average between the two limiting values mentioned in (*c*) and which gives reasonably correct results.

(*e*) For a particular machine[5] the range that this equivalent reactance may take is shown on figure 11. The values of equivalent reactance which determine the pull-out power were found to be between that given by equation 1 and equation 3, with equation 3 indicating quite definitely the lower limit of equivalent reactance.

With this as a starting point for further investigation, approximate stability limits were determined at rated kilowatt load using equations 1 and 3. The results of some of these calculations are shown by the small circles and vertical lines on figures 9 and 10. The reactance data, values of equivalent reactance, and the maximum permissible external reactance for stability at rated load for these machines are given in table I. These results, as well as other calculations not shown in the paper, bear out the previous conclusion that the value of equivalent reactance determining the pull-out lies between that given by equations 1 and 3.

Calculations were also made for the case when the ratio of the external resistance to the external reactance was equal to 0.5 ($r_e/x_e = 0.5$). For this condition the actual power limit still remained between that given by equations 1 and 3 but was in general closer to that given by equation 1 than for the corresponding case when resistance was neglected.

For accurate calculation of pull-out, it is not necessarily recommended that equation 1 be used but that it be considered only for determining the approximate stability limits of turbine generators.

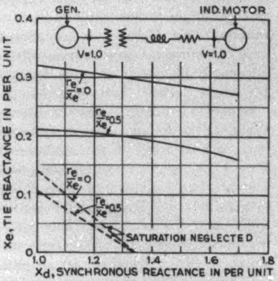

Figure 14. Permissible synchronous reactance for stability as function of tie reactance, motor delivering full load at constant shaft torque (induction motor power = 0.89 in per unit)

Generator saturation characteristics shown on figures 1, 2, and 3

A method of testing for this reactance using a zero-power volt-ampere characteristic is described in appendix C. As an approximation the value of this reactance may be determined from the open-circuit saturation curve as described in

Table I. Turbine Generator Reactance Data

Item	Power Factor	x_d	x_l	Short-Circuit Ratio	Saturation Characteristic	x_{eq} Equation 1	x_{eq} Equation 3	$\frac{x_{eq}\ \text{Equation 1}}{x_d}$	$\frac{x_{eq}\ \text{Equation 3}}{x_d}$	x_e, External Reactance* Exact	x_e, External Reactance* Equation 1	x_e, External Reactance* Equation 3
1	0.8	1.09	0.11	1.03	Figure 3	0.67	0.50	0.62	0.46	1.18	1.10	1.18
2	0.8	1.29	0.13	0.85	Figure 2	0.78	0.58	0.61	0.45	1.14	1.06	1.14
3	0.8	1.63	0.15	0.66	Figure 1	0.94	0.67	0.58	0.41	1.09	1.02	1.10
4	1.0	1.09	0.11	1.03	Figure 3	0.79	0.65	0.72	0.60	1.38	1.25	1.54
5	1.0	1.29	0.13	0.85	Figure 2	0.96	0.80	0.74	0.62	1.11	1.04	1.26
6	1.0	1.63	0.15	0.66	Figure 1	1.20	0.99	0.74	0.61	0.85	0.83	1.01

* Maximum permissible external reactance for stability at rated load.

appendix *D*. This latter method does not take into account correctly the effect of load saturation and will result in slightly lower values of equivalent reactance than that given by the test described in appendix *C*, as the no-load saturation curve has a smaller slope than the saturation curves corresponding to constant field excitation.

E. EQUIVALENT REACTANCE FOR INDUCTION MOTOR LOAD

For the case of induction motor load, as for the case of synchronous load, equations 1 and 3 were used to determine the approximate stability limit. The results of these calculations are given in figure 13. It will be noted that, as for the case of synchronous load, the actual limit lies between that given by the two formulas. In the case of induction load, however, the actual limit lies much closer to that determined by equation 3 than by equation 1. These calculations indicate a value of equivalent reactance for induction motor loads which lies between that given by equations 1 and 3, possibly:

$$x_{eq} = x_l + \frac{x_d - x_l}{k\left(1 + \frac{a}{b}\right)^{3/4}} \tag{4}$$

If it is desired to calculate the limit accurately, methods similar to that described in appendices *A* and *B* may be used, or possibly equation 4 used to obtain a check without a great deal of calculation for the case of induction motor load.

Conclusions

From the results of this study the following general conclusions have been drawn.

1. Saturation affects very appreciably the steady-state stability limits of a turbine generator and should not be neglected in a comparison of generator characteristics.

2. For a generator not equipped with voltage regulators, short-circuit ratio is an approximate measure of the relative stability characteristics.

3. A generator not equipped with a voltage regulator may require a short-circuit ratio of at least 1.0 if it is expected to pick up full load from an initially lightly loaded condition with no change in excitation.

4. A generator connected to a motor load or remote electrically from its load should be equipped with a voltage regulator; otherwise, the short-circuit ratio required in order that the generator be able to pick up full load from a lightly loaded condition with no change in field excitation is of such high order of magnitude that it becomes uneconomical to do without the regulator.

5. The reactance of a turbine generator operating at 0.8 power factor overexcited, and provided with a reliable voltage regulator and excitation system, does not influence appreciably the static stability limits, and accordingly machines which are to operate under these conditions need not necessarily be built with particularly low reactance or high short-circuit ratio.

6. The reactance of a turbine generator operating at 1.0 power factor near the static stability limit does have a considerable influence on the static stability limit, and accordingly machines which are known or are expected to operate under these conditions should be studied and, if necessary, the machines built to have a low reactance. A study of such cases should make it possible to obtain the necessary stability in the most economical manner.

7. The formula for equivalent reactance

$$x_{eq} = x_l + \frac{x_d - x_l}{k\sqrt{1 + a/b}}$$

may be used to calculate the approximate stability limits of turbine generators which are equipped with voltage regulators.

8. Induction motor load is considerably more stable than synchronous load and, except for unusual cases in which there is a large amount of line impedance, or when machines are not equipped with voltage regulators, does not constitute a stability limitation. Even for the case of a large line impedance the reactance of the generator is relatively unimportant.

9. A study and rational evaluation of the stability requirements to be demanded of a generator for a particular job should be made in order to realize the maximum savings in machine cost and improvement in system performance.

Nomenclature

All quantities are expressed in per unit on the machine normal kilovolt-ampere and voltage base.

P_{mn} = real power from point m directed toward n

Q_{mn} = reactive power from point m directed toward n. (Sign of Q_{mn} is negative when the voltage at m is "overexcited" with respect to voltage at n)

$\frac{a}{b} = \frac{e_l}{k}\frac{\partial k}{\partial e_l}$ = ratio determined from figure 18

e_e = system voltage

e_l = voltage corresponding to the air gap flux (behind leakage reactance)

e_t = terminal voltage of machine

e_d = voltage corresponding to direct-axis field magnetomotive force

i = armature current

$k = 1 + \frac{\text{iron magnetomotive force}}{\text{air gap magnetomotive force}}$

r_a = machine armature resistance

Z_e = external impedance

r_e = external resistance

x_e = external reactance

x_{eq} = equivalent reactance

x_l = leakage reactance

x_d = direct-axis synchronous reactance

θ = power-factor angle of machine (angle between e_t and i)

ϕ = angle between e_l and i

$\delta_{mn} = \delta_m - \delta_n$ = electrical angular displacement between m and n

Appendix A

Calculation of Steady-State Stability Limits for Synchronous Load

The following presents a method by which the synchronizing power coefficients $dP/d\delta$, and therefore the stability, may be determined when a synchronous machine is connected to a point of maintained voltage e_e through an impedance Z_e. This point of maintained voltage may correspond to a voltage representing that of a large system or to the voltage back of the equivalent reactance of a synchronous motor load, the impedance Z_e being the total impedance from the generator terminals to the point of maintained voltage.

In the following derivation it is assumed that the reader is familiar with the material in reference 5. The derivation given in this appendix is based on the same assumptions as that given in reference 5, and as far as possible the same nomenclature has been used.

Figure 15 is a steady-state vector diagram of a cylindrical-rotor synchronous generator with saturation and figure 16 is the corresponding equivalent circuit. From this equivalent circuit the following relations may be written for point l in a manner similar to that used in reference 7.

$$P_{ld} = -\frac{e_d e_l}{x_d - x_l}\sin\delta_{dl} \tag{5}$$

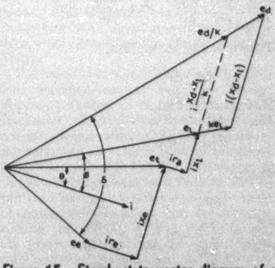

Figure 15. Steady-state vector diagram of a cylindrical-rotor synchronous generator with saturation

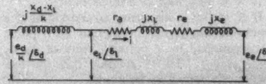

Figure 16. Equivalent circuit representing cylindrical-rotor synchronous generator with saturation

$$Q_{ld} = -\frac{ke_l^2}{x_d - x_l} + \frac{e_d e_l}{x_d - x_l}\cos\delta_{dl} \qquad (6)$$

$$P_{le} = \frac{e_l^2 R}{Z^2} + \frac{e_l e_e}{Z}\sin(\delta_{le} - \alpha) \qquad (7)$$

$$Q_{le} = -\frac{e_l^2 X}{Z^2} + \frac{e_l e_e}{Z}\cos(\delta_{le} - \alpha) \qquad (8)$$

Differentiating these four equations, the following relations are obtained.

$$\frac{dP_{ld}}{de_l} = -\frac{e_d}{x_d - x_l}\sin\delta_{dl} - \frac{e_d e_l}{x_d - x_l}\cos\delta_{dl}\frac{d\delta_{dl}}{de_l} \qquad (9)$$

$$\frac{dQ_{ld}}{de_l} = -\frac{2ke_l}{x_d - x_l} - \frac{e_l^2}{x_d - x_l}\frac{\partial k}{\partial e_l} + \frac{e_d}{x_d - x_l}\cos\delta_{dl} - \frac{e_d e_l}{x_d - x_l}\sin\delta_{dl}\frac{d\delta_{dl}}{de_l} \qquad (10)$$

$$\frac{dP_{le}}{de_l} = \frac{2e_l R}{Z^2} + \frac{e_e}{Z}\sin(\delta_{le} - \alpha) + \frac{e_l e_e}{Z}\cos(\delta_{le} - \alpha)\frac{d\delta_{le}}{de_l} \qquad (11)$$

$$\frac{dQ_{le}}{de_l} = -\frac{2e_l X}{Z^2} + \frac{e_e}{Z}\cos(\delta_{le} - \alpha) - \frac{e_l e_e}{Z}\sin(\delta_{le} - \alpha)\frac{d\delta_{le}}{de_l} \qquad (12)$$

Also

$$\frac{dP_{ld}}{de_l} = -\frac{dP_{le}}{de_l} \qquad (13)$$

$$\frac{dQ_{ld}}{de_l} = -\frac{dQ_{le}}{de_l} \qquad (14)$$

Equations 9 to 14 may be solved simultaneously to determine

$$\frac{dP_{le}}{de_l}, \frac{dQ_{le}}{de_l}, \frac{d\delta_{dl}}{de_l}, \text{ and } \frac{d\delta_{le}}{de_l}.$$

Then it can be shown that,

$$\frac{dP_{le}}{d\delta_{de}} = \frac{\dfrac{dP_{le}}{d\delta_{dl}}\dfrac{dP_{le}}{d\delta_{le}}}{\dfrac{dP_{le}}{d\delta_{dl}} + \dfrac{dP_{le}}{d\delta_{le}}} \qquad (15)$$

where,

$$\frac{dP_{le}}{d\delta_{dl}} = \frac{\dfrac{dP_{le}}{de_l}}{\dfrac{d\delta_{dl}}{de_l}} \text{ and } \frac{dP_{le}}{d\delta_{le}} = \frac{\dfrac{dP_{le}}{de_l}}{\dfrac{d\delta_{le}}{de_l}}$$

Equation 11 gives the synchronizing power coefficient corresponding to a point back of leakage reactance of the machine. If $R = 0$, $dP_{le}/d\delta_{de} > 0$ for stability.

If $R \neq 0$ it is necessary to determine the synchronizing power coefficient at a point corresponding to a voltage back of Z_e or $(-dP_{el}/d\delta_{le})$. To determine this the following relations may be used.

$$-P_{el} = P_{le} - i^2 R \qquad (16)$$

$$i^2 e_l^2 = P_{le}^2 + Q_{le}^2 \qquad (17)$$

Differentiating 16 and 17 and solving simultaneously

$$-\frac{dP_{el}}{d\delta_{le}} = \left(1 - \frac{2RP_{le}}{e_l^2}\right)\frac{dP_{le}}{d\delta_{le}} - \frac{2RQ_{le}}{e_l^2}\frac{dQ_{le}}{d\delta_{le}} + \frac{2i^2 R}{e_l}\frac{de_l}{d\delta_{le}} \qquad (18)$$

Once equations 9 to 14 have been solved simultaneously all of the quantities required by equation 18 to determine the synchronizing power coefficient at point e are available. For the system to be stable

$$-\frac{dP_{el}}{d\delta_{le}} > 0 \qquad (19)$$

Appendix B

Calculation of Steady-State Stability Limits for Induction Motor Load

The following is a method by which the stability of an induction motor load may be determined including the saturation effects in the generator. Equations 13a and 14a of reference 7 give dP_a/dE_a and dQ_a/dE_a in terms of the external system impedance and the constants of the induction motor load.

Since

$$\frac{dQ_a}{dE_a} = \frac{dQ_{ld}}{de_l}$$

dQ_a/dE_a can be substituted in equation 10 of appendix A for dQ_{ld}/de_l and then this equation solved with equation 9 to obtain the expression

$$\frac{dP_{ld}}{de_l} = -\cot\delta_{dl}\left\{\frac{e_d}{x_d - x_l}\frac{1}{\cos\delta_{dl}} - \frac{dQ_a}{dE_a} - \frac{2ke_l}{x_d - x_l} - \frac{e_l^2}{x_d - x_l}\frac{\partial k}{\partial e_l}\right\} \qquad (20)$$

If

$$+\frac{dP_{ld}}{de_l} > +\frac{dP_a}{dE_a} \text{ the system is stable.}$$

Appendix C

Method for Determining Equivalent Reactance From Test

The following describes a method of testing for the equivalent reactance as given by

$$x_{eq} = x_l + \frac{x_d - x_l}{k\sqrt{1 + \dfrac{a}{b}}} \qquad (21)$$

based on a test zero power factor volt-ampere characteristic taken with constant field current corresponding to that necessary for normal rated load.

In a manner as described by J. W. Butler[8] a value of equivalent reactance,

$$x_{eq} = \frac{c}{d} = x_l + \frac{x_d - x_l}{k\left(1 + \dfrac{a}{b}\right)} \qquad (22)$$

corresponding to normal full-load flux conditions can be obtained by taking the slope of the volt-ampere characteristic at point P. (Figure 17.)

Next a straight line tangent to this volt-ampere characteristic at zero armature voltage is drawn. This line corresponds to the unsaturated volt-ampere characteristic.

Use can be made of the following relations, for the unsaturated characteristic,

$$e_1 = e_d - i_1 x_d \qquad (23)$$

for the saturated characteristic,

$$e_2 = \frac{e_d}{k} - \left(x_l + \frac{x_d - x_l}{k}\right)i_2 \qquad (24)$$

Subtracting these two voltages for the condition when $i_1 = i_2 = i$.

$$e_1 - e_2 = \Delta e = \frac{(k-1)}{k}[e_d - (x_d - x_l)i] \qquad (25)$$

From (25)

$$k = \frac{[e_d - (x_d - x_l)i]}{[e_d - (x_d - x_l)i - \Delta e]} = \frac{e_l}{e_l - \Delta e} \qquad (26)$$

where

$$e_l = e_d - (x_d - x_l)i$$

Substituting equation 26 in 22

$$\left(1 + \frac{a}{b}\right) = \frac{(x_d - x_l)(e_l - \Delta e)}{\left(\dfrac{c}{d} - x_l\right)e_l} \qquad (27)$$

Substituting (26) and (27) in (21) and simplifying,

$$x_{eq} = x_l + \sqrt{(x_d - x_l)\left(\frac{c}{d} - x_l\right)\left(1 - \frac{\Delta e}{e_l}\right)} \qquad (28)$$

Equation 28 is the desired expression for

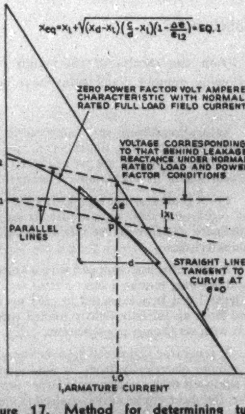

Figure 17. Method for determining turbine-generator average equivalent reactance corresponding to normal rated load and power factor conditions from zero-power-factor volt-ampere characteristic

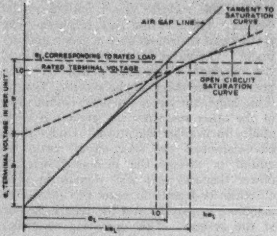

Figure 18. Graphical method for determining average equivalent reactance corresponding to normal rated load and power factor conditions from open-circuit saturation curve

$$x_{eq} = x + \frac{x_d - x_l}{k\sqrt{1+a/b}}$$

the equivalent reactance corresponding to equation 21, with c/d and Δe being obtained from the results (figure 17) of tests which can in many cases be made in the factory. The reactance is determined under the condition of the same fundamental air gap flux and field current as the machine has at normal rated full load.

Appendix D

Approximate Method for Determining Equivalent Reactance

The saturation factors k and a/b entering into the determination of the equivalent reactance defined by equation 1 can be determined approximately from the open circuit saturation curve as shown in figure 18. With these factors determined and knowing x_d and x_l, all the necessary quantities for substitution in equation 1 are available. e_l, the point on the saturation curve at which k and a/b are determined, corresponds to the voltage back of stator leakage reactance at rated load. If the leakage reactance x_l is not available Potier reactance x_p may be used. This will result in somewhat larger saturation factors and therefore a lower value of x_{eq}.

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Discussion

Sterling Beckwith (Allis-Chalmers Manufacturing Company, Milwaukee, Wis.): There has been need for a paper of just this type to present the designer's point of view to the operating engineers. To draw an analogue, the process of selecting a machine on the basis of short-circuit ratio only is like selecting a cooler on the basis of square feet of surface only without giving consideration to the effectiveness of the surface as determined by the size and shape of the fins, the air velocity over the surface, etc.

One distinct disadvantage of high-short-circuit-ratio machines which is not sufficiently emphasized by the paper is that they have inherently lower transient and subtransient reactances, thereby requiring the use of larger breakers and bus structures. On two-pole machines, especially, this is a disadvantage as normal values for these two reactances are of the order of 12 per cent and 7 per cent, respectively.

In appendix *D* the statement is made that use of X_p instead of X_l will result in somewhat larger saturation factors and therefore a lower value of X_{eq}. This is not necessarily true at higher values of saturation or for slow speed machines, as may be seen by considering k and a/b to be very large, or by considering X_p to be a large fraction of X_d.

S. H. Mortensen (Allis-Chalmers Manufacturing Company, Milwaukee, Wis.): The paper by Concordia, Crary, and Lyons is timely in its discussion of the generator short-circuit ratio and its relation to the operating stability.

In general, a generator can be designed for any required short-circuit ratio by expenditure of materials, but where large, 3,600-rpm machines are involved, short-circuit ratio may become the factor that determines the capacity for which this type of machine can safely be built.

The authors bring out the dependence of stability upon generator saturation, load, line, and exciter characteristics, and also show how the effect of automatic voltage regulators renders the use of lower-short-circuit-ratio machines practicable.

In this connection it may be mentioned that at present the short-circuit ratio called for in a majority of 3,600-rpm generator specifications is unity or better. It is not the writer's intention to imply that high short-circuit ratio may not, under circumstances, be justifiable, but as designers and operators alike are interested in producing efficient economical machines, it may not be out of order to call attention to the fact that quite a number of large, important machines with short-circuit ratios much below unity, have been in operation for years, under widely varying conditions, without introducing stability problems. As an example of this kind, may serve two large two-pole generators with short-circuit ratios of less than 0.6, which, for years, took the load swings incidental to parallel operation with gas-engine-driven generators on fluctuating steel-mill loads. Mention may also be made of a four-pole 121,000-kva 60-cycle turbine generator which is giving excellent service on a large interconnected system, yet its short-circuit ratio is less than 0.65.

Numerous other examples—both in the United States and abroad, of similar nature, could be quoted.

The main factors determining generator short-circuit ratio are saturation, reactances, and air-gap length. Inherently high short-circuit ratios require low reactances and long air gaps, which result in expensive machines and switching equipment. The most economical machines, switches, and bus structures are obtained with designs where reactances are limited by generator heating, and the air-gap length by rotor stray loss. As the latter is a function of the peripheral rotor velocity and stator slot width, large 3,600-rpm high-voltage machines with open stator slots prevent the use of short air gaps and limit the extent to which the designer can take advantage of low short-circuit ratios. This limitation applies less to machines built with closed or semiclosed stator slots which, consequently, can be of lighter design.

The maximum capacities for which 3,600-rpm machines can be built are limited through a number of factors, such as characteristics of available rotor materials, critical speeds, vibrations, high values of short-circuit ratios, etc. Most of the above factors are inherent but the elimination of the short-circuit ratio as a limiting factor, will tend to render available the full benefit of modern developments, such as hydrogen-cooling, and further extend the limits for the maximum capacities for which 3,600-rpm turbo-generators can be built with reasonable mechanical factors of safety.

In conclusion, the writer fully indorses the author's suggestion that the short-circuit ratio of 3,600-rpm machines should be based upon careful consideration of all operating requirements.

G. C. Crossman (Consolidated Edison Company of New York, Inc., New York, N. Y.): The authors have shown that short-circuit ratio for generators not equipped with voltage regulators is an approximate measure of the relative steady-state stability characteristics when supplying a synchronous motor load, and that with this type of load high short-circuit ratios are desirable for stability. However, they do not discuss this ratio as a measure of stability for generators not equipped with voltage regulators when supplying induction motor load. In large metropolitan systems, such as that of the Consolidated Edison Company, the generators are not equipped with voltage regulators and the motor load consists chiefly of induction motors with very little synchronous motor load. Since this induction motor load may be as high as 70 per cent of the total system load during the day, the inherent generator steady-state stability when carrying this load becomes of prime importance.

When troubles on such a system involve the loss or field or tripping of a loaded generator, the resulting voltage disturbance at the station in difficulty is very liable to be severe, particularly if this station separates from the rest of the system. Increase in load on the remaining generators will cause a considerable drop in bus voltage. An unexcited generator connected to the bus would depress the bus voltage very greatly. If the bus voltage drops initially to 75–80 per cent of normal, "voltage instability" is very likely to occur. This results from stalling of induction motors with a consequent increase in current which further reduces the bus voltage. Under these conditions, the bus voltage may reach a minimum of 30–40 per cent of normal before the stalled motors are disconnected from the system. At this low voltage the maintenance of synchronism between generators in the station becomes doubtful and if they pull out of step with each other, it may be necessary to shut down the station.

These conditions may be avoided by the use of generators with high inherent stability limits, a large spinning reserve of generator capacity, or generator voltage regulators. Since a large reserve is uneconomical and voltage regulators often undesirable, many specifications for new generators have called for relatively high values of short-circuit ratio as a measure of high inherent generator stability. Studies of an actual case of "voltage instability" on the Brooklyn system several years ago indicated that unity-short-circuit-ratio machines would have materially improved the situation and probably prevented the voltage collapse. Since short-circuit ratios up to unity could be obtained in new machines without involving undesirable performance characteristics or excessive cost, approximately unity short-circuit ratio has been specified for many of the Consolidated Edison system generators purchased subsequently.

Although the authors did not discuss this type of generator stability, they did include in their paper a formula, number 4, for equivalent generator reactance with induction motor load. The question then arises as to whether this reactance constitutes a better index of generator stability than the accepted short-circuit ratio. It would be interesting to know what conclusions the authors have reached on this problem.

C. F. Wagner (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The question of the proper short-circuit ratio to specify for proposed machines serving metropolitan districts is receiving more and more attention. But while the problem is important it is also difficult of exact solution. The principal difficulty lies in setting up the proper criterion to determine whether the system is stable or unstable. The mere citation of the satisfactory operation of a machine which has, say, a short-circuit ratio of 0.6 in this city or that city, is not proof that machines of this character are satisfactory for general application in metropolitan districts. System layout, external reactance, excitation systems, and the number of machines in parallel should all be considered in this problem. In the final analysis, successful system operation is the true criterion. During the past we have accumulated considerable data in the operation of systems upon which are installed machines of short-circuit ratio equal to 1.0. Until similar data are accumulated for machines with smaller short-circuit ratios, we should be cautious in the general adoption of such machines.

Messrs. Concordia, Crary, and Lyons have considered only one aspect of the problem, namely, a particular type of steady-state stability. This is a valuable contribution and represents a large amount of labor in calculating the results by the methods which they have used. The curves of figures 9 and 10 show the stability limits for a machine equipped with a voltage regulator connected through an external reactance to an infinite bus. For values of external reactance in excess of 0.5 or 0.6, the transient stability limits are usually of greater importance than steady-state stability limits. Therefore, the portion of the curves beyond these values of external reactance is of minor importance. For smaller values of external reactance the power limits are in excess of the ability of the steam turbines to deliver the power. The general conclusions from these considerations would indicate that this particular type of steady-state stability limit is not important in the choice of short-circuit ratio. While this might appear to be a negative result, still it is important to know. Aside from the conclusions to be drawn from the curves, the representation of the system by means of a generator, an external reactance, and an infinite bus is too crude an approximation for analytical purposes. This is particularly true when operating at leading power factor, the maximum power conditions for which results in very large values of voltage at the infinite bus.

From the cases of so-called instability in metropolitan systems, which have come to my attention, almost invariably the cause has been associated in some manner with the excitation system. The excitation of one machine in some cases, either intentionally or unintentionally, had been decreased continuously to the point at which the machine was no longer able to deliver the power at the reduced excitation. Of course, a high short-circuit ratio would not in such case prevent pull-out but would only reduce the value to which the excitation would have to decrease before pull-out occurred. For other cases, the excitation of a particular machine failed completely. The generator, for which the excitation failed, pulled out of synchronism with the remainder of the system and thus threw a considerable reactive power burden on the system. The question then is whether the system can withstand such a shock and demand. The larger the short-circuit ratio of the machine which has pulled out, the greater is the shock to the system. On the other hand, the larger the short-circuit ratio of the other machines the greater is their ability to weather the storm. Under these circumstances, the short-circuit ratio required to satisfy the increased reactive power demand and still remain in synchronism is dependent upon the number of machines in parallel and the general layout of the system. This operating condition rather than the simplified steady-state stability condition would appear to constitute the more important criterion for the choice of short-circuit ratio.

J. F. Calvert (Northwestern University, Evanston, Ill.): It is my understanding that the purpose of this paper is to further the study of lower-short-circuit-ratio generators for steam plants in this country. In addition to the system stability problems, discussed in this paper, there are certain design features which must be given due consideration. There are quite a number of machines in this country built to operate at a short-circuit ratio of around 0.8. It is probable that in many designs this can be reduced to 0.65 or 0.7 without encountering serious problems. However, if the short-circuit ratio is reduced appreciably below the latter values for steam turbine generators, I would anticipate considerable difficulty with pole-face losses on no load as pointed out by one of the other discussers. This assumes that the open-type armature slot and form-wound coils, which are practically universally used in this country, are to be continued.

In addition to the pole-face loss on no load, I hope the authors will discuss the matter of load losses produced in the rotor surface due to currents flowing in the armature conductors for turbine generators of low short-circuit ratio. I would like to know what limitations they feel would be encountered here, and if any methods have been devised recently for reducing this difficulty. It seems to me that this matter of pole-face losses under load conditions would be a particularly serious one in low-short-circuit-ratio generators where the machine is to carry a double-winding armature and load unbalance between the two windings is to be permitted. A discussion of this point would also be appreciated. The matter of ventilation through the air gap may also become something of a problem.

I am of the opinion that this study of lower-short-circuit-ratio generators is a particularly timely one and hope that the paper will bring out further discussion, not only from the point of view of the stability characteristics, but also from that of the design problems to be encountered in connection with such machines.

L. A. Kilgore (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): The authors of this paper have shown that if one trusts the voltage regulator "to make changes in the excitation so as to maintain terminal or system voltage following load changes," then generators with much lower than unity short-circuit ratio would seem to be adequate. This fact has been appreciated for a long time, but has not been made use of by operators because of their reluctance to rely on the regulator and excitation system. The authors have said very little about the regulator or the excitation system requirements.

If in the future operators are to "shift part of the responsibility for maintaining synchronism onto the excitation system," then they must make sure that their excitation system has the following characteristics:

1. The regulator must never stick or fail to function.

2. The regulator and exciter system must be sufficiently fast to follow within a few tenths of a second the changes in excitation required, otherwise one must calculate the transient effects introduced by the rate of response of the exciter and the lag of the voltage regulator.

3. All of the regulators on the system must be coordinated so that they divide any increased reactive kilovolt-amperes properly, and increase their excitation suitably with increased kilowatt load. This is usually accomplished by giving the regulators a drooping characteristic. However, this gets away from the ideal assumptions on which the analysis in the paper was based. In a few cases, devices have been used which tend to increase the voltage with increased kilowatt load.

4. The ceiling voltage of the exciter must be sufficient to take care of any demands, otherwise the machine cannot maintain its voltage under abnormal conditions.

The authors' analysis is based on pull-out occurring at rated load and power factor, and the approximate equivalent reactance proposed is also based on this assumption. Actually, pull-out does not occur under these conditions even on a system equipped with voltage regulators. In most cases, loss of stability occurs under abnormal conditions, such as loss of field on one machine, loss of a tie line or other switching operation, sudden increase in kilowatt load on a machine whose regulator has been adjusted to give low kilovolt-amperes at light load. Instability may also occur due to improper operation of the regulator. It may be argued that these are transient conditions or that the regulator should take care of them, but they may persist too long to be considered on a transient basis, and the regulator and excitation system may be inadequate or out of adjustment. It would seem that a detailed study of these cases should be made before applying machines of low short-circuit ratio.

The authors propose to use an equivalent reactance in calculating the stability, and give an approximate value to be used as a criterion of the machine's stability which is intended to evaluate properly the effect of saturation. Equivalent reactance would appear to be a very useful tool; however, it may be very misleading if improperly used. The equivalent reactance is intended to apply only to very small changes away from a given operating condition. It cannot be used in any simple manner to calculate the power margin with fixed excitation, since even if the power margin is only five per cent, the voltage will change considerably between the operating condition and the pull-out point and the equivalent reactance may increase as much as 50 per cent.

The equivalent reactance is intended to give the same change in terminal conditions with fixed excitation as the actual machine. However, one must realize that there will not be one equivalent reactance for each operating condition, but a different one for each direction of change away from this operating point. For example, the equivalent reactance for changing load at constant voltage will show very little effect of saturation and may be 50 per cent greater than the value applying to the change at constant load.

In the discussion of Mr. Beckwith's paper (reference 12 in this paper), I suggested that the equivalent reactance be limited to apply to changes at constant power only, since at pull-out the changes are at nearly constant power. This would simplify the concept and make it possible, at least theoretically, to accurately determine the equivalent reactance at any given operating condition by the method given in Mr. Beckwith's paper (reference 2).

A more practical approximate method based on the slope of the no-load saturation curve is given in the discussions by Mr. Beckwith and the writer on the first paper on equivalent reactance (reference 5). A slight modification of this method to determine the reactance applying to changes at constant power for turbine generators may be written:

$$X_{eq} = X_p + \frac{S(X_d - X_p)}{\cos \phi + (1 - \cos \phi)S}$$

where X_p is the Potier's reactance, S is the slope of the saturation curve on a per unit basis, and ϕ is the angle between the internal voltage and the voltage back of Potier's reactance.

It may be argued that Potier's reactance is somewhat empirical, but we have recently completed tests on turbine generators measuring the internal angle as well as terminal quantities over a wide range of loads and power factors, which show this empirical method to be quite accurate. In fact, it would seem to be fully as accurate as the more complicated method proposed by the authors. These tests showed that the quadrature-axis reactance saturated much more than the direct-axis reactance, which explains why using the voltage back of Potier's reactance rather than the direct-axis projection as a measure of saturation is a better approximation.

C. Concordia, S. B. Crary, and J. M. Lyons: In treating a subject of this nature, which must necessarily depend upon experience as well as theory, the point of view of designers and operators is most valuable. All discussers, we believe, have indicated the importance of a careful review of the problem of short-circuit ratio in the light of present-day knowledge of system performance and generator design.

Messrs. Mortensen and Beckwith point out that machines of low short-circuit ratio will develop less short-circuit current and consequently result in less bus and breaker short-circuit stresses than high-short-circuit-ratio machines. This point was not sufficiently emphasized in the paper and may be quite an important factor, particularly when considering 3,600-rpm units of large capacity, as their transient and subtransient reactances are inherently quite low. Since with the use of a voltage regulator a higher reactance machine may be justified from a stability standpoint, a reduction in maximum short-circuit current is also obtained as an additional benefit. This will reduce the bus and breaker stresses and possibly allow for a certain amount of simplification in circuit arrangement to the extent of eliminating or reducing the amount of circuit reactors in the station bus. When the additional benefit of a reduction in maximum short-circuit current is considered along with improved stability and generator characteristics, it becomes increasingly difficult to justify the exclusion of a voltage regulator, particularly for a new or proposed unit. At least, a rational approach to the problem will undoubtedly be well worth while rather than arbitrarily specifying a short-circuit ratio of unity or more.

We agree with Mr. Beckwith that the statement in appendix *D* is incorrect when applied for the high values of k and a/b; that is, the use of X_p instead of X_l does not necessarily result in lower values of X_{eq}. The statement in the appendix should be modified accordingly.

Mr. Calvert raises the question as to the feasibility or desirability of reducing the short-circuit ratio below 0.65 or 0.7, because of design limitations. As Mr. Calvert points out, there exist design features which make it impractical to reduce the short-circuit ratio below a certain amount. These factors, of course, should be considered when designing a generator. However, it is not expected that short-circuit ratios of such low values as to result in high rotor losses will be used. Rather, a machine will be built that will give the best all round performance, determined by a rational study of both generator design and system performance instead of an arbitrary specification. We believe Mr. Mortensen has summed up very well the point of view of the designer that the important thing to be accomplished at this time is "the elimination of the short-circuit ratio as a limiting factor." This will allow a more careful study and evaluation of the design factors which Mr. Calvert and Mr. Mortensen have pointed out as bearing on the problem.

Mr. Crossman has pointed out that on their system they are concerned primarily with a metropolitan type of load consisting largely of induction motors connected to generators which are under manual control of excitation. Although this case was not considered specifically in the paper, it is believed the results presented and the conclusions given apply fairly well to it also. Figures 4 and 5, which give the power limits for machines with fixed excitation connected directly to an infinite bus, give the upper limit of power which can be delivered under any condition of load. That is, under the conditions met in practice the terminal voltage will tend to drop, resulting in lower limits for the machine than those represented by figures 4 and 5. Although the load for Mr. Crossman's system is essentially induction motors, the problem, we believe, can be evaluated very well in terms of figures 4, 5, 6, and 7, given in the paper, for the first machine to pull out of synchronism with respect to the other machines of the system. When a disturbance takes place requiring additional load to be thrown on the remaining generators, the problem with which the operators are chiefly concerned is the ability of the remaining generators to stay in synchronism with each other. If one of these units is operated initially at light load with low excitation, it will tend to take a larger percentage of the load increment and, because of its low initial excitation, will be the most likely to pull out of step. This first machine which tends to lose synchronism when operating in parallel with a large number of other machines is influenced more by the characteristics of the other generators than by the characteristics of the load. We believe, therefore, that an indication of its ability to pick up load can be represented in a simple manner by operating into a certain amount of external reactance. As Mr. Wagner points out, it is more accurate to consider the actual system and individual machines operating in parallel. If this is done, a more accurate answer will, of course, be obtained for the individual case. However, we believe that the general results will not differ from those presented in the paper and that the same general conclusions would be obtained if such an analysis is made.

It is not surprising that Mr. Crossman and his associates found that machines of high short-circuit ratio would be valuable if no voltage regulators were used. This conclusion corresponds with the results of our study. Equation 4, which Mr. Crossman asked about, does not apply in general for the case when no voltage regulators are used, and is meant to apply only for small changes around an operating point. This corresponds to the condition when a voltage regulator is used or when field changes are made manually to follow load changes closely.

Mr. Kilgore points out the fact that if a lower-short-circuit-ratio machine is used a reliable excitation system is required. This is in accordance with our conclusions and, we believe, operators can justifiably place reliance on modern regulator and excitation systems. The problem is not nearly as serious as Mr. Kilgore indicates, as stability of a system is influenced by all of the machines on the system and, therefore, if one regulator or excitation system did not operate quite properly it is questionable whether the system performance would be seriously impaired. In regard to the use of equivalent reactance, it is intended that this be used only for determining the stability limits for machines equipped with voltage regulators, and is intended to apply only to small changes around an operating point. Mr. Kilgore indicates large variations in the value of equivalent reactance based upon the terminal conditions. Variations do exist and are dependent upon the connected system. However, these can be evaluated for any particular case and, as our studies have indicated, it is possible to use a value which is conservative but still evaluates the effect of saturation to a greater extent than short circuit ratio. It is hoped that Mr. Kilgore will present his ideas and results of the tests which he has mentioned more completely at some future time.

Mr. Wagner questions the use of an external reactance to represent a system. We believe that external reactance is probably one of the simplest, most practical methods of obtaining a comparison in generator characteristics. If a fault occurs on a system the generator or generators are, in effect, connected to the other sources of synchronous generation through an effectively higher reactance tie. Also, if the machine is operating with leading power factor, another case which Mr. Wagner questions, the capacitive reactance representing the line charging capacity of the system can be paralleled with the actual system line and machine reactance to obtain an equivalent higher reactance which represents the receiving system. These methods have been known and used for many years by engineers calculating stability characteristics. The voltage back of this effective or receiving system reactance is entirely a fictitious one and is used only in order to determine its effect on the generator under consideration. Mr. Wagner has questioned specifically the idealization which we have used when applied to metropolitan systems and concludes that a more serious consideration regarding the relative short-circuit ratio is the loss or reduction of excitation on a particular unit. This, of course, corresponds to our conclusion 3, that a generator not equipped with a voltage regulator may require a high short-circuit ratio. We agree that it is advisable in any particular case to make a study of the system in order to determine the stability requirements. It was not intended that this paper be used directly for specifying the short-circuit ratio of machines, but rather that it point the way in which improvements in system performance and machine characteristics, as well as reduction in costs, can be realized.

Mr. Wagner believes that lower-short-circuit-ratio machines should not be adopted until experience on actual systems is accumulated with this type of machine. We believe this is highly desirable, but also believe that certain system conditions and the use of voltage regulators can justify a reduction in short-circuit ratio if this is desirable from a generator design and system standpoint. We have attempted to show the possibility of improving system performance and inherent generator characteristics, for the same or less expenditure, when a rational approach to the problem is made. We are not by any means advocating a general adoption of low-short-circuit-ratio machines, but believe that there do exist conditions under which the use of such machines would be very well worth while.

Transmission Theory of Spherical Waves

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Synopsis: This paper formulates the essential properties of spherical waves in terms of familiar engineering concepts. The physical picture resulting from this point of view facilitates the solution of certain physical problems, notably those having to do with reflection, refraction, shielding and power absorption.

THE AIM of this paper is to formulate the essential transmission properties of spherical waves in terms of familiar engineering concepts. The chief advantages of this formulation are: (1) the subject of spherical waves is brought under the general heading of transmission-line theory and the treatment of two apparently distinct subjects is co-ordinated, (2) the solution of certain shielding problems is greatly facilitated and the physical aspects of shielding are stressed. In the latter respect this paper is complementary to a previously published[1] paper containing general formulas for shielding in terms of impedances, which, for spherical waves, are derived in this paper.

On the mathematical side this paper is brief and limited largely to final results for two reasons. Firstly, the formal mathematical treatment of the wave equation in spherical co-ordinates may be found in almost any book dealing with partial differential equations of mathematical physics; besides, Bessel functions, Legendre functions, and associated Legendre functions are incidental to our main purpose and, in practice, it is sufficient to know that these functions are tabulated. Secondly, the mathematical procedure for reducing Maxwell's equations to a particular form which is needed for our purposes is analogous to that employed for a similar purpose in connection with plane waves.[2]

Spherical Waves

A *spherical* wave is defined as a wave whose equiphase surfaces form a family of concentric spheres. Their common center is the *center of the wave*. The rays are the radii emanating from this center.

A wave is *transverse magnetic* if the magnetic intensity H is perpendicular to the ray and *transverse electric* if the electric intensity E is so disposed. If both vectors, E and H, are perpendicular to the ray, then the wave is *transverse electromagnetic*.

While we are concerned primarily with

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1. For all numbered references, see list at end of paper.

spherical waves most mathematical equations contained in this paper are equally applicable to *quasi-spherical* waves in which the phase-amplitude pattern, that is, the relative phase-amplitude distribution is independent of the distance from their center. In accordance with this definition any wave function V describing a quasi-spherical wave is of the following form

$$V(r, \theta, \phi) = T(\theta, \phi)\, R(r) \tag{A}$$

where r, θ, and ϕ are the spherical coordinates (figure 1). The function $T(\theta, \phi)$ represents the phase-amplitude pattern and $R(r)$ describes the variation of the amplitude of this pattern with the distance from the center of the wave. For spherical waves, the phases at various points (θ, ϕ) situated on the same sphere $r = a$ must be the same and $T(\theta, \phi)$ represents the relative amplitude distribution over the equiphase surfaces; hence for spherical waves $T(\theta, \phi)$ must be real. (Strictly speaking, T may differ from a real function by a constant complex factor which, however, may always be included in $R(r)$.) Spherical waves of this type appear to be more special than those defined in the first paragraph since it is conceivable that the amplitude patterns over the equiphase spheres could vary with the distance from the center. It is possible to show, however, that such more general types of spherical waves do not exist and that consequently all possible spherical waves are included in the form (A).

The amplitude pattern $T(\theta, \phi)$ may be different for different field components. For example, the radial field of the wave emitted by an electric current element varies as $\cos \theta$, where θ is the angle made by a typical radius with the axis of the element, while the transverse field varies as $\sin \theta$. The pattern associated with the radial component and with the potential function may be regarded as basic inasmuch as the remaining patterns are obtained from it by simple differentiation. Thus the transverse field is proportional to the gradient of the radial field. On this account, $T(\theta, \phi)$ will henceforward express the relative distribution of the radial current (electric current in transverse magnetic waves and magnetic current in transverse electric waves). (The *magnetic current* is defined as the time rate of change of the magnetic flux.) On a typical equiphase surface the lines

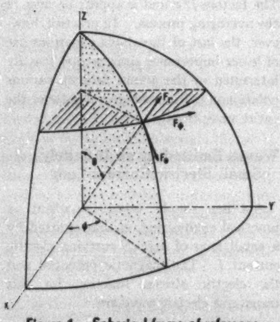


Figure 1. Spherical frame of reference

The positive directions of r-, θ-, and ϕ-components of a vector are the directions of increasing r, θ, and ϕ. OZ is the polar axis of the frame and XOY plane is the equatorial plane

$$T(\theta, \phi) = \text{constant} \tag{B}$$

represent, therefore, the lines connecting points of equal radial current density. These lines are also equipotential lines (electric equipotential lines in transverse magnetic waves and magnetic equipotential lines in transverse electric waves). Furthermore, lines (B) are magnetic lines of force in transverse magnetic waves and electric lines of force in transverse electric waves.

The amplitude distribution function T is not arbitrary but must satisfy equation 3 and certain boundary and continuity conditions. It is perfectly conceivable, of course, that with a proper distribution of radiating elements over some sphere $r = a$ we could enforce any desired distribution of radial current; but this relative distribution of current will not remain the same as we move away from the sphere $r = a$. On the other hand, if we were to enforce a distribution satisfying equation 3, we should find the same distribution at any distance from the sphere. It is only these waves that obey simple transmission laws; all other waves may be regarded as due to superposition of such simple waves.

By definition, in a transverse magnetic wave there is no radial magnetic current and, therefore, the electromotive force around any closed circuit situated completely in one equiphase sphere is zero. This means that the electromotive force from one point of the equiphase surface to another is the same irrespective of the path joining them so long as this path never leaves the equiphase surface. Thus to each point we can ascribe an *electric potential* V, representing the electromotive force from the point in question to a reference point of "zero potential," this force being taken along any path contained in the particular equiphase surface. The transverse electric field is, therefore, the negative of the gradient of this potential

$$E_t = -\text{grad } V \tag{1}$$

If du and dv are two elementary lengths in an orthogonal co-ordinate system drawn on the particular equiphase sphere, than (1) takes the form

$$E_u = -\frac{\partial V}{\partial u}, \qquad E_v = -\frac{\partial V}{\partial v}$$

Again, in a transverse magnetic wave, magnetic lines are situated on equiphase spheres and since these lines are closed, we must have a definite number of them crossing any path joining two fixed points, so long as this path is in the equiphase surface. It is possible, therefore, to define a *magnetic stream function* A as the number of magnetic lines crossing a path from the point in question to some fixed reference point. If these crossings are regarded as positive when they appear to be from right to left to an observer standing on the equiphase sphere at the point in question and looking toward the center, then the components of the magnetic field are

$$H_u = \frac{\partial A}{\partial v}, \qquad H_v = -\frac{\partial A}{\partial u}$$

If these equations are substituted in Maxwell's equations, it turns out that the total (the conduction and the displacement) radial electric current per unit solid angle is equal to the stream function A, except for a numerical factor k^2 depending upon the particular amplitude distribution function $T(\theta, \phi)$. Thus the entire field may be described by two functions A and V, the first being proportional to the radial electric current and the second being equal to the transverse electromotive force. The analogy with an ordinary transmission line composed of a pair of parallel wires, in which the waves are also described in terms of *longitudinal* current and *transverse* electromotive force, is clear. In free space spherical waves, however, the longitudinal current is a continuously distributed displacement current (in parallel with a conduction current if the medium is dissipative) while in plane waves guided by parallel wires it is the conduction current concentrated in the wires. Thus the transmission line representing a transverse magnetic spherical wave must have a distributed series capacity associated with the longitudinal electric field as well as the series inductance associated with the transverse magnetic field.

Transverse electric waves are, in a sense, reciprocal to transverse magnetic waves: the roles played by the electric and the magnetic field are interchanged. Since there is no radial electric current, the magnetomotive force around a closed circuit lying completely in an equiphase sphere is zero and it is possible, therefore, to define a *magnetic potential* U at any point as the magnetomotive force from this point to a reference point of zero potential. Similarly there exists an *electric stream function* F representing the number of electric lines crossing a path connecting a fixed reference point to a point in question. This stream function turns out to be proportional to the radial magnetic current per unit solid angle. Thus the entire field can be expressed in terms of two functions as follows

$$H_u = -\frac{\partial U}{\partial u}, \qquad H_v = -\frac{\partial U}{\partial v},$$

$$i\omega\mu r^2 H_r = k^2 F,$$

$$E_u = -\frac{\partial F}{\partial v}, \qquad E_v = \frac{\partial F}{\partial u}$$

One of these functions, U, is the transverse magnetomotive force and the other, F, is proportional to the longitudinal magnetic current.

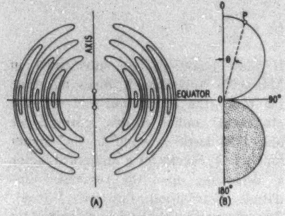


Figure 2

A—Electric lines of force in a transverse magnetic wave (in free space) produced by an electric current element. Magnetic lines are circles coaxial with the element and linking with electric lines. This is a zonal wave with $k^2 = 2$ and $T(\theta, \phi) = \cos\theta$. The same picture represents a transverse electric wave produced by an elementary electric current loop in which case the lines in the picture are magnetic lines. Electric lines are, then, circles coaxial with the loop

B—The chord $OP = \cos\theta$ represents the relative amplitude of the radial current density in the direction OP making angle θ with the axis of the source. The radial current is the electric displacement current if the source of radiation is an electric current element, and the magnetic displacement current if the source is an elementary electric current loop. The radial current flows in opposite directions in the shaded and unshaded regions. The power radiated in any given direction is proportional to the gradient of the radial current density. Thus there is no radiation in the directions of maximum flow of radial current

The nature of this type of electromagnetic transmission can also be indicated by an equivalent transmission line. The transverse magnetic field represents a series inductance and the transverse electric field a shunt capacity (in parallel with a conductance if the medium is dissipative) while the longitudinal magnetic field represents a shunt inductance in parallel with this capacity.

Transverse electromagnetic waves are obtainable from either of the two above types by letting the modular constant k be equal to zero. Such waves, however, can exist only in the presence of two or more conducting wires, emerging from the center of the waves.

All spherical waves either belong to one of the above discussed types or are decomposable into components that belong to them. In a region surrounding the sources, nonspherical waves can be decomposed into a finite or an infinite number of spherical waves. The corresponding mathematical process is the expansion of a given function into spherical harmonics.

Waves Emitted by an Electric Current Element

Before proceeding with the general treatment of transverse magnetic waves, it will be instructive to discuss from our point of view a transverse magnetic wave which is particularly familiar to us, namely the wave emitted by an oscillating charge or by an electric current element. In this wave magnetic lines are circles coaxial with the element and electric lines are in radial planes (figure 2). The expressions for the field of this wave may be found in various books dealing with the advanced topics of electricity and magnetism. From these expressions we can calculate by integration the electromotive force from any point to the equator of the particular equiphase sphere and the number of magnetic lines crossing a path from this point to the equator; thus if the element is at the origin and along the polar axis of the spherical co-ordinate system, we have

$$V = \eta \frac{i\beta I_0 l e^{-i\beta r}}{4\pi}\left[1 + \frac{1}{i\beta r} + \frac{1}{(i\beta r)^2}\right]\cos\theta$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$A = \frac{i\beta I_0 l e^{-i\beta r}}{4\pi}\left(1 + \frac{1}{i\beta r}\right)\cos\theta$$

$$\beta = \omega\sqrt{\mu\epsilon} = \frac{2\pi}{\lambda}$$

where $I_0 l$ is the moment of the element in ampere-meters, η the intrinsic impedance of the medium in ohms, β the phase constant in radians per meter, ω the frequency in radians per second, μ the permeability in henries per meter, ϵ the dielectric constant in farads per meter, and λ the wave length in meters. The radial impedance Z_r is defined as the ratio V/A which is also equal to the ratio of transverse field strengths. The resistive and the reactive components of this impedance as functions of the distance in wave lengths are shown in figure 3.

It could also be shown that V and A satisfy the following "transmission equations"

$$\frac{\partial V}{\partial r} = -\left(i\omega\mu + \frac{2}{i\omega\epsilon r^2}\right)A,$$

$$\frac{\partial A}{\partial r} = -i\omega\epsilon V$$

This equation makes it evident that we are dealing with a "transmission line" having a distributed series inductance μ henries per meter, an ever-increasing series capacity $(1/2)\,\epsilon r^2$ farad-meters, and a shunt capacity ϵ farads per meter. The analogy becomes even more striking if the equations are written in terms of the potential difference $\overline{\mathrm{V}}$ between the corresponding points on the "north" and "south" axes of the wave and the radial electric current I flowing across the northern half of the equiphase sphere

$$\frac{d\overline{\mathrm{V}}}{dr} = -\frac{1}{\pi}\left(i\omega\mu + \frac{2}{i\omega\epsilon r^2}\right)I,$$

$$\frac{dI}{dr} = -i\omega\epsilon\pi\overline{\mathrm{V}}$$

The factors $1/\pi$ and π appear because of the averaging process. In practice, however, the last of the above equations are of lesser importance since we are usually interested in the actual field at various points and not in average values over the entire wave front.

Waves Emitted by an Infinitely Small Electric Current Loop

Another simple wave important in practical engineering, is that emitted by a small loop of area S carrying electric current I. The magnetic potential and the electric stream function for this transverse electric wave are

$$U = -\frac{1}{4\pi}\beta^2 SIe^{-i\beta r}\left[1 + \frac{1}{i\beta r} + \frac{1}{(i\beta r)^2}\right]\cos\theta$$

$$F = -\frac{\eta}{4\pi}\beta^2 SIe^{-i\beta r}\left(1 + \frac{1}{i\beta r}\right)\cos\theta$$

and the transverse fields are obtainable from these by differentiation

$$H_\theta = -\frac{\partial U}{r\,\partial\theta}, \qquad E_\phi = \frac{\partial F}{r\,\partial\theta}$$

In this case the transmission equations are

$$\frac{\partial F}{\partial r} = -i\omega\mu U, \qquad \frac{\partial U}{\partial r} = -\left(i\omega\epsilon + \frac{2}{i\omega\mu r^2}\right)$$

Thus the equivalent transmission line consists of a constant series inductance, a constant shunt capacity, and a shunt inductance proportional to the square of the distance from the loop. The radial impedance of the wave is defined as the ratio F/U or the ratio $E_\phi/-H_\theta$. The product of this impedance and the corresponding impedance of the wave emitted by the current element is the square of the intrinsic impedance. In air this product is, therefore, 377^2 ohm^2 and if one impedance happens to be small, the other will be large.

Figure 3 shows that at distances from the source short compared with λ the radial impedance of the wave emitted by an electric current element in air is very large; therefore, the corresponding impedance of the wave emitted by the loop is very small. Thus near the sources the two waves in air can be characterized as a high- and a low-impedance wave. It turns out, however, that in good conductors both waves are low-impedance waves even at frequencies as high as 100 megacycles. Hence the reflection at the air-metal boundaries, whose radii are short compared with the wave length, is very much higher for waves originated by condensers than for waves originated by coils and metal shields are correspondingly more effective for the former waves.

Transverse Magnetic Spherical Waves

In dealing with transverse magnetic waves we shall choose their center as the origin of our co-ordinate system (figure 1) and set $H_r = 0$ in the general electromagnetic equations. An analysis closely parallel to that adopted in dealing with plane waves[3] shows that various field components can be expressed in terms of an electric potential V and a magnetic stream function A as follows

$$\left.\begin{aligned} rE_\theta &= -\frac{\partial V}{\partial\theta}, \quad r\sin\theta\, E_\phi = -\frac{\partial V}{\partial\phi},\\ (g+i\omega\epsilon)\,E_r &= \frac{k^2A}{r^2}\\ rH_\phi &= -\frac{\partial A}{\partial\theta}, \quad r\sin\theta\, H_\theta = \frac{\partial A}{\partial\phi},\\ H_r &= 0 \end{aligned}\right\} \qquad (2)$$

where the *modular constant k* depends upon the amplitude pattern of the wave.

The relative amplitudes of V and A over equiphase spheres are specified by a function $T(\theta, \phi)$ which must satisfy the following *amplitude equation*

$$\sin\theta\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{\partial^2 T}{\partial\phi^2} = -k^2\sin^2\theta\, T \qquad (3)$$

The solutions of this equation are known as the *spherical surface harmonics.*

The variation of V and A with the distance from the center of the wave is given by the following *transmission equations*

$$\frac{\partial V}{\partial r} = -ZA, \qquad \frac{\partial A}{\partial r} = -YV \qquad (4)$$

where the distributed series impedance and the distributed shunt admittance are, respectively

$$Z = i\omega\mu + \frac{k^2}{(g+i\omega\epsilon)r^2}, \qquad Y = g + i\omega\epsilon \qquad (5)$$

The shunt admittance is seen to consist merely of a parallel combination of conductance g and capacity ϵ; the series impedance, on the other hand, consists of an inductance μ in series with a parallel combination of a capacity $\epsilon r^2/k^2$ and a conductance gr^2/k^2.

When r is sufficiently large, Z is substantially independent of r and the transmission equations are those of a uniform line. It is seen from (2) that under these circumstances the longitudinal component E_r of the electric intensity becomes small and the wave becomes nearly transverse electromagnetic. At such distances the propagation constant and the radial characteristic impedance of the wave are approximately equal to the corresponding intrinsic constants of the medium

$$\Gamma = \sigma = \sqrt{i\omega\mu(g+i\omega\epsilon)},$$

$$Z_r = \eta = \sqrt{\frac{i\omega\mu}{g+i\omega\epsilon}} \qquad (6)$$

By (5) the distance r is "sufficiently large" if $k/|\sigma r|$ is small compared with unity. If the conductivity of the medium is high, then even short distances may be "sufficiently large." In nondissipative media $\sigma = 2\pi i/\lambda$; in this case, if the distance in wave lengths is large compared with $k/2\pi$ then the distance is sufficiently large for the wave to be regarded as substantially transverse electromagnetic.

In general, however, the transmission line described by (4) is nonuniform. Eliminating V, we have

$$\frac{\partial^2 A}{\partial r^2} - \left(\sigma^2 + \frac{k^2}{r^2}\right)A = 0 \qquad (7)$$

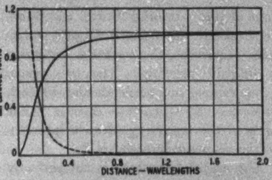


Figure 3. The ratio of the radial impedance of a wave emitted by an electric current element to the intrinsic impedance as a function of the distance from the element. The solid line represents the resistance and the dotted line the capacitive reactance. The intrinsic impedance of air is 120π ≈ 377 ohms

This equation possesses two independent solutions of the form

$$\begin{aligned} A^+ &= T(\theta,\phi)\sqrt{r}\,K_p(\sigma r),\\ A^- &= T(\theta,\phi)\sqrt{r}\,I_p(\sigma r),\\ p &= \sqrt{k^2 + {}^1/_4} \end{aligned} \qquad (8)$$

where I_p and K_p are the modified Bessel functions of the first and second kind. When $r = \infty$, A^+ vanishes and A^- becomes infinite; thus A^+ is the solution appropriate for a region extending to infinity and represents an outward-bound progressive wave. When $r = 0$, K_p is infinite and I_p is finite; hence A^- is the solution appropriate to a region which includes the origin and is bounded by an equiphase sphere. We shall designate this wave as the "internal" wave; in a nondissipative medium, such a wave is always a standing wave.

From (4) and (8) we can obtain the corresponding expressions for V and hence for the outward-looking radial impedance Z_r^+ and for the inward-looking radial impedance Z_r^-; thus we have

$$\left.\begin{aligned} Z_r^+ &= -\frac{1}{g+i\omega\epsilon}\frac{\partial}{\partial r}\log A^+\\ &= -\eta\left[\frac{1}{2\sigma r} + \frac{K_p'(\sigma r)}{K_p(\sigma r)}\right]\\ Z_r^- &= \frac{1}{g+i\omega\epsilon}\frac{\partial}{\partial r}\log A^-\\ &= \eta\left[\frac{1}{2\sigma r} + \frac{I_p'(\sigma r)}{I_p(\sigma r)}\right] \end{aligned}\right\} \qquad (9)$$

Since the transverse-field components are expressed in terms of the derivatives of V and A with respect to θ and ϕ and since V and A depend upon these variables only through the factor $T(\theta, \phi)$, we have from (2)

$$\frac{E_\theta}{H_\phi} = -\frac{E_\phi}{H_\theta} = Z_r \qquad (10)$$

Inasmuch as T is a real function, E_θ and E_ϕ are in phase and thus add into a single vector whose direction is independent of time; in other words, the transverse electric intensity is linearly polarized. Likewise, the magnetic intensity is linearly polarized. The two transverse vectors are perpendicular to each other because $E_\theta/E_\phi = -H_\phi/H_\theta$. The ratio of the transverse electric intensity to the magnetic intensity is Z_r.

The modular constant is not perfectly arbitrary. Thus in a medium which is homogeneous throughout the entire space, T must be finite for all values of the coordinates θ and ϕ and from the theory of equation 3 we know that this requirement restricts the values of k to a set given by

$$k^2 = n(n+1), \quad n = 1, 2, 3, 4, \ldots, \quad p = n + {}^1/_2 \qquad (11)$$

The integer n can be regarded as the index of the particular *transmission mode*. The wave emitted by an electric current element corresponds to $n = 1$. The general form of the solution is given by equation 12. In free space m is an integer because the field must be periodic in ϕ with the period 2π. When $m = 0$, the field is independent of the ϕ co-ordinate and the waves may be called *zonal* waves. In this case, the integer n represents the number of nodal cones, that is, conical surfaces made up of radii along which the radial current density is equal to zero. In this count the equatorial plane is to be regarded as a (degenerate) conical surface. When $m \neq 0$, the waves may be called *tesseral* waves. The integer m represents the number of nodal planes passing through the axis of the wave; the number of nodal cones is then $n-m$. When $m = n$, the wave may be called *sectorial* since the wave may be divided into m equal sectors with identical field distributions. Figures 2, 4, and 5 illustrate a few types of spherical waves in free space.

If a homogeneous medium is bounded by a perfectly conducting conical surface, then the tangential component of the electric field and hence T must vanish on the boundary. Again the "permissible" values of k will be restricted to a discreet set of numbers.

The transmission modes which are possible in regions bounded by coaxial circular cones and by planes passing through the common axis can be found with the aid of *tesseral harmonics*. These harmonics are the solutions of (3) in the form $\Theta(\theta)\,\Phi(\phi)$, namely

$$\left.\begin{aligned} T(\theta,\phi) &= [a_{n,m}P_n^m(\cos\theta) + b_{n,m}Q_n^m(\cos\theta)]\,(c_{n,m}\cos m\phi + d_{n,m}\sin m\phi)\\ k^2 &= n(n+1) \quad \text{or}\\ n &= -{}^1/_2 + \sqrt{k^2 + {}^1/_4} \end{aligned}\right\} \qquad (12)$$

The functions P_n^m and Q_n^m are *associated* Legendre functions satisfying the following differential equation

$$\sin\theta\frac{d}{d\theta}\left(\sin\theta\frac{dx}{d\theta}\right) + (k^2\sin^2\theta - m^2)x = 0 \qquad (13)$$

If we are concerned with the region defined by the inequality $\theta < \theta_0$ and bounded by a perfectly conducting conical metal sheet $\theta = \theta_0$, then m is an integer and n is a root of

$$P_n^m(\cos\theta_0) = 0 \qquad (14)$$

If the region is given by $\theta_1 \leq \theta \leq \theta_2$ and $\phi_1 \leq \phi \leq \phi_2$ and bounded by perfectly conducting cones $\theta = \theta_1$ and $\theta = \theta_2$ and by perfectly conducting planes $\phi = \phi_1$ and $\phi = \phi_2$, then T will vanish on the boundary when $m = s\pi/(\phi_2 - \phi_1)$, s being an integer, and n is a root of

$$\frac{P_n^m(\cos\theta_1)}{Q_n^m(\cos\theta_1)} = \frac{P_n^m(\cos\theta_2)}{Q_n^m(\cos\theta_2)} \qquad (15)$$

If the region is bounded merely by two conducting coaxial cones $\theta = \theta_1$ and $\theta = \theta_2$, then m in (15) is an integer since the field must remain the same if ϕ is increased by 2π, that is, if we return to the same point after making a complete turn along a circle of latitude.

For waves in free space the Bessel functions defined by (8) reduce to exponential functions multiplied by polynomials in $1/\sigma r$; thus

$$\left.\begin{aligned} A^+ &= T(\theta,\phi)e^{-\sigma r}f_n(\sigma r)\\ f_n(\sigma r) &= \sum_{s=0}^{n}\frac{(n+s)!}{s!(n-s)!(2\sigma r)^s}\\ A^- &= T(\theta,\phi)[e^{-\sigma r}f_n(\sigma r) + (-)^{n+1}e^{\sigma r}f_n(-\sigma r)] \end{aligned}\right\} \qquad (16)$$

The following are, then, the expressions for the radial impedances

$$\left.\begin{aligned} Z_r^+ &= \eta\left[1 - \frac{f_n'(\sigma r)}{f_n(\sigma r)}\right]\\ Z_r^- &= \eta\,\frac{e^{-\sigma r}[f_n'(\sigma r) - f_n(\sigma r)] + (-)^n e^{\sigma r}[f_n'(-\sigma r) - f_n(-\sigma r)]}{e^{-\sigma r}f_n(\sigma r) + (-)^{n+1}\times e^{\sigma r}f_n(-\sigma r)} \end{aligned}\right\} \qquad (17)$$

The impedances of the wave emitted by an electric current element are obtained from (17) if we assume $n = 1$; thus we have

$$Z_r^+ = \eta\frac{1 + \sigma r + (\sigma r)^2}{\sigma r(1 + \sigma r)},$$

$$Z_r^- = \eta\frac{(1 + \sigma^2 r^2)\tanh\sigma r - \sigma r}{\sigma r(\sigma r - \tanh\sigma r)} \qquad (18)$$

In a nondissipative medium $\sigma = i\beta = i\omega\sqrt{\mu\epsilon} = 2\pi i/\lambda$; hence (18) becomes

$$Z_r^+ = \eta\frac{\beta^2 r^2}{1 + \beta^2 r^2} + \frac{\eta}{i\beta r(1 + \beta^2 r^2)},$$

$$Z_r^- = i\eta\frac{(1 - \beta^2 r^2)\tan\beta r - \beta r}{\beta r(\tan\beta r - \beta r)} \qquad (19)$$

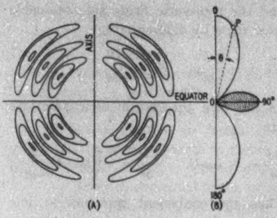


Figure 4

A—Lines of force in a zonal wave with $k^2 = 6$ and $T(\theta, \phi) = \frac{1}{2}(3\cos^2\theta - 1)$. These lines are electric if the wave is transverse magnetic and magnetic if the wave is transverse electric. Magnetic lines in the first case and electric lines in the second are coaxial circles linking with the lines shown in the figure

B—Relative distribution of the radial current density. The radial current flows in opposite directions in the shaded and unshaded regions

The outward-looking impedance consists of a resistance and a capacitive reactance; the former increases with the distance from zero to a constant value while the latter decreases from infinity to zero (figure 3). At distances short compared with the wave length we have approximately

$$Z_r^+ = \frac{1}{i\omega\epsilon r} = -i\frac{\eta}{2\pi}\left(\frac{\lambda}{r}\right)$$

The inward-looking impedance is reactive. At distances small compared with λ this impedance is nearly equal to $2/i\omega\epsilon r$; then its absolute value decreases to become equal to zero when $r = 0.44\lambda$; subsequently the reactance becomes positive and it increases to infinity at $r = 0.71\lambda$; then it changes its sign again, etc.

The average flow of power across an equiphase surface is given by

$$\left.\begin{aligned} \Psi &= ({}^1/_2)r^2\iint_{(S)}(E_\theta H_\phi^* - E_\phi H_\theta^*)\,d\Omega\\ &= ({}^1/_2)r^2 Z_r\iint_{(S)}(H_\theta H_\theta^* + H_\phi H_\phi^*)\,d\Omega\\ &= ({}^1/_2)Z_r\iint_{(S)}\left(\frac{\partial A}{\partial\theta}\frac{\partial A^*}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial A}{\partial\phi}\frac{\partial A^*}{\partial\phi}\right)d\Omega \end{aligned}\right\} \qquad (20)$$

where the asterisk is used to designate conjugate complex numbers and $d\Omega$ is the elementary solid angle defined by

$$d\Omega = \sin\theta \, d\theta \, d\phi \tag{21}$$

Substituting for A its expression in the form $T(\theta, \phi)\ R(r)$, we have

$$\Psi = (1/2) Z_r RR^* \iint_{(s)} \left[\left(\frac{\partial T}{\partial \theta}\right)^2 + \frac{1}{\sin^2\theta}\left(\frac{\partial T}{\partial \phi}\right)^2\right] d\Omega \tag{22}$$

Multiplying (3) by $T\ d\theta\ d\phi/\sin\theta$ and integrating over an equiphase surface, we obtain the following identity for a wave in free space or in a region bounded by perfectly conducting conical surfaces emerging from the center of the wave

$$\iint_{(s)} \left[\left(\frac{\partial T}{\partial \theta}\right)^2 + \frac{1}{\sin^2\theta}\left(\frac{\partial T}{\partial \phi}\right)^2\right] \times \sin\theta \, d\theta \, d\phi = k^2 \iint_{(s)} T^2 \, d\Omega \tag{23}$$

Hence (22) becomes

$$\Psi = (1/2) k^2 Z_r RR^* \iint_{(s)} T^2 \, d\Omega \tag{24}$$

Thus the power flow across an equiphase surface is proportional to the mean square of the amplitude distribution function T.

Transverse Electric Spherical Waves

In the case of transverse electric waves the field is expressed in terms of the magnetic potential U and the electric stream function F as follows

$$\left.\begin{aligned} rH_\theta &= -\frac{\partial U}{\partial \theta}, \quad r\sin\theta\, H_\phi = -\frac{\partial U}{\partial \phi}, \\ i\omega\mu\, H_r &= \frac{k^2 F}{r^2} \\ r\,E_\phi &= \frac{\partial F}{\partial \theta}, \quad r\sin\theta\, E_\theta = -\frac{\partial F}{\partial \phi}, \\ E_r &= 0 \end{aligned}\right\} \tag{25}$$

where F and U satisfy the following transmission equations

$$\left.\begin{aligned} \frac{\partial F}{\partial r} &= -ZU, \quad Z = i\omega\mu \\ \frac{\partial U}{\partial r} &= -YF, \; Y = (g + i\omega\epsilon) + \frac{k^2}{i\omega\mu r^2} \end{aligned}\right\} \tag{26}$$

These are seen to be the equations for a transmission line with constant distributed series inductance, constant shunt conductance and capacity and tapered shunt inductance. Both functions F and U are proportional to the amplitude distribution function $T(\theta, \phi)$ which satisfies equation 3.

In so far as the r co-ordinate is concerned, F satisfies the same equation as A in (7). Thus the solutions may be written as follows

$$\begin{aligned} F^+ &= T(\theta, \phi)\sqrt{r}\, K_p(\sigma r), \\ F^- &= T(\theta, \phi)\sqrt{r}\, I_p(\sigma r), \\ p &= \sqrt{k^2 + 1/4} \end{aligned} \tag{27}$$

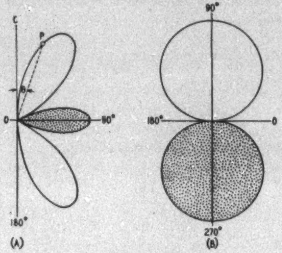
the first being appropriate to the outward-bound progressive waves and the second to the internal waves.

The product of the outward-looking impedances for the transverse magnetic and transverse electric waves having the same modular constant is equal to the square of the intrinsic impedance of the medium. The same is true of the corresponding inward-looking impedances. Thus we have

$$Z_{TM}^+ Z_{TE}^+ = \eta^2, \quad Z_{TM}^- Z_{TE}^- = \eta^2 \tag{28}$$

This result is deduced from the fact that A and F satisfy the same equation 7 so that they are represented by the same mathematical functions and from the fact that in accordance with (4) and (26) the expressions for radial impedances can be written in the following forms

$$\left.\begin{aligned} Z_{TM}^+ &= \frac{V^+}{A^+} = -\frac{1}{(g + i\omega\epsilon)A^+}\frac{\partial A^+}{\partial r} \\ Z_{TE}^+ &= \frac{F^+}{U^+} = -\frac{i\omega\mu F^+}{\dfrac{\partial F^+}{\partial r}} \end{aligned}\right\} \tag{29}$$



(A) (B)

Figure 5

A—The vertical pattern representing the radial current density in a tesseral wave with $k^2 = 12$ and $T(\theta, \phi) = \sin\theta\,(5\cos^2\theta - 1)\sin\phi$

B—The horizontal pattern of the radial current. The radial current flows in opposite directions in the shaded and unshaded regions

Equation 28 implies that if the impedance of one wave is large compared with the intrinsic impedance, the impedance of the other is low. Likewise the resonant points of transverse magnetic waves are antiresonant points of transverse electric waves, and vice versa.

In free space the modular constant of transverse electric waves satisfies (11). In the regions bounded by perfectly conducting conical surfaces, the normal component of the magnetic intensity and, therefore, the normal derivative of T must vanish on the boundary. This condition is sufficient for determination of modular constants when the boundaries are specified. Thus for a region bounded by a cone $\theta = \theta_0$, n must satisfy the following equation

$$\frac{d}{d\theta_0} P_n^m (\cos\theta_0) = 0 \tag{30}$$

where m is an integer. In a region bounded by two coaxial cones $\theta = \theta_1$ and $\theta = \theta_2$, n must satisfy

$$\frac{\dfrac{d}{d\theta} P_n^m (\cos\theta_1)}{\dfrac{d}{d\theta} Q_n^m (\cos\theta_1)} = \frac{\dfrac{d}{d\theta} P_n^m (\cos\theta_2)}{\dfrac{d}{d\theta} Q_n^m (\cos\theta_2)} \tag{31}$$

where m is still an integer. Finally, in a region bounded by two coaxial cones $\theta = \theta_1$ and $\theta = \theta_2$ and by two planes $\phi = \phi_1$ and $\phi = \phi_2$ passing through the axis of these cones, n must be a root of (31) and $m = s\pi/(\phi_2 - \phi_1)$, s being an integer.

The average power flow across an equiphase surface can be expressed in terms of the amplitude distribution function in the same manner as for transverse magnetic waves. Thus we obtain

$$\Psi = \frac{k^2}{2Z_r^*} RR^* \iint_{(S)} T^2 \, d\Omega \tag{32}$$

Transverse Electromagnetic Spherical Waves

If the modular constant is zero, there is neither electric nor magnetic radial field and the wave is strictly transverse. As may be seen from equations 4 and 5 the propagation constant and the radial impedance are, in this case, independent of the distance from the source and are equal to the corresponding intrinsic constants of the medium.

Transverse electromagnetic spherical waves cannot exist in free space. In order to prove this we observe that when $k = 0$, (3) becomes

$$\sin\theta \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{\partial^2 T}{\partial \phi^2} = 0 \tag{33}$$

Multiplying this equation by $T\ d\theta\ d\phi/\sin\theta$, supposing that T is everywhere finite and single-valued, and integrating by parts over the entire sphere, we obtain

$$\int_0^{2\pi}\int_0^{\pi}\left[\left(\frac{\partial T}{\partial \theta}\right)^2+\frac{1}{\sin^2\theta}\left(\frac{\partial T}{\partial \phi}\right)^2\right] \times \sin\theta\, d\theta\, d\phi = 0 \quad (34)$$

Since the integrand is nonnegative, (34) can be true only when T reduces to a constant and the field to zero.

Transverse electric waves may exist, however, in the presence of two or more perfectly conducting conical wires emerging from a given point. The following is the most general solution of (33)

$$T = f_1\left(e^{i\phi}\tan\frac{\theta}{2}\right)+f_2\left(e^{-i\phi}\tan\frac{\theta}{2}\right) \quad (35)$$

where f_1 and f_2 are arbitrary functions.[3] In particular we have a solution of the form

$$T = P\log\tan\frac{\theta}{2} \quad (36)$$

Hence, we can set

$$A = P_1\log\tan\frac{\theta}{2}, \quad V = P_2\log\tan\frac{\theta}{2} \quad (37)$$

where P_1 and P_2 are functions of r alone and are related as A and V in (4). From (2) we now obtain

$$\left.\begin{aligned} H_\phi &= -\frac{P_1}{r\sin\theta}, \\ E_\theta &= -\frac{P_2}{r\sin\theta}, \\ H_\theta &= E_\phi = 0 \end{aligned}\right\} \quad (38)$$

These solutions are finite everywhere except along the radii $\theta = 0$ and $\theta = \pi$. Calculating the magnetomotive force along a typical parallel, we have the expression for the total radial current flowing across the spherical segment bounded by this parallel; thus

$$I(r) = 2\pi r\sin\theta H_\phi = -2\pi P_1,$$

$$P_1 = -\frac{I(r)}{2\pi} \quad (39)$$

Since the field is transverse, there is no radial current anywhere except possibly along the singular lines $\theta = 0$ and $\theta = \pi$. Equation 39 assures us now that these lines actually do support equal and opposite currents. Since the electric field is perpendicular to the singular lines they must be perfect conductors; hence, the field under consideration is a spherical wave guided by two infinitely thin wires diverging from a fixed point in opposite directions.

We have seen that the radial impedance of an outward-bound progressive transverse electromagnetic wave is equal to the intrinsic impedance η and that the propagation constant is σ; therefore

$$P_2 = \eta P_1 = -\frac{\eta}{2\pi}I_0e^{-\sigma r} \quad (40)$$

where I_0 is the current through the generator at $r = 0$. Equations 38 assume now the following form

$$H_\phi = \frac{I_0e^{-\sigma r}}{2\pi r\sin\theta}, \quad E_\theta = \frac{\eta I_0e^{-\sigma r}}{2\pi r\sin\theta} \quad (41)$$

The field will not be disturbed if we introduce perfectly conducting conical sheets $\theta = \theta_1$ and $\theta = \theta_2$. By (37) and (40) the potential difference between these sheets is

$$V_2 - V_1 = \frac{\eta I_0e^{-\sigma r}}{2\pi}\log\left(\cot\frac{\theta_1}{2}\tan\frac{\theta_2}{2}\right) \quad (42)$$

Thus the characteristic impedance of the transmission line formed by two coaxial cones is

$$Z_0 = \frac{\eta}{2\pi}\log\left(\cot\frac{\theta_1}{2}\tan\frac{\theta_2}{2}\right) \quad (43)$$

Taking the hint from (36) it is quite easy to construct synthetically the solution for the spherical wave guided by two infinitely thin wires diverging along arbitrary radii $\theta = \theta_1$, $\phi = \phi_1$, and $\theta = \theta_2$, $\phi = \phi_2$. Thus we have

$$A = -\frac{I}{4\pi}\log\frac{\left(e^{i\phi}\tan\frac{\theta}{2}-e^{i\phi_1}\tan\frac{\theta_1}{2}\right)\left(e^{-i\phi}\tan\frac{\theta}{2}-e^{-i\phi_1}\tan\frac{\theta_1}{2}\right)}{\left(e^{i\phi}\tan\frac{\theta}{2}-e^{i\phi_2}\tan\frac{\theta_2}{2}\right)\left(e^{-i\phi}\tan\frac{\theta}{2}-e^{-i\phi_2}\tan\frac{\theta_2}{2}\right)} \quad (44)$$

The only singularities of this function occur at points where either the numerator or the denominator vanishes, that is, along the above-mentioned radii. These singularities are logarithmic as they were for the previously discussed special case. The coefficient can be verified by calculating the magnetic intensity and integrating the latter around an infinitely small circle surrounding either of the singular lines. Multiplying the quantities in the parentheses of (44), we have an expression free from complex quantities

$$A = -\frac{I}{4\pi}\log\frac{\tan^2\frac{\theta}{2}-2\tan\frac{\theta}{2}\tan\frac{\theta_1}{2}\cos(\phi-\phi_1)+\tan^2\frac{\theta_1}{2}}{\tan^2\frac{\theta}{2}-2\tan\frac{\theta}{2}\tan\frac{\theta_2}{2}\cos(\phi-\phi_2)+\tan^2\frac{\theta_2}{2}} \quad (45)$$

It is now easy to determine the characteristic impedance of a transmission line formed by two perfectly conducting conical wires, of angles (these angles are the angles made by the generator with the axes of the cones) Ψ_1 and Ψ_2 small compared with the angle ζ between the axes of the cones. The latter condition is made in order to assure the substantial absence of proximity effect. The impedance is

$$Z_0 = \frac{\eta}{\pi}\log\frac{2\sin\frac{\zeta}{2}}{\sqrt{\Psi_1\Psi_2}} \quad (46)$$

In the foregoing pages we have discussed those special types of waves which are governed by particularly simple transmission laws. The simplest of these waves are those emitted by a straight electric current element and by a small current loop; the first is a transverse magnetic wave with $n = 1$, $m = 0$ and the second is a transverse electric wave with $n = 1$, $m = 0$. But most practical antenna systems produce more complex waves, which may be regarded, however, as combinations of the simpler types. Likewise any wave emitted by a source at one point and a properly chosen group of waves with their center at some other point produce the same field in the region outside the sphere with its center at the second point and which passes through the first point. This equivalent representation is needed in dealing with the problem of shielding in the case of a source which is not centrally disposed with respect to the shield.

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An Analysis of the Induction Machine

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THERE has long been a need for a means of evaluating the performance of induction motors under transient conditions, as when driving pulsating compressor loads. The familiar equivalent-circuit analysis is inadequate to determine the effects of pulsating speeds, and the more complete analysis of S. J. Levine[1] is somewhat inconvenient, since the rotating axes assumed lead to involved equations. Probably the most convenient approach is to be found in the stationary-axis method so successfully applied by R. H. Park[2] to the analysis of salient-pole synchronous machines.

It is, therefore, the purpose of this paper to present a simplified mathematical analysis of the induction machine which is applicable to the solution of a variety of transient and steady state problems. By means of a set of linear transformations, the phase voltages, currents, and flux linkages are referred to a set of orthogonal co-ordinate axes which are stationary with respect to the stator. The resulting differential equations are not only linear but have constant coefficients for the case when the rotor speed is constant. Such equations are readily soluble by the operational methods.

The simplified equations which are derived are applicable to any machine with a uniform air gap and balanced, sinusoidally distributed rotor windings. These equations are solved in literal operational form for the cases of those current and voltage transients for which it is sufficiently accurate to treat the rotor speed as a constant.

The present treatment, however, is not limited to problems wherein constant rotor speed is maintained. Indeed, the greatest justification for its use is probably to be found in the field of variable speed problems. In this class are those steady hunting or pulsating phenomena which are met in connection with the operation of Selsyn follow-up systems and induction machines with pulsating shaft torques. As an example of the usefulness of the method in the latter field, the analysis is carried out for the case of a single-phase induction motor which is driving a load with pulsating torque, for example an air compressor. In this problem the procedure for determining the current, voltage, and speed fluctuations caused by a given pulsating torque reduces to that of simply solving two sets of three simultaneous, linear algebraic equations.

Assumptions

The analysis is based upon the following simplifying assumptions:

1. Balanced rotor windings are assumed for all cases, and the three-phase machine equations are derived upon the additional assumption that the stator windings are also balanced.
2. It is assumed that the coefficient of mutual inductance between any stator winding and any rotor winding is a cosinusoidal function of the electrical angle between the axes of the two windings.
3. It is further assumed that the rotor is smooth and that the self-inductances of all the windings are independent of the rotor position.
4. The effects of saturation, hysteresis, and eddy currents are neglected.

Part I. Three-Phase Machine Equations

A—Derivation of the Differential Equations

By summing the voltages around the closed circuit of each of the rotor and stator phases, the following fundamental voltage equations may be written:*

$$\left.\begin{aligned} e_1 &= p\psi_1 + ri_1 \\ e_2 &= p\psi_2 + ri_2 \\ e_3 &= p\psi_3 + ri_3 \end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned} e_a &= p\psi_a + Ri_a \\ e_b &= p\psi_b + Ri_b \\ e_c &= p\psi_c + Ri_c \end{aligned}\right\} \quad (2)$$

Now, by virtue of assumption number 4, the flux linkages ψ are linear functions of the various winding currents. Thus,

$$\psi_1 = li_1 + M_R(i_2 + i_3) + M\left[i_a\cos\theta + i_b\cos\left(\theta - \frac{2\pi}{3}\right) + i_c\cos\left(\theta + \frac{2\pi}{3}\right)\right] \quad (3)$$

Equation 3 follows from a consideration of the simplifying assumptions and the arrangement of the windings in the machine. The windings are depicted schematically in figure 1 where the positions of the various coils represent the geometrical positions of the magnetic axes of their respective windings and the arrows indicate assumed positive directions of currents, flux linkages, and applied voltages.

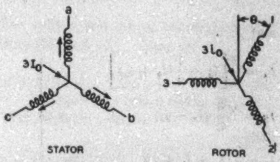

Figure 1. Relative positions of magnetic axes of stator and rotor windings

The substitution of expressions similar to (3) for the various flux linkages into equations 1 and 2 would yield six linear differential equations in the six currents i_1, i_2, i_3, i_a, i_b, i_c. Because of the form of (3), these equations would contain numerous trigonometric coefficients.** It is possible, however, by a change of reference axes, to obtain equations which do not contain these annoying trigonometric functions of θ. Indeed, it is found that the following linear transformations will lead to simpler equations in six new variables, i_α, i_β, i_0, I_α, I_β, I_0. Thus, let

$$\left.\begin{aligned} i_1 &= i_\alpha\cos\theta + i_\beta\sin\theta + i_0 \\ i_2 &= i_\alpha\cos\left(\theta + \frac{2\pi}{3}\right) + i_\beta\sin\left(\theta + \frac{2\pi}{3}\right) + i_0 \\ i_3 &= i_\alpha\cos\left(\theta - \frac{2\pi}{3}\right) + i_\beta\sin\left(\theta - \frac{2\pi}{3}\right) + i_0 \end{aligned}\right\} \quad (4)$$

and

$$\left.\begin{aligned} i_a &= I_\alpha + I_0 \\ i_b &= -\frac{1}{2}I_\alpha + \frac{\sqrt{3}}{2}I_\beta + I_0 \\ i_c &= -\frac{1}{2}I_\alpha - \frac{\sqrt{3}}{2}I_\beta + I_0 \end{aligned}\right\} \quad (5)$$

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[1] For all numbered references, see list at end of paper.

* Refer to the "Nomenclature" for a list of symbols and their definitions.

** These equations have been set up and solved for several cases by S. J. Levine. See reference 1.

When equations 4 and 5 are solved for i_α, i_β, i_0, I_α, I_β, and I_0, the expressions for the new variables in terms of the phase quantities are found to be

$$\left.\begin{aligned} i_\alpha &= \frac{2}{3}\left[i_1 \cos\theta + i_2 \cos\left(\theta + \frac{2\pi}{3}\right) + i_3 \cos\left(\theta - \frac{2\pi}{3}\right)\right] \\ i_\beta &= \frac{2}{3}\left[i_1 \sin\theta + i_2 \sin\left(\theta + \frac{2\pi}{3}\right) + i_3 \sin\left(\theta - \frac{2\pi}{3}\right)\right] \\ i_0 &= \frac{1}{3}(i_1 + i_2 + i_3) \end{aligned}\right\} \quad (6)$$

and

$$\left.\begin{aligned} I_\alpha &= \frac{2}{3}\left[i_a - \frac{1}{2}(i_b + i_c)\right] \\ I_\beta &= \frac{\sqrt{3}}{3}(i_b - i_c) \\ I_0 &= \frac{1}{3}(i_a + i_b + i_c) \end{aligned}\right\} \quad (7)$$

Equations exactly similar to (4) and (5) are introduced to define the transformation of the phase quantities e_1, e_2, e_3, e_a, e_b, e_c, ψ_1, ψ_2, ψ_3, ψ_a, ψ_b, and ψ_c to the new quantities e_α, e_β, e_0, E_α, E_β, E_0, ψ_α, ψ_β, ψ_0, Ψ_α, Ψ_β, and Ψ_0. The inverse transformations are given by equations similar to (6) and (7).

It will be noted that the stator equations 5 and 7 are similar to the rotor equations 4 and 6 when the rotor position angle θ is put equal to zero.

Taking the first time derivative of ψ_α in the equation for ψ_α corresponding to the first of equations 6,

$$p\psi_\alpha = \frac{2}{3}\left[\cos\theta p\psi_1 + \cos\left(\theta + \frac{2\pi}{3}\right)p\psi_2 + \cos\left(\theta - \frac{2\pi}{3}\right)p\psi_3\right] - \frac{2}{3}\left[\psi_1 \sin\theta + \psi_2 \sin\left(\theta + \frac{2\pi}{3}\right) + \psi_3 \sin\left(\theta - \frac{2\pi}{3}\right)\right][p\theta] \quad (8)$$

Then, substituting expressions for $p\psi$ from equations 1, and ψ_β for its expression corresponding to (6), equation 8 becomes:

$$p\psi_\alpha = e_\alpha - ri_\alpha - \psi_\beta[p\theta]$$

or

$$e_\alpha = p\psi_\alpha + ri_\alpha + \psi_\beta[p\theta] \quad (9)$$

By a similar process e_β is determined. Thus,

$$e_\beta = p\psi_\beta + ri_\beta - \psi_\alpha[p\theta] \quad (10)$$

Also, from (1) and (6),

$$e_0 = p\psi_0 + ri_0 \quad (11)$$

Following this same procedure, stator equations corresponding to the rotor equations 9, 10, and 11 are derived. Thus,

$$E_\alpha = p\Psi_\alpha + RI_\alpha \quad (12)$$
$$E_\beta = p\Psi_\beta + RI_\beta \quad (13)$$
$$E_0 = p\Psi_0 + RI_0 \quad (14)$$

Now, just as the winding flux linkages are linear functions of the various winding currents as shown in (3), the new flux linkage quantities are linear functions of the new currents. These relations are given by equations 15.

$$\left.\begin{aligned} \psi_\alpha &= l_0 i_\alpha + M_0 I_\alpha \\ \psi_\beta &= l_0 i_\beta + M_0 I_\beta \\ \psi_0 &= (l + 2M_R) i_0 \\ \Psi_\alpha &= L_0 I_\alpha + M_0 i_\alpha \\ \Psi_\beta &= L_0 I_\beta + M_0 i_\beta \\ \Psi_0 &= (L + 2M_S) I_0 \end{aligned}\right\} \quad (15)$$

When (15) are substituted in (9)–(14), the flux linkage terms are eliminated and the following equations result:

$$\left.\begin{array}{lllll} (R + L_0 p)I_\alpha + & 0 + M_0 p i_\alpha & + & & 0 = E_\alpha \\ 0 + (R + L_0 p)I_\beta & + & 0 + & & M_0 p i_\beta = E_\beta \\ M_0 p I_\alpha + M_0[p\theta]I_\beta & + (r + l_0 p)i_\alpha & + & & l_0[p\theta]i_\beta = e_\alpha \\ -M_0[p\theta]I_\alpha + M_0 p I_\beta & - l_0[p\theta]i_\alpha & + & & (r + l_0 p)i_\beta = e_\beta \\ & & [R + & & (L + 2M_S)p]I_0 = E_0 \\ & & [r + & & (l + 2M_R)p]i_0 = e_0 \end{array}\right\} \quad (16)$$

Equations 16 completely specify the relations between the currents, terminal voltages, and rotor speed. An inspection of these relations will give a clearer picture of the physical significance of the linear transformations (4) and (5). They are the equations which would result if voltages E_α, E_β were impressed at the terminals of two stator windings with resistance R and self-inductance L_0, oriented in electrical space quadrature—i. e., with their magnetic axes coincident, respectively, with the axis of phase a (α-axis) and an axis displaced 90 electrical degrees in the direction of rotation from the axis of phase a (β-axis). The corresponding rotor voltages e_α, e_β would be applied at the terminals of two rotor coils with resistance r and self-inductance l_0, oriented with their axis coincident with the α- and β-axes, respectively. To account for the appearance of the "speed-voltage" terms $\psi_\beta[p\theta]$ and $-\psi_\alpha[p\theta]$ in the rotor equations, the fictitious rotor coils are thought of as having their magnetic axes coincident with the stationary α- and β-axes, yet having an instantaneous velocity $[p\theta]$ with respect to them. Such a condition could be approximated by supplying the rotor currents through stationary brushes to commutator bars on a moving rotor. The quantities I_0 and i_0 are simply measures of the currents which flow in the stator and rotor neutral leads.

To specify completely the machine performance for a given set of applied voltages a further relation between the currents and rotor speed is sought. This relation is found in the equation for the electrical torque on the rotor which is derived in the following section.

B—Derivation of the Torque Equation

The value of the magnetic torque between two magnetically coupled, current-carrying coils is given by the following expression:[5]

$$T = \frac{1}{2} I_1^2 \frac{dL_1}{d\theta} + \frac{1}{2} I_2^2 \frac{dL_2}{d\theta} + I_1 I_2 \frac{dM}{d\theta} \quad (17)$$

where

L_1, L_2 = self-inductances of the coils
M = mutual-inductance between the coils
I_1, I_2 = coil currents
θ = relative angular displacement of coil axes

For an induction motor with a smooth rotor, the self-inductances are constant and (16) becomes, for the summation of the torques due to each pair of rotor and stator coils,

$$T = \sum I_R I_S \frac{d}{d\theta} M_{RS} \quad (18)$$

where the subscripts R and S denote rotor and stator quantities, respectively.

Or, since M_{RS} is assumed to be a cosinusoidal function of the angular displacement of the coils R and S, $d/d\theta\ M_{RS}$ is a sinusoidal function of the displacement angle and (18) becomes for the three-phase induction motor,

$$T = -(i_a i_1 + i_b i_2 + i_c i_3)M \sin\theta - (i_a i_2 + i_b i_3 + i_c i_1) M \sin\left(\theta + \frac{2\pi}{3}\right) - (i_a i_3 + i_b i_1 + i_c i_2) M \sin\left(\theta - \frac{2\pi}{3}\right) \quad (19)$$

Substituting the expressions (4) and (5) into (19),

$$T = -\frac{3}{2} I_\alpha M_0 i_\beta + \frac{3}{2} I_\beta M_0 i_\alpha \quad (20)$$

Equation 20 together with equations 16 completely specify the machine performance in terms of the impressed voltages and torque.

C—Solution of the Machine Equations

One group of transient problems is that for which it is sufficiently accurate to treat the rotor speed as a constant. With the assumption of constant speed, the machine equations (16) become linear differential equations with constant coefficients and subject to solution in the literal operational form. Thus, replacing the term $[p\theta]$ by the constant n in (16) and letting the rotor terminal voltages go to zero, the solution of (16) yields the following operational expressions for the various currents:

$$\left.\begin{aligned} I_\alpha &= \frac{V_1}{Z_1} + \frac{V_2}{Z_2} \\ I_\beta &= -j\frac{V_1}{Z_1} + j\frac{V_2}{Z_2} \\ i_\alpha &= -\frac{(p-jn)M_0}{r+(p-jn)l_0}\frac{V_1}{Z_1} - \\ &\qquad \frac{(p+jn)M_0}{r+(p+jn)l_0}\frac{V_2}{Z_2} \\ i_\beta &= j\frac{(p-jn)M_0}{r+(p-jn)l_0}\frac{V_1}{Z_1} - \\ &\qquad j\frac{(p+jn)M_0}{r+(p+jn)l_0}\frac{V_2}{Z_2} \\ I_0 &= \frac{V_0}{Z_0} \end{aligned}\right\} \quad (21)$$

These expressions, when evaluated by operational methods, yield the transient currents as functions of time. It must be remembered that these particular solutions are valid for constant rotor speed n only and when the rotor speed is a function of time equations 16 must be solved by some other means.

The steady-state currents are readily determined in the familiar vector form by expressing the applied voltages V_1, V_2, V_0 as vectors and replacing p by $j\omega$ in (21).

Part II. Two-Phase Machine Equations

A—The General Machine Equations

Assuming the direction of rotation from the positive axis of phase a toward the positive axis of phase b, the α-axis is taken as the axis of phase a and the β-axis as the axis of phase b. Then, the equations of transformation corresponding to (4), (5), (6), (7) are

$$\left.\begin{aligned} i_1 &= i_\alpha \cos\theta + i_\beta \sin\theta \\ i_2 &= -i_\alpha \sin\theta + i_\beta \cos\theta \end{aligned}\right\} \quad (22)$$

$$\left.\begin{aligned} i_a &= I_\alpha \\ i_b &= I_\beta \end{aligned}\right\} \quad (23)$$

$$\left.\begin{aligned} i_\alpha &= i_1 \cos\theta - i_2 \sin\theta \\ i_\beta &= i_1 \sin\theta + i_2 \cos\theta \end{aligned}\right\} \quad (24)$$

With similar transformations for the voltages and flux linkages, the voltage equations (9), (10), (12), (13) of the three-phase case are applicable here. Also, the two-phase flux linkage relations are similar to the three-phase relations (15) with the exception that the actual inductance coefficients are used instead of the three-phase apparent coefficients. Thus

$$\left.\begin{aligned} \psi_\alpha &= li_\alpha + MI_\alpha \\ \psi_\beta &= li_\beta + MI_\beta \\ \Psi_\alpha &= LI_\alpha + Mi_\alpha \\ \Psi_\beta &= LI_\beta + Mi_\beta \end{aligned}\right\} \quad (25)$$

The general machine equations corresponding to (16) are obtained by substituting (23) into (9), (10), (12), (13). They are

$$\left.\begin{aligned} (R+Lp)I_\alpha + 0 + Mpi_\alpha + 0 &= E_\alpha \\ 0 + (R+Lp)I_\beta + 0 + Mpi_\beta &= E_\beta \\ MpI_\alpha + M[p\theta]I_\beta + (r+lp)i_\alpha + l[p\theta]i_\beta &= e_\alpha \\ -M[p\theta]I_\alpha + MpI_\beta - l[p\theta]i_\alpha + (r+lp)\,i_\beta &= e_\beta \end{aligned}\right\} \quad (26)$$

B—The Torque Equation

The magnetic torque is, by (18),

$$\begin{aligned} T &= -(i_ai_1 + i_bi_2)\,M\sin\theta + \\ &\quad (i_bi_1 - i_ai_2)\,M\cos\theta \\ &= -i_aM(i_1\sin\theta + i_2\cos\theta) + \\ &\quad i_bM(i_1\cos\theta - i_2\sin\theta) \end{aligned}$$

Or, by (23) and (24),

$$T = -I_\alpha Mi_\beta + I_\beta Mi_\alpha \quad (27)$$

C—Solution of the Machine Equations for Constant Speed

Since the machine equations (26) are similar to (16), their solutions are given by the solutions of (16). Thus, the two-phase currents are the same as those of equations 21 with the exception that the zero subscripts referring to apparent three-phase quantities are omitted here. There are no zero-phase-sequence quantities for the two-phase case.

Part III. Single-Phase Machine Equations

A—Machine Equations

The single-phase relations are readily obtained from the two-phase equations by setting the stator current in phase b equal to zero. For simplicity of analysis a two-phase, balanced rotor is assumed. Thus, from (22), (23), (24),

$$\left.\begin{aligned} i_1 &= i_\alpha \cos\theta + i_\beta \sin\theta \\ i_2 &= -i_\alpha \sin\theta + i_\beta \cos\theta \end{aligned}\right\} \quad (28)$$

$$i_a = I_\alpha \quad (29)$$

$$\left.\begin{aligned} i_\alpha &= i_1 \cos\theta - i_2 \sin\theta \\ i_\beta &= i_1 \sin\theta + i_2 \cos\theta \end{aligned}\right\} \quad (30)$$

From (25) the flux linkage relations are

$$\left.\begin{aligned} \psi_\alpha &= li_\alpha + MI_\alpha \\ \psi_\beta &= li_\beta \\ \Psi_\alpha &= LI_\alpha + Mi_\alpha \end{aligned}\right\} \quad (31)$$

The voltage and torque equations follow from (26) and (27). They are

$$\begin{aligned} (R+Lp)I_\alpha + MpI_\alpha + 0 &= E_\alpha \\ MpI_\alpha + (r+lp)i_\alpha + l[p\theta]i_\beta &= e_\alpha \\ -M[p\theta]I_\alpha - l[p\theta]i_\alpha + (r+lp)i_\beta &= e_\beta \end{aligned} \quad (32)$$

$$T = -I_\alpha Mi_\beta \quad (33)$$

B—Solution of the Machine Equations for Constant Speed

If the rotor be short-circuited symmetrically at its terminals, the short-circuiting impedance in each phase may be included with the rotor resistance and the voltages e_α, e_β put equal to zero in (32). Assuming the rotor circuits to be closed through equal resistances which are included in r, the solutions of (32) are

$$\left.\begin{aligned} I_\alpha &= [(r+lp)^2 + n^2l^2]\,\frac{E_\alpha}{\Delta} \\ i_\alpha &= -\frac{M}{\Delta}[rp + (p^2+n^2)l]E_\alpha \\ i_\beta &= \frac{Mrn}{\Delta}E_\alpha \end{aligned}\right\} \quad (34)$$

where

$$\begin{aligned} \Delta = (R+Lp)\,[(r+lp)^2 + n^2l^2] - \\ M^2p[rp + (p^2+n^2)l] \end{aligned}$$

or in terms of the single-phase positive- and negative-phase-sequence impedances,

$$\Delta = \frac{Z_1+Z_2}{2}\,[(r+lp)^2 + n^2l^2]$$

and (34) may be written

$$\left.\begin{aligned} I_\alpha &= \frac{2E_\alpha}{Z_1 + Z_2} \\ i_\alpha &= -\frac{2M}{Z_1 + Z_2}\,\frac{rp + (p^2 + n^2)l}{(r + lp)^2 + n^2l^2}E_\alpha \\ i_\beta &= \frac{2M}{Z_1 + Z_2}\,\frac{rn}{(r + lp)^2 + n^2l^2}E_\alpha \end{aligned}\right\} \quad (35)$$

C—Steady-State Relations

Substituting $j\omega$ for p in (35), the vector expressions for the steady-state currents become

$$\left.\begin{aligned} I_\alpha &= \frac{2\mathbf{E}_\alpha}{\mathbf{Z}_1 + \mathbf{Z}_2} \\ i_\alpha &= -jX_M\frac{\mathbf{E}_\alpha}{\mathbf{Z}_1 + \mathbf{Z}_2} \times \\ &\left[\frac{1}{\frac{r}{s} + jX_r} + \frac{1}{\frac{r}{2 - s} + jX_r}\right] \\ i_\beta &= -X_M\frac{\mathbf{E}_\alpha}{\mathbf{Z}_1 + \mathbf{Z}_2} \times \\ &\left[\frac{1}{\frac{r}{s} + jX_r} - \frac{1}{\frac{r}{2 - s} + jX_r}\right] \end{aligned}\right\} \quad (36)$$

D—Single-Phase Motor Running With Small, Steady Pulsations in Speed

When a single-phase induction motor is driving a load such as a compressor which impresses a pulsating torque upon the shaft of the machine, the resulting pulsations in speed cause pulsations in the line current and possibly line voltage "flicker." When the pulsation frequency is low the rotor inertia may not be of sufficient magnitude adequately to suppress the speed and current variations, and the voltage pulsations at the motor terminals are apt to be a source of considerable annoyance, especially if there are lighting loads on the same circuit.

In the following analysis the rotor inertia is assumed to be large enough that the speed is not appreciably affected by the double-line-frequency components of the single-phase torque, and the rotor speed is assumed to consist only of a constant component plus a small deviation which varies sinusoidally at pulsation frequency. Then, the currents and load torque are broken up into two components, one the steady-state, steady-speed value and the other a small variation due to the speed pulsation. Hence

$$\left.\begin{aligned} n &= N + \Delta n \\ I_\alpha &= I_\alpha^0 + \Delta I_\alpha \\ i_\alpha &= i_\alpha^0 + \Delta i_\alpha \\ i_\beta &= i_\beta^0 + \Delta i_\beta \\ T_L &= T_L^0 + \Delta T_L \end{aligned}\right\} \quad (37)$$

$$\left.\begin{aligned} &(R + Lp)\Delta I_\alpha + Mp\,\text{[illegible]} + 0 = 0 \\ &Mp\Delta I_\alpha + (r + lp)\Delta i_\alpha + lN\Delta i_\beta = -li_\beta^0\Delta n \\ &-MN\Delta I_\alpha - lN\Delta i_\alpha + (r + lp)\Delta i_\beta = (MI_\alpha^0 + li_\alpha^0)\Delta n \\ &i_\beta^0M\Delta I_\alpha + I_\alpha^0M\Delta i_\beta + Jp(\Delta n) = \Delta T_L \end{aligned}\right\} \quad (38)$$

Assuming that the motor is supplied from a stiff voltage source through a line impedance which is included with the stator impedance, the stator voltage E_α is constant. Then, when (37) are substituted into the machine equations (32), (33) with rotor short-circuited, and the products of small pulsation terms are neglected, the equations (38) in the pulsation components result.*

In (38) the quantities with the superscript 0 are the steady-state values and are given by (36). If the currents are in amperes and the inductances in henrys, the electrical torque is in watt-seconds and the rotor intertia J must be in watt-seconds²/electrical radian; or, for a motor with q pairs of poles, $J = J'/(10^7 q)$, where J' is the inertia in gram-centimeters².

Because of the nature of the pulsation-phenomenon, the following steady-pulsation solutions of (38) are assumed:

$$\left.\begin{aligned} \Delta I_\alpha &= Re\,[\Delta I_1\epsilon^{j\omega_1 t} + \Delta I_2\epsilon^{j\omega_2 t}] \\ \Delta i_\alpha &= Re\,[\Delta i_{\alpha_1}\epsilon^{j\omega_1 t} + \Delta i_{\alpha_2}\epsilon^{j\omega_2 t}] \\ \Delta i_\beta &= Re\,[\Delta i_{\beta_1}\epsilon^{j\omega_1 t} + \Delta i_{\beta_2}\epsilon^{j\omega_2 t}] \end{aligned}\right\} \quad (39)$$

Also, by the assumption of sinusoidal speed and torque variation,

$$\left.\begin{aligned} \Delta n &= \Delta N Re\,[\epsilon^{j\omega_p t}] \\ \Delta T_L &= Re\,[\Delta T\epsilon^{j\omega_p t}] \end{aligned}\right\} \quad (40)$$

and, by (36) the steady-state components of the currents are of the form

$$\left.\begin{aligned} I_\alpha^0 &= Re\,[I_0\epsilon^{j\omega t}] \\ i_\alpha^0 &= Re\,[i_{\alpha_0}\epsilon^{j\omega t}] \\ i_\beta^0 &= Re\,[i_{\beta_0}\epsilon^{j\omega t}] \end{aligned}\right\} \quad (41)$$

Then, from (39), (40), (41)

$$\left.\begin{aligned} i_\beta^0\Delta n &= \frac{\Delta N}{2}Re\,[i_{\beta_0}\epsilon^{j\omega_1 t} + i_{\beta_0}\epsilon^{j\omega_2 t}] \\ i_\alpha^0\Delta n &= \frac{\Delta N}{2}Re\,[i_{\alpha_0}\epsilon^{j\omega_1 t} + i_{\alpha_0}\epsilon^{j\omega_2 t}] \\ I_\alpha^0\Delta n &= \frac{\Delta N}{2}Re\,[I_0\epsilon^{j\omega_1 t} + I_0\epsilon^{j\omega_2 t}] \\ I_\alpha^0\Delta i_\beta &= \frac{1}{2}Re\,[\bar{I}_0\Delta i_{\beta_1}\epsilon^{j\omega_p t} + \\ &\qquad I_0\Delta\bar{i}_{\beta_2}\epsilon^{j\omega_p t}] \\ i_{\beta_0}\Delta I_\alpha &= \frac{1}{2}Re\,[\bar{i}_{\beta_0}\Delta I_1\epsilon^{j\omega_p t} + \\ &\qquad i_{\beta_0}\Delta\bar{I}_2\epsilon^{j\omega_p t}] \\ p(\Delta n) &= \Delta N\,Re\,[j\omega_p\epsilon^{j\omega_p t}] \end{aligned}\right\} \quad (42)$$

Note that, in the evaluation of the products $I_\alpha^0\Delta i_\beta$ and $i_\beta^0\Delta I_\alpha$, the terms of frequencies $2\omega + \omega_p$ and $2\omega - \omega_p$ have been omitted because of the assumption that the torque components of these high frequencies are absorbed by the rotor inertia with changes of speed which are negligible compared with the variations at pulsation frequency.

When (39), (40), (41), (42) are substituted into (38) and coefficients of like exponentials equated to zero, the seven equations (43) result.

The first six equations of (43) are linear algebraic equations and may readily be solved for the currents ΔI_1, ΔI_2, Δi_{β_1}, Δi_{β_2} in terms of the speed variation ΔN. Then, these values may be substituted in the torque equation and the ratio $\Delta N/\Delta T$ calculated. Thus for a given amplitude of torque pulsations, the amplitude of the speed and current pulsations may be determined from (43).

Briefly, the procedure for determining the magnitudes of the speed and line current pulsations due to a pulsating torque

$$\left.\begin{aligned} &(R + j\omega_1L)\Delta I_1 + j\omega_1M\Delta i_{\alpha 1} + 0 = 0 \\ &j\omega_1M\Delta I_1 + (r + j\omega_1l)\Delta i_{\alpha'} + lN\Delta i_{\beta 1} = -li_{\beta_0}\frac{\Delta N}{2} \\ &-MN\Delta I_1 - lN\Delta i_{\alpha_1} + (r + j\omega_1l)\Delta i_{\beta_1} = (MI_0 + li_{\alpha_0})\frac{\Delta N}{2} \\ &(R + j\omega_2L)\Delta I_2 + j\omega_2M\Delta i_{\alpha_2} + 0 = 0 \\ &j\omega_2M\Delta I_2 + (r + j\omega_2l)\Delta i_{\alpha_2} + lN\Delta i_{\beta_2} = -li_{\beta_0}\frac{\Delta N}{2} \\ &-MN\Delta I_2 - lN\Delta i_{\alpha_2} + (r + j\omega_2l)\Delta i_{\beta_2} = (MI_0 + li_{\alpha_0})\frac{\Delta N}{2} \\ &\frac{M}{2}[\bar{i}_{\beta_0}\Delta I_1 + i_{\beta_0}\Delta\bar{I}_2 + \bar{I}_0\Delta i_{\beta_1} + I_0\Delta\bar{i}_{\beta_2}] + j\omega_pJ\Delta N = \Delta T \end{aligned}\right\} \quad (43)$$

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* After substitution in (32), (33), the steady-state equations must be subtracted from the resulting relations to obtain (38). The torque equation is the result of balancing the induction-motor, load and inertia torques on the rotor.

of given amplitude may be outlined as follows:

1. Determine the average speed N and calculate the steady-state vector currents I_0, $i_{\alpha 0}$, $i_{\beta 0}$ by means of equations 36.

2. Substitute these values into (43) and calculate ΔI_1, ΔI_2, $\Delta i_{\beta 1}$, $\Delta i_{\beta 2}$ in terms of ΔN. Thus,

$$\Delta I = \frac{j\omega Ml[i_{\beta 0}(r + j\omega l) + (MI_0 + li_{\alpha 0})N]}{\Delta} \frac{\Delta n}{2}$$

$$\Delta i_\beta = \frac{-Nli_{\beta 0}[(R + j\omega L)l - j\omega M^2] + (MI_0 + li_{\alpha 0})[(R + j\omega L)(r + j\omega l) + \omega^2 M^2]}{\Delta} \frac{\Delta N}{2}$$

where

$$\begin{aligned}\Delta = {} & (R + j\omega L)[(r + j\omega l)^2 + N^2 l^2] - j\omega M^2[j\omega r + (N^2 - \omega^2)l] \\ = {} & \frac{Z_1(j\omega) + Z_2(j\omega)}{2} [(r + j\omega l)^2 + N^2 l^2]\end{aligned}$$

3. Calculate the speed in terms of the torque by means of the last equation of (43). Thus

$$\Delta N = \kappa \Delta T$$

4. Determine ΔI_1 and ΔI_2 by substituting $k\Delta T$ for ΔN in the expressions already calculated in (2).

5. The line current pulsation is given by (39). Thus

$$\Delta I_\alpha = Re[\Delta I_1 \epsilon^{j\omega_1 t} + \Delta I_2 \epsilon^{j\omega_2 t}]$$

Conclusion

A method of analysis has been developed, which, based upon a relatively few simplifying assumptions, is applicable to the solution of a variety of transient and steady-state problems. The treatment has concentrated on the method and derivation of the general equations rather than the application of the equations to the solution of particular problems. It is believed that the analysis is straightforward and that, having grasped the fundamental principles involved in the derivation of the general equations, it is a relatively simple matter to apply them to the solution of special problems. Accordingly only a minimum number of actual solutions in literal form have been included. These indicate the general procedure to be followed and illustrate the relative simplicity with which results can be obtained in special cases.

The results should be especially useful in the practical solution of problems due to the excessive current pulsations and light flicker which may result when induction motors are used for driving such loads as refrigerator compressors. Also, the operational solutions which are given are in a form which is convenient for the calculation of short-circuit currents and torques. Furthermore, should it be desired to determine the performance of an induction machine with non-steady variations in speed as in the case of rotor acceleration, the general equations are in a convenient form for making the necessary calculations.

Nomenclature

L = self-inductance of one stator phase

l = self-inductance of one rotor phase

M = maximum value of mutual inductance between one stator phase and one rotor phase

L_0 = $(L - M_S)$ = apparent three-phase stator self-inductance

l_0 = $(l - M_R)$ = apparent three-phase rotor self-inductance

M_0 = $\frac{3}{2}M$ = apparent three-phase mutual inductance

M_S = mutual inductance between stator phases

M_R = mutual inductance between rotor phases

R = resistance of one stator phase

r = resistance of one rotor phase

$e_{a,b,c}$ = stator phase terminal voltages

$e_{1,2,3}$ = rotor phase terminal voltages

$i_{a,b,c}$ = stator phase currents

$i_{1,2,3}$ = rotor phase currents

$\psi_{a,b,c}$ = stator phase flux linkages

$\psi_{1,2,3}$ = rotor phase flux linkages

n = rotor speed in electrical radians per second

θ = $\int_0^t ndt$ = electrical angle between phases a and 1 at time t

ω = frequency of applied voltage when it is sinusoidal

s = $\frac{\omega - n}{\omega}$ = slip of the rotor

p = $\frac{d}{dt}$, time derivative operator

ωL_0 = X_s = stator apparent reactance per phase at frequency of applied voltage

ωl_0 = X_r = rotor apparent reactance per phase at frequency of applied voltage

ωM_0 = X_M = apparent mutual reactance per phase at frequency of applied voltage

Ψ_α = stator flux linkages in positive α axis

Ψ_β = stator flux linkages in positive β axis

Ψ_0 = stator flux linkages zero phase-sequence

I_α = stator current in positive α axis

I_β = stator current in positive β axis

I_0 = stator current zero phase-sequence

E_α = stator terminal voltage in positive α axis

E_β = stator terminal voltage in positive β axis

E_0 = stator terminal voltage zero phase-sequence

ψ_α = rotor flux linkages in positive α axis

ψ_β = rotor flux linkages in positive β axis

ψ_0 = rotor flux linkages zero phase-sequence

i_α = rotor current in positive α axis

i_β = rotor current in positive β axis

i_0 = rotor current zero phase-sequence

e_α = rotor terminal voltage in positive α axis

e_β = rotor terminal voltage in positive β axis

e_0 = rotor terminal voltage zero phase-sequence

*Z_1 = $(R + pL_0) - \frac{p(p - jn)M_0^2}{r + (p - jn)l_0}$ transient positive phase-sequence impedance

*Z_2 = $(R + pL_0) - \frac{p(p + jn)M_0^2}{r + (p + jn)l_0}$ transient negative phase-sequence impedance

Z_0 = $R + (L + 2M_S)p$ transient zero phase-sequence impedance

V_1 = $\frac{E_\alpha + jE_\beta}{2}$ positive phase-sequence stator voltage

V_2 = $\frac{E_\alpha - jE_\beta}{2}$ negative phase-sequence stator voltage

V_0 = E_0 zero phase-sequence stator voltage

T = induction motor torque

$T_{av.}$ = average value of single-phase induction-motor torque

T_L = mechanical torque on motor shaft plus windage and friction torque in watt-seconds

J = $J'/(10^7 q)$ = rotor inertia in watt-seconds^2/electrical radian

J' = rotor inertia in gram-centimeters^2

q = number of pairs of poles

ΔN = magnitude of speed pulsation due to torque ΔT_L

ΔT_L = magnitude of pulsation component of load torque

ω_p = frequency of pulsations

ω_1 = $\omega + \omega_p$

ω_2 = $\omega - \omega_p$

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* For the two-phase and single-phase motors, the inductance coefficients in these expressions are substituted by actual quantities; L, M, and l.

Discussion

C. Concordia (General Electric Company, Schenectady, N. Y.): Mr. Stanley has expressed the performance of an induction machine in terms of quantities referred to the stator. It may be pointed out that his form of the equations is particularly well adapted to the solution of problems involving unbalanced stator circuits. In general it may be stated that one can have unbalanced circuits on either the stator or the rotor of an electric machine, but not on both, and still arrive at differential equations which have constant coefficients for the case of constant speed. The variables should be referred to the side of the machine having the unbalanced circuits.

It may be helpful to those familiar with the theory of synchronous machines to show how equations similar to Mr. Stanley's, but closer in form to the conventional synchronous machine equations,[1] may be derived, and to show another application of these equations.

ANALYSIS

Consider a machine with symmetric rotor and possibly asymmetric stator. The stator equations are

$$\left.\begin{aligned} e_\alpha &= p\psi_\alpha - ri_\alpha - Z_\alpha(p)i_\alpha \\ e_\beta &= p\psi_\beta - ri_\beta - Z_\beta(p)i_\beta \\ e_0 &= p\psi_0 - ri_0 - Z_0(p)i_0 \end{aligned}\right\} \quad (1)$$

where the α, β, 0 quantities[2] are referred to axes fixed in the stator, and are related to the familiar d, q, 0 quantities[1] by the equations

$$\left.\begin{aligned} i_\alpha &= i_d \cos\theta - i_q \sin\theta \\ i_\beta &= i_d \sin\theta + i_q \cos\theta \end{aligned}\right\} \quad (2)$$

or

$$\begin{aligned} i_d &= i_\alpha \cos\theta + i_\beta \sin\theta \\ i_q &= -i_\alpha \sin\theta + i_\beta \cos\theta \end{aligned}$$

with similar relations for ψ and e. In terms of phase quantities, there are the relations

$$i_\alpha = \frac{1}{3}(2i_a - i_b - i_c)$$

$$i_\beta = \frac{1}{\sqrt{3}}(i_b - i_c)$$

$$i_0 = \frac{1}{3}(i_a + i_b + i_c)$$

so i_α, i_β may be considered as the armature currents of an equivalent two-phase machine.

$Z_\alpha(p)$, $Z_\beta(p)$, $Z_0(p)$ are the impedances[3] to α, β, 0 currents, respectively, of the circuits external to the machine.

There are also the equations[1]

$$\left.\begin{aligned} \psi_d &= G(p)E_{fd} - x_d(p)i_d \\ \psi_q &= -x_q(p)i_q \\ \psi_0 &= -x_0 i_0 \end{aligned}\right\} \quad (3)$$

the first two of which must be converted to α, β axes. If we let[3]

$$\begin{aligned} x_q(p) &= x_d(p) \\ &= x_d - \frac{p x_{afd}^2}{p x_{ffd} + R_{fd}} = \frac{x'T_0p + x}{T_0p + 1} \end{aligned} \quad (4)$$

$$\left.\begin{aligned} (T_0p + 1)\psi_\alpha + T_0\psi_\beta p\theta &= -(x'T_0p + x)i_\alpha - x'T_0 i_\beta p\theta + \frac{x_{afd}}{R_{fd}} E_{fd} \cos\theta \\ (T_0p + 1)\psi_\beta - T_0\psi_\alpha p\theta &= -(x'T_0p + x)i_\beta + x'T_0 i_\alpha p\theta + \frac{x_{afd}}{R_{fd}} E_{fd} \sin\theta \end{aligned}\right\} \quad (5)$$

and

$$G(p) = \frac{x_{afd}}{p x_{ffd} + R_{fd}} = \frac{1}{T_0p + 1}\,\frac{x_{afd}}{R_{fd}}$$

we find equation 5 from (2), (3), and (4).

For constant speed, $p\theta = \omega$, equations 5 may be solved for ψ_α, ψ_β, and these in turn substituted in equations 1. We have then:

$$\begin{aligned} e_\alpha = &-\left\{ \frac{[p(T_0p+1)(x'T_0p+x)+p(T_0\omega)^2x']}{(T_0p + 1)^2 + (T_0\omega)^2} + r + Z_\alpha(p) \right\} i_\alpha - \\ &\left[\frac{p(T_0p + 1)T_0\omega x' - T_0\omega p(x'T_0p + X)}{(T_0p + 1)^2 + (T_0\omega)^2}\right] i_\beta + \\ &\frac{x_{afd}}{R_{fd}} E_{fd} \left[\frac{p(T_0p + 1)\cos\theta - T_0\omega\, p \sin\theta}{(T_0p + 1)^2 + (T_0\omega)^2}\right] \end{aligned} \quad (6a)$$

$$\begin{aligned} e_\beta = &-\left\{ \frac{[p(T_0p + 1)(x'T_0p + x)+(T_0\omega)^2x'p]}{(T_0p + 1)^2 + (T_0\omega)^2} + r + Z_\beta(p) \right\} i_\beta + \\ &\left[\frac{T_0\omega x'p(T_0p + 1) - T_0\omega p(x'T_0p + x)}{(T_0p + 1)^2 + (T_0\omega)^2}\right] i_\alpha + \\ &\frac{x_{afd}}{R_{fd}} E_{fd} \left[\frac{T_0\omega\, p\cos\theta + p(T_0p + 1)\sin\theta}{(T_0p + 1)^2 + (T_0\omega)^2}\right] \end{aligned} \quad (6b)$$

SINGLE PHASE OPERATION OF INDUCTION MOTOR (LINE-TO-LINE)

Equations 6 may be used to determine the performance of a machine having unbalanced stator circuits. We consider the case of an induction motor with line a open and find the regions of self-excitation.[4] Since line a is open, there are

$$Z_\alpha(p) = \infty, \; i_\alpha = 0 \quad (7a)$$

Also

$$E_{fd} = 0 \quad (7b)$$

If there is a capacitor C and shunt resistor R in each line,

$$Z_\beta(p) = Z_b(p) = Z_c(p) = \frac{x_c}{p + \alpha} \quad (8)$$

where

$\alpha = (CR)^{-1}$, $x_c = C^{-1}$

Equation 6*b* then reduces to:

$$-\frac{e_\beta}{i_\beta} = \left\{ \frac{\begin{gathered}[p(T_0p + 1)(x'T_0p + x) + (T_0\omega)^2 \times \\ x'p](p + \alpha) + [r(p + \alpha) + x_c] \times \\ [(T_0p + 1)^2 + (T_0\omega)^2]\end{gathered}}{[(T_0p + 1)^2 + (T_0\omega)^2](p + \alpha)} \right\} \quad (9)$$

From (9), the operational denominator D_β of i_β is

$$\begin{aligned} D_\beta = \; & p^4 x'T_0^2 + \\ & p^3[(x+x')T_0 + x'T_0^2\alpha + T_0^2 r] + \\ & p^2[x + x'T_0^2\omega^2 + T_0^2(x_c + r\alpha) + \\ & (x + x')T_0\alpha + 2T_0 r] + \\ & p[2T_0(x_c + r\alpha) + \\ & (x + x'T_0^2\omega^2)\alpha + (1 + T_0^2\omega^2)r] + \\ & (1 + T_0^2\omega^2)(x_c + r\alpha) \end{aligned} \quad (10)$$

If we let $r = \alpha = 0$ and apply Routh's stability criterion we find that the electrical system is unstable if

$$x_c < \left(\omega^2 - \frac{1}{T_0^2}\right)\left(\frac{x + x'}{2}\right) \quad (11)$$

NUMERICAL RESULTS AND CONCLUSIONS

There is no evident simplification permitting the determination of an approximate boundary of the stable region in the range (11), so calculations have been made for the numerical cases

$x = 3.24$
$x' = 0.40$
$T_0 = 171.4$ and 17.14
$\omega^2 = 0.95$ and 0.25

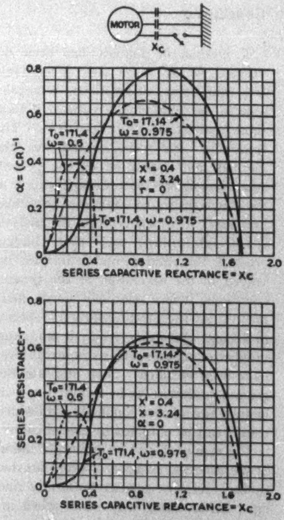

Figure 1. Negative damping of induction motors with series capacitors—single-phase operation

Series and shunt resistances required for stable operation which were previously considered in reference 4. The boundary curves obtained are shown in figure 1, and indicate that less series resistance or shunt conductance is required for stability in the single phase case than is required in the three-phase case for the same rotor speed, since the single-phase curves lie entirely within the unstable regions found[4] for three-phase operation.

SINGLE PHASE OPERATION (LINE-TO-NEUTRAL)

Another possible stator unbalance is the case of a four-wire or grounded-neutral system with lines b and c open. Then

$$\left.\begin{aligned} & i_\alpha = \frac{2}{3} i_a = 2i_0;\ i_\beta = 0 \\ & Z_\alpha(p) = Z_0(p) = Z_a(p) = \frac{x_c}{p+\alpha} \\ & e_a = e_\alpha + e_0 \end{aligned}\right\} \qquad (12)$$

and, in general $r_0 \neq r$.

From (12), (6), (3) and (1)

$$-\frac{e_a}{i_a} = \frac{\left\{\begin{aligned} & 2[p(T_0p+1)(x'T_0p+x)+ \\ & (T_0\omega)^2x'p](p+\alpha)+ \\ & [(px_0+2r+r_0)(p+\alpha)+3x_c]\times \\ & [(T_0p+1)^2+(T_0\omega)^2] \end{aligned}\right\}}{3(p+\alpha)[(T_0p+1)^2+(T_0\omega)^2]} \qquad (13)$$

and the operational denominator D_a of i_a becomes:

$$\begin{aligned} D_a = {} & p^4T^2(2x'+x_0)+ \\ & p^3[2(x+x'+x_0)T_0+ \\ & T_0^2(2x'+x_0)\alpha+T_0^2(2r+r_0)]+ \\ & p^2[2x+x_0+T_0^2\omega^2(2x'+x_0)+ \\ & T_0^2(3x_c+(2r+r_0)\alpha)+ \\ & 2T_0(x+x'+x_0)\alpha+2T_0(2r+r_0)]+ \\ & p[2T_0(3x_c+(2r+r_0)\alpha)+ \\ & (2x+x_0+T_0^2\omega^2(2x'+x_0))\alpha+ \\ & (1+T_0^2\omega^2)(2r+r_0)]+ \\ & (1+T_0^2\omega^2)(3x_c+(2r+r_0)\alpha) \end{aligned} \qquad (14)$$

For $(2r + r_0) = \alpha = 0$ the electrical system is unstable if

$$x_c < \left(\omega^2 - \frac{1}{T_0^2}\right)\left(\frac{x + x' + x_0}{3}\right) \qquad (15)$$

All of the results obtained here may be adapted to a single-phase machine by recalling that the self-inductance L of the armature winding of a single-phase machine in terms of three-phase inductance is:

$$L = \frac{1}{3}(2x + x_0)$$

if the single-phase winding is regarded as one phase of a three-phase machine, or it is:

$$L = 2x$$

if the single-phase winding is regarded as two phases of a three-phase machine in series. The steady-state reactance x_s of the single-phase machine is given by the corresponding expressions

$$x_s = \frac{1}{3}(x + x' + x_0) = x + x'$$

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2. OVERVOLTAGES CAUSED BY UNBALANCED SHORT CIRCUITS (EFFECT OF AMORTISSEUR WINDINGS), **Edith Clarke, C. N. Weygandt,** and **C. Concordia.** AIEE Technical Paper 37-74, December 1937.

3. THE OPERATIONAL IMPEDANCES OF A SYNCHRONOUS MACHINE, **M. L. Waring, S. B. Crary.** *General Electric Review*, volume 35, November 1932, page 578.

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W. V. Lyon (Massachusetts Institute of Technology, Cambridge): Mr. Stanley recommends the method Park has developed and applied to salient-pole synchronous machines. In this method currents, of their magnetomotive forces, are resolved into components acting along quadrature axes—the so-called direct-axis and quadrature-axis. I suggest it be referred to as the "method of quadrature components." I believe the method of symmetrical components is equally useful in all the problems Mr. Stanley proposes. The method of symmetrical components is just as forceful under transient conditions as under steady-state conditions. There is very little difference in the formulation of the required equations by either method. Except for nomenclature Mr. Stanley's general induction-motor equations (16) have the same appearance as the corresponding equations derived by applying the method of symmetrical components. Any advantage possessed by the method of quadrature components is most pronounced when the magnetic circuit and the windings on one side of the air gap are symmetrical about two quadrature axes, as in the salient-pole synchronous machine. This is particularly true when damper windings are being considered. Nevertheless the method of symmetrical components has been successfully applied to this case by Ku and others. Any advantage possessed by the method of symmetrical components is most pronounced when the air gap is uniform and the windings on both sides of the air gap are symmetrical.

About seven years ago we obtained solutions for the current and torque in a symmetrical induction motor when the speed was retarded linearly from synchronism and when the speed oscillated harmonically about synchronous speed. We used the method of symmetrical components and made the same assumptions Mr. Stanley lists under 1 to 4, and also assumed the stator resistance was negligibly small. When the speed oscillates about synchronism we found the relation between torque and instantaneous slip is an oval having the steady torque-slip line for its major axis.

H. C. Stanley: The author is grateful to Mr. Concordia and Professor Lyon for their discussions.

Mr. Concordia has pointed out an interesting example illustrating a practical case where unbalance in the stator circuits of an induction machine requires an analytical approach more powerful than that offered by the conventional equivalent circuit method.

Professor Lyon is quite right in his claim that the method of symmetrical components is equally as useful mathematically as the "method of quadrature components." As he points out, however, each set of transformations has its own advantages which are amplified or suppressed, depending upon the nature of the physical problem to which they are applied. It behooves the engineer to pick the particular transformations which are most advantageous in the solution of his special problem.

It might be considered as somewhat of an advantage by some that the quadrature-components transformations are real as compared with the complex transformations of the symmetrical components method.

High-Voltage Gaseous and Fluorescent Tubes for Advertising and Architectural Lighting Effects

By J. A. McDERMOTT
ASSOCIATE AIEE

Synopsis: Rapid strides have been made in the field of high-voltage electrical-discharge tubes. The most recent development has been the use of fluorescent phenomena. Fluorescent materials were first introduced as an integral part of the glass. The New York World's Fair has made a number of installations of Claude Zeon fluorescent tubes in which the fluorescent materials are placed as a coating on the inside walls of the tubing. Remarkable efficiencies and a full range of pastel shades make high-voltage fluorescent tubing a valuable tool for the use of the engineer, architectural designer, and advertiser.

ALTHOUGH gaseous tubing had been used previously in the United States, the introduction of high-voltage neon and mercury tubes by the Claude Companies in 1924 inaugurated a new industry of such tremendous proportions that the whole aspect of outdoor electrical advertising was changed. The reasons for the wide acceptance of gaseous tubing were both functional and economic. By its use advertising features could be outlined directly, thus offering better definition than the general floodlighting of those features. In addition, the life of the tubing was relatively long, the installation requirements were simple, and attractively colored light could be produced more efficiently than from incandescent sources. Rapidly the gaseous tube assumed its role as a major feature of the nation's "white ways."

Filter Glasses and Basic Colors

The light emitted due to an electrical discharge in a gas gives a line spectrum, and by the use of filter glass certain of the lines may be absorbed and others transmitted.

Neon gives a series of lines, including yellow and red, the net physiological effect of which is the characteristic orange red. If the proper red-colored tubing is employed, all but the deep red light is absorbed and a ruby tube is the result. Similarly, mercury vapor, when activated, gives blue and green lines, which result in a whitish blue light. However, by using "Novial glass," which absorbs the blue lines, a green tube may be obtained. If the proper blue glass is employed, a so-called "midnight blue" tube will result.

The use of helium in tubes was not found practical until 1933, when the Claude laboratories introduced special electrodes to permit its use. Helium gives a yellowish white light and when a yellow filter tubing is employed a gold effect can be obtained.

Elementary Use of Fluorescent Tubes

The first commercial fluorescent tubes employed uranium glass. By its use with a mercury discharge a green fluorescence was obtained and this combined with the filtering effect of the tubing gave a blue green light at an efficiency somewhat higher than that obtained from Novila glass. However, mercury tubes continued to play only a minor role, due to the comparative brilliance of neon tubes, and attempts were made to improve the efficiency of mercury tubes. It was well known that at the gas pressures used a large proportion of the energy emitted by the discharge was in the form of ultraviolet light, that was not utilized.

In 1935 "Lumophor" tubing was introduced to the United States by the Claude companies. In this tubing a fluorescent material was incorporated as an integral part of the glass walls of the tubing. A wide variety of colors was made available, including a yellow green, blue, gold, and a few shades of white. This development was important because it increased the efficiency of certain colors and introduced others heretofore not available from any type of gaseous tubes. Green tubes were produced which were 500 per cent more efficient than nonfluorescent tubes. These new tubes found wide use in the sign industry and the white tubes were used to some extent in interior decoration.

A typical example of the use of this type of white fluorescent tubing is found in the illumination of the glass dance floor (figure 1).

In this design the effect of a marine compass was obtained by coating the underside of the glass slabs with colored translucent paint.

Although Lumophor tubing represented an advance in the application of high-voltage gaseous tube phenomena, it did

Figure 1. Hotel Warwick, New York, N. Y.

Paper number 38-94, recommended by the AIEE committee on production and application of light and presented at the AIEE summer convention, Washington, D. C., June 20–24, 1938. Manuscript submitted May 24, 1938; made available for preprinting May 27, 1938.

J. A. McDermott is electrical engineer for Claude Neon Lights, Inc., New York, N. Y.

not utilize all the ultraviolet light that was available. Too much of the ultraviolet energy was lost in reflection and absorption by the glass walls of the tubing.

Coated Fluorescent Tubes

Methods were developed whereby fluorescent materials were made to adhere to the interior walls of the tubing. With this construction, the ultraviolet light fell directly upon the material to be activated and the reflection of the ultraviolet light by the interior surface of the tubing and absorption of the ultraviolet by the glass had no effect upon the efficiency of the tube.

The efficiencies obtained by this method were revolutionary, and, in the instance of green, reached 60 lumens per watt. In addition, many new colors were made available. Future research promised the development of high-efficiency white shades and a full range of pastels. However, a great amount of groundwork had to be done to reduce the laboratory samples to a practical basis. Fluorescent materials were developed that best responded to the ultraviolet radiation emitted by low-pressure tubes.

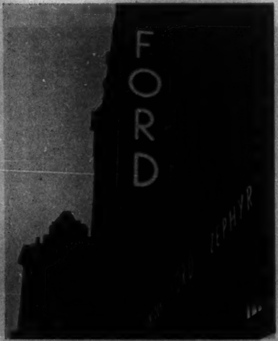
General practice in the sign industry had established the use of tubes having a life of from 3,000 to 10,000 hours and it was essential that during the major part of this period the luminescence be practically constant. The fluorescent tubes would have to possess similar characteristics so that a section replaced due to breakage would not stand in too sharp a contrast to adjacent sections of an installation.

Figure 2. Store display, New York, N. Y.

Developments in the technique of coating, evacuating, and processing the tubes, together with improvements in the phosphors, have resulted in high-voltage fluorescent tubes that meet all the rigid requirements that have been set. These tubes, known as Zeon tubes, have only recently been introduced to the American market.

Fluorescence has also been obtained where neon gas is used in the tubes without any mercury. There is a certain amount of ultraviolet emitted by low-pressure discharges in neon. A golden-colored tube is produced by using a material that gives a green fluorescence. The addition of the green light to the orange red of the neon gives the effect of a golden tube. By the use of other phosphors various neon fluorescent effects are obtainable.

Figure 3. Ford fluorescent sign, New York, N. Y.



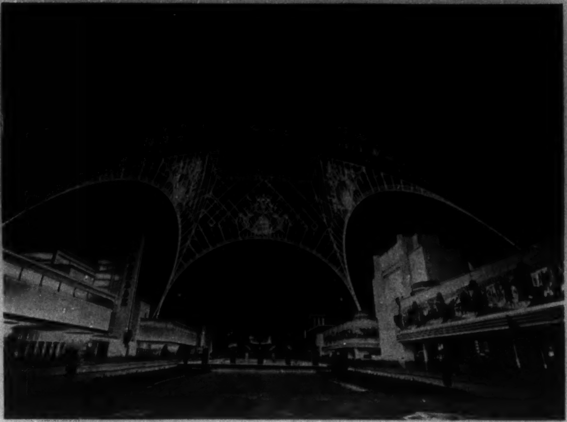


Figure 4. Outlining of Eiffel Tower base, Paris Fair 1937

Characteristics of Zeon Tubes

The efficiencies of high-voltage gaseous tubes are to some extent dependent upon the conditions of installation, lengths of tubes, diameters, operating currents, and, in the case of Zeon tubes, the point on the saturation curve on which the fluorescent coating is being activated. In general, the longer lengths of tubing will have the highest efficiencies due to the fact that a large part of the power losses occurs at the electrodes.

The following average efficiencies may be expected:

Blue	20–25 lumens per watt—activation mercury
Green	45–60 lumens per watt—activation mercury
Orchid	20–25 lumens per watt—activation mercury
Warm White	30–35 lumens per watt—activation mercury
Cold White	30–35 lumens per watt—activation mercury
Yellow	18–20 lumens per watt—activation mercury
Pink	15–20 lumens per watt—activation mercury
Gold	12–16 lumens per watt—activation neon
Salmon	12–16 lumens per watt—activation neon
Rose	12–16 lumens per watt—activation neon

Comparison

A comparison between standard nonfluorescent tubes and the new fluorescent tubes reveal the fact that the efficiencies have been increased by the following percentages:

Green 1,200 per cent
Gold (neon) 1,200 per cent
Blue 100 per cent

Comparisons of the other new colors with ordinary gaseous-tube sources are useless because the new colors were not commercially obtainable in any form previously.

A comparison of fluorescent tubes with incandescent sources for the production of colored light reveals the following increases in luminous efficiencies:

Green	4,000 per cent
Blue	3,000 per cent
Gold (neon)	100 per cent
White (warm)	250 per cent
White (cold)	350 per cent

The color of neon fluorescent tubes is to a large extent dependent upon the current, as the apparent color is a combination of the line spectrum of neon and the continuous but distributed spectrum of the fluorescent coating.

The neon emission may be roughly considered as following a straight line in the lower current values. However, the fluorescent material follows a saturation curve. For this reason, the color of any individual neon fluorescent tube changes from a hue associated with the fluorescent material to a reddish color, as the current is increased.

It is found that if the color is to be maintained and higher tube currents are indicated as a means of increasing the light output, the larger-diameter tubes must be used for neon fluorescence. In this way, the balance between the light from the neon spectrum and the light from the fluorescent materials is maintained.

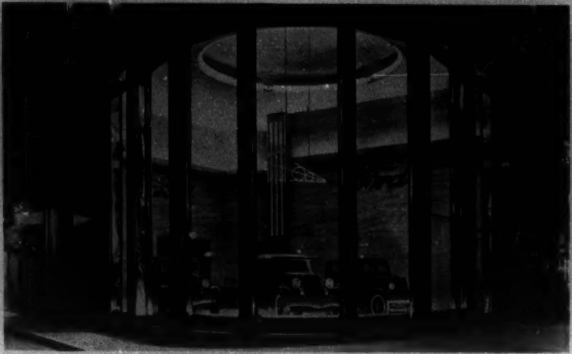


Figure 6. Ford showroom, Paris Fair 1937

Winter Operation

The question of winter operation of high-voltage gaseous tubes has always been a serious one. Although neon tubes are not subject to any ill effects, when mercury is used the cold weather causes a condensation of the mercury and a consequent dimming of the tube.

For this reason, in the colder sections of the country the main features of signs have been always outlined in either helium or neon tubes. However, with mercury fluorescent tubes the multiplicity of colors will be available for practically all uses. Although the cold weather causes a reduction of the light output, the intensity is so high that the reduced light output obtained in cold weather is acceptable. Neon fluorescent tubes are not subject to any reduced efficiency in cold weather and the new shades will allow considerable freedom from the basic colors previously obtainable.

General Practice

Fluorescent tubes will be actuated by high-voltage high-reactance transformers and the methods of connection and installation will be in general similar to those previously used in sign work.

Transformers may range from 1,000 to 15,000 volts. Currents of from 12 milliamperes up to 500 milliamperes are anticipated. General practice will fall into a range of between 12 and 60 milliamperes.

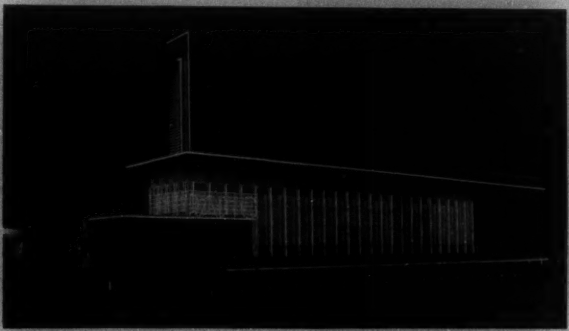
Standard transformers operate at a power factor of about 50 per cent. Transformers are obtainable with the power factor corrected to over 90 per cent by means of condensers.

Dimming

Dimming may be accomplished by means of voltage control devices in the primary circuits of the transformers or by adjustable chokes. It is not possible to get a complete variation from full intensity to darkness. At the lowest point from 15 to 25 per cent of the light remains. It has been found, however, that this is not a particularly inconvenient arrangement, due to other considerations.

The most common requirement for dimming is in theatrical work. In such locations, multiple colors are also required and by proper adjustment certain colors or groups of tubes are switched out successively, so that gradual light dimi-

Figure 5. Radio building, Paris Fair 1937



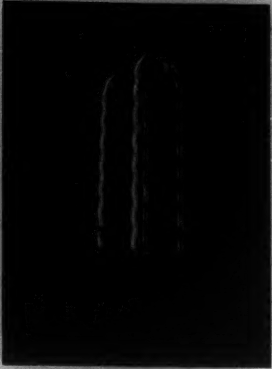


Figure 7. Pylons, New York World's Fair 1939

nution is effected. The best results where dimming is required are obtained when the tubes themselves are concealed and all other installations must be designed with special care.

The neon fluorescent tubes previously discussed would change color with change in intensity and hence they would not lend themselves readily to dimming.

Fluorescent tubing will be available in sizes from eight millimeters up to 35 millimeters. Power consumption will range from one-half to 25 watts per foot of tubing, depending upon the current, diameter of the tubing, and whether the tube is designed for use in the colder climates, or not. Where cold weather is encountered the fluorescent tubes are specially treated to give them such characteristics that temperature changes will have the least effect.

Color Combinations

It is desirable in many installations to obtain a white effect by a combination of gaseous tubes of different colors. In this way multiple effects may be obtained and if dimming is included in the installation, numerous attractive variations will be available. In addition, the more efficient green tubes may be made use of. Listed below are the possible combinations of tubing that will result in sensations of white light:

Combination	Result
Red (neon) regular-green	Slightly deficient in blue
Gold-pink	Pinkish amber
Green-gold	Deficient in blue
Pink-green	Good
Gold-blue	Good
Red (neon)-green-blue	Offers wide range of hues

Application of Zeon Tubing

Numerous uses have already been made of high-voltage fluorescent tubes both here and in Europe and these uses have not been limited to advertising, but have also included decorative lighting. A typical example of the combination of architectural effects with an advertising display is shown in figure 2. Lines of green fluorescent tubing operating at 30 milliamperes were carried up behind the joints of a glass-block wall.

The fluting on the interior surface of the blocks ran vertically so that the effect of a multiplicity of tubes was obtained. Against the glass blocks as a background was silhouetted a ruby colored neon sign. The effect of the red-colored light is augmented by the complementary value of the green background.

An outstanding effect was obtained by using a triple row of blue fluorescent tubes in a recessed letter sign (figure 3). In this installation 60 milliamperes were used on a 12-millimeter tube. The result was outstanding without following the usual trend to red neon.

Numerous installations have been made where fluorescent gold tubing has been substituted for helium gold. Such installations have resulted in power savings up to 80 per cent. However, it is felt that Zeon tubes will in general not affect loads materially because of the tendency to use higher intensities not available previously. Zeon tubing has also been used for the illumination of marquees. In an installation at the Hampshire House in New York a neon gold tube was substituted for amber bulbs and a power saving of 50 per cent was effected.

The greatest of the installations of high-voltage tubes up to the present time was at the Paris Exposition of 1937. Literally, miles of tubing were installed and such spectacular effects were obtained as the lighting of the Eiffel Tower base (figure 4) and the radio building (figure 5). In addition, a number of indoor lighting effects were obtained. Typical of these was the Ford showroom installation (figure 6).

Other installations were made for parapet lighting and in coves (figures 4, 5, 6).

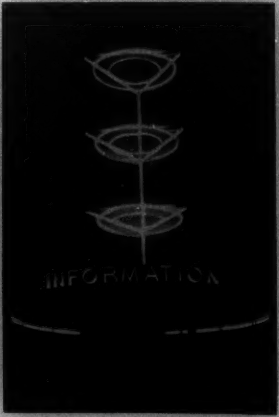
The Future Uses of High-Voltage Gaseous Tubes for Architectural and Advertising Effects

As the World of Tomorrow, the preview of the New York World's Fair 1939

Figure 8. Booth sign, New York World's Fair 1939

INFORMATION
INFORMATION

Figure 9. Information sign and pylon, New York World's Fair 1939



gives us some idea of the architectural uses to which this new medium may be adapted.

Serpentine tracery on the pylons (figure 7) gives a beautiful night-time effect. One-inch-diameter tubes were used. One hundred milliamperes flowed through two rose-colored tubes and 50 milliamperes were used on the green tubes.

Blue fluorescent tubes behind a plastic resulted in an attractively illuminated silhouette sign operating with remarkable economy (figure 8).

Circles of blue and warm white tubing provided general illumination around a booth and a neon gold tube concealed in a reflector gave a golden glow to the information sign (figure 9). The circles were of 18-millimeter tubing and operated at 60 milliamperes. The gold neon tube operated at 60 milliamperes and was 24 millimeters in diameter.

Figure 10. "Aqualons," New York World's Fair 1939

An outstanding night effect was obtained by actually installing high-voltage blue tubing below the water level in the "aqualons" shown in figure 10. This color was set into contrast by the gold tubes located in the vertical cylinders above. Bubbles rising through the water gave a shimmering effect. These units provided general street illumination in their immediate area, in addition to their purely decorative features.

The jobs indicated represent only a small part of the designs that have been completed by the World's Fair engineers, Bassett Jones and Morgan, Hamil and Engelken, and it is certain that the New York World's Fair will develop many new uses of high-voltage fluorescent tubes and will serve as a true indication of future trends.

Reactance of Square Tubular Bus Bars

By H. B. DWIGHT
FELLOW AIEE

T. K. WANG
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SQUARE tubular bus bars are coming into use to an increasing extent, as they are economical and convenient for joining to insulators and to branch circuits. The reactance of bus-bar circuits is frequently of importance, in order to compute the voltage drop under normal conditions and the current under short-circuit conditions. The purpose of this paper is to present formulas and curves by which the reactance of square tubular bus bars may be determined.

In a few cases, the inductance of bus bars can be expressed by simple formulas. For instance, the inductance of round rods is

$$L = 2 \log_\epsilon \frac{s}{r} + \frac{1}{2} \text{ abhenries per centimeter}$$

as used in power-line computations. For round tubular bus bars, expression (13), given by Clerk Maxwell[1] may be used. This is expressed in (14) in this paper in the form of a convergent series, for greater convenience with usual shapes of tubes. For copper straps, expressions are available, but if the thickness is proportionately large or the spacing small, as frequently happens, the length of the mathematical expressions is almost prohibitive, and the only practical method of computation seems to be by means of published curves, as given in reference 2. Similarly, the curves given in this paper in figures 1, 2, and 3 for square tubular bus bars may often be more convenient than the formulas 8, 11, and 15.

Commercial square tubular bus bars have rounded corners with outside radius of curvature from about three-eighths inch to three-fourths inch and they have a wall thickness from about one-eighth inch to one-half inch. However, the reactance is nearly the same as that of a very thin, hollow, square tube. The two features of rounded corners and thick walls are brought into the calculation by approximate methods in such a way that the error in the net reactance is estimated to be less than one per cent for practical cases in common use.

In order to save time in determining reactances, curves are given in figures 1, 2, and 3 which show, respectively, the reactance of thin square tubes, assuming uniform distribution of current, and the approximate corrections for rounded corners and thick walls.

Infinitely Thin Square Tubular Conductors

The inductance L of two duplicate, parallel conductors forming a return circuit, assuming uniform current distribution, is

$$L = 2 \log_\epsilon \frac{M}{T} \text{ abhenries per centimeter of conductor} \quad (1)$$

where $\log_\epsilon$ denotes natural logarithm, where M is the geometric mean distance between the two cross sections, and T is the geometric mean distance of one of the cross-sectional areas from itself, that is, the self geometric mean distance of the section.

The logarithm of the geometric mean distance between two areas is defined as the average of the logarithms of all possible distances from points on one area to those on the other, and the logarithm of the self geometric mean distance is the average of the logarithms of all possible distances between two points on the area. See Maxwell, "Electricity and Magnetism," paragraph 691.

The logarithm of the self geometric mean distance of a hollow square is derived in the appendix and is

$$\log_\epsilon T = \log_\epsilon a + \frac{1}{4} \log_\epsilon 2 + \frac{\pi}{4} - \frac{3}{2} \quad (2)$$

$$= \log_\epsilon a - 0.5413 \quad (3)$$

$$= \log_\epsilon 0.582a \quad (4)$$

where a is the length of a side of the square.

Also, as derived in the appendix,

$$\log_\epsilon M = \log_\epsilon s + \frac{1}{40}\left(\frac{a}{s}\right)^4 - \frac{17}{720}\left(\frac{a}{s}\right)^8 \dots \quad (5)$$

where s is the distance between centers of

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H. B. DWIGHT is professor of electrical machinery at Massachusetts Institute of Technology, Cambridge; T. K. WANG is a former graduate student in electrical engineering at that institution.

The calculations in this paper were made in connection with a thesis at Massachusetts Institute of Technology.

[1] For all numbered references, see list at end of paper.

two squares whose sides are parallel and perpendicular to s. If the sides are not perpendicular to s, as sometimes occurs with triangular spacing, the change in the reactance is in the terms in $(a/s)^4$ and higher powers. Their effect in usual cases is negligible, as shown in example I.

From the above equations,

$$L = 2\log_\epsilon\frac{s}{a} + 1.083 + \frac{1}{20}\left(\frac{a}{s}\right)^4 - \frac{17}{360}\left(\frac{a}{s}\right)^8 \ldots \qquad (6)$$

abhenries per centimeter of conductor

For square tubular conductors widely separated,

$$L = 2\log_\epsilon\frac{s}{a} + 1.083$$

or

$$L = 2\log_\epsilon\frac{s}{0.582a} \text{ abhenries per centimeter} \qquad (7)$$

Changing to practical units, the reactance of thin square tubular conductors is

$$X_1 = 2\pi f\, 140\,4 \times 10^{-6}\left[\log_{10}\frac{s}{a} + 0.235 + 0.0108\left(\frac{a}{s}\right)^4 - 0.0102\left(\frac{a}{s}\right)^8 \ldots\right]$$

ohms per 1,000 feet of conductor (8)

where f = frequency in cycles per second. See the curve in figure 1.

EXAMPLE I

The reactance per 1,000 feet of a thin square tube of 2.5 inches diameter, that is, the distance between opposite sides, and 10 inches spacing, center to center, at 60 cycles is, by (8),

$$X_1 = 120\pi \times 140.4 \times 10^{-6}\left(\log_{10}\frac{10}{2.5} + 0.235 + 0.00004 \ldots\right)$$

$= 0.0443$ ohm per 1,000 feet of conductor

The terms in a^4/s^4, etc., are less than 0.01 per cent of the total, in this case. For comparison, the reactance of a thin round tube with the same diameter and spacing is

$$X = 120\pi \times 140.4 \times 10^{-6}\log_{10}\frac{10}{1.25}$$

$= 0.0479$

which is approximately eight per cent greater than the other value. The reactance of a thin round tube large enough just to enclose the square tube described, is

$$X = 120\pi \times 140.4 \times 10^{-6}\log_{10}\frac{10}{1.25\sqrt{2}}$$

$= 0.0398$

that is, approximately ten per cent less than the value for the square tube.

It is seen from this example that the average of the reactances of an inscribed tube and a circumscribed tube is a fairly close approximation to the reactance of a square tube. This is equivalent to stating that the inductance of thin square tubes is approximately

$$L = 2\log_\epsilon\frac{s}{0.595a}$$

This approximation is not to be recommended for engineering calculations, since it is not more convenient in any way than the corresponding expression (7) for square tubes.

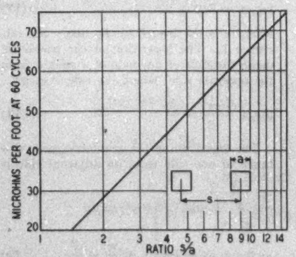


Figure 1. Reactance of thin square tubes

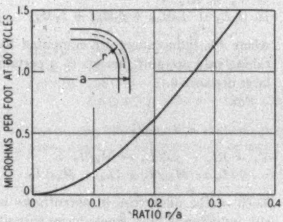


Figure 2. Increase caused by round corners

Effect of Rounded Corners

Let the thin, square tube with rounded corners of radius r be replaced by a square tube with notches of side c at the corners, as shown in figure 4. The cross-shaped figure will lie along the mean position of the curves and will have nearly the same inductance, if the two figures have the same area. Then,

$$c^2 = r^2 - \frac{\pi r^2}{4} = 0.2146r^2$$

$$c = 0.463r \qquad (9)$$

In practical cases, r is to be taken not as the outside radius but as the average of the outside and inside radii of the corners of a thick tube. See figure 2.

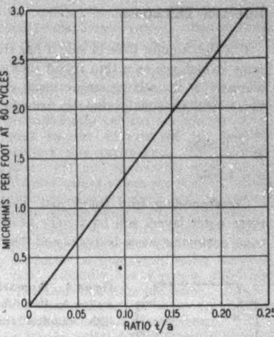


Figure 3. Increase caused by thickness of tubes

Putting the self geometric mean distance of the cross-shaped figure equal to T', there is obtained the correction

$$\log_\epsilon T - \log_\epsilon T' = \left(\frac{c}{a}\right)^2\log_\epsilon\frac{a}{c} + \left(\frac{3}{2} - \frac{\pi}{4}\right)\left(\frac{c}{a}\right)^2 + \frac{1}{2}\left(\frac{c}{a}\right)^3 \ldots\ldots \qquad (10)$$

This gives such a small correction in practical cases (see example II and figure 2) that the corresponding change in $\log_\epsilon M$ caused by rounded corners is seen to be negligible.

Then the increase in reactance due to the rounded corners is, by (10),

$$\Delta_1 X = 2\pi f\, 140.4 \times 10^{-6}\left[\left(\frac{c}{a}\right)^2\log_{10}\frac{a}{c} + 0.310\left(\frac{c}{a}\right)^2 + 0.217\left(\frac{c}{a}\right)^3 \ldots\right]$$

ohms per 1,000 feet of conductor (11)

where $c = 0.463r$, from (9).

EXAMPLE II

Let the corners of a 2.5-inch thin, square tube have a radius $r = 0.5$ inch and let the spacing s be 10 inches, center to center.

$c/a = 0.463 \times 0.5/2.5 = 0.0926$
$c^2/a^2 = 0.0086$
Reactance of tube with square corners $= 0.0443$ ohm per 1,000 feet by (8), example I.
Increase in reactance due to rounded corners
$= 120\pi \times 140.4 \times 10^{-4}(0.0089 + 0.0027 + 0.0002 \ldots)$
$= 0.0006$ ohm per 1,000 feet by (11).

The increase in reactance for this extreme case is 1.4 per cent.

Effect of Thickness

If a thin square tube of side a has the same inductance as a thin round tube of diameter d, the self geometric mean distances will be equal; that is, from (4),

$$\frac{d}{2} = 0.582a$$

$$d = 1.164a \qquad (12)$$

Corresponding thin tubes just inside these outer layers will have very nearly equal geometric mean distances and therefore the increase in inductance, due to thickness, of a commercial square tube will be approximately the same as for a round tube of the same thickness and with 1.164 times as large a diameter.

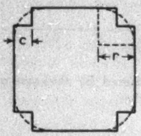


Figure 4. Approximation to thin tube with rounded corners

The logarithm of the self geometric mean distance (GMD) of a round tube of outside diameter d and inside diameter v is[1]

$$\log_\epsilon \frac{d}{2} - \frac{v^4}{(d^2 - v^2)^2}\log_\epsilon \frac{d}{v} + \frac{3v^2 - d^2}{4(d^2 - v^2)} \qquad (13)$$

Expanding in powers of the thickness $t = \dfrac{d - v}{2}$

$$\log_\epsilon GMD = \log_\epsilon \frac{d}{2} - \frac{2}{3}\frac{t}{d} + \frac{4}{15}\left(\frac{t}{d}\right)^3 + \frac{2}{5}\left(\frac{t}{d}\right)^4 \cdots \qquad (14)$$

Putting $d = 1.164\,a$, the increase in reactance, due to thickness, of a square tube of outside diameter a is

$$\Delta_2 X = 2\pi f\, 140.4 \times 10^{-6} \times \left[0.249\frac{t}{a} - 0.073\left(\frac{t}{a}\right)^3 - 0.095\left(\frac{t}{a}\right)^4 \cdots\right]$$

ohms per 1,000 feet of conductor (15)

EXAMPLE III

Let the outside diameter of a square tube be $a = 2.5$ inches, the thickness $t = 0.5$ inch, and the spacing $s = 10$ inches, center to center.

$$\frac{t}{a} = 0.20$$

Increase in reactance due to thickness

$= 120\pi \times 140.4 \times 10^{-6}\,[0.0498 - 0.0006 - 0.0002 \ldots]$

$= 0.0026$ ohm per 1,000 feet by (15)

The increase is six per cent of 0.0443 (see example I).

Groups of Conductors

The reactive drop in each conductor of a three-phase circuit or in each conductor of other groupings of parallel conductors, can be expressed in terms of single-phase reactances by first expressing them in terms of self-inductances and mutual inductances and then arranging them in pairs.

Thus, let the currents in conductors a, b, c, d, etc., in a given direction be I_a, I_b, I_c, I_d, etc. These may be polyphase currents or some of them may be the currents of branches in parallel. In any case, since a complete system of currents is specified, their sum is zero.

Voltage drop in conductor a, whose resistance is R,

$$= V_a = RI_a + j\omega\,[I_aL_a + I_bM_{ab} + I_cM_{ac} + I_dM_{ad} \ldots]$$

where the inductances are computed by taking into account flux up to a certain large distance u.

Put

$$I_a = -I_b - I_c - I_d - \ldots$$

$$V_a = RI_a - j\omega[(L_a - M_{ab})I_b + (L_a - M_{ac})\,I_c + (L_a - M_{ad})\,I_d \ldots]$$

Each of the quantities in parentheses is the inductance of a single-phase circuit of conductors a and b, a and c, etc. The quantity u cancels out in each. Thus,

$$V_a = I_aR - j[I_bX_{ab} + I_cX_{ac} + I_dX_{ad} \ldots] \qquad (16)$$

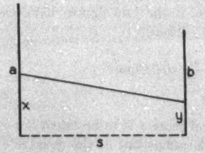
where X_{ab} is the reactance of the return circuit made up of conductors a and b, etc.

In a similar way, V_b, V_c, etc., can be computed and so the formulas and curves given in this paper are sufficient for computations for polyphase circuits and for conductors in parallel, where the conductors are alike. This has been described already for the case of solid rectangular conductors.[2]

EXAMPLE V

A three-phase bus-bar circuit is composed of eight-inch-square tubes arranged in a plane, at 24 inches spacing, center to center. The wall thickness is three-eighth inch and the outside radius of the corners is three-fourth inch. Determine the drop in 200 feet for balanced three-phase current of 5,000 amperes at 60 cycles.

Figure 5. Two parallel lines



Let the three conductors be a, b, and c.

$$\frac{r}{a} = \frac{0.75 + 0.375}{2 \times 8} = 0.0703$$

$$\frac{c}{a} = 0.0703 \times 0.463 = 0.0325$$

$$\frac{t}{a} = \frac{0.375}{8} = 0.0469$$

From (8), (11), and (15) or from figures 1, 2 and 3,

$X_{ab} = X_{bc} = 0.00754 + 0.00002 + 0.00012$

$= 0.00768$ ohms for 200 feet

$X_{ac} = 0.01073 + 0.00002 + 0.00012$

$= 0.01087$ ohms for 200 feet

Resistance $= R = 0.000138$ ohms for 200 feet.

Let

$I_a = 5{,}000$

$I_b = 5{,}000\,(-0.5 + j0.866)$

$I_c = 5{,}000\,(-0.5 - j0.866)$

From (16), $V_a = -13 + j46$

Similarly,

$V_b = -34 - j19$ $V_c = 33 - j36$

$|V_a| = 48$ volts $|V_b| = 38$ volts

$|V_c| = 49$ volts

The drop in the voltage between conductors is obtained by subtracting vectorially V_b from V_a, etc.

Appendix

The logarithm of the geometric mean distance of a straight line from itself is

$$\log_e a - 3/2 \qquad (17)$$

where a is the length of the line. See reference 1. The logarithm of the geometric mean distance of one side of a square from the opposite side, where the side is a, is[3]

$$\log_e a + \frac{\pi}{2} - \frac{3}{2} \qquad (18)$$

The logarithm of the geometric mean distance of one side from an adjacent side is

$$\frac{1}{2a^2}\int_0^a\int_0^a \log_e (x^2 + y^2)\,dx\,dy$$

$$= \log_e a + \frac{1}{2}\log_e 2 + \frac{\pi}{4} - \frac{3}{2} \qquad (19)$$

For the four sides of the square,

$$\log_e GMD = \frac{1}{4}[(17) + (18) + 2 \times (19)]$$
$$= \log_e a + \frac{1}{4}\log_e 2 + \frac{\pi}{4} - \frac{3}{2} \qquad (20)$$

as in (2).

The logarithm of the geometric mean distance of two lines (figure 5) of lengths a and b drawn perpendicular to a line of length s at its ends and on the same side of it, is

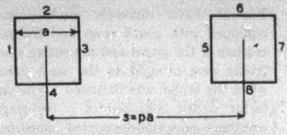


Figure 6. Two square tubular bus bars

$$\frac{1}{ab}\int_0^b\int_0^a \frac{1}{2}\log_e\{s^2 + (x - y)^2\}\,dx\,dy$$
$$= \frac{1}{ab}\left[\frac{1}{4}\{s^2 - (a - b)^2\}\log_e\{s^2 + (a - b)^2\} + \frac{1}{4}(a^2 - s^2)\log_e(a^2 + s^2) + \frac{1}{4}(b^2 - s^2)\log_e(b^2 + s^2) + \frac{1}{2}s^2\log_e s - s(a - b)\tan^{-1}\frac{a - b}{s} + as\tan^{-1}\frac{a}{s} + bs\tan^{-1}\frac{b}{s} - \frac{3}{2}ab\right] \qquad (21)$$

There appears to be a misprint in the expression in reference 1, paragraph 692. Putting $a = b$,

$$\log_e G_{15} = \frac{s^2}{b^2}\log_e s + \frac{1}{2}\left(1 - \frac{s^2}{b^2}\right) \times \log_e(b^2 + s^2) + \frac{2s}{b}\tan^{-1}\frac{b}{s} - \frac{3}{2} \qquad (22)$$

as in reference 3, equation 132. G_{15} is the geometric mean distance of sides 1 and 5 in figure 6.

Putting $s = b$, equation 18 is obtained. See reference 3, equation 133.

$$\log_e G_{27}\text{ (figure 6)} = \frac{1}{2a^2}\int_0^a\int_0^a \log_e\{y^2 + (pa + x)^2\}\,dx\,dy$$
$$= \frac{p+1}{2}\log_e\{1 + (p+1)^2\} - \frac{p}{2}\log_e(p^2 + 1) + \log_e a + \frac{1}{2}\{1 - (p + 1)^2\}\tan^{-1}(p + 1) + \frac{1}{2}(p^2 - 1)\tan^{-1}p + \frac{\pi}{2}\left(p + \frac{1}{2}\right) - \frac{3}{2} \qquad (23)$$

Log$_e$ G_{25} is obtained by putting $p = p - 1$ in (23).

$$\log_e G_{26} = \frac{(p + 1)^2}{2}\log_e\{(p + 1)a\} - p^2\log_e(pa) + \frac{(p - 1)^2}{2}\log_e\{(p - 1)a\} - \frac{3}{2} \qquad (24)$$

See reference 3, equation 130.

$$\log_e G_{28} = \frac{1}{4}\{(p + 1)^2 - 1\} \times \log_e\{(p + 1)^2 + 1\} - \frac{1}{2}(p^2 - 1) \times \log_e(p^2 + 1) + \frac{1}{4}\{(p - 1)^2 - 1\} \times \log_e\{(p - 1)^2 + 1\} + \log_e a + (p + 1) \times \tan^{-1}(p + 1) - 2p\tan^{-1}p + (p - 1)\tan^{-1}(p - 1) - \frac{3}{2} \qquad (25)$$

In averaging these and corresponding expressions, similar terms can be combined. The geometric mean distance of one hollow square from the other is given by

$$\log_e M = \frac{1}{8}[(p + 1)\log_e\{(p + 1)^2 + 1\} - (p^2 - 1)\log_e(p^2 + 1) - (p - 1) \times \log_e\{(p - 1)^2 + 1\} + (p + 1)^2 \times \log_e(p + 1) + (p - 1)^2\log_e(p - 1) + 8\log_e a - (p^2+2p)\tan^{-1}(p+1) + (p^2-2p)\tan^{-1}(p-1) - 4p\tan^{-1}p + 4\pi p - 12] \qquad (26)$$

For putting this expression in the form of a power series, the following may be used:

$$\tan^{-1}(p + 1) = \frac{\pi}{2} - \frac{1}{p} + \frac{1}{p^2} - \frac{3}{3p^3} + \frac{4}{5p^5} - \frac{4}{3p^6} + \frac{8}{7p^7} - \frac{16}{9p^9} + \frac{16}{5p^{10}}\cdots \qquad (27)$$

$$\tan^{-1}(p - 1) = \frac{\pi}{2} - \frac{1}{p} - \frac{1}{p^2} - \frac{2}{3p^3} + \frac{4}{5p^5} + \frac{4}{3p^6} + \frac{8}{7p^7} - \frac{16}{9p^9} - \frac{16}{5p^{10}}\cdots \qquad (28)$$

$$\log_e M = \log_e s + \frac{1}{40}\left(\frac{a}{s}\right)^4 - \frac{17}{720}\left(\frac{a}{s}\right)^8\cdots$$

as in (5).

References

1. Electricity and Magnetism (a book), J. Clerk Maxwell. Volume 2, paragraph 692.

2. Reactance Values for Rectangular Conductors, H. B. Dwight. *The Electric Journal*, volume 16, June 1919, page 255, examples II and III.

3. Scientific Paper No. 169 of the Bureau of Standards, E. B. Rosa and F. W. Grover. Equation 133.

Discussion

O. R. Schurig (General Electric Company, Schenectady, N. Y.): The authors have done a good piece of work in deriving formulas for calculating the reactance of square tubes. Their method is to calculate the exact reactance for hollow square conductors with infinitely thin walls and then to make correction for wall thickness and for rounded corners, on the basis of uniformly distributed current. This procedure is sufficiently accurate for most engineering calculations.

For widely spaced conductors of small or moderate wall thickness at 60 cycles, the current division within the conductors will be sufficiently uniform so that the method used by the authors will be applicable without any corrections. The proximity effect will not appreciably affect the reactance values unless the spacing between tubes is relatively small, that is, not over three or four times the tube diameter.

Nonuniform current division due to proximity effect and skin effect for the conductor arrangements under consideration, ordinarily has a greater effect upon resistance than upon reactance. This is all the more important since the resultant effect is to increase the resistance.

Mathematical determination of resistance and reactance for solid rectangular or square conductors, taking into account nonuniform current density caused by skin effect and proximity effect, would be extremely complicated, and it is more likely that the answers will be obtained by measurements during tests.

H. W. Papst (nonmember) and **L. F. Hickernell** (both of Anaconda Wire and Cable Company, Hastings-on-Hudson, N. Y.): This paper is a welcome contribution to fundamental theory for predetermining the electrical characteristics of heavy-current conductors, deriving formulas for calculating reactance of square tubular busbars.

From a practical design standpoint, we wish to point out that this type of bus is subject to the same limitations as the round tubular bus; namely, it is inefficient for heavy currents (that is, 4,000 amperes and above). This inefficiency is due to the fact that the inner surface is entirely ineffective for dissipating heat unless the tubing is cooled from the inside artificially by the use of circulating water or oil or forced air ventilation.

Another disadvantage of all tubular-type of busbars is the expense and difficulty in making joints and taps. Round tubes require expensive fittings and clamps. Square tubes require drilled and threaded holes for attachment of fishplates by means of cap screws without nuts. The bearing for the cap screw threads in the copper of the tube is inadequate for mechanical and electrical connection during load cycles resulting in local heating.

These objections are overcome by the use of channels or angles clamped in the form of an open square or rectangle. The open longitudinal slot at top and bottom of the bus provides natural ventilation for the inner surfaces due to the so-called "chimney effect." This arrangement greatly increases the current-carrying capacity per unit cross-section for a given temperature rise and hence the efficiency.

Joints and taps for the channel and angle arrangements are very simply effected by means of fishplates attached by means of bolts and nuts. This requires only that the channel or angle be drilled (not threaded). Adequate mechanical and electrical connection is assured by the use of a nut on the bolt.

Within practical design limits, the Dwight-Wang formulas can be used without modification for calculation of the reactance of channel or angle square busses although a small difference will result.

Design engineers will find the curves in the publication, "Anaconda Copper Bus Conductor" (Publication No. *C*-25, second edition, 1938) very convenient for the determination of the electrical characteristics of all types of busses.

A comparison for a typical case between the results obtained from the Dwight-Wang formulas and the above reference is as follows:

Comparison of Voltage Drop in Square Tubular Busses

Bus Size* (Inches)	Type of Copper Bus	Reactance at 60 Cycles (Ohms per 1,000 Feet)	Drop for 1,000 Amperes 10-Inch Spacing (Volts)	Authority
2.5	Square tube (rounded corners)	0.045	45	Dwight-Wang
2.5	Channel square bus		42	Anaconda Publication *C*-25, figures 18–19

* Interpolated to place on same basis (0.4-inch wall).

H. B. Dwight: Referring to O. R. Schurig's discussion, the change in resistance caused by proximity effect in round tubes when the axial spacing is greater than two times the diameter, is less than about 15 per cent, as shown by figure 3, "Skin Effect and Proximity Effect in Tubular Conductors," AIEE TRANSACTIONS, 1922, page 189. The change in inductance for such spacings is considerably less than 15 per cent, for round or square tubes, and is practically negligible in usual engineering applications, as stated by Mr. Schurig.

Formulas, some of them semi-empirical, for the a-c resistance of hollow, square busbars have been given by A. H. M. Arnold, IEE *Journal* (England), May 1938, volume 82, page 537. His formulas agree with measurements within 2.4 per cent or less, including limiting cases.

In the second paragraph of the discussion by H. W. Papst and L. F. Hickernell, it does not seem quite right to say that the inner surface of square tubular busbars is entirely ineffective for dissipating heat except for longitudinal cooling by liquids or forced air, because most of the square tubular busbars made in this country, such as those manufactured by the Chase Brass and Copper Company of Waterbury, Conn., have regularly spaced ventilating holes, which lower the temperature for a given current by providing natural cooling for the inner surface.

Also, square tubes do not necessarily require cap screws, but the standard arrangement with square tubular busbars is to provide nut plates inside the tubes which are placed there during fabrication. These plates carry the threads and give the required mechanical strength and the electrical contact inside and outside of the tube wall. Also, the square tubes are very suitable for welded or brazed connections and these are being used to an increasing extent.

The problem of the 1,000-foot circuit solved by means of figures 18 and 19 of Anaconda Publication *C*-25 is a three-phase problem, and requires the use of two single-phase reactances, one for 10-inch and one for 20-inch spacing. This would increase the calculated value of average reactance drop somewhat. While figures 18 and 19 give fairly good results, both the convenience and the accuracy would be improved by plotting reactance drop on a base of spacing divided by self-geometric mean distance and avoiding the use of the correction factor of figure 19. This would require tabulating the self-geometric mean distance of each conductor.

Absolute Power Factor of Air Capacitors

By W. B. KOUWENHOVEN
FELLOW AIEE

E. L. LOTZ
ASSOCIATE AIEE

AS THE sensitivity of measuring equipment has been improved, it has become evident that all air capacitors are not loss free. The high sensitivity bridges used today in dielectric investigations measure the difference between the power factor of the standard air capacitor and of the specimen in the test capacitor. To determine accurately the loss in the specimen the value of the absolute power factor of the standard should be known.

In this investigation seven air capacitors of different types of construction were studied. In the preliminary tests it was determined that the power factor was independent of the voltage stress and that it varied with the separation of the plates. The results of these studies led to the development of a method of determining the absolute power factor of an air capacitor. In addition, data are presented on the effects produced by the degree of polish of the electrode surfaces, by the width of the gap between the measuring electrode and its guard ring, and by the presence of water films on the plates.

Test Equipment

All of the measurements were made in a high sensitivity Schering bridge of the type developed by Kouwenhoven and Banos.[1] Each capacitor studied was equipped with guard rings and the potentials of the guard and measuring electrodes were brought to the same value when the bridge was balanced. The detector circuit consisted of a two-stage resistance-capacitance-coupled amplifier feeding into an a-c galvanometer with separately excited field. The power supply was furnished by a General Electric Company sine-wave generator.

The power factor sensitivity of the bridge circuit with an applied potential of 4,000 volts, and with capacitors of the order 20 to 100 micromicrofarads in its arms was 1×10^{-6}. When large capacitors were tested, the sensitivity was greater and differences in power factor of 0.5×10^{-6} could be detected.

Metal enclosures or boxes were provided in which the capacitors were placed while under test. These boxes were held at a constant temperature of 28 degrees centigrade by means of thermostatic control.

Capacitors

The capacitors used in this investigation are listed below, each one being designated by an appropriate symbol. The plates of all of these capacitors were of brass.

Capacitor Cp was a cylindrical pressure capacitor which was made with three coaxial brass tubes, one-eighth inch thick, mounted in a vertical position with the central tube acting as the high-voltage electrode. The entire assemblage was mounted inside a one-inch-thick iron pipe and operated at 100 pounds pressure of carefully dried clean air. The capaci-

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[1] For all numbered references, see list at end of paper.

tor which was constructed in 1931 by W. B. Kouwenhoven and Berberich[2] was shielded throughout having both guard rings and shields.

The high-voltage electrode was 23 inches in length; the low-voltage electrodes were each 9 inches in length; and the guard rings were each $4^1/_2$ inches long. The capacitance of the measuring electrode was 517 micromicrofarads and that of the guard was 527 micromicrofarads. The spacing between the high-voltage electrode and the low-voltage plates was 200 mils.

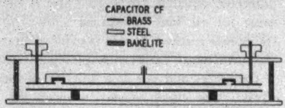


Figure 1. Schematic diagram showing construction of capacitor Cf

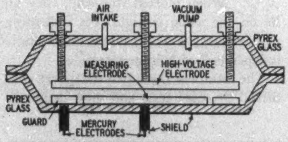
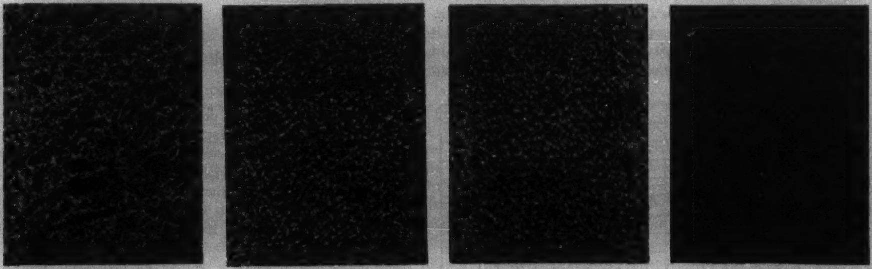


Figure 2. Construction of glass-enclosed capacitors C1, C2, C3, C4

Capacitor Cf was constructed of two one-quarter-inch-thick brass plates mounted in a horizontal position and supported on bakelite pillars. The bakelite pillars were in turn mounted on a metal plate which was grounded. A schematic diagram of the capacitor is shown in figure 1. The high-voltage plate was 24 inches square and the low-voltage electrode was 18 inches square with a guard ring three inches in width. The separation of the plates could be easily adjusted by four set screws and the capacitance thus varied from 70 to 800 micromicrofarads. The entire capacitor was completely enclosed in a sheet-metal box.

Figure 3. Photomicrographs of the capacitor plates at various degrees of polish. Magnification 100 diameters



A—Plates ground with number 200 carborundum

B—Plates ground with number 400 carborundum

C—Plates ground with number 600 carborundum

D—Plates polished with rouge

Capacitor Cu was a three-electrode capacitor consisting of a high-voltage electrode and two measuring electrodes, one mounted on either side, and all enclosed in a metal box. The plates which were mounted in a vertical position were supported from a grounded metal frame by bakelite supports. The electrodes were approximately 15 by 17 inches in size and the guard rings were $1^1/_2$ inches in width. The separation of the plates was varied by means of screw supports and the capacitance range was variable from 80 to 900 micromicrofarads.

Capacitor C1 was a small circular plate capacitor which was enclosed and sealed in a glass cell made of two Pyrex pie plates, whose lips were ground so as to form a vacuum tight joint. The capacitor was a three-electrode capacitor having a high-voltage electrode nine inches in diameter, a measuring electrode six inches in diameter and a guard ring $1^1/_2$ inches wide. All of the plates were made of one-fourth-inch brass and were polished to a mirror finish as described under the section on polishing.

The method of mounting the plates is shown in figure 2. The low-voltage electrode and the guard rings were ground to fit the bottom half of the cell and the leads taken out by means of mercury wells. Shields were provided as shown in the figure. The high-voltage electrode was supported from the upper glass dish by means of three threaded brass rods. The separation of the plates was varied by wing nuts and this variation could be made without opening the cell. The capacitance range of the capacitor was 30 to 100 micromicrofarads.

Capacitor C2 was of the same construction as capacitor *C*1 except that its plates were polished with number 200 carborundum. The condition of the plate surfaces is shown in figure 3.

Capacitor C3 was also identical in construction with capacitor *C*1 and its plates were polished to the highest degree obtainable. The surface finish of the plates is shown by the photomicrograph of figure 3.

Capacitor C4 was similar in construction to capacitor *C*1. Its plates were rough and contained grooves $^1/_{32}$ inch deep.

Surface Finish

The study of the effect of the roughness of the surface of the electrodes on the power factor of an air capacitor was carried out with the small glass-enclosed capacitors *C*1, *C*2, *C*3, and *C*4. The method of polishing was that employed by the physics department of The Johns Hopkins University in the preparation of diffraction gratings. The method consisted of using various sized carborundum particles in a water solution and grinding the surface of the plates by clamping them on a revolving spindle and using a lead lap. The initial polishing was accomplished with a coarse carborundum number 80. This was followed by using in order number 200, 400, and 600 carborundum. After grinding with the 600 carborundum, number 600 emery dust in a water solution was used. At this stage the plate surfaces appeared very smooth to the eye. The final polish was given by using rouge and a buffer wheel. This gave a very high degree of polish and nearly 100 per cent of the incident light was reflected by the mirror-like surface of the plate. In figure 3 are shown photomicrographs of the plate surface at various stages during polishing. These pictures were taken of the surfaces without any etching or further treatment other than the polishing. The magnification used was 100 diameters.

Standard Conditions of Tests

All of the measurements on the capacitors were made at room temperature (28 degrees centigrade) and extreme care was taken never to allow the plates to become heated or to be exposed to agents which would accelerate oxidation. As far as was possible oxidation of the plates was reduced to an absolute minimum. The surfaces of the plates were maintained bright and were cleaned in accordance with a standard method, which consisted of washing them in varsol, alcohol, soapy water, and distilled water in the order named; rubber sponges being used. After the distilled water treatment, dry air was blown over the surface of the plates for 15 minutes and then the plates were mounted in their respective containers and allowed to stand for 12 hours before measurements were started.

All of the measurements, except those where the effect of humidity was studied, were made in an atmosphere whose relative humidity never exceeded 35 per cent.

Experimental Results

The seven capacitors described above were used in carrying out the investigation. The first step was the study of the variation of the power factor with voltage stress. One of the small capacitors, the pressure unit, and the flat plate capacitor *Cf* were used in this study. Next in order are given the theory and the experimental results in connection with the method of determining the absolute power factor. Both large and small capacitors were used in this part of the work. The variation of the power factor with separation between the plates was then studied. Following this study, there is presented a discussion of the equivalent circuit of a low-loss air capacitor. The next steps were the investigation of the effects of the roughness of the surface and of the width of the gap between the guard and the measuring electrode. In this work, the small units were used. Data are also presented on the variation of the power factor with the relative humidity in the room and in the final section, the results are given for the condition where a water film is present on the plates of a capacitor.

VARIATION OF POWER FACTOR WITH VOLTAGE STRESS

In the measurements of the variation of power factor with stress, a number of experiments were conducted. Only two of these will be presented here. The first test was made on the small polished capacitor *C*1 and the capacitor *Cf*. These two capacitors were mounted in the bridge and their power-factor difference measured as a function of the applied voltage. Capacitor *C*1 was set at 66.2 micromicrofarads capacitance (96 mils separation) and capacitor *Cf* was adjusted to a capacitance of 65.6 micromicrofarads (1,100 mils separation). These adjustments gave voltage stresses in the two capacitors of a ratio of about ten-to-one. With the two capacitors at their respective separations, the power-factor difference was measured at four applied voltages giving stresses in capacitor *C*1 which ranged from 20 to 52 volts per mil, while the stress in capacitor *Cf* ranged from 1.8 to 4.5 volts per mil. The results of these tests are given in table I and show clearly that the power factor difference was constant and independent of the voltage stresses for the range studied.

In the second test capacitors *Cp* and *Cf* were used. Capacitor *Cp* had a fixed spacing of 200 mils between its plates. The separation between the plates of capacitor *Cf* was varied from 96.8 mils to 362.5 mils. Runs were made at four voltages ranging from 1,000 to 4,000 volts in 1,000 volt steps. The stress in the capacitor *Cp* varied from 5 to 20 volts per mil. The stress in the capacitor *Cf* was varied over wide limits ranging from 2.76 volts per mil at its maximum separation and minimum applied voltage to 41.3 volts per mil at its minimum separation and the highest applied voltage.

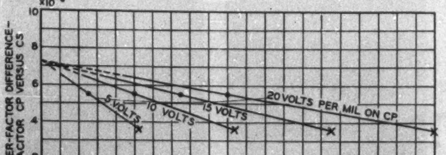


Figure 4. Stress voltage curves

The results of this test are plotted in figure 4. Four curves, one for each value of stress on *Cp*, are plotted in terms of power factor difference and the voltage stress on the variable capacitor *Cf*. At any given spacing of *Cf*, for example, 200 mils, the power factor difference equaled 5.5×10^{-6} and is the same as shown by the points marked with circles on the four curves. At another plate separation, for example, 96.8 mils, the power factor difference equaled 3.6×10^{-6} and is the same for all four curves as shown by the points marked *X*. It is evident that the power factor varies with the separation of the plates and not as a function of the stress. The effect of separation is discussed in a later section of this paper.

These four curves of figure 4 are replotted in curve *A*, figure 5 in terms of power-factor difference versus capacitance of *Cf*. When plotted this way, they all lie on the same straight line indicating clearly that the power-factor difference is independent of voltage stress.

The results of these two tests were confirmed by other tests and show definitely that the power-factor difference is independent of the voltage stress, at least up to 50 volts per mil.

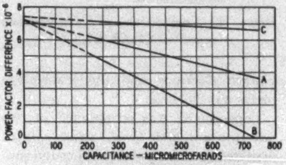


Figure 5. Method of finding the absolute power factor of capacitor Cp

Curve *A*—Power-factor difference, *Cp* versus *Cf*

Curve *B*—Power-factor difference, *Cp* versus *Cu*

Curve *C*—Power-factor difference, *Cp* versus *Cf* (after cleaning)

ABSOLUTE POWER FACTOR

The determination of the absolute power factor of an air capacitor by the usual bridge methods is difficult, because the bridge measures the difference between the power factor of the capacitor *Cx* in its unknown or specimen arm and that of the capacitor *Cs* in its standard arm. Balsbaugh and Moon[3] have published a method for computing the absolute power factor of a capacitor on the basis of experimental results.

In the method here reported the absolute power factor is determined by taking successive bridge measurements with the capacitor Cx held at a fixed value while the capacitance of Cs is varied by changing the separation between its plates. When the resulting power factor differences are plotted against the corresponding values of the capacitance Cs, the points are found to lie on a straight line. If this line be extended until it intercepts the axis (capacitance $Cs = 0$) the intercept gives the absolute power factor of the fixed capacitor Cx.

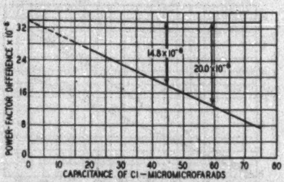


Figure 6. Power-factor difference, capacitor C2 fixed versus capacitor C1 variable

That this is the case may be shown mathematically as follows: The power factor δx of the specimen capacitor and δs of the standard capacitor may be written:

$$\delta x = 2\pi f\, CxRx$$
$$\delta s = 2\pi f\, CsRs$$

where

f = frequency
Cs = capacitance of standard in farads
Cx = capacitance of specimen in farads
Rs = equivalent series resistance corresponding to loss angle of standard capacitor
Rx = equivalent series resistance corresponding to loss angle of specimen capacitor

Table I. Variation of Power Factor With Stress

Applied Voltage	Stress in Volts Per Mil in C1	Stress in Volts Per Mil in Cf	Power-Factor Difference
2,000	20.8	1.82	0.0000165
3,000	31.2	2.72	0.0000168
4,000	41.7	3.64	0.0000156
5,000	52.0	4.54	0.0000168

The equivalent series resistances are assumed to be constant, see the section of this paper dealing with the equivalent circuit of a low-loss air capacitor. When these values are substituted in the relation for $\delta x - \delta s$, it may be readily seen that the expression for the power-factor difference takes the form of the equation of the straight line, $Ax + By = C$, where

Ax corresponds to $2\pi fRsCs$
By corresponds to $\delta x - \delta s$
C corresponds to $2\pi fCxRx$, a constant

The bridge measures $\delta x - \delta s$ and if the line is extended to the point where the capacitance of the standard equals zero then it is evident that δs must equal zero and therefore the intercept is the absolute power factor of the specimen capacitor.

Experimental tests were made to verify this method. The capacitor used as the specimen capacitor was *Cp*, the pressure capacitor. The absolute power factor of the pressure capacitor was determined by first comparing it with capacitor *Cf* and then with capacitor *Cu*. The results of these tests were given in figure 5 as curve *A*. The power factor difference was measured using capacitor *Cf* as the standard at values of capacitance varying from 225 micromicrofarads to 650 micromicrofarads. Extending curve *A* to the axis gives the absolute power factor of *Cp* as 7.2×10^{-6}. After this test the capacitor *Cf* was dismantled and given a thorough cleaning and polishing and the power-factor difference with *Cp* again measured. This test is plotted in figure 5 as curve *C* and its intercept gives the absolute power factor of *Cp* as 7.4×10^{-6}.

With the capacitor *Cu* used as the standard capacitor at values of capacitance varying from 200 micromicrofarads to 700 micromicrofarads, the power factor difference curve is plotted in figure 5 as curve *B* and the intercept of this curve gives the absolute power factor of *Cp* as 7.24×10^{-6}.

The three tests, although made with different standard capacitors, give the same value for the absolute power factor of *Cp* to within the sensitivity of the bridge.

If this method is correct, then it should be a simple matter to determine the absolute power factor of a variable capacitor at any given separation. This was checked using the small glass-enclosed capacitors *C*1 and *C*2. In the first experiment, the capacitor *C*2 was used as the specimen capacitor and the capacitor *C*1 was used as the variable capacitor. The results of this test are plotted in figure 6 and the intercept gives the absolute power factor of *C*2 at a capacitance of 63.7 micromicrofarads as 33.8×10^{-6}. Since the absolute power factor of *C*2 is now known, the absolute power factor of *C*1 can be read directly from the curve in figure 6, for any value of capacitance. If then *C*1 is held fixed and the power factor difference measured against another variable capacitor the intercept of this curve should be the value as read from figure 6.

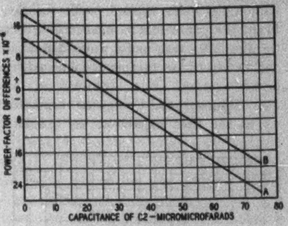


Figure 7. Power-factor difference, capacitor C1 fixed versus capacitor C2 variable

Curve *A*—C1 fixed at 44.5 micromicrofarads

Curve *B*—C1 fixed at 59.2 micromicrofarads

For example, with *C*1 set at a capacitance of 44.5 micromicrofarads, its absolute power factor as read from figure 6 is 14.8×10^{-6}. A test was now made with capacitor *C*1 set at this value of capacitance and used as the fixed capacitor in the bridge. The other capacitor was varied. The power-factor difference measured is plotted as curve *A* in figure 7. The intercept of this curve gives the absolute power factor of *C*1 at 44.5 micromicrofarads setting as 13.0×10^{-6}.

With the capacitor *C*1 set at a different value of capacitance, namely, 59.2 micromicrofarads, figure 6 gives its absolute power factor as 20.0×10^{-6}. A second run was now made with *C*1 held fixed at this capacitance. The results of this run are given as curve *B* in figure 7. When curve *B* is extended back to the axis, the intercept gives the absolute power factor of *C*1 at the given capacitance as 19×10^{-6}, which checks the value as determined from figure 6.

The experimental data checks the correctness of this method of determining the absolute power factor of an air capacitor. The method which is simple in operation requires that a variable air capacitor be available and that the bridge have the necessary sensitivity.

POWER-FACTOR VARIATION WITH SEPARATION

The variation of the absolute power factor with separation was studied using four capacitors. The capacitors measured were of different designs and consisted of the flat plate capacitor *Cf*, the upright capacitor *Cu*, and the two glass-enclosed capacitors *C*1 and *C*2.

With one capacitor held fixed, the separation of the other capacitor was varied and the power-factor difference measured with 4,000 volts applied to the bridge. From the power-factor difference curves the absolute power factor was obtained as outlined in the section on absolute power factor.

1. *Capacitor Cf*. Using the capacitor *Cp* as the fixed capacitor, the absolute power factor of capacitor *Cf* was obtained for a capacitance range from 200 micromicrofarads to 725 micromicrofarads. The absolute power factor of capacitor *Cf* is plotted in figure 8, curve *A*. It is seen that the absolute power factor varied directly with the capacitance and ranged from 1×10^{-6} to 4×10^{-6}. The absolute power factor of this capacitor immediately after polishing and cleaning is also given in figure 8 as curve *C*. Under these conditions its power factor was found to be very small.

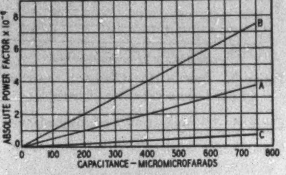


Figure 8. Variation of absolute power factor with separation

Curve *A*—Capacitor *Cf*

Curve *B*—Capacitor *Cu*

Curve *C*—Capacitor *Cf* (after cleaning)

2. *Capacitor Cu*. The absolute power factor of capacitor *Cu* was measured using capacitor *Cp* as the fixed capacitor and varying the capacitance of capacitor *Cu* from 225 micromicrofarads (400 mils separation) to 750 micromicrofarads (120 mils separation). Its absolute power factor was obtained from the power factor difference curve and is also plotted in figure 8, curve *B*. It is seen that the absolute power factor varied directly with the capacitance as was the case for the capacitor *Cf*.

3. *Capacitor C1*. The variation of the absolute power factor of the glass-enclosed capacitor *C*1 was measured using capacitor *C*2 as the fixed capacitor and varying the capacitance of *C*1 from 20 micromicrofarads (316 mils separation) to 70 micromicrofarads (90 mils separation). The absolute power factor *C*1 is plotted in figure 9. The absolute power factor of *C*1 at 20 micromicrofarads was 7.0×10^{-6} and as the plates were brought closer together the absolute power factor varied directly with the capacitance. At 70 micromicrofarads, its value had increased to 24.6×10^{-6}.

4. *Capacitor C2*. The small glass-enclosed capacitor *C*2 was measured against capacitor *C*1 as the fixed capacitor and its absolute power factor, which is plotted in figure 9, was found to vary directly with the capacitance.

In order to make certain that the measured variation in power factor with separation was not caused by an error due to changes in the bridge ratio, the following test was made: The absolute power factors of capacitors *C*1 and *C*2 at a given separation, were determined separately against the flat plate capacitor *Cf* using a one-to-one bridge. Then they were connected in parallel and the power factor of the combination measured. In this latter measurement the bridge ratio was two-to-one, and the measured power factor checked the calculated value of the parallel combination. Similar measurements which were made on a cylindrical capacitor of the sectional type also proved that the results were independent of the bridge ratio.

The curves in figures 8 and 9 show that for the four types of capacitors studied the absolute power factor varies directly with the capacitance, or expressed in terms of separation, the power factor varies inversely as the separation of the plates. In all of these four runs the voltage stress was less than 50 volts per mil.

Equivalent Circuit of a Low-Loss Air Capacitor

Two equivalent circuits[4] are used to represent capacitors with loss. In the parallel notation the loss is represented by a high resistance, *Rp*, in parallel with a pure capacitance *Cp*. The power factor of the combination is one divided by $2\pi fCpRp$ where f is the frequency. In the series notation, the loss is represented by low resistance *Rs* in series with a pure capacitor *Cs*. Its power factor is given by the expression $2\pi fCsRs$. At low values of power factor *Cp* and *Cs* are equal.

A study of the results of the measurements of the variation of the absolute power factor with separation of the plates of a capacitor, figures 8 and 9, shows clearly that the series notation is the simpler and is preferable, since a constant resistance in series with a variable pure capacitor will give identical power factor versus capacitance curves.

The use of the parallel notation to represent a low-loss air capacitor would require that as *Cp* is increased by reducing the spacing *Rp* must decrease. For example, if the value of *Cp* is doubled, then *Rp* must decrease in the ratio of four-to-one. The parallel notation does not offer a simple representation of the equivalent circuit of a low-loss air capacitor.

Effect of Surface Roughness

The variation of the absolute power factor with the roughness of the surface was measured using the small glass-enclosed capacitor. Three degrees of roughness were studied; capacitor *C*4 with its plates roughened by actual grooves cut in its surface to a depth of $1/32$ inch, capacitor *C*2 polished with number 200 carborundum, and capacitor *C*3 with a mirror-like surface. Photomicrographs of the plate surfaces of capacitor *C*1, *C*2, and *C*3 are given in figure 3. The surface of the plates of all of the capacitors were clean and bright and free from oxide.

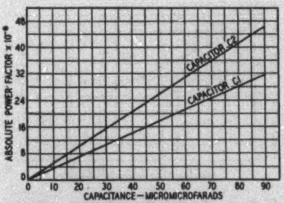


Figure 9. Variation of absolute power factor with separation of capacitor C1 and C2

During the measurements the capacitors were enclosed in the glass cells made from the Pyrex pie plates. The cells were completely shielded and were held at a constant temperature of 28 degrees centigrade during the tests. Capacitor *C*1 with a mirror-like finish was used as the fixed capacitor. Its capacitance was set at 63 micromicrofarads. The other capacitors were connected in turn in the other arm of the bridge and their absolute power factor determined at different values of capacitance. The applied potential was 4,000 volts. The results are given in table II.

It is evident from the results, table II, that the absolute power factor was independent of the roughness of the surface of the plates, provided that voltage stress was below the corona-forming value.

Variation of Absolute Power Factor With Guard Spacing

Measurements of the effect of guard spacing or gap on the absolute power factor were made using the small glass-enclosed capacitor *C*2. The power factor of *C*2 was determined with the normal gap of 0.0625 inch between the guard ring and the measuring electrode. The ratio of the plate separation to gap width was 3.2. A new guard ring was then made, which reduced the gap to 0.005 inch; this gave a ratio of plate separation to gap of 40. The results showed the absolute power factor of *C*2 was independent of the width of the gap between the guard ring and the measuring electrode within the limits used in these measurements.

Table II. Variation of Absolute Power Factor With Roughness

Capacitance (Micromicrofarads)	Absolute Power Factor × 10^{-6} Capacitor C4 $^1/_{32}$-Inch Grooves	 Capacitor C2 Medium Polish	 Capacitor C3 Mirror Surface
20	10.4	10.6	10.2
30	15.8	15.8	15.4
40	21.0	21.0	20.6
50	26.2	25.8	25.6
60	31.6	30.6	30.8
70	36.8	36.0	35.8

POWER-FACTOR VARIATION
WITH HUMIDITY

The effect of humidity on the power factor of air capacitors has been reported by several investigators.[5,6] In practice air capacitors are either operated in the open or enclosed in boxes, which are assumed to protect the capacitor from the effects of atmospheric changes. In periods of unsettled weather, erratic results were observed. These led to a study of the effect of changes in the relative humidity of the room upon the absolute power factor of an air capacitor. Capacitor *Cf* was chosen for this study. This capacitor was completely enclosed in a sturdy galvanized sheet-metal box and the box, although not air tight, was of substantial construction and its top fitted tightly in place. No attempt was made to circulate air from the room into the capacitor enclosure.

Table III. Effect of Water Films on Absolute Power Factor

Time Elapsed After Wetting Plates (Minutes)	Absolute Power Factor
0	0.000022 (dry value)
3	Too high to measure
7	Too high to measure
17	0.00154
27	0.00104
42	0.00084
60	0.00031
82	0.00015
103	0.00010
825	0.000035

The capacitance of *Cf* after cleaning and drying was adjusted to 430 micromicrofarads (174 mil separation) and kept fixed at this value. This capacitor was measured against the capacitor *Cp* which was sealed under a pressure of 100 pounds of dry air and therefore was unaffected by room conditions. At the start of the test the relative humidity of the room was 35 per cent. It gradually increased to 70 per cent and then decreased to its initial value over a period of three days due to changing weather conditions. The measurements were made at 4,000 volts and the absolute power factor of *Cf* determined as the humidity varied.

The results are plotted in figure 10. Initially the absolute power factor of Cf was 2.2 × 10^{-6}. As the relative humidity of the room air increased the power factor of *Cf* increased until at 70 per cent relative humidity, it reached a value of 70 × 10^{-6}. As the humidity of the air in the room decreased, the curve departed from the curve for increasing humidity. When, after three days, the relative humidity of the air was again 35 per cent, the power factor of *Cf* was 15 × 10^{-6}. After eight hours at this value of humidity the power factor fell to 6 × 10^{-6}. During the next eight-hour period there was no further decrease in power factor. The capacitor was then opened up and dried by circulating air with a blower. This treatment brought the power factor back to its initial value of 2.2 × 10^{-6}.

The separation of the power-factor curves for increasing and decreasing humidity show clearly that it required appreciable time before the air between the plates was affected. It is also of interest to note that it required considerable time for the moisture to disappear from the plates and insulating supports.

VARIATION OF POWER FACTOR
WITH WATER FILMS

Horst[7] has reported that the power factor of air capacitors is caused largely by the leakage over the surface of insulating supports due to the presence of adsorbed water films. It was noticed in the course of this work that immediately after the washing and the drying of a capacitor that its power factor was unduly high and that after a period of some six or eight hours, the power factor reached a constant low value. It is reasonable to assume that during the 15-minute circulation of dry air by means of the high-speed blower that all except the absorbed layers of moisture were removed from the capacitor.

In order to get a quantitative picture of the effect of the presence of water films upon the plates of a capacitor; one of the small capacitors, whose absolute power factor under normal conditions of dryness was known, was opened and the plates wiped with a damp cloth. The capacitor was reassembled and measurements started immediately. The results of this test are given in table III.

It was impossible to balance the bridge until 17 minutes after the layer of water had been placed on the plates. The power factor of that moment was 70 times its initial value. It continued to decrease with time; at first quite rapidly and then more slowly, until after approximately 14 hours it had reached a value of less than twice its initial dry value.

In this test where the water was actually put on the plates, it is natural to expect that it would begin to evaporate immediately and to fill the space between the plates with vapor. Water vapor when it evaporates under these conditions is free from ions. However, the high power factor noted is believed to have been caused by the presence of adsorbed layers of water not only on the plates but on the insulating supports as well. As this water passed off into the atmosphere of the capacitor, the power factor decreased.

Summary and Conclusions

1. A method is presented for determining the absolute power factor of an air capacitor by direct measurement. The air capacitor whose absolute power factor is desired is held fixed and the power factor difference measured as a function of the separation of the plates of the other air capacitors in the bridge. When the power-factor difference-capacitance measurements are plotted, the result is a straight line, whose intercept with the axis gives the absolute power factor of the fixed capacitor.

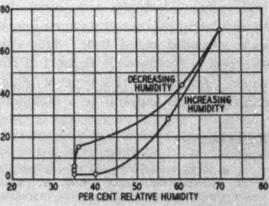


Figure 10. Variation of absolute power factor with humidity

2. The experimental results presented show that in an air capacitor with brass plates whose surfaces are clean, dry, and

free from oxides and dust that the absolute power factor is:

(*a*) Independent of the voltage stress up to 50 volts per mil,

(*b*) Varies inversely with the separation between its plates,

(*c*) Not affected by the roughness of the surface,

(*d*) Independent of the width of the gap or space between its guard ring and measuring electrodes.

3. The simplest and preferable method of representing the equivalent circuit of an air capacitor is that given by the series notation.

4. High atmospheric humidity increases the power factor of an air capacitor even when it is enclosed in a substantial metal box. The increase does not disappear immediately when the relative humidity of the air falls, as it requires several days for the adsorbed moisture to evaporate. The presence of layers of water on the plate surfaces and insulating supports produces a considerable increase in power factor. It is recommended therefore that an air capacitor be protected from the atmosphere and that the air within the enclosure be maintained at a relative humidity of not more than 35 per cent. With perfectly clean capacitors the results indicate that the observed power factor is caused by adsorbed moisture on the plates and insulating supports.

5. It is of interest to note that of the air capacitors studied, the type of construction that gave the lowest power factor was that in which both the low- and high-voltage electrodes were supported by insulating pillars resting upon a common grounded metal base. This construction is illustrated in figure 1 of this paper. Apparently this type of design reduces leakage currents to a minimum.

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Discussion

J. C. Balsbaugh (Massachusetts Institute of Technology, Cambridge): I think Professor Kouwenhoven and Mr. Lotz have made a very interesting and valuable addition to the general subject of the evaluation of losses in air capacitors. There are two points on which I should like to comment:

First, on number 4 of the "Summary and Conclusions," it is stated "The presence of layers of water on the plate surface and insulating supports produces a considerable increase in power factor." I can appreciate that the water on the plate surface of the capacitor will increase the losses, but I do not feel that layers on the insulating supports should cause an increase in the measured loss. The evaporation of any water from the surface of the insulating supports may possibly produce some effect and any effect of the change in the equivalent impedance of the supports, that is, the inherent impedance between bridge and guard circuits, should not have any effect upon the bridge balance and therefore the measured loss.

The second point has to do with number 5 of the "Summary and Conclusions" in which it is stated that "the type of construction that gave the lowest power factor was that in which both the low- and high-voltage electrodes were supported by insulating pillars resting upon a common grounded metal base."

I have in the past attempted to use the same construction in principle corresponding to figure 2 and have found that even with metallic surfaces baked on to the glass for shielding that it was impossible to eliminate completely the loss in the glass from entering into the bridge measurements. This effect could only be completely eliminated by the use of a metal-to-glass seal, as is indicated by figure 4 of the first paper under reference 3, and by figure 5 of the second paper under reference 3.

H. L. Curtis (National Bureau of Standards, Washington, D. C.): In experimental work which has been carried out by means of the method just described by Doctor A. V. Astin, results concerning the effect of humidity on the power factor of an air capacitor with brass plates have been found to be quite different from those described in the paper by Kouwenhoven and Lotz. In their paper they indicate a very rapid increase in the power factor when the humidity exceeds 50 per cent. They state that this may be in part the result of absorption of moisture on the insulators of their capacitors. This seems quite probable since the curve which they have obtained is similar to the well-known curves which represent the surface conductivity of solid insulators as a function of the humidity.

In the work at the National Bureau of Standards the effect of the losses in insulators on the value of the power factor has been completely eliminated. Under this condition, the power factor increases with humidity, reaching a maximum at 60 per cent humidity, then decreasing to practically zero power factor at about 85 per cent humidity. Moreover, the maximum value for power factor is about 10×10^{-6} with an electrode separation of one-half millimeter, whereas Professor Kouwenhoven has obtained nearly ten times that value with a somewhat larger separation.

The reason for the observed change in power factor with humidity is readily explained if one considers that the loss is in an oxide film on the surface of the electrodes. When the film is dry the oxide is a satisfactory dielectric and there is no loss in it. When the film is saturated the film becomes a sufficiently good conductor so that it is a part of the plate and again there is no loss. At intermediate points the film is a semiconductor in which there is considerable loss, which accounts for the maximum value at about 60 per cent humidity.

A. V. Astin (nonmember; National Bureau of Standards, Washington, D. C.): At the Bureau of Standards we have been using a method of determining absolute power factors which is similar to that described by Kouwenhoven and Lotz. We are able to confirm the linear variation of the power factor of a guard ring capacitor with capacitance since that is also the essential element of our method. There are, however, some differences which should be pointed out, the most important being that the same ratio of bridge resistors is used throughout the measurements, whereas in the method just described the resistance ratio is varied to compensate a changing capacitance ratio. The advantage of a constant bridge ratio is that there can be no error due to changing phase angles in the variable resistors.

The power-factor change with capacitance is determined, in an equal-ratio bridge, as follows: The power-factor differences between the guard ring capacitor and two auxiliary capacitors of equal capacitance are measured. The two auxiliary capacitors are then connected in parallel and compared to the guard ring capacitor at a new capacitance setting equal to the combined capacitance of the parallel auxiliaries. From the measured differences the change in power factor of the guard ring capacitor between the two settings is determined by a simple computation. By using auxiliary capacitors of different magnitudes the power factor variation of any capacitor over its full-scale range may be determined. In practice it has been found more convenient to calibrate, in the manner just described, a rotating plate capacitor having a third electrode in order to eliminate the effect of losses in solid insulation. Such a calibration curve is shown by *A* in figure 1 of this discussion. Power-factor values on this curve are fixed arbitrarily by the double-circled point which represents the power-factor difference between the rotating plate condenser and a cylindrical, guard ring capacitor of wide electrode spacing and which serves as a standard of power factor. Next the power-factor differences between the guard ring capacitor and the rotating electrode capacitor are measured for a range of capacitance values. Such a set of measurements is shown by curve *B*. Then the values for the rotating plate capacitor are added to these differences to give the actual values for the guard ring capacitor. The result is shown by curve *C*, which is the sum of curves *A* and *B*.

The straight line verifies the observation of Kouwenhoven and Lotz that the power factor of a guard ring capacitor is linear with capacitance. The zero intercept indicates that the standard capacitor used to determine the double-circled point on *A* had zero power factor within the limits of the measuring equipment. The curves *A* and *C* then give the absolute power factor values of the rotating electrode and guard ring capacitors, respectively. Were the intercept of the curve *C* other than zero the curves *A* and *C* would have to be shifted vertically by the amount of the intercept in order to give absolute values.

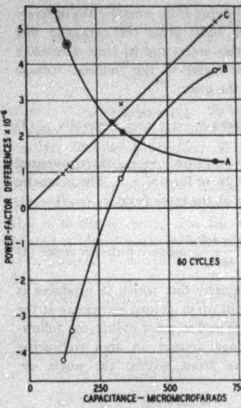

Figure 1

H. W. Bousman (General Electric Company, Schenectady, N. Y.): The authors' work on air-capacitor losses is particularly timely because of the increased use of guarded adjustable capacitors with small plate spacings. These capacitors commonly have constant spacing and variable area; hence their power factor may not be measured by comparing two of them. They are often made with aluminum plates; therefore, it would be interesting to have the magnitude of the effect described in this paper estimated for aluminum with its various surface finishes.

In this connection it is interesting to note that high-voltage standard air capacitors with nickel-plated electrodes have slightly lower power factor at high relative humidity than capacitors similar except for the use of brass electrodes, even when the brass is freshly cleaned. Because of the large spacing used at high voltage, the effect is about tenfold smaller than that reported by the authors; therefore rather difficult to measure accurately at low relative humidity.

Concerning the absolute power factor of an air capacitor, consider two capacitors, one with variable electrode spacing, both with a dielectric characterized by constant power factor at the frequency used for a series of measurements. The equivalent series resistance of the capacitor with variable spacing will be a constant plus a term inversely proportional to the capacitance. When this function is put into the authors' formula for power-factor difference, the result is a linear relationship between the power-factor difference and the capacitance of the adjustable capacitor, experimentally indistinguishable from the relationship found if the dielectric has zero power factor. The intercept at zero capacitance means not the absolute power factor but a quantity smaller by the power-factor characteristic of the dielectric. Thus, valuable as the authors' work is in finding certain losses in air capacitors, it is still necessary to rely on indirect evidence for the absolute power factor.

W. B. Kouwenhoven and E. L. Lotz: Doctor Astin's confirmation of the determination of the absolute power factor of an air capacitor is a very valuable check of our method, particularly when he used a constant-ratio bridge and we used a variable-ratio bridge. In our case, it was necessary to make a thorough study of the accuracy of the bridge at various ratios. This was done in the following manner. The absolute power factor of several similar capacitors was determined by the zero-intercept method. The capacitors were then connected in parallel and the absolute power factor of the parallel combination determined by the same method. Since the absolute power factor of the individual capacitors was known, the absolute power factor of the parallel combination could be easily calculated and this was compared with the measured value. It was found that for the ratios used, the bridge gave the correct values.

In the method employed by Doctor Astin, it was necessary for him to assume one of the capacitors as a standard and later measurements showed that it had a negligible power factor. In our case, this was not necessary as the absolute power factor of the fixed capacitor is determined by the zero intercept of the power-factor-difference curve.

The points brought out by Doctor Bousman are of value. All of our experiments were conducted with brass-electrode capacitors, and his results showing extremely low values of power factor with nickel-plated electrodes are very interesting. Doctor Balsbaugh[3] has reported also that capacitors with brass electrodes have a lower power factor than capacitors with aluminum plates and chromium-plated brass plates.

The absolute power factor of an air capacitor is referred to by Doctor Bousman as the value less the loss due to the dielectric air. Since air, or some other gas is used as a dielectric, it is our view that the absolute power factor must include the loss in the dielectric as well as any other surface phenomena losses and leakage losses. Both of the capacitors in our bridge had air as their dielectric. It is well known that due to radium emanations, cosmic rays, and other sources there are six or seven pairs of ions produced per cubic centimeter per second. The presence of these ions will naturally cause a loss in an air capacitor. Calculation of this loss in our air capacitors, one of which contained varying volumes of the dielectric air, show that the loss is far too small to be detected by our present measuring equipment, the power factor being of the order of 2×10^{-10}.

Doctors Curtis and Balsbaugh refer to the effect of humidity upon the power factor of an air capacitor. The results of Doctor Curtis wherein he obtains a maximum power factor at 60 per cent relative humidity are extremely interesting as are his comments as to the cause. In our study, we were very careful to avoid oxidation of the plates other than what would occur by the normal exposure to atmosphere air for a short period. The effect of oxidation seems to be an important one and perhaps some more work along these lines will be forthcoming.

Both Doctors Curtis and Balsbaugh recognize that the presence of moisture in a capacitor causes an increase in the loss. When we applied a layer of water to the plates, the increase in loss amounted to 22×10^{-6} watts, and there was also an increase in the capacitance of the capacitor of five parts in a thousand. The calculated thickness of the water film to account for this capacitance increase amounted to 0.00038 centimeter (81 being taken as the dielectric constant of the water). If we assume that the increase in loss is caused by the Joule effect due to the charging current flowing through this film of water, then the water must have a specific resistance of 4.8×10^{10} ohms per centimeter cubed. This value of specific resistance for water is approximately one hundred times greater than the value for pure water. The increase in loss could not be explained by calculations based on either the Maxwell's theory of a two-layer dielectric as extended by Wagner, or on the Debye theory of the loss due to the polarization of the water molecule. The calculation of the possible loss which could be attributed to conduction through the glass supports also failed to account for the observed value. A careful measurement of the resistance of the capacitor leads also eliminated that source. The quiet evaporation of water does not produce ionized water vapor, and therefore the loss cannot be attributed to this. The exact mechanism by which the presence of moisture produces an increased loss in an air capacitor is not fully understood. We believe that leakage over insulation due to adsorbed water film is a contributing factor in the increase in loss.

Basic Equations for Electric and Sound Radiations

By ALAN HAZELTINE
FELLOW AIEE

THE purpose of this discussion is to derive the basic equations for electric radiation and for sound radiation by elementary calculus, without using the notions of differential equations or of vector analysis. The essential step is the selection of an elementary wave pulse discontinuous only at its front and not involving double layers nor regions of infinite energy at the wave front.

The procedure for *electric radiation* will be: to assume an elementary wave pulse having a certain current distribution; thence to find the distributions of magnetic intensity H ampere-turns per meter and electric intensity e volts per meter, employing the law of magnetomotive force, the law that dielectric current is the time derivative of dielectric flux, and the proportionalities between the two intensities and their flux densities, as given by the proportionality factors for the medium, permittivity κ and permeability μ; and finally to show that the remaining fundamental law, that of induced voltage, is everywhere satisfied.

In figure 1*a* is represented an *open linear electric radiator*, or antenna, of differential length l meters, whose current starts from zero at the time τ seconds and increases at the constant time rate D_tI amperes per second (the symbol D being used for *derivative*). This current flows through space as a dielectric current $D_t\Psi$ coulombs per second, and consists of three superposed components: a current flowing uniformly out in all directions from the top of l for a distance r_w meters; a current flowing uniformly in to the bottom of l; and a current sheet joining these on the surface of the sphere of radius r_w. This sphere is the *wave front* of the elementary pulse and expands at some constant velocity c meters per second, to be determined. The resultant current distribution is shown in figure 1*a*, together with the magnetic flux Φ webers which it produces around the axis of the radiator, as represented by the dots and crosses. The current also builds up a dielectric flux Ψ coulombs, extending from the top of l to the bottom, as shown in figure 1*b*.

The total current at any time $t > \tau$ being $(t - \tau)D_tI$, the component current from the top of l, within the angle θ, will be $(t - \tau)D_tI(1 - \cos\theta)/2$; and the resultant current within θ at the radial distance $r \leqq r_w = c(t - \tau)$ will be:

$$D_t\Psi = -lD_z\left[(t-\tau)D_tI\,\frac{1-\cos\theta}{2}\right]_x$$

$$= \frac{l(t-\tau)D_tI}{2}\,D_z\,\frac{z}{(x^2+z^2)^{1/2}}$$

$$= \frac{l(t-\tau)D_tI}{2}\cdot\frac{x^2}{(x^2+z^2)^{3/2}}$$

$$= \frac{l\sin^2\theta\cdot(t-\tau)D_tI}{2r} \qquad (1)$$

where z is the distance along the axis, as represented in figure 2*a*. This current is equal to the magnetomotive force along the path surrounding it, of length $2\pi r \sin\theta$; so the magnetic intensity at (r,θ) is:

$$H = \frac{D_t\Psi}{2\pi r\sin\theta} = \frac{l\sin\theta\cdot(t-\tau)D_tI}{4\pi r^2}$$

ampere-turns per meter (2)

Multiplying H by μ and by the differential area $rd\theta dr$ gives the magnetic flux within this area; and its time derivative must be equal to the induced voltage around the area:

$$D_t(H\mu rd\theta dr) = \frac{\mu l\sin\theta\cdot D_tI}{4\pi r}d\theta dr \text{ volts} \qquad (3)$$

which is valid for $r = r_w$, as represented in figure 2*b*, or for $r < r_w$. The magnetic intensity at the wave front, $r_w = c(t - \tau)$, is:

$$H_w = \frac{l\sin\theta\cdot D_tI}{4\pi cr_w} \text{ ampere-turns per meter} \qquad (4)$$

The magnetic flux which it produces is being built up in an area increasing at the rate $cr_wd\theta$ and so must induce the following voltage around an area embracing the wave front within the angle $d\theta$, figure 2*b*:

$$\mu H_wcr_wd\theta = \frac{\mu l\sin\theta\cdot D_tI}{4\pi}\,d\theta \text{ volts} \qquad (5)$$

The total dielectric flux Ψ within the angle θ at the radial distance r is the integral of its derivative $D_t\Psi$, from the time when the wave front reaches r:

$$\Psi = \int_{r/c}^{t-\tau} D_t\Psi d(t-\tau)$$

$$= \frac{l\sin^2\theta\cdot D_tI}{2r}\cdot\frac{1}{2}\left[(t-\tau)^2 - \frac{r^2}{c^2}\right]$$

coulombs (6)

Its negative r-derivative—$D_r\Psi$, divided by the length $2\pi r\sin\theta$ of the differential area through which it passes, is the dielectric flux density normal to r at (r, θ). Dividing this by κ gives the electric intensity e normal to r:

$$e = \frac{1}{\kappa}\cdot\frac{-D_r\Psi}{2\pi r\sin\theta}$$

$$= \frac{l\sin\theta\cdot D_tI}{4\pi\kappa r}\cdot\frac{1}{2}\left[\frac{(t-\tau)^2}{r^2}+\frac{1}{c^2}\right]$$

$$= \frac{l\sin\theta\cdot D_tI}{4\pi\kappa r^3}\cdot\frac{1}{2}\left[(t-\tau)^2+\frac{r^2}{c^2}\right]$$

volts per meter (7)

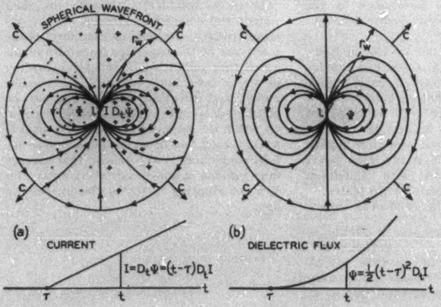

Figure 1 (left). Elementary elective wave pulse

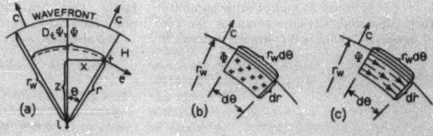

Figure 2. Electromagnetic relations in elementary wave pulse

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Just inside the wave front, the radial dielectric flux density is zero, since it has had no time to build up. So the total voltage around the differential area $r_w d\theta dr$, figure 2c, is the difference between the tangential voltages at r_w and $r_w - dr$, which is:

$$-D_r(erd\theta)dr =$$
$$-\frac{l \sin \theta \cdot D_t I}{4\pi\kappa} \cdot \frac{1}{2}(t-\tau)^2\left(\frac{-2}{r^3}\right) d\theta dr$$
$$= \frac{l \sin \theta \cdot D_t I}{4\pi\kappa c^2 r_w} d\theta dr \text{ volts} \tag{8}$$

since $r = r_w = c\,(t - \tau)$. Subsequent building up of Ψ does not alter this total voltage, because the added Ψ is distributed in the same way as in a steady field around two oppositely charged conductors at the ends of l (since the current $D_t\Psi$ is so distributed); and the voltage around a closed path in such a field is zero. Hence, if we put r for r_w in the final expression in (8), it will give the total voltage around an area $rd\theta dr$ at all times after the passing of the wave front. Just inside the wave front, the tangential electric intensity e_w is found by putting $c\,(t - \tau) = r = r_w$ in (7):

$$e_w = \frac{l \sin \theta \cdot D_t I}{4\pi\kappa c^2 r_w} \text{ volts per meter} \tag{9}$$

The tangential electric intensity outside the wave front and the radial intensities at the wave front being zero, the total voltage around the differential area embracing the wave front within the angle $d\theta$ is:

$$e_w r_w d\theta = \frac{l \sin \theta \cdot D_t I}{4\pi\kappa c^2} d\theta \text{ volts} \tag{10}$$

Comparing the modified expression (8) with (3) and (10) with (5), it will be seen that the law of induced voltage will be satisfied everywhere provided that $\mu = 1/(\kappa c^2)$, or $c = (\kappa\mu)^{-1/2}$. *Hence our elementary wave pulse satisfies all the fundamental electromagnetic laws.* Inserting the values for free space, $\kappa = 10^{-12}\,8.85$ farad in a meter cube and $\mu = 10^{-6}\,1.257$ weber per ampere-turn in a meter cube, gives the velocity of propagation in free space, $c = (10^{-12}\,8.85 \cdot 10^{-6}\,1.257)^{-1/2} = 10^8\,2.998$ meters per second, equal to the velocity of light.

Any varying radiator current I can be resolved into differential elementary wave pulses starting at successive instants τ, as represented in figure 3. The values of H and e at (r, θ) are the sums of the values due to these elementary wave pulses; but, at any instant t, only those pulses which started prior to $t - r/c$ contribute, since the wave fronts of later pulses have not yet reached (r, θ). To combine the component pulses, it is convenient to express H and e in terms of quantities existing at $t - r/c$, eliminating τ. In any elementary pulse at $t - r/c$, the current is:

$$I = \left(t - \frac{r}{c} - \tau\right)D_t I \text{ amperes} \tag{11}$$

Thence

$$(t - \tau)D_t I = \frac{r}{c} D_t I + I \tag{12}$$

which substituted in (2) gives H in terms of $D_t I$ and I at $t - r/c$:

$$H = \frac{l \sin \theta}{4\pi cr}\left(D_t I + \frac{r}{c} I\right)$$
$$\text{ampere-turns per meter} \tag{13}$$

The time integral of the current from τ (or from $-\infty$, since it is zero before τ) to $t - r/c$ is the dielectric flux Ψ of the radiator at $t - r/c$:

$$\Psi = \int_{-\infty}^{t-r/c} Id\left(t - \frac{r}{c}\right)$$
$$= \frac{1}{2}\left(t - \frac{r}{c} - \tau\right)^2 D_t I \text{ coulombs} \tag{14}$$

Thence, applying (12), we have:

$$\frac{1}{2}\left[(t-\tau)^2 + \frac{r^2}{c^2}\right] D_t I = \frac{r}{c}(t - \tau)D_t I +$$
$$\int_{-\infty}^{t-r/c} Id\left(t - \frac{r}{c}\right) = \frac{r^2}{c^2} D_t I + \frac{r}{c} I +$$
$$\int_{-\infty}^{t-r/c} Id\left(t - \frac{r}{c}\right) \tag{15}$$

which substituted in (7) gives [since $\mu = 1/(\kappa c^2)$]:

$$e = \frac{\mu l \sin \theta}{4\pi r}\left[D_t I + \frac{c}{r} I + \right.$$
$$\left.\frac{c^2}{r^2}\int_{-\infty}^{t-r/c} Id\left(t - \frac{r}{c}\right)\right]$$
$$\text{volts per meter} \tag{16}$$

Now we may sum all expressions of the form (13) or (16) for the elementary wave pulses started before $t - r/c$. The sum of the individual $D_t I$, I and Ψ will be the total $D_t I$, I and Ψ at $t - r/c$. *So* (13) *and* (16) *are the final formulas for* H *and* e *in any wave from a differential open linear radiator.* To apply these formulas to an antenna of finite length, we write dl in place of l and integrate with respect to l along the antenna, as usual, taking into account the changes in θ, r, and I with the position of dl.

Radiation from a *closed linear electric radiator*, or loop, of differential enclosed area S square meters is similar to that from an open radiator, but with an interchange of the electrostatic and the magnetic quantities. Such a loop carrying a current I amperes constitutes a magnetic doublet of moment $l\Phi = \mu SI$ weber-meters, Φ webers being the flux of a magnet of differential length l meters. So we must put for $lI = lD_t\Psi$ the expression $lD_t\Phi = \mu SD_t I$. Interchanging H and e, μ and κ, equations (16) and (13) then give, as the final formulas for H and e in any wave from a differential closed linear radiator:

$$H = \frac{S \sin \theta}{4\pi c^2 r}\left(D_t^2 I + \frac{c}{r} D_t I + \frac{c^2}{r^2} I\right)$$
$$\text{ampere-turns per meter; and} \tag{17}$$

$$e = \frac{\mu S \sin \theta}{4\pi cr}\left(D_t^2 I + \frac{c}{r} D_t I\right)$$
$$\text{volts per meter} \tag{18}$$

To apply these formulas to a coil of finite area, we write dS in place of S and integrate with respect to S over the area of the coil, as usual, taking into account the changes in θ, r, and I with the position of dS.

If the current I of a differential radiator is varying sinusoidally, as

$$I = I_m \sin \omega t = I_m \sin 2\pi ft$$
$$= I_m \sin \frac{2\pi ct}{\lambda} \text{ amperes}$$

where ω radians per second and f cycles per second represent frequency and $\lambda = c/f = 2\pi c/\omega$ meters is the wave length, then (13), (16), (17), (18) reduce to the following:

$$H = \frac{l \sin \theta}{4\pi cr} I_m\left(\omega \cos \omega t + \frac{c}{r} \sin \omega t\right)$$
$$= \frac{l \sin \theta}{2\lambda r} I_m\left(\cos \frac{2\pi ct}{\lambda} + \frac{\lambda}{2\pi r} \sin \frac{2\pi ct}{\lambda}\right) \tag{13'}$$

$$e = \frac{\mu l \sin \theta}{4\pi r} I_m\left(\omega \cos \omega t + \frac{c}{r} \sin \omega t - \right.$$
$$\left.\frac{c^2}{r^2\omega} \cos \omega t\right) = \frac{\mu cl \sin \theta}{2\lambda r} I_m\left[\cos \frac{2\pi ct}{\lambda} + \right.$$
$$\left.\frac{\lambda}{2\pi r} \sin \frac{2\pi ct}{\lambda} - \left(\frac{\lambda}{2\pi r}\right)^2 \cos \frac{2\pi ct}{\lambda}\right] \tag{16'}$$

$$H = \frac{S \sin \theta}{4\pi c^2 r} I_m\left(-\omega^2 \sin \omega t + \frac{\omega c}{r} \cos \omega t + \right.$$
$$\left.\frac{c^2}{r^2} \sin \omega t\right) = \frac{\pi S \sin \theta}{\lambda^2 r} I_m\left[-\sin \frac{2\pi ct}{\lambda} + \right.$$
$$\left.\frac{\lambda}{2\pi r} \cos \frac{2\pi ct}{\lambda} + \left(\frac{\lambda}{2\pi r}\right)^2 \sin \frac{2\pi ct}{\lambda}\right] \tag{17'}$$

$$e = \frac{\mu S \sin \theta}{4\pi cr} I_m\left(-\omega^2 \sin \omega t + \frac{\omega c}{r} \cos \omega t\right)$$
$$= \frac{\pi\mu c S \sin \theta}{\lambda^2 r} I_m\left(-\sin \frac{2\pi ct}{\lambda} + \frac{\lambda}{2\pi r} \cos \frac{2\pi ct}{\lambda}\right) \tag{18'}$$

When r is large compared with λ, only the first terms are appreciable, as given in the usual radiation formulas (e.g., Del-

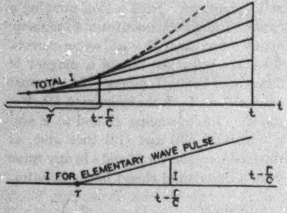

Figure 3 (left). Superposition of elementary wave pulses

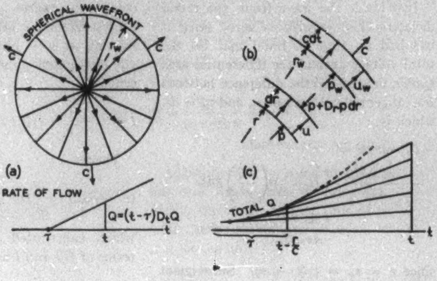

Figure 4. Sound radiation from point source

linger, "Scientific Papers of the Bureau of Standards," volume 15, pages 454, 489).

The procedure for *sound radiation* in a gas will be like that for electric radiation: to assume an elementary wave pulse having a certain distribution of rate of flow; thence to find the distributions of particle velocity u meters per second and excess pressure p joules per meter per square meter, employing the relation that velocity is rate of flow per unit sectional area, the law of continuity and the adiabatic law; and finally to show that the remaining fundamental law, Newton's law, is everywhere satisfied. As usual, the excess pressure p is supposed very small in comparison with the normal pressure p_0 of the gas.

In figure 4*a* is represented a *point source* from which gas flows at a rate Q cubic meters per second, starting from zero at the time τ and increasing at the constant time rate D_tQ. This rate of flow extends uniformly out in all directions for a distance r_w meters, beyond which the gaseous medium is quiescent. The sphere of radius r_w is the *wave front* of the elementary wave pulse and expands at some constant velocity c meters per second, to be determined. Since Q is the same at all radii $r < r_w$, the mass of gas in any region within the wave front does not change with time—that is, the excess density ρ kilograms per cubic meter at any point rises suddenly from zero when the wave front passes and thereafter does not change.

At all radii $r < r_w = c(t - \tau)$, the total rate of flow Q is:

$$Q = (t - \tau)D_tQ \text{ cubic meters per second} \quad (19)$$

Dividing this by the spherical area $4\pi r^2$ gives the particle velocity:

$$u = \frac{(t - \tau)D_tQ}{4\pi r^2} \text{ meters per second} \quad (20)$$

The actual density being very nearly the normal density ρ_0, the rate of increase in momentum per unit area of a shell of differential thickness dr is:

$$\rho_0 dr \cdot D_t u = \frac{\rho_0 D_tQ}{4\pi r^2} dr \text{ joules per meter per square meter} \quad (21)$$

This must be equal to the difference in pressure on the two sides of the shell. The particle velocity u_w at the wave front, $r = r_w = c(t - \tau)$, is:

$$u_w = \frac{D_tQ}{4\pi c r_w} \text{ meters per second} \quad (22)$$

As the wave front advances with the velocity c, the rate of increase in momentum per unit area of the wave front, which must be equal to the excess pressure p_w just inside the wave front, is:

$$\rho_0 c u_w = \frac{\rho_0 D_tQ}{4\pi r_w} \text{ joules per meter per square meter} \quad (23)$$

In an area dS of the wave front, during an interval dt, the inflow at the velocity u_w increases the mass by $\rho_0 dS \cdot u_w dt$. As the wave front then advances by the distance cdt, figure 4*b*, the excess density ρ_w will be:

$$\rho_w = \frac{\rho_0 dS \cdot u_w dt}{dS \cdot c dt} = \frac{\rho_0 u_w}{c} = \frac{\rho_0 D_tQ}{4\pi c^2 r_w} \text{ kilograms per cubic meter} \quad (24)$$

Since the excess density ρ at any radius r does not change after the wave front passes, it is:

$$\rho = \frac{\rho_0 D_tQ}{4\pi c^2 r} \text{ kilograms per cubic meter} \quad (25)$$

The accompanying excess pressure p is given by the adiabatic law:

$$\frac{p_0 + p}{p_0} = \left(\frac{\rho_0 + \rho}{\rho_0}\right)^{\gamma} \quad (26)$$

or $p/p_0 = \gamma\rho/\rho_0$, very nearly, since p/p_0 is very small. Then, by (25),

$$p = \frac{\gamma p_0 D_tQ}{4\pi c^2 r} \text{ joules per meter per square meter} \quad (27)$$

The difference in pressure acting outwardly on the two sides of a shell of thickness dr, figure 4*b*, is then:

$$-D_r p dr = \frac{\gamma p_0 D_tQ}{4\pi c^2 r^2} dr \text{ joules per meter per square meter} \quad (28)$$

Comparing (28) with (21), and (27) with (23) at $r = r_w$, it will be seen that Newton's law will be satisfied everywhere provided that $\rho_0 = \gamma p_0/c^2$, or $c = (\gamma p_0/\rho_0)^{1/2}$. *Hence our elementary wave pulse satisfies all the fundamental mechanical laws.*

Any varying rate of flow Q from the point source can be resolved into differential elementary wave pulses, as was done with electric radiation and as represented in figure 4*c*. The excess pressure p will still be given by (27), where D_tQ now represents the total value at $t - r/c$, since it is the sum of the individual D_tQ up to $t - r/c$. For the particle velocity u, we proceed as before: the rate of flow of an elementary wave pulse at $t - r/c$ is:

$$Q = \left(t - \frac{r}{c} - \tau\right)D_tQ \text{ cubic meters per second} \quad (29)$$

Thence

$$(t - \tau)D_tQ = \frac{r}{c}D_tQ + Q \quad (30)$$

which, substituted in (20), gives u in terms of D_tQ and Q at $t - r/c$:

$$u = \frac{1}{4\pi c r}\left(D_tQ + \frac{c}{r}Q\right) \text{ meters per second} \quad (31)$$

Summing up the individual D_tQ and Q gives the same formula for the resultant u. *So* (27) *and* (31) *are the final formulas for p and u in any sound wave from a point source.*

In addition to their more elementary character, the preceding derivations, as compared with those usually given, are more general, because they apply to any time variation at the radiator, not merely to sinusoidal variation; they include all terms present, not merely the "wave terms;" and they do not assume a known velocity of propagation, or even that such a velocity exists, but include a proof of the existence and the value of the velocity of propagation.

Amplitudes of Magnetomotive Force Harmonics for Fractional Slot Windings—I

By J. F. CALVERT
MEMBER AIEE

Synopsis: Fractional slot windings have certain well-known attributes which cause them to be quite generally used in synchronous-machine design. However, it is recognized that occasionally a winding will be produced which will develop harmonics in the air gap magnetomotive force that will lead to excessive losses, vibration, and noise. The published methods for computing the amplitudes of these harmonics are laborious to use and make a special case of every winding. It is the purpose of this paper to develop general formulas for computing the distribution factor and pitch factor of any magnetomotive force harmonic set-up by three-phase fractional slot windings of normal design. It is believed that these should permit in many cases a choice of winding arrangement such as either to eliminate or at least to minimize any undesirable harmonic so that its ill effects would be negligible. It is believed that the formulas are sufficiently simple to permit a complete tabulation of distribution and pitch factors over the normal range of slots per phase per pole now used in fractional slot windings.

I. Introduction

THE fractional slot winding will usually introduce magnetomotive force harmonics of pitches greater than two field poles as well as shorter ones. Each of these harmonics travels its own wave length in one electrical cycle. Some travel with and some against the direction of rotation of the rotor. The flux set up by these magnetomotive force waves tends to produce losses, reactance, mechanical vibrations, and noise. These effects may be serious or negligible depending upon design proportions. The general characteristics of these magnetomotive forces are well described in the technical literature (see bibliography), particularly in the paper by Q. Graham where a very careful analysis is given. The earlier paper by B. Hague demonstrates the existence of harmonics of greater than two pole pitches, and illustrates with certain examples.

However, at the present time it is the writer's belief that no methods have been published which permit the easy determination of the harmonic amplitudes. Consequently, it is very difficult if not somewhat impracticable to give each winding the analysis one might wish to. It is intended to present here methods which will permit the easy determinations of these amplitudes. Later the resultant flux, loss, reactances, torques, and noises might well be studied. However, the latter will not be considered as within the scope of the present investigation.

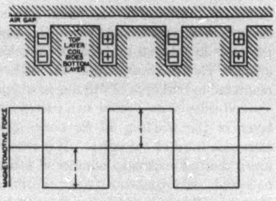

Figure 1. Magnetomotive force for one set of full-pitch coils in a two-layer winding

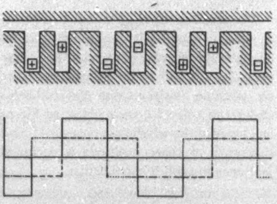

Figure 2. Magnetomotive force for one set of fractional pitch coils in a two-layer winding

— — — — Magnetomotive force of upper coil sides

- - - - - - Magnetomotive force of lower coil sides

———— Resultant magnetomotive force

II. Main Discussion

1. Review of the Problem for Integral Number of Slots per Phase per Pole

Before proceeding to a discussion of the fractional slot windings, it will probably be advisable to review very briefly the case of those with an integral number of slots per phase per pole. The total magnetomotive force per coil-side will be taken as 1.0. If a machine has a full-pitch winding then the starting coils only of each pole group will produce the rectangular magnetomotive force of figure 1. In this figure a reluctance per pole has been assumed which repeats itself cyclically in each pole pitch.

If a short pitch coil is assumed, the magnetomotive force distribution will be as shown in figure 2. So far as the air-gap flux is concerned, it is immaterial how these conductors are connected on the ends, provided the conductor currents in the slots are the same as in the actual winding. Therefore, the top layer coil-sides and the bottom layer coil-sides can be considered separately—each as a set of full-pitch coils. The magnetomotive forces of each of these are shown in broken lines as components, and the resultant is shown as a solid line. The rectangular magnetomotive force of the top layer coil-sides can be written in the form of a Fourier series as,

$$y' = \frac{2}{\pi}\left[\sin\theta + \frac{1}{3}\sin 3\theta + \frac{1}{5}\sin 5\theta + \ldots + \frac{1}{n}\sin n\theta + \ldots \text{etc.}\right] \qquad (1)$$

where $\theta = \pi$ for one rotor pole pitch and where n is always an odd integer. The magnetomotive force of the lower coil sides will be

$$y'' = \frac{2}{\pi}\left[\sin(\theta + \pi - \rho) + \frac{1}{3}\sin 3(\theta + \pi - \rho) + \ldots \text{etc.}\right] \qquad (2)$$

where ρ = the angular throw of the armature coil and is π for one rotor pole pitch.

If a complete phase winding is assumed, the top layer coil sides are displaced successively one from another by one slot pitch. If μ is the angular slot pitch in terms of the synchronous harmonic of wave length equal to two pole pitches, the magnetomotive force of the

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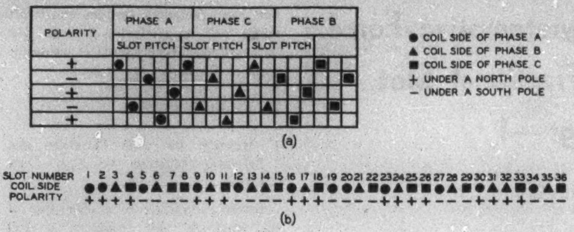


Figure 3. Winding for 6/5th slots per phase per pole with chart equal in width to one pole span. Positive and negative polarity groups alternate by rows

top coil sides only is for the synchronous harmonic

$$A_1' = \frac{2}{\pi} R' \sum_{m=0}^{m=(R'-1)} \sin(\theta - m\mu) \qquad (3)$$

$$= \frac{2}{\pi} R' K_{d1} \sin\left[\theta - \frac{(R'-1)}{2}\mu\right] \qquad (4)$$

For the top and bottom coil sides taken together the synchronous harmonic of the magnetomotive force is

$$A_1 = \frac{4}{\pi} R' K_{d1} K_{p1} \times$$
$$\sin\left[\theta - \frac{(R'-1)}{2}\mu + \frac{(\pi - \rho)}{2}\right] \qquad (5)$$

In the above,

ρ = angular throw of the coil
K_{d1} = the distribution factor for the synchronous harmonic
K_{p1} = pitch factor for the synchronous harmonic
R' = the number of top layer coil-sides per phase per pole

$$\left.\begin{aligned} K_{d1} &= \frac{\sin R'\mu/2}{R' \sin \mu/2} \quad \text{and} \\ K_{p1} &= \cos\frac{(\pi - \rho)}{2} \end{aligned}\right\} \qquad (6)$$

This method of attack will prove useful in part of the work on fractional slot windings, namely the type of winding in which the slots per phase per pole is a fraction.

2. Fractional Slot Windings With R/N Slots per Phase per Pole When N Is an Odd Integer

The problem will be restricted to three-phase machines and consequently where R/N is the slots per phase per pole, with R and N both integers, the denominator, N, must not be an integral multiple of 3 if a balanced winding occupying all slots is to be obtained. For windings of the type to be considered, the slots and consequently the coil sides in either layer of the winding, when considered with respect to the pole pitches, will repeat themselves once every N poles. The fractional slot windings will be discussed in two groups—first those where N is an odd integer, and second those where N is an even integer. A repeatable group will be defined as that winding in one layer which is included between two coil-sides that are of the same phase, carry current in the same direction, and that are an integral multiple of N poles apart. The discussion will be further restricted to that type of winding in which the coil-sides of each phase in a particular layer of the winding all lie under the same one-third of the poles. It is recognized that an enormous number of other balanced arrangements are possible. However, they are used less frequently and are much more difficult types about which to make simple and useful generalizations.

The usual method of layout for a fractional-slot winding is illustrated in figure 3 for the specific case of 6/5 slots per phase per pole. It will be noted that there are six possible positions for the coil-sides under the poles for each phase so far as one layer of the winding is concerned, and that each of these positions is occupied just once in each five-pole span.

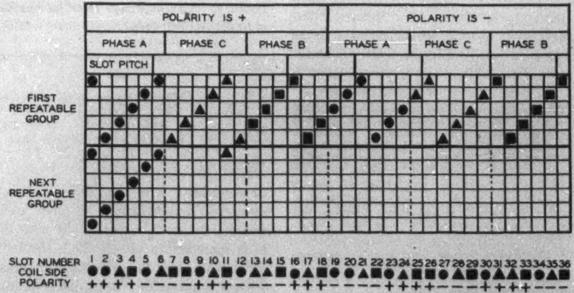
For this case (figure 3) the slot positions repeat exactly once every five poles or 18 slots, and although the currents are equal they are opposite in direction. The coil-sides 18 (or $3R$) slots apart in the top layer may be treated as though they formed coils of pitch equal to five (or N) rotor pole pitches. It can be seen that this will give rise to a first harmonic of five times the normal pole pitch of the machine. (If N is even the coil-sides N pole pitches apart carry currents in the same direction, and a somewhat different treatment will be developed for them later.)

However, it is intended now to treat the coil-sides in one layer N poles apart as an actual coil when N is an odd number. Instead of writing the magnetomotive forces for these "coils" in the sequence in which they appear in the top layer of the winding they will be taken in the sequence in which they appear when reading from left to right by columns in the winding chart (figure 3A) with due regard given to their polarity (or their relative directions of current at any one instant). The total magnetomotive force of the top layer coil-sides of phase A will produce the first harmonic

$$A_1' = \frac{2}{\pi}\left[\sin\frac{\theta}{5} - \sin\left(\frac{\theta}{5} - \frac{11}{18}\pi\right) - \sin\left(\frac{\theta}{5} - \frac{4}{18}\pi\right) + \sin\left(\frac{\theta}{5} - \frac{15}{18}\pi\right) + \sin\left(\frac{\theta}{5} - \frac{8}{18}\pi\right) + \sin\left(\frac{\theta}{5} - \frac{1}{18}\pi\right)\right] \qquad (7)$$

where $\theta = 2\pi$ for two rotor pole pitches. By suitable additions of integral multiples of π to the angular expressions,

Figure 4. Winding for 6/5th slots per phase per pole with chart extended in width to two pole spans so that the positive and negative polarity groups are separated laterally



equation 7, may be rewritten in the form

$$A_1' = \frac{2}{\pi}\left[\sin\frac{\theta}{5} + \sin\left(\frac{\theta}{5} - \frac{29}{18}\pi\right) + \sin\left(\frac{\theta}{5} - \frac{2\times 29}{18}\pi\right) + \sin\left(\frac{\theta}{5} - \frac{3\times 29}{18}\pi\right) + \sin\left(\frac{\theta}{5} - \frac{4\times 29}{18}\right)\pi + \sin\left(\frac{\theta}{5} - \frac{5\times 29}{18}\pi\right)\right] \quad (8)$$

or,

$$A_1' = \frac{2}{\pi}\left[\sin\frac{\theta}{5} + \sin\left(\frac{\theta}{5} + \frac{7}{18}\pi\right) + \sin\left(\frac{\theta}{5} + \frac{14}{18}\pi\right) + \sin\left(\frac{\theta}{5} + \frac{21}{18}\pi\right) + \sin\left(\frac{\theta}{5} + \frac{28}{18}\pi\right) + \sin\left(\frac{\theta}{5} + \frac{35}{18}\pi\right)\right] \quad (9)$$

It will be noted that

$$-\sin\left[\frac{\theta}{N} - \beta''\right] = + \sin\left[\frac{\theta}{N} - (\beta'' - \pi)\right] \quad (10)$$

and that

$$-\sin n\left[\frac{\theta}{N} - \beta''\right] = + \sin n\left[\frac{\theta}{N} - (\beta'' - \pi)\right] \quad (11)$$

when n is an odd integer, but that equation 11 is incorrect when n is even. For the case of N an odd integer a symmetrical rectangular magnetomotive force was obtained for coil-sides exactly N pole pitches apart. This magnetomotive force wave will contain nothing but odd harmonics (equation 1), hence the nth harmonic may be written from either equation 8 or 9, respectively, as:

$$A_n' = \frac{2}{\pi}\left[\sin\left(\frac{n\theta}{5}\right) + \sin n\left(\frac{\theta}{5} - \frac{29}{18}\pi\right) + \ldots + \sin n\left(\frac{\theta}{5} - \frac{5\times 29}{18}\pi\right)\right] \quad (12)$$

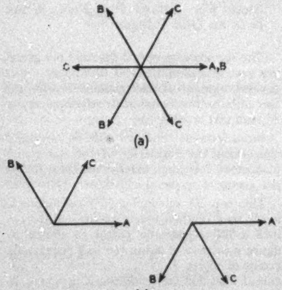


Figure 5. Possible arrangement of space vectors for the first harmonics (those of greatest wave length)

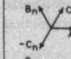
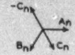
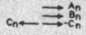
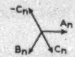
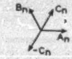
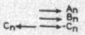
Space Arrangement	Values of n	Magnitude of the Resultant Harmonic in Terms of the Value Per Phase When Produced by Currents of Various Sequences: Positive-Sequence Current	Negative-Sequence Current	Zero-Sequence Current	Case
	1, 4, 7, 10, 13, etc., in general $1 + 3K$, where K is any positive integer or zero	3/2 negative rotation	3/2 positive rotation	0	I - 1
	2, 5, 8, 11, 14, etc. In general $2 + 3K$	3/2 positive rotation	3/2 negative rotation	0	I - 2
	3, 6, 9, 12, 15, etc. In general $3 + 3K$	0	0	3/2 positive rotation and 3/2 negative rotation	I - 3
	1, 4, 7, 10, 13, etc. In general $1 + 3K$	3/2 positive rotation	3/2 negative rotation	0	II - 1
	2, 5, 8, 11, 14, etc. In general $2 + 3K$	3/2 negative rotation	3/2 positive rotation	0	II - 2
	3, 6, 9, 12, 15, etc. In general $3 + 3K$	0	0	3/2 positive rotation and 3/2 negative rotation	II - 3

Figure 6. Chart to show the possible magnitudes and directions of travel of all magnetomotive-force harmonics for the various sequence currents

and

$$A_n' = \frac{2}{\pi}\left[\sin\left(\frac{n\theta}{5}\right) + \sin n\left(\frac{\theta}{5} + \frac{7}{18}\pi\right) + \ldots + \sin n\left(\frac{\theta}{5} + \frac{35}{18}\pi\right)\right] \quad (13)$$

For the particular case, just discussed, of 6/5 slots per phase per pole, the magnetomotive forces for these artificial coils composed of coil-sides N poles apart were made use of. If the magnetomotive forces of these were taken in the order in which the coil-sides appeared by columns on the winding chart, the angular displacement of any harmonic as produced by these successive "coils" differ by a constant angle. This would lead immediately to the determination of the distribution factor k_{dn} for the nth harmonic. That this is always true for N an odd integer, is shown in appendix I. The pitch factor for any harmonic is determined there from the addition of the magnetomotive forces of the top and bottom layers of phase A. Also relative space arrangements of the nth harmonics produced by three separate phase windings are established in appendix I. The results of the latter calculation are summarized in figure 6.

When the slots per phase per pole, R/N, is such that N is an even integer the conductors exactly N poles or $3R$ slots apart again are symmetrically arranged. However, this time the currents are all in the same direction in this group of conductors, and the arrangement does not permit treating adjacent pairs of conductors in the group as artificial coils. Nevertheless, this physical symmetry should not be overlooked in the analysis, and it can be made use of. To this end an artificial current sheet is introduced on the stator teeth tips or the bore of the stator iron (figure 7), which represents a constant amperes per inch of circumference and which is opposite in direction but equal in total magnitude to the sum of the currents in the set of uniformly spaced conductors. This current sheet may even be thought of as an artificial return for the conductor currents of the set. The air gap magnetomotive force wave for these conductors and the current sheet is saw-toothed in form, as shown in figure 7, and it may be represented by a Fourier series. In appendix II it is shown that a uniform angular displacement exists between the nth harmonics when the conductor sets with their return current sheets are considered in the order in which the conductors appear by columns on the winding chart (see figure 8 for a representative chart).

Some complications are introduced by the fact that all odd and all even harmonics appear for N an even integer whereas only odd harmonics appeared in the magnetomotive force wave when N was an odd integer. However, this may be taken care of, and constants obtained which are designated as distribution and pitch factors. The magnetomotive forces for all the phases are considered as before and the relative space arrangements of the *n*th harmonics of the three phases are determined. These also are summarized in figure 6.

In superposing or combining the effects of sets of coils with their current returns it may be shown that the effect of the current sheets will cancel leaving only the effect of the armature conductors. For instance, consider one set of conductors in the top layer of phase *A* with the current sheet return. Then consider the set of coil-sides in the lower layer of phase *A* with its current sheet return, where each coil-side of the lower layer set when paired with the corresponding coil-side in the upper layer set makes a real armature coil. If these two magnetomotive forces are superposed, it is seen that the current sheet associated with the second set of conductors will cancel that associated with the first set because the current sheets are each uniformly distributed over the stator bore, are equal in magnitude, but are opposite in direction. Therefore, this method of analysis may be used in any case where the leads are all brought out on one end of the machine so that the sum of the conductor currents in a given direction is zero per coil and per phase. Balanced currents between phases are not essential for the use of these current sheets in the manner proposed above.

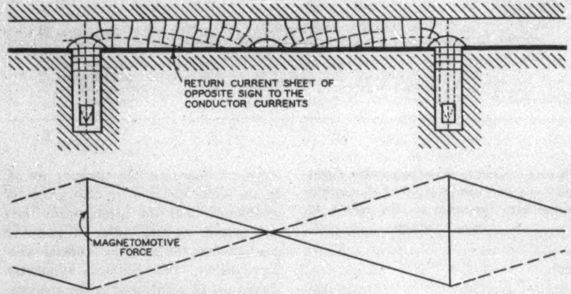

RETURN CURRENT SHEET OF OPPOSITE SIGN TO THE CONDUCTOR CURRENTS
MAGNETOMOTIVE FORCE

Figure 7. Air-gap magnetomotive force for conductor currents of one polarity with return current sheets of the opposite polarity

——— Air-gap magnetomotive force for positive conductor currents

— — — — Air-gap magnetomotive force for negative conductor currents

So far only the magnitude and space arrangement for the magnetomotive forces of the three phases have been discussed, first for *N* an odd and then for *N* an even integer. In appendix III the effect of different phase sequence currents is treated in some detail. Figure 6 indicates the results of this analysis for positive-, negative-, and zero-sequence currents. The conclusions point out the final formula or end results and discuss how these should be used in conjunction with figure 6.

III. Conclusions

1. Figure 6 and equation 61 show practically all the data necessary for computing the amplitudes of the various harmonics. It will be noted that these fall either in cases I-1, I-2, and I-3, or else in cases II-1, II-2, II-3 of figure 6. When $n = 1$, the wave is the first harmonic, that is, the longest possible wave in the machine.

2. All harmonics which exist, travel their own wave length in one electrical cycle.

3. Certain further statements are needed concerning the order of the harmonic, *n*.

(*a*) When *N* is odd. For the first harmonic, $n = 1$. This is the longest harmonic, and has a wave length of $2N$ rotor poles or a full repeatable group. When $n = N$ the wave length is two poles and for positive-sequence currents must travel synchronously with the rotor. This may be used to establish whether the harmonics come under case I or II of figure 6, and from this all the other harmonics and their performance can be found.

(*b*) When *N* is even. The first or longest harmonic $n = 1$ has a wave length of *N* poles or a full repeatable group. When $n = N/2$ the wave length is two poles and for positive-sequence currents must travel synchronously with the rotor. This establishes whether the harmonics come under case I or II of figure 6.

4. It is well known that these harmonics produce losses, reactance (of fundamental frequency), torques, radial forces, and noise. By the judicious choice of slot combinations and pitch factors through the preceding data, it is believed that in a great many cases it should be possible to avoid or at least to minimize the objectionable harmonics. It is recognized that no effort has been made to show at what points around the gap the harmonics will add to cause especially high resultant densities, such as might be expected at the ends of phase bands, but it should be possible to extend the work here to cover this feature, if needed. However, it is believed that at present it is more desirable to have data on the amplitudes of individual harmonics. For instance, if the stator frame and punchings are susceptible to vibration with a two-pole wave around the entire gap, this is one harmonic to be minimized if it can occur in this machine.

5. All possibilities of winding combinations are far from exhausted by those permitted within the laying-out methods shown here. It is not necessary to restrict the coil sides of one phase in a certain layer to a given third under each pole. It is possible to get a balanced winding by a very great variety of methods other than this and certain arrangements prove very useful (see discussion by P. L. Alger of reference 4). At the moment they are probably less used, and certainly they appear to be much less susceptible to generalization.

6. Further work on the flux setup by the type of windings discussed here and its effects on losses, forces, etc., would probably yield results which would more than justify the time invested. The same is probably true of other types of balanced windings such as were mentioned under (5) above.

Appendix I

Fractional Slot Windings With R/N Slots Per Phase Per Pole When N Is an Odd Integer

The specific case of 6/5 ths slots per phase per pole has already been discussed. It is desired now to obtain general results for three-phase fractional slot windings when *N* is an odd integer.

First, general relations will be obtained concerning the regularity of the space displacement of the magnetomotive forces per group of artificial coil-sides.

This can be accomplished most conveniently by a rearrangement of the conductors into a full repeatable group as shown in figure 4 instead of using the half repeatable group of figure 3. From figure 4 it is apparent that for any such winding where *N* is an odd integer that

$$\frac{6R\mathcal{P}_2 + 1}{N} = \gamma_2 \tag{14}$$

where

γ_2 = the number of slots spanned from the coil side in the first to that in the second column on the chart, and

$\mathcal{P}_2$ = the lowest integer which will make γ_2 an integer.

It will be noted that $\mathcal{P}_2$ is the whole number of pairs of poles spanned, and that $\mathcal{P}_2$ is less than N and is single valued. The coil-sides spanned to that of the third column is

$$\frac{2[6R\mathcal{P}_2 + 1]}{N} - K_1 6R = \gamma_3 \qquad (15)$$

where $K_1 = 0$ or 1, the largest integer which will leave γ_3 still a positive integer. If $K_1 = 1$, then the first term in equation 15 will locate the coil-side in the third column, but in the second repeatable group. As a repeatable group represents 2π radians for the first harmonic it is possible to omit the factor $K_1 6R$ and to write the series of terms with the equal angular spacing

$$\beta = \frac{\gamma_2 \pi}{3R} \qquad (16)$$

For figure 4,

$$\left.\begin{aligned} N &= 5 \\ 6R &= 36 \\ \mathcal{P}_2 &= 4 \\ \gamma_2 &= 29 \\ \beta &= \frac{29}{18}\pi \end{aligned}\right\} \qquad (17)$$

It will be seen in figure 4 that all of the positive-polarity conductors are in the left half of the first repeatable group and the negative-polarity conductors are in the right. Each of the positive-polarity conductors may be paired with a negative one $3R$ slots ahead of it (either in the first or the next repeatable group). Consequently, only the locations of positive conductors need be considered in writing the expression for the nth harmonic of top layer phase A coils, and the general expression for this is

$$A_n' = \frac{2}{n\pi} \sum_{m=0}^{m=(R-1)} \sin n\left(\frac{\theta}{N} - m\beta\right) \qquad (18)$$

Equation 18 might be written next in the usual form with a distribution factor included. However, since $n\beta$ may become quite large, it seems advisable to derive the expression without resorting to the usual geometric construction, because the signs of the final terms as well as their magnitudes will be quite significant.
Let

$$S = \sin a + \sin (a - b) + \ldots + \sin (a - rb) \qquad (19)$$

where r is a finite integer.
Multiplying through by $2 \cos b$,

$$2S \cos b = \{2 \sin a \cos b + 2 \sin (a - b) \times \cos b + \ldots + 2 \sin [a - (r - 1)b] \times \cos b + 2 \sin (a - rb) \cos b\} \qquad (20)$$

Expanding each term on the right gives,

$$2S \cos b = \{\sin (a + b) + \sin a + \sin (a - b) + \ldots + \sin [a - (r - 2)b] + \sin [a - (r - 1)b]\} + \{\sin (a - b) + \sin (a - 2b) + \sin (a - 3b) + \ldots + \sin (a - rb) + \sin [a - (r + 1)b]\} \qquad (21)$$

or

$$2S \cos b = \{S + \sin (a + b) - \sin (a - rb)\} + \{S - \sin a + \sin [a - (r + 1)b]\} \qquad (22)$$

and

$$2S[(\cos b) - 1] = \{\sin (a + b) + \sin [a - (r + 1)b]\} - \{\sin (a - rb) + \sin a\} \qquad (23)$$

Combining terms in the brackets on the right gives

$$S(1 - \cos b) = \sin \left(a - \frac{rb}{2}\right) \times \left\{\cos \frac{rb}{2} - \cos (r + 2)\frac{b}{2}\right\} \qquad (24)$$

$$S = \frac{2 \sin b/2 \sin (r + 1)b/2}{2 \sin^2 \frac{b}{2}} \sin \left(a - r\frac{b}{2}\right) \qquad (25)$$

or

$$S = (r + 1)K_d \sin \left(a - r\frac{b}{2}\right) \qquad (26)$$

where

$$K_d = \frac{\sin (r + 1)b/2}{(r + 1) \sin b/2} \qquad (27)$$

Reviewing equations 18, 19, 26, and 27, it is seen that equation 18 may be written as,

$$A_n' = \frac{2}{n\pi} R K_{dn} \sin n \left[\frac{\theta}{N} - \frac{(R - 1)}{2}\beta\right] \qquad (28)$$

where,

$$K_{dn} = \frac{\sin Rn\beta/2}{R \sin n\beta/2} \qquad (29)$$

So far only the top layer coil-sides in phase A have been considered. Let ρ = the chord or pitch angle of the coil where $\rho = \pi$ for a full-pitch coil. The magnetomotive force of the lower layer coil-sides is displaced from the upper by just that amount, and is reversed in sign. Hence the nth harmonic in the total magnetomotive force of phase A may be written as,

$$A_n = \frac{2}{n\pi} K_{dn}\left[\sin n\left\{\frac{\theta}{N} - \frac{(R - 1)}{2}\beta\right\} - \sin n\left\{\frac{\theta}{N} - \frac{(R - 1)}{2}\beta - \frac{\rho}{N}\right\}\right] \qquad (30)$$

Combining terms,

$$A_n = \frac{4R}{n\pi} K_{dn}\left[\sin \frac{n\rho}{2N} \times \cos \left\{\frac{n\theta}{N} - \frac{(R - 1)}{2} n\beta - \frac{n\rho}{2N}\right\}\right] \qquad (31)$$

or,

$$A_n = \frac{4R}{n\pi} K_{dn} K_{pn} \times \sin\left\{n\left[\frac{\theta}{N} - \frac{(R - 1)}{2}\beta - \frac{\rho}{2N}\right] + \frac{\pi}{2}\right\} \qquad (32)$$

where

$$K_{pn} = \sin \frac{n\rho}{2N} = \cos\left(\frac{\pi - n\rho/N}{2}\right) \qquad (33)$$

The next step is to combine the magnetomotive forces of the different phases. This requires certain generalizations concerning the possible arrangements of the space vectors. Referring again to the winding chart of figure 4, let,

$$\left.\begin{aligned} R\beta &= \phi_2' \\ 2R\beta &= \phi_3' \\ 3R\beta &= \phi_4' \end{aligned}\right\} \qquad (34)$$

Note that these are the angular displacements of coil-sides not necessarily in the first repeatable group, yet in the first columns of phases C, and B positive polarity sets and A of the negative polarity set of figure 4. Each angular displacement is expressed with respect to the first coil-side in the phase A positive polarity set in the first repeatable group. The angular positions for the coil-sides in these columns and actually in the first repeatable group will be called ϕ_2, ϕ_3, and ϕ_4 in the same order as above. Then,

$$\left.\begin{aligned} \phi_2 &= \phi_2' - K_2 2\pi \\ \phi_3 &= \phi_3' - K_3 2\pi \quad \text{and} \\ \phi_4 &= \phi_4' - K_4 2\pi \end{aligned}\right\} \qquad (35)$$

where K_2, K_3, and K_4 are the appropriate positive integers which make ϕ_2, ϕ_3, and ϕ_4 greater than zero, $\phi_4 = \pi$ and ϕ_2 and ϕ_3 less than 2π

It can be seen that

$$\left.\begin{aligned} \beta &= \frac{\gamma_2 \pi}{3R} \quad \text{and} \\ \phi_4' &= \gamma_2 \pi = (6R\mathcal{P}_2 + 1)\frac{\pi}{N} \end{aligned}\right\} \qquad (36)$$

It is seen that $6R\mathcal{P}_2$ must be an even integer because 6 is even. Then $(6R\mathcal{P}_2 + 1)$ is an odd integer. Also since N is odd and $N\gamma_2 = (6R\mathcal{P}_2 + 1)$ is odd, it follows that γ_2 is an odd integer. Therefore, equation 35 may be written in the form,

$$\left.\begin{aligned} \phi_2 &= \gamma_2 \pi/3 - K_2\, 2\pi \\ \phi_3 &= \gamma_2\, 2\pi/3 - K_3\, 2\pi \\ \phi_4 &= \gamma_2 \pi - K_4 2\pi \end{aligned}\right\} \qquad (37)$$

where γ_2 is an odd integer.

Placing these on a polar diagram shows the possible relative space positions of the first harmonics of the different phases. Reference to the winding chart of figure 4 will rule out the positions shown dotted in figure 5*a*. By taking differences in equation 37 it is noted that

$$\left.\begin{aligned} \phi_4 - \phi_3 &= \gamma_2\pi/3 - (K_4 - K_3)2\pi \quad \text{and} \\ \phi_3 - \phi_2 &= \gamma_2\pi/3 - (K_3 - K_2)2\pi \end{aligned}\right\} \qquad (38)$$

and it is seen that the possible arrangements are either of the sets shown in figure 5*b*. These are shown again under "Space

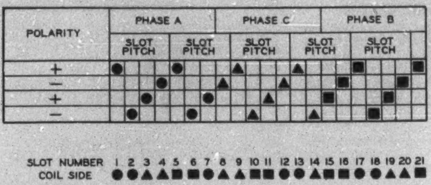


Figure 8. Winding for 7/4 slots per phase per pole—chart and coil arrangement

Arrangements" case I-1 and case II-1 of figure 6.

These possible arrangements were arrived at from the location of the conductors in the positive polarity set in figure 4. Consequently, the relative angular displacements of the third, fifth, seventh, etc., space positions for harmonics can be formed starting with case I-1, figure 6, and increasing the relative angles by three, five, seven, etc., times, respectively. A similar procedure may follow from case II-1 as shown. These diagrams show the relative positions of the space vectors for any one value of n as produced by all three phase windings. No space relations between harmonics of different frequencies are being considered.

In determining the space vectors, only direct currents, or maximum positive currents, in each phase are assumed. The positive directions have been taken as inward on all phase conductors in the top layer under the first pole. Before considering the effects of the various phase currents the arrangement of the space vectors will be established for the cases where N is even.

Appendix II

Fractional Slot Windings With R/N Slots Per Phase Per Pole When N Is an Even Integer

In the preceding discussion of fractional slot windings where N was an odd integer, use was made of the fact that the coil-sides in one layer of the winding located exactly N pole pitches (or a half repeatable group) apart carried currents of equal magnitude but in opposite directions. These were considered as coils, and a symmetrical rectangular magnetomotive force was found. As pointed out before, this is not possible when N is even, because while the coil-sides in one layer which are exactly N poles apart are of the same magnitude they are in the same direction. Figure 7 illustrates the method to be used. Thus, for all three-phase synchronous machines when N is odd, there are $2N$ rotor poles or $6R$ stator slots per repeatable group, and when N is even there are N rotor poles or $3R$ stator slots per repeatable group.

If for the set of positive currents shown in figure 7 a negative current of equal total magnitude is assumed to be uniformly distributed over the bore of the stator iron, the magnetomotive force across the gap can be determined from a field map; but since the gap periphery is large compared to the gap length, the magnetomotive force across the gap will closely approximate the saw-toothed wave shown in the figure. It will be noted here that the slot width has been treated as negligible. This is permissible in that this harmonic analysis is not proposed as accurate for magnetomotive force wave lengths approaching one slot pitch. The treatment should not be used for such short pitch until the errors involved in assuming simple rectangular and triangular magnetomotive force wave shapes have been duly considered. For currents correspondingly spaced, but in the opposite direction, the magnetomotive force wave is as shown in figure 7. That these current sheets are a valid artifice has been shown in the main body of the paper and they now will be accepted as such.

Referring to figure 7, let,

$$\left.\begin{array}{l} f(\xi) = -y/2\,(1+\xi/\pi) \text{ for } -\pi<\xi<0 \\ \text{and} \\ f(\xi) = +y/2(1-\xi/\pi) \text{ for } 0<\xi<+\pi \\ \text{where} \\ \xi = 2\pi \text{ for } N \text{ poles} \end{array}\right\} \quad (39)$$

From the above $f(\xi)$ is an odd function. Hence,

$$f(\xi) = a_1 \sin\xi + a_2 \sin 2\xi + \ldots + a_m \sin m\xi + \ldots + \text{etc.} \quad (40)$$

where

$$a_m = \frac{2}{\pi}\int_0^{\pi} y/2\,[\sin m\xi - \xi/\pi \sin m\xi]d\xi \quad (41)$$

or

$$a_m = y/m\pi \quad (42)$$

Then when the magnetomotive force per coil side, y, is taken to be unity (as was done for the case of N an odd integer), it may be written that,

$$f(\xi) = \frac{1}{\pi}\left[\sin\xi + \frac{1}{2}\sin 2\xi + \frac{1}{3}\sin 3\xi + \ldots + \frac{1}{m}\sin m\xi + \ldots +, \text{etc.}\right] \quad (43)$$

It will be necessary to consider the winding charts next for N even. Figure 8 shows one specific example. Taking the coil side in the first column of phase A as reference, then for the first repeatable group,

γ_2' = coil sides spanned to that coil side of the second column

γ_3' = coil sides spanned to that coil side of the third column

γ_4' = coil sides spanned to that coil side of the fourth column, etc.

Then

$$\gamma_2' = \frac{3RP_2+1}{N} \quad (44)$$

where P_2 = number of whole poles spanned. (Note P_2 is not the same as $\mathcal{P}_2$ used above for N an odd integer).

Similarly,

$$\left.\begin{array}{l} \gamma_3' = \dfrac{2(3RP_2+1)}{N} - K_5 3R \\ \gamma_4' = \dfrac{3(3RP_2+1)}{N} - K_6 3R \\ \quad | \qquad\qquad | \qquad\qquad | \\ \quad | \qquad\qquad | \qquad\qquad | \\ \gamma_l' = (l-1)\dfrac{(3RP_2+1)}{N} - K_{l+2} 3R, \\ \qquad\qquad\qquad\qquad \text{etc.} \end{array}\right\} \quad (44a)$$

where K_5, K_6, K_{l+2} are the largest positive integers which leave γ_3', γ_4', etc., positive integers.

It is seen next that for conductors in adjacent columns

$$\gamma_m' - \gamma_l' = \frac{(3RP_2+1)}{N} - (K_{m+2} - K_{l+2})3R \quad (45)$$

It is evident that $\gamma_2' = [(3RP_2+1)/N]$ is an integer, and since the case now under consideration is that of N an even integer, it follows that $(3RP_2+1)$ must be an even integer. Also, by definition, R is always an odd integer for N an even integer. Hence P_2 is an odd integer. The factor $(K_{m+2} - K_{l+2})3R$ always represents an integral number of repeatable slot groups, and, therefore, indicates an even number of poles spanned. Consequently, the polarity of the conductors (figure 8) taken by columns will always alternate when N is even. The number of coil-sides per repeatable group is R. For N even, R is always an odd integer, as stated above. Therefore, the coil sides in the first columns of phases A and B are of the same polarity, and that of phase C is of the opposite polarity. Taking the first coil side of phase A as positive, there will be $[(R+1)/2]$ positive and $[(R-1)/2]$ negative conductors in phase A and the same in B. Also, there will be $[(R-1)/2]$ positive and $[(R+1)/2]$ negative coil sides in phase C per layer per repeatable group. Due to the alternating polarity of the conductors it is possible to write

$$\left.\begin{array}{l} A_n' = \dfrac{1}{\pi n}\left\{\displaystyle\sum_{m=0}^{m=\frac{R-1}{2}} \sin n\,(\xi - m2\beta') - \right. \\ \displaystyle\sum_{m=0}^{m=\left(\frac{R-3}{2}\right)} \sin n\,(\xi - \beta' - m2\beta') \end{array}\right\} \quad (46)$$

By the aid of equations 19 to 27 this may be rewritten as

$$A_n' = \frac{R}{\pi n}(K_{dn}' - K_{dn}'') \times \sin n\left[\xi - \frac{(R-1)}{2}\beta'\right] \quad (47)$$

$$A_n' = \frac{2R}{\pi n} K_{dn} \sin n\left[\xi - \frac{(R-1)}{2}\beta'\right] \quad (48)$$

where

$$K_{dn}' = \frac{\sin\left(\dfrac{R+1}{2}\right)n\beta'}{R\sin n\beta'} \quad (49)$$

$$K_{dn}'' = \frac{\sin\left(\frac{R-1}{2}\right)n\beta'}{R \sin n\beta'} \qquad (50)$$

$$\left.\begin{aligned} K_{dn} &= \frac{K_{dn}' - K_{dn}''}{2} \quad \text{or} \\ K_{dn} &= \frac{\cos \frac{Rn\beta'}{2}}{2R \cos \frac{n\beta'}{2}} \end{aligned}\right\} \qquad (51)$$

where

$$\left.\begin{aligned} \xi &= 2\theta/N \\ \theta &= 2\pi \text{ for two pole pitches} \\ \beta' &= \frac{\gamma_2'}{3R} 2\pi \end{aligned}\right\} \qquad (52)$$

The pitch factor may be derived much as was done earlier for equations 32 and 33. This time the magnetomotive force of the nth harmonic for the two layers of the A phase winding may be written as

$$A_n = \frac{2R}{\pi n} K_{dn} \left\{ \sin n \left[\xi - \frac{(R-1)}{2} \beta' \right] - \sin n \left[\xi - \frac{(R-1)}{2} \beta' - \frac{2\rho}{N} \right] \right\} \qquad (53)$$

$$A_n = \frac{4R}{\pi n} K_{dn} \left\{ \sin \left[\frac{n\rho}{N} \right] \times \cos n \left[\xi - \frac{(R-1)}{2} \beta' - \frac{\rho}{N} \right] \right\}$$

$$A_n = \frac{4R}{\pi n} K_{dn} K_{pn} \times \sin \left\{ n \left[\xi - \frac{(R-1)}{2} \beta' - \frac{\rho}{N} \right] + \frac{\pi}{2} \right\} \qquad (54)$$

where the pitch factor when N is even is

$$K_{pn} = \sin \frac{n\rho}{N} = \cos \left(\frac{\pi}{2} - \frac{n\rho}{N} \right) \qquad (55)$$

The significance of K_{dn}', K_{dn}'', and K_{dn} should be given further consideration. Reference to equation 29 and its derivation will show that K_{dn} as defined for the case of N an odd integer, is in accord with the usual definition of the distribution factor for integral slot windings. It takes into account only the distributed effect of the armature coils per phase.

However, for the case of N an even integer, it was found convenient to introduce artificial current sheets on the stator bore. Each conductor of a set of conductors with exactly N poles between each adjacent pair, carries current in the same direction. The sum of these currents was assumed to return in the current sheet. Such a set of conductors and their current sheet, when considered as a unit, produced a saw-toothed magnetomotive force wave. The factor K_{dn}' is $[(R+1)/2R]$ times the distribution factor per layer per phase for all of the positive polarity conductors with their artificial current returns. The factor K_{dn}'' is $[(R-1)/2R]$ times the distribution factor for the negative polarity conductors of the same layer and phase taken with their artificial current sheet returns. The factor K_{dn} which is to be called the distribution factor takes into consideration all the conductors per layer for this phase plus just one current return, because on superposing, all such current sheets cancel except a single negative one which goes with one of the positive polarity conductor sets. Hence, the factor K_{dn} is not quite a distribution factor in the usual sense. The effect of one artificial current sheet is included. This will be cancelled out when the effect of the other layer of coil-sides in this phase is added to determine K_{pn} (all for the case of N, an even integer). The coefficients K_{dn} and K_{pn} for the case of N, an even integer, are still defined in this paper as distribution and pitch factors, respectively. While this is not quite in accord with the usual form for integral slot windings, nevertheless, these definitions will be adhered to in what is to follow because they seem to be the most usable forms.

The next step for N even is to determine the space arrangements for the harmonics when all phases are considered. From equation 52,

$$\left.\begin{aligned} R\beta' &= \gamma_2' 2\pi/3 \\ 2R\beta' &= \gamma_2' 4\pi/3 \\ 3R\beta' &= \gamma_2' 2\pi \end{aligned}\right\} \qquad (56)$$

For N an even integer and for the first repeatable group, let,

ψ_2 = angular position of the first coil-side in phase C (by columns on the winding chart) with respect to that of phase A
ψ_3 = same for phase B
ψ_4 = same for phase A of the next repeatable group

Then,

$$\left.\begin{aligned} \psi_2 &= \gamma_2' 2\pi/3 - K_7 2\pi \\ \psi_3 &= \gamma_2' 4\pi/3 - K_8 2\pi \\ \psi_4 &= \gamma_2' 2\pi - K_9 2\pi \end{aligned}\right\} \qquad (57)$$

where K_7, K_8, K_9 are the highest positive integers which will make ψ_2 and ψ_3 each greater than zero and less than 2π and which will make ψ_4 equal to 2π.

Subtracting,

$$\left.\begin{aligned} \psi_4 - \psi_3 &= \gamma_2' 2\pi/3 - (K_9 - K_8)2\pi \\ \psi_3 - \psi_2 &= \gamma_2' 2\pi/3 - (K_8 - K_7)2\pi \end{aligned}\right\} \qquad (58)$$

Equations 57 and 58 establish the two possible space positions of $+A_1$, $-C_1$ and $+B_1$ for the first harmonics as shown by case I-1, and case II-1 of figure 6. The phase C vector appears as $(-C_1)$ in these two cases because the polarity of the first coil side in the C phase by columns on the chart is always negative because of the alternating polarity of conductors in successive columns, and there is one more negative than positive polarity coil side in the phase C group. Caution must be exercised in locating the relative space arrangements of vectors for the second and higher even harmonics. The idea expressed by equations 10 and 11 is applicable here.

Suppose for N an even integer, it is desired to locate the relative positions of the space vectors An, Cn, Bn, where n is the order of the harmonic. Refer either to case I-1 or case II-1 of figure 6. Take the position of An to be in the same direction as A_1. If the angular displacement of $-C_1$ with respect to A_1 is increased by n times, this gives $(-C_n)$. Then $+C_n$, the C vector desired, is π radians from $-C_n$ where A_n is the reference vector. If the angular position of $+B_1$ is increased n times, the position of $+B_n$ is found with respect to A_n directly. The caution above applies particularly to the location of the $+C_n$ vector relative to the $+A_n$ space vector when n is an even number.

Appendix III

Resultant Magnetomotive-Force Waves for Fractional Slot Windings

So far only the space vectors have been determined for N an odd and for N an even integer. It is possible to establish, now, the resultant magnetomotive force waves when positive, or negative, or zero-sequence currents are assumed for the phase windings. These are shown in figure 6, but brief derivations will be given here.

The space vectors of figures 5 and 6 are determined on the basis that for either figure 4 or figure 8, unit positive currents flow in all the top layer conductors under the first pole. Consequently, for positive sequence currents in the lines the currents in phases C and B lag those in phase A by 60 degrees and 120 degrees, respectively. For negative-sequence currents in the line these angles of lag become angles of lead, and for zero-sequence currents in the lines the currents in windings A and B are in phase and lead those of C by 180 degrees.

Case I-1 and case II-2 of figure 6 may be treated jointly. The resultant magnetomotive force produced by the positive-sequence currents is,

$$H_n = \frac{4RIK_{dn}K_{pn}}{n\pi} \left\{ \sin n\xi' \sin \omega t + \sin (n\xi' + \pi/3) \sin (\omega t - \pi/3) + \sin \left(n\xi' + \frac{2\pi}{3} \right) \sin \left(\omega t - \frac{2\pi}{3} \right) \right\} \qquad (59)$$

where $\xi' = 2\pi$ for the wave length of the first or longest harmonic. Then

$$H_n = -\frac{3}{2} \times \frac{4RI}{\pi n} K_{dn} K_{pn} \cos (n\xi' + \omega t) \qquad (60)$$

This is a negatively rotating wave of 3/2 times the magnitude per phase traveling its own wave length in one electrical cycle.

Similar calculations were made to prove the remaining data on figure 6. It will be noted the harmonics for a winding fall either all under cases I or all under cases II. In general, the amplitude of the traveling wave, if one exists is

$$|H_n| = 1.91 \frac{RI}{n} K_{dn} K_{pn} \qquad (61)$$

In the use of equation 61 it should be noted that R/N, the slots per phase per pole, defines R.

I = the crest current per coil side
K_{dn} = the distribution factor. This is defined by equation 29 when N is odd and jointly by equation 49, and 50, and 51 when N is even
K_{pn} = the pitch factor defined by equation 33 when N is odd and by equation 55 when N is even

n = the order of the harmonic whether odd or even

It can easily be shown that with positive-sequence currents flowing in the winding, the harmonic of wave length equal to two rotor or field poles, travels with the field at synchronous speed. This will be called the synchronous harmonic. When N is an odd integer then for the synchronous harmonic $n = N$. When N is an even integer, the synchronous harmonic $n = N/2$. The value of n for this harmonic may be found under either case I or case II of figure 6 for positive-sequence currents. When this is located all the other harmonics regardless of sequence will conform to that case (I or II) in which the synchronous harmonic has been located.

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Discussion

W. V. Lyon (Massachusetts Institute of Technology, Cambridge): We are much indebted to Professor Calvert for clarifying computation regarding fractional-slot windings.

In such windings the *fundamental* poles are those having the greatest possible pole pitch. The *principal* poles are those having the greatest flux per pole. With integral-slot windings the fundamental poles are the principal poles. With fractional-slot windings the winding is arranged in repeatable groups, each group covering either an odd or an even number of principal poles. When the repeatable group covers an odd number of principal poles, only odd harmonic components of the fundamental poles are present. When the repeatable group covers an even number of principal poles, both odd and even harmonic components of the fundamental poles are present. If these windings are symmetrically arranged, balanced polyphase currents produce circular fields of each harmonic order if the air gap is uniform. With a three-phase symmetrical winding, positive- and negative-sequence currents produce no harmonic field having 3 k times as many poles as the fundamental field; where k is any integer. The other harmonic fields, taken in order, rotate oppositely; that is, the fundamental, seventh, thirteenth, and nineteenth rotate in one direction, while the fifth, eleventh, and seventeenth rotate in the opposite direction. The third, ninth, and fifteenth harmonic fields, which are produced only by zero-sequence currents, do not rotate. They are stationary fields which pulsate at the frequency of the zero-sequence current.

There are two types of symmetrical fractional-slot windings which I shall name "regular" and "irregular." Professor Calvert discusses the regular windings only. They are the ones for which general formulas can be developed. All other windings are irregular, and each one must be considered as a separate problem. With a regular winding the vectors which represent the components of the principal polar flux due to the current in the individual coils of one phase are equally spaced from each other like the sticks of an open fan. This is true whether the repeatable group covers an odd or an even number of principal poles.

The following test can be applied to determine whether the winding is a regular one. Let $x, y, z, u, \ldots$ etc., be the numbers of consecutive top-coil sides in the successive phase groups, a, b, c, etc. That is, x is the number of top-coil sides in adjacent slots of phase a; y is the number of top-coil sides in the next phase group b; z is the number of top-coil sides in the next phase group, c; u is the number of top-coil sides in the next phase group, a, etc. Let P be the number of principal poles covered by the repeatable group, and let S be the number of slots of one phase in the repeatable group. If the winding is regular, the following P fractions can be arranged in a series so that the difference between successive fractions is $1/P$: $S/P - x$, $2S/P - x - y$, $3S/P - x - y - z$, $4S/P - x - y - z - u$, etc. For example, if the repeatable group covers five principal poles and the top-coil sides in successive slots are arranged so that there are two in phase a, three in phase b, two in phase c, three in phase a, and two in phase b, these fractions are $12/5 - 2 = +2/5$, $24/5 - 5 = -1/5$, $36/5 - 7 = +1/5$, $48/5 - 10 = -2/5$, $60/5 - 12 = 0$. When arranged in order, the fractions are $-2/5$, $-1/5$, 0, $+1/5$, $+2/5$, the common difference being $1/P = 1/5$.

It follows that x, y, z, etc., must equal either n or $n + 1$, where n may be any integer. Table I shows the possible values of x, y, z, etc., when, the repeatable group covers two poles, four poles, five poles, seven poles, and eight poles. In any one case either the plus (+) signs must be chosen or the minus (−) signs must be chosen.

Table I

2 poles	$n, n + 1$
4 poles	$n, n, n, n \pm 1$
5 poles	$n, n, n, n, n \pm 1$
5 poles	$n, n \pm 1, n, n \pm 1, n$
7 poles	$n, n, n, n, n, n, n \pm 1$
7 poles	$n, n, n, n \pm 1, n, n, n \pm 1$
7 poles	$n, n \pm 1, n, n \pm 1, n, n, n \pm 1$
8 poles	$n, n, n, n, n, n, n, n \pm 1$
8 poles	$n, n \pm 1, n, n, n \pm 1, n, n, n \pm 1$

The simpler case is the one in which the repeatable group covers an odd number of principal poles. There are then only odd harmonic multiples of the fundamental poles. The pitch factor for the qth harmonic of the fundamental poles is $K_{pq} = \sin(qp/2R)\pi$; where p is the coil pitch in slots and R is the number of slots in a repeatable group. The breadth factor for the qth harmonic of the fundamental poles is

$$K_{bq} = \frac{\sin S\,q\beta}{S \sin q\beta}$$

where S is the number of slots in one phase of a repeatable group. Professor Calvert gives a simple method for computing the angle β.

When a repeatable group covers an even number of principal poles I find there is no difficulty in combining the vectors which represent the component fluxes due to the individual coils of one phase and without recourse to the fictitious current distribution that Professor Calvert uses. The pitch factor for the qth harmonic component of the fundamental poles is $K_{pq} = \sin(qp/R)\pi$, where p is the coil pitch in slots and R is the number of slots in a repeatable group. It seems best to give a separate breadth factor for each of the four cases listed in table I.

Table II

	60 Slots	57 Slots
K_{b1}	0.955	0.952
K_{b5}	0.200	0.192
K_{b7}	0.150	0.137
K_{b11}	0.110	0.088
K_{b13}	0.102	0.075

Repeatable Group Covering Two Poles.

$$K_{bq} = \frac{\sin S\,q\alpha}{S \sin q\alpha} \quad \text{if } q \text{ is odd}$$

$$K_{bq} = \frac{\cos S\,q\alpha}{S \cos q\alpha} \quad \text{if } q \text{ is even}$$

S is the number of slots per phase in a repeatable group, and α is one-fourth the angle between slots on fundamental scale, or $\alpha = 2\pi/12S$.

Repeatable Group Covering Four Poles. There are two cases, the first in which the number of consecutive upper-coil sides in successive phases are $n, n, n, n + 1$, and the second in which the numbers of consecutive upper-coil sides in successive phases are $n, n, n, n - 1$. In both cases

$$K_{bq} = \frac{\sin S\,q\alpha}{S \sin q\alpha} \quad \text{if } q = 2, 6, 10$$

$$K_{bq} = \frac{\cos S\,q\alpha}{S \cos q\alpha} \quad \text{if } q = 4, 8\ 12$$

In the first case, if q is odd,

$$K_{bq} = \frac{1}{S}\,\frac{\cos S\,q\alpha \pm \sin S\,q\alpha}{\cos q\alpha \pm \sin q\alpha}$$

upper signs if $q = 3, 7, 11$
lower signs if $q = 1, 5, 9$

In the second case the + and − signs in the denominator are interchanged. S is the number of slots per phase in a repeatable group, and α is one-eighth the angle between slots on fundamental scale, or $\alpha = 2\pi/24S$.

By this method formulas can also be derived for the breadth factor when the repeatable group covers eight principal poles.

The same pitch and breadth factors can also be used in computing the harmonic components of the open-circuit electromotive force of a synchronous machine. Due to geometrical symmetry the air-gap flux distribution usually consists of a fundamental and odd harmonics. The number of these fundamental poles, however, is the same as the number of the principal poles for which the armature is wound. Therefore the qth-harmonic breadth factor to be used in computing the open-circuit electromotive force generated in a regular fractional-slot winding is

$$K_{bq} = \frac{\sin S\,q\alpha}{S \sin q\alpha}$$

where S is the number of slots per phase in a repeatable group, α equals 30 degrees divided by S, and the principal poles are considered to be the fundamental poles ($q = 1$). For all odd harmonics, except for the third and its multiples, the numerator of K_{bq} is 0.5. The electromotive force breadth factors for a regular fractional-slot winding having S slots per phase in a repeatable group are the same as the breadth factors for an integral-slot winding having S slots per pole per phase. Since S is ordinarily greater than the usual number of slots per pole per phase, the fractional-slot winding more nearly approaches a uniformly distributed winding than does an integral-slot winding. Consequently the breadth factors for the fractional-slot winding are less than the corresponding breadth factors for an integral-slot winding. The difference between corresponding breadth factors becomes greater as the harmonic order increases. Hence a fractional-slot winding gives a smoother wave-form of generated electromotive force than an integral-slot winding. For example, a four-pole winding in 60 slots has 5 slots per pole phase, but a four-pole winding in 57 slots has 19 slots per phase. The corresponding breadth factors for these two windings are in table II.

The fundamental breadth factor for electromotive force is greater with a regular fractional-slot winding than with an irregular winding having the same number of slots.

L. A. Kilgore (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): This paper makes it possible to calculate, by substitution in a relatively simple formula, the magnetomotive-force harmonics of a fractional slot winding. The earlier method described in a paper by Mr. Q. Graham required the calculation of the harmonics for each coil and a rather laborious method of combining these to give the resultant magnetomotive force. Not only is much time saved, but the chance of an error is much less in this more direct method.

Further study on the effects of these subsynchronous harmonics on vibration and losses would be very valuable. The writer has done some work on the problem of vibration produced by the longer pitch harmonics of force resulting from the harmonics of air-gap density. If a frame is susceptible to vibration due to a four-pole force wave which pulls in on opposite sides of the frame making it elliptical, then one must not only look out for the two-pole component of the flux wave, but also any harmonic whose order differs from the synchronous harmonic by 2 (or whose pole number differs by 4), for this may contribute a large four-pole force wave since the force depends on the square of the resultant flux density wave.

Quentin Graham (Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.): Mr. Calvert has greatly simplified the calculation of the coefficients of the lower order harmonics by making use of a symmetry which has not been pointed out heretofore. Thus instead of adding together a series of unsymmetrically spaced vectors as was suggested in a previous paper he has made use of easily computed distribution factors. One practical aspect of this problem which has occurred to me is that it is often necessary to locate the phase position of the resultant nth harmonic vector of one phase with respect to particular coils so that a rearrangement or omission of coils can be made to reduce that particular harmonic. The resultant vector probably corresponds to the position of the center coil of the repeatable group but care must be taken to determine whether its value is positive or negative. Can the author clarify this point? I believe it will add to the usefulness of the paper.

J. F. Calvert: I wish to thank Professor Lyon for his very helpful discussion. The first two paragraphs of his discussion are in complete agreement with the paper. Professor Lyons' method for determining whether or not a winding is a "regular one" is a desirable addition. The formulas he presents for pitch factor and for distribution factor when the number of poles for the slots to repeat (N in the paper) is an odd integer, appear to be identical with those in the paper. For the case where N is an even integer his pitch factor seems to be identical with that shown in the paper. The development for the distribution factor which he indicates for the case of N an even integer seems to result in a variety of forms depending upon whether the harmonic is odd or even and also on the type of "regular winding" used. The only justification for this increased complexity seems to be the avoidance of the artificial current sheet introduced in the paper. As the product of pitch factor and distribution factor is to be used in computation this latter artifice is entirely justifiable. This is a method of solution which it seems to me should be resorted to freely if it simplifies either solution or result. This with the appropriate use of artificial magnetic sheets has proved most useful in magnetic force loss and reactance problems as well as in this problem where only one expression is needed for the distribution factor for N an even integer in the type of windings discussed in the paper.

I am in full agreement with Mr. Kilgore's remarks, and hope that further work can be done to apply the results of this paper to the study of forces and losses.

One possible answer to Mr. Graham's question for the case where N is an odd integer lies in equations 18, 28, 29, and 32 of the paper. (The first three of these are really sufficient.) Equation 28 shows the angular position and magnitude of the radial air-gap magnetomotive force produced by the phase A coils in the top layer of the winding. The resultant is located here with respect to the first coil side in the top left-hand corner of the winding chart of figures 3 and 4. The middle coil, if one existed, is located by the sine term. However, K_{dn} may be either plus or minus. Another way to consider the matter is to consider the series of terms in equations 18 and 28 as space vectors. The vectors are spaced out by an angle $n\beta$ from one another when the set of coil sides producing them are chosen in the order in which they appear by columns in figure 4. Thus coil sides 1 and 19 produce the first space vector, then coil sides 29 and 12 produce the second space vector, then 23 and 5, etc.

For the case where N is an even integer a similar statement may be made regarding the resultant magnetomotive force due to the top layer coils and one current sheet as expressed by equation 48. However, in this case the coil sides and their artificial current sheets always alternate in sign or polarity when considered in the order in which they appear in the chart by columns (see figure 8, and the discussion above equation 46).

This means that it may be desirable to consider these as two sets of space vectors and to make use of K_{dn}' and K_{dn}''. In either case equations 47 and 48 are available and written with respect to the first coil side in the top left-hand corner of the chart of figure 8.

I may add that it is intended to publish at an early date a table of distribution factors covering a range for N from 5 to 29 where N is an odd integer and 4 to 32 when N is even, and values of R from $(N + 1)$ to $(5\,N + 1)$ in each case. The values of n in all cases will be from 1 to 23, inclusive, where $n = 1$ is the first or longest harmonic

The Economic Status of the Engineer

By R. W. SORENSEN
FELLOW AIEE

Synopsis: This is not a committee report, though perhaps it could be made to serve as such, because its genesis was a request from the committee on the economic status of the engineer that a paper be written around the author's remark, "The economic status of the engineer is largely a matter determined by each individual engineer according to his particular personal qualifications and the relations these bear to the work he does and to the personalities of those persons with whom and by whom he is employed."

The study on which the paper is based shows conclusions which may be briefed as follows:

Engineers, through suffering considerable loss of income and employment during the recent years of business depression, on the whole have fared much better than most classification groups, be they government, capital, profession, or labor.

Preceding the depression, there was no lack of employment for engineers and their compensation for the most part was equitable in comparison with the pay for other types of service.

The verdict of users of engineering service regarding the reasons why engineers who have not made satisfactory progress professionally or in their economic status is almost unanimous that such failures are due to deficiencies in personality, general culture, tact, industry, etc., rather than for lack of technical training.

Graduation from college is prerequisite to success in engineering, but does not *per se* guarantee an engineer.

The education obtained by taking engineering courses and engaging in the practice of engineering for most of those who have chosen the routes thereof has led to "the more abundant life" and a better economic status than the lots of the families from which engineers have come.

If the writer were not a pedagogue, this summary and the list of references would probably constitute the entire paper, because most of the readers will probably have time for reading only the summary. The readers whose engineering conscience will not permit ready acceptance of unsupported statements, will be burdened with the urge to read not only the entire paper but will have a desire to consider all the information available publications afford. In writing the paper, much use has been made of data in bulletins published by the Bureau of Labor Statistics. In this paper these are called bulletins or noted by numbers. For a complete list of these bulletins see footnote to article, "Sources of Engineering Income, 1929–1934," page 1353 in Electrical Engineering, volume 56, November 1937.

THE ECONOMIC status of the engineer is bifunctional in its scope—one part deals with the value of the integrated work which all engineers have contributed to the progress and welfare of mankind; the other part has to do with economic recognition in the way of social position and salary which engineers and their families receive in return for service rendered.

Electrical Engineering in the September 1932 number announced for the first time the appointment of a committee to be known as the "committee on the economic status of the engineer." The function of that committee is set forth in Institute by-laws, article III, section 83, which says this committee "shall consist of five members, and shall consider matters relating to the position, function, and responsibility of the engineer in the development of human welfare, and make reports and recommendations to the board of directors thereupon. The committee shall co-operate with similar committees of other engineering societies, and shall also consider and report upon all matters referred to it by the board of directors, the president, and the national secretary." The language of this article shows wisdom in its formulation and clearly charges the committee with the duty of keeping informed and advising the members of the Institute, through its board of directors, of ways and means whereby they may be of service to mankind. Such responsibility would be overwhelming, were it not for the fact that all engineering aims at exactly the goal specified.

The world's economic evolution has resulted in much classification of the workers responsible for changing the habits of its citizens from those of the jungle to our present complex but regal standard of living, though jumbled be its attendant economic program under which we are muddling along.

One group of these workers is known as the professional men's group. Professional men are, perhaps, best defined by saying they are men who have professional education; that is, "the training that fits men for special vocations in which science is applied to the practical purposes of life. It supposes, as its basis, the knowledge and discipline which general culture affords."

Many attempts have been made to write an all-inclusive definition for the engineer, but the rapid march of time has made each effort obsolete, even as the progressive science of engineering rather than the wearing out of machines has relegated many engineering products to the oblivion of the obsolete.

Engineers qualify as professional men by having professional education. Government research as to the "Educational Qualifications in the Engineering Profession," shows, as set forth in Bulletin R-400:

> "A first degree in engineering is now almost a prerequisite in order to obtain professional status and a position. Postgraduate work, however, is important only in a few of the professional classes. The tendency of engineers to transfer from the course of college specialization to other classes of work is negligible. These are a few of the facts developed in the survey of the engineering profession, which was undertaken by the Bureau of Labor Statistics in May 1935, at the request of the American Engineering Council."

Figures on which the above statements are based show that only 1.52 per cent of the engineers who began practice between 1930 and 1934 were not graduates. For all years up to 1929, 27.6 per cent of all engineers were not graduates. For all years up to 1935, 17.7 per cent of all engineers were not graduates. For all years up to 1935, 13.3 per cent of all electrical engineers were not graduates. Also the number of engineering graduates with more than one degree is very small, *viz.*, $^1/_2$ per cent and $^1/_{10}$ per cent only having respectively masters' and doctorate degrees.

The doubt expressed as to the importance of postgraduate work is challenged, because graduate work in engineering colleges is too new to provide enough statistical data to draw conclusions as to its value. There is strong evidence, however, that men who are qualified for and have completed graduate courses which are provided in properly manned and well-equipped colleges for the study of modern science and mathematics as applied to engineering, have, for the most part, advanced in professional status at rates which show justification for graduate work. In the author's opinion, the limited number of men who have the special scientific and mathematical ability to warrant the continuation of postgraduate work unto the earning of a doctorate degree (and no others) should be encouraged toward that end.

A poet scanning these data and writing in Biblical language might well say, "It is easier for a camel to go through the eye of a needle than for a man to become an engineer without the advantage of graduation from an engineering college," or expressed in current language of the street, it may be said, "The odds are better than 98 to one you can't be an engineer without graduation."

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In fact the engineer finds that graduation does not end study, but that he must supplement his practice by continued study else he will lag behind just in proportion as his interest in research and study wanes. Perhaps, therefore, it will be easier to determine who are engineers by the manner in which they do their work, rather than to judge by graduation, license to practice, by passing examinations, often irrelevant to the kind of engineering done.

The law says a boy becomes a man the day he is 21 years old, but, except for legal privileges and voting, no change takes place on the 21st birthday, but rather a boy becomes a man when he puts away childish things and meets his problems in a manly way.

So with becoming an engineer. A man does not become an engineer because he graduates, or because he completes a test course and becomes a good draughtsman, mechanic, calculator, designer, or professor of electrical engineering. He becomes an engineer when he diligently and intelligently uses his God-given and hereditary talents, his education, his environment, his background, and his personality to produce new ideas and, through the medium of the crafts mentioned, finds ways and means for putting these ideas to work and makes a scientific analysis of his procedure in order that he may proceed by the engineering method rather than by cut-and-try or empirical methods.

Engineers, perforce, must at once be very co-operative and very individualistic. The co-operative characteristic is necessary because engineering problems of today are too large for one man to solve, and must be worked out by groups of men working in such close relationship as often to make practically impossible any acknowledgment as to the source of key ideas which unlock the problem solutions. Engineers must be individualists in order that each may contribute his share to the profession, by discovering in the daily tasks performed, new problems and their solutions. As illustrative of the latter point, consider the 27 Edison Medalists, beginning with Elihu Thompson and including the most recent one, Gano Dunn. No two of the entire 27 have traveled even similar paths to success. In fact, in all engineering history there are probably no case records which show that two engineers have done identical work, even in instances where promotions and other causes have made vacancies in organization that result in succession appointments.

Thus we see that engineers in their relations with their work and fellow men have true professional status in that the particular contributions of each engineer to society are unique.

The other phase of the economic status of engineers, bluntly stated, is to what extent does all this work enable them to provide themselves and their families with good social positions and the use of the facilities for human betterment which engineers have made available. This part of the question, while not specifically mentioned in the Institute by-laws, is implied by the name of the committee, and committee consideration thereof is expected by the Institute members.

Statistical data regarding employment and income are now available in bulletins published by the United States Department of Labor, Bureau of Labor Statistics. Parts of the data in these bulletins have been published in ELECTRICAL ENGINEERING.

A study of these data and much other information obtained from the printed page, by discussion with others, and by experience on the part of the author, seems to warrant the conclusion that the economic status of engineers in comparison to that of other citizens is for the most part reasonably equitable, though many engineers are of the opinion that the members of the engineering profession have received less reward than their work warrants.

Bulletin No. *R*-497 opens with the sentence: "As far as is known, the recent depression was unique in its disastrous repercussions upon professional groups." This, after all, is just another way of saying the depression was terribly severe, extending even into the professional groups to such an extent as to show a simultaneous unemployment of about 11 per cent of all engineers, all of the several engineering classifications suffering to about the same degree. As might be expected, there was a greater percentage of unemployment for engineers over 53 or under 27 years of age. Summary analysis number 9 of this bulletin says:

> "9. The type of education the professional engineer had received did effect variations in both the incidence and severity of unemployment. These factors were very much less for postgraduates than for engineers with other types of education. But as between engineers with first degrees in engineering and those whose college course was incomplete or who had attended noncollegiate technical schools, the differentials were very slight."

Other information shows less than three per cent of the graduates of some engineering colleges unemployed at any one time during the depression years 1930–1935. No comparable data being available for the other professions, it is difficult to know just what a reasonable standard of depression unemployment should be, but there is every reason to believe that lawyers, physicians, dentists, though busy did not fare any better than engineers in regard to net income received for service rendered. It is also very certain that engineers, in that respect, fared better than skilled mechanics and other craftsmen who constitute much of our working citizenry.

Bulletin No. *R*-543, "Employment in the Engineering Profession, 1929–1934," shows among other things that during the five-year period of rampant unemployment, the number of engineers graduating into the profession was 25.3 per cent the number engaged in engineering in 1929. Since the nation-wide unemployment of engineers at any one time reached a maximum of 17.7 per cent, there was, even during unemployment times, a considerable amount of employment for the beginner in engineering. The number of engineering graduates, therefore, probably was not too high for normal times and the indication is that all should be needed by industry, if only persons well qualified for engineering work choose to enter the profession.

Bulletin No. *R*-588, "Income and Earnings in the Engineering Profession, 1929–1934," presents much very interesting data. Table I is part of table 3, page 7, of Bulletin No. *R*-588.

The table shows that in 1929 all but ten per cent of the engineers received wages equal to or better than the wages of skilled mechanics, as published in Bulletin No. 616, "Wages and Hours of Labor," which lists the pay for skilled and unskilled labor in many industries and shows mechanics' wages are $25 to $35 per week with occasional skilled occupations paying larger amounts.

First-degree engineering graduates start work at about $25 to $30 per week. Graduates with master's and doctorate degrees start at $30 to $50 per week, the

Table I. Comparison of Five Levels of Annual Earnings for All Professional Engineers Reporting in 1929, 1932, and 1934

(Figures derived from adjusted data as explained on page 4 of bulletin, and without regard to employment status reported or type of education)

Per cent at Specified Income Level	Annual Earnings of More Than Specified Amount (Dollars)		
	1929	1932	1934
10	7,466	5,605	5,138
25	5,012	3,827	3,429
50	3,412	2,574	2,286
75	2,509	1,698	1,473
90	1,878	889	872

latter amount being reserved for men of special ability. These rates of pay appear just—the mechanic or skilled laborer being paid a premium over the common labor wage in recognition of his skill and the cost of its acquirement. The wage premium for neophite engineers is a recognition of the fact that time and money have been required for college training which will enable them to become engineers rapidly, rather than as recognition of acquired proficiency as in the case of mechanics. The pay men just out of college receive, provided it keeps them from want, is relatively unimportant, as compared to opportunity for advancement in responsibility and salary. Salary advances for engineers of the United States and for lawyers of California, so far as the latter information is available, are compared in figure 1. The curves applying to engineers are taken from chart 1, Bulletin *R*-588. The data pertaining to the lawyers of California is from an unpublished "Digest of a Survey of the Economic and Professional Status of California Lawyers during the First Five Years of Practice," prepared in 1937 by the committee for co-operation between the law schools and the state bar.

The curves for the engineers show favorable advancement in salary with age and experience for the upper half of those in the profession, as also does the rather limited data for the lawyers. It is interesting to note the long rise in earning capacity extending unto a man's 60th year of age and 37th year of practice. Observation, without confirming data, creates the opinion that lawyers and physicians follow the same laws in this respect. No actual data have been made available for physicians, but an oral check with a number of them brought forth statements, all in agreement, to the effect that in the opinions of those interviewed, the curves for the engineers were, on the whole, indicative of the net salaries for physicians—the average for the physicians being perhaps a little better than the average for engineers; but less than ten per cent of the physicians have net incomes above the 10 per cent curve for engineers.

All this information seems to indicate equity in the income of engineers as compared to skilled laborers, lawyers, and physicians.

The professional men are indeed fortunate in having occupations which provide for increasing service to fellow men as years add to skill and experience, and also in having at the same time increasing incomes which grow apace with the family expenses and often continue to grow beyond the period required to get the children established in their own homes and occupations.

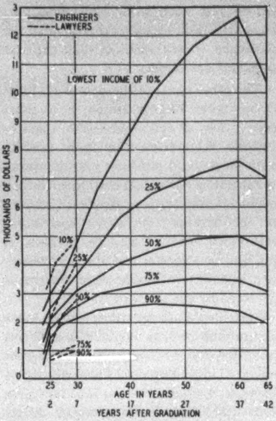

Figure 1. Earned annual income of engineers and lawyers according to age

The curves for engineers represent the findings for all the engineers of the United States. The curves for lawyers represent the findings for California lawyers admitted to the bar in 1931

Those of our profession who disagree with these findings, particularly as they apply to engineers, have two arguments against them—one the apparent greater expenditure of money by lawyers and physicians as compared to that of the engineers, which they have witnessed. It must be borne in mind that some of the expenditure made by physicians and lawyers is for office equipment and automobiles that must be used in connection with the practice of these professions. Also it is worthy of note that the expenditures which attract attention are usually those made by the more prosperous, rather than by the representative members of the profession.

Moreover, comparisous which men make purely by observation, rather than on the basis of exact data, between the spheres of their own activity and those of others, generally result in optimistic interpretations regarding the outside spheres, with a simultaneous pessimistic appraisal of their own. "The grass on the other side of the fence is always greener."

The second objection has to do with the exactness of the data obtained from returned questionnaires as compared to that which would have been available, had every engineer in the land returned a complete questionnaire. A complete report, would, of course, be impossible except by absolute governmental decree ordering a census of all engineers, but the data available which came from the 52,589 engineers who properly filled out and returned the questionnaire should give a fair cross-section of the 167,268 engineers who received them.

Assuming the data used to be representative, the analysis has narrowed to two questions: (1) Why is there such great spread in income for men in the same profession who have gone through the same training courses, been subject to the same tests as to ability, and have survived the same processes of selection? (2) What are we going to do about it?

Many educators and others have often asked these questions and made surveys of industry, hoping thereby to find the answer, but all have ended with only a variety of general but authoritative statements by those who employ engineers. Some of these statements are:

Graduates of engineering colleges do not fail to reach expected goals because of lack of technical education but rather because of deficiencies in those qualities described by such terms as:

Personality
Loyalty
Patience
Humility
Breadth of interest
Business ability
Leadership
Aptitude
Promptness
Accuracy
Judgment
Proper estimate of own value
Executive ability

with sometimes the comment that there is lack of education in basic science and mathematics. All of which takes us back to a part of the definition with which we started, "it supposes as its basis the knowledge and discipline which general culture affords."

Only the last of these objections; *viz.*, the lack of education in basic science and mathematics, will, at first thought, be charged to weakness in engineering-college curricula and training—but there is a fourfold responsibility involved in the making of an engineer which must be assumed jointly by the young man, his family, the college, and industry.

America, the land of opportunity, provides many avenues for rendering service and at the same time improving standards of living. Not the least of these are the engineering professions. Via engineering and the education provided by the engineering colleges, many men have reached social positions and attained economic rewards far better than those of the families from which they came. In fact, there are so few exceptions to this order of things, even among the poorer paid engineers, as to make perfectly valid some such declaration as: "Engineering is a profession through which the sons of small merchants, farmers, and laborers, as well as those of professional men and the prosperous in industry find golden opportunities to high living standards."

A large portion of the enrolled students in engineering colleges in part or entirely "work their way through." The author has, for more than a quarter of a century, co-operated with these men to make work, college courses, family budgets, and loan funds blend to the best advantage of all concerned. This blending process is not always easy, nor, though a certain amount of labor experience is desirable, is it advantageous for a student to be compelled to allot a very large part of his college time to earning money. In one college where the tuition is $300 per year and there are relatively few scholarships, $^1/_6$ of the undergraduate student body and $^1/_{10}$ of the graduate students are using NYA assistance. Nearly all the families to which these young men belong have incomes $1,200.00 per year or less. According to the catalogue of the college in question, the minimum estimated cost per student for board and room, books, tuition, etc., but with no allowance for entertainment and clothing, is:

Students taking 21 meals in student houses per week—$840 per year
Students taking 15 meals (going home weekends)—$740 per year
Nonresident students—$390 per year

The cost per student in many free-tuition colleges, when all factors are considered is practically the same. If a $1,200.00-per-year-income family (and there are those with less) must, with the aid of the student apportion an amount equal to $^2/_3$ the family income for the bare essentials of being in college, it is obvious there is little money available for travel, hotel life, theater, dances, and other social functions, or even for church activities—all of which have great bearing upon the phases of life which employers have declared are deficient in engineers to an extent which impairs engineering careers.

These deficiencies fortunately can all be remedied by any normal young man with capacity to complete an engineering course, if he is made aware of them, and will make an honest effort to know himself and apply the needed corrections. Keeping well is always simpler and better than curing illness, but continuing in illness is infinitely worse and sometimes inexcusable.

Being born and nurtured in an atmosphere of culture where all the graces of life are daily habits which can be acquired with little conscious effort has its advantages. When this experience has been denied the engineering students, colleges should provide clinics for correcting the deficiencies. Some of the more progressive engineering colleges have made progress in the right direction by having in their curricula a goodly proportion of cultural courses with the consequent necessity for postponement of the more special technical courses to graduate years. Industry also should not limit all its training courses for young engineers to the technique of the business, but should provide opportunity for them to learn of and correct faults which impair the rendering of the highest possible type of engineering service. College and industry together must show interest in our educational program from kindergarten on and co-operate with our engineering societies and the Engineers Council for Professional Development in extending their program for educating the public as to the requirements for being an engineer.

Industry should see to it that all who qualify as engineers be paid all the work done will warrant and should not designate as engineering, work which is not engineering, but is only high-class clerical calculating, draughting, or skilled machine operation.

Engineers should make themselves thoroughly conversant and be sympathetic with all the problems of labor, skilled and unskilled, preferably through having had actual experience as workers in both classifications. They should not make entangling commitments to either capital or labor which may interfere with their great opportunities to correlate these two great industrial factors into teams that, working together at the business of applying engineering methods, cannot be defeated.

Fortunate indeed are the youths who find their talents and choices leading them into engineering; they can have a lot of fun following one of the many paths leading to enjoyable service for their fellow men, and at the same time providing so well a means of livelihood for themselves and their families.

Discussion

L. A. Doggett (Pennsylvania State College, State College): In commenting on Professor Sorensen's paper, I would like to point out that the economic status of the engineer is different from that of either the doctor or the lawyer. The engineer's economic status is that of the hired man. By that I mean that his economic status is seldom that of a member of a board of directors (a sampling of 209 directors, 19 electrical companies, in Moody's 1933 Utilities shows 17 AIEE members). The engineer in the last analysis takes his orders from those who are not engineers. Why is this? Primarily I believe it is due to a MYTH that has been fostered for the last 25 years, a myth which has as one of its tenets that the engineer must stick strictly to his volts and amperes. For example, it is perfectly orthodox for the engineer to study and investigate switches, bushings, power factor, line interruptions, etc. It is entirely orthodox to study line interruptions due to lightning, but when a certain switch was opened on the Connecticut state line the day after the holding company bill was signed, if thereupon we study the reasons for this line interruption, if we study this holding company bill, if we study the 70 odd volumes of the report of the Federal Trade Commission on utility corporations; then we are no longer orthodox, we are heretics; for does not the code of the AIEE technical program committee, 1934 edition, say "rate making, project financing, ..., utility regulation" are "subjects *not suitable* for Institute presentation." Since reading that statement these subjects have taken on for me the glamour of the forbidden. I have taken a peculiar pleasure in studying these subjects which are supposed to be taboo for engineers. With the same wicked pleasure that a boy might take in slipping out of church, I "snuk" out of the Tuesday morning AIEE meeting and went down to 18th and Pennsylvania avenues to listen in on two Securities and Exchange Commission hearings. In each hearing some sort of an exploratory operation was being performed on a holding company. There I could hear all sorts of "verboten" words, such as voting trust agreement, underwriters commissions, voting stock, etc., etc.

In conclusion I want to recommend a wider freedom for AIEE members, a freedom which would allow them, when studying any electrical problem, to follow up the investigation regardless of where it leads them, whether into the field of law, history, economics, medicine, politics, or archaeology, a freedom to make their investigations and publish their results in ELECTRICAL ENGINEERING. For example, if an engineer starts out to study filters and networks, goes thence into telephone traffic problems, thence into commercial phases of the telephone art, and finally into a study of the recent Government investigation of the telephone industry; if he finds this investigation unfair in its procedures and in its conclusions, he ought to have the same right to publish these findings as to publish something about filters. And these latter findings deal with a subject which has a very powerful effect on the economic status of the electrical engineer. I would, however, qualify this freedom by specifying that the main title of any AIEE article should be of an electrical engineering nature and that the nonengineering matter might appear as paragraphs subsidiary to the main title.

When the electrical engineer regains his freedom of speech, then will his economic status far transcend his present status which as Professor Sorensen points out isn't so bad.

E. E. George (Tennessee Electric Power Company, Chattanooga): The paper by Professor Sorensen is of unusual interest to the engineering profession at the present time. There have been very few papers on this subject presented to the AIEE. Professor Sorensen's statements appeal to the reader on account of the broad viewpoint taken and the supporting facts presented. It is interesting and encouraging to note that the engineer both in normal times and during depression fares as well as other professional men.

Few people would contest Professor Sorensen's comment that the career of many engineers would be smoother if they were more experienced in co-operation and were better equipped with the social graces. Nevertheless, it would be extremely discouraging to see more emphasis placed on co-operation and culture unless these are subordinated to learning "the value of a dollar." Notwithstanding the current political tendency to consider hard work and meager income as an unfair handicap and a social injustice, it is more and more becoming apparent that practically every great contribution to civilization has come from those who were required early in life to work hard through necessity or who had the responsibility of providing jobs for others (without impairment of capital) before they reached middle age. Most of the greatest technical improvements have been made by those who took up engineering in order to make a living and not as a hobby or to keep out of idleness. It is believed that the majority of engineers feel that the most important qualification an engineer can have is the breadth of economic judgment and the innate sense of fairness which is quickly acquired by those who have to earn their own living or to have to meet a payroll from which others earn a living.

It is becoming more and more apparent that if engineers are to improve their economic status both business and government must be controlled and operated by men who "know the value of a dollar."

It is misplaced emphasis for the technical school or AIEE to encourage the young engineer to develop a better induction motor if there is no market for that motor when it is built and if inferior motors built several years before and with their cost already amortized are idle because of governmental interference with economics. It is likewise futile to expect engineers to make technical advances if the majority of them have to be more concerned with continuity of employment than with any other problem. The AIEE cannot maintain its high position as a leading professional body if it continues to take no important position toward fact finding in the public ownership controversy and in similar industrial problems which are now occupying the minds of a large percentage of its members. To say that public enlightenment on the technical and economic phases of these problems must come from the action of engineers as *individuals* is to deny the value of *co-operation* and *organization*. Without imposing on the rights of any minority or being unfair to Institute members who are public employees the Institute ought to be able to find a middle-of-the-road course with committees of highly respected engineers assigned to a program of fact finding which would help the majority of its members immediately and ultimately further the program of technical development to the final benefit of all its members.

In view of the conservative attitude taken by the AIEE in dealing with this and with related problems, it is suggested that readers should note the widely different attitude prevailing in the American Society of Civil Engineers, as reported in the *Bulletin* of the American Engineering Council for June 1938.

R. W. Sorensen: The remarks made by Professor L. A. Doggett and by E. E. George are very expressive of the opinion of many Institute members and are, indeed, valuable additions to the paper. I think, however, that Professor Doggett has missed a point regarding the distinction between the business which uses the products of engineering and the economic phases of that business which have to do with the work of an engineer. I do not interpret any AIEE code to mean that the Institute publications are closed to a discussion of scientific principles of rate making, utility regulation, or perhaps even project financing, if those discussions adhere to Professor Doggett's own limitation that any AIEE article should be of an electrical engineering nature.

Professor Doggett seems to think there is something wrong because the 209 directors of 19 companies in Moody's 1934 Utilities include only 17 AIEE members. This statement in itself does not indicate, per se, there is something wrong without further analysis which would include the proportion of AIEE members to non-AIEE members who are interested in the utility business as stockholders and who have, therefore, the right to determine how the business shall be conducted. In fact I would like to cite from memory some approximate figures published in Forbes Magazine in response to an urgent request for an analysis of leaders in industry according to professions. After a careful canvass of all factors involved, Forbes published the following figures: About 21 out of 59 leading men of the world, in this respect, were trained as business men, about 17 arose to their positions through the legal profession, about 12 through the engineering profession and the balance, 5, 3, and 1, respectively, through training in other professions which were listed, but the names of which I cannot recall. Considering the relative newness of the engineering profession to the field of business and the legal profession, I am inclined to think the proper number of engineers have achieved places among this limited group of the world's outstanding men selected by the Forbes Magazine. Furthermore I have personally never discovered any evidence of lack of freedom of speech for engineers. Is it any more reasonable to use the time of engineering meetings and the pages of engineering publications for the preaching and publication of sermons and political speeches than it is to use the pulpit for the presentation of technical papers? On the other hand I am sure any engineer who wished to preach a sermon in keeping with the purpose of a house of worship, would not be forbidden freedom of speech.

I think, as an engineer, I would disagree with Mr. George and say it is the business of technical schools or the AIEE to encourage the young engineer to develop a better induction motor even if induction motors were never used; but, of course, I must acknowledge quite readily that much of the enthusiasm for such work or the opportunity for such work will be lost if such development cannot receive recognition in the way of finding purchasers for the improved induction motors.

Again Mr. George said "Without imposing on the rights of any minority or being unfair to Institute members who are public employees, the Institute ought to be able to find the middle-of-the-road course with committees of highly respected engineers assigned to a program of fact finding which would help the majority of its members immediately, and ultimately further the progress of technical development to the final benefit of all its members." I, speaking as an individual, am of the opinion that whenever engineers on opposite sides of controversial subjects can agree as to what are the facts, the technical papers committee will not stand in the way of the publication of such facts.

Selection and Design of the Electrification of the San Francisco-Oakland Bay Bridge Railway

By WENDELL P. MONROE
MEMBER AIEE

THE San Francisco-Oakland Bay Bridge was built by the State of California not only for heavy automobile traffic but also to benefit the 30,000,000 suburban railway passengers per year who are now transported across San Francisco Bay by passenger ferry boats belonging to the railway companies. The railway passengers, who are practically all commuters living in the east-bay cities and working in San Francisco, board the electric trains on branch lines near their homes and are taken to the piers by rail. They then transfer to the ferry boats which require about 20 minutes to cross the bay to the ferry building in San Francisco where the passengers disembark and walk or take street cars to their places of business. When the Bridge Railway is completed, passengers will board the electric trains near their homes in the east-bay cities and will be transported by rail, without transfer, to the new railway terminal in San Francisco near the business district of the city.

Two principal competing suburban electric railways will operate the state-owned Bridge Railway: the Key System and the Interurban Electric Railway (formerly the Southern Pacific Company electric lines). The suburban traffic is about equally divided between these two railways. A third company, the Sacramento Northern, an interurban line, will also operate over the Bridge Railway about 18 trains per day between Sacramento and San Francisco.

Figure 1 shows the general layout of the Bridge Railway, including profile, and figure 2 a typical cross section of the San Francisco-Oakland Bay Bridge showing the location of the two tracks on the lower deck of the bridge.

Selection of System

The Key System and Interurban Electric Railway, both standard-gauge railroads, now operate at 600 volts direct current and 1,200 volts direct current, respectively. The Sacramento Northern, also standard gauge, can operate at either 600 or 1,200 volts direct current. It was necessary to select an electrical distribution system for the Bridge Railway which would be satisfactory for all three railways using the same tracks. The following four systems were thoroughly investigated:

(*A*) The 600-volt system, using overhead catenary and requiring conversion of the Interurban Electric Railway cars to operate on 600 volts on the Bridge Railway and 1,200 volts on their own lines.

(*B*) The 1,200-volt system, using overhead catenary and requiring conversion of the Key System cars to operate on 1,200-volts on the Bridge Railway and 600 volts on their own lines.

(*C*) The dual-voltage system, using 1,200-volt overhead catenary for the Interurban Electric Railway and Sacramento Northern trains and 600-volt contact rail for the Key System trains.

(*D*) The "three wire" system, using overhead catenary and contact rail with 1,200 volts between them and 600 volts between catenary and running rails, the Key System and Sacramento Northern trains operating from the overhead catenary and the Interurban Electric Railway operating from both contact rail and catenary simultaneously.

In studying these systems for the Bridge Railway, the following premises were set up and adhered to:

(*a*) Keeping the first cost to a minimum is of prime importance because the state, in financing the Bridge Railway, must borrow the money from the Reconstruction Finance Corporation and justify the loan by showing that it can be repaid in a reasonable time from tolls tentatively set at $2^{1}/_{2}$ cents per railway passenger.

(*b*) For reasons of safety the 80 wooden cars comprising a portion of the Key System rolling stock must be replaced by steel cars at the state's expense, but using the old electrical equipment if practical.

(*c*) The substations required for the Bridge Railway can be financed by the Pacific Gas & Electric Company which will supply the power at their published rate for alternating current for traction purposes.

(*d*) Additional cars necessitated by the longer train trips will be furnished by the state.

(*e*) The state will retain title to the Bridge Railway and any equipment furnished by it, but will permit the railroads to operate them as a public convenience under certain conditions involving fares, maintenance, etc.

(*f*) Future consolidation of the two competing railroads is not contemplated and is not to be considered near in the investigation.

The studies resulted in the recommendation that the dual-voltage system with 1,200-volt catenary and 600-volt contact rail be adopted for the Bridge Railway. The reasons for this recommendation were as follows:

1. The dual-voltage system would require the lowest total initial investment by all concerned, especially by the state. Table I gives the preliminary estimate of costs of the three principal systems considered; the "three wire" system being discarded in the early stages of the investigation because of its complexity and difficulty of balancing the system.

2. An estimate of the annual costs of the three principal systems considered, as shown in table II, gave indication that the dual-voltage system would be as economical as any system considered. In common with the 1,200-volt system, it would have the advantage of permitting the Interurban Electric Railway to supply 1,200-volt energy to the Bridge Railway from the excess capacity of its steam generating station and its substations, this excess capacity being released by abandonment of its Oakland Pier service when the Bridge Railway is put into operation. The utilization of this available capacity would save approximately $18,000 per year in power bill as shown in table II.

3. The dual-voltage system would not require extensive and costly car equipment changes, such as conversion of the electrical equipment of one railroad to operate on the voltage of the other. It avoids the expenditure of a large amount of money on conversion of car electrical equipment already approaching obsolescence.

4. The operation and maintenance problems of the competing railroads operating on the same tracks is simplified by the dual-voltage system because the Key System would operate and maintain the 600-volt contact rail, feeders, and substation apparatus, and the Interurban Electric Railway would operate and maintain the 1,200-volt catenary, feeders, and substation apparatus, each without mixed responsibility.

5. The dual-voltage system would not involve contact rails in the car storage yards, since each railroad will have its own yard and 600-volt catenary can be used in the Key System yard in lieu of contact rail.

6. The dual-voltage system would enable the two competing suburban railroads to negotiate their power contracts separately with the power company, greatly expediting

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Paper number 38-99, recommended by the AIEE committee on transportation and presented at the AIEE Pacific Coast convention, Portland, Ore., August 9-12, 1938. Manuscript submitted April 8, 1938; made available for preprinting July 15, 1938.

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Table I. Preliminary Estimate of First Costs of Bridge Railway Facilities for Three Electrification Systems

	600-Volt System	1,200-Volt System	Dual-Voltage System
I. Fixed property			
(a) Previously estimated amounts:			
Real estate	$ 3,600,000	$ 3,600,000	$ 3,600,000
Grading, trackwork, off-line changes, separation structures, inspection buildings, contingencies, etc	2,280,000	2,280,000	2,280,000
San Francisco approach viaduct	1,100,000	1,100,000	1,100,000
San Francisco terminal	1,700,000	1,700,000	1,700,000
Signals and interlocking	1,200,000	1,200,000	1,200,000
Engineering	300,000	300,000	300,000
Subtotal of previous estimates	10,180,000	10,180,000	10,180,000
(b) Power distribution system:			
Catenary system	280,461	280,461	280,461
Track rail bonding	34,649	34,649	34,649
Feeders	96,425	5,760	19,205
Connections to substations	16,340	11,820	23,640
Sectionalizing equipment	19,965	24,960	41,596
Contact-rail installation			204,339
Yard lighting	5,700	5,700	5,700
Contingencies, ten per cent	45,360	36,335	60,959
Subtotal of distribution system	498,950	399,685	670,549
(c) Interest during construction, fixed property	367,929	364,817	374,157
II. Car equipment			
Key System, financed by state	2,470,000	3,321,894	2,470,000
Key System, financed by Key System	882,041	1,075,642	1,075,642
Interurban Electric Railway, financed by state	1,924,674	1,330,000	1,330,000
Sacramento Northern Railway, financed by state	54,400	54,400	54,400
Subtotal of car equipment	5,331,115	5,781,936	4,930,042
Interest during construction of car equipment, financed by state	153,416	162,286	132,910
Interest during construction of car equipment, financed by Key System	30,415	37,091	37,091
III. Substations, financed by Pacific Gas and Electric Company	1,014,200	789,500	1,186,000
Total cost to state	15,649,369	15,813,082	15,212,016
Grand total cost	17,576,025	17,715,315	17,510,749

the whole project. The total cost of power would not be materially affected, as the 30-minute peak demands of the two railroads will be practically coincident (see figures 3 and 4).

7. The dual-voltage system would make possible the operation of the traction motors of the Key System and the Interurban Electric Railway at separate voltages, each suitable to give equal speeds to the trains of both companies, thus speeding up schedules. The advantage of each railroad operating on the voltage best suited for it also avoids certain difficulties such as in the lighting of the cars. The Key System requires a lower voltage on the lighting circuits than would be suitable for the desired speeds on the Interurban Electric Railway if both were to operate on 600 volts. Moreover, flickering of lights on Interurban Electric Railway cars would occur if they were connected directly to a 600-volt catenary, but would be avoided by retaining the 1,200 volts to operate these trains.

8. The recommended system has the advantage that the railroads would require a much shorter time for preparing their equipment for bridge operation, since much less equipment changes would be necessary. The construction of the contact rail on the bridge could be accomplished in a shorter time than the changes in equipment if either 600 or 1,200 volts were used exclusively.

9. The dual-voltage system offers two types of distribution systems to possible future railways which might use the Bay Bridge. The lower voltage, 600 volts, is more suitable to possible light-weight streamline trains, hauled as far as the bridge by Diesel motive power, provided such trains were equipped with traction motors throughout their length. Also 600 volts is probably more suitable for city subway trains, which might later use the bridge. The higher voltage, 1,200 volts, is more suitable for multiple-unit trains of electrified railroads serving distant suburbs, if these lines are built.

10. The adoption of the dual-voltage system would not seriously affect any possible consolidation of the railroads.

No changes are necessary to Sacramento Northern Railway car equipment for any of the four systems studied, aside from the installation of cab signals and other equipment required in any case. In the dual-voltage system the Sacramento Northern trains will operate from the catenary on the joint-operated tracks of the Bridge Railway and on the Key System tracks.

Design of Dual Voltage Electrification

The dual-voltage system having been adopted for the Bridge Railway, the electrical distribution system was designed in detail and is now under construction. Approximately 31 miles of track are being built and electrified, including 19 miles of main-line track (14 miles of which are jointly used by the Key System and Interurban) and 12 miles of yard track. The main line is double-track with a six-track loop terminal in San Francisco. The general location is shown by figure 1.

The distribution system was designed with sufficient conductivity to limit the total average voltage drop over the maximum one-minute peak to 75 volts for the

Table II. Approximate Estimate of Comparative Annual Costs of Bridge Railway as Affected by the System of Electrification

	600-Volt System	1,200-Volt System	Dual-Voltage System
I. Fixed charges on investment (not including substations):			
Interest, 5 per cent, and insurance, 0.25 per cent	$ 869,496	$ 888,605	$ 857,049
Amortization of car equipment	145,006	157,269	134,097
Amortization of sectionalizing equipment	543	679	1,131
II. Cost of power for Bridge Railway:			
A-c power demand charge	202,000	181,000	181,000
A-c energy charge	0	0	0
D-c energy from Interurban Electric Railway substation		3,000	3,000
Substation fixed charges	116,633	90,793	136,390
Substation operation and maintenance	46,000	36,000	46,000
III. Maintenance of car equipment (motors and control, only):			
Key System and Interurban cars, on and off Bridge Railway	458,000	461,500	455,500
IV. Maintenance of distribution system:			
Catenary	8,000	8,000	8,000
Contact rail			3,200
Feeders and connections	950	160	660
Sectionalizing equipment	1,080	1,350	2,160
V. Cost per year of hauling extra weight of apparatus necessary for particular electrification system (on all lines)	4,920	4,180	2,550
Total (items affected, only)	1,852,628	1,832,538	1,830,737

600-volt system and 200 volts for the 1,200-volt system, the substation bus voltages being assumed to be 625 and 1,300, respectively.

The main features of the design will be briefly described:

SUBSTATIONS

The 1,200-volt* catenaries will be energized from two substations, the San Francisco substation and the Mole substation, approximately 24,000 feet apart. The 600-volt* contact rails will be energized from three substations, the San Francisco, Island, and Mole substations, the first and last being in the same buildings with the 1,200-volt substations of the same names. The Island substation, exclusively for the 600-volt contact rails, will be located on Yerba Buena Island approximately half way between San Francisco and Mole substations. The substations are being designed and built by the Pacific Gas & Electric Company, but will be owned and operated by the railroad companies. These three substations (two of which house both 1,200- and 600-volt conversion equipments) have the following capacities, including spares:

San Francisco substation—
- two 2,500-kw 1,300-volt mercury arc rectifiers
- two 2,000-kw 625-volt mercury arc rectifiers

Island substation—
- two 2,000-kw 625-volt mercury arc rectifiers

Mole substation—
- two 2,500-kw 1,300-volt mercury arc rectifiers
- two 1,000-kw 625-volt mercury arc rectifiers
- one existing 1,000-kw 625-volt motor-generator

The substations' locations and their connections to the Bridge Railway are shown in figure 5. Their capacities were determined from estimated load curves in the usual manner, allowing one machine per substation as spares.

SECTIONALIZING

The catenaries and contact rails will be sectionalized in accordance with figure 5. In order to utilize as fully as practical the conductivity of the catenaries and contact rails on both tracks and at the same time secure protection from short circuits and isolate the faults, tie stations will be installed between substations as shown. The tie stations will be located at points where emergency crossovers in the tracks occur. In the 1,200-volt catenaries, the substations will be completely sectionalized, that is, a short circuit on a section of catenary between a substation and a tie station will cause the circuit breakers feeding the section to open, thus isolating the section affected. A sectionalizing tie station requires four circuit breakers for a double-track railroad such as this. The substations feeding the 600-volt contact rails are only one-half as far apart as the 1,200-volt substations, and, although tie stations at midpoints are necessary to obtain sufficient conductivity, the sections are so short that the tie stations were not designed to be completely sectionalizing on the bridge, but

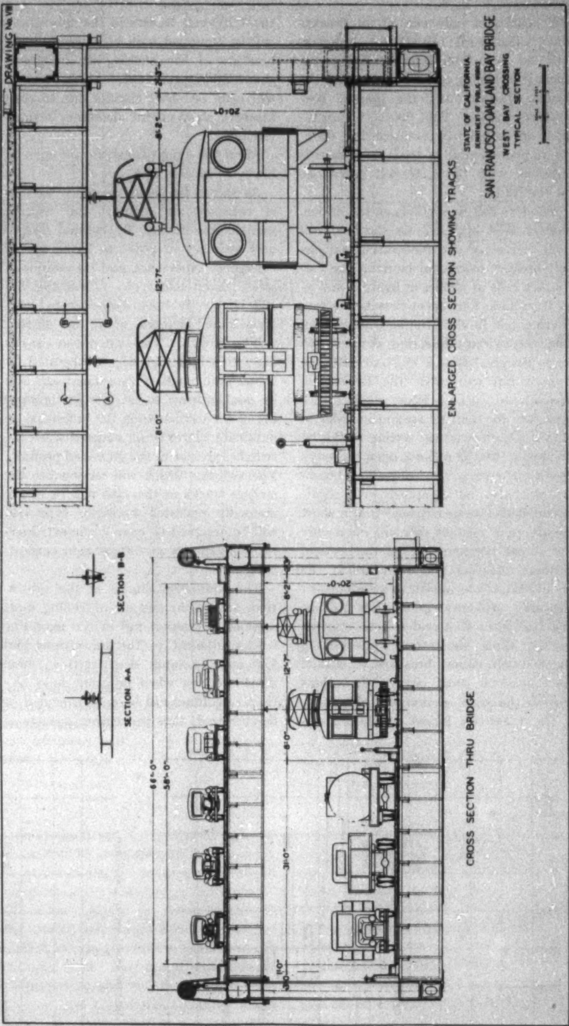

Figure 2

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* Most of the descriptive voltages given in this paper are nominal.

will consist of only one circuit breaker each. The yard tie station, however, will be completely sectionalizing because it forms the connection between the Bridge Railway and the present Key System tracks. The 600-volt substations are completely sectionalizing, forming the same length of track sections as is obtained with the 1,200-volt catenaries on the bridge.

The 600-volt tie stations on the bridge, namely *W*-4 and *E*-7 tie stations, will each consist of one moderate-speed circuit breaker connected between the two contact rails as shown in figure 5 and so as to include four short-circuit-detecting devices, one in each contact-rail section. The short-circuit-detecting devices will be so designed that a short circuit on a contact rail will cause the tie station breaker to open. Four devices are necessary for each tie station in order to obtain a high overload setting of the tie breaker so that it will not open on heavy useful currents. The substation breakers will also be equipped with short-circuit detectors so adjusted that a short circuit on a contact rail will open only the circuit breakers feeding the section, without affecting the other track. As an additional precaution to insure proper clearance of faults, pilot wires will be installed from *W*-4 and *E*-7 tie stations to the three substations, interlocking the 600-volt circuit breakers so that if one opens on short circuit, the others feeding the same section will also open.

The 1,200-volt Island tie station and the 1,200-volt feeders in the substations will be equipped with high-speed circuit breakers of the bucking-bar inductive-shunt holding-coil type. These breakers will open on short circuits due to their inherent short-circuit detecting characteristics.

All of the circuit breakers will open on heavy overloads, also.

As shown by figure 5, many sections of catenary and contact rail will be sectionalized further by manual disconnect switches in order to make use of emergency crossovers, and for reasons of safety, maintenance, etc. These switches will be mostly hook-stick-operated outdoor knife switches where no current will be broken. The switches in the car inspection buildings will be operated by a link and handle arrangement and will be double-throw to provide for grounding of the catenaries in the buildings, an automatic alarm being sounded when the switches are not in the grounded position. The switches which will sectionalize the storage tracks in the yard will be of the manually operated horn-gap type and will be designed to have a current interruption capacity of 150 per cent current-carrying capacity.

The contactors shown at the substations are for the purpose of feeding short sections of contact rail in the main-line tracks adjacent to the substations and for opening and de-energizing these short sections when adjacent long sections of contact rail become grounded or de-energized, thus preventing passage of short-circuit current through the contact-rail shoe bus cable on a Key System train, should the train run into the grounded section.

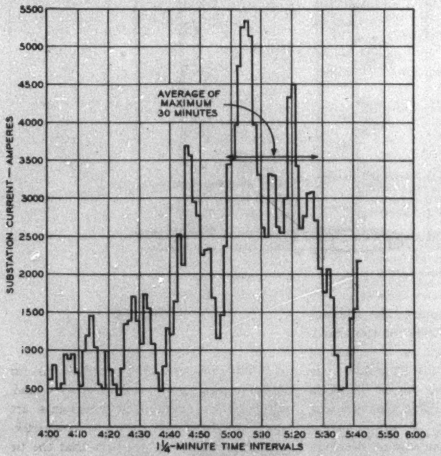
CATENARY

The catenary for the Bridge Railway will be of the chord type. Most of the main-line catenary will consist of one or two main messengers of 500,000-circular-mil hard-drawn stranded copper cable, an intermediate messenger of 4/0 hard-drawn stranded copper cable, and two solid grooved contact wires of high-conductivity bronze. The yard catenary will consist of a high-strength stranded bronze messenger cable and one hard-drawn copper grooved contact wire.

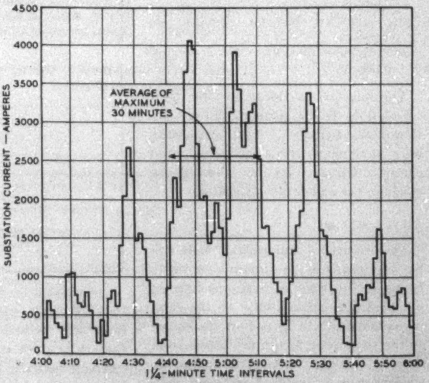
One feature of the catenary that may be of special interest is the use of floating "bridle" beams for pull-offs on sharp curves on viaducts where there is not sufficient room for frequent pull-off poles or "back bone" construction. These "bridle" beams are steel compression members suspended across the tracks by high-strength bronze cables from the adjacent catenary bridges. The pull-offs for the multiple-track catenaries are secured to these floating beams.

Two-unit suspension insulators with a wet flashover rating of 70 kv per string are used for ordinary catenary insulation. Dead-end insulators are somewhat larger. The salt air and fog conditions of the San Francisco Bay area prompted the selection of the insulators.

The section insulators in the catenaries



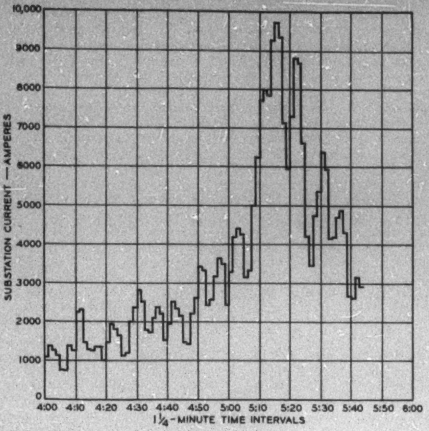
Part 1. San Francisco substation



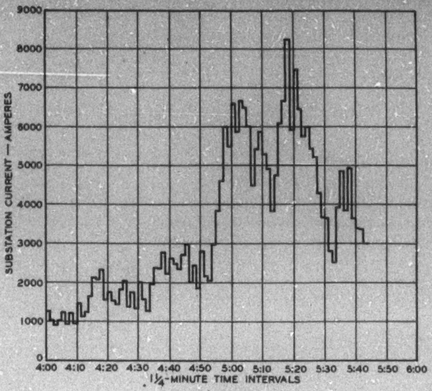
Part 2. Mole substation

Figure 3. San Francisco-Oakland Bay Bridge Railway, evening peak 1,200-volt substation load curves

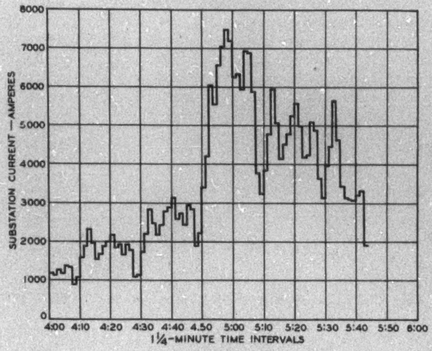
Southern Pacific loads and Sacramento Northern loads, 1938 plus 25 per cent traffic



Part 1. San Francisco substation



Part 2. Island substation



Part 3. Mole substation

Figure 4. San Francisco-Oakland Bay Bridge Railway, evening peak 600-volt substation load curve

Key system loads

will be of the wood stick type throughout. Air-gap construction was not considered to be necessary for the 35-mile-per-hour train speeds.

Contact Rails and Shoes

The contact-rail system will be of the over-running type with top and side protection boards and will be located on porcelain insulators at the side of the tracks on long ties. The contact rails will be rolled iron of conductivity ratio to copper: 1/6.85, and will be of standard cross section, weighing 150 pounds per yard in heavy-duty territory and 55 pounds per yard in light-duty areas. At one location on a 3 per cent grade, the 150-pound contact rail will be reinforced with a 1,000,000-circular-mil bare copper feeder cable attached to the rail.

The Key System cars will be equipped with shoe collectors of the folding type. Wayside ramps will be provided to fold these shoes when the Key trains leave the Bridge Railway for the streets of Oakland and Berkeley, and to unfold them when the trains enter the Bridge Railway. The pantographs on the Key cars will be raised and lowered automatically when leaving and entering the Bridge Railway, respectively, by means of control circuits interlocked with the contact-rail shoes and a wayside tripping arm engaging a pneumatic valve on the top of the car.

Return Circuit and Bonding

The current returning to the substations will be conducted by the 90-pound running rails and 90-pound guard rails of the tracks. Two guard rails per track will be available for conductivity in addition to the running rails at all locations on the main bridge and viaducts. On account of the a-c signal circuits in the tracks, impedance bonds will be installed at the insulating joints in the running rails. The guard rails will be connected to the impedance bond neutrals at the cross-bond locations which occur at approximately 2,500-foot intervals.

The bonding of rail joints on the Bridge Railway is very important because of the absence of ballast on the bridge and the necessity of preventing stray currents from leaking to the bridge steel and causing electrolysis. The standard joint bonds will be two 250,000-circular-mil $13^1/_2$-inch copper cable bonds per joint, with terminals welded to the rails by an oxyacetylene flame.

Special trackwork bonds will be two 350,000-circular-mil copper cables per joint, also gas welded to the rails. The proper location of long bonds at track switches is of special importance because of the inductive type of signal system used.

Contact-rail jumpers will be multiple 500,000-circular-mil flexible stranded copper cables insulated with 30 per cent rubber and provided with a 60 per cent rubber protecting sheath. These jumpers will be cut at the factory and have standard gas-weld bond terminals for welding to the contact rails.

Feeder terminals at contact rails will be copper cones welded to the contact rails and provided with cable sockets to which the cables will be both clamped and soldered.

The joints in the 150-pound contact rails will be arc welded at the top and flanges and also bonded to full conductivity with hard-drawn copper splice bars bolted and soldered in place. The 55-pound contact rails will be bonded with two 11-inch 4/0 stranded copper gas-weld bonds per joint.

Expansion Joints

The extraordinary length of the bridge and the flexibility inherent in suspension bridges entailed special problems in expansion joints, movements amounting to four feet per joint in some cases. The catenaries will be dead ended at these joints, a sliding copper bar being used to guide the pantographs across, the current being carried across through flexible overhead jumpers. The contact rails and guard rails will simply have gaps at these expansion joints, the current in them being conducted across through flexible insulated cable loops suspended beneath the bridge. The running rails will, of course, be continuous across the bridge expansion joints, the movement being taken care of by a series of sliding joints in the rails, the proportioning of the total bridge movement among them being governed by a link arrangement. These running-rail expansion joints will be bonded with insulated cable loops provided with gas-weld terminals.

Discussion

G. R. McDonald (General Electric Company, Erie, Pa.): We have read Mr. Monroe's paper with a great deal of interest and believe it an excellent paper and very useful as a record of the investigations made and the reasons for the selection of the particular systems being installed for the San Francisco-Oakland Bay Bridge Railway.

We believe a few words about the mercury arc rectifiers being installed in the substations would be of interest in supplementing Mr. Monroe's paper. The number of rectifiers and their nominal rating are as listed by Mr. Monroe. However, in order to take care of the exacting service, the rectifiers were given a special rating; following continuous operation at full load they will be capable of carrying 150 per cent current for two hours, or 200 per cent current for 30 minutes, or 300 per cent current for five minutes. With the exception of the Mole substation, two rectifiers operating on each voltage were selected for each of the substations. Normally both rectifiers will be operated during the peak load periods, but in an emergency one rectifier operating alone will be able to carry the load through the peak. At the Mole substation, advantage was taken of an existing 1,500-kw motor generator set already installed at this point operating at 625 volts direct current. For this reason two 1,000-kw rectifiers were installed to operate in multiple with this motor generator set on the basis that two units would be available at all times for carrying the peak loads. All of the rectifiers are of the 12-anode 12-phase type with the exception of the two 1,000-kw equipments operating at 625 volts in the Mole substation.

The natural output characteristic of a mercury arc rectifier is similar to that of a shunt machine; that is, the output voltage is reduced as the load current is increased. The voltages used, 625 and 1,300, are the maximum which it is desired to apply to the distribution system and traction equipment. If, therefore, the rectifiers were selected to provide this voltage under full load conditions, too high a voltage would result at light load. To insure good operating voltage and prevent excessive voltage at light loads, the rectifiers are provided with control grids which in conjunction with a voltage regulator serve to maintain constant bus voltage from zero to 150 per cent load on the rectifiers. Above 150 per cent load, the rectifiers operate on their natural characteristic, having approximately 5 per cent regulation. Consequently, in assuming load from 150 per cent to 300 per cent, there will be a drop in voltage of some $7^{1}/_{2}$ per cent.

Voltage control of these rectifiers is obtained in the usual manner by delaying the firing of the main anodes at loads below 150 per cent in order to reduce the output voltage to the required value. The regulator varies the phase angle of the voltage applied to the grids by applying a d-c bias to the neutral of the grid supply transformers. This d-c bias is obtained from a small motor generator set, the output voltage of which is controlled by the regulator. The regulators are also equipped with load balance coils for the purpose of insuring equal loading between rectifiers operating in multiple. In the case of the Mole substation, 625-volt equipment, load balance is insured in a similar manner between the two 1,000-kw rectifiers and the 1,500-kw motor generator set. The regulators are fast in operation so that the voltage will be maintained approximately constant regardless of rapid variations in load.

At the Mole substation, the two 6-phase 1,000-kw, 625-volt rectifiers are supplied by transformers having their high-voltage windings connected wye and delta, respectively, and, in addition, an interphase transformer is connected between the two units. When both six-phase rectifier units are in operation, their combined output will therefore be the equivalent of a single 2,000-kw 12-phase rectifier.